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MA 444

Explanation of Problem:

For this project, we were asked to solve a problem involving the scheduling of plane landing times. We are told that any given plane has a target time it is expected to land at, but could land early or late for a fixed fee that varies per plane. We are also told that the only things stopping planes from landing at their target times is that when a given plane lands on a runway there is a certain amount of time all other planes must wait to land after it, which varies whether they are landing on the same or a different runway. The problem being solved is what is the minimum cost that we can land all the planes for a day at and still maintain the constraints surrounding their landing times.

Mathematical Model:

Variables:

$$Landing_i = Time\ That\ Plane\ i\ Lands,\ Landing_i\ \in\ \mathbb{R}.$$

$$Early_i = \begin{cases} 1\ if\ Plane\ i\ Is\ Early \\ 0\ Otherwise \end{cases}$$

$$Late_i = \begin{cases} 1\ if\ Plane\ i\ Is\ Late \\ 0\ Otherwise \end{cases}$$

$$PrecedesS_{i,j} = \begin{cases} 1\ if\ Plane\ i\ Lands\ Immediately\ Before\ Plane\ j\ on\ Same\ Runway \\ 0\ Otherwise \end{cases}$$

$$PrecedesD_{i,j} = \begin{cases} 1\ if\ Plane\ i\ Lands\ Immediately\ Before\ Plane\ j\ on\ Different\ Runway \\ 0\ Otherwise \end{cases}$$

$$FirstOrLast_i = \begin{cases} 1\ if\ Plane\ i\ is\ First\ or\ Last\ to\ Land \\ 0\ Otherwise \end{cases}$$

Parameters:

$$MinTime_i = Minimum\ Time\ Plane\ i\ Can\ Land$$

$$TargetTime_i = Target\ Time\ \ For\ Plane\ i\ To\ Land$$

$$MaxTime_i = Maximum\ Time\ Plane\ i\ Can\ Land$$

$$EarlyCost_i = Cost\ For\ Plane\ i\ To\ Land\ Early$$

$$LateCost_i = Cost\ For\ Plane\ i\ To\ Land\ Late$$

$$SameDifference_{i,j} = Time\ After\ Plane\ i\ Plane\ j\ Must\ Wait\ To\ Land\ On\ Same\ Runway$$

$$DifferentDifference_{i,j} = Time\ Plane\ j\ Must\ Wait\ After\ Plane\ i\ to\ Land\ On\ Different\ Runway$$

Objective Function:

$$Cost = \sum_{i=Planes} Early_i * EarlyCost_i + Late_i * LateCost_i$$

Constraints:

$$\sum_{i=Planes} (FirstOrLast_i) = 2$$

For Any Given Plane i,

$$Early_i + Late_i \leq 1$$

$$MinTime_i + (1 - Early_i) * (TargetTime_i - MinTime_i) + Late_i \leq Landing_i$$

$$MaxTime_i - (1 - Late_i) * (MaxTime_i - TargetTime_i) - Early_i \geq Landing_i$$

$$FirstOrLast_i + \sum_{j=Planes} (PrecedesS_{i,j} + PrecedesS_{j,i} + PrecedesD_{i,j} + PrecedesD_{j,i}) = 2$$

$$\sum_{j=Planes} (PrecedesS_{i,j} + PrecedesD_{i,j}) \leq 1$$

$$\sum_{j=Planes} (PrecedesS_{j,i} + PrecedesD_{j,i}) \leq 1$$

$$(PrecedesS_{i,i}) = 0$$

$$(PrecedesD_{i,i}) = 0$$

For Any Given Planes i, j

$$Landing_i - M * \left(1 - PrecedesS_{i,j}\right) + PrecedesS_{i,j} * SameDifference_{i,j} \leq Landing_j$$

$$Landing_i - M * \left(1 - PrecedesS_{i,j}\right) + PrecedesS_{i,j} * SameDifference_{i,j} \leq Landing_j$$

Explanation of Mathematical Model:

To start off this model we have the variables of landing time which is a Real Number and Early and Late which are both binary variables. These variables are responsible for keeping track of which ranges of time the planes can land at, and the costs that would be associated with landing at those times. The ranges of time a plane can land during are controlled by two constraints, one for the upper bound and one for the lower. These constraints will push the landing time to be less than the target time in the case that the Early variable is 1, and greater than the target time in the case that the Late variable is 1. The sum of the Early and Late variables for a given plane must be less than or equal to 1, because a plane can't be both early and late, but it can be neither early or late.

The cost for a given landing schedule can be easily calculated by multiplying the binary variables for landing early and late by the costs for each plane associated with landing early or late. This cost will be our objective function that we are attempting to minimize.

Next we have the two sets of variables PrecedesS and PrecedesD. In order to schedule a plane the proper amount of time after another plane lands, you have to keep track of what the most recent plane to land was and that is what these variables are for. PrecedesS is a binary variable relating two planes that is 1 in the situation that a plane i was the plane that landed immediately before plane j and they landed on the same runway and 0 otherwise. PrecedesD is similar, as it is a binary variable that is 1 in the situation that a plane i was the plane that landed immediately before plane j and they landed on different runways. The PrecedesS and PrecedesD relating a plane to itself must be 0 since a plane can't land before itself, so we need constraints forcing these to be zero. The landing time for any given plane is affected by these two variables in the form of two constraints. Landing time for a plane j is forced to be greater than the landing time for a plane i plus the time after plane i plane j must land multiplied by the given Precedes to make this only applicable if this plane i was the one that landed immediately before it. An arbitrary large number M is then multiplied by 1 minus the given Precedes and subtracted from this constraint so that the constraint does not affect the landing time of plane j in the case that plane i was not the plane that landed immediately before it. The way PrecedesS and PrecedesD are set up with the constraints, a given plane i can only have at most one PrecedesS or PrecedesD indicating a plane came before it, and one PrecedesS and PrecedesD indicating it preceded a plane. It is at most one because this is where our last variable FirstOrLast comes in.

There must be one plane that comes in first, and there must be one plane that comes in last and so FirstOrLast is 1 in the situation that a given plane i comes in first or last. The constraints where PrecedesS + PrecedesD <= 1 for both plane i in the position of having a plane before it and a plane after it are <= 1 because there will be two planes that only have 1 PrecedesS or PrecedesD variable equal to 1 relating to it. FirstOrLast must also have a constraint that sets the sum of FirstOrLast for all planes i equal to two so that the model is forced to have one plane come in first and one come in last.

Now that all our variables have been explained, the final constraint can be as well. The last constraint is the FirstOrLast for a given plane i added to the sum of PrecedesS and PrecedesD for both the order i, j and j, i for all planes j is equal to 2. This constraint ensures that for each plane there is a plane that comes both before and after it or it was first or last so there will only be a plane either before or after it. This is the last constraint necessary for this problem, so it can now be setup and solved in our linear solver.

Parameter Values:

	Planes									
	1	2	3	4	5	6	7	8	9	10
Earliest Time	45	30	120	100	90	75	140	145	130	110
Target Time	75	65	160	125	120	110	200	180	175	150
Latest Time	150	150	220	200	180	190	275	250	260	225
Early Penalty	25	25	45	50	40	40	50	45	50	35
Late Penalty	50	50	70	75	90	100	85	65	65	60

Same Difference

	1	2	3	4	5	6	7	8	9	10
1	-	5	10	8	4	7	10	6	10	7
2	5	-	15	15	10	8	10	15	10	15
3	10	15	-	10	5	15	10	7	15	10
4	8	15	10	-	7	10	5	8	8	10
5	4	10	5	7	-	10	15	10	15	8
6	7	8	15	10	10	-	8	10	6	9
7	10	10	15	8	15	6	-	10	10	8
8	6	15	7	8	10	10	10	-	8	10
9	10	10	15	8	15	6	10	8	-	7
10	7	15	10	10	8	9	8	10	7	-

Different Difference

	1	2	3	4	5	6	7	8	9	10
1	-	3	5	4	4	7	5	6	5	7
2	3	-	9	9	5	4	5	9	5	9
3	5	9	-	5	3	9	5	7	9	5
4	4	9	5	-	7	5	3	4	4	5
5	4	5	3	7	-	5	9	5	9	4
6	7	4	9	5	5	-	4	5	6	9
7	5	5	9	4	9	6	-	5	5	4
8	6	9	7	4	5	5	5	-	4	5
9	5	5	9	4	9	6	5	4	-	7
10	7	9	5	5	4	9	4	5	7	-

Solution:

Landing Times:	Target Time	Plane Before This Plane	Plane After This Plane
Plane 1 : 75.0	75	2 ON DIFFERENT RUNWAY	6 ON DIFFERENT RUNWAY
Plane 2 : 65.0	65	FIRST	1 ON DIFFERENT RUNWAY
Plane 3 : 160.0	160	10 ON DIFFERENT RUNWAY	9 ON DIFFERENT RUNWAY
Plane 4 : 125.0	125	5 ON DIFFERENT RUNWAY	10 ON DIFFERENT RUNWAY
Plane 5 : 118.0	120	6 ON DIFFERENT RUNWAY	4 ON DIFFERENT RUNWAY
Plane 6 : 110.0	110	1 ON DIFFERENT RUNWAY	5 ON DIFFERENT RUNWAY
Plane 7 : 200.0	200	8 ON SAME RUNWAY	LAST
Plane 8 : 180.0	180	9 ON DIFFERENT RUNWAY	7 ON SAME RUNWAY
Plane 9 : 175.0	175	3 ON DIFFERENT RUNWAY	8 ON DIFFERENT RUNWAY
Plane 10 : 150.0	150	4 ON DIFFERENT RUNWAY	3 ON DIFFERENT RUNWAY

Objective Function : Value = 40

This is an interesting Result, as a person looking at this problem would not be able to tell that by simply alternating most planes it is possible to get nearly every plane to land on their target time except for 1 of them. This is also a very low objective function value relative to how many costs they give in the problem and the idea that even though they give all these parameters and costs only one cost needs to be incurred for the planes to land in an optimized scenario.