**Supporting Information**. Drees, T.H. and K. Shea, 2022. *Elevated temperatures shift flower head height distributions and seed dispersal patterns in two invasive thistle species*. Ecology.

**Appendix S2.** Brief description of methodology used to estimate invasion wavespeeds, and tables for select wavespeed statistics.

Demographic Models

The demographic models used in the wavespeed simulations are identical to those used in Zhang *et al*. (2011) and Teller *et al*. (2016). Like these previous studies, we use a four-stage matrix model with a seed bank and small, medium, and large size classes to represent annual demographic transitions. Demographic rates used to define the demographic matrix include For more information on the demographic model structure, refer to Text S1 in Zhang *et al*. (2011); for a table of the vital rates used, refer to Table S2 in Zhang *et al*. (2011) or Table S1 in Teller *et al*. (2016).

Dispersal Models

Refer to the main text for information about the WALD dispersal model used in our analyses. Our dispersal modelling methods are similar to those used in previous work by Zhang *et al*. (2011) and Teller *et al*. (2016), with three notable differences where we instead use a) a distribution of flower head heights rather than a single point source, b) the empirical distributions of wind speeds and terminal velocities rather than a lognormal approximations, and c) a surrounding vegetation height of 0.15 m rather than 0.5 m.

Spread Models

**Table S1**. Demographic parameters used in the matrix model, sourced from Zhang *et al*. (2011) and Teller *et al*. (2016). Note that growth (not from seed), retrogression, and bolting probabilities are conditional on surviving individuals only. An asterisk indicates that quantities are different between the warmed and unwarmed treatments.

|  |  |  |  |
| --- | --- | --- | --- |
| **Parameter** | **Description** | **Unwarmed** | **Warmed** |
|  | Prob. of survival of seed in seed bank | 0.2597 | 0.2597 |
|  | Prob. of survival of small rosettes (S)\* | 0.2619 | 0.2864 |
|  | Prob. of survival of median rosettes (M)\* | 0.6761 | 0.7393 |
|  | Prob. of survival of large rosettes (L)\* | 0.8971 | 0.9810 |
|  | Prob. of growth of establishing seed to M | 0.2076 | 0.2076 |
|  | Prob. of growth of establishing seed to L | 0.0911 | 0.0911 |
|  | Prob. of growth of non-bolting S to M | 0.8028 | 0.8028 |
|  | Prob. of growth of non-bolting S to L | 0.1268 | 0.1268 |
|  | Prob. of growth of non-bolting M to L | 0.3824 | 0.3824 |
|  | Prob. of retrogression of non-bolting M to S | 0.0000 | 0.0000 |
|  | Prob. of retrogression of non-bolting L to S | 0.0000 | 0.0000 |
|  | Prob. of retrogression of non-bolting L to M | 0.0000 | 0.0000 |
|  | Prob. of bolting of S | 0.1932 | 0.1932 |
|  | Prob. of bolting of M | 0.7143 | 0.7143 |
|  | Prob. of bolting of L | 1.0000 | 1.0000 |
|  | Potential seed production by S\* | 5443 | 7809 |
|  | Potential seed production by M\* | 6150 | 8145 |
|  | Potential seed production by L\* | 12446 | 16483 |
|  | Potential seed escaping from floral herbivory | 0.8500 | 0.8500 |
|  | Prob. of new seed entering seed bank | 0.2333 | 0.2333 |
|  | Prob. of new seed establishing seedling\* | 0.2333 | 0.3022 |
|  | Prob. seed from seed bank establishing seedling\* | 0.2333 | 0.3022 |

**Table S2**. Dispersal statistics from the second set of simulations comparing warmed/unwarmed outcomes and maximum/distributed flower head heights for *C. nutans* (CN) and *C. acanthoides* (CA), with values given for the mean and lower/upper values of the 95% bootstrap interval (BI).

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
|  |  | **95% BI Lower** | **Median** | **Mean** | **95% BI Upper** |
| **Flower height shifts only** | | |  |  |  |
| Height Dist. | Unwarmed |  |  |  |  |
|  | Warmed |  |  |  |  |
| Max. Height | Unwarmed |  |  |  |  |
|  | Warmed |  |  |  |  |
|  | | | | | |
| **All demographic shifts** | | |  |  |  |
| Height Dist. | Warmed |  |  |  |  |
| Max. Height | Warmed |  |  |  |  |