**Supporting Information**. Drees, T.H. and K. Shea, 2023. *Elevated temperatures shift flower head height distributions and seed dispersal patterns in two invasive thistle species*. Ecology.

**Appendix S2.** Brief description of methodology used to estimate invasion wavespeeds, and tables for select wavespeed statistics.

**Demographic Models**

The demographic models used in the wavespeed simulations are based on those used in previous studies on *C. nutans* dispersal and invasion speeds by Jongejans *et al*. (2008), Zhang *et al*. (2011) and Teller *et al*. (2016). Here, our baseline is a four-stage matrix model used by Jongejans *et al*. (2008) and Zhang *et al*. (2011) with a seed bank and small, medium, and large classes to represent annual demographic transitions:

|  |  |
| --- | --- |
|  | [1] |

Demographic rates used in the matrix above can be found in Table S1. The class boundaries are based on probability of flowering as estimated by Shea and Kelly (1998), with small plants having <20% flowering probability, medium plants 20-80% flowering probability, and large plants >80% flowering probability. We then expand this matrix to 11x11 based on the analyses by Teller *et al*. (2016), allowing for reproductive classes to be broken down by whether they dispersed via wind or via fallen capitula.

**Dispersal Models**

For information about the WALD dispersal model used in our analyses, refer to the dispersal methodology outlined in the main text. Our dispersal modelling methods are similar to those used in previous work by Zhang *et al*. (2011) and Teller *et al*. (2016), with three notable differences where we instead use a) an empirical distribution of flower head heights rather than a single point source, b) empirical distributions of wind speeds and seed terminal velocities rather than approximating their distributions as lognormal, and c) a surrounding vegetation height of 0.15 m rather than 0.5 m.

**Spread Models**

Population spread was modelled in an identical manner to the analyses performed in Zhang *et al*. (2011) and Teller *et al*. (2016), based on the wavespeed analyses for structured populations outlined in Neubert and Caswell (2000). Movement of the population was modelled as a travelling wave in one dimension; as such, dispersal events were initially simulated as a two-dimensional kernel with angles randomly sampled from a uniform distribution between 0 and 2, but were then marginalized onto a single spatial axis. The simulated dispersal distances were then used to calculate the empirical moment generating function (MGF)

|  |  |  |
| --- | --- | --- |
|  |  | [2] |

where is the total number of dispersal events, is dispersal distance for a given dispersal event, and is an auxiliary variable that describes the steepness of the wave. We then calculate as the dominant eigenvalue of the Hadamard product , where **A** is the demographic matrix, and is a matrix containing the MGF for dispersing stages and the Dirac delta MGF of 1 for non-dispersing stages. The wavespeed can then be found by minimizing

|  |  |  |
| --- | --- | --- |
|  |  | [3] |

over a range of *s*. For each combination of warmed/unwarmed treatment and distributed/maximum height, we simulated 100000 times; summary results can be found in Table S2.

**References**

Jongejans, E., Shea, K., Skarpaas, O., Kelly, D., Sheppard, A. W., & Woodburn, T. L. (2008). Dispersal and demography contributions to population spread of *Carduus nutans* in its native and invaded ranges. *Journal of Ecology*, 96(4), 687-697.

Neubert, M.G. & Caswell, H. (2000). Demography and dispersal: calculation and sensitivity analysis of invasion speed for structured populations. *Ecology*, 81(6), 1613-1628.

Shea, K. & Kelly, D. (1998). Estimating biocontrol agent impact with matrix models: *Carduus nutans* in New Zealand. *Ecological Applications*, 8(3), 824-832.

Teller, B.J., Zhang, R., & Shea, K. (2016). Seed release in a changing climate: initiation of movement increases spread of an invasive species under simulated climate warming. *Diversity and Distributions*, 22(6), 708-716.

Zhang, R., Jongejans, E., & Shea, K. (2011). Warming increases the spread of an invasive thistle. *PLoS One*, 6(6), e21725.

**Table S1**. Demographic parameters used in the matrix model, sourced from Zhang *et al*. (2011) and Teller *et al*. (2016). Note that growth (not from seed), retrogression, and bolting probabilities are conditional on surviving individuals only. An asterisk indicates that quantities are different between the warmed and unwarmed treatments.

|  |  |  |  |
| --- | --- | --- | --- |
| **Parameter** | **Description** | **Unwarmed** | **Warmed** |
|  | Prob. of survival of seed in seed bank | 0.2597 | 0.2597 |
|  | Prob. of survival of small rosettes (S)\* | 0.2619 | 0.2864 |
|  | Prob. of survival of median rosettes (M)\* | 0.6761 | 0.7393 |
|  | Prob. of survival of large rosettes (L)\* | 0.8971 | 0.9810 |
|  | Prob. of growth of establishing seed to M | 0.2076 | 0.2076 |
|  | Prob. of growth of establishing seed to L | 0.0911 | 0.0911 |
|  | Prob. of growth of non-bolting S to M | 0.8028 | 0.8028 |
|  | Prob. of growth of non-bolting S to L | 0.1268 | 0.1268 |
|  | Prob. of growth of non-bolting M to L | 0.3824 | 0.3824 |
|  | Prob. of retrogression of non-bolting M to S | 0.0000 | 0.0000 |
|  | Prob. of retrogression of non-bolting L to S | 0.0000 | 0.0000 |
|  | Prob. of retrogression of non-bolting L to M | 0.0000 | 0.0000 |
|  | Prob. of bolting of S | 0.1932 | 0.1932 |
|  | Prob. of bolting of M | 0.7143 | 0.7143 |
|  | Prob. of bolting of L | 1.0000 | 1.0000 |
|  | Potential seed production by S\* | 5443 | 7809 |
|  | Potential seed production by M\* | 6150 | 8145 |
|  | Potential seed production by L\* | 12446 | 16483 |
|  | Potential seed escaping from floral herbivory | 0.8500 | 0.8500 |
|  | Prob. of new seed entering seed bank | 0.2333 | 0.2333 |
|  | Prob. of new seed establishing seedling\* | 0.2333 | 0.3022 |
|  | Prob. seed from seed bank establishing seedling\* | 0.2333 | 0.3022 |

**Table S2**. Mean and median simulated wavespeeds for *C. nutans*, with lower/upper values of the 95% bootstrap interval (BI). The “flower height shifts only” grouping contains wavespeeds accounting for only the warming-induced increase in distributed or maximum flower height, while the “all demographic shifts” grouping contains the aforementioned increase while also accounting for warming-induced increases in survival, reproduction, and establishment.

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
|  |  | **95% BI Lower** | **Median** | **Mean** | **95% BI Upper** |
| **Control** | | |  |  |  |
| Height Dist. | Unwarmed |  |  |  |  |
| Max. Height | Unwarmed |  |  |  |  |
|  | | | | | |
| **Flower height shifts only** | | |  |  |  |
| Height Dist. | Warmed |  |  |  |  |
| Max. Height | Warmed |  |  |  |  |
|  | | | | | |
| **All demographic shifts** | | |  |  |  |
| Height Dist. | Warmed |  |  |  |  |
| Max. Height | Warmed |  |  |  |  |