Lab 6 - Trevor Callow - MAT 275 Lab

Forced Equations and Resonance

Exercise 1

Part (a)

Period of the forced oscillation: 4.19 second

Numerical value of the angle α: .71 rads (2pi modulo)

Part (b)

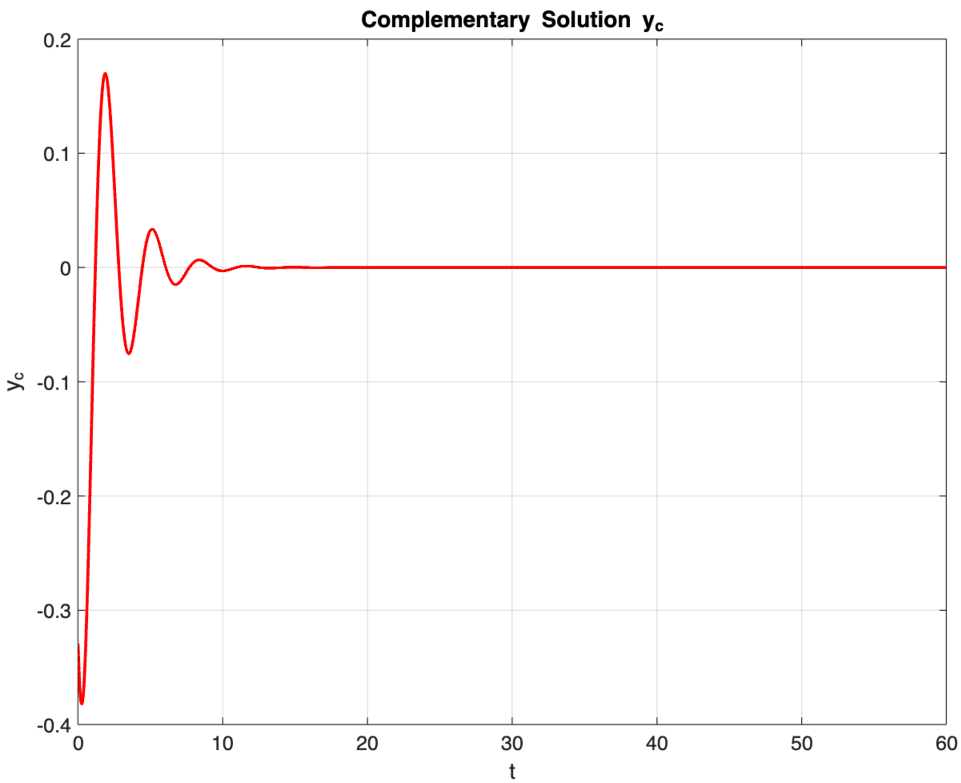
NOTE: *Modify the file LAB06ex1.m according to the instructions. Then print it and run it. Delete this note upon submission.*

```
type LAB06ex1bt.m
```

```
clear all;

omega0 = 2;
c = 1;
omega = 1.5;

param = [omega0, c, omega];

t0 = 0;
y0 = 0;
v0 = 0;
Y0 = [y0; v0];
tf = 60;

options = odeset('AbsTol', 1e-10, 'RelTol', 1e-10);
[t, Y] = ode45(@f, [t0, tf], Y0, options, param);

y = Y(:, 1);
v = Y(:, 2);

if omega0 > omega
    alpha = atan(c * omega / (omega0^2 - omega^2));
else
    alpha = pi + atan(c * omega / (omega0^2 - omega^2));
end

C = 1 / sqrt((omega0^2 - omega^2)^2 + (c * omega)^2);

yc = y - C * cos(omega * t - alpha);

figure;
plot(t, yc, 'r-', 'LineWidth', 1.5);
ylabel('y_c');
xlabel('t');
title('Complementary Solution y_c');
grid on;

function dYdt = f(t, Y, param)
    y = Y(1);
    v = Y(2);
    omega0 = param(1);
    c = param(2);
```

```
    omega = param(3);
    dYdt = [v; cos(omega * t) - omega0^2 * y - c * v];
end
```

```
LAB06ex1bt
```



Does it look like an exponentially decreasing oscillation? Why or why not?

yes the plot seems to be ending oscilation and endoing near zero due to the damping of the function

Exercise 2

Part (a)

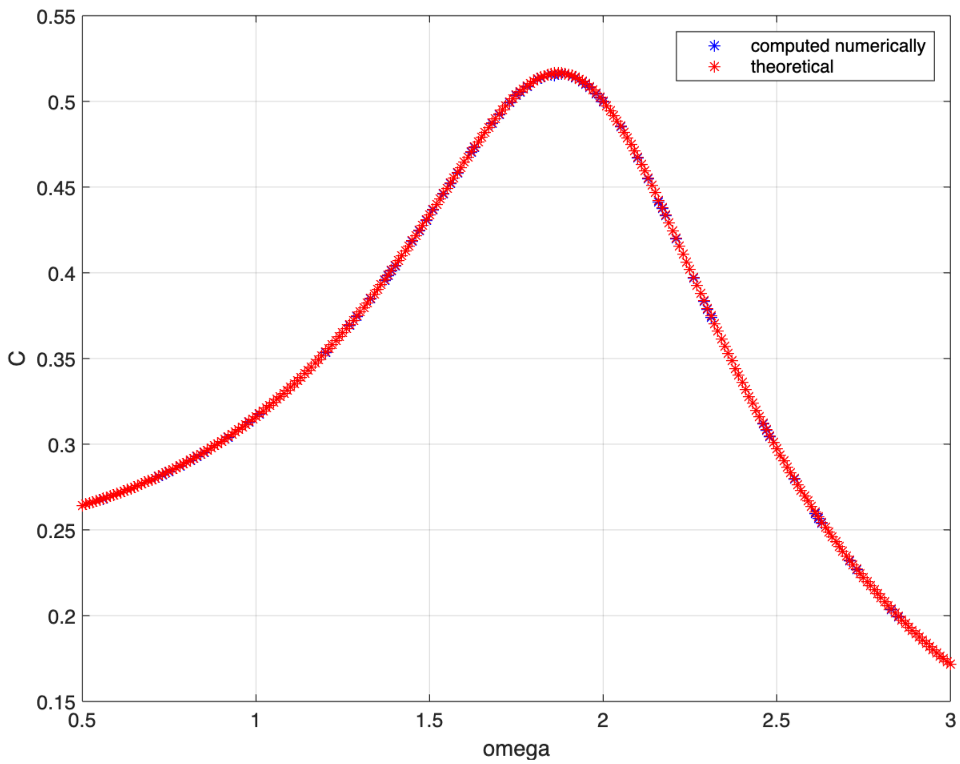
NOTE: *Fill in the missing parts in LAB06ex2.m. Then print it and run it. Delete this note upon submission.*

```
type LAB06ex2a.m
```

```
clear all; %this deletes all variables
omega0 = 2; c = 1;
OMEGA =0.5:0.01:3;
C = zeros(size(OMEGA));
Ctheory = zeros(size(OMEGA));
t0 = 0; y0 = 0; v0 = 0; Y0 = [y0;v0]; tf =60; t1 = 25;
for k = 1:length(OMEGA)
   omega = OMEGA(k);
   param = [omega0,c,omega];
   [t,Y] = ode45(@f,[t0,tf],Y0,[],param);
   i = find(t>t1);
```

```
   C(k) = (max(Y(i,1))-min(Y(i,1)))/2;
   Ctheory(k) = 1 / sqrt((omega0^2 - omega^2)^2 + (c * omega)^2); % FILL-IN the formula for Ctheory
end
figure(2)
plot(OMEGA, C, 'b*', OMEGA, Ctheory, 'r*'); grid on; % FILL-IN to plot C and Ctheory as a function of OMEG
xlabel('omega'); ylabel('C');
legend('computed numerically','theoretical')
%------------------------------------------------------
function dYdt = f(t,Y,param)
y = Y(1); v = Y(2);
omega0 = param(1); c = param(2); omega = param(3);
dYdt = [ v ; cos(omega*t)-omega0^2*y-c*v ];
end
```

```
LAB06ex2a
```



Part (b)

Value of ω that gives the practical resonance frequency: 1.732

Coresponding maximum value of the amplitude C: .577

Part (c)

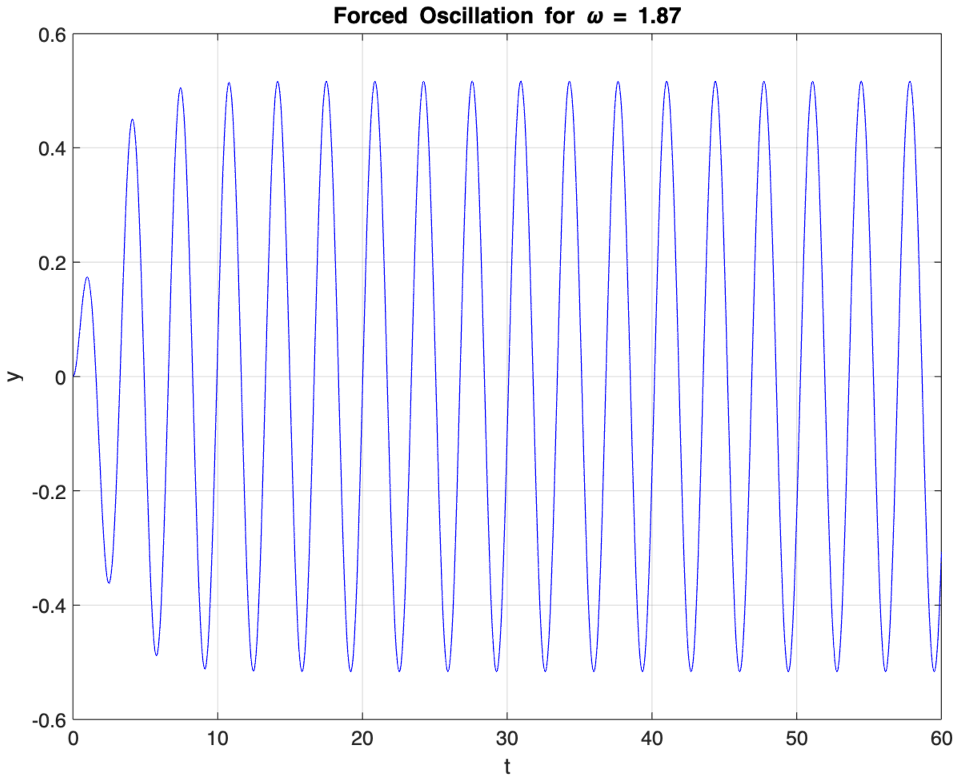
Compute analytically the value of ω that gives the practical resonance frequency by differentiating C: 1.87

Part (d)

Run LAB06ex1 with the value of ω found in part (c). Here, it is convenient to create a new M-file for this problem, e.g. LAB06ex1_d.m. You should create a copy of LAB06ex1.m and then change the value of ω.

```
LAB06ex1_d
```

```
Amplitude of forced oscillation = 0.5164
```



Amplitude of the forced oscillation: .5164

How does it compare with the amplitude of the forced oscillation found in problem 1? it is greater then the amplitude in problem 1

If you run LAB06ex1.m with any other value of ω, how do you expect the amplitude of the solution to be?

The amplitude is higher at the resonance frequency because the forcing frequency matches the system's natural response, maximizing energy transfer. The closer to resonance frequency the greater the amplitude.

Part (e)

NOTE: *Modify the initial conditions y_0 and v_0 in LAB06ex2.m and run the file with the new initial conditions. Then answer the following question and justify your answer theoretically (by analyzing the formula for C). Delete this note upon submission. Don't forget to reset the initial conditions to the original values once you are done experiementing.*

Is the amplitude of the forced oscillation, C, affected by the initial conditions? The amplitude is not affected by the intial conditions. the steady-state amplitude depends only on the system parameters and the forcing frequency.

Exercise 3

Set $c = 0$ in LAB06ex2.m and run it

Part (a)

What is the maximal amplitude? infintity

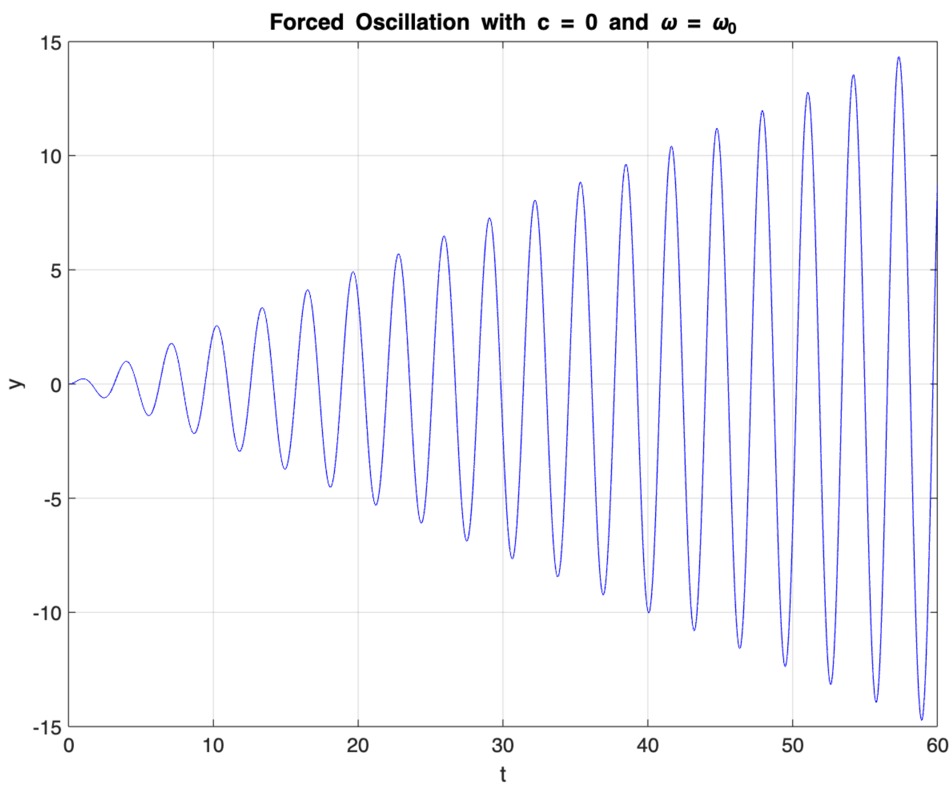
What is the value of ω yielding the maximal amplitude in the forced oscillation? 2

How does this value compare to ω_0? They are the same

Part (b)

Set $c = 0$ and ω equal to the value found in part (a) in LAB06ex1.m and run it. Here, it is convenient to create a new M-file for this problem, e.g. LAB06ex3_b.m. This file will be the same as LAB06ex1.m except for the values of c and ω.

```
LAB06ex3_b
```

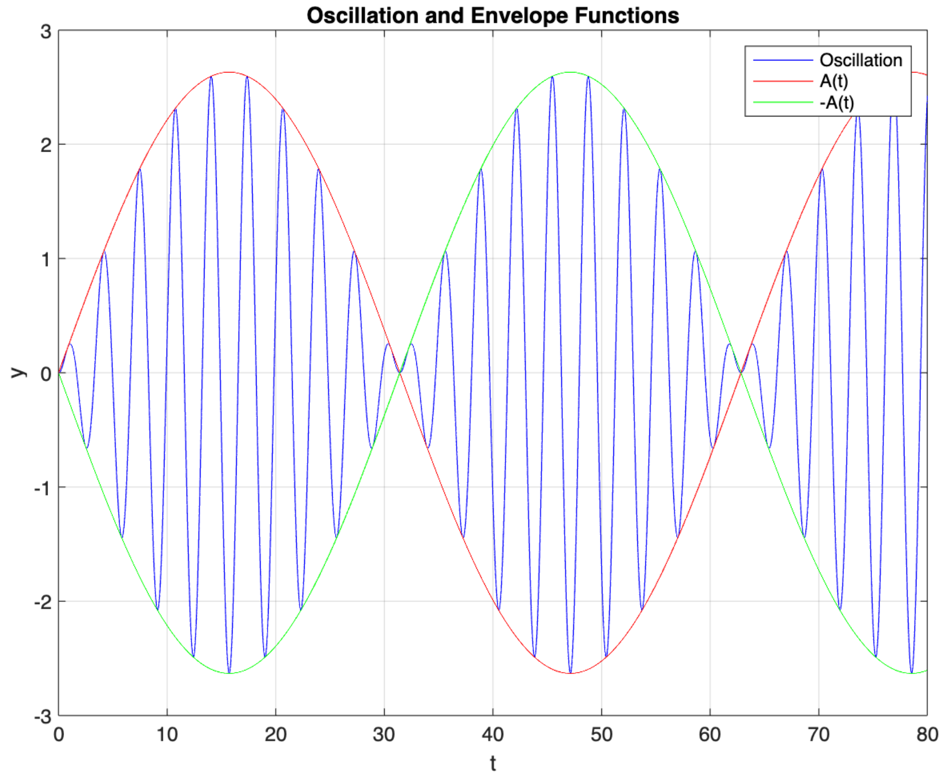


Comment on the behavior of the solution:When C=0 and ω=ω0�, the system enters resonance. In the absence of damping, the oscillations become unbounded as the system continuously absorbs energy from the forcing term. This demonstrates the theoretical concept of infinite amplitude during resonance in undamped systems

Exercise 4

Set $c = 0$ and ω equal to the given value in LAB06ex1.m . Extend the interval of simulation and run it. Here, it is convenient to create a new M-file for this problem, e.g. LAB06ex4.m. This file will be the same as LAB06ex1.m except for the values of c , ω and for the value of `tf`. *You can delete lines 10-14 in your new file as they are no longer relevant.*

```
LAB06ex4
```



Part (a)

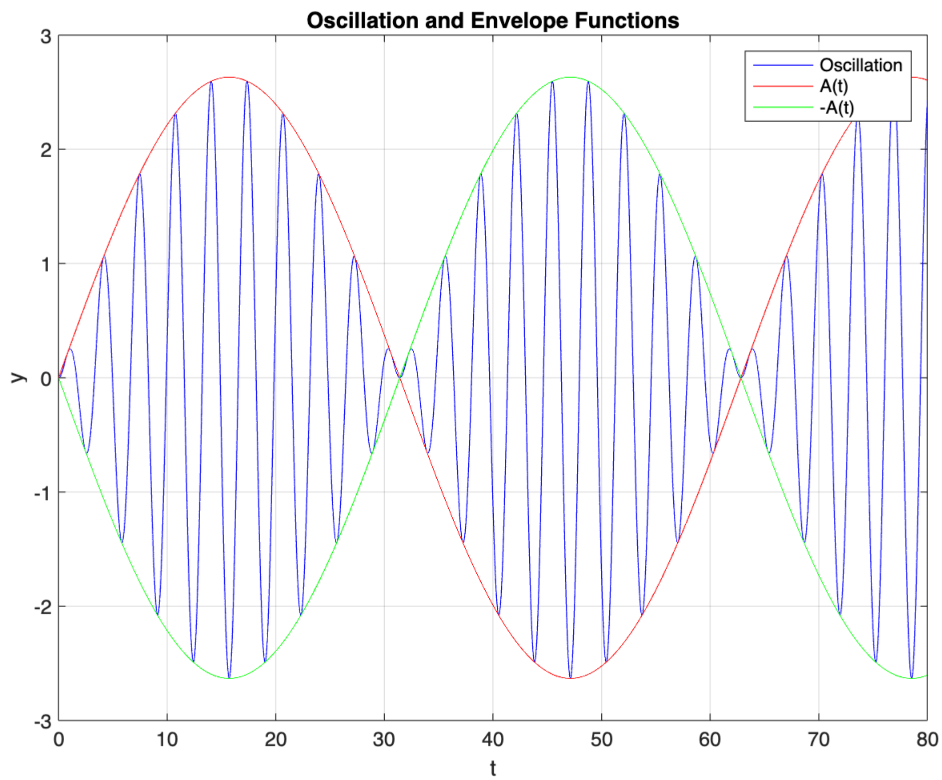
Define the "envelope" functions A and $-A$ in LAB06ex4.m

```
type LAB06ex4.m
```

```
clear all;
omega0 = 2;
c = 0;  % No damping
omega = 1.8;  % Frequency slightly different from omega0
param = [omega0, c, omega];
t0 = 0; y0 = 0; v0 = 0;
Y0 = [y0; v0];
tf = 80;  % Extended simulation time
options = odeset('AbsTol', 1e-10, 'RelTol', 1e-10);
[t, Y] = ode45(@f, [t0, tf], Y0, options, param);
y = Y(:, 1);
% Define envelope functions A(t) and -A(t)
C = 1 / abs(omega0^2 - omega^2);
A = 2 * C * sin(0.5 * abs(omega0 - omega) * t); % Envelope amplitude
negA = -A;
```

```
figure;
plot(t, y,'-b', t, A,'-r',t ,negA, '-g');
ylabel('y');
xlabel('t');
legend('Oscillation', 'A(t)', '-A(t)');
title('Oscillation and Envelope Functions');
grid on;
function dYdt = f(t, Y, param)
    y = Y(1);
    v = Y(2);
    omega0 = param(1);
    c = param(2);
    omega = param(3);
    dYdt = [v; cos(omega * t) - omega0^2 * y - c * v];
end
```

```
LAB06ex4
```



Part (b)

Period of the fast oscillation computed analytically: 3.31 seconds

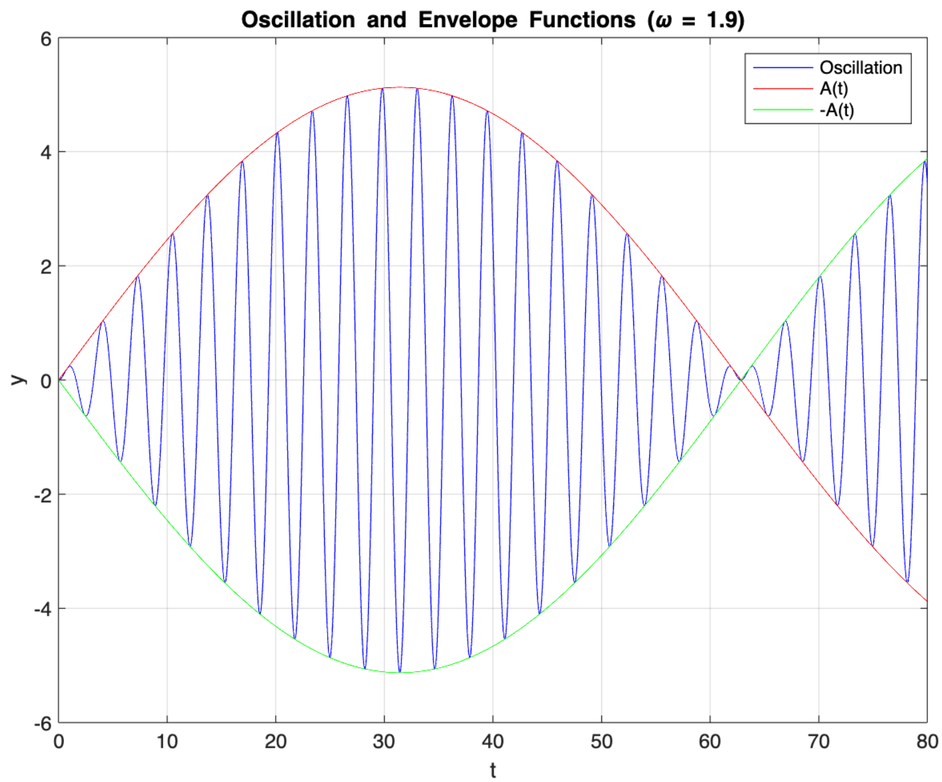
Part (c)

Length of beats: 62.83 seconds

Part (d)

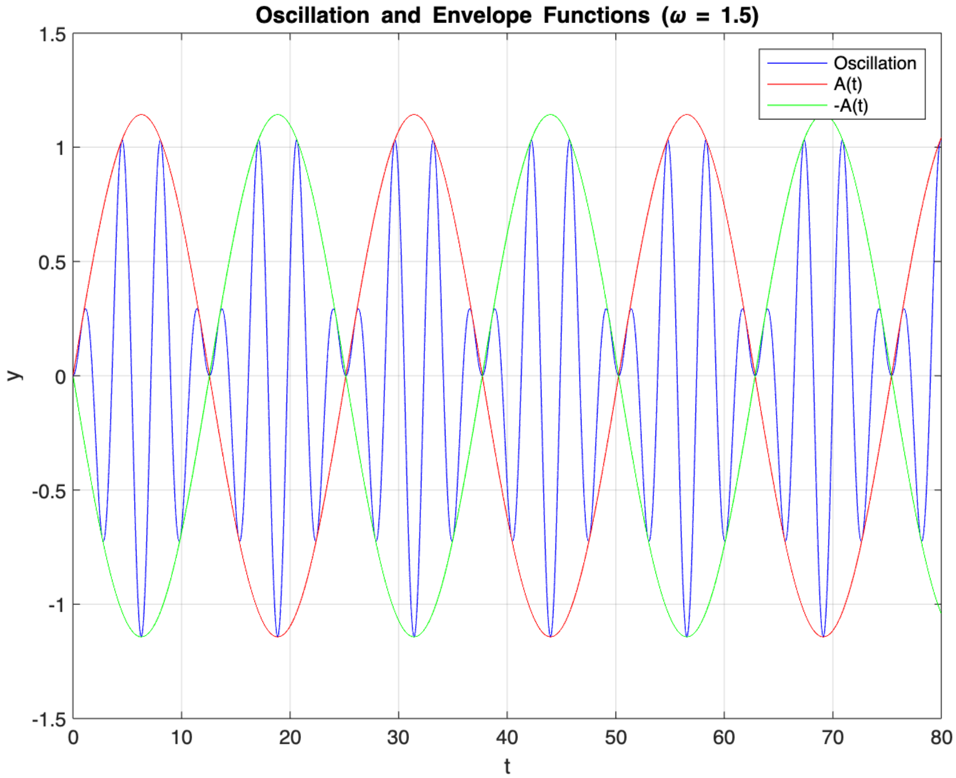
Change the value of ω in LAB06ex4.m and run it (use a new M-file)

```
LAB06ex4_d1
```



Change the value of ω in LAB06ex4.m and run it (use a new M-file)

```
LAB06ex4_d2
```



Period of the fast oscillation and length of beats for the two values of ω:

1.5 - Period:3.59 seconds , Beats: 25.13 seconds

1.9- period:3.22 seconds , beats:125.66 seconds

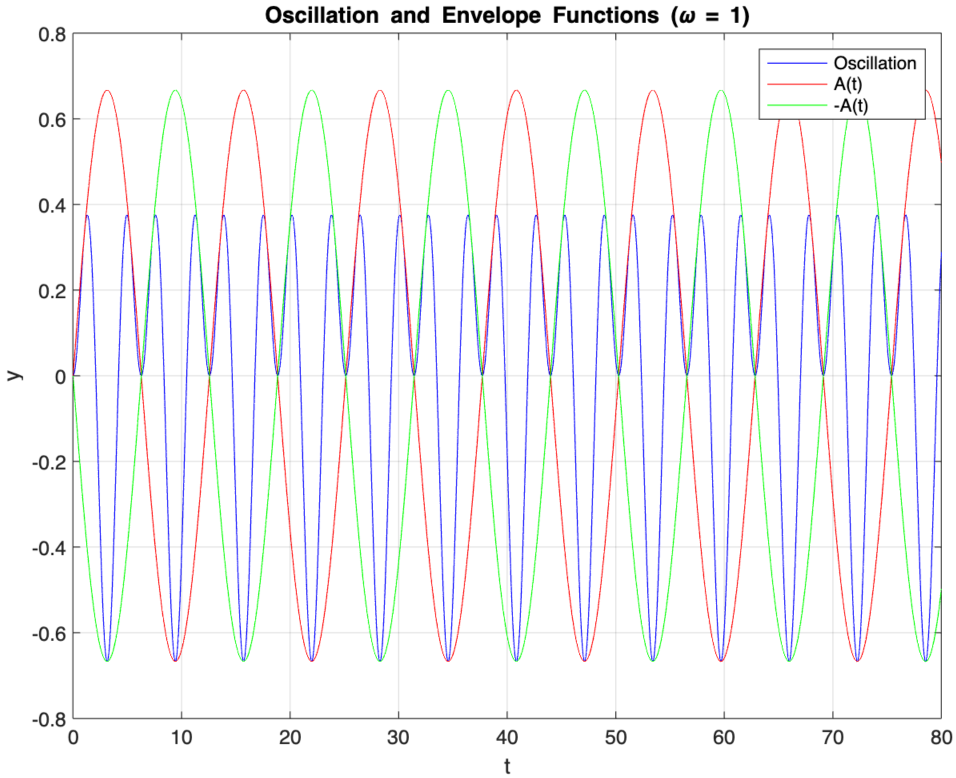
How do the periods compare to parts (a) and (c) ? That is, does the period increase or decrease as ω gets farter away from ω_0? Does the length of beats increase or decrease as as ω gets farter away from ω_0?

The periods got longer and increased as a result. The length of the beats increased fro 1.9 and decreased for 1.5

Part (e)

Change the value of ω in LAB06ex1.m (save it to a new file) and run it

```
LAB06ex4_e
```



Is the beats phenomemon still present? Why or why not? Yes, however the waves are to slow to be seen in the graph, however you can see from the graph