Welcome To Math 34A!
Differential Calculus

Instructor:

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Office Hours:

MTWR after class 2:00-3:00, and by appointment. Details on Gauchospace.

A nice thing about derivatives...

$$\begin{aligned}\frac{d}{dx}(a \cdot f(x) + b \cdot g(x)) &= a\frac{d}{dx}f(x) + b\frac{d}{dx}g(x) \\ &= a \cdot f'(x) + b \cdot g'(x)\end{aligned}$$

For example...

A nice thing about derivatives...

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$$= a \cdot f'(x) + b \cdot g'(x)$$

For example...

$$\begin{aligned} \tfrac{d}{dx}(3x^2 + 5x) &= 3\tfrac{d}{dx}x^2 + 5\tfrac{d}{dx}x \\ &= 3(2x) + 5(1) \\ &= 6x + 5 \end{aligned}$$

A Warning!

$$\frac{d}{dx}\left(f(x)g(x)\right) \neq f'(x) \times g'(x)$$

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Example: $$5x^4 = \frac{d}{dx}\left(x^5\right) = \frac{d}{dx}\left(x^2 \cdot x^3\right) \neq (2x)(3x^2) = 6x^3$$

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Example: Find the derivative of $(x+1)(2x+3)$

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Example: Find the derivative of $(x+1)(2x+3)$

Question: $\frac{d}{dx}\left((x^2+1)(x^3+1)\right) = ?$

A$= 6x^3$ B$= 5x^4 + 3x^2 + 2x$ C$= x^5 + x^3 + x^2 + 1$ D$=$ Other

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Answer: B

Review Examples:

(1) What is the x-coordinate of the point on the graph of $y = 4x^2 - 3x + 7$ where the graph has slope 13?

A= 0 B= 1 C= 2 D= 3 E= 4

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A$= 0$ B$= 1$ C$= 2$ D$= 3$ E$= 4$ C

(2) A circle is expanding so that after R seconds it has radius R cm. What is the rate of increase of area inside the circle after 2 seconds?

A$= 4\pi$ B$= 2\pi R^2$ C$= 2$ D$= 2\pi R$ E$= \pi R^2$

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Differentiating $f(x) = e^{kx}$

$$\frac{d}{dx}\left(e^{kx}\right) = ke^{kx}$$

versus

$$\frac{d}{dx}\left(x^{n}\right) = nx^{n-1}$$

⚠ Do not get confused and write $\dfrac{d}{dx}\left(e^{kx}\right) = ke^{(k-1)x}$. ⚠

Question: Find $\dfrac{d}{dx}\left(4e^{3x} + 5x^3\right)$

A$= 12e^{2x} + 15x^2$ B$= 12e^{3x} + 15x^3$ C$= 4e^{3x} + 15x^2$
D$= 12e^{3x} + 15x^2$ E= Other

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Example

$$\boxed{\frac{d}{dx}\left(e^{kx}\right) = ke^{kx}}$$

The temperature (in $^\circ$C) of a cup of coffee t hours after it is made is $f(t) = 50 + 40e^{-2t}$.

(a) What is the initial temperature when the coffee is made?

A= 40 B= 50 C= 90 D= 100

Example

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(b) How quickly is the coffee cooling down initially? This means how many degrees per hour is the temperature going down instantaneously at $t = 0$?

A= 20 B= 40 C= 60 D= 80 E= 100

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More Examples

$$\boxed{\frac{d}{dx}\left(e^{kx}\right) = ke^{kx}}$$

(1) $\dfrac{d}{dx}\left(\dfrac{3}{e^{2x}}\right) = ?$

A $= \frac{3}{2e^{2x}}$ B $= \frac{3}{2e^{x}}$ C $= \frac{6}{e^{2x}}$ D $= \frac{-6}{e^{2x}}$

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$$\frac{d}{dx}\left(e^{kx}\right) = ke^{kx}$$

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(2) The number of grams of Einsteinium-253 after t days is $m(t) = 10e^{-t/30}$. How quickly is the mass changing (in grams per day) when $t = 0$?

A $= -1/30$ B $= -1/3$ C $= -10e^{-t/30}$ D $= -\frac{1}{3}\,e^{t/30}$

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§8.12: The Second Derivative

Today: We can take the derivative of a function repeatedly!

Example: If $f(x) = x^3 - 3x + 2$, then

- $\frac{df}{dx} = f'(x) =$

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- $\dfrac{df}{dx} = f'(x) = 3x^2 - 3$
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- Keep Going! The fourth derivative is $\dfrac{d^4 f}{dx^4} = f''''(x) = 0$.

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- The fun ends here, for this $f(x)$ all higher derivatives are zero.

Examples

General idea: Differentiating the function n times gives us the nth derivative of f. It is written as

$$f''''\cdots''''(x) = f^{(n)}(x) = \frac{d^n f}{dx^n}.$$

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(2) $\dfrac{d^2}{dx^2}\left(x^5\right) =?$

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(3) $\dfrac{d^2}{dx^2}\left(\sqrt{x}\right) =?$

A$= \frac{1}{4}x^{-3/2}$ B$= \frac{-1}{4}x^{-1/2}$ C$= \frac{-1}{4}x^{-3/2}$ D$= \frac{1}{2}x^{-1/2}$ E$= 0$

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More Examples

(4) $\dfrac{d^2}{dt^2}\left(e^{3t}\right) = ?$

A$= e^{3t}$ B$= 3e^{2t}$ C$= 9e^{3t}$ D$= 3e^{3t}$ E$= 9e^{t}$

More Examples

(4) $\dfrac{d^2}{dt^2}\left(e^{3t}\right) = ?$

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(5) Find $f'''(x)$ when $f(x) = x^3$.

A$= 6x^2$ B$= 0$ C$= 3x$ D$= 3x^2$ E$= 6$

More Examples

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(5) Find $f'''(x)$ when $f(x) = x^3$.

A$= 6x^2$ B$= 0$ C$= 3x$ D$= 3x^2$ E$= 6$ E

(6) If $f(x) = x^3 - 4x^2 + 7x - 31$, then $f''(10) =?$

A$= 6$ B$= 3x^2 - 8x$ C$= 6x$ D$= 60$ E$= 52$

More Examples

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Example: Acceleration

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$$\begin{aligned} \text{acceleration} &= \text{rate of change of velocity} &&= \text{derivative of velocity}. \\ \text{velocity} &= \text{rate of change of distance} &&= \text{derivative of distance}. \end{aligned}$$

Therefore

> acceleration = second derivative of distance

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Example: Height of ball is $h(t) = 20t - 5t^2$ meters after t seconds.
(a) Velocity of ball after t seconds is

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Example: Height of ball is $h(t) = 20t - 5t^2$ meters after t seconds.

(a) Velocity of ball after t seconds is $h'(t) = 20 - 10t$ m/sec

(b) Acceleration of ball after t seconds is $h''(t) = -10$ m/sec^2

It's not the speed that kills

Suppose you hit a brick wall at 60 mph.

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$$\left(\begin{array}{c}\text{Average rate of}\\ \text{change of velocity}\\ \text{in stopping}\end{array}\right) = \frac{\Delta \text{ velocity}}{\Delta \text{ time}} = \frac{-60 \text{ mph}}{1/10 \text{ sec}}$$

$$\approx \frac{-88 \text{ ft/sec}}{1/10 \text{ sec}} = -880 \text{ ft/sec}^2.$$

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Since 1 gravity $= 32$ ft/sec^2, this is about

$$880 \text{ ft/sec}^2 = \left(880 \text{ ft/sec}^2\right) \times \frac{1 \text{ gravity}}{32 \text{ ft/sec}^2}$$

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$$880 \text{ ft/sec}^2 = \left(880 \text{ ft/sec}^2\right) \times \frac{1 \text{ gravity}}{32 \text{ ft/sec}^2} \approx 28 \text{ ``g''}.$$

The force at which the brick wall pushes you is 28 times your weight.

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$$880 \text{ ft/sec}^2 = \left(880 \text{ ft/sec}^2\right) \times \frac{1 \text{ gravity}}{32 \text{ ft/sec}^2} \approx 28 \text{ “g”}.$$

The force at which the brick wall pushes you is 28 times your weight.
If you weigh 110 pounds, this force is about 3000 pounds = 1.5 tons.

A Rocket

A rocket is fired vertically upwards. The height after t seconds is $2t^3 + 5t^2$ meters.

Question: What is the acceleration in m/sec^2 after t seconds?

A$= 2t^3 + 5t^2$ B$= 6t^2 + 10t$ C$= 12t + 10$ D$= 12$ E$= 0$

A Rocket

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- $h(t)$ = height in meters at time t seconds
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More Questions:

(a) What can we say about $f(t)$ if $f'(t) = 0$ for all t?

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$$\text{and so } \frac{d^2f}{dx^2} = \frac{d}{dx}\left(\frac{df}{dx}\right) = \text{rate of change of } \frac{df}{dx}$$

Conclusion:

> The second derivative tells you how quickly the rate of change is changing.

Uses of second derivative:

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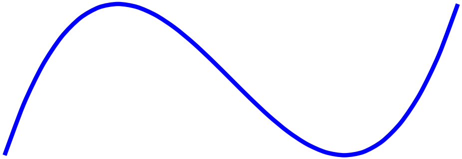
- Is the graph concave up or concave down?
- Are things changing for better or worse?

Meanings: The First Derivative



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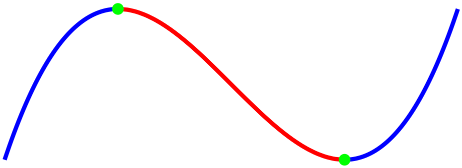

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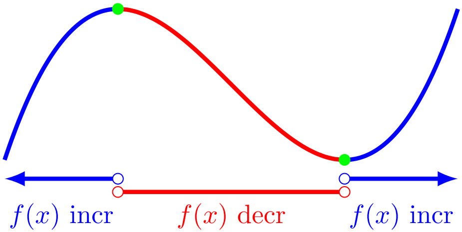

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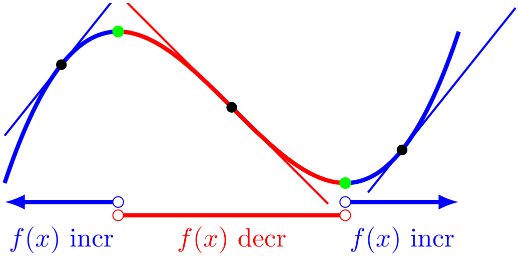

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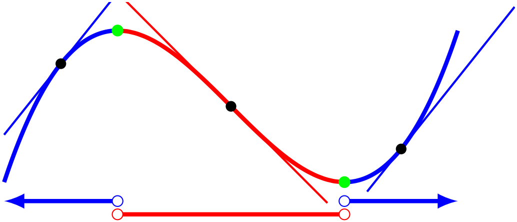

$f(x)$ incr | $f(x)$ decr | $f(x)$ incr

$f'(x) > 0$ | $f'(x) < 0$ | $f'(x) > 0$

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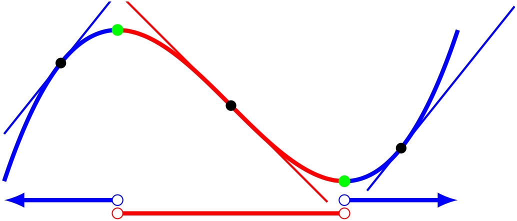

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$f'(x) > 0$

$f(x)$ decr
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$f(x)$ incr
$f'(x) > 0$

Point:

$$f'(x) > 0 \iff f(x) \text{ is increasing}$$
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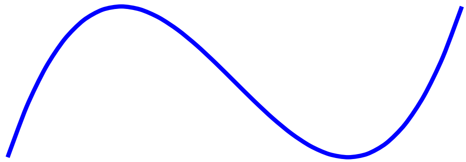
Meanings: The Second Derivative



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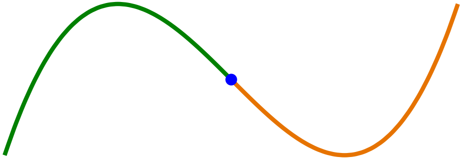

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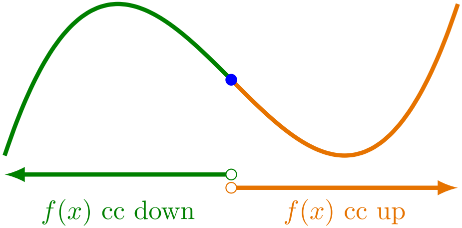

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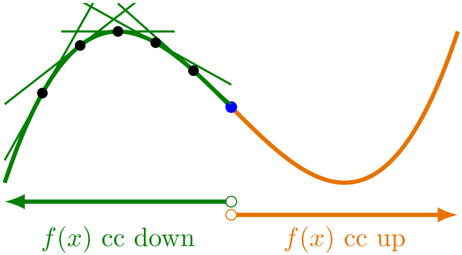

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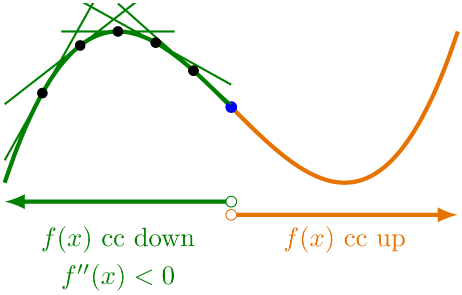

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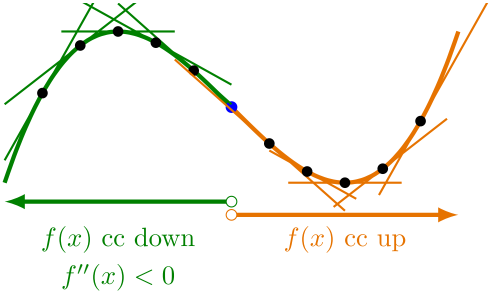

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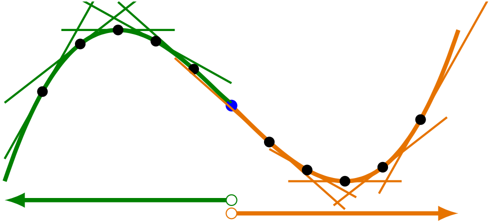


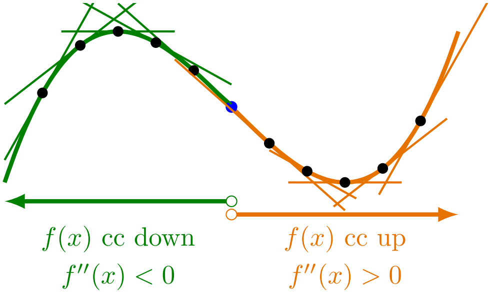
$f(x)$ cc down

$f''(x) < 0$

$f(x)$ cc up

$f''(x) > 0$

Meanings: The Second Derivative



Point:

$$
\begin{aligned}
f''(x) > 0 &\iff f'(x) \text{ is increasing} \\
&\iff f(x) \text{ is concave up} \\[1ex]
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\end{aligned}
$$

Concavity

$$f''(x) > 0 \iff f(x) \text{ is concave up}$$
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(1) For which values of x is $f(x) = x^3 - 6x^2 + 3x + 2$ concave up?

A when $x = 0$ B when $x < 6$ C when $x > 6$
D when $x < 2$ E when $x > 2$

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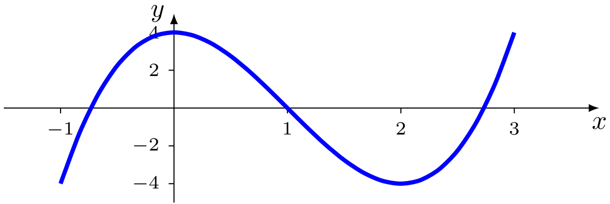
(2) Where is $f''(x) > 0$?



A when $x < 2$ B when $x > 2$ C when $x < 1$
D when $x > 1$ E when $-0.7 < x < 1$

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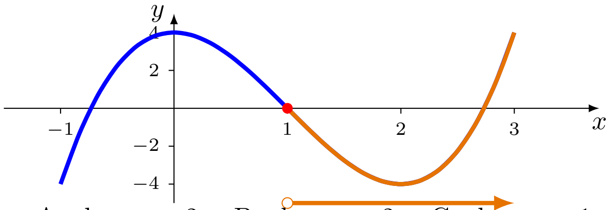

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