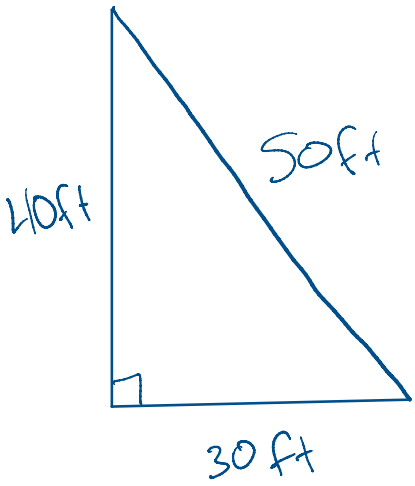
4-5 More Review

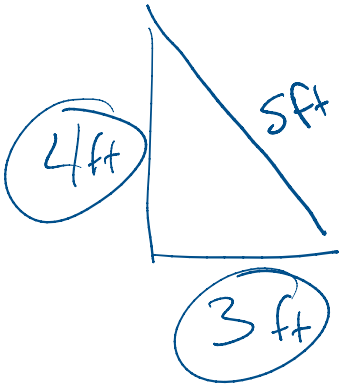
Tuesday, January 11, 2022 1:53 PM

A word problem to start off

1. The vertical mast of a yacht is 40 feet high. A rope runs in a straight line from the top to a pulley 30 feet horizontally from the base of the mast. How many feet long is the rope?
Hint: Draw a picture!

A = 30 B = 40 C = 50 D = 60 E = 70





$30^2+40^2=900+1600=2500$

50

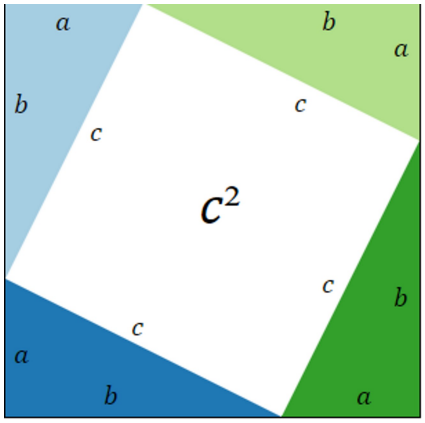
$\sqrt{1600+900}$

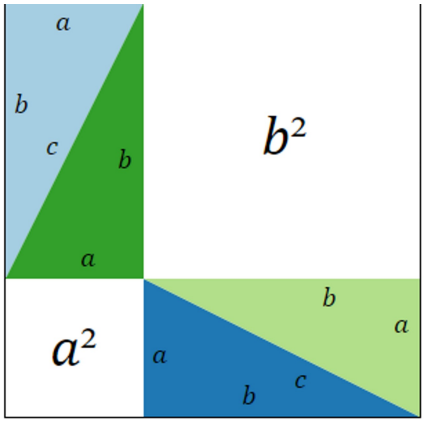
$\sqrt{2500}$

$\sqrt{25}\cdot 10$

50

Why Pythagorean Theorem works





$$c^2 = a^2 + b^2$$

Here is a better picture than the one I drew in lecture. You can find the animation that slides the triangles at this link: https://upload.wikimedia.org/wikipedia/commons/9/9e/Pythagoras-proof-anim.svg

Applications

2. A rectangular parking lot is to be made in the shape of a rectangle. It will have an area of 2000 square meters. Express the length of the parking lot in terms of the W = width.

A $= (2000 - 2W)/2$ B $= 2000/W$ C $= 2000 - W$

D = Other

Area $= 2{,}000\ m^2$, $l = ?$ $w = ?$ but...

area $= l \cdot w$, so $l \cdot w = 2{,}000$. $l = \frac{2{,}000}{w}$

3. The parking lot will be surrounded by a fence. Express the total length of the fence in terms of W.

A $= 2000 + 2W$ B $= L + W$ C $= 4000W^{-1} + 2W$

Fence length = perimeter = $2l$ & $2w$, so

$2(l) + 2w = 2\left(\frac{2000}{w}\right) + 2w$

$= \frac{4000}{w} + 2w$

5. A rectangular poster is to have a total area of 500 cm^2. There is an empty margin where nothing is printed 6 cm wide at the top and 4 cm wide along the sides and bottom. The rest is the printed area.

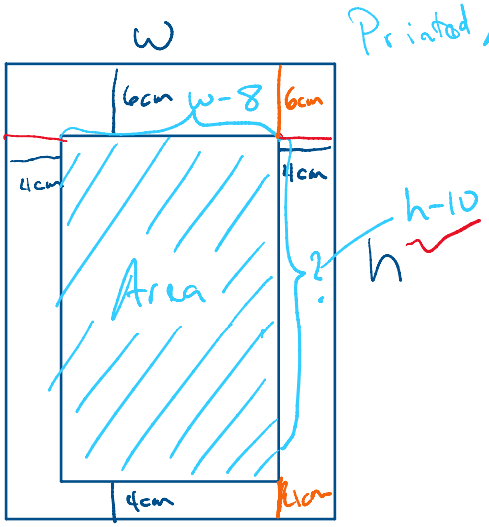
Hint: Draw a picture! Name your unknowns!

- Express printed area in terms of width W and height H of the poster.

  A $= HW$ B $= (H - 8)(W - 8)$ C = Other

H of the poster.

A $= HW$ B $= (H-8)(W-8)$ C $=$ Other



Printed Area $= (h-10)(w-8)$ $\quad h = \frac{500}{w}$

$= \left(\frac{500}{w} - 10\right)(w-8)$

$= 10\left(50w^{-1} - 1\right)(w-8)$

$= 10\left(50w^{-1}\cdot(w-8) - 1(w-8)\right)$

$= 10\left(50 - 400w^{-1} - w + 8\right)$

$= 500 - 4000w^{-1} - 10w + 80$

$= 580 - 4000w^{-1} - 10w$

Area $= 500\,cm^2 = h\cdot w$

$h = \frac{500}{w}\,cm$

- Express the area of the printed part in terms of the width W of the poster.

Printed area $= (w-8)(h-10) = (w-8)\left(\frac{500}{w} - 10\right)$

3.2.41 Express the total surface area of a cube in terms of its volume V.

Draw a picture! Name the unknowns!

$V = w^3$

$w = \sqrt[3]{V}$

$SA = 6\cdot w^2$

$= 6\left(\sqrt[3]{V}\right)^2$

8. An oil leak!

- Oil is leaking from an oil tanker at the rate of 4000 liters per hour.
- 8 liters of oil spread out over 10 square meters of ocean surface.
- A SQUARE oil slick forms.

- Express the length, X, of one side of the square oil slick as a function of the time t (in hours) the tank has been leaking.
- After how many hours will the oil slick be a square with side length 2 kilometers?

Every hour, 4,000 L spills.

8L of oil $\longleftrightarrow$ $10m^2$ surface area

After t=1 hour? 4,000L of oil, that's 500·8L of oil, so $500\cdot 10m^2$ of surface area. So $5{,}000m^2$ after 1 hour.

This means 10,000 after 2 hours... and $5{,}000\cdot t\,m^2$ after t hours.

What is X? $X^2 = 5{,}000\cdot t^2$

$X = \sqrt{5{,}000}\,t$ (in meters).

What is X? $X^2 = 5{,}000 \cdot t^2$,

so $x = \sqrt{5{,}000\,t^2} = \sqrt{5{,}000}\,t$ (in meters).

9. When you substitute $x = y + 3$ into $x^2 - 6x + 8$ you get...

A $= y^2 - 6y - 1$ B $= y^2 + 35$ C $= y^2 - 6y + 35$ D $= y^2 - 1$

$x^2 + 6x + 8 = (y+3)^2 \; 6(y+3) + 8$

$= y^2 + 6y + 9 - 6y - 18 + 8 = y^2 - 1$