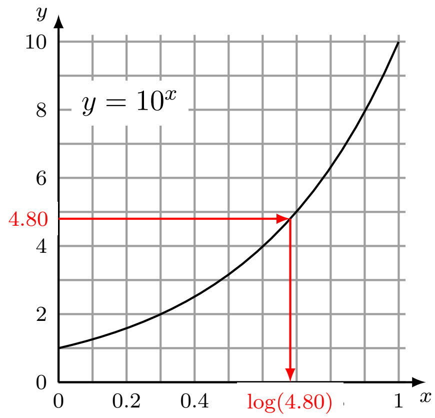
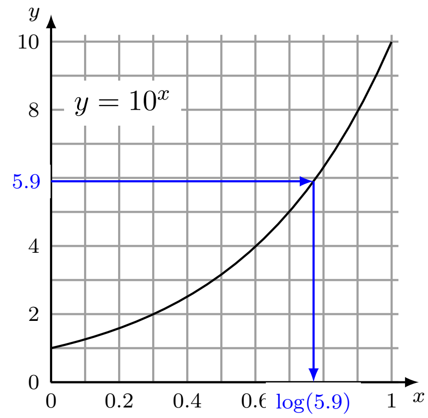
Math 34A Winter 2020
Old Midterm 3 #2 Solutions

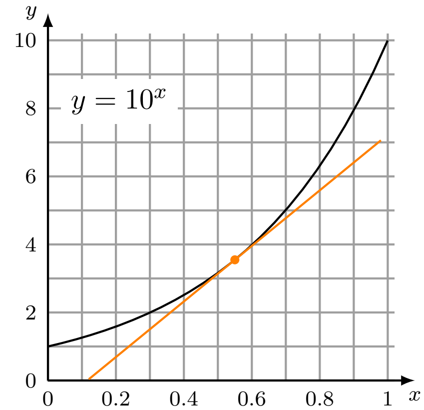
1. Here are the three graphs we'll use in solving these problems:



Part (a)



Parts (a) and (b)



Part (c)

(a) We use the rules of logs to write

$$\log(\sqrt{480}) = \log(480^{1/2}) = \frac{1}{2}\log(480).$$

Now we use the "move the decimal point" trick to compute $\log(480)$:

$$\log(480) = \log(10^2 \times 4.80) = \log(10^2) + \log(4.80) = 2 + \log(4.80).$$

The graph tells us that $\log(4.80) \approx 0.68$. Thus $\log(480) = 2 + 0.68 = 2.68$ and $\log(\sqrt{480}) = \frac{1}{2}(2.68) = \boxed{1.34}$.

(Mathematica tells me that $\log(\sqrt{480}) \approx 1.340\,620\,6\ldots$)

(b) The solution to $10^x = 1/59$ is $x = \log(1/59)$, which by rules of logs is $x = \log(1) - \log(59) = -\log(59)$ (since $\log(1) = 0$). Again we use the "move the decimal point" trick to see that is what we need here:

$$x = -\log(59) = -\log(10^1 \times 5.9) = -\big(\log(10^1) + \log(5.9)\big) = -1 - \log(5.9).$$

We use the graph to find that $\log(5.9) \approx 0.77$, so $x = -\log(59) \approx -1 - 0.77 = \boxed{-1.77}$. (Mathematica tells me that $\log(1/59) \approx -1.770\,852\,0\ldots$, so we're pretty close.)

(c) We've drawn the tangent line at $x = 0.55$ on the third graph, above. The slope of this line is about

$$m = \frac{6.4 - 0.7}{0.9 - 0.2} = \frac{5.7}{0.7} \approx \boxed{8.14}.$$

The actual slope of the tangent line to $y = 10^x$ at $x = 0.55$ is $m = 10^{0.55}\ln(10) \approx 8.169\,880\,2\ldots$, so as usual we're pretty close.

2. We write down the answers without much commentary:

(a) $\dfrac{d}{dx}(7x^3 + 3x - 4) = 21x^2 + 3$

(b) $\dfrac{d^2}{dx^2}(9x^2 + 5e^{3x}) = \dfrac{d}{dx}(18x + 15e^{3x}) = 18 + 45e^{3x}$

(c) $\dfrac{d}{dx}(x^{k-1} + k^2) = (k-1)x^{k-2} + 0 = (k-1)x^{k-2}$

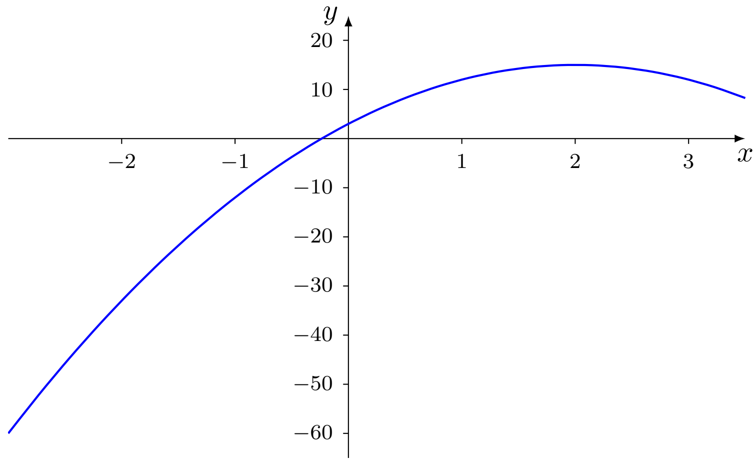
3. The tangent line is the line through $(t, f(t)) = (5, f(5)) = (5, 88)$ with slope $f'(5) = -2$. Thus the line has equation

$$y - 88 = -2(t-5) \qquad \text{or, equivalently,} \qquad y = -2t + 98.$$

(a) When $t = 7$, the temperature of the coffee is $f(7) \approx -2(7) + 98 = -14 + 98 = \boxed{84° \text{ C}}$.

(b) This question can be re-phrased as: what is t when the temperature is 70° C: $f(t) = 70$? We have estimated the temperature as $f(t) \approx -2t + 98$, so this is 70 when $-2t + 98 = 70$. Solving, we get $t = \boxed{14 \text{ minutes}}$.

4. Here is a picture of the graph of $y = -3x^2 + 12x + 3$:



(a) The slope of the graph is the derivative, $\frac{dy}{dx}$. Since $\frac{dy}{dx} = -6x + 12$, the slope of the graph at $x = 1$ is $-6(1) + 12 = \boxed{6}$.

(b) The tangent line at $x = 1$ has slope 6 (from part (a)) and passes through the point $(x, y) = (1, -3(1)^2 + 12(1) + 3) = (1, 12)$. Thus the equation of the tangent line is

$$y - 12 = 6(x-1) \qquad \text{or, equivalently} \qquad \boxed{y = 6x + 6}.$$

(c) The slope is $\frac{dy}{dx} = -6x + 12$, so this is zero when $-6x + 12 = 0$; that is, when $x = 2$. The y-coordinate at this point is

$$y = -3\,(2)^2 + 12\,(2) + 3 = -12 + 24 + 3 = \boxed{15}.$$

(d) The slope is $\frac{dy}{dx} = -6x + 12$, which is 11 when $-6x + 12 = 11$. The solution to this is $\boxed{x = 1/6}$.

5. (a) The velocity of the rocket is $\boxed{h'(t) = -6t + 60 \text{ m/s}}$.

(b) The acceleration of the rocket is $\boxed{h''(t) = -6 \text{ m/s}^2}$.

(c) The velocity is 18 m/s when $-6t + 60 = 18$ (where this formula is from part (a)). Solving, we get $\boxed{t = 7 \text{ seconds}}$.

(d) The rocket is rising (going up) when the velocity $h'(t)$ is positive, and similarly the rocket is falling when $h'(t)$ is negative. Since $h'(t) = -6t + 60$, this velocity is zero only when $t = 10$ seconds. When $t < 10$, $h'(t)$ is positive (so the rocket is rising). When $t > 10$, $h'(t)$ is negative (so the rocket is falling). Thus the rocket reaches its maximum height when $t = 10$ seconds. This height is

$$h(10) = 500 - 3(10)^2 + 60(10) = 500 - 300 + 600 = \boxed{800 \text{ meters}}.$$

(e) The height of the rocket above the ground at time $t = 0$ seconds is $h(0) = 500 - 3(0)^2 + 60(0) = 500$ meters. Similarly, the height of the rocket above the ground at time $t = 2$ seconds is $h(2) = 500 - 3(2)^2 + 60(2) = 608$ meters. Thus the rocket has traveled $h(2) - h(0) = \boxed{108 \text{ meters}}$ between $t = 0$ and $t = 2$ seconds.