# Office Hours!

Instructor:
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Office Hours:
    ~~Mondays 1–2PM~~ Not Today!
    Tuesdays 10:30–11:30AM
    Thursdays 1–2PM
    or by appointment

Office:
    South Hall 6431X (Grad Tower, 6th floor, blue side, first door on the right)

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Have you used OnLine Math Lab (there is a link on Gauchospace)?
A Yes
B No
C What is it?
D Prefer not to say

# Today: Start Chapter 7 (Logs)

**Applications:**

- **Chemistry**: alkalinity and acidity, pH scale

- **Finance**: compound interest (get rich slow)

- **Geology**: Richter scale for earthquakes (did you feel the earth move too ?)

- **Archeology**: radio carbon dating (how old is that bone ?)

- **Astronomy**: stellar magnitude (brightness of stars)

- **Sound**: decibels (what did you say?, the music is too loud)

- **Math**: solving equations with exponents (includes all of the above)

# Today: Start Chapter 7 (Logs)

Main Idea of Chapter 7:

> $\log(x)$ is how many tens you multiply to get $x$

Conclusion:

Before we do logs we should be really good at powers of 10.

# Powers of Ten

1 meter $\approx$ 3 feet
1 centimeter $= 0.01$ meters $= 10^{-2}$ meters $\approx 1/2$ inch
1 kilometer $= 1,000$ meters $= 10^3$ meters $\approx 1/2$ mile

Approximate distance (in meters), to nearest power of 10

| | |
|---|---|
| $10^7$ meters | Size of Earth |
| $10^9$ meters | Distance to moon |
| $10^{14}$ meters | Size of our solar system |
| $10^{16}$ meters | One light-year |
| $10^{21}$ meters | Size of the Milky Way galaxy |
| $10^{27}$ meters | Size of the universe (about 93 billion light-years) |
| $10^{80}$ | number of protons in the observable universe? |
| $10^{100}$ | 1 googol |
| $10^{1000}$ meters | ??? |

# Exponential Basics

$$10^4 = 10 \times 10 \times 10 \times 10 \ = \ 10,000$$
$$= 4 \text{ lots of } 10 \text{ multiplied together}$$
$$= 1 \text{ followed by } 4 \text{ zeroes}$$

$$10^x = \underbrace{10 \times 10 \times \cdots \times 10}_{x \text{ lots of } 10} = 1\underbrace{00000\cdots0}_{x \text{ zeros}}$$
$$= 1 \text{ followed by } x \text{ zeroes}$$

Ex: 
$$10^2 \times 10^3 = (10 \times 10) \times (10 \times 10 \times 10)$$
$$= 10^{2+3} = 10^5.$$

$$10^x \times 10^y = 10^{x+y} \qquad \text{First Law of Exponents}$$

Why?    We can work it out!

# Exponential Basics (cont'd)

$$10^x \times 10^y = 10^{x+y} \qquad \text{First Law of Exponents}$$

**Why?**    <u>We can work it out:</u>

($x$ lots of 10 multiplied together) × ($y$ lots of 10 multiplied together)
$= (x + y)$ lots of 10 multiplied together

For now $x$ and $y$ are positive whole numbers.

# More Exponentiation

$$\left(10^2\right)^3 = (10 \times 10)^3$$
$$= (10 \times 10) \times (10 \times 10) \times (10 \times 10)$$
$$= 10^6$$

$$\left(10^a\right)^b = 10^{ab} \qquad \text{Fourth Law of Exponents}$$

**Why?**  We can work it out:

$$10^a = \underbrace{10 \times 10 \times \cdots \times 10}_{a \text{ times}}$$

$$\left(10^a\right)^b = \underbrace{(10 \times \cdots \times 10) \times \cdots \times (10 \times \cdots \times 10)}_{b \text{ times}}$$
$$= 10^{ab}.$$

Just count the zeros!

# When the power is 0 or negative

What is $10^0$?$= 1$    But why?    <u>We can work it out:</u>

$$10^0 \times 10^1 = 10^{0+1}$$

$$\text{so} \quad 10^0 \times 10 = 10$$

$$\text{and therefore} \quad 10^0 = 10/10 = 1$$

Summary: we used the first law of exponents to figure out what $10^0$ must be.

There is a second explanation in the book!

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What is $10^{-2}$?$= 1/100 = 0.01$    But why?    <u>We can work it out:</u>

$$10^{-2} \times 10^2 = 10^{-2+2} = 10^0 = 1$$

$$\text{therefore} \quad 10^{-2} = \frac{1}{10^2} \quad \text{and} \quad 10^{-a} = \frac{1}{10^a}$$

There is a second explanation in the book

# The Five Laws of Exponents

(1) $10^a \times 10^b = 10^{a+b}$        (2) $10^0 = 1$

(3) $10^{-a} = 1/10^a$        (4) $(10^a)^b = 10^{ab}$

(5) $10^a/10^b = 10^{a-b}$

**1.** What is $10^3 \times 10^4$?

A= $10^{12}$    B= $10^7$    C= $10^{34}$    D= $10^0$    E= $10^{-7}$    $\boxed{B}$

**2.** Find $10^3/10^4$

A= $10^7$    B= $10^1$    C= $10^{-4}$    D= $10^{-1}$    E= $10^{-7}$    $\boxed{D}$

**3.** Find $(10^3)^4$.

A= $10^7$    B= $10^1$    C= $10^{12}$    D= $10^{-1}$    E= $10^0$    $\boxed{C}$

# The Five Laws of Exponents

(1) $10^a \times 10^b = 10^{a+b}$  (2) $10^0 = 1$

(3) $10^{-a} = 1/10^a$  (4) $(10^a)^b = 10^{ab}$

(5) $10^a/10^b = 10^{a-b}$

**4.** What is $\left(10^2 \times 10^3\right)^4$?

A= $10^8$   B= $10^9$   C= $10^{12}$   D= $10^{20}$   E= $10^{24}$   $\boxed{D}$

**5.** What is $(10^2 \times 10^6)/(10^2 \times 10^3)$?

A= $10^2$   B= $10^3$   C= $10^{-1}$   D= $10^7$   E= $10^6$   $\boxed{B}$

**6.** What is $\left(10^2/10^5\right)^{-2}$?

A= $10^{-6}$   B= $10^{-5}$   C= $10^6$   D= $10^4$   $E = 10^5$   $\boxed{C}$

# Non-Integer Powers

We can work them out!

**7.** What is $10^{0.5} = 10^{1/2}$? Answer: $10^{0.5} = \sqrt{10} \approx 3.16288$

**8.** What is $10^{0.1} = 10^{1/10}$? Answer: $10^{0.1} = \sqrt[10]{10} \approx 1.258926$

**9.** Similarly: $10^{0.01} = \sqrt[100]{10} \approx 1.02329$

$10^{0.001} = \sqrt[1000]{10} \approx 1.00231$

**10.** What is $10^{0.27}$? Answer:

$$10^{0.27} = 10^{27/100} = \sqrt[100]{10^{27}} = \left(\sqrt[100]{10}\right)^{27} \approx 1.862$$

# Moving to Logarithms

$\log(y)$ is how may tens you multiply together to get $y$

$$10^{\log(y)} = y$$

$\log(10) = ?1$    because    $10^1 = 10$

$\log(100) = ? = 2$    because    $10^2 = 100$

$\log(1000) = ? = 3$    because    $10^3 = 1000$

$\log(100,000) =$

A= 2    B= 3    C= 4    D= 5    E= 6    $\boxed{D}$

# Still moving to Logarithms

$\log(y)$ is how may tens you multiply together to get $y$

$$10^{\log(y)} = y$$

$\log(0.1) = ? -1$    because   $10^{-1} = 1/10 = 0.1$

$\log(0.01) = ? = -2$    because   $10^{-2} = 1/100 = 0.01$

$\log(10^x) = ? = x$    duh?

How confused are you?

A=not at all    B=a bit    C=a lot    D=totes confused

# Next Time: How To Find Logs

(1) Use a calculator: efficient but not good for learning

(2) Use the graph on page 290 of textbook / handout (see GS)

(3) Use table of logarithms on page 289 of textbook / handout (GS)

Our goal: use (2) and (3) to understand:

<div style="text-align:center">logs, functions and inverse functions.</div>

Our main use of logs: solving certain kinds of equation.
Mistakes will follow unless you <u>practice</u> finding logs the old fashioned way.

antilog

$$y = \log(x) \qquad 10^y = x$$

log
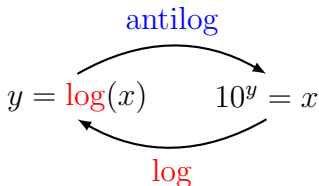
log is the inverse function of antilog
antilog is another name for
the 10-to-the-power function:
$$\text{antilog}(y) = 10^y.$$