Welcome To Math 34A! Differential Calculus

Instructor:

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South Hall 6701

Office Hours:

T R 11-11:50, T 3:45-4:35 Details on Gauchospace.

Warm-up

How many times do we need to double 1 to get the following numbers?

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- 4
- 8
- 32
- 1
- $\frac{1}{2}$

Warm-up

How many times do we need to double 1 to get the following numbers?

- 4 $\boxed{2}$
- 8 $\boxed{3}$
- 32 $\boxed{5}$
- 1 $\boxed{0}$
- $\frac{1}{2}$ $\boxed{-1}$

Straight Lines (§6.1)

Calculus is about derivatives (Math 34A) and integrals (Math 34B).

A derivative is the slope of a line.

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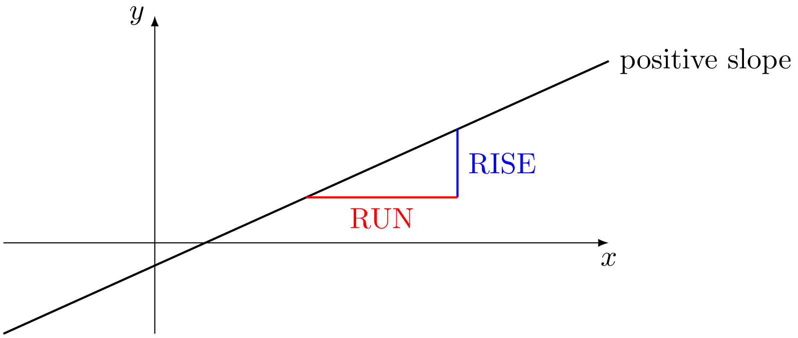

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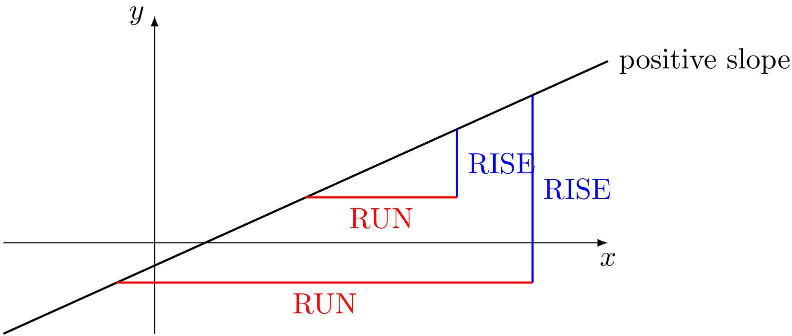

Examples

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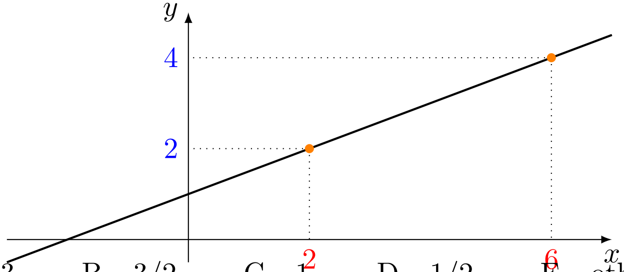

A= 3 B= 3/2 C= 1 D= 1/2 E= other

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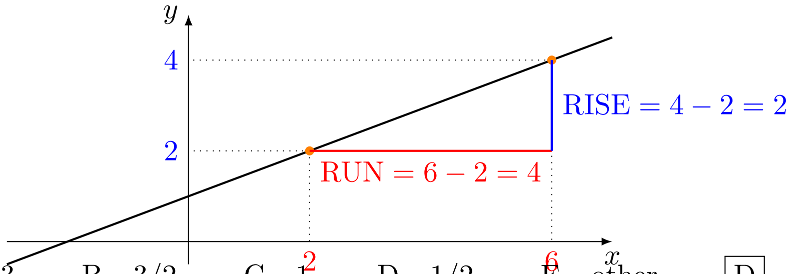

A= 3 B= 3/2 C= 1 D= 1/2 E= other $\boxed{\text{D}}$

slope = # units UPWARDS you move for each unit you move TO THE RIGHT

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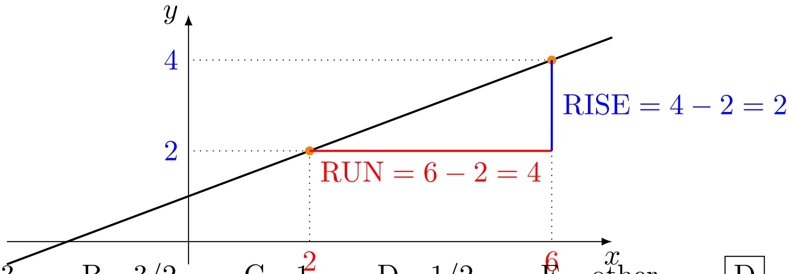

A$=3$ B$=3/2$ C$=1$ D$=1/2$ E$=$ other $\boxed{\text{D}}$

slope = # units UPWARDS you move for each unit you move TO THE RIGHT

Idea: RISE = slope $\times$ RUN

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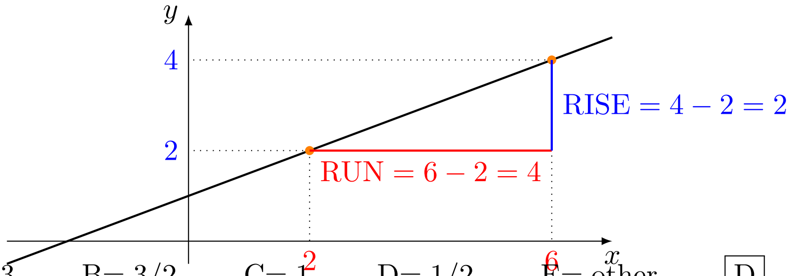

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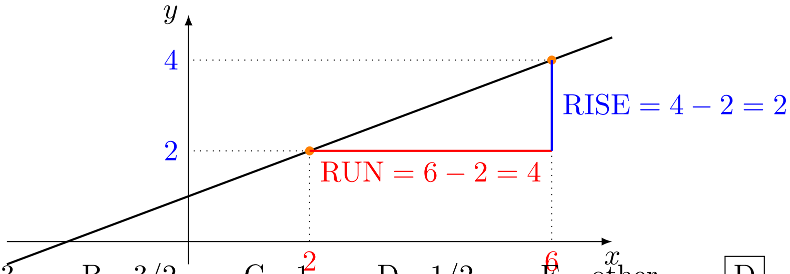

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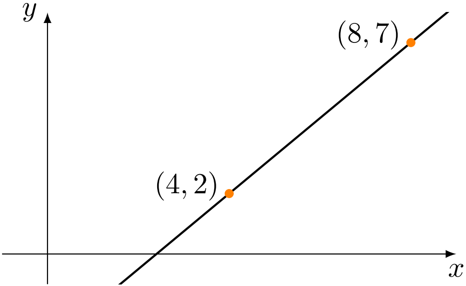
A 10% gradient on a mountain road is a slope of 1/10. It means for every 10 feet you move horizontally you go up (or down) 1 foot

Examples (page 2)

2. What is the slope here:



A$= 5/4$ B$= 4/5$ C$= 1/4$ D$= 4$ E$= 5$

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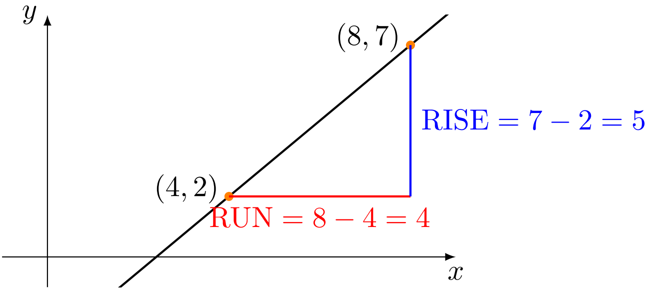

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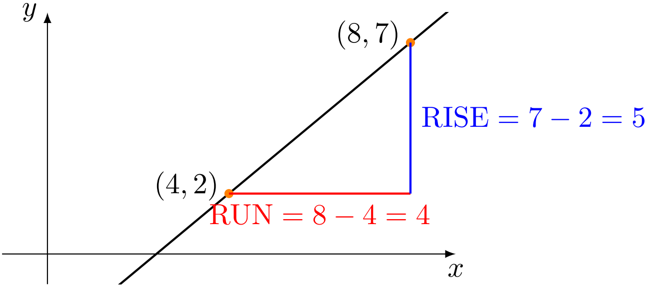

A= 5/4 B= 4/5 C= 1/4 D= 4 E= 5 $\boxed{\text{A}}$

Examples (page 3)

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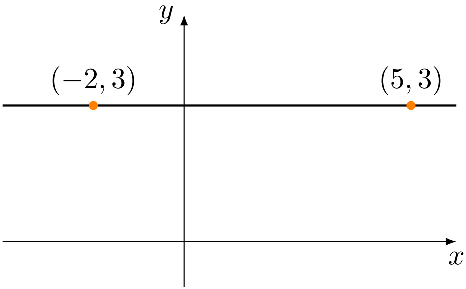

A$= 0$ B$= 7$ C$= 5/3$ D$= \infty$ E$= 3/5$

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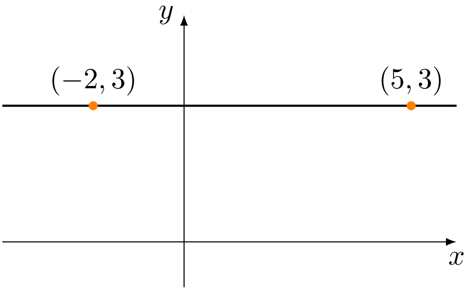

RISE $= 3 - 3 = 0$

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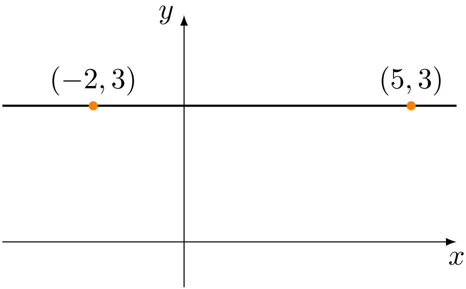

RISE $= 3 - 3 = 0$

RUN $=$ who cares?

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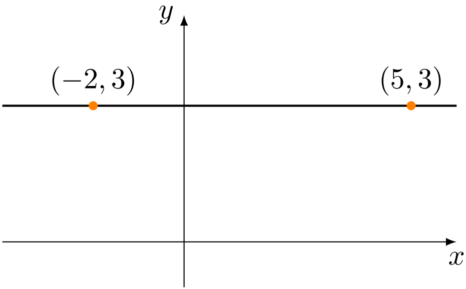


$\color{blue}{\text{RISE} = 3 - 3 = 0}$

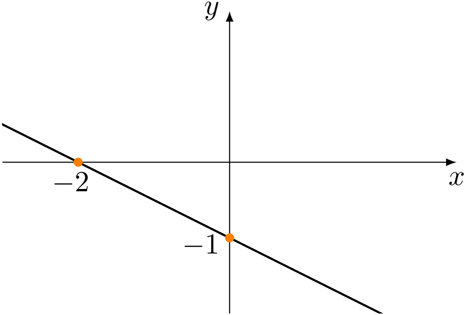
$\color{red}{\text{RUN} = \text{who cares?}}$

$$\text{slope} = \frac{0}{\text{something}} = 0$$

A$= 0$ B$= 7$ C$= 5/3$ D$= \infty$ E$= 3/5$ $\boxed{\text{A}}$

Examples (big finish!)

4. What is the slope here:



A$= -1$ B$= 1$ C$= 1/2$ D$= -1/2$ E$= -2$

Examples (big finish!)

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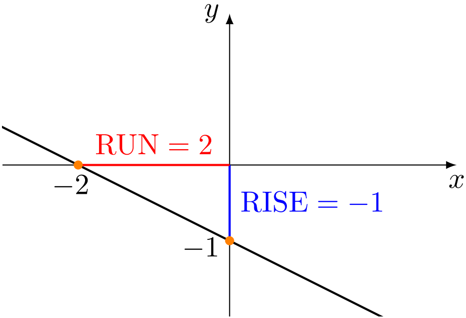

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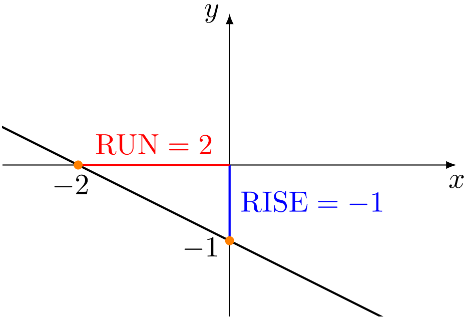

A= -1 B= 1 C= $1/2$ D= $-1/2$ E= -2 $\boxed{\text{D}}$

General Case

5. A line goes through two points: (x_0, y_0) and (x_1, y_1). Find the slope of this line. Draw a picture!

A= $y_1 - y_0$ B= $(y_1 - x_1)/(y_0 - x_0)$
C= $(y_1 - y_0)(x_1 - x_0)$

D= $(y_1 - y_0)/(x_1 - x_0)$ E= Shirley you're joking

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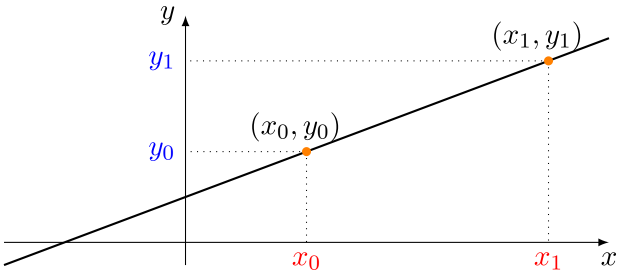

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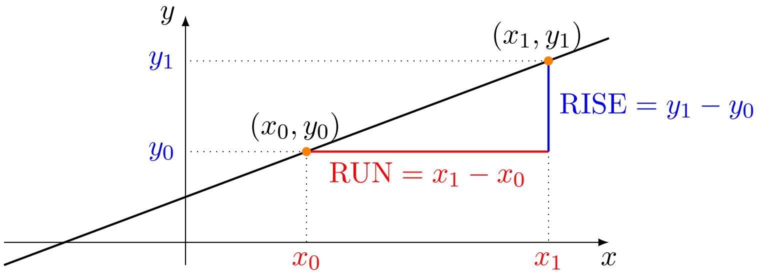

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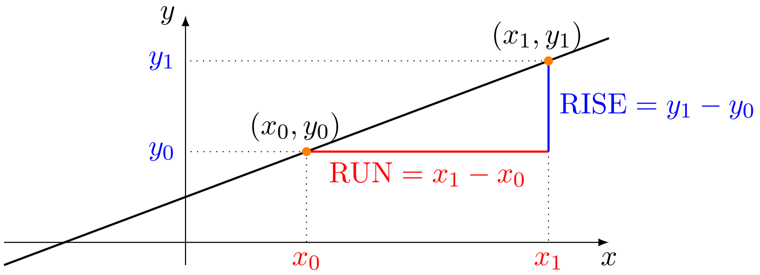


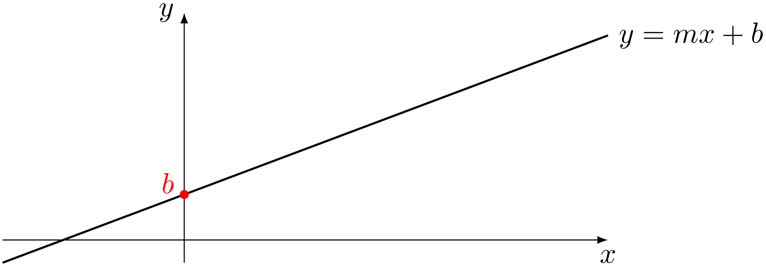
$$\text{Slope} = \frac{\text{RISE}}{\text{RUN}} = \frac{y_1 - y_0}{x_1 - x_0}$$

The Equation of a Line

The Slope Intercept Form

The **slope intercept** equation of a straight line is

$\boxed{y = {\color{blue}m}x + {\color{red}b}}$.



${\color{blue}m}$ = the **slope**. CRUCIAL for calculus.

${\color{red}b}$ = where the line crosses the y-axis (the "y-intercept").

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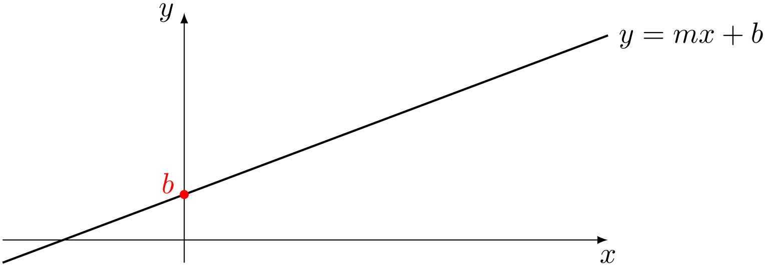

$m =$ the **slope**. CRUCIAL for calculus.

$b =$ where the line crosses the y-axis (the "y-intercept").

WHY? Because when you plug in $x = 0$, you get $y = b$.

Example

6. Find the equation of the line $\boxed{y = mx + b}$ through the points $(1, 3)$ and $(7, 5)$.

Plan: Find m, then find b.

- What is m?

  A$= 1$ B$= 3$ C$= 5$ D$= 1/3$ E$= 2$

Example

6. Find the equation of the line $\boxed{y = mx + b}$ through the points $(1, 3)$ and $(7, 5)$.

Plan: Find m, then find b.

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Plan: Find m, then find b.

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  So $y = \dfrac{1}{3}x + b$. What is b? Plug in either point!

- What do you get for b?

  A $= 1/3$ B $= 4/3$ C $= 7/3$ D $= 8/3$ E $= 10/3$

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You Try It

7. A line has slope $1/2$ and goes through the point $(2, 5)$. What is the y-coordinate of the point on this line where $x = 6$?

A$= 3$ B$= 4$ C$= 5$ D$= 6$ E$= 7$

You Try It

7. A line has slope $1/2$ and goes through the point $(2, 5)$. What is the y-coordinate of the point on this line where $x = 6$?

A$= 3$ B$= 4$ C$= 5$ D$= 6$ E$= 7$

Plan: 1. Find equation of the line.
2. Plug in $x = 6$ to find y.

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A$= 3$ B$= 4$ C$= 5$ D$= 6$ E$= 7$ $\boxed{\text{E}}$

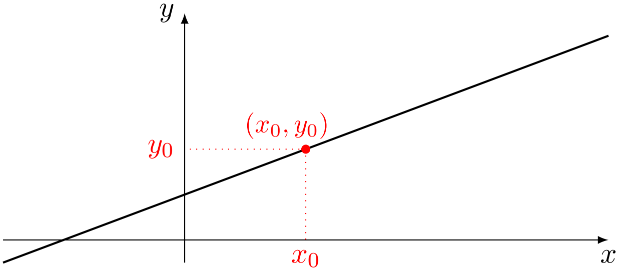
Plan: 1. Find equation of the line.
2. Plug in $x = 6$ to find y.

Another Equation of a Line

The Point-Slope Form

The **point slope** equation of a straight line is

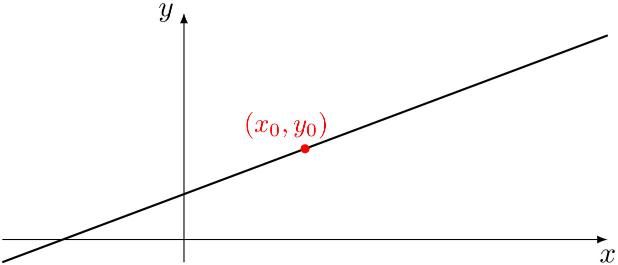
$$\boxed{y = y_0 + m(x - x_0)}.$$



m = the **slope**. Still CRUCIAL for calculus.

(x_0, y_0) = any point on the line.

Why Does This Work?



(x, y) lies on the line exactly when

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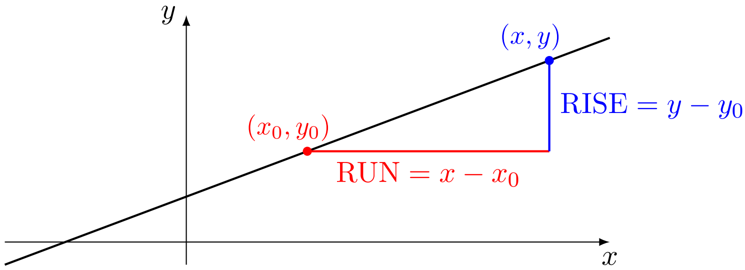

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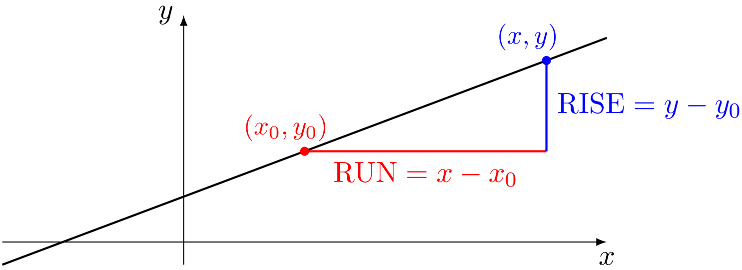

(x, y) lies on the line exactly when $\dfrac{y - y_0}{x - x_0} = m$

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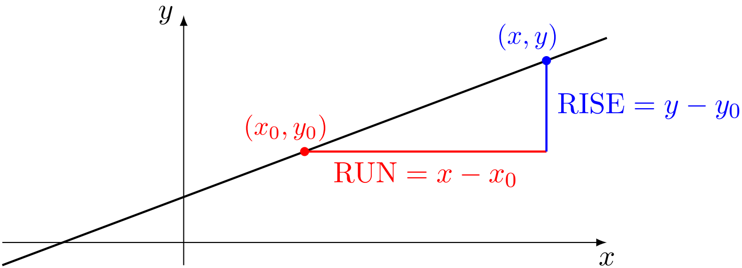

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$$y - y_0 = m(x - x_0)$$

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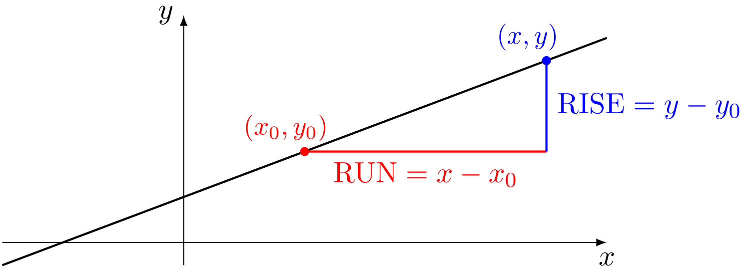
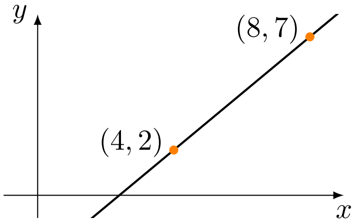


(x, y) lies on the line exactly when

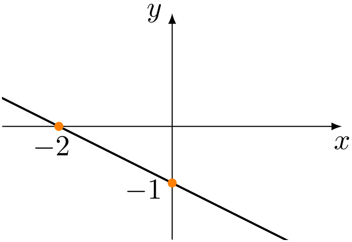
$$\frac{y - y_0}{x - x_0} = m$$
$$y - y_0 = m(x - x_0)$$
$$y = y_0 + m(x - x_0)$$

Examples

8. Find the equations of these lines (whose slopes we've already found):



$m = 5/4$



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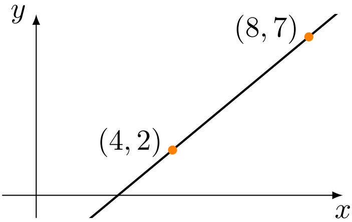

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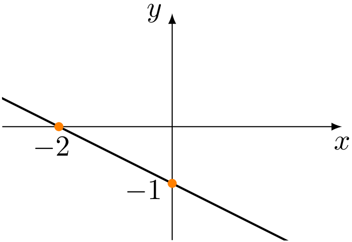

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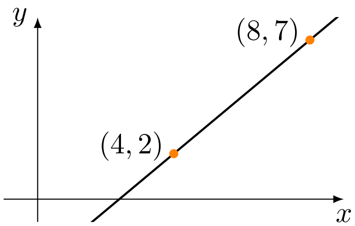

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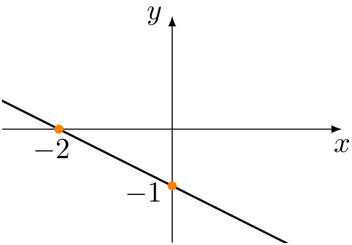

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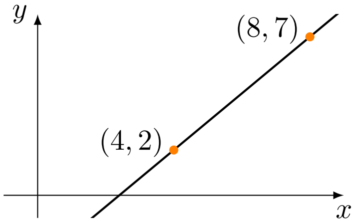

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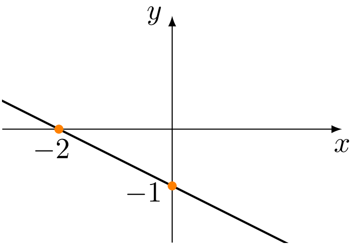

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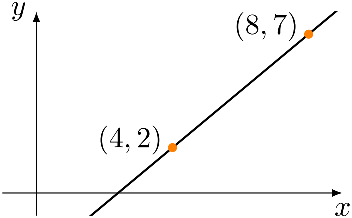

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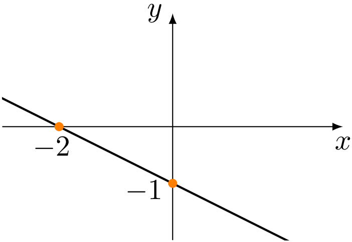

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$$y = -\tfrac{1}{2}x - 1$$

And. . . ?

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Derivatives are about **rate of change** and that is what **slope** is!

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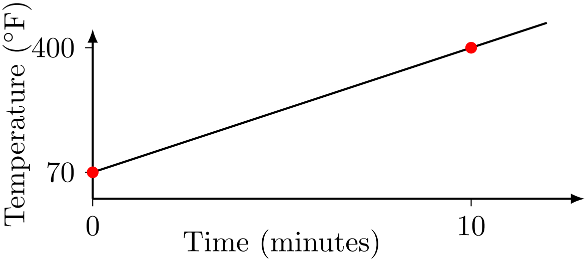
Example: This graph shows the temperature in an oven as it heats up:



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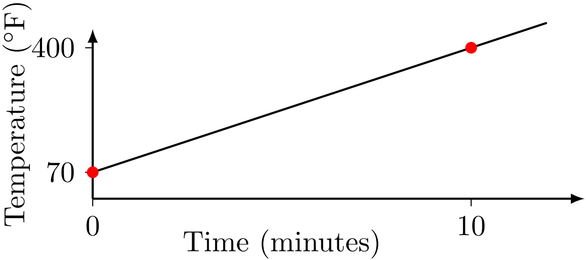

9. How quickly (in °F/min) is the oven heating up?

A= 70 B= 10 C= 40 D= 33 E=Other

And. . . ?

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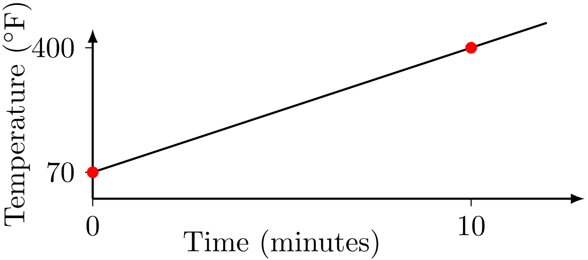

9. How quickly (in °F/min) is the oven heating up?

A= 70 B= 10 C= 40 D= 33 E=Other $\boxed{\text{D}}$

And. . . ?

Yes, but what's this got to do with calculus?

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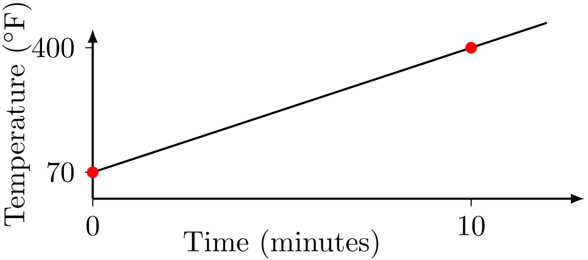

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3. THINK why this gives the answer!

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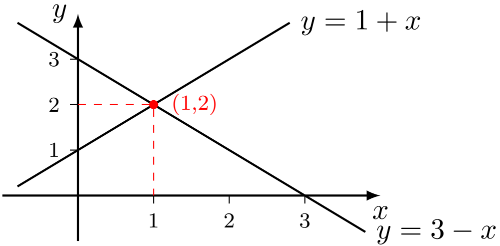

That's it. Thanks for being here.