Office Hours!

Instructor:

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Office Hours:

~~Mondays 1–2PM~~ Not Today!

Tuesdays 10:30–11:30AM

Thursdays 1–2PM

or by appointment

Office:

South Hall 6510

Have you used OnLine Math Lab (there is a link on Gauchospace)?
A Yes
B No
C What is it?
D Prefer not to say

Today: Start Chapter 7 (Logs)

Applications:

- Chemistry: alkalinity and acidity, pH scale
- Finance: compound interest (get rich slow)
- Geology: Richter scale for earthquakes (did you feel the earth move too ?)
- Archeology: radio carbon dating (how old is that bone ?)
- Astronomy: stellar magnitude (brightness of stars)
- Sound: decibels (what did you say?, the music is too loud)
- Math: solving equations with exponents (includes all of the above)

Today: Start Chapter 7 (Logs)

Main Idea of Chapter 7:

> $\log(x)$ is how many tens you multiply to get x

Conclusion:

Before we do logs we should be really good at powers of 10.

Powers of Ten

1 meter $\approx$ 3 feet
1 centimeter $= 0.01$ meters $= 10^{-2}$ meters $\approx 1/2$ inch
1 kilometer $= 1,000$ meters $= 10^{3}$ meters $\approx 1/2$ mile

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Approximate distance (in meters), to nearest power of 10

10^{7} meters	Size of Earth
10^{9} meters	Distance to moon
10^{14} meters	Size of our solar system
10^{16} meters	One light-year
10^{21} meters	Size of the Milky Way galaxy
10^{27} meters	Size of the universe (about 93 billion light-years)
10^{80}	number of protons in the observable universe?
10^{100}	1 googol
10^{1000} meters	???

Exponential Basics

$$10^4 = 10 \times 10 \times 10 \times 10 = 10,000$$
$$= 4 \text{ lots of } 10 \text{ multiplied together}$$
$$= 1 \text{ followed by } 4 \text{ zeroes}$$

$$10^x = \underbrace{10 \times 10 \times \cdots \times 10}_{x \text{ lots of } 10} = 1\underbrace{00000\cdots0}_{x \text{ zeros}}$$
$$= 1 \text{ followed by x zeroes}$$

Ex: $10^2 \times 10^3 = (10 \times 10) \times (10 \times 10 \times 10)$
$$= 10^{2+3} = 10^5.$$

$$10^x \times 10^y = 10^{x+y} \qquad \text{First Law of Exponents}$$

Why? We can work it out!

Exponential Basics (cont'd)

$$10^x \times 10^y = 10^{x+y} \qquad \text{First Law of Exponents}$$

Why? We can work it out:

(x lots of 10 multiplied together) $\times$ (y lots of 10 multiplied together)
$= (x + y)$ lots of 10 multiplied together

For now x and y are positive whole numbers.

More Exponentiation

$$\begin{aligned}(10^2)^3 &= (10 \times 10)^3 \\ &= (10 \times 10) \times (10 \times 10) \times (10 \times 10) \\ &= 10^6\end{aligned}$$

$$(10^a)^b = 10^{ab} \qquad \text{Fourth Law of Exponents}$$

Why? We can work it out:

$$10^a = \underbrace{10 \times 10 \times \cdots \times 10}_{a \text{ times}}$$

$$\begin{aligned}(10^a)^b &= \underbrace{(10 \times \cdots \times 10) \times \cdots \times (10 \times \cdots \times 10)}_{b \text{ times}} \\ &= 10^{ab}.\end{aligned}$$

Just count the zeros!

When the power is 0 or negative

What is $10^0 ?= 1$ But why? We can work it out:

$$10^0 \times 10^1 = 10^{0+1}$$

$$\text{so} \quad 10^0 \times 10 = 10$$

$$\text{and therefore} \quad 10^0 = 10/10 = 1$$

Summary: we used the first law of exponents to figure out what 10^0 must be.

There is a second explanation in the book!

What is $10^{-2} ?= 1/100 = 0.01$ But why? We can work it out:

$$10^{-2} \times 10^2 = 10^{-2+2} = 10^0 = 1$$

$$\text{therefore} \quad 10^{-2} = \frac{1}{10^2} \quad \text{and} \quad 10^{-a} = \frac{1}{10^a}$$

There is a second explanation in the book

The Five Laws of Exponents

(1) $10^a \times 10^b = 10^{a+b}$

(2) $10^0 = 1$

(3) $10^{-a} = 1/10^a$

(4) $(10^a)^b = 10^{ab}$

(5) $10^a/10^b = 10^{a-b}$

1. What is $10^3 \times 10^4$?

A= 10^{12} B= 10^7 C= 10^{34} D= 10^0 E= 10^{-7} B

2. Find $10^3/10^4$

A= 10^7 B= 10^1 C= 10^{-4} D= 10^{-1} E= 10^{-7} D

3. Find $\left(10^3\right)^4$.

A= 10^7 B= 10^1 C= 10^{12} D= 10^{-1} E= 10^0 C

The Five Laws of Exponents

(1) $10^a \times 10^b = 10^{a+b}$ (2) $10^0 = 1$

(3) $10^{-a} = 1/10^a$ (4) $(10^a)^b = 10^{ab}$

(5) $10^a/10^b = 10^{a-b}$

4. What is $\left(10^2 \times 10^3\right)^4$?

A= 10^8 B= 10^9 C= 10^{12} D= 10^{20} E= 10^{24} D

5. What is $(10^2 \times 10^6)/(10^2 \times 10^3)$?

A= 10^2 B= 10^3 C= 10^{-1} D= 10^7 E= 10^6 B

6. What is $\left(10^2/10^5\right)^{-2}$?

A= 10^{-6} B= 10^{-5} C= 10^6 D= 10^4 $E = 10^5$ C

Non-Integer Powers

We can work them out!

7. What is $10^{0.5} = 10^{1/2}$? Answer: $10^{0.5} = \sqrt{10} \approx 3.16288$

8. What is $10^{0.1} = 10^{1/10}$? Answer: $10^{0.1} = \sqrt[10]{10} \approx 1.258926$

9. Similarly: $10^{0.01} = \sqrt[100]{10} \approx 1.02329$

$$10^{0.001} = \sqrt[1000]{10} \approx 1.00231$$

10. What is $10^{0.27}$? Answer:

$$10^{0.27} = 10^{27/100} = \sqrt[100]{10^{27}} = \left(\sqrt[100]{10}\right)^{27} \approx 1.862$$

Moving to Logarithms

$\log(y)$ is how may tens you multiply together to get y

$$10^{\log(y)} = y$$

$$\log(10) = ?1 \quad \text{because} \quad 10^1 = 10$$
$$\log(100) = ? = 2 \quad \text{because} \quad 10^2 = 100$$
$$\log(1000) = ? = 3 \quad \text{because} \quad 10^3 = 1000$$
$$\log(100,000) =$$

A= 2 B= 3 C= 4 D= 5 E= 6 D

Still moving to Logarithms

$\log(y)$ is how may tens you multiply together to get y

$$10^{\log(y)} = y$$

$\log(0.1) = ? -1$ because $10^{-1} = 1/10 = 0.1$

$\log(0.01) = ? = -2$ because $10^{-2} = 1/100 = 0.01$

$\log(10^x) = ? = x$ duh?

How confused are you?

A=not at all B=a bit C=a lot D=totes confused

Next Time: How To Find Logs

(1) Use a calculator: efficient but not good for learning

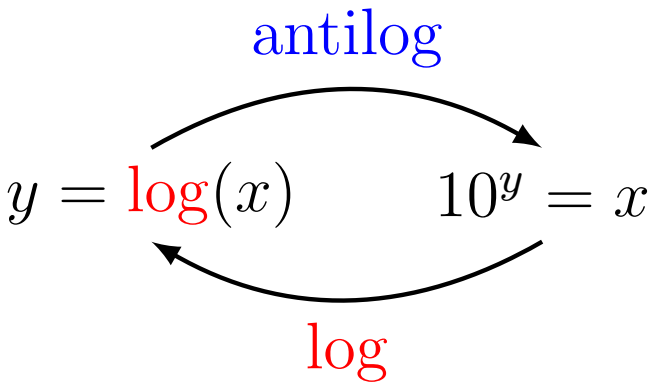
(2) Use the graph on page 290 of textbook / handout (see GS)

(3) Use table of logarithms on page 289 of textbook / handout (GS)

Our goal: use (2) and (3) to understand:

logs, functions and inverse functions.

Our main use of logs: solving certain kinds of equation.
Mistakes will follow unless you practice finding logs the old fashioned way.



log is the inverse function of antilog

antilog is another name for the 10-to-the-power function:

$$\text{antilog}(y) = 10^y.$$