Welcome Back!
Differential Calculus

Instructor:

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Office Hours:

T R 11-11:50, T 3:45-4:35 Details on Gauchospace.

Counting and Our Logarithmic Perception of the World

Vsauce:
1,2,3,4,5,6,7,8,9,10,11,12,13,14,15,16,17,18,19,20,21,22,23,24,25,26,27,
28,29,30,31,32,33,34,35...
https://www.youtube.com/watch?v=Pxb5lSPLy9c

Warm-up

- $\log_9(9^4) = \boxed{4}$
- $\log_3(9^4) = \boxed{8}$
- $\log_{27}(27^5) = \boxed{5}$
- $\log_3(27^5) = \boxed{15}$
- $\log_5(25^{17}) = \boxed{34}$
- $\log_3(27^0) = \boxed{0}$
- $\log_8(2^{12}) = \boxed{4}$
- $\log_8(2) = \boxed{1/3}$

Warm-up Part II

- $\log_{100}(100^7) = \boxed{7}$
- $\log_{10}(100^7) = \boxed{14}$
- $\log_{10}(1000000^{-4}) = \boxed{-24}$
- $\log_{10}(2) =$ about $\boxed{.3}$ Still warm-up?
- $\log_{10}(2^{11}) =$ about $.3 \cdot 11 = \boxed{3.3}$

logs are "opposite " of exponents (inverse function of antilog)
So every fact about exponents corresponds to a fact about logs:

	laws of exponents	corresponding law of logs
(1)	$10^{a} \times 10^{b} = 10^{a+b}$	$\log(xy) = \log(x) + \log(y)$
(2)	$10^{0} = 1$	$\log(1) = 0$
(3)	$10^{-a} = 1/10^{a}$	$\log(1/x) = -\log(x)$
(4)	$(10^{a})^{p} = 10^{ap}$	$\log(x^{p}) = p\log(x)$
(5)	$10^{a}/10^{b} = 10^{a-b}$	$\log(x/y) = \log(x) - \log(y)$

Example: $\log(x^a/y^b) =?$

$$A = a\log(x)/(b\log(y)) \qquad B = a\log(x) + b\log(y)$$
$$C = a\log(x) - b\log(y) \quad D = (a + \log(x)) - (b + \log(y)) \quad \boxed{\text{C}}$$

Rule (4): $\log(x^p) = p\log(x)$

Explanation of (4)

$\log(a \times a) = \log(a) + \log(a) = 2\log(a)$
$\log(a \times a \times a) = \log(a) + \log(a) + \log(a) = 3\log(a)$

In general: the number of tens you multiply to get x^p is p times as many tens as you multiply to get x.

What is $\log\left(\sqrt{\frac{1}{x^7}}\right)$?

$A = 7 - \log(x)$ $B = (7/2) - \log(x)$ $C = -7/2$ $D = -(7/2)\log(x)$

$\boxed{\text{D}}$

Find x by solving $3^x = 5$.

A $\log(5)/\log(3)$
B $\log(3)/\log(5)$
C $\log(5)^3$
D $\log(3) - \log(5)$
E $\log(5) - \log(3)$ $\boxed{\text{A}}$

§7.5: Using logs to multiply

First rule of logs: $\log(a \times b) = \log(a) + \log(b)$

Example: Find 2.7×1.6 using logs
Given info: $\log(2.7) \approx 0.43$ and $\log(1.6) \approx 0.20$

Method
(i) Look up $\log(2.7)$ and $\log(1.6)$
(ii) Add these
(iii) Take the antilog of result from (ii)
(iv) Think: Is the answer reasonable or did I goof up?

§7.5: Using logs to multiply

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Example: Find 2.7×1.6 using logs
Given info: $\log(2.7) \approx 0.43$ and $\log(1.6) \approx 0.20$

Look how I write the answer.

- $\log(2.7 \times 1.6) = \log(2.7) + \log(1.6)$
- We are told $\log(2.7) \approx 0.43$ and $\log(1.6) \approx 0.20$, so $\log(2.7 \times 1.6) \approx 0.43 + 0.20 = 0.63$
- Is this the answer ? Heck No! It is the log of the answer
- $2.7 \times 1.6 \approx \text{antilog}(0.63) = 10^{0.63}$
- $10^{0.63} \approx 4.3$
- Is my answer $\boxed{4.3}$ reasonable? Yes, about $2 \times 2 = 4$.

§7.5: Using logs to divide

Remember Log Rule (5): $\boxed{\log(a \div b) = \log(a) - \log(b)}$

Example: Use this rule to find 38.2/1.77

Given info: $\log(3.82) \approx 0.58$ and $\log(1.77) \approx 0.25$

Method

(i) Look up $\log(3.82)$ and $\log(1.77)$, find $\log(38.2)$

⋆You can find $\log(38.2)$ by adding 1 to $\log(3.82)$ because 38.2 is 3.82 times one more power of 10.⋆

(ii) Subtract!

(iii) Take the antilog of result from (ii)

(iv) Think: Is the answer reasonable or did I goof up?

A= done B= confused

Powers Using Logs

Or, exploting Log Rule (4):

$$\boxed{\log(a^p) = p\log(a)}$$

Use this and the graph of $y = 10^x$ to find $\sqrt{70}$.

One Approach:

(i) Use graph and move decimal point trick to find $\log(70)$
⋆I will show the graph of the exponential function 10^x and talk about this method on Thursday.⋆

(ii) $\log(\sqrt{70}) = \log(70^{1/2}) = (1/2)\log(70)$

(iii) Take the antilog of result from (ii)

(iv) Think: Is the answer reasonable or did I goof up?

Hint: $\log(7) \approx 0.84$

A= done B= working C= confused

Answer: $\sqrt{70} \approx 8.3$. Is that reasonable?

Computer Applications

One kilobyte (1 KB) is 2^{10}.

Problem: Calculate 2^{10} using logs. **Hint:** $\log(2) \approx 0.3$

A$\approx$ 3 B$\approx$ 10.3 C$\approx$ 30 D$\approx$ 1000 E$\approx$ 1100 $\boxed{\text{D}}$

So: $2^{10} \approx 10^3 = 1000$ (really $2^{10} = 1024$).

1KB is really $2^{10} = 1024 \approx 10^3$ (K is Kilo = thousand)

1MB is really $2^{20} = \left(2^{10}\right)^2 \approx (10^3)^2 = 10^6$ (M is Mega = million)

1GB is really $2^{30} = \left(2^{10}\right)^3 \approx (10^3)^3 = 10^9$ (G is Giga = billion)

1TB is really $2^{40} = \left(2^{10}\right)^4 \approx (10^3)^4 = 10^{12}$ (T is Tera = trillion)

Example: suppose on a certain island the population of rabbits doubles every generation. After 20 generations it multiplies by…
$2^{20} \approx 1$ million.

Powers of 2 are easy to do, even in your head. To work out 2^n the log of the answer is approximately $0.3n$, so 2^n is 1 followed by $0.3n$ zeroes.

§7.7: Solving Exponential Eq'ns

1. Find x by solving $10^x = 5$.

A$= 5$ B$= 0.5$ C$= \log(5)$ D$= \log(0.5)$
E$= \log(5) - \log(10)$ $\boxed{\text{C}}$

Look how I write the answer!

$$\log(10^x) = \log(5) \quad \text{Take logs of both sides}$$
$$x = \log(10^x) = \log(5) \quad \text{Using } \log(a^p) = p\log(a) \text{ and } \log(10) = 1$$

Examples:

Use the Fourth Law:

$$\log(a^{x}) = x\log(a)$$

Slogan: Logs bring exponents down to ground level.

2. Solve $3^{x} = 7$

A$=\log(7/3)$ B$=\log(7)-\log(3)$ C$=\log(7)+\log(3)$
D$=\log(3)/\log(7)$ E$=\log(7)/\log(3)$ $\boxed{\text{E}}$

Look how I write the answer:

$$\log(3^x) = \log(7) \qquad \text{Take logs of both sides}$$

$$x\log(3) = \log(3^x) = \log(7) \qquad \text{Using } \log(a^p) = p\log(a)$$

$$\text{So:} \qquad x = \log(7)/\log(3)$$

Examples:

Use the Fourth Law:

$$\boxed{\log(a^{x}) = x\log(a)}$$

Slogan: Logs bring exponents down to ground level.

3. Solve $7^{x+2} = 30$.

$$\text{A}= \frac{\log(30) - 2\log(7)}{\log(7)} \qquad \text{B}= \frac{\log(30)}{\log(7)} - 2 \qquad \text{C}= \frac{\log(30) - \log(49)}{\log(7)}$$

$$\text{D}= \frac{\log(30/49)}{\log(7)} \qquad \text{E}\approx -0.25213$$

All are correct!

Examples:

Use the Fourth Law:

$$\log(a^{x}) = x\log(a)$$

Slogan: Logs bring exponents down to ground level.

4. Solve $7 \times 3^y = 2^{4y+3}$

A= $\dfrac{3\log(2) - \log(7)}{\log(3) - 4\log(2)}$ B= $\dfrac{3\log(2)}{7\log(3)}$ C= $\dfrac{3\log(2)}{7\log(3) - 4\log(2)}$

D= $\dfrac{7\log(3) - 4\log(2)}{3\log(2)}$ E=none of the above

$\boxed{\text{A}}$

Compound Interest

At the end of each year a bank pays 7% interest into your account. Initially have \$10,000 in account. How much after 10 years?

Think $10 \times 7\% = 70\%$ in 10 years, so have \$17,000 but that is wrong.

$$\begin{aligned} &\text{After 1 year:} && \$10,000 \times 1.07 = \$10,700 \\ &\text{After 2 years:} && \$10,700 \times 1.07 = \$10,000 \times 1.07 \times 1.07 = \$11,449 \\ &\text{After 3 years:} && \$11,449 \times 1.07 = \$10,000 \times (1.07)^3 = \$12,250.40 \end{aligned}$$

Each year what you had before is multiplied by 1.07. Thus compound interest.
So after 10 years have

$$\$10,000 \times \underbrace{1.07 \times 1.07 \times \cdots \times 1.07}_{\text{10 times}} = 10,000 \times (1.07)^{10} \approx \boxed{\$20,000}$$

Conclusion: Money approximately doubles in 10 years!
So in 20 years multiplies by 4, in 30 years by 8,...

That's it. Thanks for being here.

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