

Name:

Solutions

Perm:

Math 34A Midterm 2

(100 pts total)

1. (6 pts) Use the log table provided with this exam to find

(a) (4 pts) $10^{1.820}$

$$\begin{aligned} 10^{1.820} &= 10^1 \cdot 10^{0.820} \\ &= (10)(6.61) \\ &= 66.1 \end{aligned}$$

66.1

(b) (8 pts) $\log(9.973/8.980)$

$$\begin{aligned} \log(9.973/8.980) &= \log(9.973) - \log(8.980) \\ &= .9987 - .9833 \\ &= .0454 \end{aligned}$$

.0454

(c) (12 pts) $\log(\sqrt[5]{537})$

$$\begin{aligned} \log(\sqrt[5]{537}) &= \log(537^{\frac{1}{5}}) \\ &= \frac{1}{5} \log(537) \\ &= \frac{1}{5} \log(100 \times 5.37) \\ &= \frac{1}{5} [\log(100) + \log(5.37)] \\ &= \frac{1}{5}[2 + .73] \\ &= 2.73 \div 5 \\ &= .546 \end{aligned}$$

.546

2. (8 pts) Use properties of logs to solve for x . Use the log table to evaluate any logs in your answer and simplify completely.

$$10^{2x-3} = 2^{10}$$

$$\log(10^{2x-3}) = \log(2^{10})$$

$$2x-3 = 10\log(2)$$

$$(0.301)2x = [10\log(2) + 3](0.5)$$

$$x = 5\log(2) + 1.5$$

$$x = 5(.301) + 1.5$$

$$x = .005$$

(4) Partial credit

$x = \boxed{3.005}$

3. (a) (4 pts) Find the equation of the line that passes through the points $(-4, 3)$ and $(-2, 5)$. Give the answer in the form $y = mx + b$.

Find slope:

$$m = \frac{\Delta y}{\Delta x} = \frac{5-3}{-2-(-4)} = \frac{2}{2} = 1$$

Find b :

$$3 = 1(-4) + b$$

$$7 = b$$

$$y = \boxed{x + 7}$$

- (b) (4 pts) Find the equation of the line that has slope $\frac{3}{5}$ and goes through the point $(4, 10)$. Give the answer in the form $y = mx + b$.

Point-slope form:

$$y - y_1 = m(x - x_1)$$

$$y - 10 = \frac{3}{5}(x - 4)$$

Simplify to $y = mx + b$:

$$y - 10 = \frac{3}{5}x - \frac{12}{5}$$

$$\begin{array}{r} 9 \\ -12 \\ \hline 38 \end{array}$$

$$y = \frac{3}{5}x - \frac{12}{5} + 10$$

$$y = \frac{3}{5}x - \frac{12}{5} + \frac{50}{5}$$

$$y = \boxed{\frac{3}{5}x + \frac{38}{5}}$$

3. (c) (4 pts) What are the coordinates of the point where the lines $y = x - 8$ and $y = 1 - \frac{1}{2}x$ intersect?

$$x - 8 = 1 - \frac{1}{2}x$$

mult. by 2 to clear the fraction

$$2x - 16 = 2 - x$$

$$3x = 18$$

$$x = 6$$

Find y by substituting:

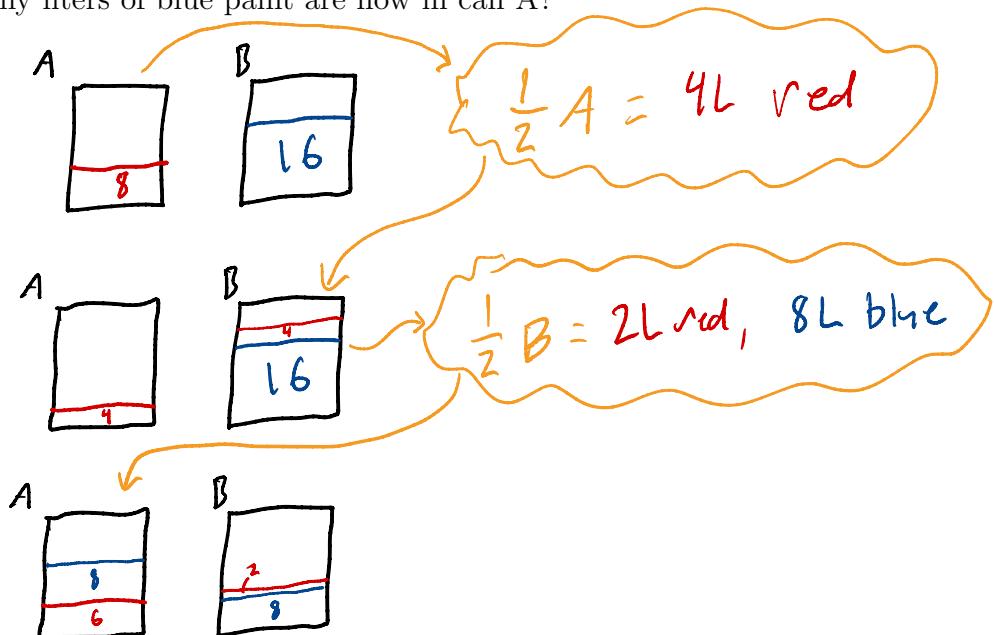
$$y = (6) - 8$$

$$y = -2$$

$$(x, y) = (6, -2)$$

4. (8 pts) Initially, can A contains 8 liters of red paint and can B contains 16 liters of blue paint. I pour half of the red paint into can B. After mixing the paint in can B, I pour half of the paint in can B into can A.

How many liters of blue paint are now in can A?



$$8$$

L

5. (12 pts) The distance traveled by a rising helium balloon t seconds after leaving a child's hand is modeled by $f(t) = t^2$ meters.

- (a) (4 pts) Find the average speed of the balloon over the time period from 1 second to 1.2 seconds.

$$\text{Average Speed} = \text{Slope} = \frac{\Delta f}{\Delta t}$$

$$\begin{aligned} & \frac{f(1.2) - f(1)}{1.2 - 1} = \frac{(1.2)^2 - 1^2}{0.2} \\ & \quad = \frac{1.44 - 1}{0.2} \\ & \quad = \frac{0.44}{0.2} \\ & \quad = \frac{4.4}{2} \\ & \quad = 2.2 \end{aligned}$$

2.2 m/s

- (b) (4 pts) Find the average speed of the balloon over the time period from 1 second to $1+h$ seconds.

$$\text{Average Speed} = \text{Slope} = \frac{\Delta f}{\Delta t}$$

$$\begin{aligned} & \frac{f(1+h) - f(1)}{1+h - 1} = \frac{(1+h)^2 - 1^2}{h} \\ & \quad = \frac{1+2h+h^2 - 1}{h} \\ & \quad = \frac{h(2+h)}{h} \\ & \quad = 2+h \end{aligned}$$

2 + h m/s

- (c) (4 pts) In your own words, what would we do to find the instantaneous speed of the balloon exactly 1 second after it was dropped?

Take the limit as $h \rightarrow 0$ of our answer to part (b).	Take the derivative $f'(t)$ and plug in $t=1$.
--	---

\uparrow \nwarrow
 Both are correct

6. (8 pts) An enterprising 34A student observes that many students forgot to bring 3x5 note cards with them to Midterm 1. She has plenty of packages of note cards lying around, so she decides to sell them to students all over campus. If she charges 10 cents a card, y she will sell 500 cards. But for each cent she increases the price, the number of cards she sells will decrease by 10. \rightarrow slope x

- (a) (4 pts) If she picks a price of $(10 + h)$ ¢ per card, find the number of cards she will sell (in terms of h).

Since the rate of change is constant (-10 cards per cent), this situation has a linear equation.

$$y = mx + b \quad y: \text{number of cards sold} \quad x: \text{price in cents}$$

Reading the story, we see that $m = -10$, and $y = 500$ if $x = 10$.

Solve for b :

$$\begin{aligned} y &= mx + b \\ 500 &= -10(10) + b \\ 500 &= -100 + b \end{aligned}$$

$$b = 600$$

Write the equation:

$$y = -10x + 600$$

Plug in $x = 10 + h$:

$$\begin{aligned} y &= -10(10 + h) + 600 \\ &= -100 - 10h + 600 \\ &= 500 - 10h \end{aligned}$$

$500 - 10h$

cards

- (b) (4 pts) Use your answer from part (a) to find the total amount of money (in cents) she will receive from selling cards if her price is $(10 + h)$ ¢ each. Please simplify your answer (it should be in terms of h).

Revenue = Price \times Number sold

$$\begin{aligned} r &= (10 + h)(500 - 10h) \\ &= 5000 - 100h + 500h - 10h^2 \quad \text{combine like terms} \\ &= 5000 + 400h - 10h^2 \end{aligned}$$

$5000 + 400h - 10h^2$

¢

7. (8 pts) Compute the following sum (your answer should be a number).

$$\sum_{n=4}^6 \frac{n(n+1)}{2}$$

$$\begin{aligned}\frac{4(5)}{2} + \frac{5(6)}{2} + \frac{6(7)}{2} \\ = \frac{20}{2} + \frac{30}{2} + \frac{42}{2} \\ = 10 + 15 + 21 \\ = 46\end{aligned}$$

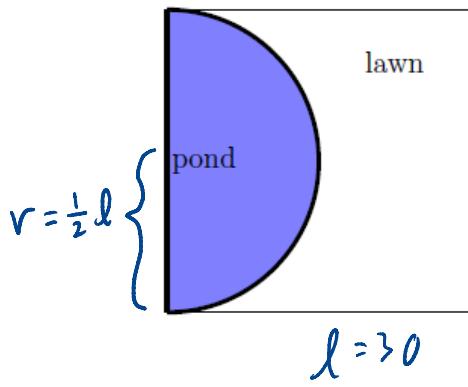
46

8. (8 pts) Find $\lim_{h \rightarrow 0} \frac{48+24h+3h^2-48}{h}$

$$\begin{aligned}&= \lim_{h \rightarrow 0} \frac{\cancel{48} + \cancel{24h} + \cancel{3h^2} - \cancel{48}}{h} \quad \text{factor out } h \\ &= \lim_{h \rightarrow 0} \frac{h(24+3h)}{h} \quad \text{as } h \rightarrow 0, \quad 3h \rightarrow 0 \\ &= \boxed{24}\end{aligned}$$

24

9. (12 pts) A square garden consists of a semicircular pond and the rest is a lawn. The length of each side of the square garden is ℓ . If the area of the square is 900 m^2 , then find the area of the lawn.



$$\text{Area of square} = \ell^2$$

$$900 = \ell^2$$

$$\ell = 30$$

$$\text{Area of pond} = \frac{1}{2} \pi r^2$$

$$r = \frac{1}{2} \ell = 15$$

$$A = \frac{1}{2} \pi (15)^2 \\ = 112.5 \pi$$

$$\begin{array}{r} 15 \\ \times 15 \\ \hline 175 \\ 150 \\ \hline 225 \end{array}$$

$$\begin{array}{r} 112.5 \\ \times 225 \\ \hline 1125 \\ 225 \\ \hline 24375 \end{array}$$

$$\text{Area of lawn} = \text{Square} - \text{Pond}$$

$$A = 900 - 112.5 \pi$$

lawn area =

$$900 - 112.5 \pi$$

m^2