

Welcome To Math 34A!

Differential Calculus

Instructor:

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MTWR after class 2:00-3:00, and by appointment. Details on Gauchospace.

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A nice thing about derivatives...

$$\begin{aligned}\frac{d}{dx}(a \cdot f(x) + b \cdot g(x)) &= a \frac{d}{dx} f(x) + b \frac{d}{dx} g(x) \\ &= a \cdot f'(x) + b \cdot g'(x)\end{aligned}$$

For example...

$$\begin{aligned}\frac{d}{dx}(3x^2 + 5x) &= 3 \frac{d}{dx} x^2 + 5 \frac{d}{dx} x \\ &= 3(2x) + 5(1) \\ &= 6x + 5\end{aligned}$$

A Warning! (Again)



$$\frac{d}{dx} (f(x)g(x)) \neq f'(x) \times g'(x)$$



Example: What is the derivative of $(x^3 + 1)(2x^2 - 3x + 5)$?

Question: Find $\frac{d}{dx} ((x^3 + 1)(2x^2 - 3x + 5))$.

- A $= 10x^4 - 8x^3 + 10x^2 + 12x - 3$
- B $= 10x^4 - 12x^3 + 15x^2 + 4x + 5$
- C $= 10x^4 - 12x^3 + 15x^2 + 4x - 3$
- D $= \text{Other}$

Hint: $2x^5 - 3x^4 + 5x^3 + 2x^2 - 3x + 6$

Answer: C

Differentiating $f(x) = e^{kx}$

$$\frac{d}{dx} (e^{kx}) = k e^{kx}$$

versus

$$\frac{d}{dx} (x^n) = n x^{n-1}$$



These are not polynomials.

$$\frac{d}{dx} (e^{kx}) \neq k e^{(k-1)x}.$$



Question: Find $\frac{d}{dx} (4e^{3x} + 5x^3)$

$$A = 12e^{2x} + 15x^2$$

$$B = 12e^{3x} + 15x^3$$

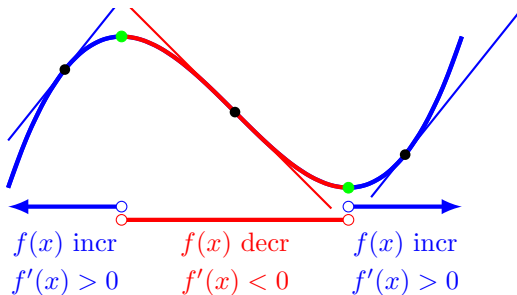
$$C = 4e^{3x} + 15x^2$$

$$D = 12e^{3x} + 15x^2$$

$$E = \text{Other}$$

$$D$$

Meanings: The First Derivative

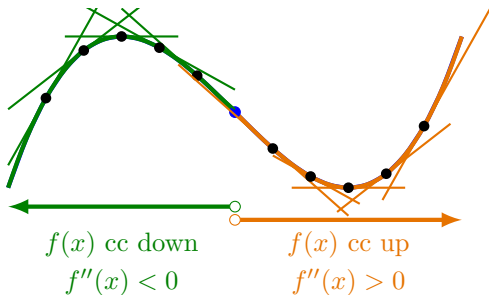


Point:

$$f'(x) > 0 \iff f(x) \text{ is increasing}$$

$$f'(x) < 0 \iff f(x) \text{ is decreasing}$$

Meanings: The Second Derivative



Point:

$$f''(x) > 0 \iff f'(x) \text{ is increasing}$$

$$\iff f(x) \text{ is concave up}$$

$$f''(x) < 0 \iff f'(x) \text{ is decreasing}$$

$$\iff f(x) \text{ is concave down}$$

Concavity

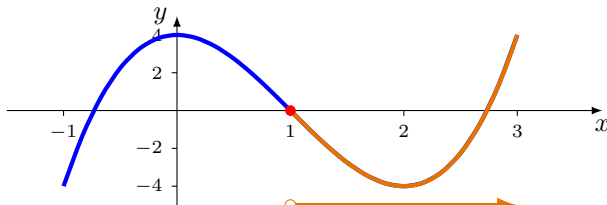
$$f''(x) > 0 \iff f(x) \text{ is concave up}$$

$$f''(x) < 0 \iff f(x) \text{ is concave down}$$

(1) For which values of x is $f(x) = x^3 - 6x^2 + 3x + 2$ concave up?

A when $x = 0$ B when $x < 6$ C when $x > 6$
D when $x < 2$ E when $x > 2$ E

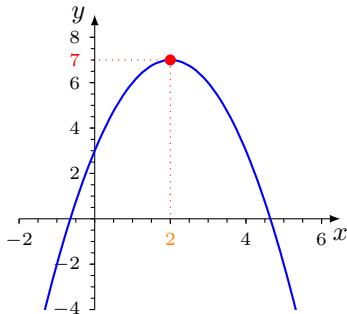
(2) Where is $f''(x) > 0$?



A when $x < 2$ B when $x > 2$ C when $x < 1$
D when $x > 1$ E when $-0.7 < x < 1$ D

§8.13: Max/Min problems

Often want to find the biggest, smallest, most, least, maximum, minimum of something.



Here's the graph of
 $y = f(x) = -x^2 + 4x + 3$

The maximum value or just maximum of the function is **7**.

The value of x which gives the maximum of $f(x)$ is $x = \mathbf{2}$

We write $f(\mathbf{2}) = \mathbf{7}$.

For this example you can see this is the maximum because

$$f(x) = -x^2 + 4x + 3 = -(x - \mathbf{2})^2 + \mathbf{7}$$

$(x - \mathbf{2})^2$ is always positive except when $x = \mathbf{2}$

so the maximum must be at $x = \mathbf{2}$. But there is an easier way.

How To Find A Maximum

- (1) At the highest point, it's not going up or down. So find $f'(x)$ to look for the flat part.
- (2) Solve $f'(x) = 0$ for x . The x value that gives the max must be one of these! (Usually there is just one.)
- (3) To find the maximum for $f(x)$, use the x -value you just found...because it gives you the maximum!

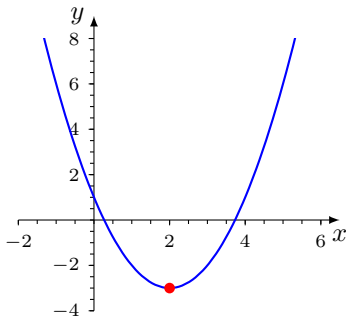
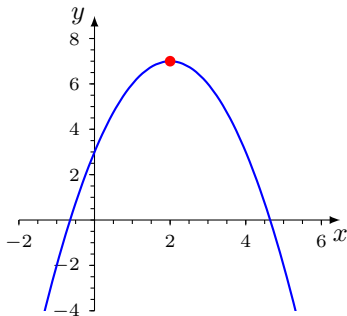
1. Use this method to find the maximum of $f(x) = -x^2 + 8x + 5$.
The maximum value is...

$$A = 4 \quad B = 5 \quad C = -2x + 8 \quad D = 21 \quad E = 15 \quad D$$

2. Find the value of x which makes $f(x) = (2 - x)(x + 6)$ a maximum.

$$A = 16 \quad B = 1 \quad C = -1 \quad D = 2 \quad E = -2 \quad E$$

How To Find A Minimum?



What this technique **actually does** is find both maxima and minima. In Math 34A a problem will have either a maximum **or** a minimum, **but not both**. So the technique will find what you want. In Math 34B you discover how to do problems which have both a maximum and a minimum and find out which is which.

More Examples

3. What is the minimum of $f(x) = (x + 2)(x + 4) + 3$?

A = 0 B = 1 C = 2 D = 3 E = 4

Answer: C

4. What is minimum of $f(x) = x^2 + 16x^{-2}$?

A = 2 B = 4 C = 6 D = 8 E = 16

Answer: D

5. Find the value of x which makes $f(x) = -e^x - e^{-2x}$ a maximum.

A = 0 B = $\ln(2)$ C = $-\ln(2)$ D = $\ln(2)/3$ E = $\ln(2)/3$

Answer: E

Word Problem #1

A ball is thrown into the air. After t seconds the height in meters above the ground of the ball is $h(t) = 40t - 10t^2$. How many meters high did the ball go?

$$A = 2 \quad B = 40 - 20t \quad C = 20 \quad D = 40 \quad D$$

Word Problem #2

If an airline sells tickets at a price of $\$200 + 5x$ each the number of tickets it sells is $1000 - 20x$. What price should the tickets be if the airline wants to get the most money?

$$A = 5 \quad B = 25 \quad C = 175 \quad D = 200 \quad E = 225 \quad E$$

Word Problem #3

A fenced garden with an area of 100 m^2 will be made in the shape of a rectangle. It will be surrounded on all four sides by a fence. What length and width should be used so the least amount of fence is needed?

Approach:

- (1) Express the total length of fence in terms of only one variable, either L = length of field, or W = width of field. This gives a formula for P = (total length of fence) involving, say, W .
- (2) Find minimum by solving $\frac{dP}{dW} = 0$.

Students always find (1) the hardest part.

You have been prepared for this by word problems from chapter 3!