Midterm Practice Problems

1. Find the general solution to the following equations

   (a) $y'' + 3y' + 2y = 0$

   (b) $\dfrac{dr}{d\theta} = \dfrac{r^2}{\theta}$

   (c) $y' + y^2 \sin x = 0$

   (d) $w' + w = 3t$

   (e) $(2xy - 3x^2) + (x^2 + 1)y' = 0$

   (f) $u' = u^2 e^x$

   (g) $tv' - v = t^2 e^{-t}$

   (h) $(2t - 2y)y' = 2y - 2t$

   (i) $9z'' + 6z' + z = 0$

2. Solve the following IVPs

   (a) $xdx + ye^{-x}dy = 0$; $y(0) = 1$

   (b) $y' = xe^{\sin x} + y\cos x$; $y(0) = 3$

   (c) $y'' - 2y' + 5y = 0$; $y\left(\frac{\pi}{2}\right) = 0$, $y'\left(\frac{\pi}{2}\right) = 2$

3. Prove that t^a and t^b are linearly independent functions if $a \neq b$.

4. Given that $y_1(x) = \sin(x^2)$ is a solution to

   $$xy'' - y' + 4x^3 y = 0, x > 0,$$

   find a second solution $y_2(x)$.

1. Find the general solution to the following equations

   (a) $y = c_1 e^{-t} + c_2 e^{-3t}$

   (b) $r = (c - \ln\theta)^{-1}$

   (c) $y^{-1} + \cos x = c$ if $y \neq 0$, also $y = 0$

   (d) $w = 3e^{-t}(\displaystyle\int te^t)$ (do IBP to finish)

   (e) exact

   (f) $u = (c - e^x)^{-1}$

   (g) $v = t(-e^{-t} + c)$

   (h) exact

   (i) $z = c_1 e^{-t/3} + c_2 t e^{-t/3}$

2. Solve the following IVPs

   (a) $y = (2(1-x)e^x - 1)^{1/2}$

   (b) $y = e^{\sin x}\left(\dfrac{x^2}{2} + 3\right)$

   (c) $y = -e^{t-(\pi/2)}\sin(2t)$

3. $W[t^a, t^b] = (b-a)t^{a+b-1} = 0$ if and only if $a = b$

4. $y_2 = \cos(x^2)$