Name:	
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Math 34A Midterm 1, Spring 2022

1. (*2pts*) Solve the system of equations below for s and t. Your answers should be in terms of x and y.

$$3s - 2t = 14x$$
$$s + 4t = 14y$$

(1) $3s - 2t = 14x$

(2) $s + 4t = 14y$ $\xrightarrow{\cdot 3}$ $3s + 12t = 3 \cdot 14y$

subtract

$14t = 3 \cdot 14y - 14x$ $\xrightarrow{\div 14}$ $t = 3y - x$

substitute t

$3s - 2(3y - x) = 14x$

cleanup

$3s = 6y - 2x + 14x$

$3s = 6y + 12x$ $\xrightarrow{\div 3}$ $s = 2y + 4x$

$s =$ $\boxed{2y + 4x}$

$t =$ $\boxed{3y - x}$

2. (*2pts*) Multiply out and simplify. Your answer should have no negative exponents in it.

$$\frac{\sqrt[3]{x^{-3}y^{-6}}}{a^{-1}b} \cdot \left(\frac{xy}{a^{-1}b}\right)^{-2}$$

$x^{-3} = \left(\frac{1}{x}\right)^3$ $\qquad \sqrt[3]{\left(\frac{1}{x}\right)^3} = \frac{1}{x} = x^{-1}$

$y^{-6} = \left(\frac{1}{y^2}\right)^3$ $\qquad \sqrt[3]{\left(\frac{1}{y^2}\right)^3} = \frac{1}{y^2}$

$$= \frac{x^{-1} \cdot y^{-2}}{a^{-1}b} \cdot \frac{a^{-2}b^2}{x^2y^2} = \frac{a \cdot b^2}{xy^2b\, a^2x^2y^2} = \frac{b}{x^3y^4a}$$

$\boxed{\dfrac{b}{x^3y^4a}}$

3. (*2pts*) Substitute $x = a + b$ into the expression below and simplify completely. There should be no parentheses in your answer.

$$x(a^4 - a^3b + a^2b^2 - ab^3 + b^4)(x - 2b)$$

$= (a+b)(a^4 - a^3b + a^2b^2 - ab^3 + b^4)(a-b)$

$= (a^5 - b^5)(a-b)$

$= a^5 \cdot a - a^5 \cdot b - b^5 \cdot a + b^5 \cdot b$

$= a^6 - a^5b - ab^5 + b^6$

★ Sum of fifths computation

$(a+b)(a^4 - a^3b + a^2b^2 - ab^3 + b^4)$

$= \underbrace{a^5 + a^4b}_{(a+b)a^4} \underbrace{- a^4b - a^3b^2}_{(a+b)(-a^3b)} \underbrace{+ a^3b^2 + a^2b^3}_{(a+b)(a^2b^2)} \underbrace{- a^2b^3 - ab^4}_{(a+b)(-ab^3)} \underbrace{+ ab^4 + b^5}_{(a+b)\cdot b^4}$

$= a^5 + b^5$

$\boxed{a^6 - a^5b - ab^5 + b^6}$

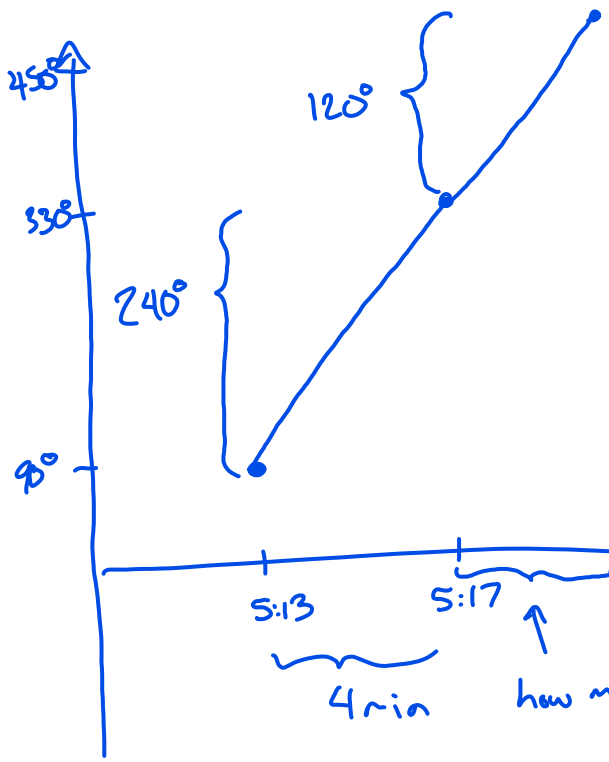
4. (*4pts*) You discovered this week that if you microwave frozen pizza to bring it above room temperature, you can finish it in the oven at 450° to retain more moisture and have the pizza ready faster. You set your oven to preheat to 450°. You notice that at exactly 5:13PM, the oven's temperature is 90°. You check back at exactly 5:17PM and the oven's temperature is now 330°. Assuming linear growth for the temperature (so using linear extrapolation), at what time will the oven be preheated to 450°?

90° @ 5:13PM

330° @ 5:17PM

↑240° over 4 minutes

slope = $\frac{\text{rise}}{\text{run}} = \frac{240}{4} = 60$, so 60° per minute.



$\frac{450° - 330°}{t} = \frac{120°}{t} = \frac{60°}{1}$

So $t = 2$ more minutes.

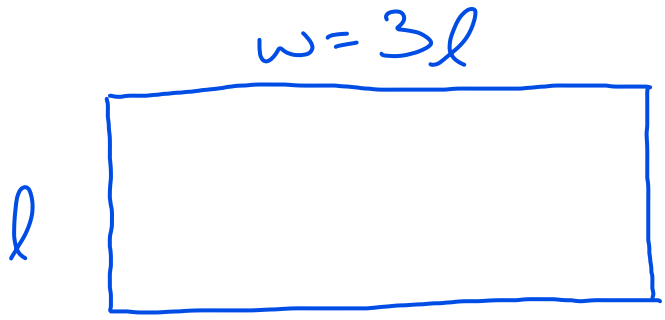
5:17 + 2 minutes = 5:19 PM

5:19 PM

~~minutes.~~

(2 minutes)

5. (*4pts*) A farmer wants to raise cattle in his pasture, but he first needs fencing. The field is three times as wide as it is long. Fencing purchases are \$35 plus \$5 per foot of fencing. Express the costing of fencing for the perimeter in terms of the length of the field.



(in \$)

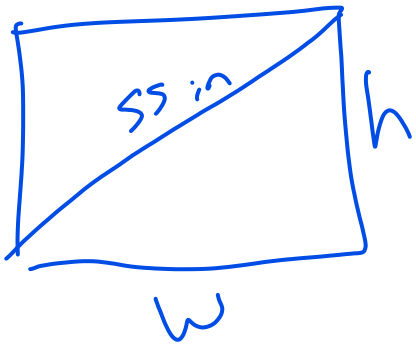
$$\text{Cost} = 35 + 5 \cdot (l + l + w + w)$$

$$= 35 + 5(2l + 2(3l)) = 35 + 5(8l)$$

$$= 35 + 40l$$

Fencing Cost = \$ $35 + 40l$

6. (*5 points*) The aspect ratio of a screen is the ratio of the width to the height. You are considering the purchase of a 55in TV (TV sizes are measured by the diagonal, not the length or width). In addition to this information, the manufacturer will only disclose the height of the TV, not the width. Express the aspect ratio in terms of the height h of the TV in inches.



$$h^2 + w^2 = 55^2$$

$$w^2 = 55^2 - h^2$$

$$w = \sqrt{55^2 - h^2}$$

$$\frac{w}{h} = \frac{\sqrt{55^2 - h^2}}{h}$$

Aspect Ratio = $\boxed{\dfrac{\sqrt{55^2 - h^2}}{h}}$

7. (*3 points*) What are the following limits?

(a) $\lim_{x\to 3} \; 5x - 10$

$15-10=5$

approaches 15

5

(b) $\lim_{x\to\infty} \; \frac{2x-3}{6x+17}$

These don't have an impact on the limit because x gets really big

$\frac{2x}{6x} = \frac{2}{6} = \frac{1}{3}$

$\frac{1}{3}$

(c) $\lim_{x\to 0} \; \frac{2x^2+6x}{2x}$

Both go to 0 in

$\frac{2x^2+6x}{2x} = \frac{x(2x+6)}{x\cdot 2}$

$\lim_{x\to 0} \frac{2x+6}{2}$ ← This approaches 6

← always 2

$\frac{6}{2} = 3$

3

8. (*3 points*) Compute the logarithms below.

(a) $\log_2(16)$ $16 = 1\cdot 2\cdot 2\cdot 2\cdot 2$

4 2's

or $\log_2(2^4) = 4$

4

(b) $\log_{10}(.1) = \log_{10}(10^{-1}) = -1$

or $.1 = 1 \div 10$

1 and it's division, so -1

-1

(c) $\log_5(125) = \log_5(5^3) = 3$

or $125 = 1\cdot 5\cdot 5\cdot 5$

3 5's

3