Welcome To Math 34A!
Differential Calculus

Instructor:

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Office Hours:

MTWR after class 2:00-3:00, and by appointment. Details on Gauchospace.

Quiz Corrections

While doing well on the quizzes is important, what is more important to me is that you learn the material so that you can do well on the Midterm Exams and the Final. To that end, you can optionally do **Quiz Corrections** after your quiz is graded. To get credit back on your quizzes, please answer the following in your own **words**:

- What was the problem was asking you to do?
- What was the mistake(s) in your work?
- Correctly and completely rework the problem, explaining your steps as you go.
- We know that mistakes are simply an opportunity to learn; what did you learn from this mistake?

then **for each problem you correct, you will earn 50% of the missing points back on the corresponding Quiz.** This means a 50% can be corrected to a 75%, a 90% to a 95%, etc. Quiz corrections are due 1 week after the graded quiz is posted.

Announcements

- Thursday is the last day to drop a class in Session A.
- I will have the exams graded ASAP (hopefully today or tomorrow morning).

Counting and Our Logarithmic Perception of the World

Vsauce:
1,2,3,4,5,6,7,8,9,10,11,12,13,14,15,16,17,18,19,20,21,22,23,24,25,26,27,
28,29,30,31,32,33,34,35...
`https://www.youtube.com/watch?v=Pxb5lSPLy9c`

Warm-up

- $\log_9(9^4) = \boxed{4}$
- $\log_3(9^4) = \boxed{8}$
- $\log_{27}(27^5) = \boxed{5}$
- $\log_3(27^5) = \boxed{15}$
- $\log_5(25^{17}) = \boxed{34}$
- $\log_3(27^0) = \boxed{0}$
- $\log_8(2^{12}) = \boxed{4}$
- $\log_8(2) = \boxed{1/3}$

Warm-up

- $\log_9(9^4) = \boxed{4}$
- $\log_3(9^4) = \boxed{8}$ $\quad \log_3(9^4) = \log_3([3^2]^4) = \log_3(3^8) = 8$
- $\log_{27}(27^5) = \boxed{5}$
- $\log_3(\overset{\text{5th power}}{2\overset{\curvearrowright}{7}{}^5}) = \boxed{15}$ $\quad$ 3rd power $\quad 3 \times 5 = 15$
- $\log_5(25^{17}) = \boxed{34}$
- $\log_3(27^0) = \boxed{0}$ $\quad \log_3(1) = 0$
- $\log_8(2^{12}) = \boxed{4}$ $\quad$ 12, 1/3 $\quad \log_8(2^{12}) = \log_8([2^3]^4) = \log_8(8^4) = 4$
- $\log_8(2) = \boxed{1/3}$ $\quad \sqrt[3]{8} = 2$ because $2^3 = 8$

Warm-up Part II

- $\log_{100}(100^7) = \boxed{7}$
- $\log_{10}(100^7) = \boxed{14}$
- $\log_{10}(1{,}000{,}000^{-4}) = \boxed{\text{-24}}$
- $\log_{10}(2) =$ about $\boxed{.3}$ Still warm-up?
- $\log_{10}(2^{11}) =$ about $.3 \cdot 11 = \boxed{3.3}$

Logs are "opposite" of exponentials (that's why we sometimes call 10^x as antilog). So every fact about exponents corresponds to a fact about logs:

	laws of exponents	corresponding law of logs
(1)	$10^a \times 10^b = 10^{a+b}$	$\log(xy) = \log(x) + \log(y)$
(2)	$10^0 = 1$	$\log(1) = 0$
(3)	$10^{-a} = 1/10^a$	$\log(1/x) = -\log(x)$
(4)	$(10^a)^p = 10^{ap}$	$\log(x^p) = p\log(x)$
(5)	$10^a/10^b = 10^{a-b}$	$\log(x/y) = \log(x) - \log(y)$

Example: $\log(x^a/y^b) =?$

$A = a\log(x)/(b\log(y))$ $\qquad$ $B = a\log(x) + b\log(y)$

$C = a\log(x) - b\log(y)$ $\qquad$ $D = (a + \log(x)) - (b + \log(y))$

$\boxed{\text{C}}$

Logs are "opposite" of exponentials (that's why we sometimes call 10^x as antilog). So every fact about exponents corresponds to a fact about logs:

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(4)	$(10^a)^p = 10^{ap}$	$\log(x^p) = p\log(x)$
(5)	$10^a/10^b = 10^{a-b}$	$\log(x/y) = \log(x) - \log(y)$

Example: $\log(x^a/y^b) = \log(x^a) - \log(y^b)$

$= a\log(x) - b\log(y)$

Rule (4): $\log(x^p) = p\log(x)$

Explanation of (4)

$\log(a \times a) = \log(a) + \log(a) = 2\log(a)$
$\log(a \times a \times a) = \log(a) + \log(a) + \log(a) = 3\log(a)$

In general: the number of tens you multiply to get x^p is p times as many tens as you multiply to get x.

What is $\log\left(\sqrt{\frac{1}{x^7}}\right)$?

$A = 7 - \log(x)$ $B = (7/2) - \log(x)$ $C = -7/2$ $D = -(7/2)\log(x)$

$\boxed{\text{D}}$

Find x by solving $3^x = 5$.

A $\log(5)/\log(3)$
B $\log(3)/\log(5)$
C $\log(5)^3$
D $\log(3) - \log(5)$
E $\log(5) - \log(3)$ $\boxed{\text{A}}$

Rule (4): $\log(x^p) = p\log(x)$

Explanation of (4)

$\log(a \times a) = \log(a) + \log(a) = 2\log(a)$
$\log(a \times a \times a) = \log(a) + \log(a) + \log(a) = 3\log(a)$

In general: the number of tens you multiply to get x^p is p times as many tens as you multiply to get x.

What is $\log\left(\sqrt{\frac{1}{x^7}}\right)$? $= \log\left(\sqrt{x^{-7}}\right) = \log\left([x^{-7}]^{1/2}\right) = \log\left(x^{-7/2}\right)$

$= \frac{-7}{2}\log(x)$

Administration Log Rules Log Arithmetic Solving Equations Word problems

Find x by solving $3^x = 5$.

$3^x = 5$

$\log(3^x) = \log(5)$

$x\log(3) = \log(5)$

$x = \frac{\log(5)}{\log(3)}$

A $\log(5)/\log(3)$
B $\log(3)/\log(5)$
C $\log(5)^3$
D $\log(3) - \log(5)$
E $\log(5) - \log(3)$

note: $\frac{\log 5}{\log 3} \neq \log 5 - \log 3$

July 6, 2022: Logarithm Applications Trevor Klar, UCSB Mathematics

§7.5: Using logs to multiply

First rule of logs: $\log(a \times b) = \log(a) + \log(b)$

Example: Find 2.7×1.6 using logs

Method

(i) Look up $\log(2.7)$ and $\log(1.6)$

(ii) Add these

(iii) Take the antilog of result from (ii)

(iv) Think: Is the answer reasonable or did I goof up?

§7.5: Using logs to multiply

First rule of logs: $\log(a \times b) = \log(a) + \log(b)$

Example: Find 2.7×1.6 using logs

2.7×1.6

$= 10^\wedge(\log(2.7 \times 1.6))$ rules of logs

$= 10^\wedge(\log(2.7) + \log(1.6))$ look up in chart

$= 10^\wedge(0.4314 + 0.2041)$ add $\quad .4314 + .2041 = .6355$

$= 10^\wedge(0.6355)$ use the chart backwards to find the anti log

$= 4.32$ seems reasonable? yes! $2.7 \times 1.6 \approx 2 \times 2 = 4 \approx 4.32$

x	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07
1.0	0.0000	0.0043	0.0086	0.0128	0.0170	0.0212	0.0253	0.0294
1.1	0.0414	0.0453	0.0492	0.0531	0.0569	0.0607	0.0645	0.0682
1.2	0.0792	0.0828	0.0864	0.0899	0.0934	0.0969	0.1004	0.1038
1.3	0.1139	0.1173	0.1206	0.1239	0.1271	0.1303	0.1335	0.1367
1.4	0.1461	0.1492	0.1523	0.1553	0.1584	0.1614	0.1644	0.1673
1.5	0.1761	0.1790	0.1818	0.1847	0.1875	0.1903	0.1931	0.1959
1.6	0.2041	0.2068	0.2095	0.2122	0.2148	0.2175	0.2201	0.2227
1.7	0.2304	0.2330	0.2355	0.2380	0.2405	0.2430	0.2455	0.2480
1.8	0.2553	0.2577	0.2601	0.2625	0.2648	0.2672	0.2695	0.2718
1.9	0.2788	0.2810	0.2833	0.2856	0.2878	0.2900	0.2923	0.2945
2.0	0.3010	0.3032	0.3054	0.3075	0.3096	0.3118	0.3139	0.3160
2.1	0.3222	0.3243	0.3263	0.3284	0.3304	0.3324	0.3345	0.3365
2.2	0.3424	0.3444	0.3464	0.3483	0.3502	0.3522	0.3541	0.3560
2.3	0.3617	0.3636	0.3655	0.3674	0.3692	0.3711	0.3729	0.3747
2.4	0.3802	0.3820	0.3838	0.3856	0.3874	0.3892	0.3909	0.3927
2.5	0.3979	0.3997	0.4014	0.4031	0.4048	0.4065	0.4082	0.4099
2.6	0.4150	0.4166	0.4183	0.4200	0.4216	0.4232	0.4249	0.4265
2.7	0.4314	0.4330	0.4346	0.4362	0.4378	0.4393	0.4409	0.4425
2.8	0.4472	0.4487	0.4502	0.4518	0.4533	0.4548	0.4564	0.4579
2.9	0.4624	0.4639	0.4654	0.4669	0.4683	0.4698	0.4713	0.4728

x	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.0
1.0	0.0000	0.0043	0.0086	0.0128	0.0170	0.0212	0.0253	0.0294	0.0
1.1	0.0414	0.0453	0.0492	0.0531	0.0569	0.0607	0.0645	0.0682	0.0
1.2	0.0792	0.0828	0.0864	0.0899	0.0934	0.0969	0.1004	0.1038	0.1
1.3	0.1139	0.1173	0.1206	0.1239	0.1271	0.1303	0.1335	0.1367	0.1
1.4	0.1461	0.1492	0.1523	0.1553	0.1584	0.1614	0.1644	0.1673	0.1
1.5	0.1761	0.1790	0.1818	0.1847	0.1875	0.1903	0.1931	0.1959	0.1
1.6	0.2041	0.2068	0.2095	0.2122	0.2148	0.2175	0.2201	0.2227	0.2
1.7	0.2304	0.2330	0.2355	0.2380	0.2405	0.2430	0.2455	0.2480	0.2
1.8	0.2553	0.2577	0.2601	0.2625	0.2648	0.2672	0.2695	0.2718	0.2
1.9	0.2788	0.2810	0.2833	0.2856	0.2878	0.2900	0.2923	0.2945	0.2
2.0	0.3010	0.3032	0.3054	0.3075	0.3096	0.3118	0.3139	0.3160	0.3
2.1	0.3222	0.3243	0.3263	0.3284	0.3304	0.3324	0.3345	0.3365	0.3
2.2	0.3424	0.3444	0.3464	0.3483	0.3502	0.3522	0.3541	0.3560	0.3
2.3	0.3617	0.3636	0.3655	0.3674	0.3692	0.3711	0.3729	0.3747	0.3
2.4	0.3802	0.3820	0.3838	0.3856	0.3874	0.3892	0.3909	0.3927	0.3
2.5	0.3979	0.3997	0.4014	0.4031	0.4048	0.4065	0.4082	0.4099	0.4
2.6	0.4150	0.4166	0.4183	0.4200	0.4216	0.4232	0.4249	0.4265	0.4
2.7	0.4314	0.4330	0.4346	0.4362	0.4378	0.4393	0.4409	0.4425	0.4
2.8	0.4472	0.4487	0.4502	0.4518	0.4533	0.4548	0.4564	0.4579	0.4
2.9	0.4624	0.4639	0.4654	0.4669	0.4683	0.4698	0.4713	0.4728	0.4

x	0.00	0.01	0.02	0.0
1.0	0.0000	0.0043	0.0086	0.01
1.1	0.0414	0.0453	0.0492	0.053
1.2	0.0792	0.0828	0.0864	0.089
1.3	0.1139	0.1173	0.1206	0.123
1.4	0.1461	0.1492	0.1523	0.155
1.5	0.1761	0.1790	0.1818	0.184
1.6	0.2041	0.2068	0.2095	0.212
1.7	0.2304	0.2330	0.2355	0.238
1.8	0.2553	0.2577	0.2601	0.262
1.9	0.2788	0.2810	0.2833	0.285
2.0	0.3010	0.3032	0.3054	0.307
2.1	0.3222	0.3243	0.3263	0.328
2.2	0.3424	0.3444	0.3464	0.348
2.3	0.3617	0.3636	0.3655	0.367
2.4	0.3802	0.3820	0.3838	0.385
2.5	0.3979	0.3997	0.4014	0.403
2.6	0.4150	0.4166	0.4183	0.420
2.7	0.4314	0.4330	0.4346	0.436
2.8	0.4472	0.4487	0.4502	0.451
2.9	0.4624	0.4639	0.4654	0.466
3.0	0.4771	0.4786	0.4800	0.481
3.1	0.4914	0.4928	0.4942	0.495
3.2	0.5051	0.5065	0.5079	0.509
3.3	0.5185	0.5198	0.5211	0.522
3.4	0.5315	0.5328	0.5340	0.535
3.5	0.5441	0.5453	0.5465	0.547
3.6	0.5563	0.5575	0.5587	0.559
3.7	0.5682	0.5694	0.5705	0.57
3.8	0.5798	0.5809	0.5821	0.583
3.9	0.5911	0.5922	0.5933	0.594
4.0	0.6021	0.6031	0.6042	0.605
4.1	0.6128	0.6138	0.6149	0.615
4.2	0.6232	0.6243	0.6253	0.626
4.3	0.6335	0.6345	0.6355	0.636
4.4	0.6435	0.6444	0.6454	0.646
4.5	0.6532	0.6542	0.6551	0.656

§7.5: Using logs to multiply

First rule of logs: $\log(a\times b) = \log(a)+\log(b)$

Example: Find 2.7×1.6 using logs

Look how I write the answer.

- $\log(2.7\times 1.6)=\log(2.7)+\log(1.6)$
- On the table we see that $\log(2.7)\approx 0.43$ and $\log(1.6)\approx 0.20$, so $\log(2.7\times 1.6)\approx 0.43+0.20 = 0.63$
- Is this the answer ? Heck No! It is the log of the answer
- $2.7\times 1.6 \approx \text{antilog}(0.63) = 10^{0.63}$
- $10^{0.63} \approx 4.3$
- Is my answer $\boxed{4.3}$ reasonable? Yes, about $2 \times 2 = 4$.

§7.5: Using logs to divide

Remember Log Rule (5): $\boxed{\log(a \div b) = \log(a) - \log(b)}$

Example: Use this rule to find 38.2/1.77

Method

(i) Look up log(3.82) and log(1.77), find log(38.2)

⋆You can find log(38.2) by adding 1 to log(3.82) because 38.2 is 3.82 times one more power of 10.⋆

(ii) Subtract!

(iii) Take the antilog of result from (ii)

(iv) Think: Is the answer reasonable or did I goof up?

A= done B= confused

§7.5: Using logs to divide

Remember Log Rule (5): $\boxed{\log(a \div b) = \log(a) - \log(b)}$

Example: Use this rule to find 38.2/1.77

38.2/1.77

= 10^log(38.2/1.77) rules of logs

38.2 = 10×3.82, so add 1

= 10^[log(38.2) − log(1.77)] look up in table

= 10^[1.5821 − .2480] subtract

$$\begin{array}{r} 1.5821 \\ -\ 0.2480 \\ \hline 1.3341 \end{array}$$

= 10^1.3341

= 10 × 10^0.3341 look up using table backwards

= 10 × 2.16

= 21.6

reasonable? yes! 38/1.7 ≈ 40/2 = 20 ≈ 21.6

x	0.00	0.01	0.02
1.0	0.0000	0.0043	[illegible]
1.1	0.0414	0.0453	[illegible]
1.2	0.0792	0.0828	[illegible]
1.3	0.1139	0.1173	[illegible]
1.4	0.1461	0.1492	[illegible]
1.5	0.1761	0.1790	[illegible]
1.6	0.2041	0.2068	[illegible]
1.7	0.2304	0.2330	[illegible]
1.8	0.2553	0.2577	[illegible]
1.9	0.2788	0.2810	[illegible]
2.0	0.3010	0.3032	[illegible]
2.1	0.3222	0.3243	[illegible]
2.2	0.3424	0.3444	[illegible]
2.3	0.3617	0.3636	[illegible]
2.4	0.3802	0.3820	[illegible]
2.5	0.3979	0.3997	[illegible]
2.6	0.4150	0.4166	[illegible]
2.7	0.4314	0.4330	[illegible]
2.8	0.4472	0.4487	[illegible]
2.9	0.4624	0.4639	[illegible]
3.0	0.4771	0.4786	[illegible]
3.1	0.4914	0.4928	[illegible]
3.2	0.5051	0.5065	[illegible]
3.3	0.5185	0.5198	[illegible]
3.4	0.5315	0.5328	[illegible]
3.5	0.5441	0.5453	[illegible]
3.6	0.5563	0.5575	[illegible]
3.7	0.5682	[illegible]	[illegible]
3.8	[illegible]	[illegible]	0.5821
3.9	0.5911	[illegible]	0.5933

x	0.00	0.01	0.02	0.03	0.04	0.05	0.06
1.0	0.0000	0.0043	0.0086	0.0128	0.0170	0.0212	[illegible]
1.1	0.0414	0.0453	0.0492	0.0531	0.0569	0.0607	[illegible]
1.2	0.0792	0.0828	0.0864	0.0899	0.0934	0.0969	[illegible]
1.3	0.1139	0.1173	0.1206	0.1239	0.1271	0.1303	[illegible]
1.4	0.1461	0.1492	0.1523	0.1553	0.1584	0.1614	[illegible]
1.5	0.1761	0.1790	0.1818	0.1847	0.1875	0.1903	[illegible]
1.6	0.2041	0.2068	0.2095	0.2122	0.2148	0.2175	[illegible]
1.7	0.2304	0.2330	0.2355	0.2380	0.2405	0.2430	[illegible]
1.8	0.2553	0.2577	0.2601	0.2625	0.2648	0.2672	[illegible]
1.9	0.2788	0.2810	0.2833	0.2856	0.2878	0.2900	[illegible]
2.0	0.3010	0.3032	0.3054	0.3075	0.3096	0.3118	[illegible]
2.1	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	0.3345
2.2	0.3424	0.3444	0.3464	0.3483	0.3502	0.3522	0.3541

0.3341 is between these two numbers, so we choose the one that's closest to 0.3341.

x	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07
1.0	0.0000	0.0043	0.0086	0.0128	0.0170	0.0212	0.0253	[illegible]
1.1	0.0414	0.0453	0.0492	0.0531	0.0569	0.0607	0.0645	[illegible]
1.2	0.0792	0.0828	0.0864	0.0899	0.0934	0.0969	0.1004	[illegible]
1.3	0.1139	0.1173	0.1206	0.1239	0.1271	0.1303	0.1335	[illegible]
1.4	0.1461	0.1492	0.1523	0.1553	0.1584	0.1614	0.1644	[illegible]
1.5	0.1761	0.1790	0.1818	0.1847	0.1875	0.1903	0.1931	[illegible]
1.6	0.2041	0.2068	0.2095	0.2122	0.2148	0.2175	0.2201	[illegible]
1.7	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	0.2480
1.8	0.2553	0.2577	0.2601	0.2625	0.2648	0.2672	0.2695	0.2718

§7.5: Powers Using Logs

Or, exploiting Log Rule (4): $\boxed{\log(a^p) = p\log(a)}$

Use this to find $\sqrt{70}$.

One Approach:

(i) Use table and move decimal point trick to find log(70)
⋆I will show the graph of the exponential function 10^x and talk about the graph method next lecture.⋆

(ii) $\log(\sqrt{70}) = \log(70^{1/2}) = (1/2)\log(70)$

(iii) Take the antilog of result from (ii)

(iv) Think: Is the answer reasonable or did I goof up?

A= done B= working C= confused

Answer: $\sqrt{70} \approx 8.32$. Is that reasonable?

§7.5: Powers Using Logs

Or, exploiting Log Rule (4): $\boxed{\log(a^{p}) = p\log(a)}$

Use this to find $\sqrt{70}$.

$\sqrt{70}$

$= 10^\wedge \log(\sqrt{70})$

$= 10^\wedge \log(70^{1/2})$

$= 10^\wedge \frac{1}{2}\log(70)$ $= 1 + \log 7$

$= 10^\wedge \frac{1}{2}(1.8451)$

$= 10^\wedge(.92255)$

$= 8.37$

$2 \overline{)1.8451}$ = .92255

-18, 4, 4, 5, 4, 11, 10, 10

x	0.00	0.01
5.5	[illegible]	0.7412
5.6	[illegible]	0.7490
5.7	[illegible]	0.7566
5.8	[illegible]	0.7642
5.9	[illegible]	0.7716
6.0	[illegible]	0.7789
6.1	[illegible]	0.7860
6.2	[illegible]	0.7931
6.3	[illegible]	0.8000
6.4	[illegible]	0.8069
6.5	[illegible]	0.8136
6.6	[illegible]	0.8202
6.7	[illegible]	0.8267
6.8	[illegible]	0.8331
6.9	[illegible]	0.8395
7.0 →	0.8451	0.8457
7.1	0.8513	0.8519
7.2	0.8573	0.8579

$\sqrt{64} = 8,$

$\sqrt{81} = 9,$

So $\sqrt{70} \approx 8.37$ seems reasonable.

x	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08
5.5	0.7404	0.7412	0.7419	0.7427	0.7435	0.7443	0.7451	[illegible]	0.746
5.6	0.7482	0.7490	0.7497	0.7505	0.7513	0.7520	0.7528	[illegible]	0.754
5.7	0.7559	0.7566	0.7574	0.7582	0.7589	0.7597	0.7604	[illegible]	0.761
5.8	0.7634	0.7642	0.7649	0.7657	0.7664	0.7672	0.7679	[illegible]	0.769
5.9	0.7709	0.7716	0.7723	0.7731	0.7738	0.7745	0.7752	[illegible]	0.776
6.0	0.7782	0.7789	0.7796	0.7803	0.7810	0.7818	0.7825	[illegible]	0.783
6.1	0.7853	0.7860	0.7868	0.7875	0.7882	0.7889	0.7896	[illegible]	0.791
6.2	0.7924	0.7931	0.7938	0.7945	0.7952	0.7959	0.7966	[illegible]	0.798
6.3	0.7993	0.8000	0.8007	0.8014	0.8021	0.8028	0.8035	[illegible]	0.804
6.4	0.8062	0.8069	0.8075	0.8082	0.8089	0.8096	0.8102	[illegible]	0.811
6.5	0.8129	0.8136	0.8142	0.8149	0.8156	0.8162	0.8169	[illegible]	0.818
6.6	0.8195	0.8202	0.8209	0.8215	0.8222	0.8228	0.8235	[illegible]	0.824
6.7	0.8261	0.8267	0.8274	0.8280	0.8287	0.8293	0.8299	[illegible]	0.831
6.8	0.8325	0.8331	0.8338	0.8344	0.8351	0.8357	0.8363	[illegible]	0.837
6.9	0.8388	0.8395	0.8401	0.8407	0.8414	0.8420	0.8426	[illegible]	0.843
7.0	0.8451	0.8457	0.8463	0.8470	0.8476	0.8482	0.8488	[illegible]	0.850
7.1	0.8513	0.8519	0.8525	0.8531	0.8537	0.8543	0.8549	[illegible]	0.856
7.2	0.8573	0.8579	0.8585	0.8591	0.8597	0.8603	0.8609	[illegible]	0.862
7.3	0.8633	0.8639	0.8645	0.8651	0.8657	0.8663	0.8669	[illegible]	0.868
7.4	0.8692	0.8698	0.8704	0.8710	0.8716	0.8722	0.8727	[illegible]	0.873
7.5	0.8751	0.8756	0.8762	0.8768	0.8774	0.8779	0.8785	[illegible]	0.879
7.6	0.8808	0.8814	0.8820	0.8825	0.8831	0.8837	0.8842	[illegible]	0.885
7.7	0.8865	0.8871	0.8876	0.8882	0.8887	0.8893	0.8899	[illegible]	0.891
7.8	0.8921	0.8927	0.8932	0.8938	0.8943	0.8949	0.8954	[illegible]	0.896
7.9	0.8976	0.8982	0.8987	0.8993	0.8998	0.9004	0.9009	[illegible]	0.902
8.0	0.9031	0.9036	0.9042	0.9047	0.9053	0.9058	0.9063	[illegible]	0.907
8.1	0.9085	0.9090	0.9096	0.9101	0.9106	0.9112	0.9117	[illegible]	0.912
8.2	0.9138	0.9143	0.9149	0.9154	0.9159	0.9165	0.9170	[illegible]	0.918
8.3	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	0.9222	0.9227	0.923
8.4	0.9243	0.9248	0.9253	0.9258	0.9263	0.9269	0.9274	0.9279	0.928
8.5	0.9294	0.9299	0.9304	0.9309	0.9315	0.9320	0.9325	0.9330	0.933

$0.9222 < 0.92255 < 0.9227$

closer to this one

Computer Applications

One kilobyte (1 KB) is 2^{10}.

Problem: Calculate 2^{10} using logs. **Hint:** $\log(2) \approx 0.3$

A≈ 3 B≈ 10.3 C≈ 30 D≈ 1000 E≈ 1100 $\boxed{\text{D}}$

So: $2^{10} \approx 10^3 = 1000$ (really $2^{10} = 1024$).

$$
\begin{aligned}
&1\text{KB is really } 2^{10} = 1024 \approx 10^3 && (\text{K is Kilo} = \text{thousand})\\
&1\text{MB is really } 2^{20} = \left(2^{10}\right)^2 \approx (10^3)^2 = 10^6 && (\text{M is Mega} = \text{million})\\
&1\text{GB is really } 2^{30} = \left(2^{10}\right)^3 \approx (10^3)^3 = 10^9 && (\text{G is Giga} = \text{billion})\\
&1\text{TB is really } 2^{40} = \left(2^{10}\right)^4 \approx (10^3)^4 = 10^{12} && (\text{T is Tera} = \text{trillion})
\end{aligned}
$$

Example: suppose on a certain island the population of rabbits doubles every generation. After 20 generations it multiplies by...
$2^{20} \approx 1$ million.

Powers of 2 are easy to do, even in your head. To work out 2^n the log of the answer is approximately $0.3n$, so 2^n is 1 followed by $0.3n$ zeroes.

§7.7: Solving Exponential Eq'ns

1. Find x by solving $10^x = 5$.

A$= 5$ B$= 0.5$ C$= \log(5)$ D$= \log(0.5)$
E$= \log(5) - \log(10)$ $\boxed{\text{C}}$

Look how I write the answer!

$$\log(10^x) = \log(5) \quad \text{Take logs of both sides}$$
$$x = \log(10^x) = \log(5) \quad \text{Using } \log(a^p) = p\log(a) \text{ and } \log(10) = 1$$

Examples:

Use the Fourth Law:

$$\log(a^{x}) = x\log(a)$$

Slogan: Logs bring exponents down to ground level.

2. Solve $3^{x} = 7$

A$= \log(7/3)$ B$= \log(7) - \log(3)$ C$= \log(7) + \log(3)$
D$= \log(3)/\log(7)$ E$= \log(7)/\log(3)$ $\boxed{\text{E}}$

Look how I write the answer:

$$\log(3^{x}) = \log(7)$$ Take logs of both sides

$$x\log(3) = \log(3^{x}) = \log(7)$$ Using $\log(a^{p}) = p\log(a)$

So: $x = \log(7)/\log(3)$

Examples:

Use the Fourth Law:

$$\log(a^{x}) = x\log(a)$$

Slogan: Logs bring exponents down to ground level.

3. Solve $7^{x+2} = 30$.

$$A=\frac{\log(30)-2\log(7)}{\log(7)} \qquad B=\frac{\log(30)}{\log(7)}-2 \qquad C=\frac{\log(30)-\log(49)}{\log(7)}$$

$$D=\frac{\log(30/49)}{\log(7)} \qquad E\approx -0.25213$$

All are correct!

Examples:

Use the Fourth Law:

$$\boxed{\log(a^{x}) = x\log(a)}$$

Slogan: Logs bring exponents down to ground level.

3. Solve $7^{x+2} = 30$.

$$7^{x+2} = 30$$

$$\log(7^{x+2}) = \log(30)$$

$$(x+2)\log(7) = \log(30)$$

$$x+2 = \frac{\log(30)}{\log(7)}$$

$$x = \frac{\log(30)}{\log(7)} - 2$$

Examples:

Use the Fourth Law:

$$\log(a^{x}) = x\log(a)$$

Slogan: Logs bring exponents down to ground level.

4. Solve $7 \times 3^y = 2^{4y+3}$

A= $\dfrac{3\log(2) - \log(7)}{\log(3) - 4\log(2)}$ B= $\dfrac{3\log(2)}{7\log(3)}$ C= $\dfrac{3\log(2)}{7\log(3) - 4\log(2)}$

D= $\dfrac{7\log(3) - 4\log(2)}{3\log(2)}$ E=none of the above

$\boxed{\text{A}}$

Examples:

Use the Fourth Law:

$$\boxed{\log(a^{x}) = x\log(a)}$$

Slogan: Logs bring exponents down to ground level.

4. Solve $7 \times 3^y = 2^{4y+3}$

$$\log(7 \times 3^y) = \log(2^{4y+3})$$

$$\log(7) + \log(3^y) = (4y+3)\log(2)$$

$$\log(7) + y\log(3) = 4y\log(2) + 3\log(2)$$

$$y\log(3) - 4y\log(2) = 3\log(2) - \log(7)$$

$$y\left(\log(3) - 4\log(2)\right) = 3\log(2) - \log(7)$$

$$y = \frac{3\log(2) - \log(7)}{\log(3) - 4\log(2)}$$

Compound Interest

At the end of each year a bank pays 7% interest into your account. Initially have \$10,000 in account. How much after 10 years?

Think $10 \times 7\% = 70\%$ in 10 years, so have \$17,000 but that is wrong.

$$\begin{aligned} \text{After 1 year:} \quad & \$10,000 \times 1.07 = \$10,700 \\ \text{After 2 years:} \quad & \$10,700 \times 1.07 = \$10,000 \times 1.07 \times 1.07 = \$11,449 \\ \text{After 3 years:} \quad & \$11,449 \times 1.07 = \$10,000 \times (1.07)^3 = \$12,250.40 \end{aligned}$$

Each year what you had before is multiplied by 1.07. Thus compound interest.
So after 10 years have

$$\$10,000 \times \underbrace{1.07 \times 1.07 \times \cdots \times 1.07}_{\text{10 times}} = 10,000 \times (1.07)^{10} \approx \boxed{\$20,000}$$

Conclusion: Money approximately doubles in 10 years!
So in 20 years multiplies by 4, in 30 years by 8,...

That's it. Thanks for being here.

That's it. Thanks for being here.