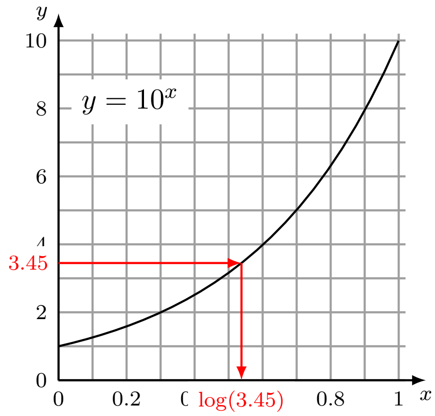
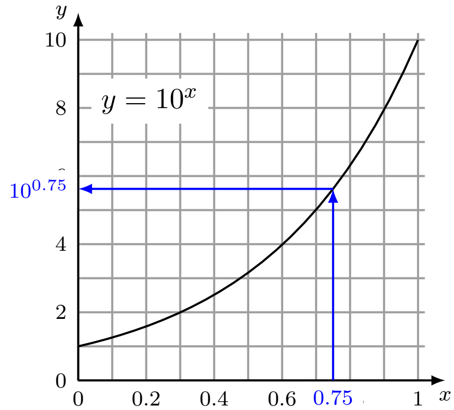
Math 34A Winter 2020
Old Midterm 2 #2 Solutions

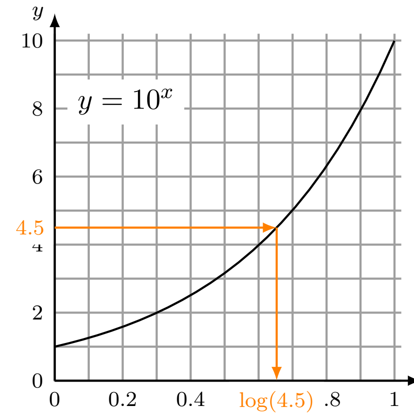
1. Here are the three graphs we'll use in solving these problems:



Part (a)



Part (b)



Part (c)

(a) This part is based on the move the decimal point trick:

$$\log(3450) = \log(10^3 \times 3.45) = \log(10^3) + \log(3.45) = 3 + \log(3.45).$$

Now we can use the graph to find that $\log(3.45) \approx 0.54$, and so $\log(3450) \approx \boxed{3.54}$. (Mathematica tells me that $\log(3450) \approx 3.537\,819\,095\ldots$, so we're pretty close.)

(b) The reverse version of the "move the decimal point trick" is what we need here:

$$10^{-2.25} = 10^{-3+0.75} = 10^{-3} \times 10^{0.75}.$$

(We can only find 10^x for values of x between 0 and 1, which is why we didn't write $-2.25 = -2 - 0.25$. In that case we'd need to find $10^{-0.25}$, but our graph doesn't show this.) We know that $10^{-3} = 0.001$, and we use the graph to find that $10^{0.75} \approx 5.62$. Thus $10^{-2.25} \approx 0.001 \times 5.62 = \boxed{0.005\,62}$. (Mathematica tells me that $10^{-2.25} \approx 0.005\,623\,413\ldots$, so as usual we're very close.)

(c) First we use the rules of logarithms and the move the decimal point trick to write

$$\log(0.45^{10}) = 10\log(0.45) = 10\log\left(4.5 \times 10^{-1}\right) = 10\left(\log(4.5) + \log(10^{-1})\right) = 10\left(\log(4.5) - 1\right).$$

Now we can use the graph to find that $\log(4.5) \approx 0.65$. Thus

$$\log(0.45^{10}) = 10\left(\log(4.5) - 1\right) \approx 10\,(0.65 - 1) = 10(-0.35) = \boxed{-3.5}.$$

(Mathematica tells me that $\log(0.45^{10}) \approx -3.467\,874\,862\ldots$)

2. Let's start with this equation slightly simplified as

$$5^{3x+1} = 9 - 2 = 7.$$

Now take the logarithm of both sides to get

$$\log(5^{3x+1}) = \log(7).$$

We simplify this using rules of logs to

$$(3x+1)\log(5) = \log(7) \qquad \text{since } \log(a^p) = p\log(a).$$

Now divide both sides by $\log(5)$ to get

$$3x + 1 = \frac{\log(7)}{\log(5)}.$$

Now subtract 1 from both sides, then divide by 3 to get

$$3x = \frac{\log(7)}{\log(5)} - 1 \qquad \text{and then} \qquad x = \boxed{\frac{\log(7)}{3\log(5)} - \frac{1}{3}}.$$

We could combine this answer to a single fraction by finding a common denominator. This will give us the equivalent answers

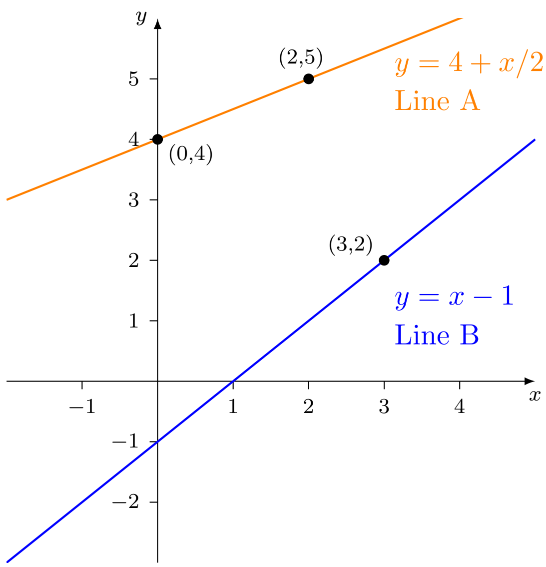
$$x = \boxed{\frac{\log(7) - \log(5)}{3\log(5)}} \qquad \text{or} \qquad x = \boxed{\frac{\log(7/5)}{3\log(5)}}.$$

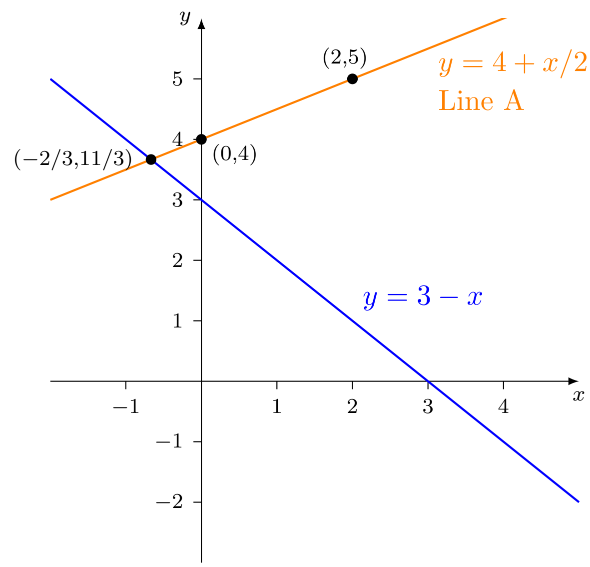
3. (a) The slope of Line A is

$$m = \frac{5-4}{2-0} = \frac{1}{2}.$$

Thus Line A has equation $y = \frac{1}{2}x + b$ for some b. We find that value by plugging in either point $(x, y) = (2, 4)$ or $(4, 5)$. If we plug in the second point, we get $4 = \frac{1}{2}(0) + b$, or $b = 4$. Thus Line A has equation $\boxed{y = \frac{1}{2}x + 4}$ or $\boxed{y = 4 + x/2}$. Line A is shown on the left, below.

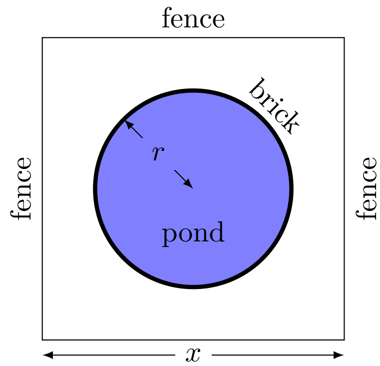
(b) Line B has slope twice the slope of Line A, or $2 \times \frac{1}{2} = 1$, so it has equation $y = x + b$. We plug in $(x, y) = (3, 2)$ to find b: $2 = (3) + b$, or $b = -1$. Thus the equation of Line B is $\boxed{y = x - 1}$. Line B is shown with Line A on the left, below.





(c) The point of intersection of Line A (which is $y = 4 + x/2$) and the line $y = 3 - x$ is where $3 - x = 4 + x/2$. Adding x and subtracting 4 from both sides, we get $3x/2 = -1$. Multiplying both sides by $2/3$ gives us that $x = -2/3$. Plugging $x = -2/3$ into either line gives us $y = 11/3$. Thus the point of intersection is $(x, y) = \boxed{(-\frac{2}{3}, \frac{11}{3})}$. (The lines and the point of intersection are shown above on the right.)

4. We reproduce the picture of the garden here, with an added-in notation showing that the radius of the pond is r and a length of the square is x:



The cost of the fence is \$17/ft and the length of the fence is the perimeter of the square: $4x$ ft. Thus the cost of the fence is $\$17(4x) = \$68x$. Similarly, the cost of the brick is \$33/ft and the length of the brick is the perimeter of the circle: $2\pi r$ ft. Thus the cost of the brick is $\$33(2\pi r) = \$66\pi r$. So the total cost is

$$\boxed{68x + 66\pi r \text{ dollars}}.$$

But this is in terms of r and x, not r and A (the area of the square). We need to write x in terms of A. But the area of the square is both A and x^2, so $A = x^2$. Thus $x = \sqrt{A}$, and the total cost is

$$\boxed{68\sqrt{A} + 66\pi r \text{ dollars}}.$$

5.

(a) I've mixed 7 liters of red paint into the blue paint, so the mixture contains 5 liters of blue paint and 7 liters of red paint. What is the percentage of this mixture is blue paint? It's simply

$$\frac{\text{amount of blue paint in mixture}}{\text{total amount of paint in mixture}} \times 100\% = \frac{5 \text{ liters}}{5+7 \text{ liters}} \times 100\% = \frac{5}{12} \times 100\% = \frac{125}{3}\%.$$

That is, the mixture is $\boxed{(125/3)\%}$ blue.

(b) Let's figure out the percentage of blue paint in a mixture of 5 liters of blue and R liters of red paint. This percentage is

$$\frac{\text{amount of blue paint in mixture}}{\text{total amount of paint in mixture}} \times 100\% = \frac{5 \text{ liters}}{5+R \text{ liters}} \times 100\% = \frac{5}{5+R} \times 100\%.$$

If we want this percentage to be 10%, we get the equation

$$\frac{5}{5+R} \times 100\% = 10\%.$$

Dividing by 10% and multiplying by $5 + R$ gives us

$$5 \times 10 = 5 + R \qquad \text{which simplifies to} \qquad R = \boxed{45 \text{ liters}}.$$

(c) This is the same as part (b), but now the "10%" is being replaced by "x%" in our calculation. The percentage of blue paint in a mixture of 5 liters of blue and R liters of red paint is again

$$\frac{\text{amount of blue paint in mixture}}{\text{total amount of paint in mixture}} \times 100\% = \frac{5 \text{ liters}}{5+R \text{ liters}} \times 100\% = \frac{5}{5+R} \times 100\%.$$

Now we want this percentage to be $x\%$, so we get the equation

$$\frac{5}{5+R} \times 100\% = x\%.$$

Dividing by 1% and multiplying by $5+R$ gives us

$$5 \times 100 = (5+R)x = 5x + Rx \qquad \text{which simplifies to} \qquad R = \boxed{\frac{500-5x}{x} \text{ liters}}.$$

We can check our answer in a couple of situations:

- If $x = 10$, then we're in the same case as part (b), and our formula should give us the same answer $R = 45$ liters. It does.
- If $x = 100$, then we're asked how much red paint should be added to make a mixture that is 100% blue. The answer, obviously, is no red paint. Sure enough, plugging in $x = 100$ into our expression gives us an answer of $R = 0$ liters.

Thus our answer seems reasonable.