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More: Spot The Difference!

1. $\lim_{x \to 1} \left(\frac{x-1}{x^2-1} \right) = \frac{1}{2}$

2. $\lim_{x \to 1} \left(\frac{x+3}{x^2+1} \right) = ?$

(A) 3 (B) 1 (C) 4 (D) 2 (E) 0 $\boxed{\text{D}}$

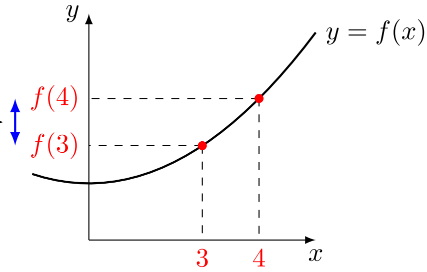
3. $\lim_{x \to 0} \left(\frac{3x+x^2}{2x} \right) = ?$

(A) 0 (B) $\frac{0}{0}$ (C) $\frac{1}{0}$ (D) $\frac{1}{2}$ (E) $\frac{3}{2}$ $\boxed{\text{E}}$

§5.2: Change in $f(x)$

$f(4) - f(3)$

$=$ change in $f(x)$
when x changes from 3 to 4



Example: $f(x) =$ stock value x years after 2010
Ex: $f(3) =$ stock value in 2013

$f(4) - f(3) =$?change in stock value from 2013 to 2014

Calculus is about change

The calculations involve limits.

4. What is the change in $f(x) = x^2$ between 2 and 3?

(A) 1 (B) 4 (C) 5 (D) 6 (E) 9 C

5. What is the change in $f(x) = x^2$ between 2 and $2 + h$?

(A) 2 (B) $h^2 - 2$ (C) $4h$ (D) h^2 (E) $4h + h^2$ E

Note: This exact example comes up when we do calculus.

§5.3: Summation Notation

$$\sum_{n=1}^{7} n = 1 + 2 + 3 + 4 + 5 + 6 + 7$$

Read aloud: "The sum from n equals 1 up to 7 of n"

$$\sum_{n=1}^{4} n^2 = 1^2 + 2^2 + 3^2 + 4^2$$

$$\sum_{n=1}^{5} 2^n = 2^1 + 2^2 + 2^3 + 2^4 + 2^5$$

Σ is the Greek version of S

...as in Summation

...and the integral sign $\int$ (Math 34B)

Examples:

6. $$\sum_{k=100}^{150} (k^2 + k) \;=\; (100^2 + 100) + (101^2 + 101) \cdots + (150^2 + 150)$$

7. Summing entries in a table of data (or in a spreadsheet program)

$$\sum_{p=5}^{9} x_p \;=\; x_5 + x_6 + x_7 + x_8 + x_9$$

8. Summing values of a function

$$\sum_{i=-2}^{1} f(i) \;=\; f(-2) + f(-1) + f(0) + f(1)$$

Examples 2: Averages

The average of $5, 1, 4, 14$ is

$$\frac{5+1+4+14}{4}$$

Add up the numbers you have then divide by how many numbers you had.

Average of $x_1, x_2, \cdots, x_N$ is

$$\frac{1}{N} \sum_{i=1}^{N} x_i = \frac{x_1 + x_2 + \cdots + x_N}{N}.$$

Examples 3: Cool Sum Formulas

9. $$\left(\sum_{k=1}^{15} a_k\right) + \left(\sum_{k=16}^{35} a_k\right) = \sum_{k=1}^{35} a_k$$

To see why this works, just write it out!

$$(a_1 + \cdots + a_{15}) + (a_{16} + \cdots + a_{35}) = (a_1 + \cdots + a_{35})$$

10. $$\left(\sum_{k=1}^{50} f(k)\right) - \left(\sum_{k=20}^{50} f(k)\right) = \sum_{k=1}^{19} f(k)$$

This just says

$$(f(1) + \cdots + f(50)) - (f(20) + \cdots + f(50)) = (f(1) + \cdots + f(19))$$

And More Cool Sum Formulas

11. $$\left(\sum_{i=1}^{7} a_i\right) + \left(\sum_{i=1}^{7} b_i\right) = \sum_{i=1}^{7} (a_i + b_i)$$

This just says that

$$(a_1 + \cdots + a_7) + (b_1 + \cdots + b_7) = (a_1 + b_1) + \cdots + (a_7 + b_7)$$

12. $$\left(\sum_{i=1}^{100} p_i\right) - \left(\sum_{i=1}^{50} p_i\right) =$$

(A) $\displaystyle\sum_{i=50}^{100} p_i$ (B) $\displaystyle\sum_{i=1}^{50} p_i$ (C) $\displaystyle\sum_{i=1}^{150} p_i$ (D) $\displaystyle\sum_{i=51}^{100} p_i$

Hint: Just write it out! D

$$(p_1 + \cdots + p_{100}) - (p_1 + \cdots + p_{50}) = (p_{51} + \cdots + p_{100})$$

One Last Question

13. What is

$$\sum_{n=1}^{3} n+1 \ = ?$$

(A) 6 (B) 7 (C) 8 (D) 9 (E) 10

WRONG!

It is ambiguous because it could mean two different things:

$$\left(\sum_{n=1}^{3} n\right) + 1 = 7 \qquad \text{or} \qquad \sum_{n=1}^{3} (n+1) = 9.$$

Without parentheses, you get into trouble.

Today: Start Chapter 7 (Logs)

Applications:

- Chemistry: alkalinity and acidity, pH scale
- Finance: compound interest (get rich slow)
- Geology: Richter scale for earthquakes (did you feel the earth move too?)
- Archeology: radio carbon dating (how old is that bone?)
- Astronomy: stellar magnitude (brightness of stars)
- Sound: decibels (what did you say? the music is too loud)
- Math: solving equations with exponents (includes all of the above)

Today: Start Chapter 7 (Logs)

Main Idea of Chapter 7:

> $\log(x)$ is how many tens you multiply to get x

Conclusion:

Before we do logs we should be really good at powers of 10.

Powers of Ten

1 meter $\approx$ 3 feet
1 centimeter $= 0.01$ meters $= 10^{-2}$ meters $\approx 1/2$ inch
1 kilometer $= 1,000$ meters $= 10^3$ meters $\approx 1/2$ mile

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Approximate distance (in meters), to nearest power of 10

10^7 meters	Size of Earth
10^9 meters	Distance to moon
10^{14} meters	Size of our solar system
10^{16} meters	One light-year
10^{21} meters	Size of the Milky Way galaxy
10^{27} meters	Size of the universe (about 93 billion light-years)
10^{80}	number of protons in the observable universe?
10^{100}	1 googol
10^{1000} meters	???

Exponential Basics

$$
\begin{aligned}
10^{4} &= 10 \times 10 \times 10 \times 10 \;=\; 10,000 \\
&= 4 \text{ lots of 10 multiplied together} \\
&= 1 \text{ followed by } 4 \text{ zeroes}
\end{aligned}
$$

$$
\begin{aligned}
10^{x} &= \underbrace{10 \times 10 \times \cdots \times 10}_{x \text{ lots of } 10} = 1\,\underbrace{00000\cdots 0}_{x \text{ zeros}} \\
&= 1 \text{ followed by x zeroes}
\end{aligned}
$$

Ex:
$$
\begin{aligned}
10^{2} \times 10^{3} &= (10 \times 10) \times (10 \times 10 \times 10) \\
&= 10^{2+3} = 10^{5}.
\end{aligned}
$$

$$\boxed{10^{x} \times 10^{y} = 10^{x+y} \qquad \text{First Law of Exponents}}$$

Why? We can work it out!

Exponential Basics (cont'd)

$$10^{x} \times 10^{y} = 10^{x+y} \qquad \text{First Law of Exponents}$$

Why? We can work it out:

(x lots of 10 multiplied together) $\times$ (y lots of 10 multiplied together)
$= (x + y)$ lots of 10 multiplied together

For now x and y are positive whole numbers.

More Exponentiation

$$\left(10^2\right)^3 = (10 \times 10)^3 = (10 \times 10) \times (10 \times 10) \times (10 \times 10) = 10^6$$

$$\boxed{\left(10^{a}\right)^{b} = 10^{ab} \qquad \text{Fourth Law of Exponents}}$$

Why? We can work it out:

$$10^a = \underbrace{10 \times 10 \times \cdots \times 10}_{a \text{ times}}$$

$$\left(10^a\right)^b = \underbrace{(10 \times \cdots \times 10) \times \cdots \times (10 \times \cdots \times 10)}_{b \text{ times}} = 10^{ab}.$$

Just count the zeros!