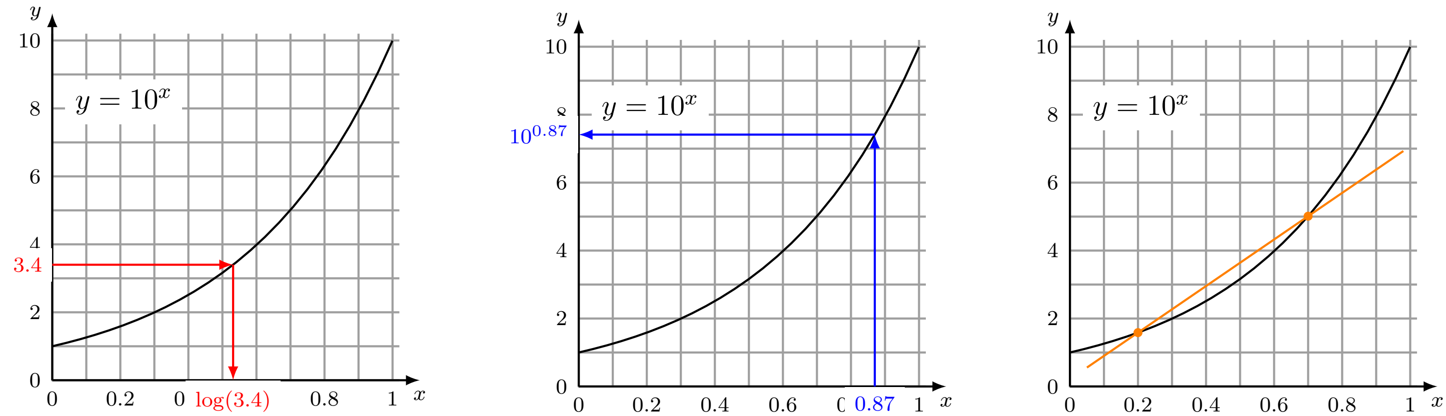
Math 34A Winter 2020
Old Midterm 3 #1 Solutions

1. Here are the three graphs we'll use in solving these problems:



Part (a) Part (b) Part (c)

(a) We know $\log(1000) = \log(10^3) = 3$, so this is really about finding $\log(0.34)$. We use the "move the decimal point" trick to compute $\log(0.34)$:

$$\log(0.34) = \log(10^{-1} \times 3.4) = \log(10^{-1}) + \log(3.4) = -1 + \log(3.4).$$

The graph tells us that $\log(3.4) \approx 0.53$. Thus $\log(0.34) \approx -1 + 0.53 = -0.47$, and so $\log(1000) + \log(0.34) \approx 3 - 0.47 = \boxed{2.53}$.

(Mathematica tells me that $\log(1000) + \log(0.34) \approx 2.531\,478\,917\ldots$)

(b) The solution to $\log(y) = 1.87$ is $y = \text{antilog}(1.87)$; that is, $y = 10^{1.87}$. By the rules of exponenents, we may write this as $y = 10^{1+0.87} = 10^1 \times 10^{0.87}$. Of course $10^1 = 10$, and we find $10^{0.87} \approx 7.4$ by reading the graph, above. Thus $10^{1.87} \approx 10 \times 7.4 = \boxed{74}$. (Mathematica tells me that $10^{1.87} \approx 74.131\,024\ldots$.)

(c) We've drawn the secant line to $y = 10^x$ between the two points $x = 0.2$ and $x = 0.7$ on the third graph, above. The slope of this line is about

$$m = \frac{5.0 - 1.6}{0.7 - 0.2} = \frac{3.4}{0.5} \approx \boxed{6.8}.$$

The actual average rate of change, computed from the actual values $(x, y) = (0.2, 10^{0.2})$ and $(0.7, 10^{0.7})$, is

$$m = \frac{10^{0.7} - 10^{0.2}}{0.7 - 0.2} \approx 6.853\,958\,287\,623\ldots,$$

so as usual we're pretty close.

2. We write down the answers without much commentary:

(a) $\dfrac{d}{dx}(2e^{7x} + 5x^3 - 7) = \boxed{14e^{7x} + 15x^2}$

(b) $\dfrac{d^2}{dx^2}(3x^4 + 12\sqrt{x}) = \dfrac{d^2}{dx^2}(3x^4 + 12x^{1/2}) = \dfrac{d}{dx}\left(12x^3 + 6x^{-1/2}\right) = \boxed{36x^2 - 3x^{-3/2}}$

(c) $f'(x) = 2cx - 16/x^2$, so $f'(2) = 2c(2) - 16/(2^2) = \boxed{4c - 4}$

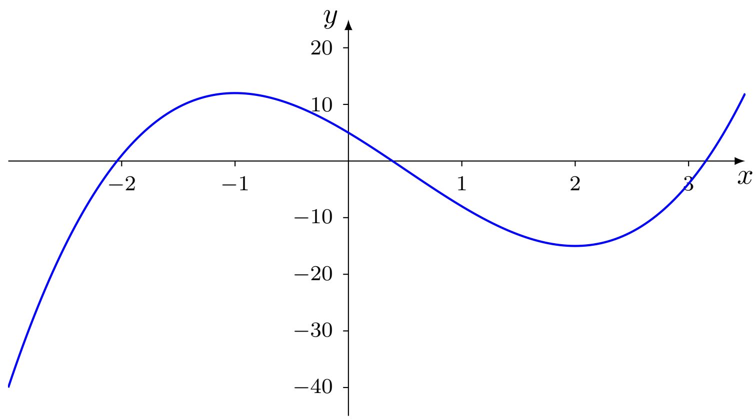
3. The tangent line is the line through $(t, h(t)) = (5, h(5)) = (5, 40)$ with slope $h'(5) = 2$. Thus the line has equation

$$y - 40 = 2(t - 5) \qquad \text{or, equivalently,} \qquad y = 2t + 30.$$

(a) When $t = 25$ (that is, in 1975, 25 years after 1950), the height of the tree is $h(25) \approx 2(25) + 30 = \boxed{80 \text{ feet}}$.

(b) We start by asking: what is t when the height of the tree is 200 feet? Thta is, when is $h(t) = 200$? We have estimated the tree height as $h(t) \approx 2t + 30$, so this is 200 when $2t + 30 = 200$. Solving, we get $t = 85$, which corresponds to the year $\boxed{2035}$ (85 years after 1950).

4. Here is a picture of the graph of $y = 2x^3 - 3x^2 - 12x + 5$:



(a) The slope of the graph is the derivative, $\frac{dy}{dx}$. Since $\frac{dy}{dx} = 6x^2 - 6x - 12$, the slope of the graph at $x = 1$ is $6(1)^2 - 6(1) - 12 = \boxed{-12}$.

(b) The tangent line at $x = 0$ has slope -12 (since $y'(0) = 6(0)^2 - 6(0) - 12 = -12$) and passes through the point $(x, y) = (0, 2(0)^3 - 3(0)^2 - 12(0) + 5) = (0, 5)$. Thus the equation of the tangent line is

$$y - 5 = -12(x - 0) \qquad \text{or, equivalently} \qquad \boxed{y = -12x + 5}.$$

(c) Remember that a curve is concave up exactly where the second derivative $y'' = \frac{d^2y}{dx^2}$ is positive. Since $y'' = 12x - 6$, this is positive (and so the graph is concave up) when $\boxed{x > 1/2}$.

(d) The slope is $\frac{dy}{dx} = 6x^2 - 6x - 12$, which is zero when $6x^2 - 6x - 12 = 0$. We factor this as $6(x^2 - x - 2) = 6(x - 2)(x + 1) = 0$, so the derivative is zero when $\boxed{x = -1 \text{ and } x = 2}$.

5. (a) The velocity of the rocket is $\boxed{h'(t) = -6t + 30 \text{ m/s}}$.

(b) The initial speed – that is, the speed at time $t = 0$ – is $\boxed{h'(0) = 30 \text{ m/s}}$.

(c) The acceleration of the rocket is $h''(t) = -6 \text{ m/s}^2$. This is a constant function, and in particular after 2 seconds the acceleration is $\boxed{h''(2) = -6 \text{ m/s}^2}$.

(d) The velocity is zero when $-6t + 30 = 0$ (where this formula is from part (a)). Solving, we get $\boxed{t = 5 \text{ seconds}}$.

(e) This asks for the height of the rocket at the time we found in part (d). This is

$$h(5) = -3(5)^2 + 30(5) = -75 + 150 = \boxed{75 \text{ meters}}.$$