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Last Time: General Rule

$$\frac{d}{dx}\left(x^{n}\right) = nx^{n-1}$$

Some Examples

$$\frac{d}{dx}\left(x^n\right) = nx^{n-1}$$

1. $\frac{d}{dx}\left(x^{-1/2}\right) =$

(A) $-\frac{1}{2}x^{1/2}$ (B) $-\frac{1}{2}x^{-3/2}$ (C) $-\frac{3}{2}x^{-3/2}$ B

Rule: ALWAYS rewrite the thing you want derivative of as x^n

2. $\frac{d}{dx}\left(\frac{1}{x^2}\right) =$

(A) $\frac{1}{2x}$ (B) $-2x^{-3}$ (C) $-2x^{-1}$ B

3. $\frac{d}{dx}\left(\frac{1}{\sqrt{x}}\right) =$

(A) $-\frac{1}{2}\sqrt{x}$ (B) $-\frac{1}{2}x^{-3/2}$ (C) $-\frac{3}{2}x^{-3/2}$ B

Polynomials

$$\frac{d}{dx}\left(4x^5 + 7x^2 - 5x + 7\right) = 4(5)x^4 + 7(2)x^1 - 5 + 0$$

Special cases

- $\frac{d}{dx}\left(-5x\right) = -5$
- $\frac{d}{dx}\left(7\right) = 0$

4. $\frac{d}{dx}\left(3x^4 + 9x^3 + 7\right) = ?$

(A) I have an answer (B) I am working on it (C) Help!

Answer: $12x^3 + 27x^2$

(A) I got it (B) I nearly got it (C) I want my mommy!

The Meanings of Derivatives

The derivative of $f(x) = x^2 + 3x + 1$ is $f'(x) = \frac{df}{dx} = 2x + 3$. This means:

- This is the slope of the graph $y = x^2 + 3x + 1$ at the point x
- It is the instantaneous rate of change of $f(x)$ at x.

That $f'(2) = 7$ means:

- The slope of the graph $y = f(x)$ at $x = 2$ is 7.
- The slope of the tangent line to the graph at $x = 2$ is 7.
- The instantaneous rate of change of $f(x)$ at $x = 2$ is 7.
- At $x = 2$ the output (value of $f(x)$) changes 7 times as fast as the input (value of (x)).
- $\Delta f \approx 7\Delta x$ near $x = 2$.
- $f(2 + \Delta x) \approx f(2) + 7\Delta x$.

Applications

5. What is the slope of the graph $y = 3x^2 - 7x + 5$ at $x = 1$?

(A) -2 (B) -1 (C) 0 (D) 1 (E) 2 B

6. What is the instantaneous rate of change of $f(x) = x^3 - 2x + 3$ at $x = 1$?

(A) -2 (B) -1 (C) 0 (D) 1 (E) 2 D

7. After t seconds a hamster on a skate board is $4t^2 + 2t$ cm from the origin on the x-axis. What is the exact speed of the hamster (in cm/sec) after 2 seconds?

(A) 10 (B) 16 (C) 18 (D) 20 (E) 14 C

Why This Works (§8.9)

$$\frac{d}{dx}\left(x^n\right) = nx^{n-1}$$

Example: $n = 3$: Calculate the average rate of change of x^3 between x and $x + h\Delta x$ then take limit as $h\Delta x \to 0$.

$$\left(\begin{array}{c}\text{average rate}\\ \text{of change between}\\ x \text{ and } x+h\end{array}\right) = \frac{(x+h)^3 - x^3}{(x+h) - x}$$

$$= 3x^2 + 3xh + h^2$$

Limit as $h \to 0$ is $3x^2$

A similar calculation works for x^n for any n.

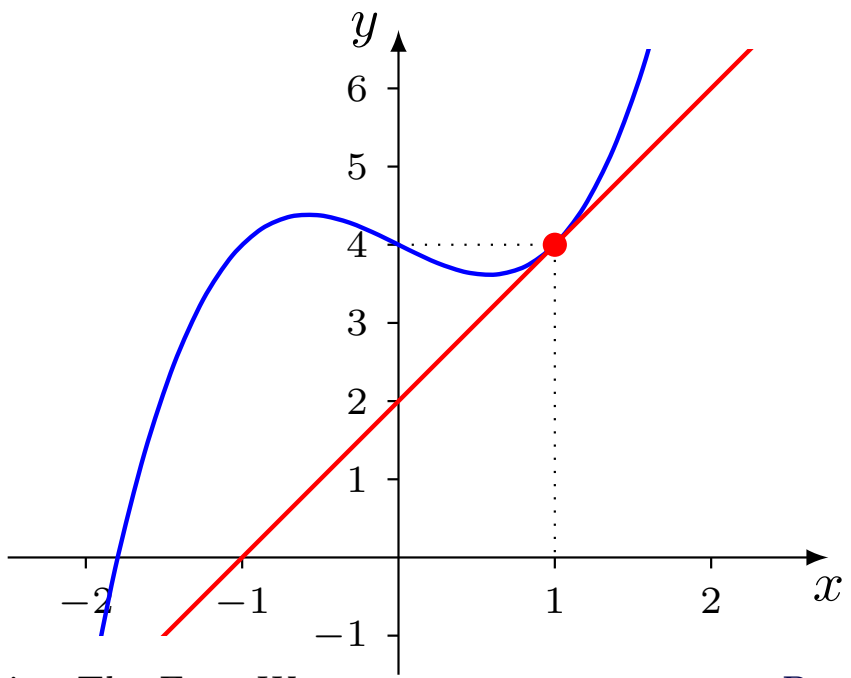
More Applications

8. What is the equation of the tangent line at $x = 1$ to the graph of $y = x^3 - x + 4$? The tangent line is $y = \ldots$?

(A) $x + 3$ (B) $3x + 1$ (C) $2x - 2$ (D) $2x + 2$ (E) $6x - 2$

Answer: D

Here's a picture:



Another Example

9. The temperature in an oven after t minutes is $50 + t^3$ °F. How quickly is the temperature rising after 2 minutes?

(A) 58 (B) 3 (C) 12 (D) 50 (E) 8

Answer: C

A Warning!

$$\frac{d}{dx}\left(f(x)g(x)\right) \neq f'(x) \times g'(x)$$

Notice: $5x^4 = \dfrac{d}{dx}\left(x^5\right) = \dfrac{d}{dx}\left(x^2 \cdot x^3\right) \neq (2x)(3x^2) = 6x^3$

Notice again: The derivative of $(x+1)(2x+3)$ is NOT $1 \cdot 2 = 2$.

10. $\dfrac{d}{dx}\left((x^2+1)(x^3+1)\right) = ?$

(A) $6x^3$ (B) $5x^4 + 3x^2 + 2x$ (C) $x^5 + x^3 + x^2 + 1$ (D) Other

Answer: B

Once upon a time. . .

There was a happy math professor and he told his happy students:

"When you work out derivatives ALWAYS write the $\frac{d}{dx}$ part so you write something like

$$\frac{d}{dx}\left(3x^2+5x+2\right)=6x+5$$

and you never-ever-ever write

$3x^2+5x+2 \qquad 6x+5$ or even worse $3x^2+5x+2=6x+5.$

Because if you don't do as I say I will become a sad math professor and you will repeat this class."

A Few More Examples:

11. If $f(x) = \sqrt{x}$, what is $f'(16)$?

(A) $\frac{1}{2}$ (B) $\frac{1}{4}$ (C) $\frac{1}{8}$ (D) $\frac{1}{16}$ (E) $\frac{1}{32}$ C

12. What is the x-coordinate of the point on the graph of $y = 4x^2 - 3x + 7$ where the graph has slope 13?

(A) 0 (B) 1 (C) 2 (D) 3 (E) 4 C

13. A circle is expanding so that after R seconds it has radius R cm. What is the rate of increase of area inside the circle after 2 seconds?

(A) 4π (B) $2\pi R^2$ (C) 2 (D) $2\pi R$ (E) πR^2 A