Math 4B **Name:** _____
Summer Session A
Midterm
9 July 2020

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1. (10 points each) Find the general solution of the given differential equation:

   (a) $w'' + 4w = 0$

   (b) $y' = \dfrac{x(x^2+1)}{4y^3}$

2. (10 points each) Solve the given initial value problem:

   (a) $u'' - 6u' + 9u = 0, \quad u(0) = 0, \quad u'(0) = 2$

(b) $(1+2y)y' - 2x = 0, \quad y(2) = 0$

(c) $v' + 2v = xe^{-2x}, \quad v(1) = 0$

3. Given that $y_1(t) = t^2$ is a solution of

$$t^2y'' - 4ty' + 6y = 0, \quad t > 0,$$

(a) (15 points) Find a second solution $y_2(t)$ that is linearly independent from $y_1(t)$.

(b) (5 points) Prove that y_1 and y_2 form a fundamental set of solutions.

(c) (5 points) Write an expression for the general solution $y(t)$ to the differential equation.

(d) (BONUS - 10 points) Let's define a third solution to the differential equation to be $y_3(t) = y_2(t) + y_1(t)$ (where $y_1(t) = t^2$ and $y_2(t)$ is whatever you got in part (a)). Without calculating any specific Wronskians, do you think that the pair $\{y_3, y_1\}$ would form a fundamental set of solutions, yes or no? What about $\{y_3, y_2\}$ - would they form a fundamental set of solutions, yes or no? Explain your reasoning and thoughts. You may use linear algebra theory.

4. (5 points) So far, what topic or homework problem(s) from the class have you found the most challenging? Give some explanation why. (There is no right or wrong answer.)