Math 34A - Practice Final

1. Short Answers

1.1. Problem 1. Solve for x:

$$5x^2 + 5x - 5 = 2x^2 - 7x + 226$$

Scratch Work

Answer:

1.2. **Problem 2.** Solve for x:

$$(x-1)(x^2+x+1) = x^3+4x-3$$

Scratch Work

Answer:

1.3. Problem 3.

1.3. Problem 3. Solve for x:

$$3^{2x+1} - 3^x = 3^{3+x} - 9$$

Scratch Work

Answer:

1.4. Problem 4. Solve for x:

$$3 \cdot 7^{3x+4} = 4$$

Scratch Work

Answer:

1.5. Problem 5.

Simplify the following expression as much as possible:

$$(1+x)(1+x^2) - x(x+1)(x^2-x+1)$$

Scratch Work

Answer:

1.6. Problem 6. Compute the derivative of $f(x) = 2(x+1)^2 + 3x^3 + \pi$.

Scratch Work

Answer:

1.7. Problem 7. Let $f(x) = -4x^2 + 7x + 8$. What is the largest value that f can take on?

Scratch Work

Answer:

1.8. Problem 8.

Solve the system of equations:

$$12x + 7y = 64$$
$$5x + 2y = 23$$

Scratch Work

Answer:

1.9. Problem 9. Let $g(x) = x^3 + 11x^2 - 576x - 6336$. Find the equation of the tangent line to $y = g(x)$ at $x = -18$.

Scratch Work

Answer:

2. Top Chef: Calzones

2.1. Units. It's impossible to find a good calzone in IV, so we're going to make one. The recipe calls for the following:

- (2 cups minus 3 tablespoons) of water.
- For every tablespoon water, we need 18 grams of flour.

Note: There are 16 tablespoons in a cup.

The recipe will make enough dough for 2 calzones, but we want to make 7 calzones.

(1) How many **cups** of water do you need? Round your answer to the nearest decimal place (e.g. if the answer is 23424.2342341354565 cups, just write 23424.2.)
(2) How much flour do we need?
(3) One tablespoon of water weighs 17.5 grams. What is the total weight of the dough we just made, in grams?

2.1.1. *Scratch Work.*

2.1.2. *Answers.*

(1)

(2)

(3)

2.2. The dough rises. we've mixed our flour and water, and also added some yeast to make our calzone dough. Yeast is alive! It makes the dough rise if you just let it sit. To start, we have 4 cups of dough.

- Every 5 minutes, the volume of the dough grows by 4%.
- We need to let the dough sit until the volume of the dough is at least 3 times what is was when we started.

(1) Write down a formula for the volume of the dough t minutes after we started.
(2) What is the volume of the dough one hour after we start?
(3) How many minutes should the dough sit to reach the target volume?

2.2.1. *Scratch Work.*

2.2.2. *Answers.*

(1)	
(2)	
(3)	

2.3. Selling the calzones. We're now going to sell our calzones. To make a single calzone, we need to spend \$ 3.50 on ingredients.

If the price of a calzone is set at \$ 10 dollars, we will sell 120 calzones. If we increase the price by a dollar, we will sell 5 fewer calzones. So if we charge \$11 dollars, we will sell 115 calzones; if we charge \$9, we will sell 125.

(1) If the price is set at d dollars, how many calzones will we sell? (Your answer should be a function involving d).
(2) How many much should we charge to maximize profit?
(3) How much profit do we make if we sell at the optimal price?

2.3.1. *Scratch Work.*

2.3.2. *Answers.*

(1)

(2)

(3)

3. Geometry

3.1. Linear meets quadratic. Let $q(x)$ be the quadratic polynomial:

$$q(x) = -(x+1)^2 + 4$$

- The graph of $q(x)$ looks like an upside-down U-shape.
- The graph of $q(x)$ contains the points $(-2,-5)$ and $(-3,0)$.

(*See Figure 1.*)

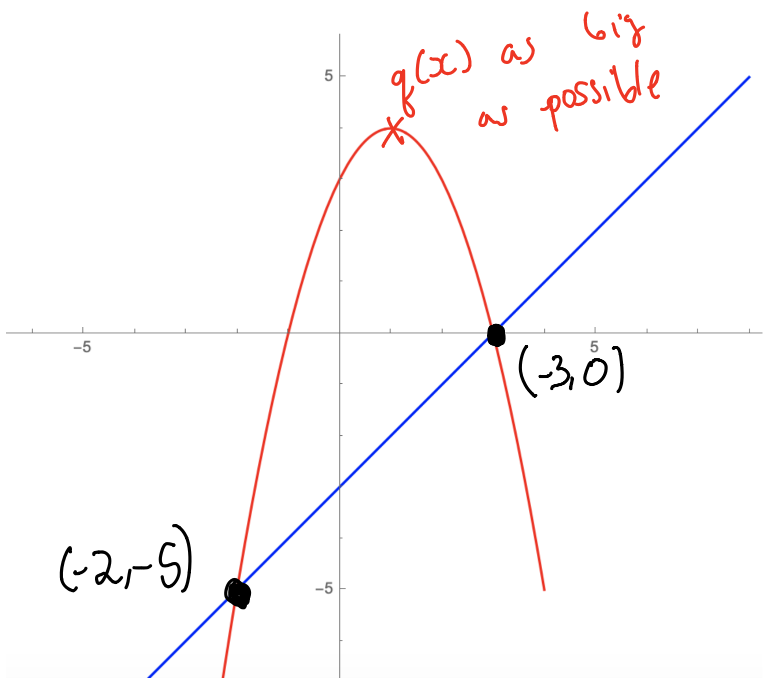


FIGURE 1. Problem 3.1

(1) For which value of x is $q(x)$ as big as possible?
(2) Find the equation of the line which is tangent to $q(x)$ at the point that makes $q(x)$ as big as possible.
(3) Find the equation of the line that passes through $(-2,5),(-3,0)$.
(4) Compute the intersection point of those two lines.

3.1.1. *Scratch Work.*

3.1.2. *Answers.*

(1)

(2)

(3)

(4)

3.2. Cubic. Let f be the cubic:

$$f(x) = x^3 + 69x^2 - 441x - 30429$$

(*See Figure 2.*)

FIGURE 2. Problem 3.2

(1) Compute the derivative of f.
(2) Find the solutions to $f'(x) = 0$.
(3) Find the coordinates of the local min and the local max.

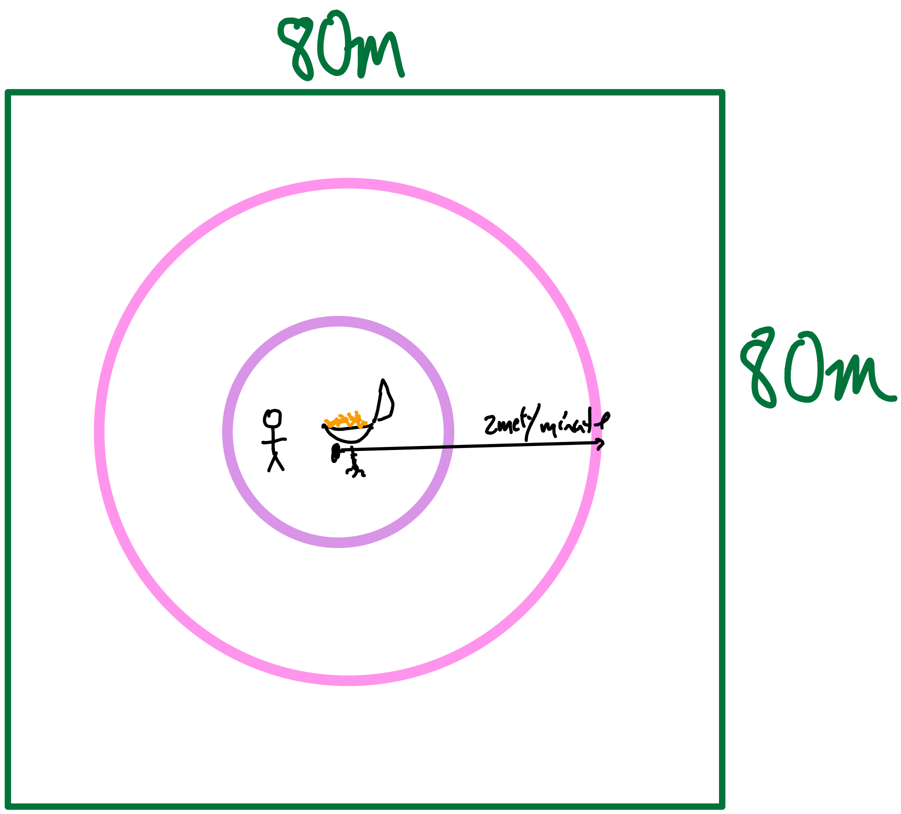
3.2.1. *Scratch Work.*

3.2.2. *Answers.*

(1)	
(2)	
(3)	

3.3. Disks and Squares. We're going to grill in the middle of a square park of side length 80 m. The aroma of the food forms a disk around us, and the radius of that disk is increasing at a rate of 2 meters per minute.

(1) After t minutes, what is the area of the disk where you can smell what we're cooking?
(2) How quickly is the area of the disk growing after 15 minutes?
(3) Tricky: How long will it take until everybody in the park can smell the food?



3.3.1. *Scratch Work.*

3.3.2. *Answers.*

(1)

(2)

(3)