Office Hours!

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Remember: Logarithms

$\log(y)$ is how may tens you multiply together to get y

$$10^{\log(y)} = y$$

$\log(100) =? = 2$ because $10^2 = 100$

You Try It: $\log(100,000) =?$

A$= 2$ B$= 3$ C$= 4$ D$= 5$ E$= 6$ $\boxed{\text{D}}$

A Few More:

$\log(0.001) =?$

A$= 3$ B$= 0$ C$= 0.001$ D$= -2$ E$= -3$ $\boxed{\text{E}}$

$\log(100 \times 1000) =?$

A$= 6$ B$= 5$ C$= 3$ D$= 9$ E$= -5$ $\boxed{\text{B}}$

$\log(100/1000) =?$

A$= -1$ B$= 0$ C$= 1$ D$= -3$ E$= -5$ $\boxed{\text{A}}$

How confused are you ?

A = not at all B = a bit C = a lot D = completely

How To Find Logarithms

(1) Use a calculator: efficient but not good for learning

(2) Use the graph on page 290 of textbook

(3) Use table of logarithms on page 289 of textbook

Our goal: use (2) and (3) to understand:

logs, functions and inverse functions.

Our main use of logs: solving certain kinds of equation.
Mistakes will follow unless you practice finding logs the old fashioned way.

$$\begin{tikzcd} y = \log(x) \arrow[r, bend left, "\text{antilog}"] & 10^y = x \arrow[l, bend left, "\log"] \end{tikzcd}$$

log is the inverse function of antilog

antilog is another name for the 10-to-the-power function:

$$\text{antilog}(y) = 10^y.$$

x	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
1.0	0.0000	0.0043	0.0086	0.0128	0.0170	0.0212	0.0253	0.0294	0.0334	0.037
1.1	0.0414	0.0453	0.0492	0.0531	0.0569	0.0607	0.0645	0.0682	0.0719	0.075
1.2	0.0792	0.0828	0.0864	0.0899	0.0934	0.0969	0.1004	0.1038	0.1072	0.110
1.3	0.1139	0.1173	0.1206	0.1239	0.1271	0.1303	0.1335	0.1367	0.1399	0.143
1.4	0.1461	0.1492	0.1523	0.1553	0.1584	0.1614	0.1644	0.1673	0.1703	0.173
1.5	0.1761	0.1790	0.1818	0.1847	0.1875	0.1903	0.1931	0.1959	0.1987	0.201
1.6	0.2041	0.2068	0.2095	0.2122	0.2148	0.2175	0.2201	0.2227	0.2253	0.227
1.7	0.2304	0.2330	0.2355	0.2380	0.2405	0.2430	0.2455	0.2480	0.2504	0.252
1.8	0.2553	0.2577	0.2601	0.2625	0.2648	0.2672	0.2695	0.2718	0.2742	0.276
1.9	0.2788	0.2810	0.2833	0.2856	0.2878	0.2900	0.2923	0.2945	0.2967	0.298
2.0	0.3010	0.3032	0.3054	0.3075	0.3096	0.3118	0.3139	0.3160	0.3181	0.320
2.1	0.3222	0.3243	0.3263	0.3284	0.3304	0.3324	0.3345	0.3365	0.3385	0.340
2.2	0.3424	0.3444	0.3464	0.3483	0.3502	0.3522	0.3541	0.3560	0.3579	0.359
2.3	0.3617	0.3636	0.3655	0.3674	0.3692	0.3711	0.3729	0.3747	0.3766	0.378
2.4	0.3802	0.3820	0.3838	0.3856	0.3874	0.3892	0.3909	0.3927	0.3945	0.396
2.5	0.3979	0.3997	0.4014	0.4031	0.4048	0.4065	0.4082	0.4099	0.4116	0.413
2.6	0.4150	0.4166	0.4183	0.4200	0.4216	0.4232	0.4249	0.4265	0.4281	0.429
2.7	0.4314	0.4330	0.4346	0.4362	0.4378	0.4393	0.4409	0.4425	0.4440	0.445
2.8	0.4472	0.4487	0.4502	0.4518	0.4533	0.4548	0.4564	0.4579	0.4594	0.460
2.9	0.4624	0.4639	0.4654	0.4669	0.4683	0.4698	0.4713	0.4728	0.4742	0.475
3.0	0.4771	0.4786	0.4800	0.4814	0.4829	0.4843	0.4857	0.4871	0.4886	0.490
3.1	0.4914	0.4928	0.4942	0.4955	0.4969	0.4983	0.4997	0.5011	0.5024	0.503
3.2	0.5051	0.5065	0.5079	0.5092	0.5105	0.5119	0.5132	0.5145	0.5159	0.517
3.3	0.5185	0.5198	0.5211	0.5224	0.5237	0.5250	0.5263	0.5276	0.5289	0.530
3.4	0.5315	0.5328	0.5340	0.5353	0.5366	0.5378	0.5391	0.5403	0.5416	0.542

Use table forwards to find logs: $\log(2.73) \approx 0.4362$

Use table backwards to find powers of 10: $10^{0.2923} \approx 1.96$

x	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
1.0	0.0000	0.0043	0.0086	0.0128	0.0170	0.0212	0.0253	0.0294	0.0334	0.037
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1.6	0.2041	0.2068	0.2095	0.2122	0.2148	0.2175	0.2201	0.2227	0.2253	0.227
1.7	0.2304	0.2330	0.2355	0.2380	0.2405	0.2430	0.2455	0.2480	0.2504	0.252
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2.2	0.3424	0.3444	0.3464	0.3483	0.3502	0.3522	0.3541	0.3560	0.3579	0.359
2.3	0.3617	0.3636	0.3655	0.3674	0.3692	0.3711	0.3729	0.3747	0.3766	0.378
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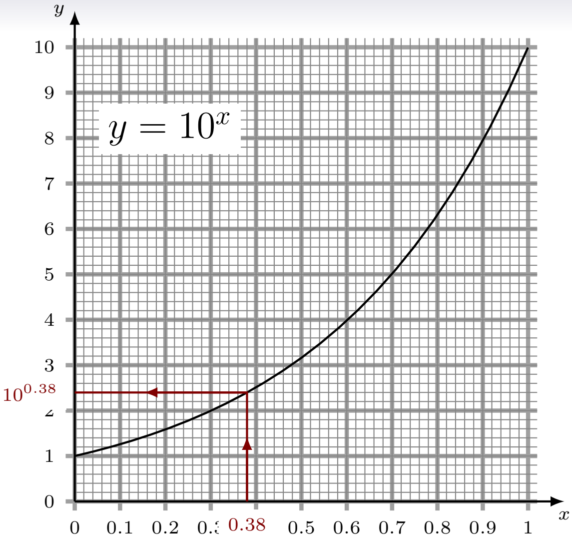
3.4	0.5315	0.5328	0.5340	0.5353	0.5366	0.5378	0.5391	0.5403	0.5416	0.542

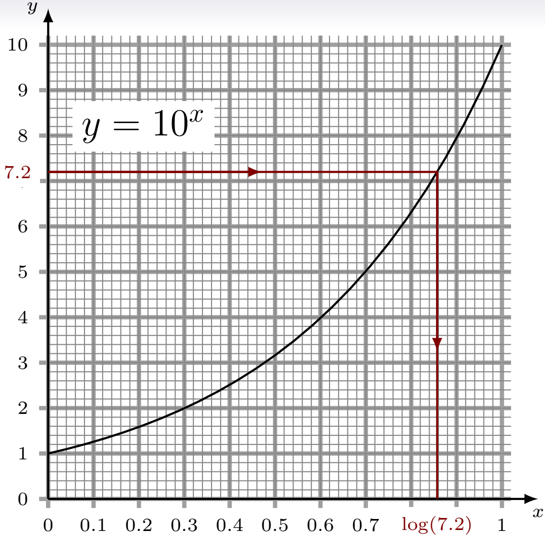
$\log(2.372)$ is

A ≈ 0.3729 B ≈ 0.3747 C ≈ 0.3766 $\boxed{\text{B}}$



Use the graph and a ruler to find $10^{0.38}$:

A$\approx$ 2.1 B$\approx$ 2.2 C$\approx$ 2.3 D$\approx$ 2.4 E$\approx$ 2.6 $\boxed{\text{D}}$



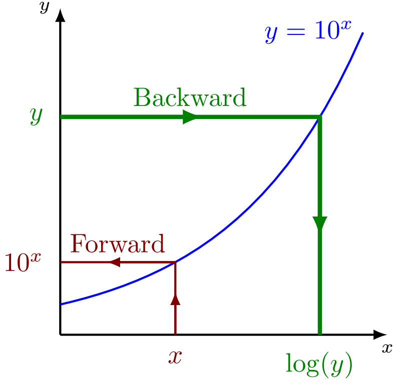
Use the graph backwards and a ruler to find log(7.2):

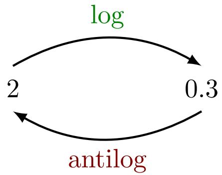
A≈ 0.81 B≈ 0.82 C≈ 0.83 D≈ 0.84 E= 0.86 $\boxed{\text{E}}$

Do you have questions about using the graph of $y = 10^x$?
A = lots B = a few C = one D = none E = move on already!

Do you have questions about using the table of logs on page 289 ?
A = lots B = a few C = one D = none E = move on already!

Using the Graph Forward and Backward





Summary

- $\log(y)$ is how many tens you multiply to get y
- log is the inverse function to antilog.
- So: method to find $x = \log(y)$ is the opposite of method to find $y = 10^x$

Use the graph $y = 10^x$ forwards to find 10^x. This means to find $10^{0.38}$ start at $x = 0.38$ on x-axis. Using graph as intended.

Use graph backwards to find $\log(y)$. This means to find $\log(7.2)$ start at $y = 7.2$ on the y-axis.

Same deal with log tables. Use the tables forwards to find $log(2.73)$. This means find 2.73 in the left column/top row and the answer $log(2.73) = 0.4362$ is in the middle of the table.

Use the table backwards to find $10^{0.2923}$. This means hunt through the middle of the table until you find 0.2932 then $10^{0.2932} = 1.96$ is in the left column/top row.

Key Fact Of Logs

First Law of Logs $\qquad \log(a \times b) = \log(a) + \log(b)$

This means logs convert multiplication into addition.

Example: $\qquad \log(100 \times 1000) = \log(100) + \log(1000) = 2 + 3 = 5$

It is easy to understand why the first law works:

log(a) = (how many 10's you multiply to get a)
log(b) = (how many 10's you multiply to get b)

THEREFORE multiplying ALL these 10s gives $a \times b$

CONCLUDE $\log(a \times b)$ is this number of 10s: that is, $\log(a) + \log(b)$.

Does this make sense to you?

A = Completely $\quad$ B = mostly $\quad$ C = a glimmer $\quad$ D = no!

Consquences of the Key Fact

We are told: $\log(2) \approx 0.3$ (from table page 289)

$$\begin{aligned}\log(20) &= \log(10\times 2)\\ &= \log(10)+\log(2) && \text{we know } \log(10)=1\\ &\approx 1+0.3\\ &\approx 1.3\end{aligned}$$

Use this method to find $\log(200)$

A= 30 B= 3 C= 2.3 D= 30 $\boxed{\text{C}}$

A few more

We are still told $\log(2) \approx 0.3$

Find $\log(0.002)$

A$= -3.3$ B$= -2.3$ C$= -2.7$ D$= -3.7$ $\boxed{\text{C}}$

Find $\log(2 \times 10^x)$

A$= 2x$ B$= 2 + x$ C$= x\log(2)$ D$= 10x + \log(2)$ E$= x + \log(2)$

$\boxed{\text{E}}$

A Trick!

The graph and the table can both be used to find logs of numbers between 1 and 10.

To find the log of ANY number, we move the decimal point:

$$\boxed{\log(10^{n} \times x) = n + \log(x)}$$

Example:

$$\log(275.67) = \log(10^{2} \times 2.7567) = 2 + \underbrace{\log(2.7567)}_{\text{look this up!}}$$

Its called the MOVING DECIMAL POINT TRICK because 2 is how many places you need to move the decimal point of 275.67 to obtain a number between 1 and 10.

You Try It!

Use the log tables on page 289 to find $\log(5.73)$

A = I have done it B = I am confused C = I don't have the texthere

$\boxed{\log(5.73) \approx 0.7582}$ Did you get this?

A = YES B = No

What is $\log(57.3) \approx$?

A= 7.582 B= $10 + 0.7582$ C= $1 + 0.7582$ D= OTHER $\boxed{\text{C}}$

Inverses!

logs are "opposite" of exponents (inverse function of antilog)
So every fact about exponents corresponds to a fact about logs:

	laws of exponents	corresponding law of logs
(1)	$10^a \times 10^b = 10^{a+b}$	$\log(xy) = \log(x) + \log(y)$
(2)	$10^0 = 1$	$\log(1) = 0$
(3)	$10^{-a} = 1/10^a$	$\log(1/x) = -\log(x)$
(4)	$(10^a)^p = 10^{ap}$	$\log(x^p) = p\log(x)$
(5)	$10^a/10^b = 10^{a-b}$	$\log(x/y) = \log(x) - \log(y)$

Example: $\log(x^a/y^b) =?$

$$A = a\log(x)/(b\log(y)) \qquad B = a\log(x) + b\log(y)$$
$$C = a\log(x) - b\log(y) \quad D = (a+\log(x)) - (b+\log(y)) \quad \boxed{\text{C}}$$

Rule (4): $\log(x^p) = p\log(x)$

Explanation of (4)

$\log(a \times a) = \log(a) + \log(a) = 2\log(a)$
$\log(a \times a \times a) = \log(a) + \log(a) + \log(a) = 3\log(a)$

In general: the number of tens you multiply to get x^p is p times as many tens as you multiply to get x.

What is $\log(\sqrt{x^7})$?

$A = 7 + \log(x)$ $\quad B = (7/2) + \log(x)$ $\quad C = 7/2$ $\quad D = 7/2\log(x)$ $\quad \boxed{\text{D}}$