

# Welcome To Math 34A!

## Differential Calculus

Instructor:

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South Hall 6701

Office Hours:

T R 11-11:50, T 3:45-4:35 Details on Gauchospace.

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& Nathan Schley

Please do not distribute outside of this course.

# Warm-up

How many times do we need to double 1 to get the following numbers?

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- 4

# Warm-up

How many times do we need to double 1 to get the following numbers?

- 4    2

# Warm-up

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- 4
- 8

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# Straight Lines (§6.1)

Calculus is about **derivatives** (Math 34A) and **integrals** (Math 34B).

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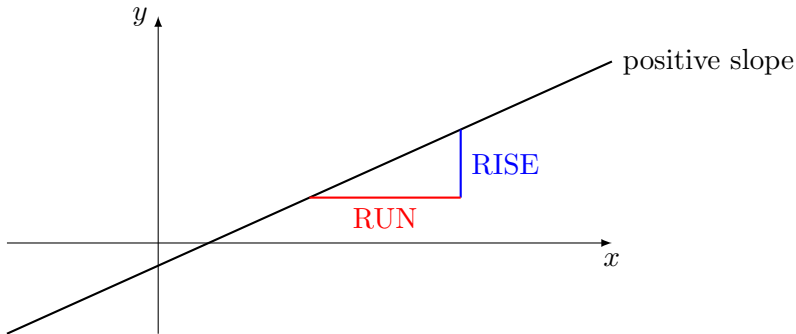
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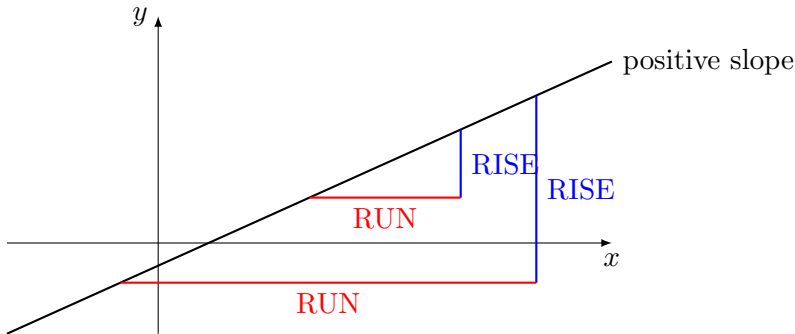
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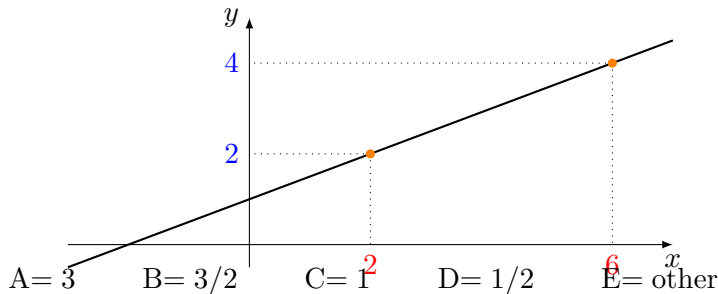
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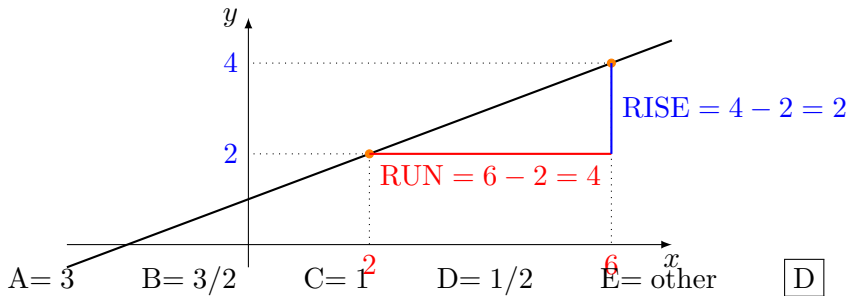
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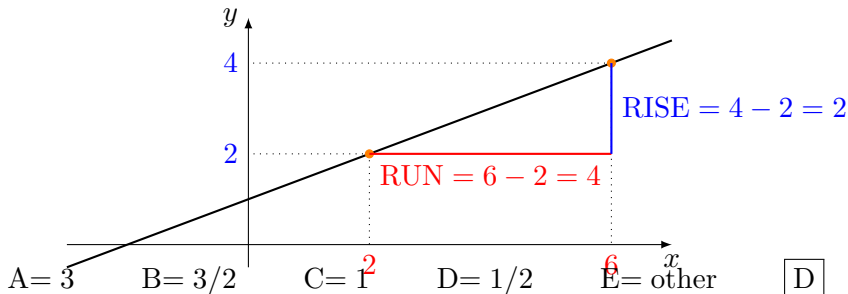


slope = # units **UPWARDS** you move for each unit you move  
**TO THE RIGHT**

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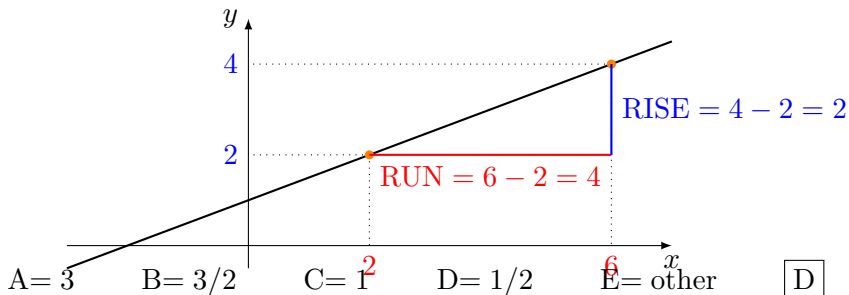


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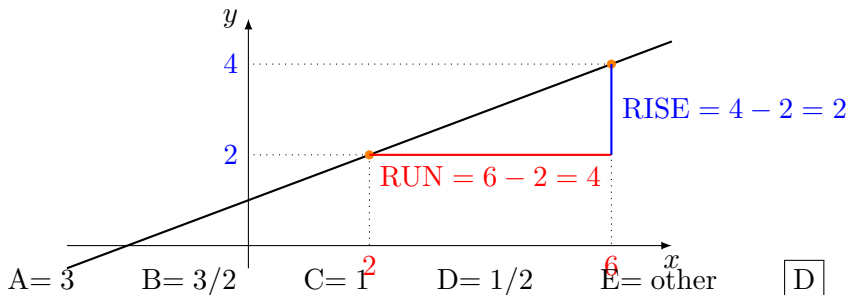


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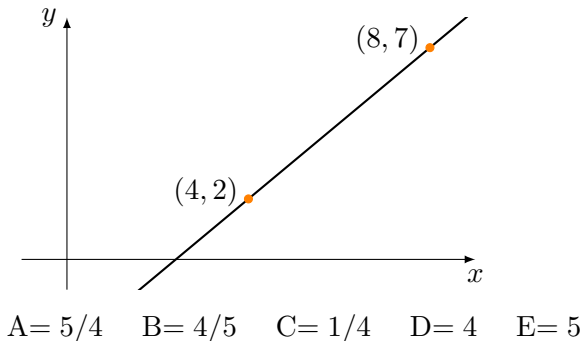
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Idea:  $\text{RISE} = \text{slope} \times \text{RUN}$  So if  $\text{RUN} = 1$  then  $\text{RISE} = \text{slope}$ .

A 10% gradient on a mountain road is a **slope** of 1/10. It means for every 10 feet you move horizontally you go up (or down) 1 foot

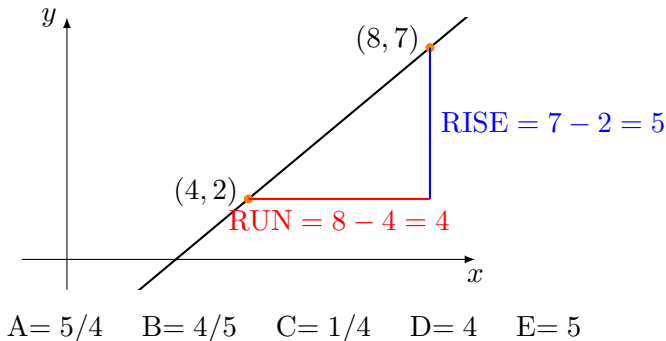
# Examples (page 2)

**2.** What is the slope here:



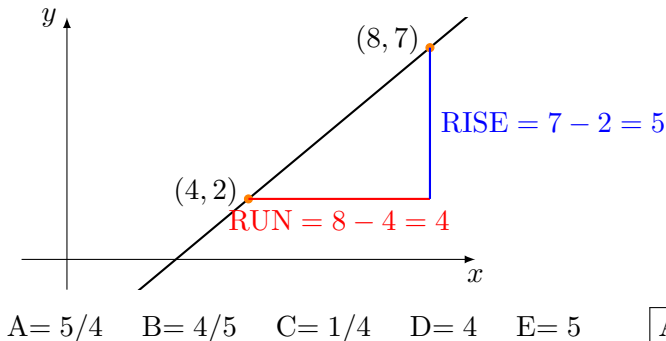
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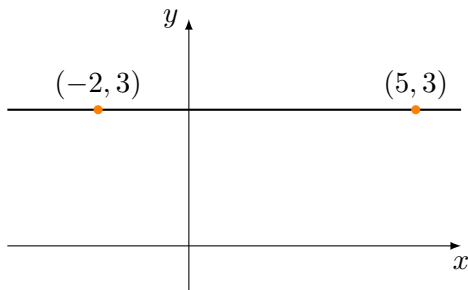
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**3.** What is the slope here:



$A = 0$

$B = 7$

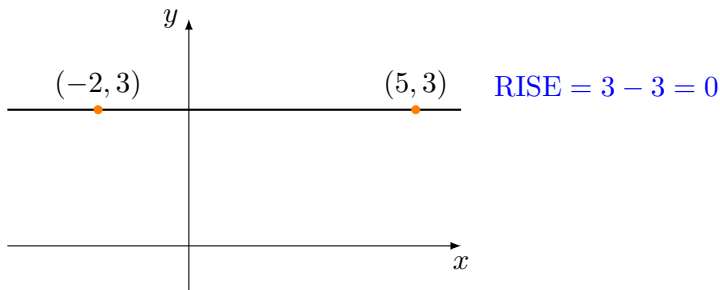
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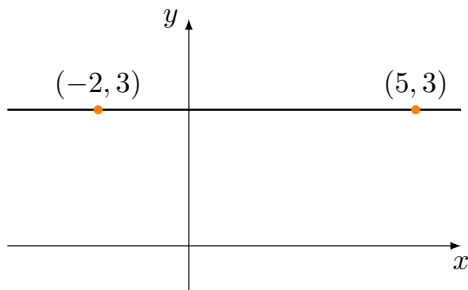


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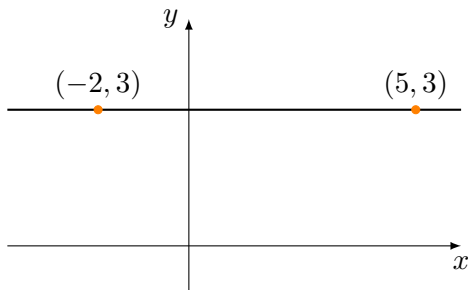
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$$\text{slope} = \frac{0}{\text{something}} = 0$$

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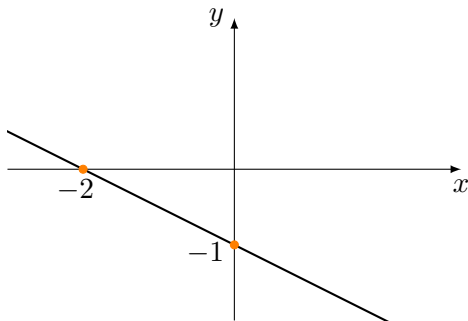
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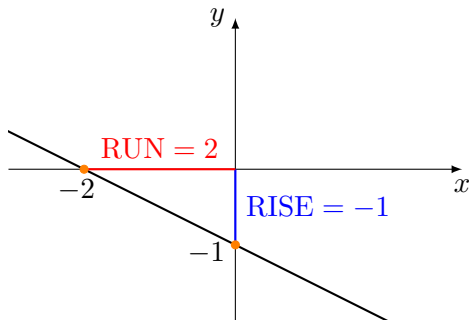
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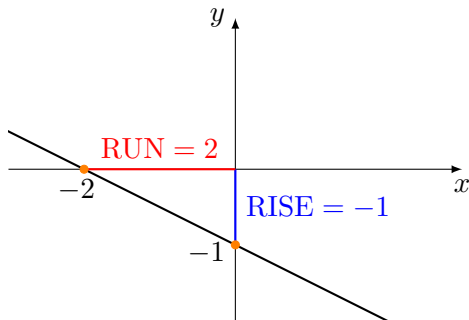
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# General Case

- 5.** A line goes through two points:  $(x_0, y_0)$  and  $(x_1, y_1)$ . Find the slope of this line. Draw a picture!

$$A = y_1 - y_0 \quad B = (y_1 - x_1)/(y_0 - x_0)$$

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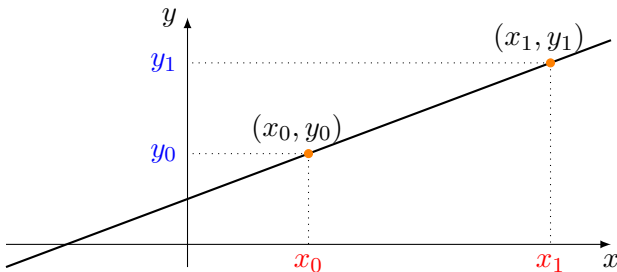
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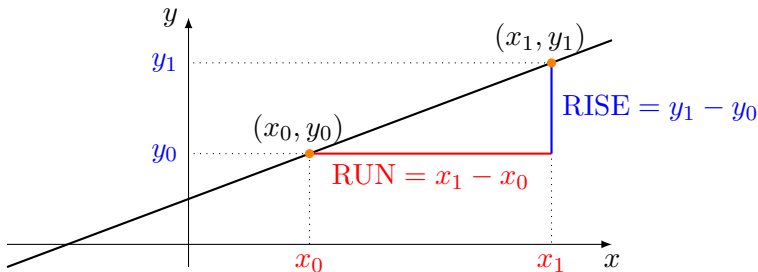
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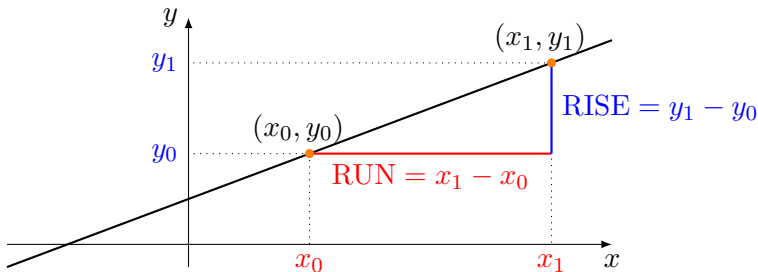
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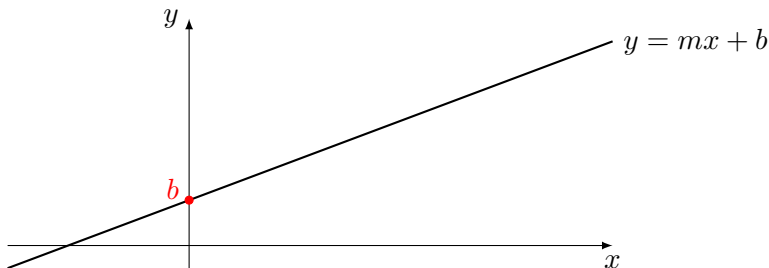
$$\text{Slope} = \frac{\text{RISE}}{\text{RUN}} = \frac{y_1 - y_0}{x_1 - x_0}$$

# The Equation of a Line

## The Slope Intercept Form

The **slope intercept** equation of a straight line is

$$y = mx + b.$$



$m$  = the **slope**. CRUCIAL for calculus.

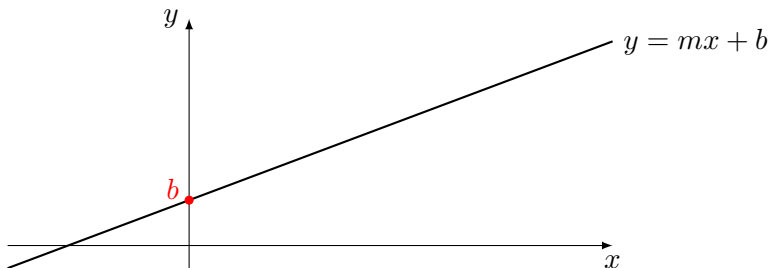
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WHY? Because when you plug in  $x = 0$ , you get  $y = b$ .

# Example

6. Find the equation of the line  $y = mx + b$  through the points  $(1, 3)$  and  $(7, 5)$ .

Plan: Find  $m$ , then find  $b$ .

- What is  $m$ ?

$$A = 1 \quad B = 3 \quad C = 5 \quad D = 1/3 \quad E = 2$$

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So  $y = \frac{1}{3}x + b$ . What is  $b$ ? Plug in either point!

- What do you get for  $b$ ?

$$A = 1/3 \quad B = 4/3 \quad C = 7/3 \quad D = 8/3 \quad E = 10/3$$



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# You Try It

- 7.** A line has slope  $1/2$  and goes through the point  $(2, 5)$ .  
What is the  $y$ -coordinate of the point on this line where  $x = 6$ ?

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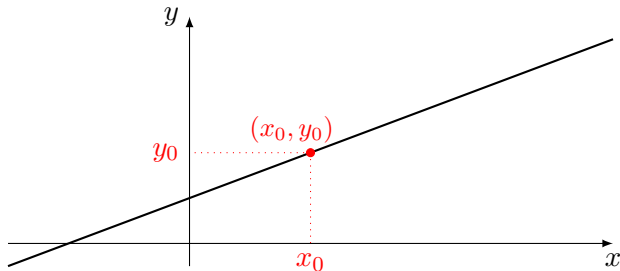
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# Another Equation of a Line

## The Point-Slope Form

The **point slope** equation of a straight line is

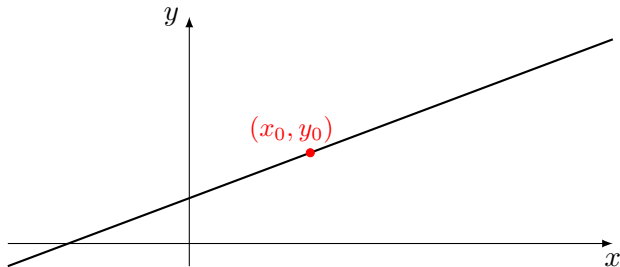
$$y = y_0 + m(x - x_0) .$$



$m$  = the **slope**. Still CRUCIAL for calculus.

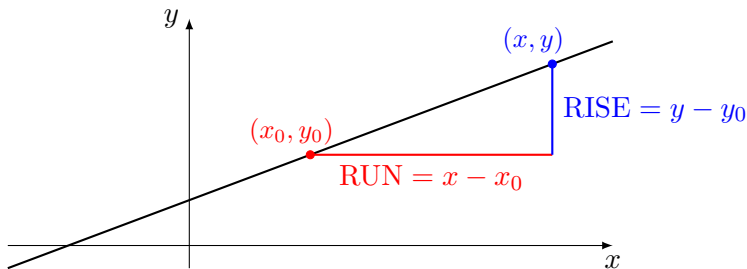
$(x_0, y_0)$  = any point on the line.

# Why Does This Work?



$(x, y)$  lies on the line exactly when

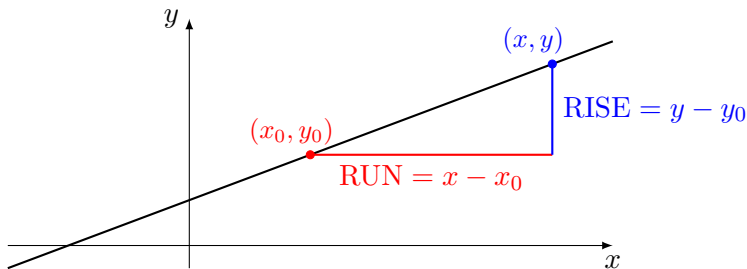
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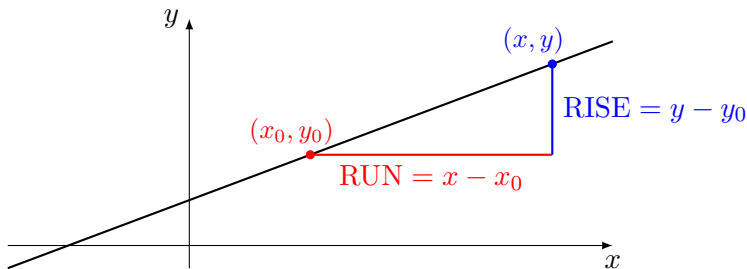


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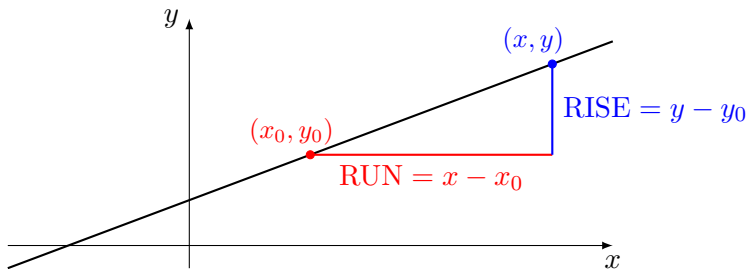
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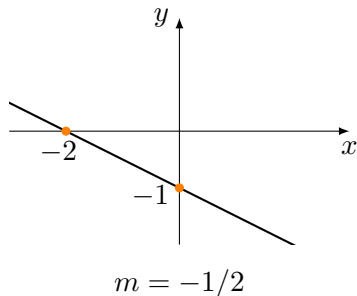
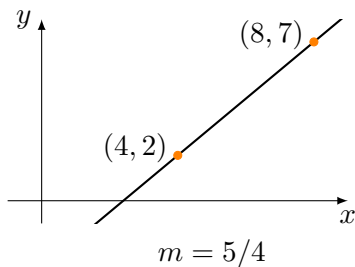
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$$y - y_0 = m(x - x_0)$$

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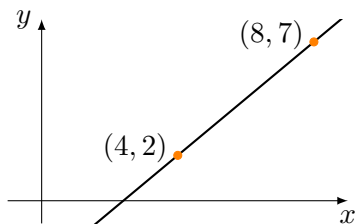
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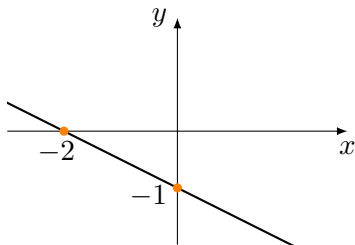
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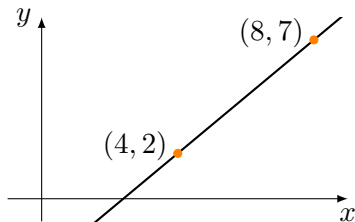
$$y - 2 = \frac{5}{4}(x - 4)$$



$$m = -1/2$$

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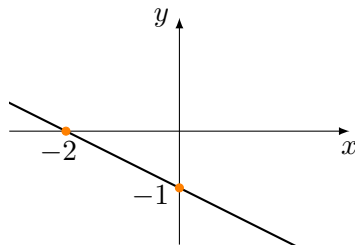
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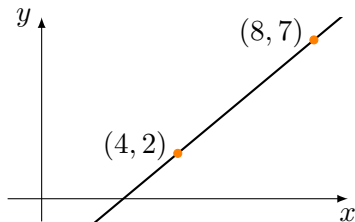
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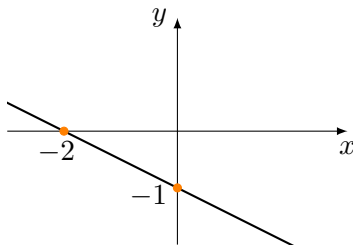
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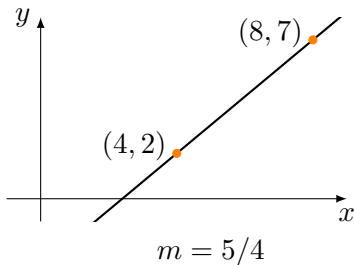


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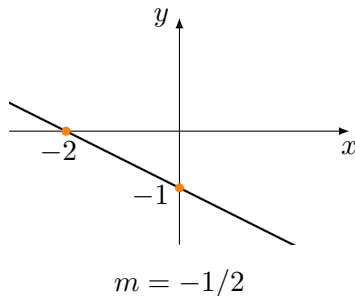
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$$y = -\frac{1}{2}x - 1$$



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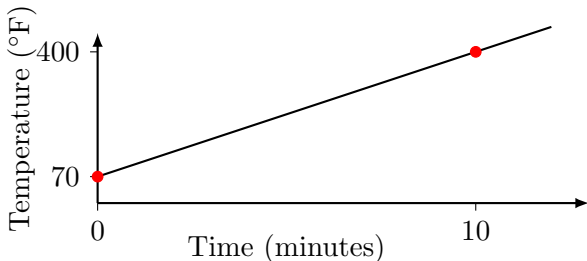
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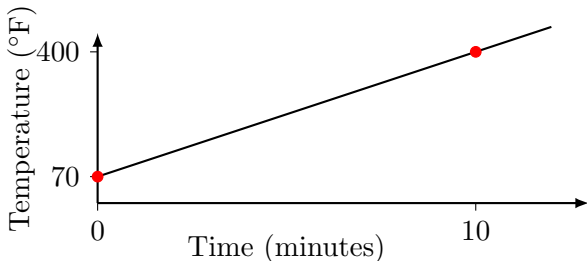


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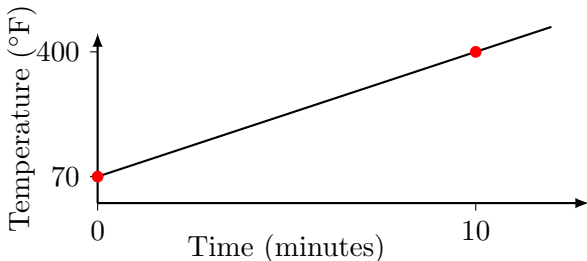
A = 70    B = 10    C = 40    D = 33    E = Other

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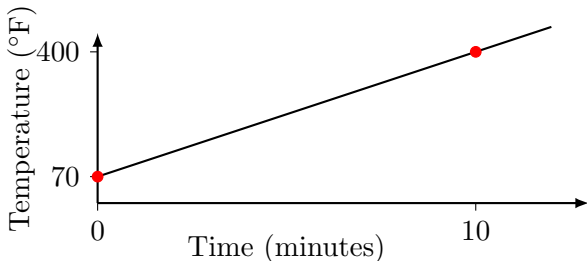
D

# And...?

Yes, but what's this got to do with calculus?

**Derivatives** are about **rate of change** and that is what **slope** is!

**Example:** This graph shows the temperature in an oven as it heats up:



**9.** How quickly (in  $^{\circ}\text{F}/\text{min}$ ) is the oven heating up?

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# One More Example

- 10.** Where does the line  $y = 1 + x$  cross the line  $y = 3 - x$ ?  
Find both the  $x$  and  $y$  coordinates of the crossing point.

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Plan:

1. **Draw a picture!** showing two straight lines crossing.
2. Solve the **two simultaneous equations**
3. **THINK** why this gives the answer!

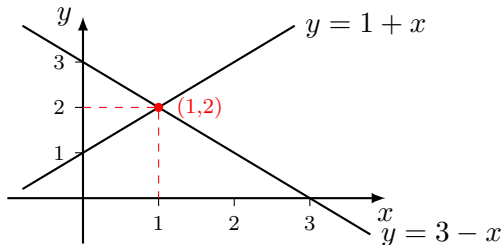


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That's it. Thanks for being here.

