Math 4B **Name:** _____
Summer Session B
Midterm
21 August 2020

Note: every problem has its own page, regardless of how long the solution would be.

1. (10 points each) Find the general solution of the given differential equation. Write your answer in explicit form.

   (a) $y' = t^2 e^y$

   (b) $2u'' + 4u = 0$

   (c) $y' = \dfrac{x^{-6}(x-1)}{5y^4}$

2. (10 points each) Solve the given initial value problem.

   (a) $-w'' + 10w' - 25w = 0, \quad w(0) = 1, \quad w'(0) = -1$

   (b) $xy' + (x+1)y = x^2 e^{-x}, \quad x > 0, \quad y(3) = 0$

3. Given that $y_1(t) = t^{-3}$ is a solution of

$$t^2 y'' + 2ty' - 6y = 0, \quad t > 0,$$

   (a) (15 points) Use the reduction of order method to find a second solution of the form $y_2(t) = t^k$.

   (b) (5 points) Prove that y_1 and y_2 form a fundamental set of solutions.

   (c) (5 points) Find the solution which satisfies the initial conditions $y(1) = 1$ and $y'(1) = 12$.

4. (10 points) Suppose y_1 and y_2 form a fundamental set of solutions to some differential equation. Let $y_3 = y_2 + y_1$. Do you think that the pair $\{y_3, y_1\}$ would also form a fundamental set of solutions, yes or no? Explain your reasoning and thoughts. You may use theory from linear algebra and/or differential equations.