Office Hours!

Instructor:

Peter M. Garfield, `garfield@math.ucsb.edu`

Office Hours:

Mondays 2–3PM

Tuesdays 10:30–11:30AM

Thursdays 1–2PM

or by appointment

Office:

South Hall 6510

Summary of calculations with logs

[Courtesy of Daryl Cooper]

Calculate the log of the thing you want then take antilog of the result.

Example: To calculate $puppy = 17^{3.1}$

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Example: To calculate $puppy = 17^{3.1}$
(i) doggy $=$ log($puppy$)
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(iv) Now have numerical value for doggy.
(v) so $puppy$ = antilog($doggy$) is the answer.

Make sure you never jot down a number on its own. It should always be part of an equation like $\log(945 \times 32) \approx 4.48$ This way one can read and understand what is written. Otherwise you get gibberish

Write math the way I do. With words and equations. One should be able to read and understand what is on the paper without being telepathic. Imagine it is a report for your employer. In reality you are explaining it to yourself.

§7.7: Solving Exponential Eq'ns

1. Find x by solving $10^x = 5$.

A$=5$ B$=0.5$ C$=\log(5)$ D$=\log(0.5)$
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A= 5 B= 0.5 C= log(5) D= log(0.5)
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Look how I write the answer!

$\log(10^x) = \log(5)$ Take logs of both sides

$x = \log(10^x) = \log(5)$ Using $\log(a^p) = p\log(a)$ and $\log(10) = 1$

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I have written equations so I can see what each thing I write means. I can see that I've found x and so don't need to take antilog.

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How do you know?

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I have written equations so I can see what each thing I write means. I can see that I've found x and so don't need to take antilog.

How do I know when to take antilog?

How do you know? My answer: If you write the problem the way I do, so it makes sense, you can see what to do.

Examples:

Use the Fourth Law:

$$\boxed{\log(a^{x}) = x\log(a)}$$

Slogan: Logs bring exponents down to ground level.

2. Solve $3^{x} = 7$

A$= \log(7/3)$ B$= \log(7) - \log(3)$ C$= \log(7) + \log(3)$

D$= \log(3)/\log(7)$ E$= \log(7)/\log(3)$

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Look how I write the answer:

$\log(3^x) = \log(7)$ Take logs of both sides

$x\log(3) = \log(3^x) = \log(7)$ Using $\log(a^p) = p\log(a)$

So: $x = \log(7)/\log(3)$

Examples:

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3. Solve $7^{x+2} = 30$.

$$\text{A}= \frac{\log(30) - 2\log(7)}{\log(7)} \qquad \text{B}= \frac{\log(30)}{\log(7)} - 2 \qquad \text{C}= \frac{\log(30) - \log(49)}{\log(7)}$$

$$\text{D}= \frac{\log(30/49)}{\log(7)} \qquad \text{E}\approx -0.25213$$

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All are correct!

Examples:

Use the Fourth Law:

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4. Solve $7 \times 3^y = 2^{4y+3}$

A= $\dfrac{3\log(2)-\log(7)}{\log(3)-4\log(2)}$ B= $\dfrac{3\log(2)}{7\log(3)}$ C= $\dfrac{3\log(2)}{7\log(3)-4\log(2)}$

D= $\dfrac{7\log(3)-4\log(2)}{3\log(2)}$ E=none of the above

Examples:

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D= $\dfrac{7\log(3) - 4\log(2)}{3\log(2)}$ E=none of the above

$\boxed{\text{A}}$

Compound Interest

At end of each year a bank pays 7% interest into your account. Initially have \$10,000 in account. How much after 10 years?

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After 1 year: $\$10,000 \times 1.07 = \$10,700$

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After 2 years: $\$10,700 \times 1.07 = \$10,000 \times 1.07 \times 1.07 = \$11,449$

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Each year what you had before is multiplied by 1.07. Thus compound interest.

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$$\$10,000 \times \underbrace{1.07 \times 1.07 \times \cdots \times 1.07}_{\text{10 times}} = 10,000 \times (1.07)^{10} \approx \boxed{\$20,000}$$

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Conclusion: Money approximately doubles in 10 years!
So in 20 years multiplies by 4, in 30 years by 8,...

General Compound Interest

If the interest rate is r%, then each year money multiplies by

$$m = 1 + \frac{r}{100}.$$

If you start with an initial amount A of money then after t years you have

$$A \times m^t = A \times \left(1 + \frac{r}{100}\right)^t$$

5. If you invest \$1000 at 14% interest, how much will you have 5 years later? (Guess!)

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How much is this?

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After 5 years, you have

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How much is this? Smart way: 14% in 1 year ≈ 7% per year for 2.

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If the interest rate is $r\%$, then each year money multiplies by

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If you start with an initial amount A of money then after t years you have

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6. If you invest $\$1,000$ at 14% interest, how many years until you have $\$7,000$?

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§7.9: Population Growth

Assume each generation of bunnies has 3 times as many bunnies as previous one. Initially have 100 bunnies. How many bunnies after n generations?

$$\text{A}= 100 \times 3n \quad \text{B}= 100 + 3n \quad \text{C}= 100(1 + 3n)$$
$$\text{D}= 100^{3n} \quad \text{E}= 100 \times 3^n$$

Answer: $\boxed{\text{E}}$

Start with 100
After 1 generation have 100×3 bunnies

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So. . .after n generations have

$$100 \times \underbrace{3 \times 3 \times \cdots \times 3}_{n \text{ times}} = 100 \times 3^n \text{ bunnies.}$$

More Bunnies

We saw that:

- if we start with 100 bunnies, and
- the bunny population triples every generation,

then we have 100×3^n bunnies after n generations.

7. How many generations until there are $10^7 = 10,000,000$ bunnies?

A$= \log(5/3)$ B$= 5 - \log(3)$ C$= 5/\log(3)$
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Flu Outbreak

At the start of an outbreak of H1N1 flu in a large herd of cattle, there were 5 infected individuals. The numbers doubles every 3 days. How many days until there are 80 infected cows?

$$A = \log(16)/\log(2) \qquad B = \log(16/2) \qquad C = 16/\log(2)$$

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