Office Hours!

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Mondays 1–2PM
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Exercise

1. When you substitute $x = y + 3$ into $x^2 - 6x + 8$ you get...

A $= y^2 - 6y - 1$ B $= y^2 + 35$ C $= y^2 - 6y + 35$ D $= y^2 - 1$

Answer: D

2. Can you check your answer to the previous question?

Hint: Plug in, say, $y = 1$. What is x?

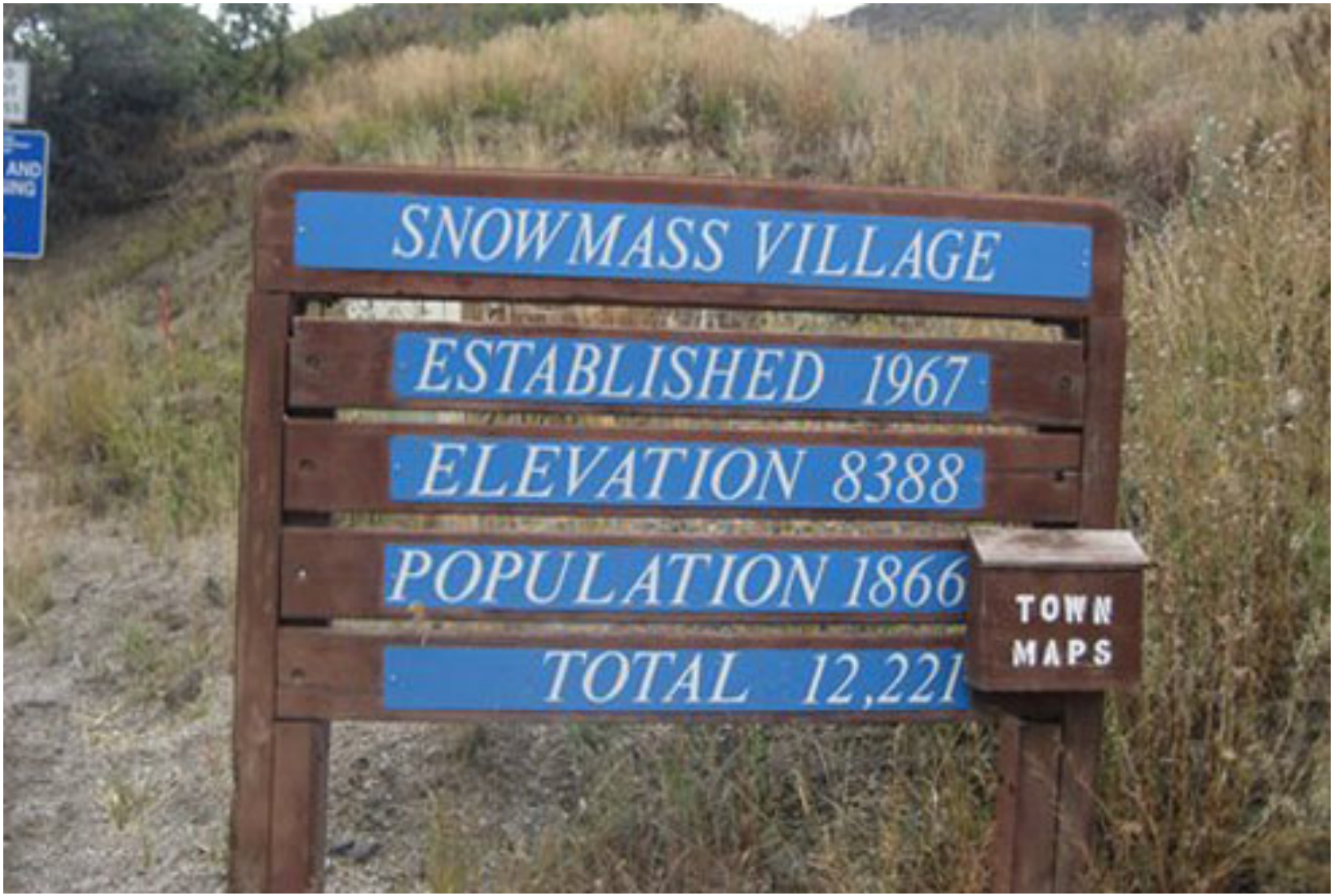
When $y = 1$, $x = 4$ so $x^2 - 6x + 8 = 4^2 - 6(4) + 8 = 0$.

The other expressions are...

A $= y^2 - 6y - 1 = -6$ B $= y^2 + 35 = 36$

C $= y^2 - 6y + 35 = 30$ D $= y^2 - 1 = 0$

Units: A Meaningless Calculation



Units: A Meaningless Calculation

Rule: Only add or subtract things measured in same units

- 3 meters + 7 inches is NOT 10 of anything
- 2 days + 5 hours $\neq 7$
- 3 nickels + 2 dimes $\neq 5$

BUT! You can multiply or divide things in different units:

$$\boxed{\text{average speed} = (\text{distance gone})/(\text{time taken})}$$

(50 miles)/(1 hour) = 50 (miles/hours) = 50 miles per hour = 50 mph
You must multiply or divide the units too !
miles divided by hours is miles per hour

When a problem has mixed units like miles and feet or years and seconds decide what units you will use (like miles and seconds) and convert everything into those units, or

SUFFER

Units conversions

3. How fast does your hair grow. . . in mph?

A= 10^{-3} B= 10^{-4} C= 10^{-5} D= 10^{-6} E= 10^{-8} ???

I don't know either.

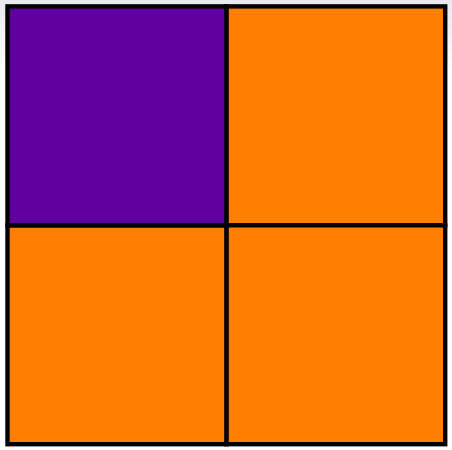
4. How fast does your hair grow. . . in cm/month?

A=faster B= 10 C= 1 D= 1/10 E=slower C

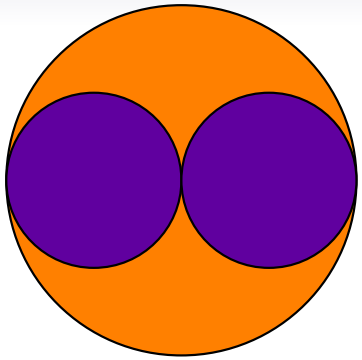
Conversions:

2.54 cm = 1 inch 12 inches = 1 foot 5280 feet = 1 mile

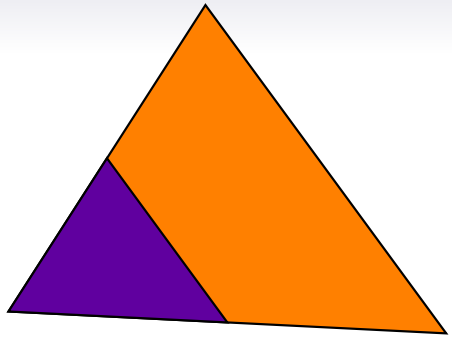
30 days = 1 month 24 hours = 1 day



The large square is 2 times the base of the small square. It has $2 \times 2 = 4$ times the area.



The large circle is 2 times the size of the small circle. It has 4 times the area.



The large triangle is 2 times the size of the small triangle. It has 4 times the area.

When you double the size of a shape the area is multiplied by 4
If you make a shape 3 times larger the area is 9 times as much
x times larger gives x^2 times as much area

area grows as the square of the linear dimensions

When you double the size of a solid object,
the volume is 8 times as much

What is going on?

An area has two dimensions : length and width.
Both of these get doubled so area is doubled twice so multiplied by 2^2

A solid object has three dimensions : length, width and height.
Each dimension is doubled so volume doubled three times : multiplied by 2^3
Make a solid object x times bigger, volume is x^3 times as much.

volume grows as the cube of the linear dimensions

Conclusion Volume and area grow at different rates
As you make an object bigger the volume gets bigger faster (cubing) than the area (only squaring). Opposite effect when you make it smaller: volume gets smaller faster than area.

Consequences!

Many important consequences read section 4.4

Why do babies get cold faster than adults?
Why can an ant pick up something weighing 10 times its own weight?
Why are humans 60 feet tall mathematically impossible?
Why can't you build a jumbo jet twice as big?
Why are my lungs crinkly?
A planet made of rock behaves like a liquid
Why can a fly walk on the ceiling, but I cant?
Why is water so dangerous to an insect but not gravity?

Paraphrasing J.B.S.Haldane Falling down a thousand yard mine shaft
A mouse walks away
A rat is killed
A man is broken
A horse splashes

5. An oil leak!

- Oil is leaking from an oil tanker at the rate of 4000 liters per hour.
- 8 liters of oil spread out over 10 square meters of ocean surface.
- A SQUARE oil slick forms.

(a) Express the length, X, of one side of the square oil slick as a function of the time t (in hours) the tank has been leaking.

(b) After how many hours will the oil slick be a square with side length 2 kilometers?

PLAN:

(i) How many liters of oil on ocean after t hours?

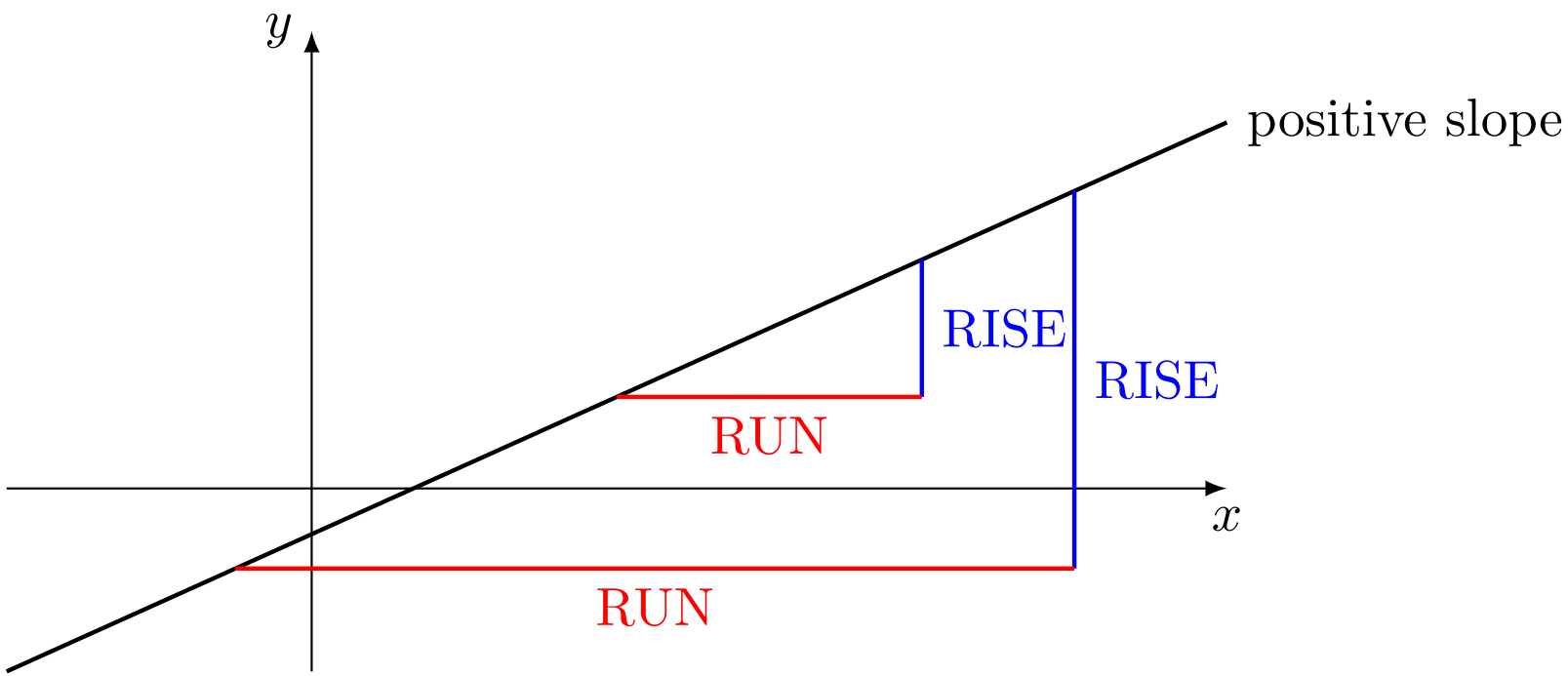
(ii) How much area does this oil cover?

Answert to (b): $\boxed{t = 800 \text{ hours}}$

Straight Lines (§6.1)

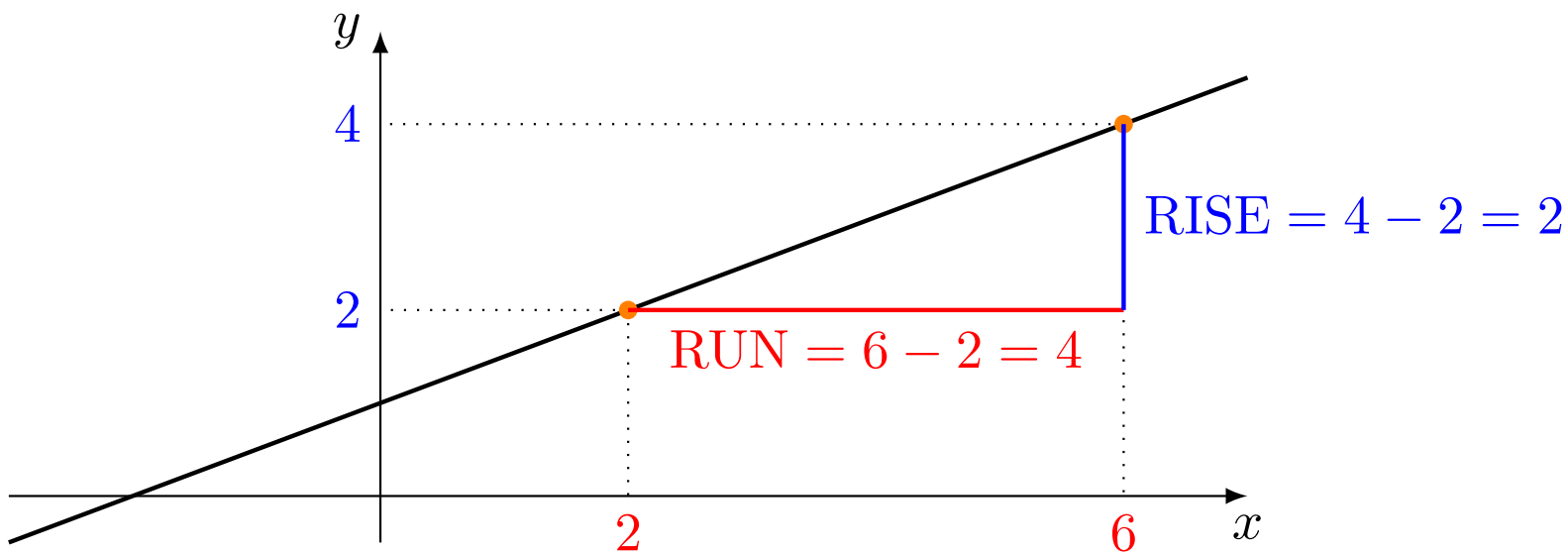
Calculus is about derivatives (Math 34A) and integrals (Math 34B).

A derivative is the slope of a line. = RISE/RUN



Examples

6. What is the slope here:



A= 3 B= 3/2 C= 1 D= 1/2 E= other **D**

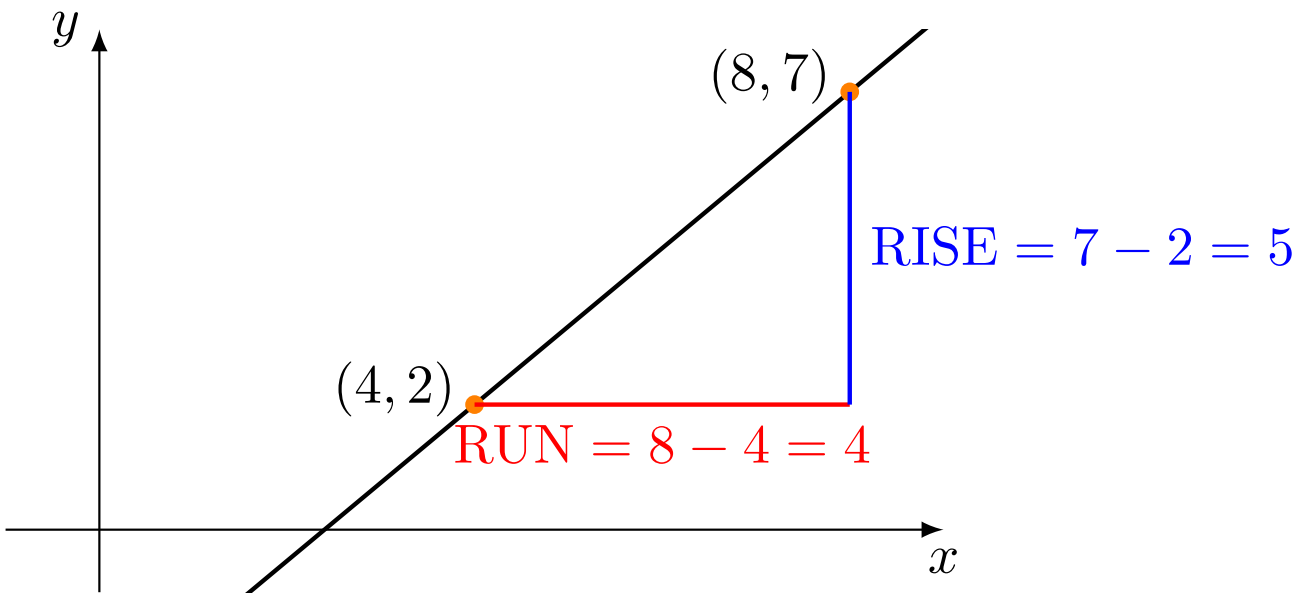
slope = # units UPWARDS you move for each unit you move TO THE RIGHT

Idea: RISE = slope × RUN So if RUN = 1 then RISE=slope.

A 10% gradient on a mountain road is a slope of 1/10. It means for every 10 feet you move horizontally you go up (or down) 1 foot.

Examples (page 2)

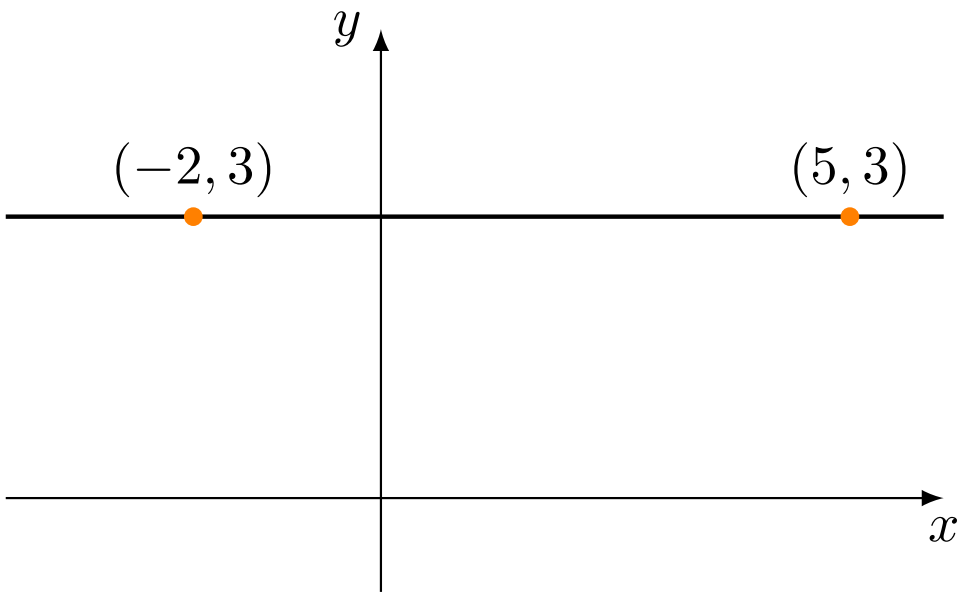
7. What is the slope here:



A= 5/4 B= 4/5 C= 1/4 D= 4 E= 5 A

Examples (page 3)

8. What is the slope here:



RISE $= 3 - 3 = 0$

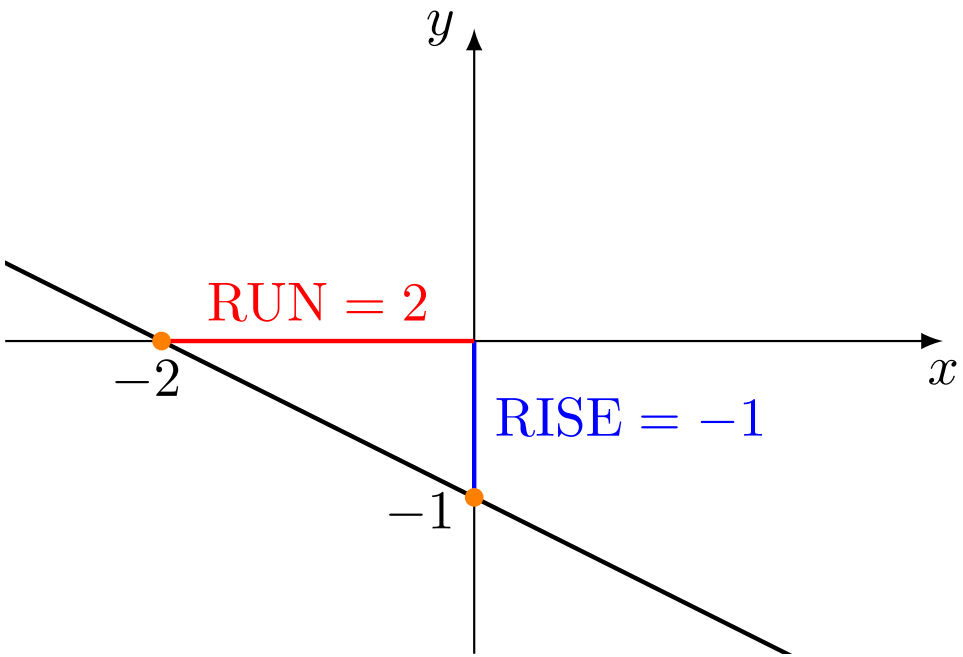
RUN $=$ who cares?

$$\text{slope} = \frac{0}{\text{something}} = 0$$

A$= 0$ B$= 7$ C$= 5/3$ D$= \infty$ E$= 3/5$ A

Examples (big finish!)

9. What is the slope here:



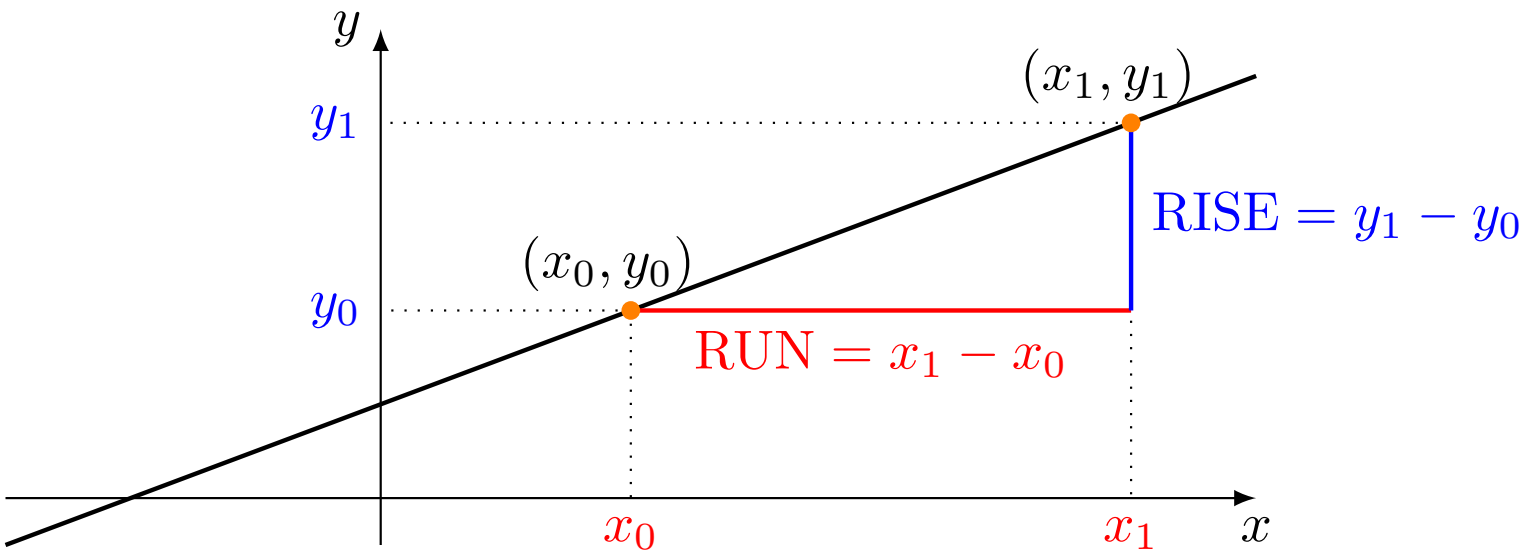
A= -1 B= 1 C= $1/2$ D= $-1/2$ E= -2 D

General Case

10. A line goes throught two points: (x_0, y_0) and (x_1, y_1). Find the slope of this line. Draw a picture!

A= $y_1 - y_0$ B= $(y_1 - x_1)/(y_0 - x_0)$ C= $(y_1 - y_0)(x_1 - x_0)$

D= $(y_1 - y_0)/(x_1 - x_0)$ E= Shirley you're joking D

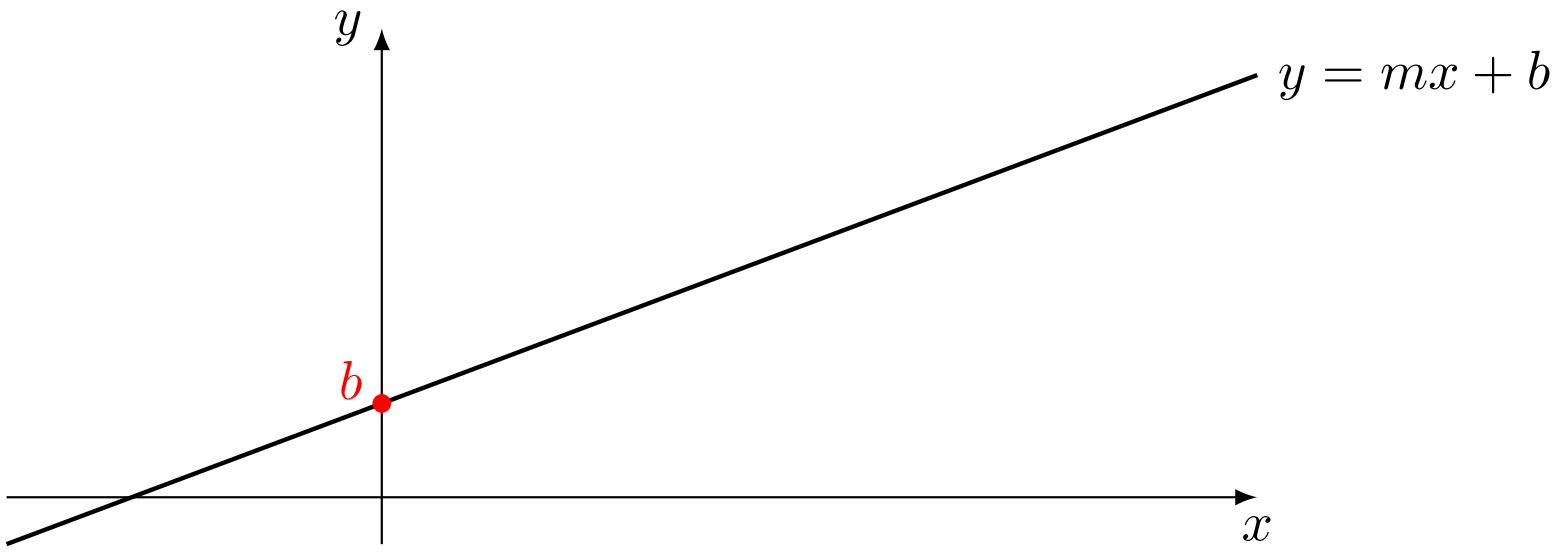


$$\text{Slope} = \frac{\text{RISE}}{\text{RUN}} = \frac{y_1 - y_0}{x_1 - x_0}$$

The Equation of a Line

The Slope Intercept Form

The **slope intercept** equation of a straight line is $\boxed{y = mx + b}$.



m = the **slope**. CRUCIAL for calculus.

b = where the line crosses the y-axis (the "y-intercept").

WHY? Because when you plug in $x = 0$, you get $y = b$.

Example

10. Find the equation of the line $\boxed{y = mx + b}$ through the points $(1, 3)$ and $(7, 5)$.

Plan: Find m, then find b.

(a) What is m?

A= 1 B= 3 C= 5 D= 1/3 E= 2 D

So $y = \frac{1}{3}x + b$. What is b? Plug in either point!

(b) What do you get for b?

A= 1/3 B= 4/3 C= 7/3 D= 8/3 E= 10/3 D

Can we check?

You Try It

11. A line has slope $1/2$ and goes through the point $(2, 5)$. What is the y-coordinate of the point on this line where $x = 6$?

A= 3 B= 4 C= 5 D= 6 E= 7 E

Plan: 1. Find equation of the line.
2. Plug in $x = 6$ to find y.