

# Math 360

## Section 1.4 Exercises

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**In 22 and 24, describe all the elements in the cyclic subgroup of  $GL(2, \mathbb{R})$  generated by the given  $2 \times 2$  matrix.**

22.  $\begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix}$

**Answer:** Let  $A = \begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix}$ . The elements of the subgroup are:

$$A = \begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix} \quad \text{and} \quad A^2 = I_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

24.  $\begin{bmatrix} 3 & 0 \\ 0 & 2 \end{bmatrix}$

**Answer:** Let  $A = \begin{bmatrix} 3 & 0 \\ 0 & 2 \end{bmatrix}$ . The elements of the subgroup are:

$$A = \begin{bmatrix} 3 & 0 \\ 0 & 2 \end{bmatrix} \quad \text{and} \quad A^2 = I_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

27. Find the order of the cyclic subgroup of  $\mathbb{Z}_4$  generated by 3.

**Answer:** The elements are 3, 2, 1, 0, so the order is 4. (also, 3 and 4 are relatively prime, so the group cannot have less than 4 elements. I claim this intuitively; I expect we will prove it as a theorem in a later section, so I omit the proof here.)

36. Complete Table 1.4.25 (below) and use it for this problem.

a. Complete the table to give a cyclic group  $\mathbb{Z}_6$  of 6 elements (you need not prove the associative law.)

**Answer:**

$$\mathbb{Z}_6 : \begin{array}{c|c|c|c|c|c|c} + & 0 & 1 & 2 & 3 & 4 & 5 \\ \hline 0 & 0 & 1 & 2 & 3 & 4 & 5 \\ \hline 1 & 1 & 2 & 3 & 4 & 5 & 0 \\ \hline 2 & 2 & 3 & 4 & 5 & 0 & 1 \\ \hline 3 & 3 & 4 & 5 & 0 & 1 & 2 \\ \hline 4 & 4 & 5 & 0 & 1 & 2 & 3 \\ \hline 5 & 5 & 0 & 1 & 2 & 3 & 4 \end{array}$$

b. Compute the subgroups  $\langle 0 \rangle$ ,  $\langle 1 \rangle$ ,  $\langle 2 \rangle$ ,  $\langle 3 \rangle$ ,  $\langle 4 \rangle$ , and  $\langle 5 \rangle$  of the group given in part (a).

**Answer:** We will list the elements of the cyclic groups in order, by multiples of the generator.

$$\begin{array}{lll} \langle 0 \rangle = \{0\} & \langle 1 \rangle = \{1, 2, 3, 4, 5, 0\} & \langle 2 \rangle = \{2, 4, 0\} \\ \langle 3 \rangle = \{3, 0\} & \langle 4 \rangle = \{4, 2, 0\} & \langle 5 \rangle = \{5, 4, 3, 2, 1, 0\} \end{array}$$

c. Which elements are generators for  $\mathbb{Z}_6$ ?

**Answer:** 1 and 5.

d. Following is a list of which cyclic groups are contained in other cyclic groups.  $\mathbb{Z}_6 = \langle 1 \rangle =$   
 $\langle 5 \rangle \left\{ \begin{array}{l} \geq \langle 2 \rangle = \langle 4 \rangle \\ \geq \langle 3 \rangle \end{array} \right\} \geq \langle 0 \rangle$

1.