Welcome To Math 34A! Differential Calculus

Instructor:

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Office Hours:

MTWR after class 2:00-3:00, and by appointment. Details on Gauchospace.

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How many times do we need to triple 1 to get the following numbers?

• 9

How many times do we need to triple 1 to get the following numbers?

• 9 2

- 9 2
- 81

- 9 2
- 81 <u>4</u>

- 9 2
- 81 **4**
- 1

- 9 2
- 81 **4**
- 1 0

- 9 2
- 81 **4**
- 1 <u>0</u>
- $\frac{1}{3}$

- 9 2
- 81 <u>4</u>
- 1 0
- $\frac{1}{3}$ -1

- 9 2
- 81 **4**
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- $\frac{1}{3}$ -1
- $\frac{1}{2}$

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- 81 <u>4</u>
- 1 0
- $\frac{1}{3}$ $\boxed{-1}$
- $\frac{1}{2}$???

- 9 2
- 81 <u>4</u>
- 1 0
- $\frac{1}{3}$ -1
- $\frac{1}{2}$??? ...something between -1 and 0.

How many times do we need to decuple 1 (multiply 1 by 10) to get the following numbers?

• 100

How many times do we need to decuple 1 (multiply 1 by 10) to get the following numbers?

• 100 <u>2</u>

- 100 <u>2</u>
- 1000

- 100 <u>2</u>
- 1000 3

- 100 <u>2</u>
- 1000 3
- 1

- 100 <u>2</u>
- 1000 3
- 1 0

- 100 <u>2</u>
- 1000 3
- 1 0
- .0001

- 100 <u>2</u>
- 1000 3
- 1 0
- .0001 -4

- 100 <u>2</u>
- 1000 3
- 1 0
- .0001 | -4
- A Googol

- 100 <u>2</u>
- 1000 3
- 1 0
- .0001 -4
- A Googol 100

- 100 <u>2</u>
- 1000 3
- 1 0
- .0001 -4
- A Googol 100 A Googol is...

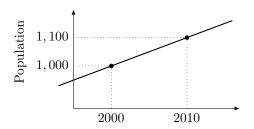
- 100 <u>2</u>
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1. In 2000, a population was 1000. In 2010, it was 1100. What would you guess the population was in 2005?

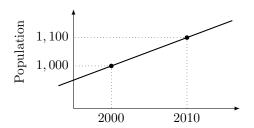
1. In 2000, a population was 1000. In 2010, it was 1100. What would you guess the population was in 2005?

$$A = 1005$$
 $B = 1020$ $C = 1050$ $D = 2050$ $E = 2010$ C



This is a guess

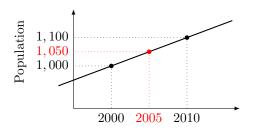
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This is a guess based on the assumption that population grows at a constant rate.

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This is a guess based on the assumption that population grows at a constant rate.

"Constant rate" means that the graph of population is a straight line.

Linear Extrapolation

2. In 2000, a population was 1000. In 2010, it was 1100. What would you guess the population will be in 2020?

$$A = 1150$$
 $B = 1200$ $C = 1250$ $D = 2020$ $E = Other$

В

Linear Extrapolation

2. In 2000, a population was 1000. In 2010, it was 1100. What would you guess the population will be in 2020?

Again: a guess

В

Linear Extrapolation

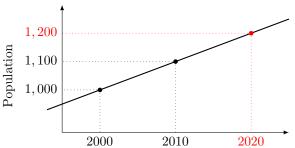
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$$\boxed{\text{B}}$$



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A Problem

You can't tell someone you just "guessed" the answer or just "drew a straight line". You need to make it <u>sound</u> more "scientific" so give it a complicated sounding name to impress people.

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Linear Interpolation and Linear Extrapolation.

Linear means straight line inter means between like intercity extra means beyond like extraordinary

The idea is to assume the population (or whetever) grows at a constant rate.

Then use this to predict.

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Method:

- (1) Use given data to draw a straight line and find equation y = mx + b
- (2) Use the equation to make predictions.

$$x=$$
 number of years after 2000 (Ex: $x=3$ is the year 2003) $y=$ population in the year x

Find the equation of a line y = mx + b:

A: 2000 + 1000x B: 1000 + 2000x C: 1000 + 100x D: 1000 + 10x

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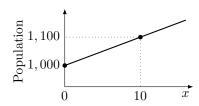
A:
$$2000 + 1000x$$
 B: $1000 + 2000x$ C: $1000 + 100x$ D: $1000 + 10x$

Answer: D

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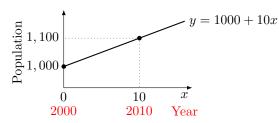
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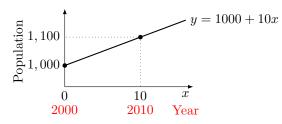
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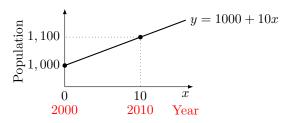
9. When will population be 1350?

$$A = 2015$$
 $B = 2025$ $C = 2035$ $D = 3350$ $E = Other$

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Find the equation of a line y = mx + b:

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9. When will population be 1350?

$$A = 2015$$
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- **5.** The number of unemployed in LA on January 1, 2015 was 50,000. After 100 days, it was 45,000.
 - Estimate the number of unemployed 300 days after Jan. 1.

$$A = 40,000$$
 $B = 35,000$ $C = 30,000$ $D = 25,000$ $E = 300$

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• Suppose x = the number of days after January 1 y = number of unemployed people on day x.

Then the equation of the line used for this linear extrapolation is y =

$$A = -100 + 50,000x$$
 $B = 50,000 - 100x$ $C = 45,000 - 100x$
 $D = 50,000 - 50x$ $E = 45,000 - 50x$

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 A= 40 B= 140 C= 200 D= 300 E= 400

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Simple Idea: $y \propto x$ "y is proportional to x" means:

If you double x, then y doubles. Triple x then y triples. And so on.

Example: If you are paid by the hour then

(amount you earn) \propto (number of hours you work)

If you work for 10 hours, then you are paid \$50. How much are you paid if you work for 20 hours?

A = \$20 B = \$50 C = \$200 D = \$100 E = Other

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If you work for t hours, how much are you paid?

$$A = \$50$$
 $B = \$50t$ $C = \$10t$ $D = \$20t$ $E = \$5t$

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If you work for t hours, how much are you paid?

$$A = \$50$$
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Because you are paid \$5/hour (or \$50 for 10 hours). The number "5" is called the constant of proportionality.

Suppose $y \propto x$ and y = 15 when x = 4.

(a) What is y when x = 8?

$$A = 15$$
 $B = 4$ $C = 8$ $D = 30$ $E = 60$

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(a) What is y when x = 8?

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(b) What is y when x = 12?

$$A = 15$$
 $B = 45$ $C = 30$ $D = 36$ $E = 12$

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(b) What is y when x = 12?

$$A = 15$$
 $B = 45$ $C = 30$ $D = 36$ $E = 12$ B

(c) What is x when y = 150?

$$A = 14$$
 $B = 1500$ $C = 40$ $D = 450$

Suppose $y \propto x$ and y = 15 when x = 4.

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Constant of Proportionality

"y is proportional to x" means y = Kx, where K is called the constant of proportionality.

Example: We are told

- Tax is proportional to income, and
- The tax on \$1,000 is \$280.

Express y = amount of tax paid in terms of x = the income. Then y =

A=
$$1000x$$
 B= $280x$ C= $\frac{1,000}{280}x$
D= $2.8x$ E= $0.28x$

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Proportionality

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Question: What does the constant of proportionality K = 0.28 mean? Answer: It is the tax on one dollar.

Example

For this question, we assume:

- The weight of an elephant is proportional to its <u>height cubed</u>, and
- An elephant 1 meter high weighs 1/3 tons.

How many tons does an elephant h meters tall weigh?

$$A = h/3$$
 $B = h^3$ $C = h^3/3$ $D = (h/3)^3$ $E = (3h)^3$

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Question: What does the constant of proportionality K = 1/3 mean?

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Question: What does the constant of proportionality K=1/3 mean? Answer: It is the weight of 1 cubic meter of elephant.

y is inversely proportional to x means $y \propto 1/x$

Proportionality

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Proportionality

Example:

- I have \$300
- N = number of apples I can buy
- p = price per apple

Then N is inversely proportional to p: $N \propto 1/p$.

What is the constant of proportionality?

Strength of Light

• P =strength of light (power per unit area)

Strength of Light

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Inverse Square Law: $P \propto 1/R^2$

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Same idea for heat, gravity, sound, and many others...

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Newton's Law of Gravity: $F \propto \frac{m_1 m_2}{r^2}$

Constant of proportionality: $G \approx 6.67 \times 10^{-11} \text{ m}^3/(\text{kg}\,\text{s}^2)$ (the Gravitational constant)

Review

1. Solve for x in the equation

$$\frac{3}{x+a} = \frac{a}{x+2}.$$

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$$x = \frac{6 - a^2}{a - 3}$$

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$$(a-3b)(4a+2b)+6ab$$

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$$x = \frac{6 - a^2}{a - 3}$$

2. Multiply out and simplify. Check your answer.

$$(a-3b)(4a+2b)+6ab$$

$$= 4a^2 - 4ab - 6b^2$$

3. Substitute x = 3t - 4 into

$$2x(x+1)$$
.

Simplify the result as much as possible.

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Simplify the result as much as possible.

$$=6\left(3t^2-7t+4\right)$$

(you don't <u>have</u> to pull out the 6)

Review

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Simplify the result as much as possible.

$$= 6 \left(3t^2 - 7t + 4 \right)$$

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4. Solve for x and y in the simultaneous equations

$$x + 2y = p \qquad \qquad x + y = 4.$$

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4. Solve for x and y in the simultaneous equations

$$x + 2y = p \qquad \qquad x + y = 4.$$

$$x = 8 - p, \quad y = p - 4$$

- 5. Marie leaves Santa Barbara at 10am, driving to Bakersfield on a route which is 150 miles long. Jason leaves Bakersfield at 11am driving the same route to Santa Barbara. Marie's speed is 40 miles/hr and Jason's speed is 60 miles/hr.
 - (a) How far apart are they at noon?

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(c) How many hours has <u>Jason</u> been driving when they meet?

[leave your answers as <u>fractions</u>]

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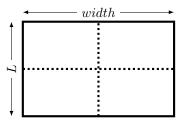
(c) How many hours has <u>Jason</u> been driving when they meet?

[leave your answers as <u>fractions</u>]

- 6. A farmer wants to partition a rectangular field into quarters, as shown. The total area of the field is 500 square meters. Suppose the length of the field is L meters.
 - (a) Express the width of the field in terms of L.

Review

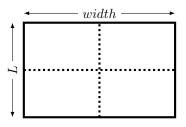
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(b) The outer boundary fence (on the perimeter of the field, shown solid) costs \$4 per meter, and the inside fence (shown dotted) costs \$3 per meter. Express the total cost of the fence needed in terms of L.

Review

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(b) The outer boundary fence (on the perimeter of the field, shown solid) costs \$4 per meter, and the inside fence (shown dotted) costs \$3 per meter. Express the total cost of the fence needed in terms of L. (Answer: $\frac{11(L^2+500)}{L}$)

That's it. Thanks for being here.

