

Welcome Back!

Differential Calculus

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Counting and Our Logarithmic Perception of the World

Vsauce:

1,2,3,4,5,6,7,8,9,10,11,12,13,14,15,16,17,18,19,20,21,22,23,24,25,26,27,
28,29,30,31,32,33,34,35...

<https://www.youtube.com/watch?v=Pxb5lSPLy9c>

Warm-up

- $\log_9(9^4) = \boxed{4}$
- $\log_3(9^4) = \boxed{8}$
- $\log_{27}(27^5) = \boxed{5}$
- $\log_3(27^5) = \boxed{15}$
- $\log_5(25^{17}) = \boxed{34}$
- $\log_3(27^0) = \boxed{0}$
- $\log_8(2^{12}) = \boxed{4}$
- $\log_8(2) = \boxed{1/3}$

Warm-up Part II

- $\log_{100}(100^7) = \boxed{7}$
- $\log_{10}(100^7) = \boxed{14}$
- $\log_{10}(1000000^{-4}) = \boxed{-24}$
- $\log_{10}(2) = \text{about } \boxed{.3}$ Still warm-up?
- $\log_{10}(2^{11}) = \text{about } .3 \cdot 11 = \boxed{3.3}$

logs are “**opposite**” of exponents (inverse function of antilog)
So every fact about exponents corresponds to a fact about logs:

	laws of exponents	corresponding law of logs
(1)	$10^a \times 10^b = 10^{a+b}$	$\log(xy) = \log(x) + \log(y)$
(2)	$10^0 = 1$	$\log(1) = 0$
(3)	$10^{-a} = 1/10^a$	$\log(1/x) = -\log(x)$
(4)	$(10^a)^p = 10^{ap}$	$\log(x^p) = p \log(x)$
(5)	$10^a/10^b = 10^{a-b}$	$\log(x/y) = \log(x) - \log(y)$

Example: $\log(x^a/y^b) = ?$

$$\begin{aligned} A &= a \log(x)/(b \log(y)) & B &= a \log(x) + b \log(y) \\ C &= a \log(x) - b \log(y) & D &= (a + \log(x)) - (b + \log(y)) \end{aligned} \quad \boxed{C}$$

Rule (4): $\log(x^p) = p \log(x)$

Explanation of (4)

$$\log(a \times a) = \log(a) + \log(a) = 2 \log(a)$$

$$\log(a \times a \times a) = \log(a) + \log(a) + \log(a) = 3 \log(a)$$

In general: the number of tens you multiply to get x^p is p times as many tens as you multiply to get x .

What is $\log\left(\sqrt{\frac{1}{x^7}}\right)$?

$$A = 7 - \log(x) \quad B = (7/2) - \log(x) \quad C = -7/2 \quad D = -(7/2) \log(x)$$

D

Find x by solving $3^x = 5$.

A $\log(5)/\log(3)$

B $\log(3)/\log(5)$

C $\log(5)^3$

D $\log(3) - \log(5)$

E $\log(5) - \log(3)$

A

§7.5: Using logs to multiply

First rule of logs: $\log(a \times b) = \log(a) + \log(b)$

Example: Find 2.7×1.6 using logs

Given info: $\log(2.7) \approx 0.43$ and $\log(1.6) \approx 0.20$

Method

- (i) Look up $\log(2.7)$ and $\log(1.6)$
- (ii) Add these
- (iii) Take the **antilog** of result from (ii)
- (iv) Think: Is the answer **reasonable** or did I goof up?

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Look how I write the answer.

- $\log(2.7 \times 1.6) = \log(2.7) + \log(1.6)$
- We are told $\log(2.7) \approx 0.43$ and $\log(1.6) \approx 0.20$, so
 $\log(2.7 \times 1.6) \approx 0.43 + 0.20 = 0.63$
- Is this the answer ? Heck No! It is the **log** of the answer
- $2.7 \times 1.6 \approx \text{antilog}(0.63) = 10^{0.63}$
- $10^{0.63} \approx 4.3$
- Is my answer 4.3 reasonable? Yes, about $2 \times 2 = 4$.

§7.5: Using logs to divide

Remember Log Rule (5): $\log(a \div b) = \log(a) - \log(b)$

Example: Use this rule to find $38.2/1.77$

Given info: $\log(3.82) \approx 0.58$ and $\log(1.77) \approx 0.25$

Method

(i) Look up $\log(3.82)$ and $\log(1.77)$, find $\log(38.2)$

★ You can find $\log(38.2)$ by adding 1 to $\log(3.82)$ because 38.2 is 3.82 times one more power of 10.★

(ii) Subtract!

(iii) Take the **antilog** of result from (ii)

(iv) Think: Is the answer **reasonable** or did I goof up?

A= done

B= confused

Powers Using Logs

Or, exploiting Log Rule (4):

$$\log(a^p) = p \log(a)$$

Use this and the graph of $y = 10^x$ to find $\sqrt{70}$.

One Approach:

- (i) Use graph and move decimal point trick to find $\log(70)$
★I will show the graph of the exponential function 10^x and talk about this method on Thursday.★
- (ii) $\log(\sqrt{70}) = \log(70^{1/2}) = (1/2) \log(70)$
- (iii) Take the **antilog** of result from (ii)
- (iv) Think: Is the answer **reasonable** or did I goof up?

Hint: $\log(7) \approx 0.84$

A= done B= working C= confused

Answer: $\sqrt{70} \approx 8.3$. Is that reasonable?

Computer Applications

One kilobyte (1 **KB**) is 2^{10} .

Problem: Calculate 2^{10} using logs. **Hint:** $\log(2) \approx 0.3$

$A \approx 3$ $B \approx 10.3$ $C \approx 30$ $D \approx 1000$ $E \approx 1100$

D

So: $2^{10} \approx 10^3 = 1000$ (really $2^{10} = 1024$).

1KB is really $2^{10} = 1024 \approx 10^3$ (**K** is **Kilo** = thousand)

1MB is really $2^{20} = (2^{10})^2 \approx (10^3)^2 = 10^6$ (**M** is **Mega** = million)

1GB is really $2^{30} = (2^{10})^3 \approx (10^3)^3 = 10^9$ (**G** is **Giga** = billion)

1TB is really $2^{40} = (2^{10})^4 \approx (10^3)^4 = 10^{12}$ (**T** is **Tera** = trillion)

Example: suppose on a certain island the population of rabbits doubles every generation. After 20 generations it multiplies by...
 $2^{20} \approx 1$ million.

Powers of 2 are easy to do, even in your head. To work out 2^n the **log** of the answer is approximately $0.3n$, so 2^n is 1 followed by $0.3n$ zeroes.

§7.7: Solving Exponential Eq'ns

1. Find x by solving $10^x = 5$.

$$\begin{array}{llll} A = 5 & B = 0.5 & C = \log(5) & D = \log(0.5) \\ E = \log(5) - \log(10) & & \boxed{C} & \end{array}$$

Look how I write the answer!

$$\begin{array}{ll} \log(10^x) = \log(5) & \text{Take logs of both sides} \\ x = \log(10^x) = \log(5) & \text{Using } \log(a^p) = p \log(a) \text{ and } \log(10) = 1 \end{array}$$

Examples:

Use the Fourth Law:

$$\log(a^x) = x \log(a)$$

Slogan: Logs bring exponents down to ground level.

2. Solve $3^x = 7$

$$A = \log(7/3) \quad B = \log(7) - \log(3) \quad C = \log(7) + \log(3)$$

$$D = \log(3)/\log(7) \quad E = \log(7)/\log(3) \quad \boxed{E}$$

Look how I write the answer:

$$\log(3^x) = \log(7)$$

$$x \log(3) = \log(3^x) = \log(7)$$

$$\text{So:} \quad x = \log(7)/\log(3)$$

Take logs of both sides

Using $\log(a^p) = p \log(a)$

Examples:

Use the Fourth Law:

$$\log(a^x) = x \log(a)$$

Slogan: Logs bring exponents down to ground level.

3. Solve $7^{x+2} = 30$.

$$A = \frac{\log(30) - 2\log(7)}{\log(7)} \quad B = \frac{\log(30)}{\log(7)} - 2 \quad C = \frac{\log(30) - \log(49)}{\log(7)}$$

$$D = \frac{\log(30/49)}{\log(7)} \quad E \approx -0.25213$$

All are correct!

Examples:

Use the Fourth Law:

$$\log(a^x) = x \log(a)$$

Slogan: Logs bring exponents down to ground level.

4. Solve $7 \times 3^y = 2^{4y+3}$

$$A = \frac{3 \log(2) - \log(7)}{\log(3) - 4 \log(2)} \quad B = \frac{3 \log(2)}{7 \log(3)} \quad C = \frac{3 \log(2)}{7 \log(3) - 4 \log(2)}$$

$$D = \frac{7 \log(3) - 4 \log(2)}{3 \log(2)}$$

E = none of the above

A

Compound Interest

At the end of each year a bank pays 7% interest into your account. Initially have \$10,000 in account. How much after 10 years?

Think $10 \times 7\% = 70\%$ in 10 years, so have **\$17,000** but that is **wrong**.

After 1 year: $\$10,000 \times 1.07 = \$10,700$

After 2 years: $\$10,700 \times 1.07 = \$10,000 \times 1.07 \times 1.07 = \$11,449$

After 3 years: $\$11,449 \times 1.07 = \$10,000 \times (1.07)^3 = \$12,250.40$

Each year **what you had before** is **multiplied** by 1.07. Thus **compound interest**.

So after 10 years have

$$\$10,000 \times \underbrace{1.07 \times 1.07 \times \cdots \times 1.07}_{10 \text{ times}} = 10,000 \times (1.07)^{10} \approx \boxed{\$20,000}$$

Conclusion: Money approximately doubles in 10 years!

So in 20 years multiplies by 4, in 30 years by 8,...

That's it. Thanks for being here.



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