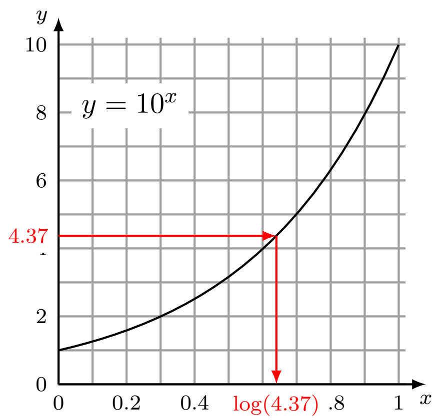
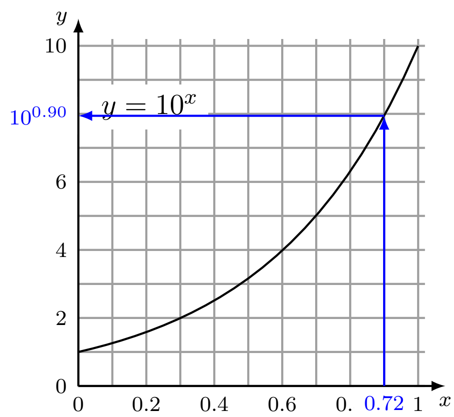
Math 34A Winter 2020
Old Midterm 2 #5 Solutions

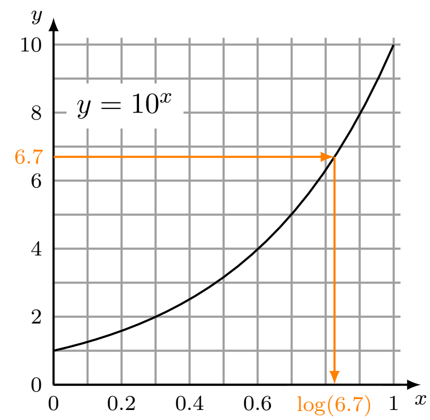
1. Here are the three graphs we'll use in solving these problems:



Part (a)



Part (b)



Part (c)

(a) Remember that we can use the move the decimal point trick:

$$\log(437) = \log(10^2 \times 4.37) = \log(10^2) + \log(4.37) = 2 + \log(4.37).$$

Now we can use the graph to find that $\log(4.37) \approx 0.64$, and so $\log(437) \approx \boxed{2.64}$. (Mathematica tells me that $\log(437) \approx 2.640\,481\,436\,97\ldots$)

(b) The reverse version of the "move the decimal point trick" is what we need here:

$$10^{-2.10} = 10^{-3+0.90} = 10^{-3} \times 10^{0.90}.$$

We know that $10^{-3} = 0.001$, and we use the graph to find that $10^{0.90} \approx 7.9$. Thus $10^{-2.10} \approx 0.001 \times 7.9 = \boxed{0.0079}$. (Mathematica tells me that $10^{-2.10} \approx 0.007\,943\,282\ldots$, so we're within $0.000\,05$ or so.)

(c) First we use the rules of logarithms to write

$$\log(100/6.7) = \log(100) - \log(6.7) = 2 - \log(6.7).$$

Now we can use the graph to find that $\log(6.7) \approx 0.83$. Thus

$$\log(100/6.7) = \log(100) - \log(6.7) \approx 2 - 0.83 = \boxed{1.17}.$$

(Mathematica tells me that $\log(100/6.7) \approx 1.173\,925\,197\ldots$, so as usual we're pretty close.)

2. Let's start with this equation slightly simplified as

$$7^{3x+2} = 8 - 2 = 6.$$

Now take the logarithm of both sides to get

$$\log(7^{3x+2}) = \log(6).$$

We simplify this using more rules of logs:

$$(3x+2)\log(7) = \log(6)$$

since $\log(a^p) = p\log(a)$. Now divide by $\log(7)$ to get

$$3x + 2 = \log(6)/\log(7).$$

Now subtract 2 and divide by 3 to get

$$3x = \log(6)/\log(7) - 2 \qquad \text{and then} \qquad x = \boxed{\frac{\log(6)/\log(7) - 2}{3}}.$$

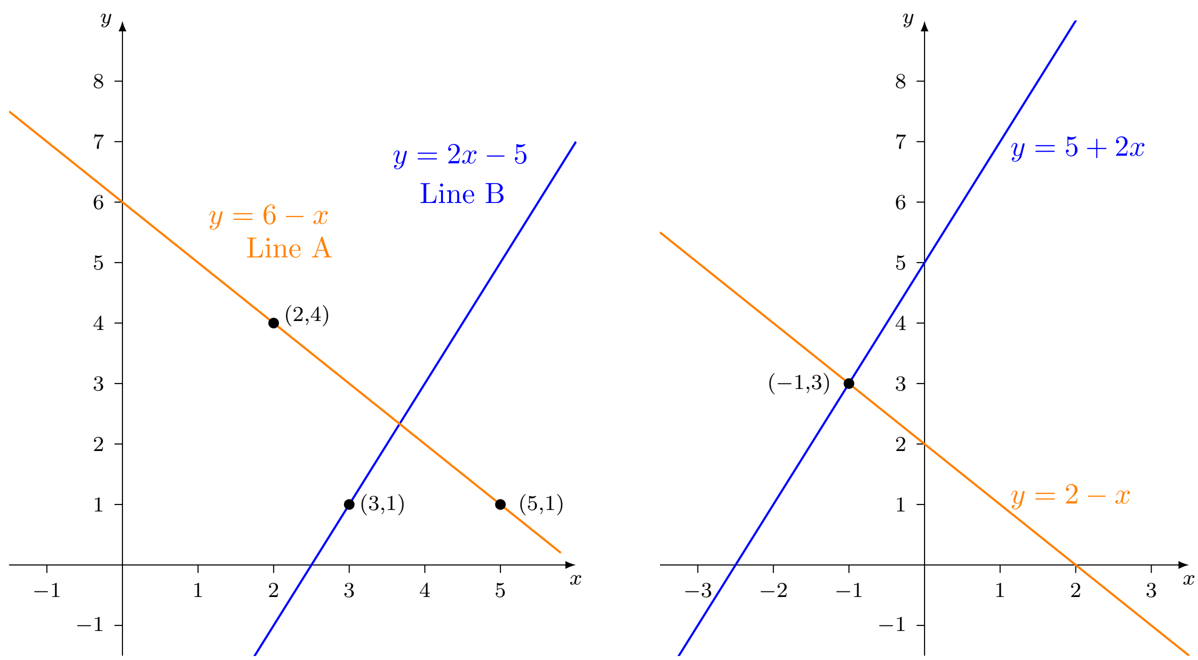
Multiplying both top and bottom gives us a slightly cleaner-looking answer, $x = \boxed{\dfrac{\log(6) - 2\log(7)}{3\log(7)}}$. Since $\log(a) - \log(b) = \log(a/b)$ and $p\log(a) = \log(a^p)$, we can "simplify" the numerator to $\log(6) - \log(7^2) = \log(6/49)$, and so another acceptable answer is $x = \boxed{\dfrac{\log(6/49)}{3\log(7)}}$.

3. (a) The slope of Line A is

$$m = \frac{4-1}{2-5} = \frac{3}{-3} = -1.$$

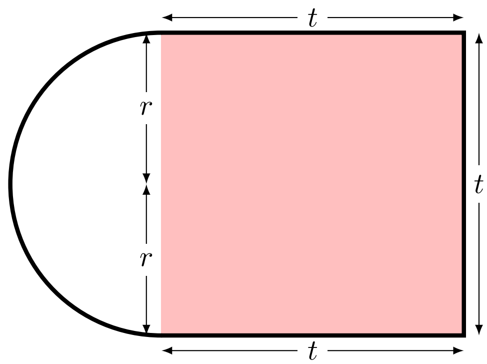
Thus Line A has equation $y = -x + b$ for some b. We find that value by plugging in either point $(x, y) = (5, 1)$ or $(2, 4)$. If we plug in the first point, we get $1 = -1(5) + b$, or $b = 6$. Thus Line A has equation $\boxed{y = -x + 6}$. Line A is shown on the left, below.

(b) Line B has slope 2, so it has equation $y = 2x + b$. We plug in $(x, y) = (3, 1)$ to find b: $1 = 2(3) + b$, or $b = -5$. Thus the equation of Line B is $\boxed{y = 2x - 5}$. Line B is shown with Line A on the left, below.



(c) The point of intersection of the lines $y = 5 + 2x$ and $y = 2 - x$ is where $5 + 2x = 2 - x$. By adding x and subtracting 5 from both sides, we get $3x = -3$. Then dividing by 3, we see that $x = -1$. Plugging $x = -1$ into either line gives us $y = 3$. Thus the point of intersection is therefore $(x, y) = \boxed{(-1, 3)}$. (The lines and the point of intersection are shown above on the right.)

4. We reproduce the picture of the garden here:



The length of each side of the square is t, so the radius of the semicircular part is $r = t/2$.

(a) The area of the garden is half the area of a circle of radius $r = t/2$ plus the area of a square of side length t. That is, the total area is $A = (1/2)\pi(t/2)^2 + t^2 = \boxed{\pi t^2/8 + t^2}$. (Here we've used the fact that the area of a circle is πr^2.)

(b) The perimeter of the garden is half the circumference of a circle of radius $r = t/2$ PLUS three sides of the square. Thus the perimeter of the garden is $P = (1/2)(2\pi(t/2)) + 3t = \boxed{\pi t/2 + 3t}$. (Here we've used the fact that the perimeter of a circle is $2\pi r$.)

(c) If the area of the square is 100, then since the area of the square is t^2, we get the length of each side is $t = 10$ (we just took the square root of $t^2 = 100$). Then from part (b), the perimeter of the garden is $\pi(10)/2 + 3(10) = \boxed{5\pi + 30}$.

5. (a) We're told that the mixture is 12 grams: 4 grams of gold and 8 grams of silver. Thus the percentage that is gold is

$$100\% \times \frac{4 \text{ grams}}{12 \text{ grams}} = \boxed{\frac{100}{3}\% \approx 33.33\%}.$$

(b) Now we have 4 grams of gold and some unknown amount of silver – call it S grams. Thus the mixture is $4 + S$ grams in total, and we want the percentage gold to be 20%:

$$100\% \times \frac{4 \text{ grams}}{4 + S \text{ grams}} = 20\%.$$

Dividing by 20% and multiplying by $4 + S$, we get

$$\frac{100\%}{20\%} \times 4 = 4 + S \qquad \text{or} \qquad 20 = 4 + S.$$

Solving for S, we find that we should use $S = \boxed{16 \text{ grams}}$ of silver.

(c) This is very similar to the previous part; our equation is now

$$100\% \times \frac{4 \text{ grams}}{4 + S \text{ grams}} = x\%,$$

where again we've used S to represent the number of grams of silver in the mixture (so the mixture has a total of $4 + S$ grams of metal). Multiplying this equation by $4 + S$ and cancelling the "%" signs (and the units "grams"), we find that

$$400 = x(4 + S) \qquad \text{or} \qquad 400 = 4x + xS.$$

We solve for S by subtracting $4x$, then dividing both sides by x. We get $xS = 400 - 4x$ and so $S = \boxed{\frac{400 - 4x}{x} \text{ grams}}$ of silver. We could also write this as $S = \boxed{\frac{400}{x} - 4 \text{ grams}}$ of silver. (We can check this by plugging in $x = 20$ to see that we get $S = 16$, as in part (b).)