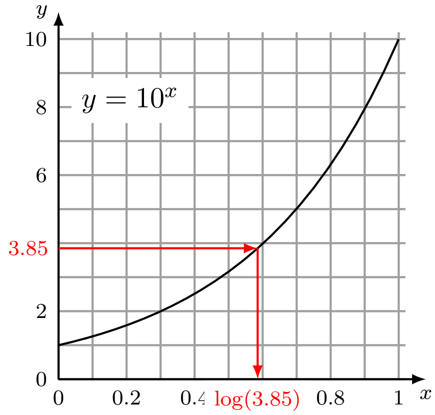
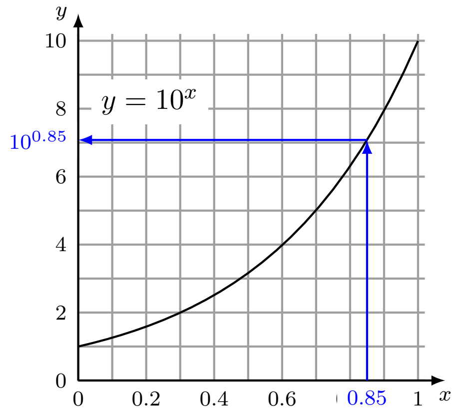
Math 34A Winter 2020
Old Midterm 2 #3 Solutions

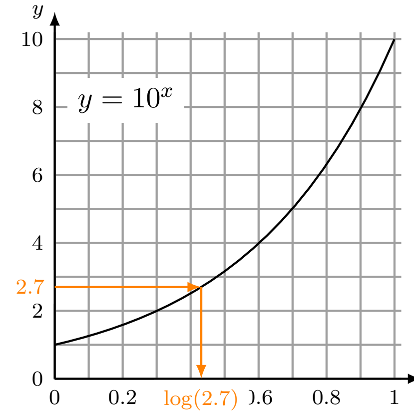
1. Here are the three graphs we'll use in solving these problems:



Part (a)



Part (b)



Part (c)

(a) Remember that we can use the move the decimal point trick:

$$\log(385) = \log(10^2 \times 3.85) = \log(10^2) + \log(3.85) = 2 + \log(3.85).$$

Now we can use the graph to find that $\log(3.85) \approx 0.59$, and so $\log(385) \approx \boxed{2.59}$. (Mathematica tells me that $\log(385) \approx 2.585\,460\,730$)

(b) The reverse version of the "move the decimal point trick" is what we need here:

$$10^{4.85} = 10^{4+0.85} = 10^4 \times 10^{0.85}.$$

We know that $10^4 = 10,000$, and we use the graph to find that $10^{0.85} \approx 7.08$. Thus $10^{4.85} \approx 10,000 \times 7.08 = \boxed{70{,}800}$. (Mathematica tells me that $10^{4.85} \approx 70,794.578\,438\ldots$, so we're within 6 out of more than $70,000$.)

(c) First we use the rules of logarithms to write

$$\log(1/2.7) = \log(1) - \log(2.7) = 0 - \log(2.7).$$

Now we can use the graph to find that $\log(2.7) \approx 0.43$, and so $\log(1/2.7) = -\log(2.7) \approx \boxed{-0.43}$.

2. The equation we wish to solve is

$$7^{3x+4} = 17.$$

As is usual in these sorts of problems, we take the logarithm of both sides to get

$$\log(7^{3x+4}) = \log(17),$$

which simplifies using the rule of logs to

$$(3x+4)\log(7) = \log(17).$$

Now we distribute the product on the left to get

$$3\log(7)x + 4\log(7) = \log(17),$$

and subtract $4\log(7)$ from each side to find that

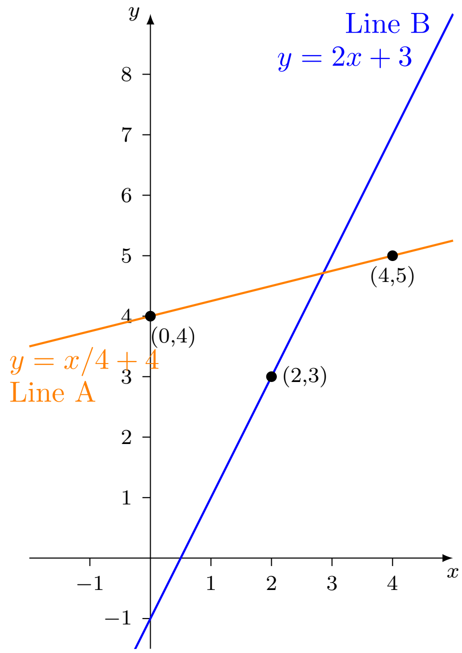
$$3\log(7)x = \log(17) - 4\log(7).$$

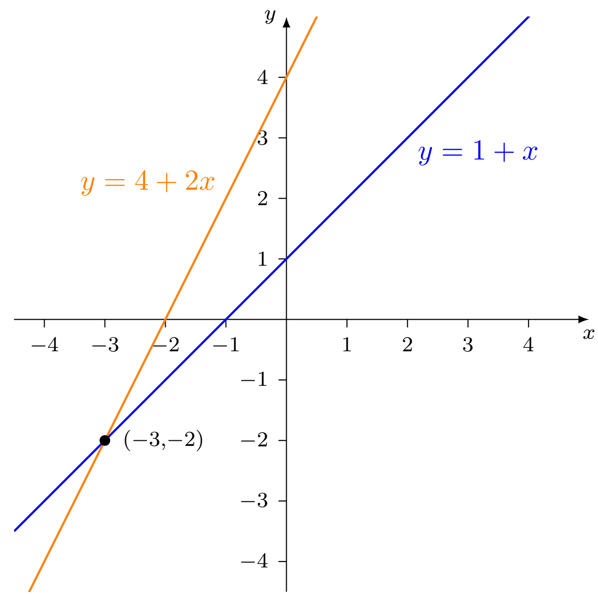
Now divide by $3\log(7)$ to find that

$$x = \boxed{\frac{\log(17) - 4\log(7)}{3\log(7)}} \qquad \text{or} \qquad x = \boxed{\frac{\log(17)}{3\log(7)} - \frac{4}{3}}.$$

3. (a) The slope of Line A is
$$m = \frac{5-4}{4-0} = \frac{1}{4}.$$
Thus Line A has equation $y = (1/4)x + b$ for some b. We find that value by plugging in either point $(x, y) = (0, 4)$ or $(4, 5)$. If we plug in the first point, we get $4 = 0 + b$, or $b = 4$. Thus Line A has equation $\boxed{y = x/4 + 4}$. Line A is shown on the left, below.

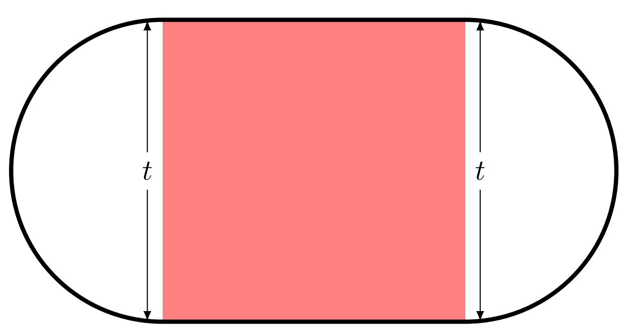
(b) Line B has slope 2, so it has equation $y = 2x + b$. We plug in $(x, y) = (2, 3)$ to find b: $3 = 2(2) + b$, or $b = -1$. Thus the equation of Line B is $\boxed{y = 2x - 1}$. Line B is shown with Line A on the left, below.





(c) The point of intersection of the lines $y = 1 + x$ and $y = 4 + 2x$ is where $1 + x = 4 + 2x$. Subtracting x and 4 from both sides, we get $x = -3$. Plugging $x = -3$ into either line gives us $y = -2$. Thus the point of intersection is $(x, y) = \boxed{(-3, -2)}$. (The lines and the point of intersection are shown above on the right.)

4. We reproduce the picture of the garden here:



The length of each semicircle's diameter is t, so the sides of the square are all length t.

(a) The area of the garden is two halves of the area of a circle (so the whole area of the circle) plus the area of the square. The circle has radius $r = t/2$ and the square has side length t, so the total area is $A = \pi(t/2)^2 + t^2 = \boxed{\pi t^2/4 + t^2}$. (Here we've used the fact that the area of a circle is πr^2.)

(b) The perimeter of the garden is two halves of the circumference of the circle (so the whole circumference of the circle) plus two sides of the square. The circle has radius $r = t/2$, so the total perimeter of the garden is $P = (2\pi(t/2)) + 2t = \boxed{\pi t + 2t}$. (Here we've used the fact that the perimeter of a circle is $2\pi r$.)

(c) If the area of the square is 400, then since the area of the square is t^2, we get the length of each side is $t = 20$ (we just took the square root of $t^2 = 400$). Then from part (b), the perimeter of the garden is $\pi(20) + 2(20) = \boxed{20\pi + 40}$.

5. (a) We're told that 50% of the 50 grams of Alloy A is silver, and that 20% of the 50 grams of Alloy B is silver. Thus the amount of silver in the $50 + 50 = 100$ total grams of the mixture is

$$50\% \times 50 + 20\% \times 50 = (0.50)(50) + (0.20)(50) = 35 \text{ grams}\,.$$

As a percentage, there is

$$\frac{35 \text{ grams of silver}}{100 \text{ total grams}} \cdot 100\% = \boxed{35\% \text{ silver}}.$$

(b) This is very similar to part (a). We're told that 50% of the x grams of Alloy A is silver, and that 20% of the $100 - x$ grams of Alloy B is silver. Thus the amount of silver in the $x + (100 - x) = 100$ total grams of the mixture is

$$(0.50)(x) + (0.20)(100 - x) = 0.5x + 20 - 0.2x = 0.3x + 20 \text{ grams.}$$

As a percentage, there is

$$\frac{0.3x + 20 \text{ grams of silver}}{100 \text{ total grams}} \cdot 100\% = \boxed{(0.3x + 20)\% \text{ silver}}.$$

Does this answer make sense? We can check three values of x pretty easily:

- When $x = 50$, this is simply part (a). Our answer was $0.3(50) + 20 = 15 + 20 = 35\%$, so this agrees.
- When $x = 100$, all the mixture is from Alloy A, so we should get paint that is 50% silver. Sure enough, plugging in $x = 100$ gives us a value of $(0.3)(100) + 20 = 50\%$, so the mixture is 50% silver.
- Similarly, when $x = 0$, all the mixture is from Alloy B, so we should get a mixture that is 20% silver. We do, which I'll leave you to check.

(c) If the resulting 100 grams is 40% silver, then this means there is $40\% \times 100 = (0.4)(100) = 40$ grams of silver in the mixture. So we have to find the value of x in part (b) (remember, x is the amount of Alloy A) so that

$$0.3x + 20 = 40 \text{ grams.}$$

Solving, we get $\boxed{x = 200/3 \text{ grams}}$ from Alloy A.

This answer is reasonable. When the split is 50 grams from each can, we end up with 35 grams of silver in the mixture. To get 40 ounces, we'd need to increase the amount of Alloy A (the alloy with a higher percent of silver).