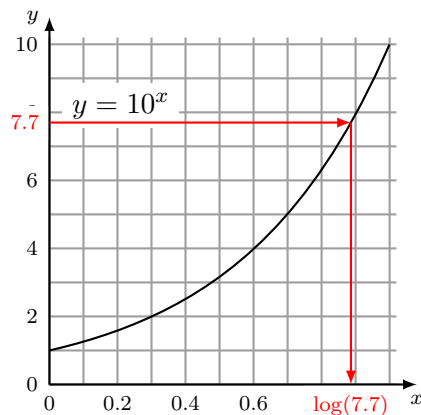
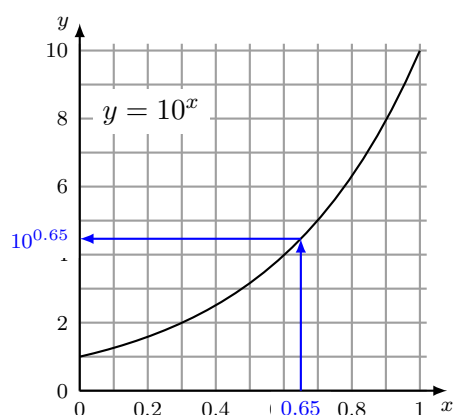


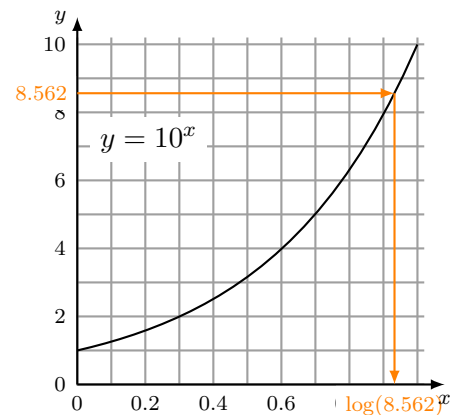
1. Here are the three graphs we'll use in solving these problems:



Part (a)



Part (b)



Part (c)

- (a) Remember that there is no way to simplify $\log(a + b)$, so instead we just add $35 + 42 = 77$ to realize we just need to find $\log(77)$. We can use the move the decimal point trick:

$$\log(77) = \log(10 \times 7.7) = \log(10) + \log(7.7) = 1 + \log(7.7).$$

Now we can use the graph to find that $\log(7.7) \approx 0.89$, and so $\log(77) \approx \boxed{1.89}$. (Mathematica tells me that $\log(77) \approx 1.886490725\dots$, so we're pretty close.)

- (b) The reverse version of the “move the decimal point trick” is what we need here:

$$10^{3.65} = 10^{3+0.65} = 10^3 \times 10^{0.65}.$$

We know that $10^3 = 1,000$, and we use the graph to find that $10^{0.65} \approx 4.47$. Thus $10^{3.65} \approx 1,000 \times 4.47 = \boxed{4,470}$. (Mathematica tells me that $10^{3.65} \approx 4,466.835921509631\dots$, so we're within 4 out of more than 4,400.)

- (c) First we use the rules of logarithms and the move the decimal point trick to write

$$\log(\sqrt{8562}) = \frac{1}{2} \log(8562) = \frac{1}{2} \log(10^3 \times 8.562) = \frac{1}{2} (3 + \log(8.562)).$$

Now we can use the graph to find that $\log(8.562) \approx 0.93$. Thus

$$\log(\sqrt{8562}) = \frac{1}{2} (3 + \log(8.562)) \approx \frac{1}{2} (3 + 0.93) = \frac{3.93}{2} \approx \boxed{1.96}.$$

(Mathematica tells me that $\log(\sqrt{8562}) \approx 1.96628761\dots$)

2. Let's start with this equation slightly simplified as

$$7^{4x+1} = 5.$$

Now take the logarithm of both sides to get

$$\log(7^{4x+1}) = \log(5).$$

We simplify this using rules of logs to

$$(4x + 1) \log(7) = \log(5)$$

$$\text{since } \log(a^p) = p \log(a).$$

Now distribute the product on the left to get

$$4x \log(7) + \log(7) = \log(5).$$

Now subtract $\log(7)$ from both sides, then divide by $4 \log(7)$ to get

$$4x \log(7) = \log(5) - \log(7) \quad \text{and then} \quad x = \boxed{\frac{\log(5) - \log(7)}{4 \log(7)}}.$$

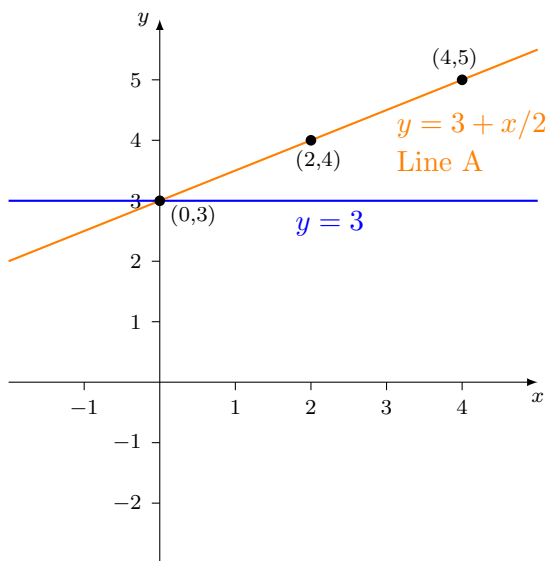
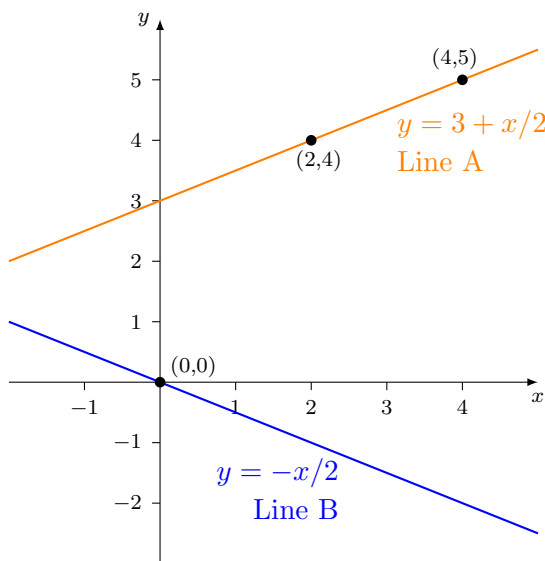
Since $\log(a) - \log(b) = \log(a/b)$, we can simplify the numerator to $\log(5) - \log(7) = \log(5/7)$. Thus we can write this as $x = \boxed{\frac{\log(5/7)}{4 \log(7)}}$.

3. (a) The slope of Line A is

$$m = \frac{5 - 4}{4 - 2} = \frac{1}{2}.$$

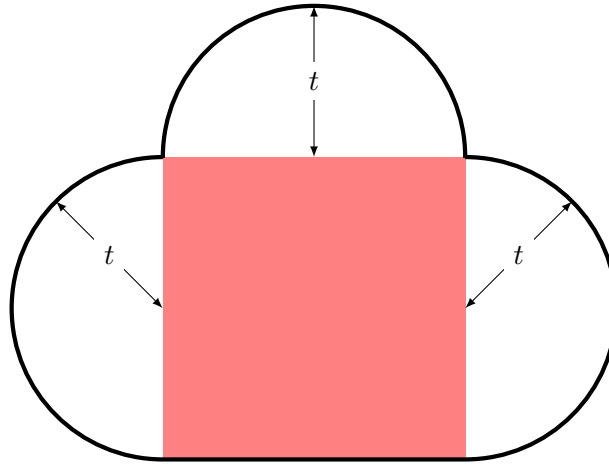
Thus Line A has equation $y = \frac{1}{2}x + b$ for some b . We find that value by plugging in either point $(x, y) = (2, 4)$ or $(4, 5)$. If we plug in the first point, we get $4 = \frac{1}{2}(2) + b$, or $b = 3$. Thus Line A has equation $\boxed{y = \frac{1}{2}x + 3}$ or $\boxed{y = 3 + x/2}$. Line A is shown on the left, below.

- (b) Line B has slope minus the slope of Line A, or $-\frac{1}{2}$, so it has equation $y = -\frac{1}{2}x + b$. We plug in $(x, y) = (0, 0)$ to find b : $0 = -\frac{1}{2}(0) + b$, or $b = 0$. Thus the equation of Line B is $\boxed{y = -x/2}$. Line B is shown with Line A on the left, below.



- (c) The point of intersection of Line A and the line $y - 3 = 0$ is where $y = 3$ (this is just the second line, slightly re-arranged). Solving $3 = 3 + x/2$ (since Line A is $y = 3 + x/2$), we get $x = 0$. Thus the point of intersection is $(x, y) = \boxed{(0, 3)}$. (The lines and the point of intersection are shown above on the right.)

4. We reproduce the picture of the garden here, with an added-in notation showing that the radius of the circles is t :



The radius of each semicircle is t , so the length of each side of the square is $2t$.

- (a) The area of the garden is the area of three semicircles of radius r plus the area of a square of side length $2r$. The area of a semicircle is half the area of a circle; that is, a semicircle of radius $r = t$ has area $\pi r^2/2 = \pi t^2/2$. Thus the area of the garden is

$$A = 3(\pi t^2/2) + (2t)^2 \quad \text{or} \quad \boxed{A = 3\pi t^2/2 + 4t^2}.$$

- (b) The perimeter of the garden is the perimeter of three semicircles of radius t plus the length of one side of the square (and we know the side length is $2t$). The perimeter of a semicircle is half the perimeter of a circle; that is, a semicircle of radius $r = t$ has perimeter $2\pi r/2 = \pi r = \pi t$. Thus the perimeter of the garden is $P = \boxed{3\pi t + 2t}$.

- (c) If the area of the square is 100, then since the area of the square is $(2t)^2 = 4t^2$, we get the radius of each semicircle is $t = 5$ (we just took the square root of $t^2 = 100/4 = 25$). Then from part (b), the perimeter of the garden is $3\pi(5) + 2(5) = \boxed{15\pi + 10}$.

5. (a) I've poured half of the 6 liters of red paint into can B, so can B now contains 9 liters of blue paint and 3 liters of red paint. What is the percentage of paint that is red in can B? It's simply

$$\frac{\text{amount of red paint in can B}}{\text{total amount of paint in can B}} \times 100\% = \frac{3 \text{ liters}}{9 + 3 \text{ liters}} \times 100\% = \frac{3}{12} \times 100\% = 25\%.$$

That is, can B is now $\boxed{25\%}$ red.

- (b) There were 12 liters in can B after my mixing experiment in part (a). Half of this is 6 liters, which I now add to the 3 liters of red paint remaining in can A. Thus there are $6 + 3 = \boxed{9 \text{ liters}}$ of paint now in can A.
- (c) So how much red paint is now in can A? The 3 liters from can A were 100% red, and the 6 liters coming from can B were 25% red (as we found in part (a)). Thus the amount of red paint in can A is

$$100\%(3 \text{ liters}) + 25\%(6 \text{ liters}) = (1)(3) + (0.25)(6) \text{ liters} = \boxed{4.5 \text{ liters}}.$$