



# Welcome To Math 34A!

## Differential Calculus

Instructor:

Trevor Klar, `trevorklar@math.ucsb.edu`

South Hall 6431X (Grad Tower, 6th floor, blue side, first door on the right)

Office Hours:

MTWR after class 2:00-3:00, and by appointment. Details on Gauchospace.

© 2017-22 Daryl Cooper, Peter Garfield, Ebrahim Ebrahim, Nathan Schley, and Trevor Klar

Please do not distribute outside of this course.

# A word problem to start off

1. The vertical mast of a yacht is 40 feet high. A rope runs in a straight line from the top to a pulley 30 feet horizontally from the base of the mast. How many feet long is the rope?

**Hint:** Draw a picture!

$A = 30$

$B = 40$

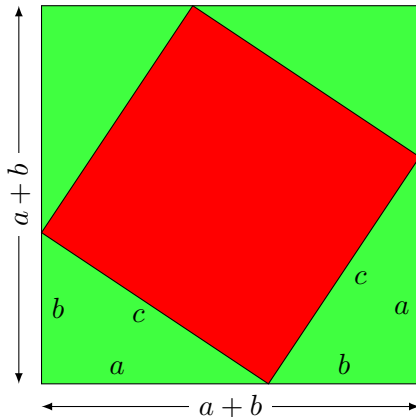
$C = 50$

$D = 60$

$E = 70$



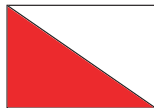
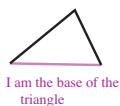
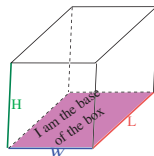
# Why Pythagorean Theorem works



## §4.2: Area and Volume

You need to know:

- Area of a rectangle = length  $\times$  width
- Area of a circle =  $\pi R^2$  ( $R$  = radius)
- Circumference of a circle =  $2\pi R$
- Area of a triangle = half base  $\times$  height =  $\frac{1}{2}bh$
- volume of rectangular box = (length  $\times$  width)  $\times$  height  
= (area of **base**)  $\times$  height



triangle = half a rectangle



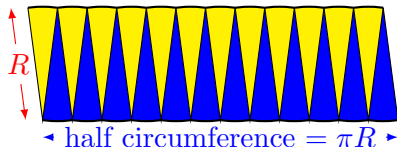
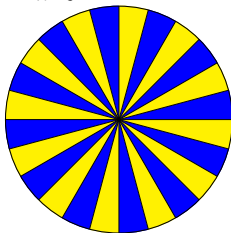
What is the (circumference of a circle) divided by the diameter?

$$A = R \quad B = 2\pi \quad C = \pi \quad D = \text{the what now?} \quad \boxed{C}$$

The definition of  $\pi$  is

$$\pi = \frac{\text{circumference of circle}}{\text{diameter}} = \frac{C}{2R},$$

so  $C = 2\pi R$ .



$$\text{Thus Area} = (R)(\pi R) = \pi R^2$$

# Applications

- 2.** A rectangular parking lot is to be made in the shape of a rectangle. It will have an area of 2000 square meters. Express the length of the parking lot in terms of the  $W = \text{width}$ .

$$A = (2000 - 2W)/2 \quad B = 2000/W \quad C = 2000 - W$$

$$D = \text{Other} \quad \boxed{B}$$

- 3.** The parking lot will be surrounded by a fence. Express the total length of the fence in terms of  $W$ .

$$A = 2000 + 2W \quad B = L + W \quad C = 4000W^{-1} + 2W \quad \boxed{C}$$

- 4.** The fence costs \$7 per meter. Express the total cost of all the fence in terms of  $W$ .

$$A = 7 \times 2000 \quad B = 7 \times 4000W^{-1} + 2W$$

$$C = 28000W^{-1} + 14W \quad \boxed{C}$$

# Applications II

- 5.** A rectangular poster is to have a total area of  $500 \text{ cm}^2$ . There is an empty margin where nothing is printed 6 cm wide at the top and 4 cm wide along the sides and bottom. The rest is the printed area.

**Hint:** Draw a picture! Name your unknowns!

- Express printed area in terms of width  $W$  and height  $H$  of the poster.

$$A = HW \quad B = (H - 8)(W - 8) \quad C = \text{Other}$$



- Express the area of the printed part in terms of the width  $W$  of the poster.

$$A = \text{got it!} \quad B = \text{working on it} \quad C = \text{help}$$

**Hint:** Express  $H$  in terms of  $W$ .

**3.2.41** Express the **total surface area** of a cube in terms of its **volume  $V$** .

Draw a picture! Name the unknowns!

$x$  = length of one side of cube



(area of each side) =  $x^2$

There are 6 sides so

$$\text{total surface area} = 6x^2$$

**Plan:**

As a first step find **total area** in terms of  $x$   
**total surface area** is

$$A = x^2 \quad B = 6x \quad C = x^3 \quad D = 6x^3 \quad E = 6x^2 \quad \boxed{E}$$

Now express  $x$  in terms of  $V$

$V$  = volume of cube =  $x^3$  so solve for  $x$

$$x = \sqrt[3]{V}$$

$$\text{sub for } x \text{ in } \boxed{1} \text{ get total surface area} = \boxed{6(\sqrt[3]{V})^2}$$



# Units: A Meaningless Calculation



# Units: A Meaningless Calculation

**Rule:** Only **add or subtract** things measured in same units

- 3 meters + 7 inches is NOT 10 of **anything**
- 2 days + 5 hours  $\neq$  7
- 3 nickels + 2 dimes  $\neq$  5

**BUT!** You can multiply or divide things in different units:

$$\text{average speed} = (\text{distance gone})/(\text{time taken})$$

$$(50 \text{ miles})/(1 \text{ hour}) = 50 \text{ (miles/hours)} = 50 \text{ miles per hour} = 50 \text{ mph}$$

You must **multiply or divide** the **units** too !  
**miles divided** by **hours** is **miles per hour**

When a problem has **mixed units** like **miles** and **feet** or **years** and **seconds** **decide** what units you will use (like **miles** and **seconds**) and convert everything into those units, or

# SUFFER

## Units conversions

**6.** How fast does your hair grow... in mph?

$$A = 10^{-3} \quad B = 10^{-4} \quad C = 10^{-5} \quad D = 10^{-6} \quad E = 10^{-8}$$

???

I don't know either.

**7.** How fast does your hair grow... in cm/month?

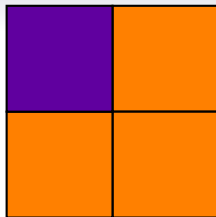
$$A = \text{faster} \quad B = 10 \quad C = 1 \quad D = 1/10 \quad E = \text{slower}$$

C

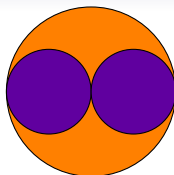
## Conversions:

$$2.54 \text{ cm} = 1 \text{ inch} \quad 12 \text{ inches} = 1 \text{ foot} \quad 5280 \text{ feet} = 1 \text{ mile}$$

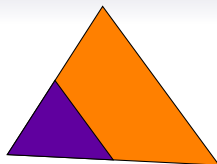
$$30 \text{ days} = 1 \text{ month} \quad 24 \text{ hours} = 1 \text{ day}$$



The **large square** is 2 times the base of the **small square**. It has  $2 \times 2 = 4$  times the area.



The **large circle** is 2 times the size of the **small circle**. It has 4 times the area.



The **large triangle** is 2 times the size of the **small triangle**. It has 4 times the area.

When you **double** the size of a shape the **area** is multiplied by 4  
If you make a shape **3** times larger the area is **9** times as much  
 **$x$**  times larger gives  **$x^2$**  times as much area

When you **double** the size of a solid object,  
the volume is **8** times as much

What is going on?

An area has **two dimensions** : **length** and **width**.

Both of these get doubled so area is doubled **twice** so multiplied by  $2^2$

A solid object has **three dimensions** : **length**, **width** and **height**.

Each dimension is doubled so volume doubled **three times** :  
multiplied by  $2^3$

Make a solid object  **$x$**  times bigger, volume is  **$x^3$**  times as much.

volume grows as **the cube** of the linear dimensions

**Conclusion** Volume and area grow at **different rates**

As you make an object bigger the volume gets bigger faster

# Consequences!

Many important consequences read section 4.4

Why do babies get cold faster than adults?

Why can an ant pick up something weighing 10 times its own weight?

Why are humans 60 feet tall mathematically impossible?

Why can't you build a jumbo jet twice as big?

Why are my lungs crinkly?

A planet made of rock behaves like a liquid

Why can a fly walk on the ceiling, but I can't?

Why is water so dangerous to an insect but not gravity?

Paraphrasing J.B.S. Haldane Falling down a thousand yard mine shaft

A mouse walks away

A rat is killed

A man is broken

**8.** An oil leak!

- Oil is leaking from an oil tanker at the rate of 4000 liters per hour.
- 8 liters of oil spread out over 10 square meters of ocean surface.
- A SQUARE oil slick forms.
- Express the length,  $X$ , of one side of the square oil slick as a function of the time  $t$  (in hours) the tank has been leaking.
- After how many hours will the oil slick be a square with side length 2 kilometers?

## PLAN:

- (i) How many liters of oil on ocean after  $t$  hours?
- (ii) How much area does this oil cover?

Answer:  $t = 800$  hours

# Exercise

**9.** When you substitute  $x = y + 3$  into  $x^2 - 6x + 8$  you get...

$$A = y^2 - 6y - 1 \quad B = y^2 + 35 \quad C = y^2 - 6y + 35 \quad D = y^2 - 1$$

Answer: D

**10.** Can you check your answer to the previous question?

**Hint:** What are the expressions when  $y = 1$ ?

What is  $x$  when  $y = 1$ ?

When  $y = 1$ ,  $x = 4$  so  $x^2 - 6x + 8 = 4^2 - 6(4) + 8 = 0$ .

The other expressions are...

$$A = y^2 - 6y - 1 = -6 \qquad B = y^2 + 35 = 36$$

$$C = y^2 - 6y + 35 = 30 \qquad D = y^2 - 1 = 0$$



That's it. Thanks for being here.

