Welcome To Math 34A!
Differential Calculus

Instructor:

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South Hall 6701

Office Hours:

T R 11-11:50, T 3:45-4:35 Details on Gauchospace.

8. An oil leak!

- Oil is leaking from an oil tanker at the rate of 4000 liters per hour.
- 8 liters of oil spread out over 10 square meters of ocean surface.
- A SQUARE oil slick forms.

- Express the length, X , of one side of the square oil slick as a function of the time t (in hours) the tank has been leaking.
- After how many hours will the oil slick be a square with side length 2 kilometers?

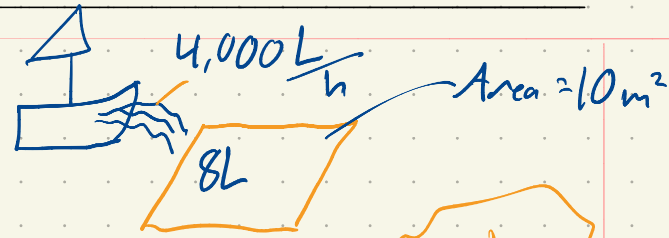
solve for T

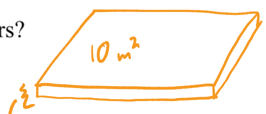
$\underbrace{\text{side length 2 kilometers}}_{x = 2000}$

PLAN:

(i) How many liters of oil on ocean after t hours?

(ii) How much area does this oil cover?





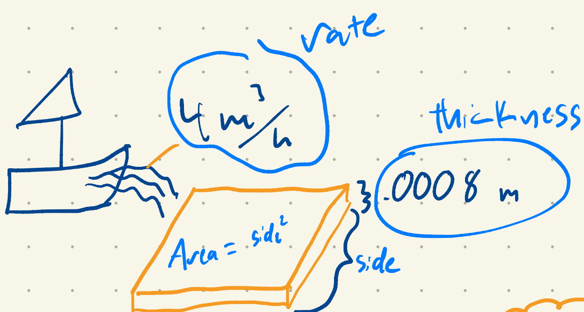
$1 m^3 = 1000 L$ = 1

$$\frac{4{,}000 \cancel{L}}{h} \cdot \frac{1 m^3}{1000 \cancel{L}} = \frac{4 m^3}{h}$$

convert 8L to m³

$$8 L \cdot \frac{1 m^3}{1000 L} = .008 m^3$$

$$\frac{.008 m^3}{10 m^2} = .0008 m$$ = .8mm



total = Area x thickness

Area = side²

$$\text{rate} = \text{total} \div \text{time} = \text{Area} \times \text{thickness} \div \text{time} = \underbrace{\text{side}^2 \times \text{thickness} \div \text{time}}$$

$$\frac{4 \cancel{m^3}}{\cancel{h}} = \frac{(x\,m)^2(.0008\ m)}{(t\ h)} = \frac{.0008x^2}{t} \cdot \frac{\cancel{m^3}}{\cancel{h}}$$

$$4 = \frac{.0008x^2}{t}$$

$$4t = .0008x^2$$

$$x^2 = \frac{4t}{.0008} = \frac{1}{.0002}t$$

$$\boxed{x = \sqrt{\frac{t}{.0002}}}$$

2×10^3

$$2000 = \sqrt{\frac{t}{.0002}}$$

$$(2 \times 10^3)^2 = \frac{t}{2 \times 10^{-4}}$$

$$(4 \times 10^6)(2 \times 10^{-4}) = t$$

$$8 \times 10^2 = t = \boxed{800}$$

Warm-up

How many times do we need to double 1 to get the following numbers?

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- 4
- 8
- 32
- 1
- $\frac{1}{2}$

Warm-up

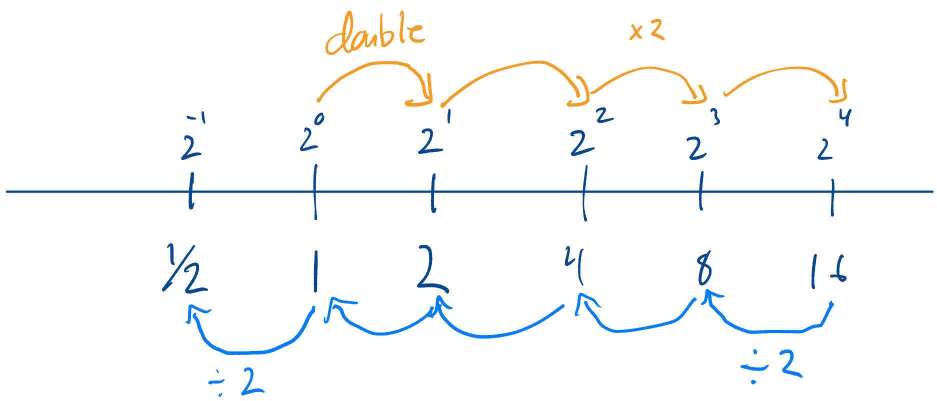
How many times do we need to double 1 to get the following numbers?

- 4 $\boxed{2}$
- 8 $\boxed{3}$
- 32 $\boxed{5}$
- 1 $\boxed{0}$
- $\frac{1}{2}$ $\boxed{-1}$

Warm-up

How many times do we need to double 1 to get the following numbers?

- 4
- 8
- 32
- 1
- $\frac{1}{2}$



Straight Lines (§6.1)

Calculus is about derivatives (Math 34A) and integrals (Math 34B).

A derivative is the slope of a line.

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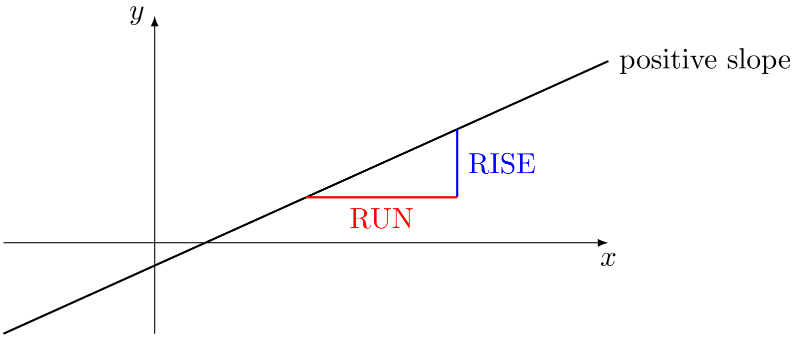

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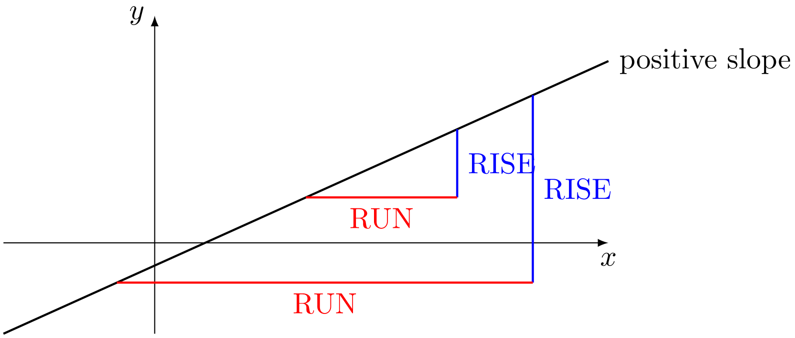

Examples

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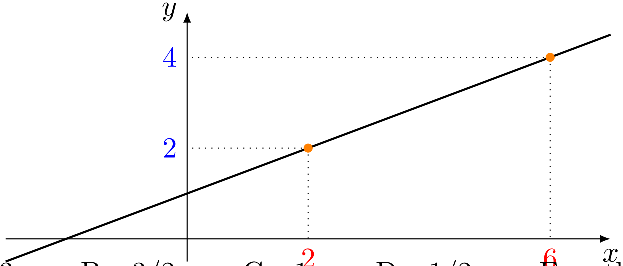

A= 3 B= 3/2 C= 1 D= 1/2 E= other

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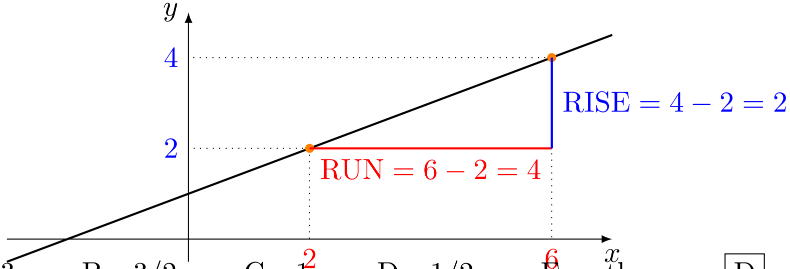

A= 3 B= 3/2 C= 1 D= 1/2 E= other $\boxed{\text{D}}$

slope = # units UPWARDS you move for each unit you move TO THE RIGHT

Examples

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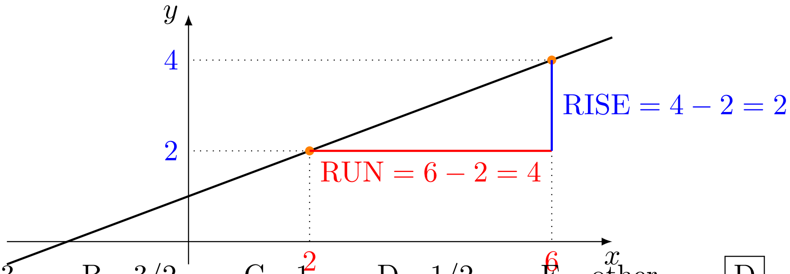

A= 3   B= 3/2   C= 1   D= 1/2   E= other $\boxed{\text{D}}$

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Idea: RISE $=$ slope $\times$ RUN

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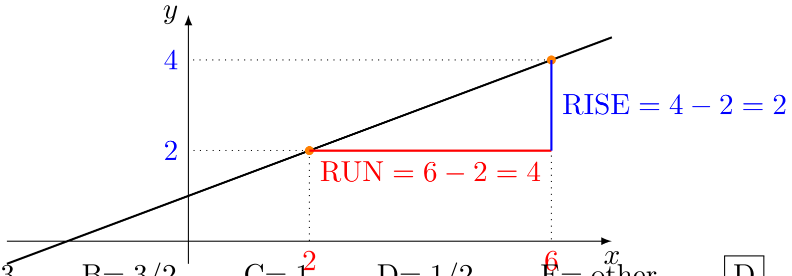

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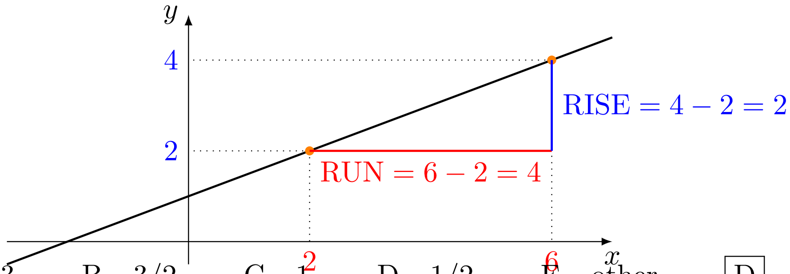

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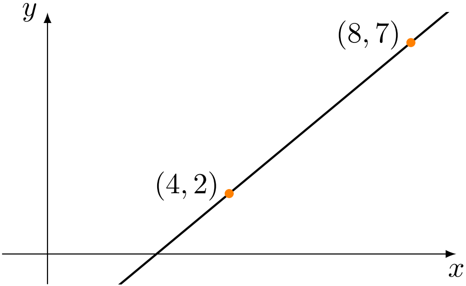
slope = # units UPWARDS you move for each unit you move TO THE RIGHT

Idea: RISE = slope × RUN So if RUN = 1 then RISE=slope.

A 10% gradient on a mountain road is a slope of 1/10. It means for every 10 feet you move horizontally you go up (or down) 1 foot

Examples (page 2)

2. What is the slope here:



A$=5/4$ B$=4/5$ C$=1/4$ D$=4$ E$=5$

Examples (page 2)

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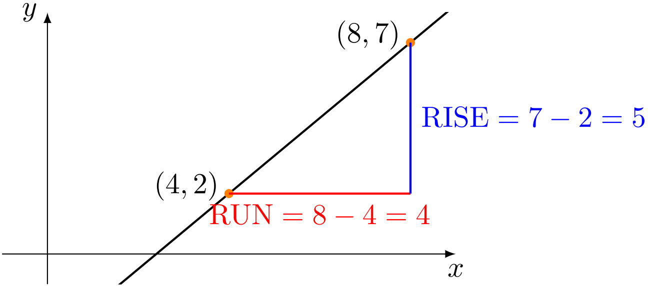

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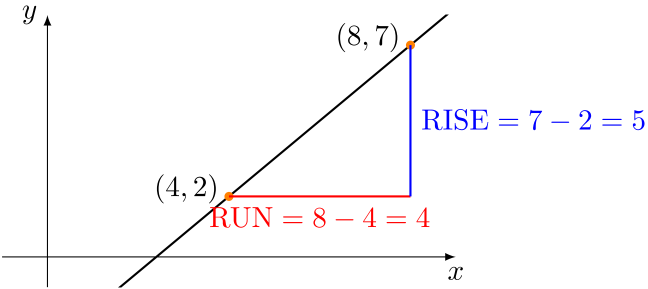

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$\boxed{\text{A}}$

Examples (page 3)

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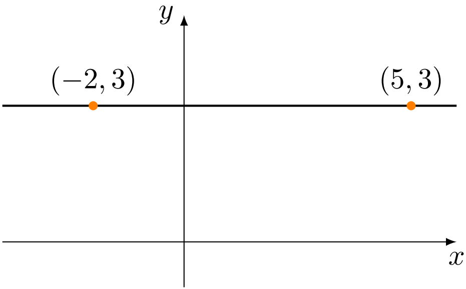

A$=0$ B$=7$ C$=5/3$ D$=\infty$ E$=3/5$

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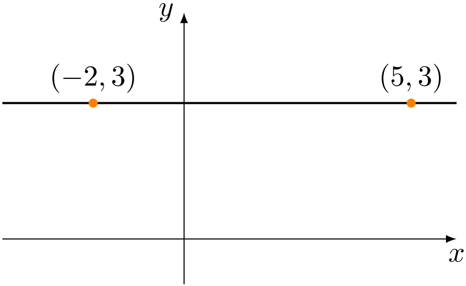

RISE $= 3 - 3 = 0$

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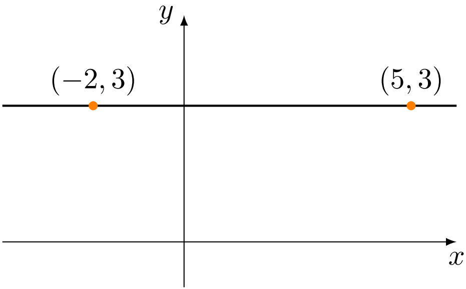

$\text{RISE} = 3 - 3 = 0$

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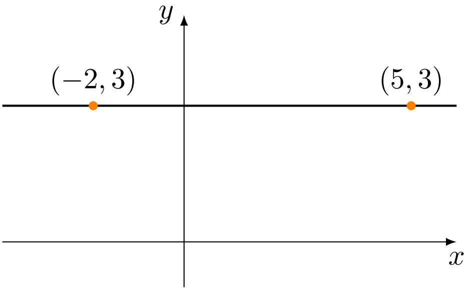


$\text{RISE} = 3 - 3 = 0$

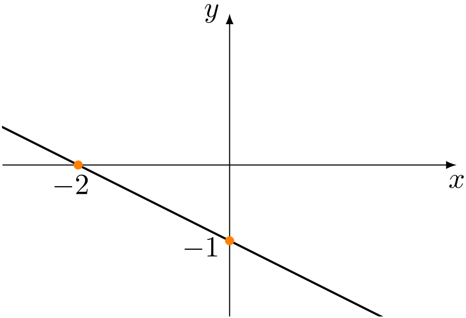
$\text{RUN} = \text{who cares?}$

$$\text{slope} = \frac{0}{\text{something}} = 0$$

A$= 0$ B$= 7$ C$= 5/3$ D$= \infty$ E$= 3/5$ $\boxed{\text{A}}$

Examples (big finish!)

4. What is the slope here:



A$= -1$ B$= 1$ C$= 1/2$ D$= -1/2$ E$= -2$

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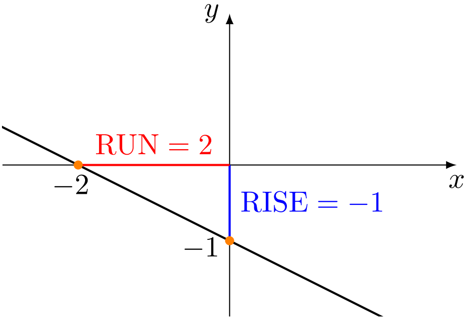

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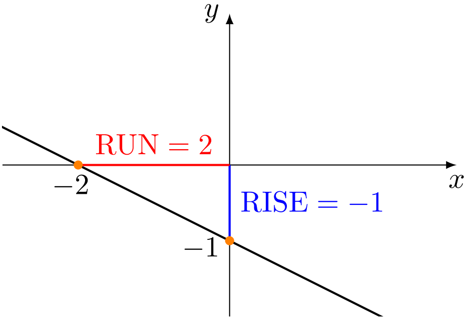

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General Case

5. A line goes through two points: (x_0, y_0) and (x_1, y_1). Find the slope of this line. Draw a picture!

A= $y_1 - y_0$ B= $(y_1 - x_1)/(y_0 - x_0)$

C= $(y_1 - y_0)(x_1 - x_0)$

D= $(y_1 - y_0)/(x_1 - x_0)$ E= Shirley you're joking

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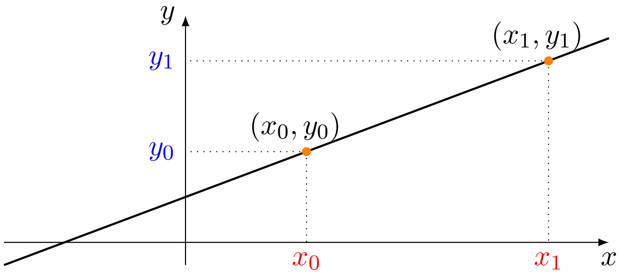

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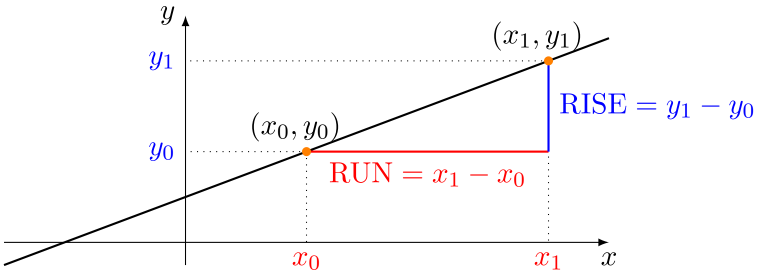

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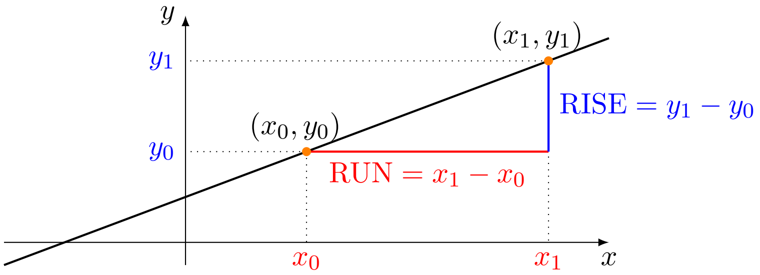
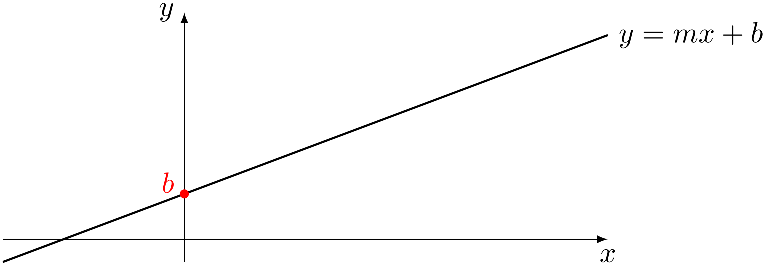


$$\text{Slope} = \frac{\text{RISE}}{\text{RUN}} = \frac{y_1 - y_0}{x_1 - x_0}$$

The Equation of a Line

The Slope Intercept Form

The **slope intercept** equation of a straight line is
$\boxed{y = mx + b}$.



m = the **slope**. CRUCIAL for calculus.

b = where the line crosses the y-axis (the "y-intercept").

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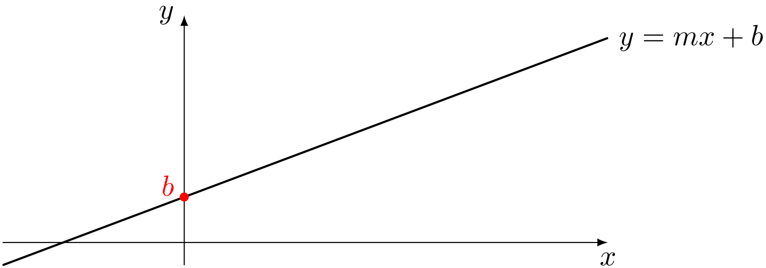

$m =$ the **slope**. CRUCIAL for calculus.

$b =$ where the line crosses the y-axis (the "y-intercept").

WHY? Because when you plug in $x = 0$, you get $y = b$.

Example

6. Find the equation of the line $\boxed{y = mx + b}$ through the points $(1, 3)$ and $(7, 5)$.

Plan: Find m, then find b.

- What is m?

  A$= 1$ B$= 3$ C$= 5$ D$= 1/3$ E$= 2$

Example

6. Find the equation of the line $\boxed{y = mx + b}$ through the points $(1, 3)$ and $(7, 5)$.

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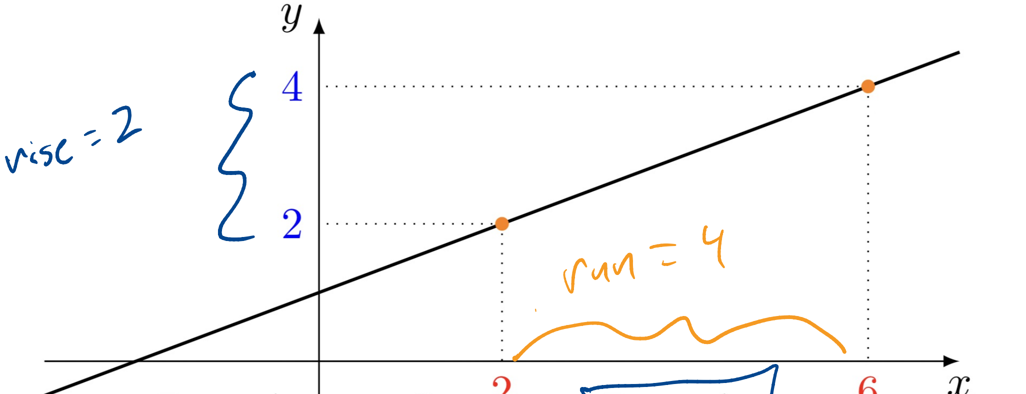

A= 3 B= 3/2 C= 1 $\boxed{\text{D}= 1/2}$ E= other

$$\frac{\text{rise}}{\text{run}} = \frac{2}{4} = \frac{1}{2}$$

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Plan: Find m, then find b.

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  So $y = \frac{1}{3}x + b$. What is b? Plug in either point!

- What do you get for b?

  A= 1/3 B= 4/3 C= 7/3 D= 8/3 E= 10/3

$3 = \frac{1}{3}(1) + b$

$3 = \frac{1}{3} + b$

$3 - \frac{1}{3} = b$

$\frac{9}{3} - \frac{1}{3} = b$

$\boxed{\frac{8}{3} = b}$

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  Can we check?

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You Try It

7. A line has slope $1/2$ and goes through the point $(2, 5)$. What is the y-coordinate of the point on this line where $x = 6$?

A$= 3$ B$= 4$ C$= 5$ D$= 6$ E$= 7$

You Try It

7. A line has slope $1/2$ and goes through the point $(2, 5)$. What is the y-coordinate of the point on this line where $x = 6$?

A$= 3$ B$= 4$ C$= 5$ D$= 6$ E$= 7$

Plan: 1. Find equation of the line.
2. Plug in $x = 6$ to find y.

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7. A line has slope $1/2$ and goes through the point $(2, 5)$. What is the y-coordinate of the point on this line where $x = 6$?

A= 3 B= 4 C= 5 D= 6 E= 7 $\boxed{\text{E}}$

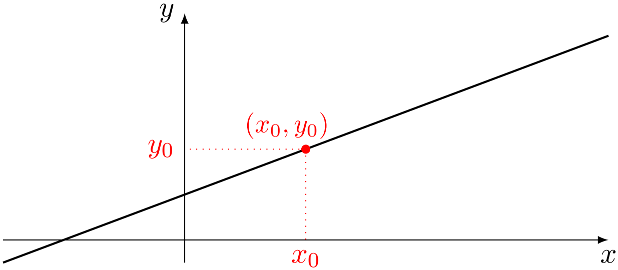
Plan: 1. Find equation of the line.
2. Plug in $x = 6$ to find y.

Another Equation of a Line

The Point-Slope Form

The **point slope** equation of a straight line is

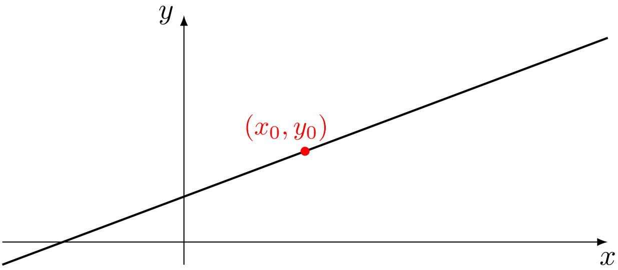
$\boxed{y = y_0 + m(x - x_0)}$.



m = the **slope**. Still CRUCIAL for calculus.

(x_0, y_0) = any point on the line.

Why Does This Work?



(x, y) lies on the line exactly when

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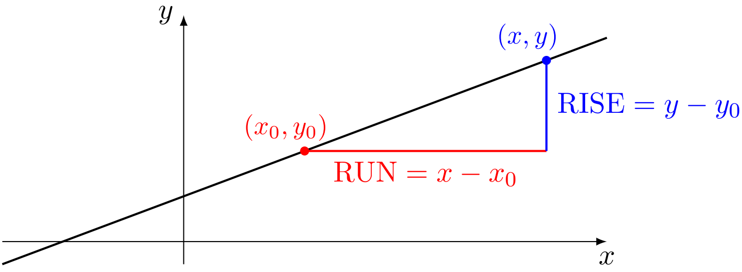

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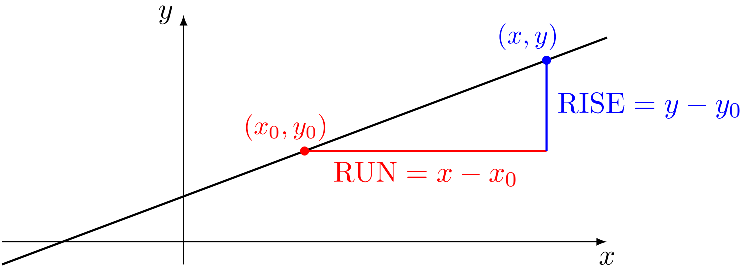

(x, y) lies on the line exactly when $\dfrac{y - y_0}{x - x_0} = m$

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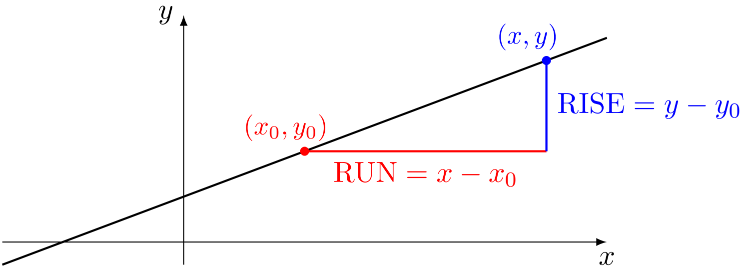

(x, y) lies on the line exactly when
$$\frac{y - y_0}{x - x_0} = m$$
$$y - y_0 = m(x - x_0)$$

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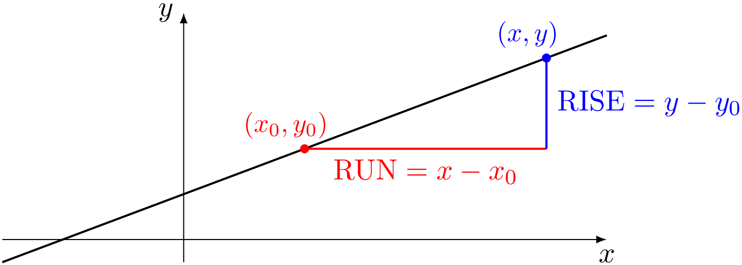


(x, y) lies on the line exactly when

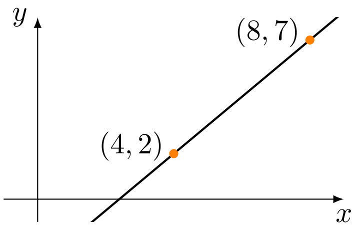
$$\frac{y - y_0}{x - x_0} = m$$

$$y - y_0 = m(x - x_0)$$

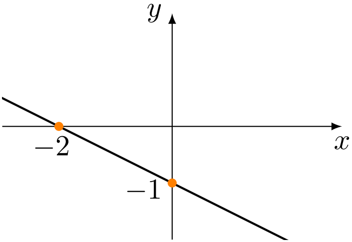
$$y = y_0 + m(x - x_0)$$

Examples

8. Find the equations of these lines (whose slopes we've already found):



$m = 5/4$



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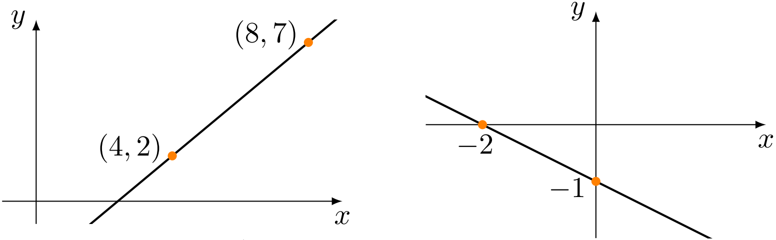

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$$y - 2 = \frac{5}{4}(x - 4)$$

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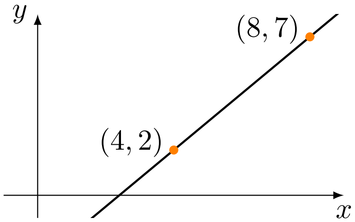

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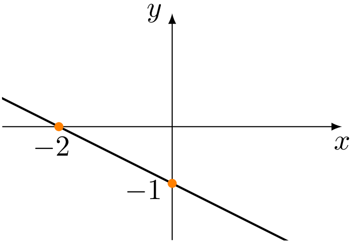
$$y - 2 = \frac{5}{4}(x - 4)$$

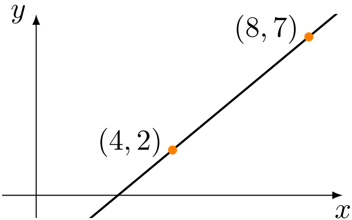
$$y = \frac{5}{4}x - 3$$



$$m = -1/2$$

Examples

8. Find the equations of these lines (whose slopes we've already found):



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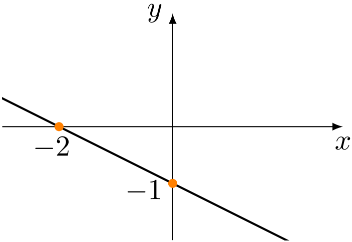

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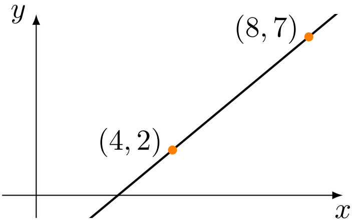

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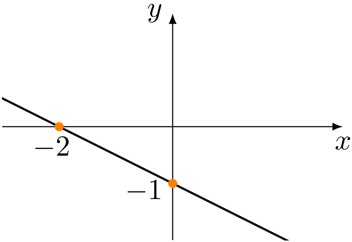

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$$y = -\frac{1}{2}x - 1$$

Examples

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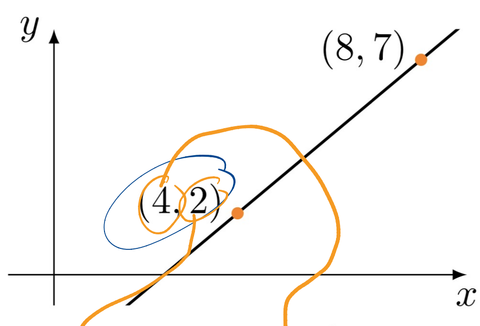

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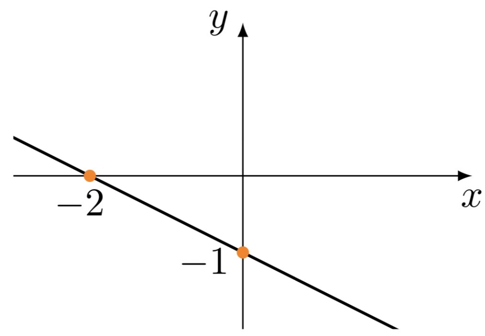

$m = -1/2$

$y = y_0 + m(x - x_0)$

$\boxed{y = 2 + \frac{5}{4}(x - 4)}$ Point slope

$y = 2 + \frac{5}{4}x - 5$

$\boxed{y = \frac{5}{4}x - 3}$ y-int

And. . . ?

Yes, but what's this got to do with calculus?

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Derivatives are about **rate of change** and that is what **slope** is!

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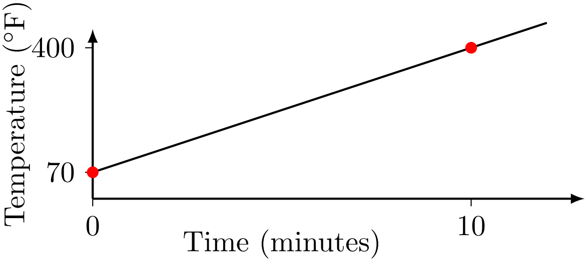
Example: This graph shows the temperature in an oven as it heats up:



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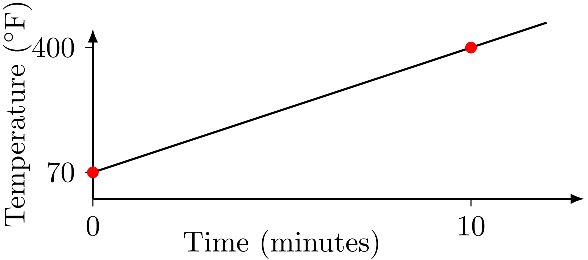

9. How quickly (in °F/min) is the oven heating up?

A= 70 B= 10 C= 40 D= 33 E=Other

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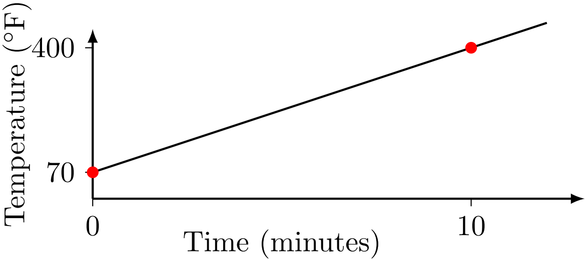

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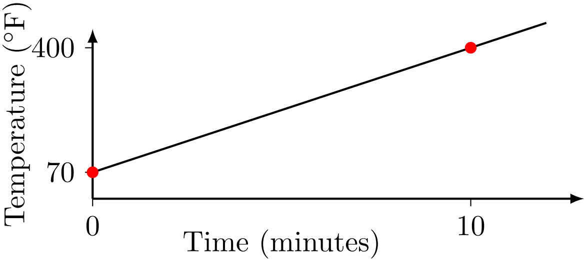

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One More Example

10. Where does the line $y = 1 + x$ cross the line $y = 3 - x$? Find both the x and y coordinates of the crossing point.

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Plan:

1. Draw a picture! showing two straight lines crossing.
2. Solve the two simultaneous equations
3. THINK why this gives the answer!

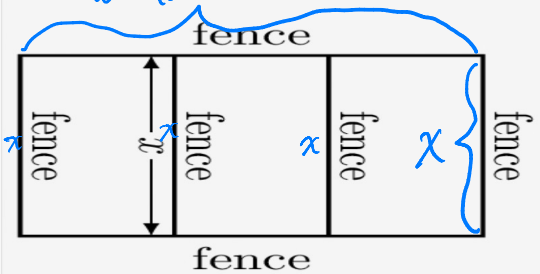
HW03: Problem 8

(1 point) local/HW04/Cooper_3_2_30-revised.pg

This set is **visible to students.**

Cooper 3.2.30

$w = \frac{x}{1000}$



$\text{Area} = 1000\ m^2$

$1000 = wx$

$w = \frac{1000}{x}$

A farmer wants to make a rectangular field with a total area of 1000 square meters. It is surrounded by a fence. It is divided into 3 equal areas by fences as shown. Express the total length L of all the fence required in terms of the length x of one of the fences which divide the field.

L(x)=

$L = 2w + 4x$

Online Math Lab resources for this problem:

- Word Problems
- High School Math Review

$L = 2\left(\frac{1000}{x}\right) + 4x$

$\boxed{L = \frac{2000}{x} + 4x}$

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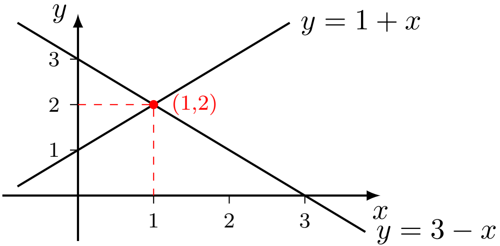

That's it. Thanks for being here.