Peaks and Valleys

Key Idea: What must the slope of the graph be at the highest and lowest points?

1. Find the coordinates of the vertex (lowest point) of the parabola $y = x^2 + 4x - 72$.

$f(x) = y = x^2 + 4x - 72$. x when $f'(x) = 0$

$f'(x) = 2x + 4 = 0$

$x = -2$

find y at this x

$y = f(2) = 4 - 8 - 72 = -76$

$x =$ | -2

$y =$ | -76

2. You have 100m of fencing to make a pen on a farm, and one of sides of your pen is provided by the wall of your barn. So you only need fencing for three sides. If ℓ and w are the dimensions (length and width) of your pen, the total fencing equation below relates the length and width

$$2\ell + w = 100$$ ← $w = ?$ $\quad w = 100 - 2\ell$

Here there are two lengths and only one width because the barn wall serves as the second width.

(a) You know the area of the pen in terms of ℓ and w. Express the area of the pen in terms of ℓ only.

$A = \ell \cdot w = \ell(100 - 2\ell) = 100\ell - 2\ell^2$

$A(\ell) =$ | $100\ell - 2\ell^2$ m^2

(b) Find the length that results in the largest area $A(\ell)$ for your pen.

$A'(\ell) = 100 - 4\ell = 0$ ← finding the best ℓ

$\ell = 25$

$\ell =$ | 25 m

(c) Use your answer in part (b) to find the maximum area for your pen.

Best $\ell = 25$, so best $w = 100 - 2(25) = 50$

$A = \ell \cdot w = 25 \cdot 50 = 1250$

$A_{\max} =$ | 1250 m