

## Homework 1

**Definition.** Let  $\Sigma$  be a subset of  $(X, d)$ . We say  $\Sigma$  is **bounded** if there exists  $x_0 \in X$  and  $0 < r < \infty$  such that  $\Sigma \subset B_r(x_0)$ .

1. Prove that  $\Sigma$  is bounded if and only if there exists  $L > 0$  such that

$$d(x, x') \leq L$$

for any  $x, x' \in \Sigma$ .

2. Suppose  $\Sigma$  is bounded and  $A \subset \Sigma$ .

- (a) Prove that  $A$  is bounded.

**Definition.** Define  $\text{diam}(\Sigma) = \sup_{\delta, \delta' \in \Sigma} d(\delta, \delta')$ .

- (b) Prove that  $\text{diam}(A) \leq \text{diam}(\Sigma)$ .

3. Let  $(X, d)$  be a metric space, and let

$$d_p((x, y), (x', y')) = d(x, x') + d(y, y').$$

- (a) Show that  $d_p$  is a metric on  $X^2$ .

- (b) Prove that  $d_p : X^2 \times X^2 \rightarrow (\mathbb{R}, \text{MKM})$  is continuous.

4. Give examples to show that if  $B_r(x) = B_s(y)$ , it need not be true that  $r = s$  or  $x = y$ .