Office Hours!

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Mondays 2–3PM
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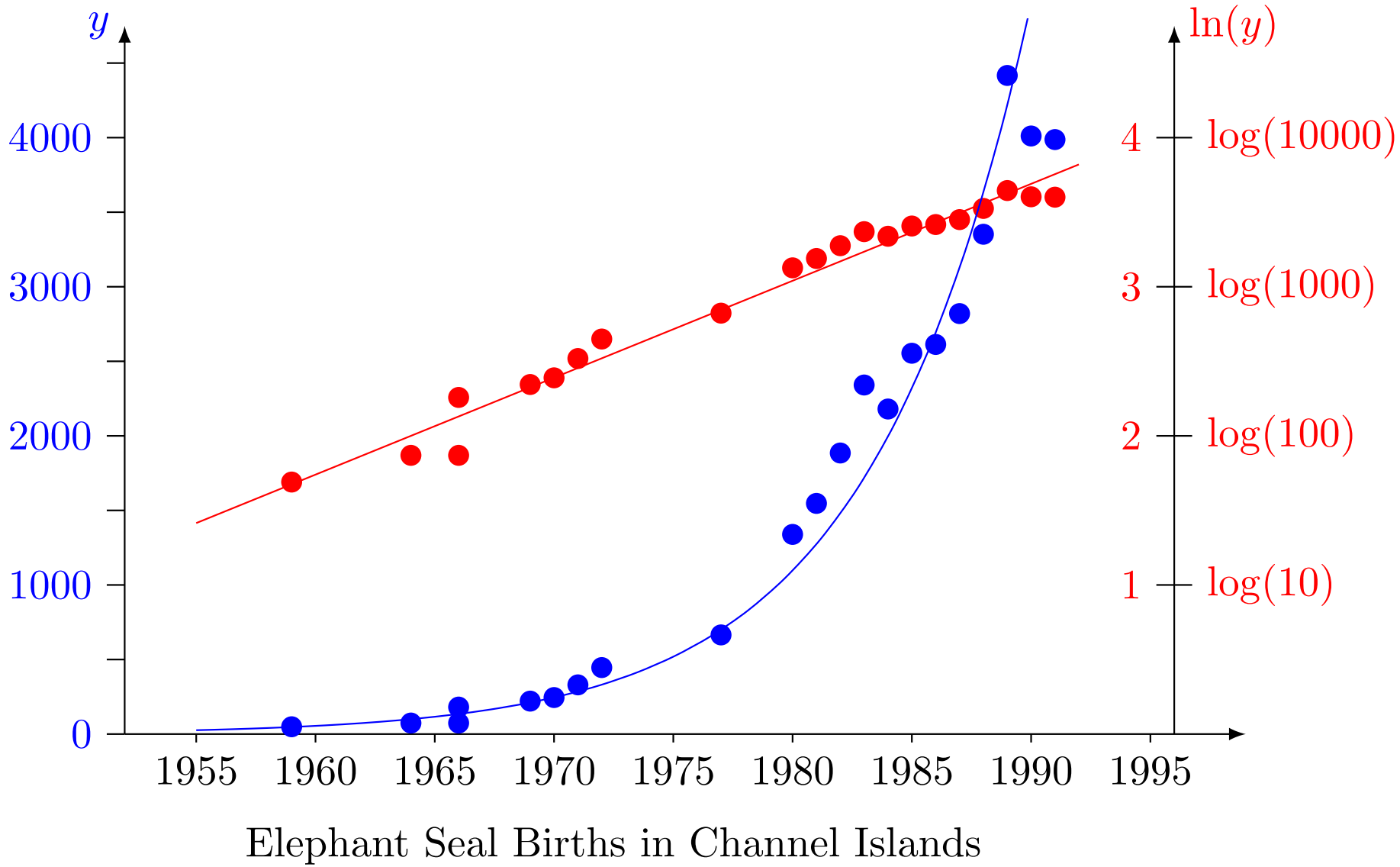
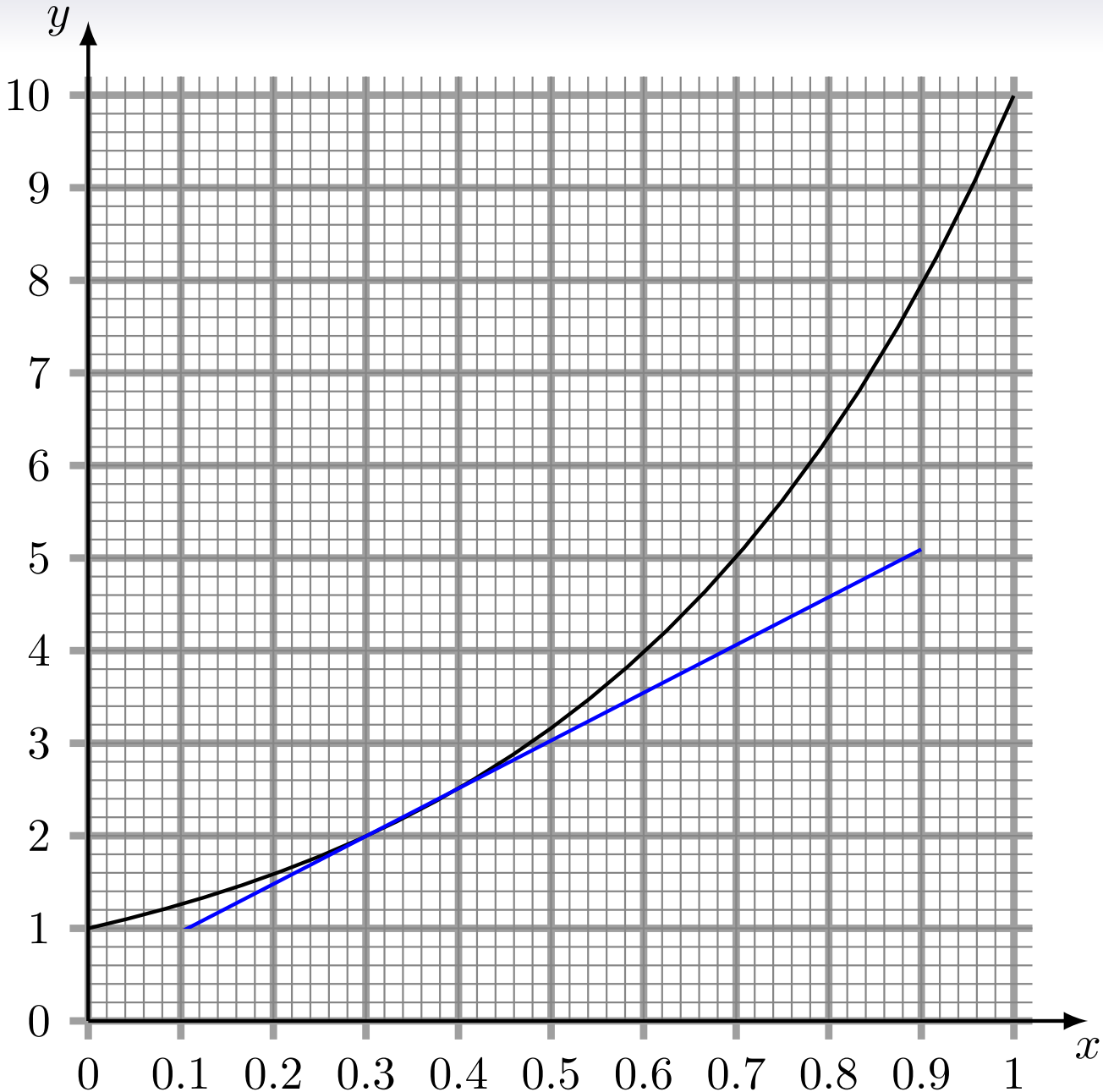


Elephant Seal Births in Channel Islands

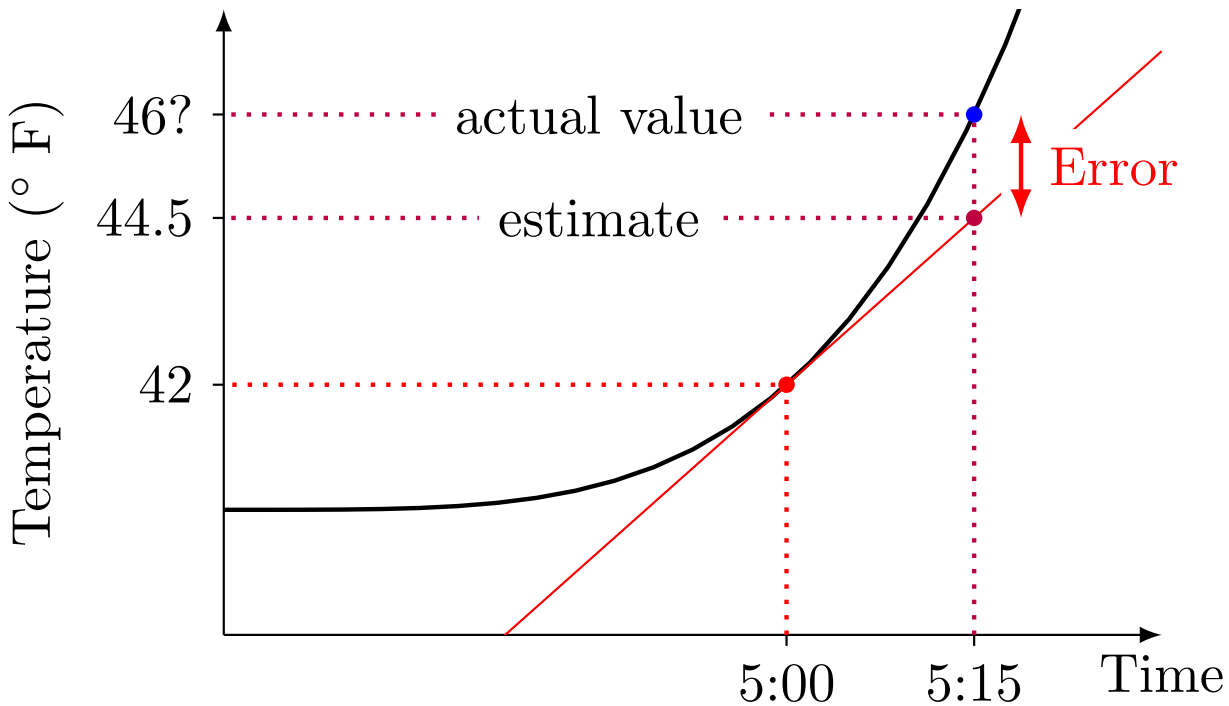

y
10
9
8
7
6
5
4
3
2
1
0
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
x

§8.6: Tangent Line Approximation

Question: At 5am the temperature is 42° F and increasing at a rate of 10° F per hour. Which of the following do you think is closest to the temperature at 5:15am?

A= 2.5° F B= 52° F C= 43.5° F D= 44.5° F E= 5.15° F

Answer: D



Continuing this example

Same set-up:

- $f(x)$ = temperature at time x hours after midnight
- $f(5) = 42$ (42° F at 5:00am)
- $f'(5) = 2$

(1) Find the equation of tangent line to $y = f(x)$ at $x = 5$.

A $y = 5x + 42$ B $y = 2x + 5$ C $y = 2(x - 5) + 42$
D $y - 5 = 2(x - 42)$ E $y - 42 = 2x - 5$

Answer: C

(2) Use this to predict the approximate temperature at 4am.

A= 40 B= 41 C= 42 D= 43 E= 44 A

(3) The tangent line approximation is used to estimate the temperature at the following times. Which do you think is most accurate?

A 4am B 4:50am C 5:25am D 6am E midnight B

Tangent Line Approximation

To do a tangent line approximation:

(i) Find the equation of the tangent line.

(ii) Plug in the required value(s) into this equation.

Suppose $f(4) = 2$ and $f'(4) = 3$.

(a) The equation of the tangent line to $y = f(x)$ at $x = 4$ is $y =$?

A= $4x - 14$ B= $3x - 10$ C= $2x - 6$
D= $3x - 4$ E= $2x - 5$

B

(b) Use this tangent line approximation to estimate $f(4.1)$.

A= 2.3 B= 1.7 C= 2.6 D= 1.4 E= 2 A

(c) Use the tangent line approximation to estimate the value of x which gives $f(x) = 2.9$.

A= 4.9 B= 4.1 C= 2.9 D= 4.1 E= 4.3 E

Standard Estimation Problem

Question: Approximate $\sqrt{26}$.

A= 0.1 B= 5.01 C= 5.05 D= 5.1 E= 5.2 $\boxed{\text{D}}$

Hint: If $g(x) = \sqrt{x}$, then $g'(25) = 1/10$ and $g(25) = \sqrt{25} = 5$.

Better estimate: $\sqrt{26} \approx 5.09902$, so the error in the tangent line approximation here is

$$\text{error} \approx 5.1 - 5.09902 \approx 0.001$$

This is a percentage error of only 0.02%.

Another Example:

- $f(t) =$ number of grams of a chemical reagent after t seconds
- We're told $f(0) = 20$ and $f'(0) = -3$

Question: Roughly how many grams are there after t seconds?

A= $4 - 3t$ B= $20 - 3t$ C= $20 - 4t$ D= $20 + 4t$ E= $32 - 3t$

Answer: B

Lake Cachuma (a linear approximation)

- Lake Cachuma was completed in 1950. *really completed 1953*
- It originally had a capacity of 205,000 acre feet (this is volume).
- In 2010 it has a capacity of approximately 190,000 acre-feet as a result of the accumulation of silt in the reservoir.
- $f(t) =$ capacity in acre-feet of Cachuma lake t years after 1950.

(1) Write down a linear approximation from this information for $f(t)$.

A= $205,000 - 15,000t$ B= $190,000 + 250t$ C= $205,000 - 250t$

D= $190,000 - 250t$ E= $190,000 - 125t$ **C**

(2) Which of the following years is the best estimate for when 10% of its original capacity will have been lost due to silt?

A= 2027 B= 2032 C= 2037 D= 2042 E= 2047 **B**