

Midterm Practice Problems

1. Find the general solution to the following equations

(a) $y'' + 3y' + 2y = 0$

(b) $\frac{dr}{d\theta} = \frac{r^2}{\theta}$

(c) $y' + y^2 \sin x = 0$

(d) $w' + w = 3t$

(e) $(2xy - 3x^2) + (x^2 + 1)y' = 0$

(f) $u' = u^2 e^x$

(g) $tv' - v = t^2 e^{-t}$

(h) $(2t - 2y)y' = 2y - 2t$

(i) $9z'' + 6z' + z = 0$

2. Solve the following IVPs

(a) $x dx + y e^{-x} dy = 0; y(0) = 1$

(b) $y' = x e^{\sin x} + y \cos x; y(0) = 3$

(c) $y'' - 2y' + 5y = 0; y\left(\frac{\pi}{2}\right) = 0, y'\left(\frac{\pi}{2}\right) = 2$

3. Prove that t^a and t^b are linearly independent functions if $a \neq b$.

4. Given that $y_1(t) = t^2$ is a solution to

$$t^2 y'' - 4t y' + 6y = 0, \quad t > 0,$$

find a second solution $y_2(t)$.

5. Given that $y_1(t) = e^t$ is a solution to

$$(t - 1)y'' - ty' + y = 0, \quad t > 1,$$

find a second solution $y_2(t)$.