Student Name: _____

PERM: _____

Circle the section you ATTEND (if you are enrolled a different section, note which one):

Kyle:	**Tue:8am**	**Tue:4pm**	**Tue:7pm**	
David:	**Tue:5pm**	**Tue:6pm**		
Yihan:	**Mon:4pm**	**Mon:5pm**	**Mon:6pm**	**Mon:7pm**
Tom:	**Tue:8am**	**Tue:4pm**	**Wed:8am**	
Matt:	**Tue:5pm**	**Tue:6pm**	**Tue:7pm**	

Math 4B, Midterm 2, Spring 2017
Version D

Instructions: Read the instructions for each question carefully. No calculators, cell phones, or other electronic devices are permitted. No notes or textbooks. Academic dishonesty will not be tolerated. Show your work, write legibly, and circle your answers.

Question	Points	Score
1	10	
2	10	
3	9	
4	9	
5	3	
Total:	41	

I understand UCSB's policies regarding academic dishonesty, and I certify that this test was taken with academic integrity.

Sign and date: _____

1. (10 points) Find the general solution to the ODE:

$$y'' + 6y' + 10y = 13e^{2t}$$

2. (10 points) Solve the initial value problem:

$$\begin{aligned} x' &= 2x + 4y \\ y' &= 3x + 3y \end{aligned}, \qquad x(0) = 1, \quad y(0) = 8$$

3. (9 points) A 10 kg mass is attached to the end of a spring with spring constant $k = 150$ N/m.

   (a) If there is no damping force, at what frequency will the mass naturally oscillate?

   (b) If we add a dashpot to the system with a damping constant of $b = 80$ N/(m/s), will the system be over-damped or under-damped?

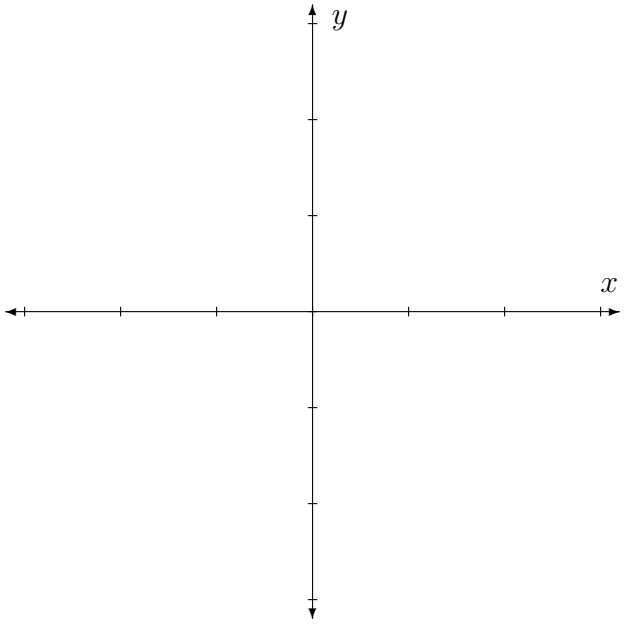
   (c) If we keep the damping consant of $b = 80$ N/(m/s) from part (b) but replace the spring, what new spring constant k will cause the system to be critically damped?

4. (9 points) Consider the non-linear system

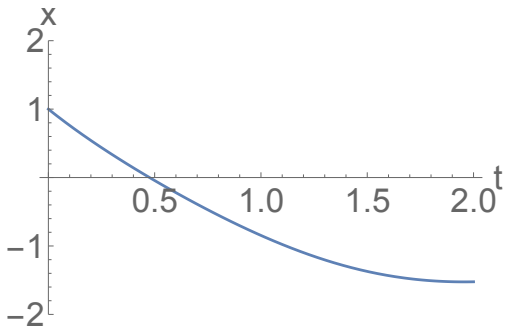
$$\frac{dx}{dt} = -x - y$$
$$\frac{dy}{dt} = 1 - x^2$$

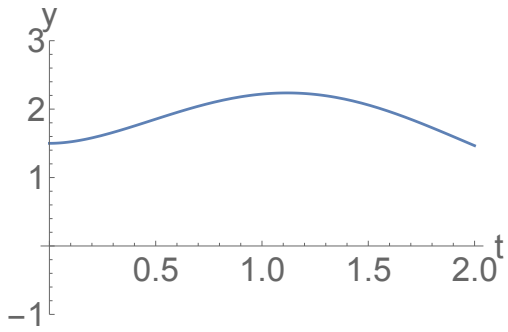
(a) List all equilibrium points for this system.

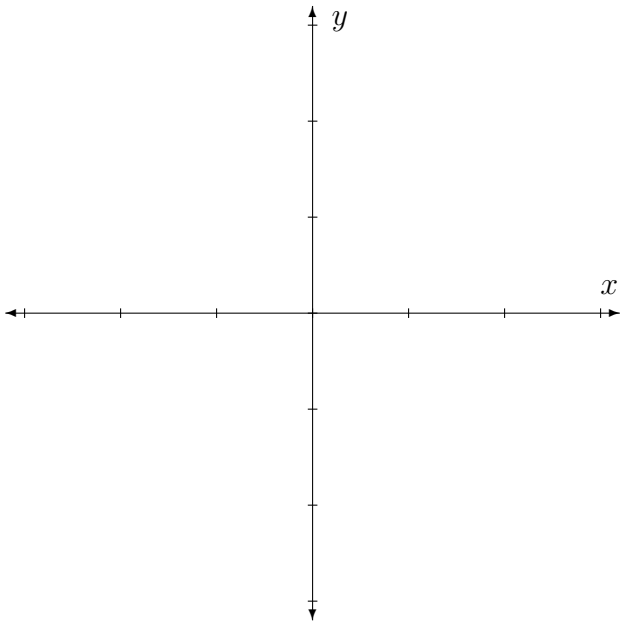
(b) Shade the part of the phase plane in which both x and y are increasing.



(c) The graphs of $x(t)$ and $y(t)$ for a solution $\begin{bmatrix} x \\ y \end{bmatrix}$ to the system are shown below. Sketch the solution curve in the phase plane.







5. (3 points) The real 2×2 matrix A has eigenvector $\vec{v}_1 = \begin{bmatrix} 2 \\ -1+i \end{bmatrix}$ for eigenvalue $\lambda_1 = 1+3i$; and A has eigenvector $\vec{v}_2 = \begin{bmatrix} 2 \\ -1-i \end{bmatrix}$ for eigenvalue $\lambda_2 = 1-3i$. Given this information, write down the general solution to the system of ODEs

$$\vec{x}\,' = A\vec{x}$$

If you finish early, you must stay in your seat until the end. You should check your work, but if you are done, you can amuse yourself by coloring in these regular pentagonal tilings.