

Limits

1. $\lim_{x \rightarrow 0} 2 + x = \boxed{2}$

2. $\lim_{x \rightarrow 0} 2 + 3x = \boxed{2}$

3. $\lim_{x \rightarrow 0} \frac{2x}{3x} = \boxed{2/3}$

4. $\lim_{x \rightarrow \infty} \frac{1}{x} = \boxed{0}$

5. $\lim_{x \rightarrow \infty} 2 + \frac{3}{x} = \boxed{2}$

6. $\lim_{x \rightarrow 0} \frac{2x+x^2}{3x-6x^2} = \boxed{2/3}$

7. $\lim_{x \rightarrow 1} \frac{x-1}{(x-1)(x+1)} = \boxed{1/2}$

Logs

8. $\log_2(4) = \boxed{2}$

9. $\log_2(8) = \boxed{3}$

10. $\log_2(16) = \boxed{4}$

11. $\log_2(2) = \boxed{1}$

12. $\log_2(\frac{1}{2}) = \boxed{-1}$

13. $\log_3(9) = \boxed{2}$

14. $\log_3(81) = \boxed{4}$

15. $\log_3(\frac{1}{27}) = \boxed{-3}$

16. $\log_4(16) = \boxed{2}$

17. $\log_4(64) = \boxed{3}$

18. $\log_5(25) = \boxed{2}$

19. $\log_5(125) = \boxed{3}$

20. $\log_{10}(100) = \boxed{2}$

21. $\log_{10}(\frac{1}{10}) = \boxed{-1}$

22. $\log_{10}(.1) = \boxed{-1}$

23. $\log_{10}(.001) = \boxed{-3}$

24. $\log_{10}(1,000,000) = \boxed{6}$

Fractional/Negative Exponents

1. $9^{\frac{1}{2}} = 3$

2. $9^{-1} = \frac{1}{9}$

3. $9^{-\frac{1}{2}} = \frac{1}{\sqrt{9}} = \frac{1}{3}$

4. $8^{\frac{1}{3}} = 2$

5. $8^{-\frac{1}{3}} = \frac{1}{2}$

6. $8^{\frac{4}{3}} = 16$

7. $8^{-\frac{2}{3}} = \frac{1}{4}$

8. $64^{\frac{1}{2}} = 8$

9. $64^{\frac{1}{3}} = 4$

10. $64^{\frac{2}{3}} = 16$

Fractions and Reciprocals

60 works well!

$$\begin{aligned}
 1. \quad (10^{-1} + 15^{-1} + 6^{-1})^{-1} &= \left(\frac{1}{10} + \frac{1}{15} + \frac{1}{6} \right)^{-1} \\
 &= \left(\frac{6}{60} + \frac{4}{60} + \frac{10}{60} \right)^{-1} \\
 &= \left(\frac{20}{60} \right)^{-1} = \left(\frac{1}{3} \right)^{-1} = \frac{3}{1} = \boxed{3}
 \end{aligned}$$

6·12 works well

$$\begin{aligned}
 2. \quad (12^{-1} - 24^{-1} + 36^{-1})^{-1} &= \left(\frac{1}{12} - \frac{1}{24} + \frac{1}{36} \right)^{-1} \\
 &= \left(\frac{6}{6 \cdot 12} - \frac{3}{6 \cdot 12} + \frac{2}{6 \cdot 12} \right)^{-1} = \left(\frac{5}{6 \cdot 12} \right)^{-1} \\
 &= \frac{6 \cdot 12}{5} = \boxed{\frac{72}{5}}
 \end{aligned}$$

Decimals

Write each the following as a decimal, then write its equivalent value as a percent.

$$1. \frac{1}{10} + \frac{2}{100} + \frac{3}{1000} = .1 + .02 + .003 = .123 = 12.3\%$$

$$2. \frac{1}{5} + \frac{1}{20} + \frac{3}{500} = \frac{2}{10} + \frac{5}{100} + \frac{6}{1000} = .256 = 25.6\%$$

$$3. \frac{3}{4} - \frac{1}{5} + \frac{1}{200} = \frac{75}{100} - \frac{20}{100} + \frac{5}{1000} = \frac{55}{100} + \frac{5}{1000} = .55 + .005 = .555 = 55.5\%$$

$$4. \frac{4}{100} + \frac{5}{10,000} + \frac{6}{1,000,000} = .04 + .0005 + .000006 = .040506 = 4.0506\%$$

$$5. \frac{4}{10} - \frac{5}{100} + \frac{6}{1,000} = .4 - .05 + .006 = .35 + .006 = .356 = 35.6\%$$

Distributing

Multiply out the following

$$1. (a+b)(c+d)(e+f) = a(c+d)(e+f) + b(c+d)(e+f) \\ = ac(e+f) + ad(e+f) + bc(e+f) + bd(e+f) \\ = ace + acf + ade + adf + bce + bcf + bde + bdf$$

$$2. (a-b)(c-d)(e-f) = ace - acf - ade + adf - bce + bcf + bde - bdf$$

$$3. (x+2)(x-5) = x^2 - 3x - 10$$

$$4. (a+b+c)(d+e+f) = ad + ae + af + bd + be + bf + cd + ce + cf$$

$$5. (k+c)(k^4 - k^3c + k^2c^2 - kc^3) = k^5 - c^5$$

+c⁴ This was a typo, sorry!

$$6. (4xy^2k^{-2} + 3x^{-1})(xk^3 - yk) = 4xy^2k^{-2} \cdot xk^3 - 4xy^2k^{-2} \cdot yk \\ + 3x^{-1} \cdot xk^3 - 3x^{-1} \cdot yk \\ = 4x^2y^2k - 4xy^3k^{-1} + 3k^3 - 3x^{-1}yk$$

Factoring

Factor the following polynomials

1. $x^3 - 1 = (x-1)(x^2+x+1)$

2. $x^3 + y^3 = (x+y)(x^2-xy+y^2)$

3. $x^2 + 15x + 50 = (x+5)(x+10)$

4. $x^2 - 15x + 50 = (x-5)(x-10)$

5. $x^2 + 5x - 50 = (x+10)(x-5)$

6. $x^2 - 5x - 50 = (x-10)(x+5)$

7. $12x^2 - 7x + 1 = (3x-1)(4x-1)$
 $\uparrow \quad \uparrow$
 $(-3)(-4) \quad -3 + -4$

Canceling Linear Factors

Simplify the following rational functions. They should all end up as polynomials.

1. $\frac{x^2-1}{x-1} = \frac{(x-1)(x+1)}{(x-1)} = x+1$

2. $\frac{x^3-8}{x-2} = \frac{(x-2)(x^2+2x+4)}{(x-2)} = x^2+2x+4$

3. $\frac{3x^2+6x+3}{x+2} = \frac{3(x^2+2x+1)}{x+2} = \frac{3(x+1)^2}{x+2}$

this should have been a 1, sorry!

Fractional/Negative Exponents (Algebra)

For each expression, simplify and write the result as a fraction using only positive exponents. #3 is a challenge.

1. $\frac{12a^{-2}b^3c^{-4}}{16a^{-3}b^{-1}c^3} = \frac{3ab^4}{4c^7}$

2. $\frac{\frac{x^3y^2k^5}{x^2y^3k^2} \cdot \frac{10kxy-4k^{-1}x^2y}{6k^{-2}xy+12kxy}}{\frac{10k^3x^3y^3-4kx^4y^3}{6+12k^3x^3y^3}} = \frac{10k^3x^3y^3-4kx^4y^3}{6+12k^3x^3y^3}$

3. $\left((a^{12}b^6)^{-\frac{1}{2}}\right)^{-\frac{1}{3}} = \left(\frac{1}{\sqrt{a^{12}b^6}}\right)^{-\frac{1}{3}} = \left(\frac{1}{a^6b^3}\right)^{-\frac{1}{3}} = \sqrt[3]{a^6b^3} = \boxed{a^2b}$