Welcome To Math 34A! Differential Calculus

Instructor:

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Chapter 7 (Logs)

§7.0 Logarithms and Exponentials

Applications:

- Chemistry: alkalinity and acidity, pH scale
- Finance: compound interest (get rich slow)
- Geology: Richter scale for earthquakes (did you feel the earth move too ?)
- Archeology: radio carbon dating (how old is that bone ?)
- Astronomy: stellar magnitude (brightness of stars)
- Sound: decibels (what did you say?, the music is too loud)
- Math: solving equations with exponents ...by performing an arithmetic operation to both sides (all of the above are examples of the use of this operation)

Chapter 7 (Logs)

Main Idea of Chapter 7:

> $\log(x)$ is how many times you multiply 1 by 10 to get x

Conclusion:

Before we do logs we should be really good at powers of 10.

Powers of Ten

1 meter $\approx$ 3 feet
1 centimeter $= 0.01$ meters $= 10^{-2}$ meters $\approx 1/2$ inch
1 kilometer $= 1,000$ meters $= 10^{3}$ meters $\approx 1/2$ mile

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Approximate distance (in meters), to nearest power of 10

10^{7} meters	Size of Earth
10^{9} meters	Distance to moon
10^{14} meters	Size of our solar system
10^{16} meters	One light-year
10^{21} meters	Size of the Milky Way galaxy
10^{27} meters	Size of the universe (about 93 billion light-years)
10^{80}	number of protons in the observable universe?
10^{100}	1 googol
10^{1000} meters	???

Exponential Basics

$$\begin{aligned} 10^4 &= 10 \times 10 \times 10 \times 10 \;=\; 10,000 \\ &= 4 \text{ lots of } 10 \text{ multiplied together} \\ &= 1 \text{ followed by } 4 \text{ zeroes} \end{aligned}$$

$$\begin{aligned} 10^x &= \underbrace{10 \times 10 \times \cdots \times 10}_{x \text{ lots of } 10} = 1\underbrace{00000\cdots0}_{x \text{ zeros}} \\ &= 1 \text{ followed by x zeroes} \end{aligned}$$

Ex: $10^2 \times 10^3 = (10 \times 10) \times (10 \times 10 \times 10)$
$= 10^{2+3} = 10^5.$

$$10^x \times 10^y = 10^{x+y} \qquad \text{First Law of Exponents}$$

Why? We can work it out!

Exponential Basics (cont'd)

$$10^x \times 10^y = 10^{x+y} \qquad \text{First Law of Exponents}$$

Why? We can work it out:

(x lots of 10 multiplied together) $\times$ (y lots of 10 multiplied together)
$= (x + y)$ lots of 10 multiplied together

For now x and y are positive whole numbers.

More Exponentiation

$$\begin{aligned}(10^2)^3 &= (10 \times 10)^3 \\ &= (10 \times 10) \times (10 \times 10) \times (10 \times 10) \\ &= 10^6\end{aligned}$$

$$(10^a)^b = 10^{ab} \qquad \text{Fourth Law of Exponents}$$

Why? We can work it out:

$$10^a = \underbrace{10 \times 10 \times \cdots \times 10}_{a \text{ times}}$$

$$\begin{aligned}(10^a)^b &= \underbrace{(10 \times \cdots \times 10) \times \cdots \times (10 \times \cdots \times 10)}_{b \text{ times}} \\ &= 10^{ab}.\end{aligned}$$

Just count the zeros!

When the power is 0 or negative

What is $10^0 ?= 1$ But why? We can work it out:

$$10^0 \times 10^1 = 10^{0+1}$$

$$\text{so} \quad 10^0 \times 10 = 10$$

$$\text{and therefore} \quad 10^0 = 10/10 = 1$$

Summary: we used the first law of exponents to figure out what 10^0 must be.

There is a second explanation in the book!

What is $10^{-2} ?= 1/100 = 0.01$ But why? We can work it out:

$$10^{-2} \times 10^2 = 10^{-2+2} = 10^0 = 1$$

$$\text{therefore} \quad 10^{-2} = \frac{1}{10^2} \quad \text{and} \quad 10^{-a} = \frac{1}{10^a}$$

There is a second explanation in the book

The Five Laws of Exponents

(1) $10^a \times 10^b = 10^{a+b}$ (2) $10^0 = 1$

(3) $10^{-a} = 1/10^a$ (4) $(10^a)^b = 10^{ab}$

(5) $10^a/10^b = 10^{a-b}$

1. What is $10^3 \times 10^4$?

A= 10^{12} B= 10^7 C= 10^{34} D= 10^0 E= 10^{-7} B

2. Find $10^3/10^4$

A= 10^7 B= 10^1 C= 10^{-4} D= 10^{-1} E= 10^{-7} D

3. Find $(10^3)^4$.

A= 10^7 B= 10^1 C= 10^{12} D= 10^{-1} E= 10^0 C

The Five Laws of Exponents

(1) $10^a \times 10^b = 10^{a+b}$ (2) $10^0 = 1$

(3) $10^{-a} = 1/10^a$ (4) $(10^a)^b = 10^{ab}$

(5) $10^a/10^b = 10^{a-b}$

4. What is $\left(10^2 \times 10^3\right)^4$?

A= 10^8 B= 10^9 C= 10^{12} D= 10^{20} E= 10^{24} D

5. What is $(10^2 \times 10^6)/(10^2 \times 10^3)$?

A= 10^2 B= 10^3 C= 10^{-1} D= 10^7 E= 10^6 B

6. What is $\left(10^2/10^5\right)^{-2}$?

A= 10^{-6} B= 10^{-5} C= 10^6 D= 10^4 $E = 10^5$ C

§7.1 Fractional Exponents

We can work them out!

7. What is $10^{0.5} = 10^{1/2}$? Answer: $10^{0.5} = \sqrt{10} \approx 3.16288$

8. What is $10^{0.1} = 10^{1/10}$? Answer: $10^{0.1} = \sqrt[10]{10} \approx 1.258926$

9. Similarly: $10^{0.01} = \sqrt[100]{10} \approx 1.02329$

$$10^{0.001} = \sqrt[1000]{10} \approx 1.00231$$

10. What is $10^{0.27}$? Answer:

$$10^{0.27} = 10^{27/100} = \sqrt[100]{10^{27}} = \left(\sqrt[100]{10}\right)^{27} \approx 1.862$$

§7.2 Logarithms

$\log(y)$ is how many tens you multiply together to get y

$$10^{\log(y)} = y$$

$$\log(10) = ?1 \quad \text{because} \quad 10^1 = 10$$
$$\log(100) = ? = 2 \quad \text{because} \quad 10^2 = 100$$
$$\log(1000) = ? = 3 \quad \text{because} \quad 10^3 = 1000$$
$$\log(100,000) =$$

A= 2 B= 3 C= 4 D= 5 E= 6 D

Still moving to Logarithms

$\log(y)$ is how may tens you multiply together to get y

$$10^{\log(y)} = y$$

$\log(0.1) =? -1$ because $10^{-1} = 1/10 = 0.1$

$\log(0.01) =? = -2$ because $10^{-2} = 1/100 = 0.01$

$\log(10^x) = ? = x$ duh?

How confused are you?

A=not at all B=a bit C=a lot D= :'(

You try it:

$\log(100,000) = ?$

A= 2 B= 3 C= 4 D= 5 E= 6 **D**

$\log(0.001) = ?$

A= 3 B= 0 C= 0.001 D= -2 E= -3 **E**

$\log(100 \times 1000) = ?$

A= 6 B= 5 C= 3 D= 9 E= -5 **B**

$\log(100/1000) = ?$

A= -1 B= 0 C= 1 D= -3 E= -5 **A**

Key Fact Of Logs

First Law of Logs $\log(a \times b) = \log(a) + \log(b)$

This means logs convert multiplication into addition.

Example: $\log(100 \times 1000) = \log(100) + \log(1000) = 2 + 3 = 5$

It is easy to understand why the first law works:

log(a) = (how many 10's you multiply to get a)
log(b) = (how many 10's you multiply to get b)

THEREFORE multiplying ALL these 10s gives $a \times b$

CONCLUDE $\log(a \times b)$ is this number of 10s: that is, $\log(a) + \log(b)$.

7.3 Using Log Tables

We are told: $\log(2) \approx 0.3$ (from table page 289)

$$\begin{aligned} \log(20) &= \log(10 \times 2) \\ &= \log(10) + \log(2) \qquad \text{we know } \log(10) = 1 \\ &\approx 1 + 0.3 \\ &\approx 1.3 \end{aligned}$$

Use this method to find $\log(200)$

A= 30 B= 3 C= 2.3 D= 30 $\boxed{\text{C}}$

7.3 Using Log Tables

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$$\begin{aligned} \log(20) &= \log(10 \times 2) \\ &= \log(10) + \log(2) \qquad \text{we know } \log(10) = 1 \\ &\approx 1 + 0.3 \\ &\approx 1.3 \end{aligned}$$

Use this method to find $\log(200)$

A= 30 B= 3 C= 2.3 D= 30

$\log(100 \times 2) = \log(100) + \log(2)$

$= 2 + 0.3$

$= 2.3$

$$\begin{array}{r} 2.0 \\ +\ .3 \\ \hline 2.3 \end{array}$$

A few more

We are still told $\log(2) \approx 0.3$

Find $\log(0.002)$

A= -3.3 B= -2.3 C= -2.7 D= -3.7 $\boxed{\text{C}}$

Find $\log(2 \times 10^x)$

A= $2x$ B= $2 + x$ C= $.3x$ D= $10x + \log(2)$ E= $x + .3$ $\boxed{\text{E}}$

A few more

We are still told $\log(2) \approx 0.3$

Find $\log(0.002)$ $0.002 = 2 \times 10^{-3}$

A= -3.3 B= -2.3 C= -2.7 D= -3.7

$\log(0.002) = \log(2 \times 10^{-3})$

look up $= \log 2 + \log(10^{-3})$

$= 0.3 - 3$

$= -2.7$

$$\begin{array}{r} 2.1 \\ 3.0 \\ -0.3 \\ \hline -2.7 \end{array}$$

Find $\log(2 \times 10^x)$

A= $2x$ B= $2 + x$ C= $.3x$ D= $10x + \log(2)$ E= $x + .3$

$\log(2) + \log(10^x)$

$0.3 + x$

A Trick!

The graph and the table can both be used to find logs of numbers between 1 and 10.

To find the log of ANY number, we move the decimal point:

$$\log(10^n \times x) = n + \log(x)$$

Example:

$$\log(275.67) = \log(10^2 \times 2.7567) = 2 + \underbrace{\log(2.7567)}_{\text{look this up!}}$$

Its called the MOVING DECIMAL POINT TRICK because 2 is how many places you need to move the decimal point of 275.67 to obtain a number between 1 and 10.

Inverses!

logs are "opposite " of exponents (inverse function of antilog)
So every fact about exponents corresponds to a fact about logs:

	laws of exponents	corresponding law of logs
(1)	$10^a \times 10^b = 10^{a+b}$	$\log(xy) = \log(x) + \log(y)$
(2)	$10^0 = 1$	$\log(1) = 0$
(3)	$10^{-a} = 1/10^a$	$\log(1/x) = -\log(x)$
(4)	$(10^a)^p = 10^{ap}$	$\log(x^p) = p\log(x)$
(5)	$10^a/10^b = 10^{a-b}$	$\log(x/y) = \log(x) - \log(y)$

Example: $\log(x^a/y^b) = ?$

$$A = a\log(x)/(b\log(y)) \qquad B = a\log(x) + b\log(y)$$
$$C = a\log(x) - b\log(y) \quad D = (a + \log(x)) - (b + \log(y)) \quad \boxed{\text{C}}$$

Rule (4): $\log(x^p) = p\log(x)$

Explanation of (4)

$\log(a \times a) = \log(a) + \log(a) = 2\log(a)$
$\log(a \times a \times a) = \log(a) + \log(a) + \log(a) = 3\log(a)$

In general: the number of tens you multiply to get x^p is p times as many tens as you multiply to get x.

What is $\log(\sqrt{x^7})$?

$A = 7 + \log(x)$ $B = (7/2) + \log(x)$ $C = 7/2$ $D = 7/2\log(x)$ D

Find x by solving $10^x = 5$.

A= 5 B= 0.5 C= $\log(5)$ D= $\log(0.5)$ E= $\log(5) - \log(10)$

C

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Rule (4): $\log(x^p) = p\log(x)$

Explanation of (4)

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In general: the number of tens you multiply to get x^p is p times as many tens as you multiply to get x.

What is $\log(\sqrt{x^7})$?

$A = 7 + \log(x)$ $B = (7/2) + \log(x)$ $C = 7/2$ $D = 7/2\log(x)$

$\log(\sqrt{x^7}) = \log(x^{7/2}) = \frac{7}{2}\log(x)$

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Rule (4): $\log(x^p) = p\log(x)$

Explanation of (4)

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In general: the number of tens you multiply to get x^p is p times as many tens as you multiply to get x.

What is $\log(\sqrt{x^7})$?

$A = 7 + \log(x)$ $B = (7/2) + \log(x)$ $C = 7/2$ $D = 7/2\log(x)$ D

Find x by solving $10^x = 5$.

A= 5 B= 0.5 C= $\log(5)$ D= $\log(0.5)$ E= $\log(5) - \log(10)$

$10^x = 5$
$\log(10^x) = \log(5)$
$x = \log(5)$

what if $2^x = 5$?
$\log(2^x) = \log(5)$
$x \log 2 = \log 5$
$x = \frac{\log 5}{\log 2}$

That's it. Thanks for being here.