Practice Problems 4

Math 4B, Spring 2017, Dr. Paul

Practice problems are for your own benefit. You won't turn them in or have them graded, but I have the expectation that you have done these when I write my tests. You can check answers with a TA, in Math Lab, or with the professor.

1. Work through the details of our in-class derivation of the logistic equation.

2. A Chipotle sofritas burrito's temperature will change at a rate of 10% of the difference between its temperature and the temperature of its surrounding per minute.

   Such a burrito is placed in an oven, which is also at 70°F initially, but then the temperature of the oven increases by 20°F per minute for the first 10 minutes, then remains at a constant temperature of 270°F for an additional 10 minutes. After this, the burrito is taken out of the oven, and left on the counter in a 70° room for 5 minutes to cool.

   (a) Write down a differential equation modeling the burrito's temperature.

   (b) Along what interval does the existence and uniqueness theorem guarantee that there a unique solution to the initial value problem in which the burrito's initial temperature is 50°F?

   (c) If the burrito's initial temperature is 50°, find the burrito's final temperature, and sketch a graph of the burrito's temperature as a function of time.

3. Consider the differential equation $x\dfrac{dy}{dx} = 2y$. Consider solutions of the form $y = Cx^2$ and variations as possible solutions (there may be others!). Is there one, more than one, or no solution(s) with the following initial conditions:

   (a) $y(0) = 1$

   (b) $y(0) = 0$

   (c) $y(-1) = 1$

4. On what interval can we be sure there is a unique solution to the initial value problem

$$y' = \sqrt{x} + \frac{y}{x-3}, \quad y(1) = 1$$

5. On what interval can we be sure there is a unique solution to the initial value problem

$$y' = x^{2/3} + \frac{y}{x-3}, \quad y(1) = 1$$

6. Solve the ODE $y' = \sqrt{x+y+1}$ using the substitution $v = x + y + 1$.

7. Solve the ODE by reducing the order: $xy'' = y'$ with $v = y'$, $\dfrac{dv}{dx} = y''$.

8. Solve the ODE by reducing the order: $y'' = y(y')^3$ with $v = y'$, $\dfrac{dv}{dy}v = y''$.
9. Solve the ODE by noticing that it is exact: $2x + 3y + (3x + 2y)y' = 0$.
10. Come up with a new example of an exact ODE.