4/26 More on Logs

Monday, April 25, 2022 11:30 AM

- Reminder: Follow along in the book. The lectures are meant to **supplement** the book's reading, not replace it.

From Page 100

It is amazing just how important logarithms and exponentials are. A calculator will tell you $log(71)$ or $10^{2.7}$ very quickly. But a calculator does not help you to understand logarithms and exponentials. This is a bit like music. You can turn on the radio and hear music, but that is not the same as being able to play a musical instrument. Why should anyone want to play a musical instrument?. The music the person plays has probably been recorded by a better musician and can be heard with much less effort by playing a CD. Why should anyone actually calculate with logarithms? Calculators do it much more quickly and accurately. However, you will not understand logarithms and exponentials if you just use a calculator. And understanding is very important! Without understanding, the instant answers provided by the calculator really have no value to you at all. So in this class you will get practise calculating by hand. In the process, the mystery of logarithms will disappear.

There are two ways you will be asked to calculate logarithms and exponentials. The first method is graphical. The second way uses a table of logarithms. These two methods are designed to teach you several skills. In particular, you will gain experience at using the laws of logarithms and exponents in many cunning ways. You will also come to understand the meaning of the phrase "the logarithm is just the inverse function of the exponential function." At the back of these notes is a table of logarithms. Of course, the table does not give the logarithms of all numbers since that is impossible. However, it does give the logarithms of numbers between 1 and 10. The nice thing is that we can use the table to find the logarithm of ANY number.

Example 7.3.1. *Find $log(20)$ using the table.*

Solution: The first thing to realize is that the rules of logarithms are very helpful:

$$log(20) = log(2 \times 10) = log(2) + log(10).$$

Now we can look up $log(2)$ in the table: it is 0.3010. We do not need to look up $log(10)$ because we know $log(10) = 1$. Think about this! Thus, $log(20) \approx 1.3010$.

I often hear comments like
- "The book isn't helpful"
- "They don't tell us how to solve these problem".

This might be true in other parts of the course, and I hope you will tell me when this happens, but in the case of using log tables and exponential graphs there are **helpful instructions** on the same page this exert was taken from.

Log example

$2^x = 7$

$\log_2 \qquad \log_2$

$\log_2(2^x) = x$

$x = \log_2(7) = \dfrac{\log_{10}(7)}{\log_{10}(2)}$

Magic base-change property

$2^x = 7$

$\log \qquad \log$

$\log(2^x) = \log(7)$

$x \cdot \log(2) = \log(7)$

$x = \dfrac{\log(7)}{\log(2)}$

From Lecture

Warm-up Part II

1, 100, 10000, 1000000, ...

how many zeros? 14 zeros

- $\log_{100}(100^7) = \boxed{7}$
- $\log_{10}(100^7) =$

Warm-up Part II

$1000000 = 10^6$

$(10^6)^{-4} = 10^{-24}$

using exponent properties

- $\log_{100}(100^7) = \boxed{7}$
- $\log_{10}(100^7) = \boxed{14}$
- $\log_{10}(1000000^{-4}) =$ $\log_{10}(10^{-24}) = -24$

$= -4 \cdot \log_{10}(10^6)$

$= -4 \cdot 6 = -24$

using logarithm properties

Warm-up Part II

$\log_{10}(2^{11}) = 11 \cdot \log_{10}(2)$

$= 11 \cdot .3 = 3.3$

- $\log_{100}(100^7) = \boxed{7}$
- $\log_{10}(100^7) = \boxed{14}$
- $\log_{10}(1000000^{-4}) = \boxed{-24}$
- $\log_{10}(2) =$ about $\boxed{.3}$ Still warm-up?
- $\log_{10}(2^{11}) =$ $\log((10^{.3})^{11}) = \log(10^{.3 \cdot 11})$

$= \cancel{\log_{10}}(\cancel{10}^{3.3}) = 3.3$

Rule (4): $\log(x^p) = p\log(x)$ . .

Explanation of (4)

$\log(a \times a) = \log(a) + \log(a) = 2\log(a)$
$\log(a \times a \times a) = \log(a) + \log(a) + \log(a) = 3\log(a)$

In general: the number of tens you multiply to get x^p is p times as many tens as you multiply to get x.

$\sqrt{\frac{1}{x^7}} = x^{-\frac{7}{2}}$

What is $\log\left(\sqrt{\frac{1}{x^7}}\right)$?

$A = 7 - \log(x)$ $B = (7/2) - \log(x)$ $C = -7/2$ $D = -(7/2)\log(x)$

Computer Applications

One kilobyte (1 KB) is 2^{10}.

Problem: Calculate 2^{10} using logs. **Hint:** $\log(2) \approx 0.3$

A≈ 3 B≈ 10.3 C≈ 30 D≈ 1000 E≈ 1100

$2^{10} \approx (10^{.3})^{10} = 10^3$

$2^{10} = 1024$

Compound Interest

At the end of each year a bank pays 7% interest into your account. Initially have \$10,000 in account. How much after 10 years?

Think $10 \times 7\% = 70\%$ in 10 years, so have \$17,000 but that is wrong.

$\$10{,}000 \cdot 1.07 \cdot 1.07 \cdot 1.07$

$\$10{,}000 \cdot 1.07^t$

Everything is a multiplier