Definition

CW-complex

Anything that can be constructed with the following type of construction:

- Start with any set of points X^0 with the discrete topology.
- Form $X^n = D^n_\alpha \sqcup_{\varphi_\alpha} X^{n-1}$ by attaching n-cells to the $(n-1)$-skeleton.
- If you go infinitely, use the weak topology; where $A \underset{\text{open}}{\subset} X$ if $A \underset{\text{open}}{\subset} X^n$ for all n.

Definition

g Homotopic to h rel A

$g \simeq h \operatorname{rel} A$ if $\exists$ a homotopy F s.t.

- $f_0 = g$
- $f_1 = h$
- $f_{t_1}(a) = f_{t_2}(a) \quad \forall a \in A$

Definition

Homotopy Equivalent rel A

$\exists f : X \to Y, g : Y \to X$ such that

- $gf \simeq \mathbb{1} \operatorname{rel} A$
- $fg \simeq \mathbb{1} \operatorname{rel} A$

Definition

Homotopy Extension Property

The following are equivalent:

- $\forall F : A \times I \to Y$ and
  $f : X \to Y$ s.t. f extends F_0,
  $\exists \bar{F} : X \times I$ which extends F and f.
- $X \times \{0\} \cup A \times I$ is a retract of $X \times I$.

DEFINITION

Smash Product

$$X \wedge Y = {}^{X \times Y}/_{X \vee Y},$$

where we wedge X and Y at their respective base points x_0, y_0, usually given.