Math 34A Winter 2020
Old Midterm 1 #1 Solutions

1. Solve for x in the equation $\dfrac{2}{k} - \dfrac{3}{x+k} = 0$.

Solution: We simplify the equation

$$\frac{2}{k} - \frac{3}{x+k} = 0$$

by multiplying the entire equation by $k(x+k)$. We get

$$\frac{2}{k} \cdot k(x+k) - \frac{3}{x+k} \cdot k(x+k) = 0 \qquad \text{or} \qquad 2(x+k) - 3k = 0.$$

Expanding this, we get $2x + 2k - 3k = 0$, which simplifies to $2x - k = 0$. Solving for x, we find $\boxed{x = k/2}$.

2. Multiply out and simplify $(5a + 3b)\left(4a^{-1} - 2b^{-1}\right)$. Check your answer.

Solution: We multiply out to get

$$\begin{aligned}(5a + 3b)\left(4a^{-1} - 2b^{-1}\right) &= 5a\left(4a^{-1} - 2b^{-1}\right) + 3b\left(4a^{-1} - 2b^{-1}\right) \\ &= 20 - 10ab^{-1} + 12a^{-1}b - 6.\end{aligned}$$

This simplifies to $\boxed{14 - 10ab^{-1} + 12a^{-1}b}$ or, put another way, $\boxed{14 - 10a/b + 12b/a}$.

We check our answer by plugging in specific values of a and b. Let's try $a = 1$ and $b = 2$. The original expression is then

$$(5a + 3b)\left(4a^{-1} - 2b^{-1}\right) = (5 \cdot 1 + 3 \cdot 2)\left(4 \cdot 1^{-1} - 2 \cdot 2^{-1}\right) = (5 + 6)(4/1 - 2/2) = 11 \cdot 3 = 33.$$

The simplified expression is

$$14 - 10a/b + 12b/a = 14 - 10 \cdot 1/2 + 12 \cdot 2/1 = 14 - 5 + 24 = 33.$$

Thus the two expressions agree, at least for these particular values of a and b.

3. Substitute $x = kt + p$ into

$$x^2 - 2px + 7$$

Simplify the result as much as possible. Write the result of this simplification here:

Solution: We replace x with $kt + p$ in $x^2 - 2px + 7$ and get

$$x^2 - 2px + 7 = (kt + p)^2 - 2p(kt + p) + 7 = k^2t^2 + 2ktp + p^2 - 2pkt - 2p^2 + 7.$$

The two "p^2" terms combine, and the two "$2ktp = 2pkt$" terms cancel. We get

$$x^2 - 2px + 7 = \boxed{k^2t^2 - p^2 + 7}.$$

4. Solve for u and v in the simultaneous equations

$$2u + v = p + 1 \qquad v = u + 3p$$

Your answers will involve p only.

Solution: From the two equations

$$2u + v = p + 1 \qquad v = u + 3p,$$

we substitute the expression for v into the first equation. We get

$$2u + (u + 3p) = p + 1 \qquad \text{or} \qquad 3u = 1 - 2p.$$

Solving, we get $u = (1 - 2p)/3$. Plugging this back into the original second equation, we get

$$v = u + 3p = \frac{1 - 2p}{3} + 3p = \frac{1 - 2p}{3} + \frac{9p}{3} = \frac{1 + 7p}{3}.$$

Thus the final answers are $\boxed{u = (1 - 2p)/3}$ and $\boxed{v = (1 + 7p)/3}$.

5. Marie started driving from Stockton towards Isla Vista at noon at a speed of 50 mph. At 1pm Jason started driving from Isla Vista towards Stockton at 100 mph. Jason and Marie met at 3pm. Meanwhile a bad guy follows Marie. He leaves Stockton at 2pm and drives along the same route at 90 mph.

How many miles from Isla Vista was the bad guy when Jason and Marie met?

Draw a diagram here. ***Briefly*** explain what you did.

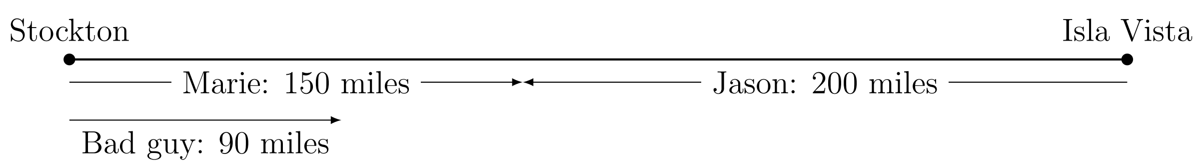
Solution: Marie has driven for 3 hours at 50 miles/hour, so she has traveled

$$(3 \text{ hours})\left(50\ \frac{\text{miles}}{\text{hour}}\right) = 150 \text{ miles.}$$

Similarly, Jason drove for 2 hours at 100 miles/hour, so he has traveled

$$(2 \text{ hours})\left(100\ \frac{\text{miles}}{\text{hour}}\right) = 200 \text{ miles.}$$

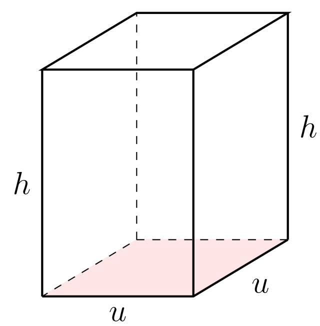
Thus we have a diagram like this:



The bad guy drives for one hour – from 2pm until Jason and Marie meet at 3pm – at 90 miles/hour, and so travels 90 miles from Stockton. Since Jason and Marie traveled a combined total of 350 miles, the bad guy is $350 - 90 = 260$ miles from Isla Vista when Marie and Jason meet.

6. A rectangular box has four sides and a bottom but no top. The volume is 7. The bottom is square and the length of each side is u. Draw a labelled diagram here:

Solution: Here is a picture of the rectangular box. We've labeled the sides of the square base u (as indicated) and the height h:



(a) Express the height of the box in terms of u.

Solution: The volume of the box is the length times the width times the height; in this case, $V = u \cdot u \cdot h = u^2 h$. We are also told that $V = 7$. Thus $u^2 h = 7$, and so we can write the height as $h = \boxed{7/u^2}$.

(b) Express the total area of the cardboard making up the box in terms of u.

Solution: The area of the bottom is u^2. The sides are all rectangles of dimensions u by h, so each of them has area $u \cdot h$. There are four sides, so the total area of these sides is $4uh$. Thus (since there is no top), the total area is $A = u^2 + 4uh$. We substitute in $h = 7/u^2$ (from part (a)) to see that $A = \boxed{u^2 + 28/u}$ or $A = \boxed{u^2 + 28u^{-1}}$.