Welcome To Math 34A! Differential Calculus

Instructor:

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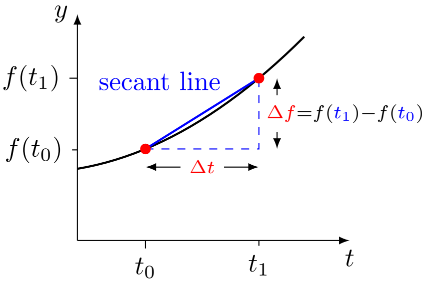
South Hall 6431X (Grad Tower, 6th floor, blue side, first door on the right)

Office Hours:

MTWR after class 2:00-3:00, and by appointment. Details on Gauchospace.

Graphical Approach



Δf = change in f
Δt = change in t

Many ways to say same thing:

$$\left(\begin{array}{c}\text{average rate of} \\ \text{change of } f\end{array}\right) = \frac{\text{change in } f}{\text{change in } t}$$

$$= \frac{\Delta f}{\Delta t}$$

$$= \text{slope of secant line} = \frac{f(t_1) - f(t_0)}{t_1 - t_0}$$

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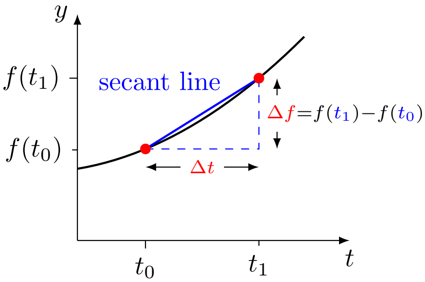

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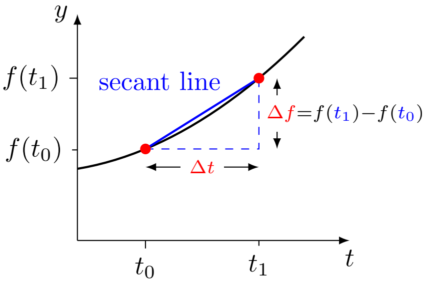

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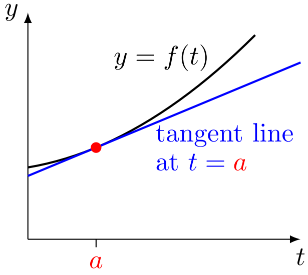
Idea: As t_1 moves closer to t_0 the secant line approaches the tangent line at t_0. This is the line with the same slope as the graph at t_0.

Understanding Derivatives

There are many ways to think about derivatives. You need to understand these to apply to problems.



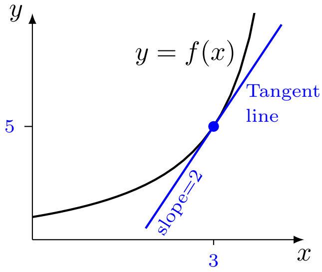
slope of graph at a
$=$ slope of tangent line
$=$ instantaneous rate of change of f at a

$$= \left(\begin{array}{c} \text{limit of average rate of change} \\ \text{of } f \text{ over shorter and shorter} \\ \text{time intervals starting at } a \end{array} \right)$$

$=$ limit of slopes of secant lines

$$= f'(a) = \left.\frac{df}{dt}\right|_{t=a}$$

The Importance of Units



Told $f(3) = 5$ and $f'(3) = 2$

This means the slope of the tangent line to the graph $y = f(x)$ at $x = 3$ is 2.

The derivative is this slope, so…

$$\boxed{\text{The units of } \frac{dy}{dx} \text{ are } \frac{\text{units of y}}{\text{units of x}}}$$

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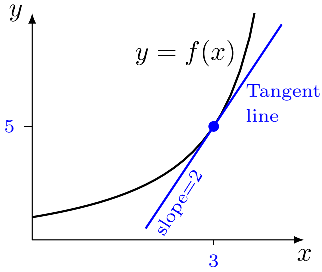

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Examples:
Heating: derivative units are $/° F = dollars per degree F
Adrenaline: bpm/mg = beats per minute per mg of adrenaline.

Units help you understand the meaning of the derivative.

Interpretation of Derivatives I

Suppose $f(x)$ = the percentage of children who still get measles when x% of children are inoculated.

Question: Which of the following is a plausible value for $f'(40)$?

A$= 0$ B$= 2$ C$= 50$ D$= -2$ E$= -50$

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Question: If $f(40)=20$ and $f'(40)=-2$, which must be true?

A when 20% of children are inoculated the percentage who gets measles decreases by 2%

B when 20% of children are inoculated then inoculating an extra 1% of children would reduce the number of measles cases by another 2%

C If the inoculation rate is 41% then 18% of children gets measles

D If the inoculation rate is 20% then 2% fewer cases of measles arise if an extra 1% of children can be inoculated

E none of the above

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Answer: $\boxed{\text{C}}$

Interpretation of Derivatives II

Air temperature gets colder the higher you go.

$T(x)$ = air temperature in $^\circ C$ at a height x meters above sea level.

Question: Which of these is a plausible value for $T'(2000)$?

A$= -1$ B$= 1$ C$= 0$ D$= 1/200$ E$= -1/200$

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Question: If $T(2000) = 10$ and $T'(2000) = -1/200$, which is most plausible?

- A the temperature at sea level is 16^oC
- B the temperature 2400 meters above sea level is 8^oC
- C the temperature 10 meters above sea level is 2000^oC
- D 2000 meters above sea level the temperature is decreasing at a rate of $1/200^oC$ per minute.
- E none of these are plausible

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Answer: $\boxed{\text{B}}$

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Question: If $f(20) = 60$ and $f'(20) = 3$ which must be true?

A When the sales of the company are 20 thousand dollars in one month the amount spent on advertising is increasing at a rate of 3 thousand dollars per month

B When the company spends 20 thousand dollars per month on advertising the sales rise at a rate of 3 thousand dollars per month

C When the company spends 20 thousand dollars per month on advertising each extra dollar a month spent on advertising generates an extra 3 dollars of sales.

D When the company spends 3 thousand dollars per month on advertising the sales are increasing at a rate of 20 thousand dollars per month

E None of the above

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E None of the above

Answer: $\boxed{\text{C}}$

Midterm 2: Next Tuesday

Bring:

- A pen or sharp pencil.
- A 3" $\times$ 5" card with your notes.
- Student ID. (Verify your midterm1 ID if haven't)

Don't bring:

- A calculator

Please Be Early!
There are sample exam questions on Gauchospace.
Some of them are from an old Midterm 2, and the rest are a few extra to round out your practice.
The exam is cumulative.

Write out the following sum:

$$\sum_{i=2}^{7}(i^2+3).$$

These review problems put together different ideas you have been reviewing.

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- What is the limit as h approaches 0?

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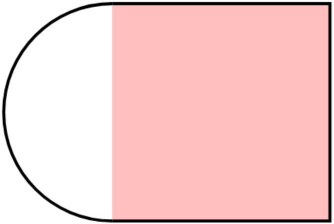
- For the function $f(t)$, find the average rate of change between $t = 3$ and $t = 5$.
- Find the average rate of change between $t = 3$ and $t = 3 + h$.
- Simplify your answer to the last problem.
- What is the limit as h approaches 0?
- what is the instantaneous speed of f at 3?

Given that $\log(2)$ is approximately 0.3, try to estimate $\log(5)$.
(Hint: $5 = 10/2$).

There are 4 grams of gold.

How many grams of silver should be mixed with this gold to create a mixture that is x% gold?

A garden is in the shape of a square and with a semicircle on one side. The length of each side of the square is t, and the area of the garden is A. Express t in terms of A, then express the perimeter of the garden in terms of A.



A growing rectangle doubles its length and width every three hours. By what factor has its area changed after one day?

A population of bacteria is growing exponentially. At one time it is found to be 10 mg in mass, and exactly one day later it is found to be 100 mg in mass. What is the doubling time in hours? That is, how long does it take for the bacteria population to double? (Hint: $\log(2) \approx .3$)

How long after there are 100 mg will there be 1 gram of bacteria?

A population of mice is getting killed off by a disease. The disease is dormant in all mice of the population, and it seems to kill them off at random, giving each mouse an equal chance of dying each day. Sadly, the mouse population decays exponentially. At one time there were a million mice, and 20 days later there are only 100 remaining. What is the half-life of the mouse population? That is, how long does it take for half of the mice to die?

How long after the 20 days will there be only one mouse left?

Solve for x:

$$\log(-x+5) = 23$$