Math 4B **Name:** _____
Summer Session B
Midterm
21 August 2020

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1. (1 point) Please write the following sentence:

   "I, [your name], understand that if I get the answer and show no work, it will be assumed that I copied off someone else and may be reported for cheating."

2 (10 points each) Find the general solution of the given differential equation. Write your answer in explicit form.

(a) $y' = t^2 e^y$

2. (10 points each) Find the general solution of the given differential equation. Write your answer in explicit form.

   (b) $2u'' + 4u = 0$

2. (10 points each) Find the general solution of the given differential equation. Write your answer in explicit form.

   (c) $y' = \dfrac{x^{-6}(x-1)}{5y^4}$

3. (10 points each) Solve the given initial value problem.

   (a) $-w'' + 10w' - 25w = 0, \quad w(0) = 1, \quad w'(0) = -1$

3. (10 points each) Solve the given initial value problem.

   (b) $xy' + (x+1)y = x^2 e^{-x}, \quad x > 0, \quad y(3) = 0$

4. Given that $y_1(t) = t^{-3}$ is a solution of

$$t^2y'' + 2ty' - 6y = 0, \quad t > 0,$$

(a) (15 points) Use the reduction of order method to find a second solution of the form $y_2(t) = t^k$.

(b) (5 points) Do y_1 and y_2 from part (a) form a fundamental set of solutions? Why?

(c) (5 points) Using your answers from parts (a) and (b), find the solution which satisfies the initial conditions $y(1) = 1$ and $y'(1) = 12$.

5. (10 points) Suppose y_1 and y_2 form a fundamental set of solutions to some differential equation. Let $y_3 = y_2 + y_1$. Do you think that the pair $\{y_3, y_1\}$ would also form a fundamental set of solutions, yes or no? Explain your reasoning and thoughts. You may use theory from linear algebra and/or differential equations.