# Welcome Back!
# Differential Calculus

## Instructor:

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## Office Hours:

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A nice thing about derivatives...

$$\begin{aligned}\frac{d}{dx}(a \cdot f(x) + b \cdot g(x)) &= a\frac{d}{dx}f(x) + b\frac{d}{dx}g(x) \\ &= a \cdot f'(x) + b \cdot g'(x)\end{aligned}$$

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For example...

$$\begin{array}{rcccc}\frac{d}{dx}(3x^2+5x) & = & 3\frac{d}{dx}x^2 & + & 5\frac{d}{dx}x\\ & = & 3(2x) & + & 5(1)\\ & = & 6x & + & 5\end{array}$$

# A Warning!

⚠ $$\frac{d}{dx}\left(f(x)g(x)\right) \neq f'(x) \times g'(x)$$ ⚠

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Question: $\frac{d}{dx}\left((x^3+1)(2x^2-3x+5)\right) =?$

A$= 10x^4 - 8x^3 + 10x^2 + 12x - 3$ B$= 10x^4 - 12x^3 + 15x^2 + 4x + 5$
C$= 10x^4 - 12x^3 + 15x^2 + 4x - 3$ D= Other

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Hint: $2x^5 - 3x^4 + 5x^3 + 2x^2 - 3x + 6$

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Hint: $2x^5 - 3x^4 + 5x^3 + 2x^2 - 3x + 6$

Answer: $\boxed{\text{C}}$

# Differentiating $f(x) = e^{kx}$

$$\frac{d}{dx}\left(e^{kx}\right) = ke^{kx}$$

versus

$$\frac{d}{dx}\left(x^{n}\right) = nx^{n-1}$$

⚠ These are not polynomials. $\frac{d}{dx}\left(e^{kx}\right) \neq ke^{(k-1)x}$. ⚠

Question: Find $\frac{d}{dx}\left(4e^{3x} + 5x^3\right)$

A= $12e^{2x} + 15x^2$ B= $12e^{3x} + 15x^3$ C= $4e^{3x} + 15x^2$

D= $12e^{3x} + 15x^2$ E= Other

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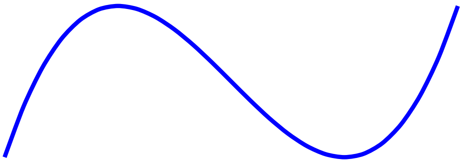

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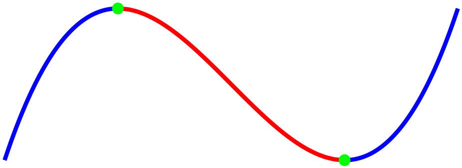

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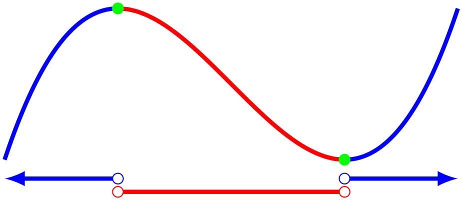

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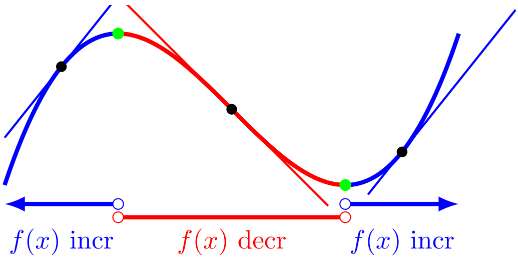

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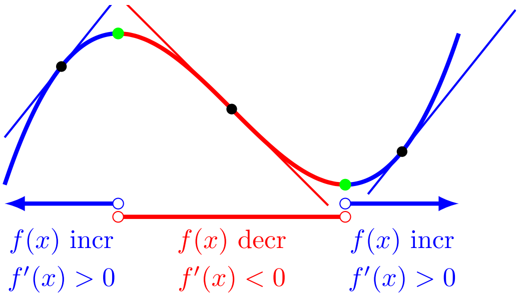

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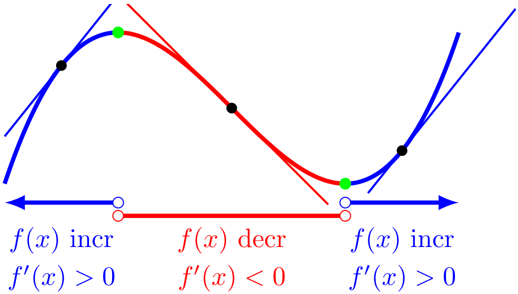


**Point:**

$$f'(x) > 0 \iff f(x) \text{ is increasing}$$
$$f'(x) < 0 \iff f(x) \text{ is decreasing}$$

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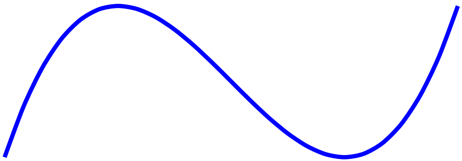

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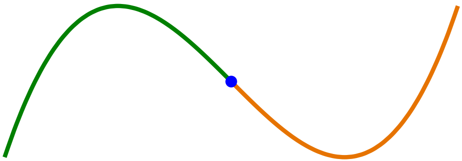

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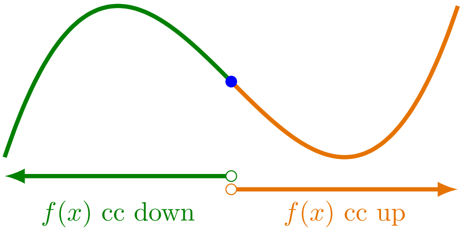

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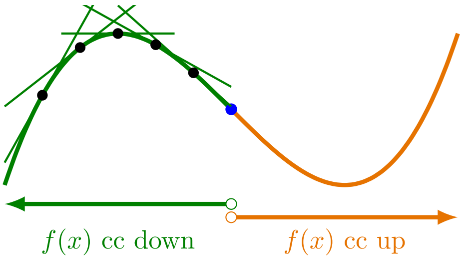

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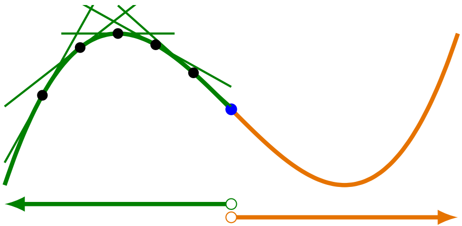

$f(x)$ cc down

$f''(x) < 0$

$f(x)$ cc up

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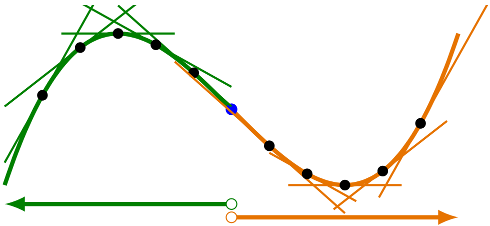

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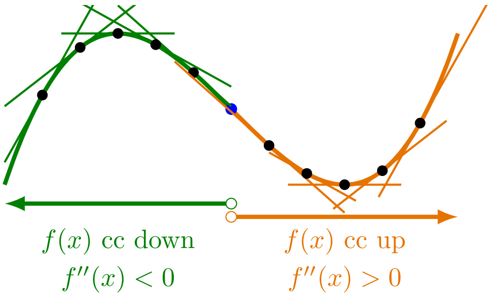

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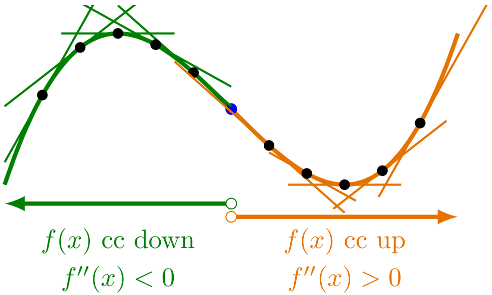


**Point:**

$$\begin{aligned} f''(x) > 0 &\iff f'(x) \text{ is increasing} \\ &\iff f(x) \text{ is concave up} \\[1ex] f''(x) < 0 &\iff f'(x) \text{ is decreasing} \\ &\iff f(x) \text{ is concave down} \end{aligned}$$

# Concavity

$$f''(x) > 0 \iff f(x) \text{ is concave up}$$
$$f''(x) < 0 \iff f(x) \text{ is concave down}$$

(1) For which values of $x$ is $f(x) = x^3 - 6x^2 + 3x + 2$ concave up?

A when $x = 0$ B when $x < 6$ C when $x > 6$

D when $x < 2$ E when $x > 2$

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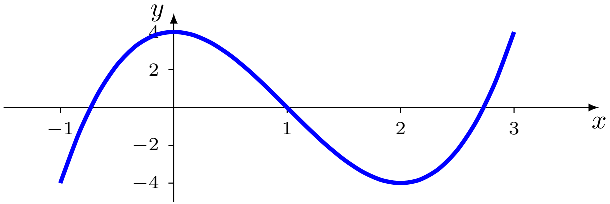
(2) Where is $f''(x) > 0$?




A when $x < 2$ B when $x > 2$ C when $x < 1$
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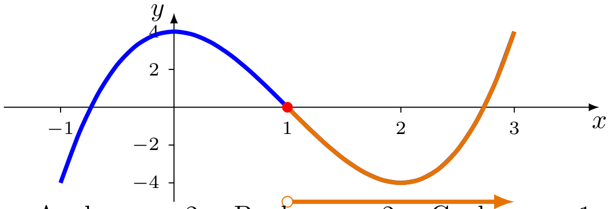


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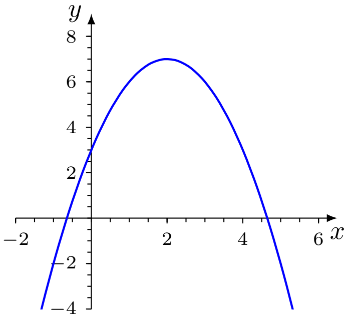
# §8.13: Max/Min problems

Often want to find the biggest, smallest, most, least, maximum, minimum of something.

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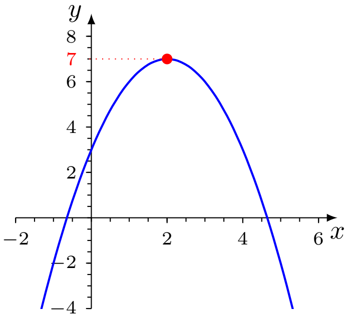


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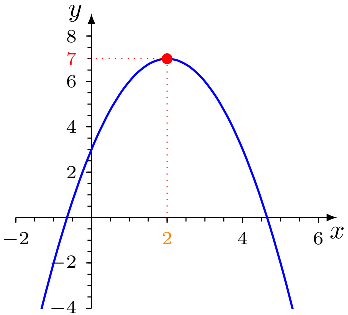


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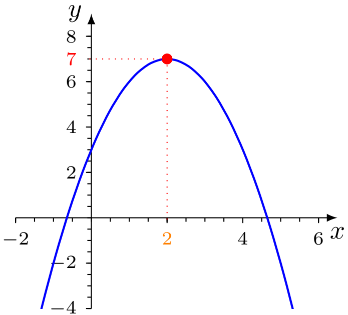


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The <u>maximum value</u> or just <u>maximum</u> of the function is 7.

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We write $f(2) = 7$.

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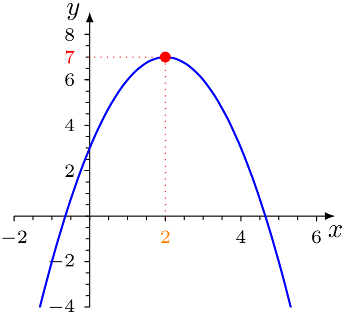


Here's the graph of $y = f(x) = -x^2 + 4x + 3$

The maximum value or just maximum of the function is $7$.

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We write $f(2) = 7$.

For this example you can see this is the maximum because

$$f(x) = -x^2 + 4x + 3 = -(x-2)^2 + 7$$

$(x-2)^2$ is always positive except when $x = 2$
so the maximum must be at $x = 2$. But there is an easier way.

# How To Find A Maximum

(1) At the highest point, it's not going up or down. So find $f'(x)$ to look for the flat part.

(2) Solve $f'(x) = 0$ for $x$. The $x$ value that gives the max must be one of these! (Usually there is just one.)

(3) To find the maximum for $f(x)$, use the $x$-value you just found...because it gives you the maximum!

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**1.** Use this method to find the maximum of $f(x) = -x^2 + 8x + 5$. The maximum value is. . .

$\mathrm{A} = 4 \quad \mathrm{B} = 5 \quad \mathrm{C} = -2x + 8 \quad \mathrm{D} = 21 \quad \mathrm{E} = 15$

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**2.** Find the value of $x$ which makes $f(x) = (2 - x)(x + 6)$ a maximum.

A $= 16$ B $= 1$ C $= -1$ D $= 2$ E $= -2$

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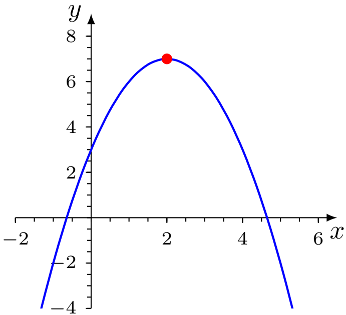
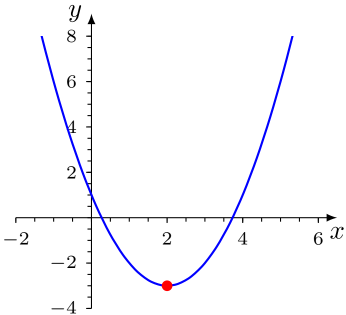
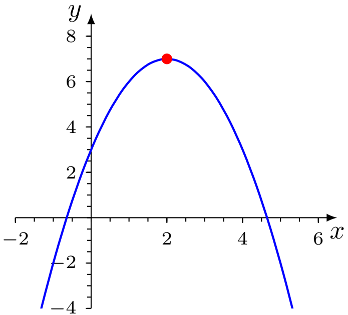
$\text{A} = 16 \quad \text{B} = 1 \quad \text{C} = -1 \quad \text{D} = 2 \quad \text{E} = -2 \quad \boxed{\text{E}}$

# How To Find A Minimum?



# How To Find A Minimum?




What this technique actually does is find both maxima and minima

In Math 34A a problem will have either a maximum or a minimum, but not both. So the technique will find what you want. In Math 34B you discover how to do problems which have both a maximum and a minimum and find out which is which.

# More Examples

**3.** What is the minimum of $f(x) = (x+2)(x+4)+3$?

$$\text{A} = 0 \quad \text{B} = 1 \quad \text{C} = 2 \quad \text{D} = 3 \quad \text{E} = 4$$

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Answer: $\boxed{\text{C}}$

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**4.** What is minimum of $f(x) = x^2 + 16x^{-2}$?

$$\text{A} = 2 \quad \text{B} = 4 \quad \text{C} = 6 \quad \text{D} = 8 \quad \text{E} = 16$$

# More Examples

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**5.** Find the value of $x$ which makes $f(x) = -e^x - e^{-2x}$ a maximum.

$$\text{A} = 0 \quad \text{B} = \ln(2) \quad \text{C} = -\ln(2) \quad \text{D} = \ln(2)/3 \quad \text{E} = \ln(2)/3$$

# More Examples

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Answer: $\boxed{\text{C}}$

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$$A = 0 \quad B = \ln(2) \quad C = -\ln(2) \quad D = \ln(2)/3 \quad E = \ln(2)/3$$

Answer: $\boxed{\text{E}}$

# Word Problem #1

A ball is thrown into the air. After $t$ seconds the height in meters above the ground of the ball is $h(t) = 40t - 10t^2$. How many meters high did the ball go?

$$\text{A} = 2 \quad \text{B} = 40 - 20t \quad \text{C} = 20 \quad \text{D} = 40$$

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# Word Problem #2

If an airline sells tickets at a price of $\$200 + 5x$ each the number of tickets it sells is $1000 - 20x$. What price should the tickets be if the airline wants to get the most money?

$$\text{A} = 5 \quad \text{B} = 25 \quad \text{C} = 175 \quad \text{D} = 200 \quad \text{E} = 225$$

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A fenced garden with an area of 100 $m^2$ will be made in the shape of a rectangle. It will be surrounded on all four sides by a fence. What length and width should be used so the least amount of fence is needed?

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Approach:

(1) Express the total length of fence in terms of only one variable, either $L$ = length of field, or $W$ = width of field. This gives a formula for $P$ = (total length of fence) involving, say, $W$.

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Students always find (1) the hardest part.
You have been prepared for this by word problems from chapter 3!