$(t-1)y'' - ty' + y = 0 \qquad t > 1$

$y_1 = e^t$

- Let $y_2 = v y_1$.

v is a fn of t

- Take derivatives: $y_2 = ve^t$

(use the product rule)

$$y_2' = v'e^t + ve^t$$

$$y_2'' = (v''e^t + v'e^t) + (v'e^t + ve^t)$$

$$= (v'' + 2v' + v)e^t$$

- Plug in: $(t-1)[(v'' + 2v' + v)e^t] - t[v'e^t + ve^t] + [ve^t] = 0$

never zero

$$[(t-1)(v'' + 2v' + v) - tv' - tv + v]e^t = 0$$

$$(t-1)v'' + [2(t-1) - t]v' + [\underbrace{\cancel{t-1-t+1}}_{0}]v = 0$$

$$(t-1)v'' + (t-2)v' = 0$$

- Let $w = v'$ and solve.

$$(t-1)w' + (t-2)w = 0$$

$$(t-1)\frac{dw}{dt} = -(t-2)w$$

$$\frac{1}{w}\,dw = \frac{-t+2}{t-1}\,dt$$

- $\int \frac{1}{w}\,dw = \ln|w|$

u-sub, $u = t-1$

- $\int \frac{-t+2}{t-1}\,dt = \int \frac{-u+1}{u} = \int -1 + u^{-1}\,du = -u + \ln|u| + C$

$$= -t + 1 + \ln|t-1| + C$$

$$\ln|w| = -t \overset{\text{absorb this into } C}{+1} + \ln|t-1| + C$$

$$|w| = C e^{-t} |t-1|$$

always positive since $t>1$

comes from abs. val.
absorb into C.

$$w = \pm C e^{-t}(t-1)$$

$$w = C e^{-t}(t-1)$$

- Integrate w to find v: (Integration by parts ... ew)

$$\int \underbrace{Ce^{-t}}_{dv} \underbrace{(t-1)}_{u} = -Ce^{-t}(t-1) + \int Ce^{-t}\,dt$$

$$= -Ce^{-t}(t-1) - Ce^{-t} + D$$

absorb

$$= -Ce^{-t}[(t-1)+1] + D$$

$$= Cte^{-t} + D$$

- Now $y_2 = v y_1$:

$$e^t(Cte^{-t} + D) = Ct + De^t$$

  - We just need an independent solution from y_1, so choose convenient values for C and D. [I choose 1 and 0]

I can do this because De^t is a multiple of $e^t = y_1$.

- My final answer: $\boxed{y_2 = t}$