Math 550
Homework 2
Dr. Fuller
Solutions

1. Change variables into cylindrical coordinates, using $g(r,\theta,z) = (r\cos\theta, r\sin\theta, z)$. This gives $\int_A f = \int_{-1}^{1}\int_0^{2\pi}\int_0^1 (r^2z^2)(r)\;dr\;d\theta\;dz = \frac{\pi}{3}$.

6. (a) 5 (b) 0 (c) 18

7. If we write $\vec{v} = (v_1, v_2, v_3)$ and $\vec{w} = (w_1, w_2, w_3)$, then both expressions evaluate to $v_1w_2 - v_2w_1$. Similarly, $(dx_p \wedge dz_p)(v,w) = -dy_p(v \times w)$.

9. If φ is alternating, then for all $\vec{v} \in V$ we have $\varphi(\vec{v},\vec{v}) = -\varphi(\vec{v},\vec{v})$, so $\varphi(\vec{v},\vec{v}) = 0$.

   Conversely, if $\varphi(\vec{v},\vec{v}) = 0$ for all $\vec{v} \in V$, then for any $\vec{v},\vec{w} \in V$ we have

   $$0 = \varphi(\vec{v}+\vec{w},\vec{v}+\vec{w}) = \varphi(\vec{v},\vec{v}) + \varphi(\vec{v},\vec{w}) + \varphi(\vec{w},\vec{v}) + \varphi(\vec{w},\vec{w}) = \varphi(\vec{v},\vec{w}) + \varphi(\vec{w},\vec{v}),$$

   which implies $\varphi(\vec{v},\vec{w}) = -\varphi(\vec{w},\vec{v})$.