Differential Geometry - Lee

Math 240, UCSB
Trevor Klar

October 17, 2019

Contents

Note: If you find any typos in these notes, please let me know at trevorklar@math.ucsb.edu. If you could include the page number, that would be helpful.

1

2

3 Tangent Vectors

3.1 Tangent Vectors

Definition. Let $a \in \mathbb{R}^n$. A **geometric tangent vector** in $\mathbb{R}^n$ is

$$v_a = (a, v), \text{ for some } v \in \mathbb{R}^n$$

(we sometimes use the notation $v|_a$). We interpret v_a as a vector v whose initial point is at a.

Definition. The **geometric tangent space to** $\mathbb{R}^n$ **at** a is the set

$$\mathbb{R}^n_a = \{v_a | v \in \mathbb{R}^n\},$$

where a is a fixed element of $\mathbb{R}^n$.

Remark. The set $\mathbb{R}^n_a$ is a vector space, with the natural operations of addition and scalar multiplication where a always remains fixed:

$$v_a + w_a = (v + w)_a, \qquad c(v_a) = (cv)_a.$$