Office Hours!

Instructor:

Trevor Klar, `trevorklar@math.ucsb.edu`

Office Hours:

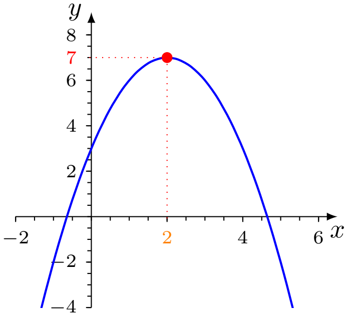
Mondays 2–3PM
Tuesdays 10:30–11:30AM
Thursdays 1–2PM
or by appointment

Office:

South Hall 6431X (Grad Tower, 6th floor, blue side, first door on the right)

§8.13: Max/Min problems

Often want to find the biggest, smallest, most, least, maximum, minimum of something.



Here's the graph of
$y = f(x) = -x^2 + 4x + 3$

The maximum value or just maximum of the function is 7.

The value of x which gives the maximum of $f(x)$ is $x = 2$

We write $f(2) = 7$.

For this example you can see this is the maximum because

$$f(x) = -x^2 + 4x + 3 = -(x-2)^2 + 7$$

$(x-2)^2$ is always positive except when $x = 2$
so the maximum must be at $x = 2$.

How To Find A Maximum

(1) Find $f'(x)$

(2) Solve $f'(x) = 0$. This is the x value that gives the max.

(3) To find the maximum plug the value of x found in (2) back into $f(x)$.

1. Use this method to find the maximum of $f(x) = -x^2 + 8x + 5$. The maximum value is...

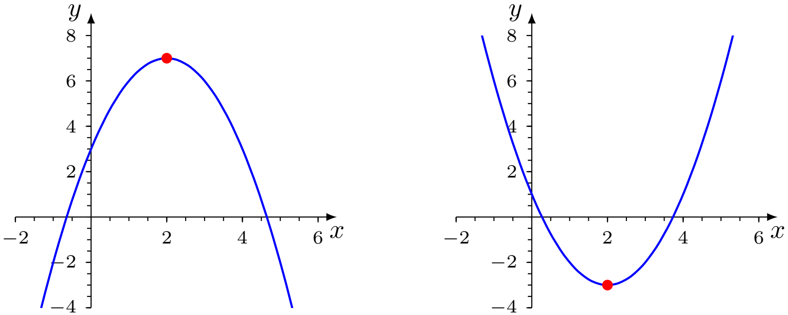
A $= 4$ B $= 5$ C $= -2x + 8$ D $= 21$ E $= 15$ $\boxed{\text{D}}$

2. Find the value of x which makes $f(x) = (2 - x)(x + 6)$ a maximum.

The value of x is...

A $= 16$ B $= 1$ C $= -1$ D $= 2$ E $= -2$ $\boxed{\text{E}}$

How To Find A Minimum?



What this technique actually does is find both maxima and minima

In Math 34A a problem will have either a maximum or a minimum, but not both. So the technique will find what you want. In Math 34B you discover how to do problems which have both a maximum and a minimum and find out which is which.

More Examples

3. What is the minimum of $f(x) = (x+2)(x+4)+3$?

$$\text{A} = 0 \quad \text{B} = 1 \quad \text{C} = 2 \quad \text{D} = 3 \quad \text{E} = 4$$

Answer: $\boxed{\text{C}}$

4. What is minimum of $f(x) = x^2 + 16x^{-2}$?

$$\text{A} = 2 \quad \text{B} = 4 \quad \text{C} = 6 \quad \text{D} = 8 \quad \text{E} = 16$$

Answer: $\boxed{\text{D}}$

5. Find the value of x which makes $f(x) = -e^x - e^{-2x}$ a maximum.

$$\text{A} = 0 \quad \text{B} = \ln(2) \quad \text{C} = -\ln(2) \quad \text{D} = \ln(2)/3 \quad \text{E} = \ln(2)/3$$

Answer: $\boxed{\text{E}}$

Word Problem #1

A ball is thrown into the air. After t seconds the height in meters above the ground of the ball is $h(t) = 40t - 10t^2$. How many meters high did the ball go?

A $= 2$ B $= 40 - 20t$ C $= 20$ D $= 40$ $\boxed{\text{D}}$

Word Problem #2

If an airline sells tickets at a price of $\$200 + 5x$ each the number of tickets it sells is $1000 - 20x$. What price should the tickets be if the airline wants to get the most money?

$$\mathrm{A} = 5 \quad \mathrm{B} = 25 \quad \mathrm{C} = 175 \quad \mathrm{D} = 200 \quad \mathrm{E} = 225 \quad \boxed{\mathrm{E}}$$

Word Problem #3

A fenced garden with an area of 100 m^2 will be made in the shape of a rectangle. It will be surrounded on all four sides by a fence. What length and width should be used so the least amount of fence is needed?

Approach:

(1) Express the total length of fence in terms of only one variable, either L = length of field, or W = width of field. This gives a formula for P = (total length of fence) involving, say, W.

(2) Find minimum by solving $\dfrac{dP}{dW} = 0$.

Students always find (1) the hardest part.
You have been prepared for this by word problems from chapter 3!

Word Problem #4 (a sequel!)

A fenced garden with an area of 1000 m^2 will be made in the shape of a rectangle. It will be surrounded on all four sides by a fence. Three sides are wood fence, and the remaining side is a brick wall.

- The wood fence costs \$5 per meter length.
- The brick wall costs \$20 per meter length.
- C = total cost of the fence and brick wall
- L = length of the brick wall
- W = width of the other side

(a) Find a formula for C in terms of only L.

$$\text{A} = 2W + 2L \quad \text{B} = 2000L^{-1} + 2L \quad \text{C} = 25L + 10000L^{-1}$$

$$\text{D} = 20L + 10000WL^{-1} \quad \text{E} = 5L + 3000 \quad \boxed{\text{C}}$$

(b) What length of brick wall gives lowest cost?

$$\text{A} = 20 \quad \text{B} = 40 \quad \text{C} = 50 \quad \text{D} = 100 \quad \text{E} = 25 \quad \boxed{\text{A}}$$