Welcome To Math 34A! Differential Calculus

Instructor:

Trevor Klar, trevorklar@math.ucsb.edu
South Hall 6431X (Grad Tower, 6th floor, blue side, first door on the right)

Office Hours:

MTWR after class 2:00-3:00, and by appointment. Details on Gauchospace.

A nice thing about derivatives...

$$\frac{d}{dx}(a \cdot f(x) + b \cdot g(x)) = a\frac{d}{dx}f(x) + b\frac{d}{dx}g(x)$$
$$= a \cdot f'(x) + b \cdot g'(x)$$

For example...

A nice thing about derivatives...

$$\frac{d}{dx}(a \cdot f(x) + b \cdot g(x)) = a\frac{d}{dx}f(x) + b\frac{d}{dx}g(x)$$
$$= a \cdot f'(x) + b \cdot g'(x)$$

For example...

$$\begin{aligned} \tfrac{d}{dx}(3x^2 + 5x) &= 3\tfrac{d}{dx}x^2 + 5\tfrac{d}{dx}x \\ &= 3(2x) + 5(1) \\ &= 6x + 5 \end{aligned}$$

A Warning! (Again)

⚠ $$\frac{d}{dx}\left(f(x)g(x)\right) \neq f'(x) \times g'(x)$$ ⚠

A Warning! (Again)

$$\frac{d}{dx}\left(f(x)g(x)\right) \neq f'(x) \times g'(x)$$

Example: What is the derivative of $(x^3+1)(2x^2-3x+5)$?

A Warning! (Again)

⚠ $$\frac{d}{dx}\left(f(x)g(x)\right) \neq f'(x) \times g'(x)$$ ⚠

Example: What is the derivative of $(x^3+1)(2x^2-3x+5)$?

Question: Find $\frac{d}{dx}\big((x^3+1)(2x^2-3x+5)\big)$.

$$\begin{aligned} \text{A} \quad &= 10x^4 - 8x^3 + 10x^2 + 12x - 3 \\ \text{B} \quad &= 10x^4 - 12x^3 + 15x^2 + 4x + 5 \\ \text{C} \quad &= 10x^4 - 12x^3 + 15x^2 + 4x - 3 \\ \text{D} \quad &= \text{Other} \end{aligned}$$

A Warning! (Again)

⚠ $$\frac{d}{dx}\left(f(x)g(x)\right) \neq f'(x) \times g'(x)$$ ⚠

Example: What is the derivative of $(x^3+1)(2x^2-3x+5)$?

Question: Find $\frac{d}{dx}\left((x^3+1)(2x^2-3x+5)\right)$.

$$\begin{aligned} \text{A} &= 10x^4 - 8x^3 + 10x^2 + 12x - 3 \\ \text{B} &= 10x^4 - 12x^3 + 15x^2 + 4x + 5 \\ \text{C} &= 10x^4 - 12x^3 + 15x^2 + 4x - 3 \\ \text{D} &= \text{Other} \end{aligned}$$

Hint: $2x^5 - 3x^4 + 5x^3 + 2x^2 - 3x + 6$

A Warning! (Again)

⚠ $$\frac{d}{dx}\left(f(x)g(x)\right) \neq f'(x) \times g'(x)$$ ⚠

Example: What is the derivative of $(x^3+1)(2x^2-3x+5)$?

Question: Find $\frac{d}{dx}\left((x^3+1)(2x^2-3x+5)\right)$.

$$\begin{aligned} \text{A} \quad &= 10x^4 - 8x^3 + 10x^2 + 12x - 3 \\ \text{B} \quad &= 10x^4 - 12x^3 + 15x^2 + 4x + 5 \\ \text{C} \quad &= 10x^4 - 12x^3 + 15x^2 + 4x - 3 \\ \text{D} \quad &= \text{Other} \end{aligned}$$

Hint: $2x^5 - 3x^4 + 5x^3 + 2x^2 - 3x + 6$

Answer: C

Differentiating $f(x) = e^{kx}$

$$\frac{d}{dx}\left(e^{kx}\right) = ke^{kx}$$

versus

$$\frac{d}{dx}\left(x^{n}\right) = nx^{n-1}$$

⚠ These are not polynomials. $\frac{d}{dx}\left(e^{kx}\right) \neq ke^{(k-1)x}$. ⚠

Question: Find $\frac{d}{dx}\left(4e^{3x} + 5x^3\right)$

A$= 12e^{2x} + 15x^2$ B$= 12e^{3x} + 15x^3$ C$= 4e^{3x} + 15x^2$

D$= 12e^{3x} + 15x^2$ E$=$ Other

Differentiating $f(x) = e^{kx}$

$$\frac{d}{dx}\left(e^{kx}\right) = ke^{kx}$$

versus

$$\frac{d}{dx}\left(x^{n}\right) = nx^{n-1}$$

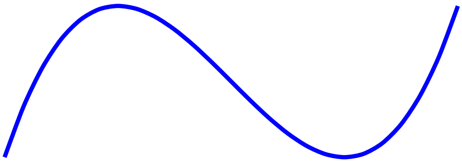
⚠ These are not polynomials. $\frac{d}{dx}\left(e^{kx}\right) \neq ke^{(k-1)x}$. ⚠

Question: Find $\frac{d}{dx}\left(4e^{3x} + 5x^3\right)$

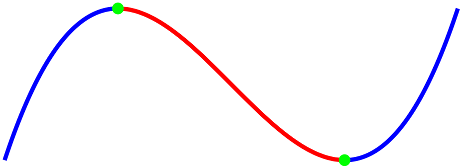
A$= 12e^{2x} + 15x^2$ B$= 12e^{3x} + 15x^3$ C$= 4e^{3x} + 15x^2$

D$= 12e^{3x} + 15x^2$ E$=$ Other D

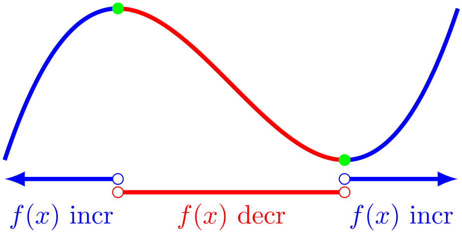
Meanings: The First Derivative



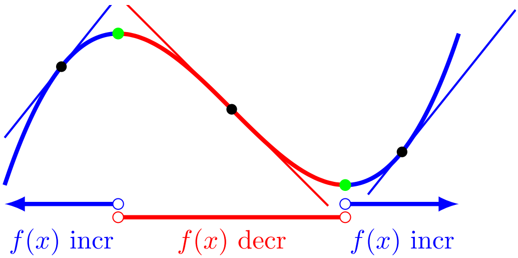
Meanings: The First Derivative



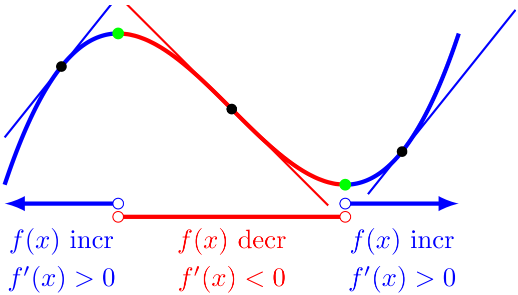
Meanings: The First Derivative



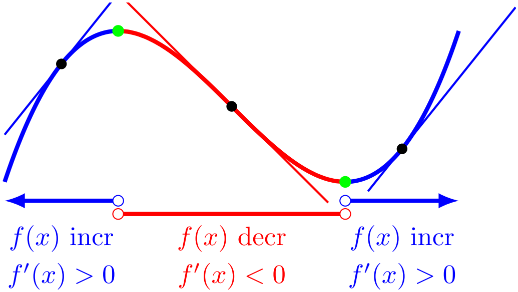
Meanings: The First Derivative



Meanings: The First Derivative



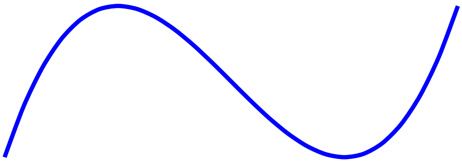
Meanings: The First Derivative



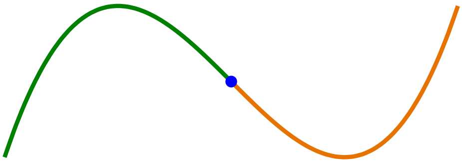
Point:

$$f'(x) > 0 \iff f(x) \text{ is increasing}$$
$$f'(x) < 0 \iff f(x) \text{ is decreasing}$$

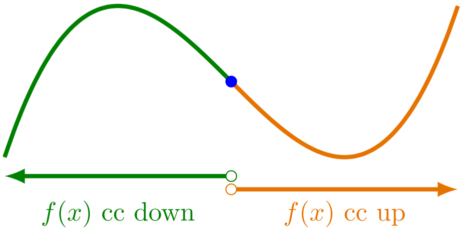
Meanings: The Second Derivative



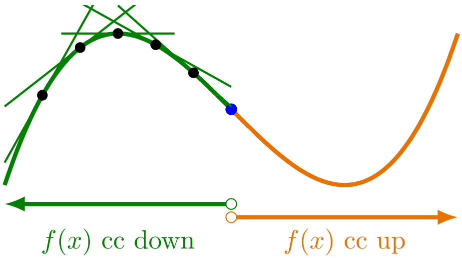
Meanings: The Second Derivative



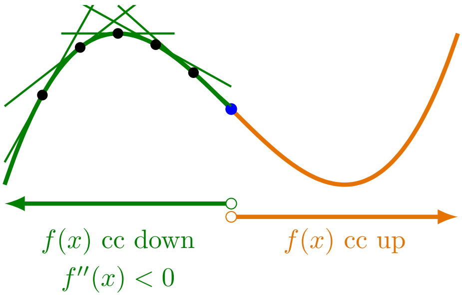
Meanings: The Second Derivative



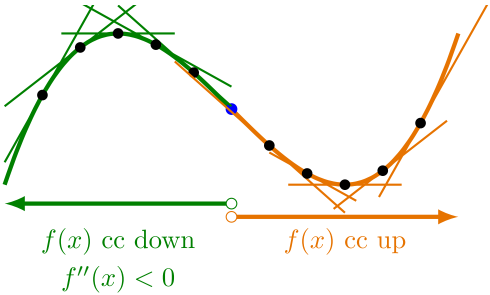
Meanings: The Second Derivative



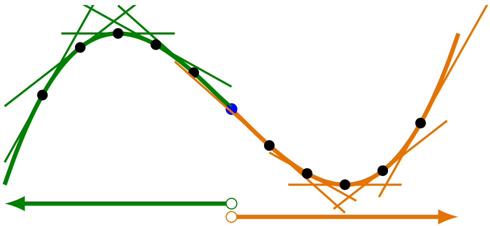
Meanings: The Second Derivative



Meanings: The Second Derivative



Meanings: The Second Derivative



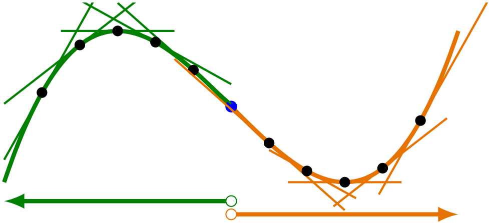
$f(x)$ cc down

$f''(x) < 0$

$f(x)$ cc up

$f''(x) > 0$

Meanings: The Second Derivative



$f(x)$ cc down

$f''(x) < 0$

$f(x)$ cc up

$f''(x) > 0$

Point:

$$\begin{aligned} f''(x) > 0 &\iff f'(x) \text{ is increasing} \\ &\iff f(x) \text{ is concave up} \\ f''(x) < 0 &\iff f'(x) \text{ is decreasing} \\ &\iff f(x) \text{ is concave down} \end{aligned}$$

Concavity

$$f''(x) > 0 \iff f(x) \text{ is concave up}$$
$$f''(x) < 0 \iff f(x) \text{ is concave down}$$

(1) For which values of x is $f(x) = x^3 - 6x^2 + 3x + 2$ concave up?

A when $x = 0$ B when $x < 6$ C when $x > 6$
D when $x < 2$ E when $x > 2$

Concavity

$$f''(x) > 0 \iff f(x) \text{ is concave up}$$
$$f''(x) < 0 \iff f(x) \text{ is concave down}$$

(1) For which values of x is $f(x) = x^3 - 6x^2 + 3x + 2$ concave up?

A when $x = 0$ B when $x < 6$ C when $x > 6$
D when $x < 2$ E when $x > 2$ E

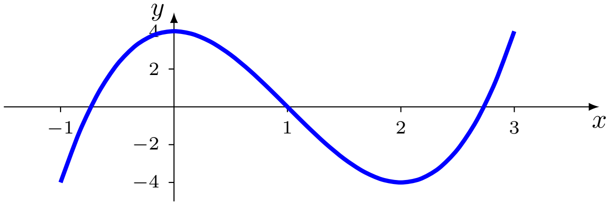
Concavity

$f''(x) > 0 \iff f(x)$ is concave up
$f''(x) < 0 \iff f(x)$ is concave down

(1) For which values of x is $f(x) = x^3 - 6x^2 + 3x + 2$ concave up?

A when $x = 0$ B when $x < 6$ C when $x > 6$
D when $x < 2$ E when $x > 2$ E

(2) Where is $f''(x) > 0$?



A when $x < 2$ B when $x > 2$ C when $x < 1$
D when $x > 1$ E when $-0.7 < x < 1$

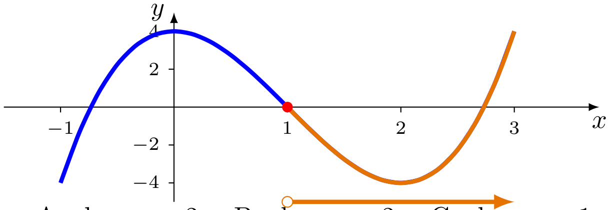
Concavity

$$f''(x) > 0 \iff f(x) \text{ is concave up}$$
$$f''(x) < 0 \iff f(x) \text{ is concave down}$$

(1) For which values of x is $f(x) = x^3 - 6x^2 + 3x + 2$ concave up?

A when $x = 0$ B when $x < 6$ C when $x > 6$
D when $x < 2$ E when $x > 2$ E

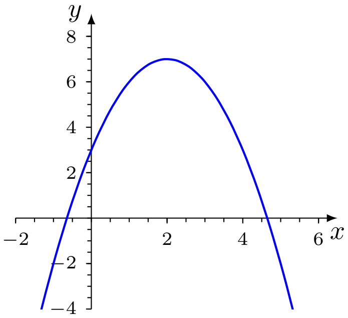
(2) Where is $f''(x) > 0$?



A when $x < 2$ B when $x > 2$ C when $x < 1$
D when $x > 1$ E when $-0.7 < x < 1$ D

§8.13: Max/Min problems

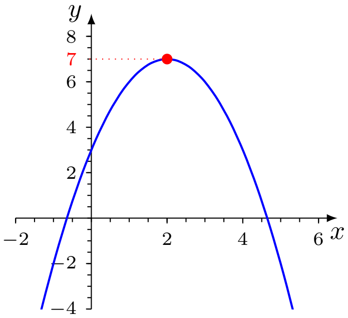
Often want to find the biggest, smallest, most, least, maximum, minimum of something.



Here's the graph of
$y = f(x) = -x^2 + 4x + 3$

§8.13: Max/Min problems

Often want to find the biggest, smallest, most, least, maximum, minimum of something.

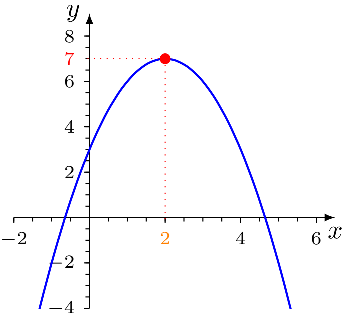


Here's the graph of
$y = f(x) = -x^2 + 4x + 3$

The maximum value or just maximum of the function is 7.

§8.13: Max/Min problems

Often want to find the biggest, smallest, most, least, maximum, minimum of something.



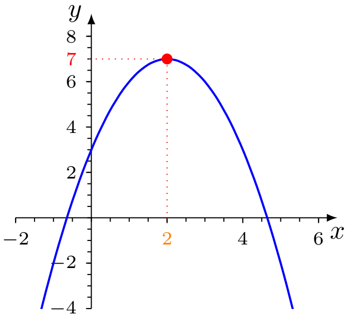
Here's the graph of
$y = f(x) = -x^2 + 4x + 3$

The maximum value or just maximum of the function is 7.

The value of x which gives the maximum of $f(x)$ is $x = 2$

§8.13: Max/Min problems

Often want to find the biggest, smallest, most, least, maximum, minimum of something.



Here's the graph of
$y = f(x) = -x^2 + 4x + 3$

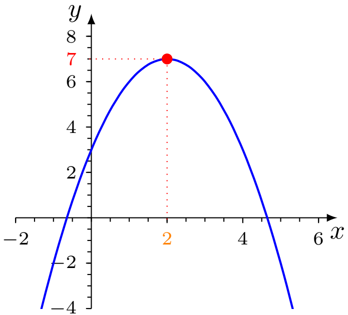
The maximum value or just maximum of the function is 7.

The value of x which gives the maximum of $f(x)$ is $x = 2$

We write $f(2) = 7$.

§8.13: Max/Min problems

Often want to find the biggest, smallest, most, least, maximum, minimum of something.



Here's the graph of $y = f(x) = -x^2 + 4x + 3$

The maximum value or just maximum of the function is 7.

The value of x which gives the maximum of $f(x)$ is $x = 2$

We write $f(2) = 7$.

For this example you can see this is the maximum because

$$f(x) = -x^2 + 4x + 3 = -(x - 2)^2 + 7$$

$(x - 2)^2$ is always positive except when $x = 2$

so the maximum must be at $x = 2$. But there is an easier way.

How To Find A Maximum

(1) At the highest point, it's not going up or down. So find $f'(x)$ to look for the flat part.

(2) Solve $f'(x) = 0$ for x. The x value that gives the max must be one of these! (Usually there is just one.)

(3) To find the maximum for $f(x)$, use the x-value you just found...because it gives you the maximum!

How To Find A Maximum

(1) At the highest point, it's not going up or down. So find $f'(x)$ to look for the flat part.

(2) Solve $f'(x) = 0$ for x. The x value that gives the max must be one of these! (Usually there is just one.)

(3) To find the maximum for $f(x)$, use the x-value you just found...because it gives you the maximum!

1. Use this method to find the maximum of $f(x) = -x^2 + 8x + 5$. The maximum value is...

$$\text{A} = 4 \quad \text{B} = 5 \quad \text{C} = -2x + 8 \quad \text{D} = 21 \quad \text{E} = 15$$

How To Find A Maximum

(1) At the highest point, it's not going up or down. So find $f'(x)$ to look for the flat part.

(2) Solve $f'(x) = 0$ for x. The x value that gives the max must be one of these! (Usually there is just one.)

(3) To find the maximum for $f(x)$, use the x-value you just found...because it gives you the maximum!

1. Use this method to find the maximum of $f(x) = -x^2 + 8x + 5$. The maximum value is...

A $= 4$ B $= 5$ C $= -2x + 8$ D $= 21$ E $= 15$ D

How To Find A Maximum

(1) At the highest point, it's not going up or down. So find $f'(x)$ to look for the flat part.

(2) Solve $f'(x) = 0$ for x. The x value that gives the max must be one of these! (Usually there is just one.)

(3) To find the maximum for $f(x)$, use the x-value you just found...because it gives you the maximum!

1. Use this method to find the maximum of $f(x) = -x^2 + 8x + 5$. The maximum value is...

$$\text{A} = 4 \quad \text{B} = 5 \quad \text{C} = -2x + 8 \quad \text{D} = 21 \quad \text{E} = 15 \quad \text{D}$$

2. Find the value of x which makes $f(x) = (2 - x)(x + 6)$ a maximum.

$$\text{A} = 16 \quad \text{B} = 1 \quad \text{C} = -1 \quad \text{D} = 2 \quad \text{E} = -2$$

How To Find A Maximum

(1) At the highest point, it's not going up or down. So find $f'(x)$ to look for the flat part.

(2) Solve $f'(x) = 0$ for x. The x value that gives the max must be one of these! (Usually there is just one.)

(3) To find the maximum for $f(x)$, use the x-value you just found...because it gives you the maximum!

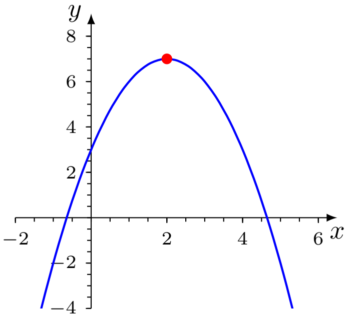
1. Use this method to find the maximum of $f(x) = -x^2 + 8x + 5$. The maximum value is...

$A = 4 \quad B = 5 \quad C = -2x + 8 \quad D = 21 \quad E = 15 \quad D$

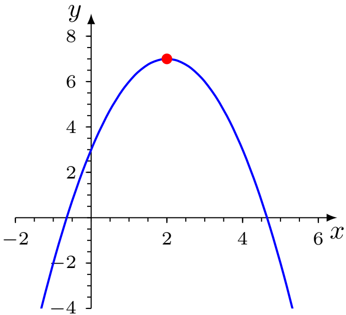
2. Find the value of x which makes $f(x) = (2 - x)(x + 6)$ a maximum.

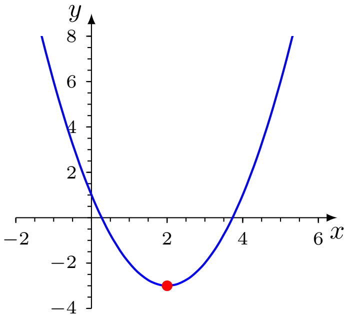
$A = 16 \quad B = 1 \quad C = -1 \quad D = 2 \quad E = -2 \quad E$

How To Find A Minimum?



How To Find A Minimum?





What this technique actually does is find both maxima and minima

In Math 34A a problem will have either a maximum or a minimum, but not both. So the technique will find what you want. In Math 34B you discover how to do problems which have both a maximum and a minimum and find out which is which.

More Examples

3. What is the minimum of $f(x) = (x+2)(x+4)+3$?

$$A = 0 \quad B = 1 \quad C = 2 \quad D = 3 \quad E = 4$$

More Examples

3. What is the minimum of $f(x) = (x+2)(x+4)+3$?

$$\mathrm{A} = 0 \quad \mathrm{B} = 1 \quad \mathrm{C} = 2 \quad \mathrm{D} = 3 \quad \mathrm{E} = 4$$

Answer: C

More Examples

3. What is the minimum of $f(x) = (x+2)(x+4)+3$?

$$\mathrm{A} = 0 \quad \mathrm{B} = 1 \quad \mathrm{C} = 2 \quad \mathrm{D} = 3 \quad \mathrm{E} = 4$$

Answer: C

4. What is minimum of $f(x) = x^2 + 16x^{-2}$?

$$\mathrm{A} = 2 \quad \mathrm{B} = 4 \quad \mathrm{C} = 6 \quad \mathrm{D} = 8 \quad \mathrm{E} = 16$$

More Examples

3. What is the minimum of $f(x) = (x+2)(x+4)+3$?

$$\mathrm{A} = 0 \quad \mathrm{B} = 1 \quad \mathrm{C} = 2 \quad \mathrm{D} = 3 \quad \mathrm{E} = 4$$

Answer: C

4. What is minimum of $f(x) = x^2 + 16x^{-2}$?

$$\mathrm{A} = 2 \quad \mathrm{B} = 4 \quad \mathrm{C} = 6 \quad \mathrm{D} = 8 \quad \mathrm{E} = 16$$

Answer: D

More Examples

3. What is the minimum of $f(x) = (x+2)(x+4)+3$?

$$\text{A} = 0 \quad \text{B} = 1 \quad \text{C} = 2 \quad \text{D} = 3 \quad \text{E} = 4$$

Answer: C

4. What is minimum of $f(x) = x^2 + 16x^{-2}$?

$$\text{A} = 2 \quad \text{B} = 4 \quad \text{C} = 6 \quad \text{D} = 8 \quad \text{E} = 16$$

Answer: D

5. Find the value of x which makes $f(x) = -e^x - e^{-2x}$ a maximum.

$$\text{A} = 0 \quad \text{B} = \ln(2) \quad \text{C} = -\ln(2) \quad \text{D} = \ln(2)/3 \quad \text{E} = \ln(2)/3$$

More Examples

3. What is the minimum of $f(x) = (x+2)(x+4)+3$?

$$A = 0 \quad B = 1 \quad C = 2 \quad D = 3 \quad E = 4$$

Answer: C

4. What is minimum of $f(x) = x^2 + 16x^{-2}$?

$$A = 2 \quad B = 4 \quad C = 6 \quad D = 8 \quad E = 16$$

Answer: D

5. Find the value of x which makes $f(x) = -e^x - e^{-2x}$ a maximum.

$$A = 0 \quad B = \ln(2) \quad C = -\ln(2) \quad D = \ln(2)/3 \quad E = \ln(2)/3$$

Answer: E

Word Problem #1

A ball is thrown into the air. After t seconds the height in meters above the ground of the ball is $h(t) = 40t - 10t^2$. How many meters high did the ball go?

$$\text{A} = 2 \quad \text{B} = 40 - 20t \quad \text{C} = 20 \quad \text{D} = 40$$

Word Problem #1

A ball is thrown into the air. After t seconds the height in meters above the ground of the ball is $h(t) = 40t - 10t^2$. How many meters high did the ball go?

$$\text{A} = 2 \quad \text{B} = 40 - 20t \quad \text{C} = 20 \quad \text{D} = 40 \quad \text{D}$$

Word Problem #2

If an airline sells tickets at a price of $\$200 + 5x$ each the number of tickets it sells is $1000 - 20x$. What price should the tickets be if the airline wants to get the most money?

$$\text{A} = 5 \quad \text{B} = 25 \quad \text{C} = 175 \quad \text{D} = 200 \quad \text{E} = 225$$

Word Problem #2

If an airline sells tickets at a price of $\$200 + 5x$ each the number of tickets it sells is $1000 - 20x$. What price should the tickets be if the airline wants to get the most money?

$$\text{A} = 5 \quad \text{B} = 25 \quad \text{C} = 175 \quad \text{D} = 200 \quad \text{E} = 225 \quad \text{E}$$

Word Problem #3

A fenced garden with an area of 100 m^2 will be made in the shape of a rectangle. It will be surrounded on all four sides by a fence. What length and width should be used so the least amount of fence is needed?

Word Problem #3

A fenced garden with an area of 100 m^2 will be made in the shape of a rectangle. It will be surrounded on all four sides by a fence. What length and width should be used so the least amount of fence is needed?

Approach:

(1) Express the total length of fence in terms of only one variable, either L = length of field, or W = width of field. This gives a formula for P = (total length of fence) involving, say, W.

Word Problem #3

A fenced garden with an area of 100 m^2 will be made in the shape of a rectangle. It will be surrounded on all four sides by a fence. What length and width should be used so the least amount of fence is needed?

Approach:

(1) Express the total length of fence in terms of only one variable, either L = length of field, or W = width of field. This gives a formula for P = (total length of fence) involving, say, W.

(2) Find minimum by solving $\dfrac{dP}{dW} = 0$.

Word Problem #3

A fenced garden with an area of 100 m^2 will be made in the shape of a rectangle. It will be surrounded on all four sides by a fence. What length and width should be used so the least amount of fence is needed?

Approach:

(1) Express the total length of fence in terms of only one variable, either L = length of field, or W = width of field. This gives a formula for P = (total length of fence) involving, say, W.

(2) Find minimum by solving $\frac{dP}{dW} = 0$.

Students always find (1) the hardest part.
You have been prepared for this by word problems from chapter 3!