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§7.9: Population Growth

Assume each generation of bunnies has 3 times as many bunnies as previous one. Initially have 100 bunnies. How many bunnies after n generations?

(A) $100 \times 3n$ (B) $100 + 3n$ (C) $100(1 + 3n)$

(D) 100^{3n} (E) 100×3^n

Answer: $\boxed{\text{E}}$

Start with 100
After 1 generation have 100×3 bunnies
After 2 generations have $100 \times 3 \times 3$ bunnies
After 3 generations have $100 \times 3 \times 3 \times 3$ bunnies
So. . .after n generations have

$$100 \times \underbrace{3 \times 3 \times \cdots \times 3}_{n \text{ times}} = 100 \times 3^n \text{ bunnies.}$$

More Bunnies

We saw that:

- if we start with 100 bunnies, and
- the bunny population triples every generation,

then we have 100×3^n bunnies after n generations.

1. How many generations until there are $10^7 = 10,000,000$ bunnies?

(A) $\log(5/3)$ (B) $5 - \log(3)$ (C) $5/\log(3)$ (D) $5/3$ (E) $10^5/3$

(A) ≈ 0.22 (B) ≈ 4.52 (C) ≈ 10.48 (D) ≈ 1.67 (E) $\approx 3,333$ $\boxed{\text{C}}$

Flu Outbreak

2. At the start of an outbreak of H1N1 flu in a large herd of cattle, there were 5 infected individuals. The numbers doubles every 3 days. How many days until there are 80 infected cows?

(A) $\log(16)/\log(2)$ (B) $\log(16/2)$ (C) $16/\log(2)$
(D) $3\log(16)/\log(2)$ (E) $\log(48/2)$

2. At the start of an outbreak of H1N1 flu in a large class of students, there were 5 infected individuals. The numbers doubles every 3 days. How many days until there are 80 infected students?

(A) $\log(16)/\log(2)$ (B) $\log(16/2)$ (C) $16/\log(2)$
(D) $3\log(16)/\log(2)$ (E) $\log(48/2)$

Answer: D

Doubling Time Formula

Suppose something doubles every K minutes*. If there is a mass of A at time $t = 0$, how much is there at time t minutes?

$$\text{mass after t minutes} = A \times 2^{(t/K)}$$

Idea: t/K is number of doubling periods in t minutes.

3. A disease spreads through a community. On March 1 there were 100 infected people. The number of people doubles in a 3 days. How many infected people are there t days after March 1?

(A) 2^t (B) $3 \times 2^{t/100}$ (C) 100×2^t (D) $100 \times 2^{t/3}$ D

4. How many days until there are $1,000$ infected people?

(A) $\log(10)/\log(2)$ (B) $3\log(10)/\log(2)$ (C) $3\log(5)$

(D) $3(\log(10) - \log(2))$ (E) $3\log(20)$ B

*Any time unit will work, not just minutes. Just be consistent!

A More Complicated Example

$$\text{mass after t minutes} = A \times 2^{(t/K)}$$

where

- K is the doubling time, and
- t/K is the number of doubling periods in t minutes.

4. A colony of mold is growing on a cheeseburger in the back of a dorm refrigerator. When discovered it has a mass of 10 mg. One week later it was found to have a mass of 30 mg. What is the doubling time measured in days?

(A) $\log(2)/\log(3)$ (B) $7\log(2)/\log(3)$

(C) $7\log(2/3)$ (D) $7\log(3/2)$

Hint: We know A and the mass t days after discovery (for some t).

Solving $30 = 10 \times 2^{7/K}$ gives $\boxed{\text{B}}$

§7.11: Half-Life, Doubling Time

The half-life of a radioactive isotope is the time it takes for half of the isotope to decay.

Example: Isotope W has a half-life of 10 years. How much remains after 20 years? None?

$$\frac{1}{2} \times \frac{1}{2} \times (\text{amount you start with})$$

Idea: In half-life problems, convert time into half-lives.

In this problem, the half-life is 10 years. Therefore, 20 years is two half-lives.

In general: After n half-lives,

$$\text{remaining amount} = \left(\frac{1}{2}\right)^{n} \times (\text{amount started with})$$

5. Start with 120 grams of an isotope with a half-life of 12 years. How many grams remains after 36 years?

(A) 0 (B) 10 (C) 15 (D) 20 (E) 40 C

Another Example

In general: After n half-lives,

$$\text{remaining amount} = \left(\frac{1}{2}\right)^{n} \times (\text{amount started with})$$

6. An isotope has a half-life of 5 years.

(a) If we start with 70 grams, how many grams will be left after t years?

(A) $= 70\left(\frac{1}{2}\right)^{t}$ (B) $= 5\left(\frac{1}{2}\right)^{70t}$ (C) $= 70\left(\frac{1}{2}\right)^{5t}$

(D) $= 70\left(\frac{1}{2}\right)^{t/5}$ (E) 0 $\boxed{\text{D}}$

(b) How many years until 10 grams remain?

(A) $5(\log(7) - \log(2))$ (B) $\log(7)/\log(2)$ (C) $5\log(7/2)$

(D) $5\log(7)/\log(2)$ (E) $\log(7)/(5\log(2))$ $\boxed{\text{D}}$

Half-Life Formula

Suppose something has a half-life of K years†. If there is a mass of A at time $t = 0$, how much is there at time t years?

$$\text{mass after } t \text{ years} = A \times \left(\tfrac{1}{2}\right)^{(t/K)}$$

Idea: t/K is number of half-lives in t years.

7. (Radiocarbon Dating) A bone is found with 2% of the usual amount of carbon-14 in it. The half-life of carbon-14 is 5730 years. How old (in years) is the bone?

(A) $5730 \log(.01)/\log(2)$

(B) $5730 \log(50)/\log(2)$

(C) 5730×50

(D) wicked old

Answer: $\boxed{\text{B}}$ $\approx 32,000$ years

† Any time unit will work, not just years. Just be consistent!

Summary of Logs

$\log(y)$ is how many tens you multiply together to get y.

	laws of exponents	corresponding law of logs
(1)	$10^a \times 10^b = 10^{a+b}$	$\log(xy) = \log(x) + \log(y)$
(2)	$10^0 = 1$	$\log(1) = 0$
(3)	$10^{-a} = 1/10^a$	$\log(1/x) = -\log(x)$
(4)	$(10^a)^p = 10^{ap}$	$\log(x^p) = p\log(x)$
(5)	$10^a/10^b = 10^{a-b}$	$\log(x/y) = \log(x) - \log(y)$

Each of these pairs of equalities says one thing!