Problem 1.

I will use for today this model for disease spread, here S is the number of susceptible, I the number of infected and R is the number of recovered/resistant, note that this are functions of time so it is legit to consider:

$$\begin{aligned} S' &= -10^{-5}SI \\ I' &= 10^{-5}SI - \tfrac{1}{14}I \\ R' &= \tfrac{1}{14}I \end{aligned}$$

1. (5 points)Even though S and R are functions of time, is still makes sense to think about how these two quanities relate to one another. Using chain rule from Calc 1, we know $\frac{dS}{dR}\frac{dR}{dt} = \frac{dS}{dt}$. Assuming $I \neq 0$, solve for $\frac{dS}{dR}$ and then find S as a function of R.

$$\frac{dS}{dR}\frac{dR}{dt} = \frac{dS}{dt}$$

$$\frac{dS}{dR} \cdot \frac{1}{14} I = -10^{-5} S I$$

$$\frac{1}{14}\frac{dS}{dR} = -10^{-5} S$$

$$\frac{dS}{dR} = -1.4 \times 10^{-4} S$$

$$\frac{1}{S}\frac{dS}{dR} = -1.4 \times 10^{-4}$$

$$\int \frac{1}{S}\frac{dS}{dR} = \int -1.4 \times 10^{-4}$$

$$\ln(S) = -1.4 \times 10^{-4} R + C$$

$$S(R) = e^{-1.4\times10^{-4}R + C}$$

2. (5 points) Does everyone on the island eventually get sick? Or do some susceptible people remain?

Yes, some susceptible people will remain, according to the graph of nxt question.

To solve with more logicality.

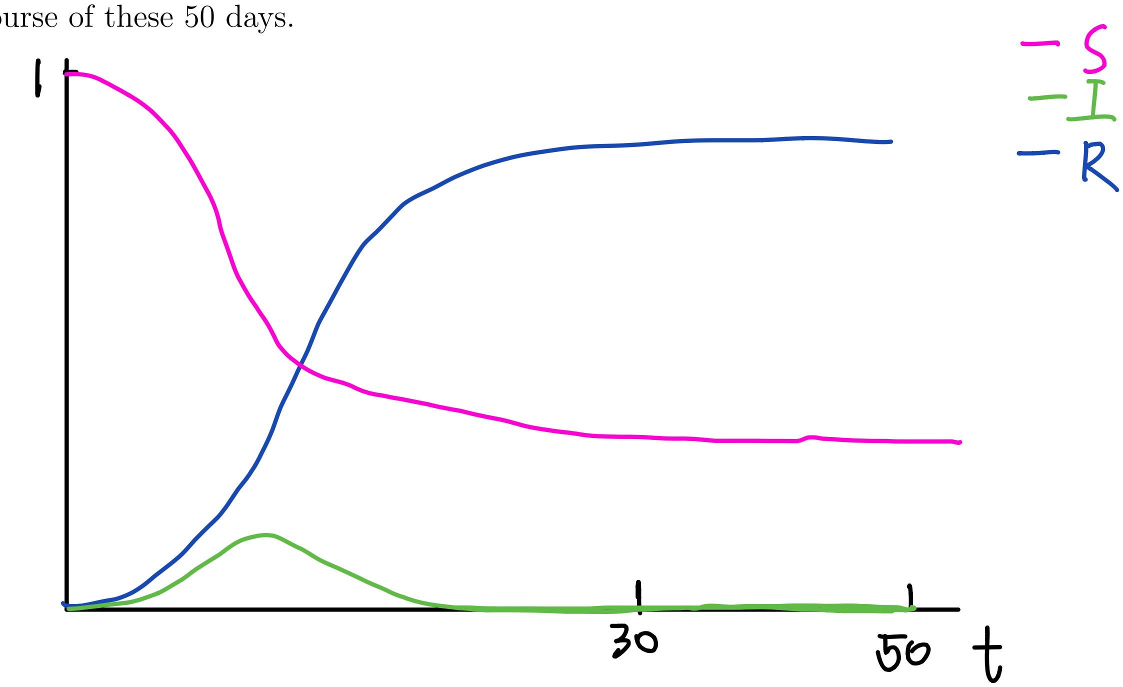
let's assume $S=0$ at some t.

$$e^{-1.4\times10^{-4}R+C}=0.$$

however. $e^{-1.4\times10^{-4}R+C} \neq 0$.

so. some susceptible people will remain

3. (OPTIONAL, THIS WONT BE GRADED) With the initial data $S(0) = 45400$, $I(0) = 2100$, $R(0) = 2500$, use a computer program like Excel, Google Sheets, or Python to implement Euler's method and predict how many people are susceptible, infected, and resistant after 30 days. Create a graphic showing what happens over the course of these 50 days.



Problem 2. We now consider a NEW model for disease spread with *immunity loss*. We use the same model as before, with transmission coefficient 4×10^{-5} and recovery coefficient 0.2, but with additional provision that on any particular day, a Resistant person has a 3% chance of becoming Susceptible.

1. (10 points) Adapt the previous model to include the effect of immunity loss.
2. (10 points)Under what circumstances will the number of recovered individuals decrease?
3. (5 points)Is there a set of initial data (with 50,000 people total) for which the numbers of Susceptible, Infected, and Resistant people stay constant (i.e. an *equilibrium* or *steady state*)?

Create a system of differential equations that would model a zombie outbreak. You can use the S-I-R model as a starting point, but the rules will probably be different (e.g. there may be an "undead" category). Write a few sentences explaining your model. Which rates of change are affected by which variables? What is the long-term behavior of your system?

1) $S' = -4\times 10^{-5} SI - \frac{3}{100}R$
$I' = 4\times 10^{-5} SI + \frac{6}{100}R - \frac{1}{5}I$
$R' = \frac{1}{5}I - \frac{3}{100}R$

2). Reeoverd Individual decrease.

R decrease.

$R' < 0$.

$\frac{1}{5}I < \frac{3}{100}R$

$I < \frac{3}{20}R$

3) Stay constant:

$S + I + R = 50{,}000$

$S' = I' = R' = 0$

$$R' = \frac{1}{5}I - \frac{3}{100}R = 0.$$

$$\frac{1}{5}I = \frac{3}{100}R.$$

$$S' = -4\times10^{-5}SI - \frac{3}{100}R = 0$$

$$4\times10^{-5}SI = \frac{3}{100}R = \frac{1}{5}I$$

$$4\times10^{-5}SI = \frac{1}{5}I$$

$$4\times10^{-5}S = \frac{1}{5}$$

$$S = 5000$$

$$I + R = 45000$$

$$I = (45000 - R)$$

$$\frac{1}{5}(45000 - R) = \frac{3}{100}R$$

$$R = 39130.4$$

$$\underline{I} = 50000 - 5000 - 39130.4 = 5869.6.$$

$$\begin{cases} S = 5000 \\ I = 5869.6 \\ R = 39130.4. \end{cases}$$