Math 4B Midterm Review Problems
October 30, 2014

These are practice problems to help you prepare for your midterm, you do not need to turn in solutions. You should think of this as a starting point for organizing your study plan. You should also review your DPs, old homework problems and problems from the lecture slides.

Your midterm will cover material from sections 1.1, 1.2, 2.1-2.5, 2.9, 3.1-3.4 and 3.7

4. A deposit into a savings account earns interest, which is just a fraction of your deposit added to the total at regular intervals.

   (a) Suppose your account earns 8% each year and that interest is compounded once a year, i.e. 8% of the amount is added each year. How much money will you have after 5 years with an initial deposit of \$100? After $N$ years?

   (b) Now suppose the interest is compounded monthly. How much will you have in the account after 5 years?

   (c) Write down a *difference equation* that describes how the account value is changing. Suppose the annual interest rate $r$ is compounded $n$ times per year. Your difference equation should look something like

   $$A_{k+1} - A_k = ??$$

   (d) Now suppose the bank makes it's payments more and more often: daily, hourly, every minute, every second... continuously. What will your difference equation look like if interest is compounded continuously? HINT: Let $A(t) = A_k$ and let $\Delta t = \frac{1}{n}$, then find an expression for $\Delta A = A(t + \Delta t) - A(t)$. In the limit as $\Delta t \to 0$, you should get a *differential equation*.

   (e) Compare the return after 5 years on two accounts with $A_0 = \$100$ and $r = 8\%$ - one compounded monthly and one compounded continuously. What kind of account do you want to invest in?

13. Make a substitution to solve the following DEs.

   (a) $y' = \dfrac{3y^2 - x^2}{2xy}$, let $v = \frac{y}{x}$.

   (b) $y' + 1 = (y + x)^2$, let $v = x + y$.

   (c) $2yy' = \cos(y^2)$, let $v = y^2$.

14. Consider the following autonomous DE:

   $$y' = y^2(4 - y^2).$$

   Use qualitative information to sketch solution curves to this equation:

(a) Find the equilibrium solutions and classify them as stable, semistable or unstable.

(b) Find a formula for $y''$ and use this to determine the concavity of solutions for certain values of $y$.

(c) Sketch several graphs of solutions in the $ty$-plane.

18. Consider the IVP $t^2y'' - t(t+2)y' + (t+2)y = 0$, $y(1) = a$, $y'(1) = b$

(a) Verify that $y_1(t) = t$ is a solution to the DE. For which values of $a$ and $b$ is the solution to the IVP a scalar multiple of $y_1$?

(b) Explain how you know that the fundamental set for this DE will have at least one other solution. Use Theorem 3.2.1 from your book in your argument.

(c) Use reduction of order to find the second solution for the fundamental set for this DE.

(d) Solve the IVP for $a = 1$ and $b = 0$.

19. Consider the IVP $y'' - 2y' + 2y = 0$, $y(0) = 2$ and $y'(0) = 0$.

(a) Find a fundamental set of *complex* solutions to the DE. Then find a solution to the IVP from among those complex solutions.

(b) Use Euler's Formula to find a fundamental set of *real* solutions to the DE.

(c) Find a solution to the IVP from among your real solutions. Then use Euler's formula to show that this solution is the same as the solution you found in part (a).