Welcome To Math 34A! Differential Calculus

Instructor:

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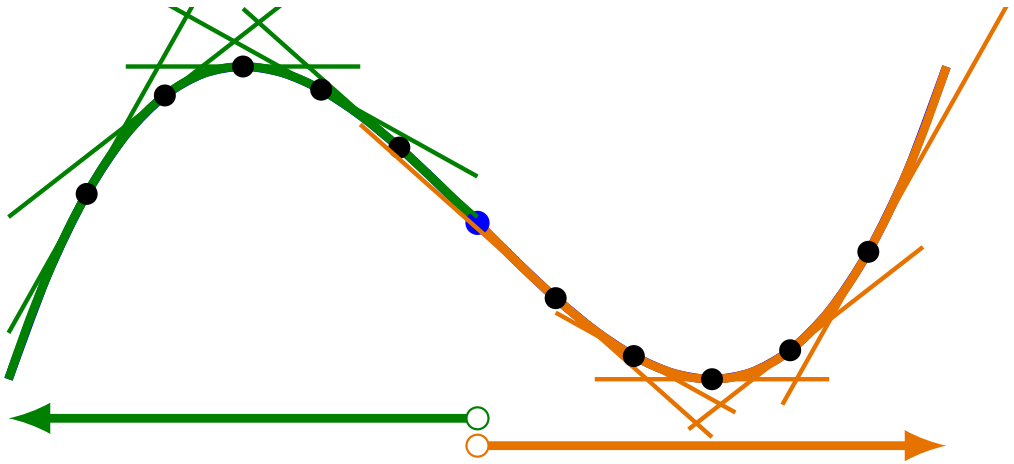
Office Hours:

MTWR after class 2:00-3:00, and by appointment. Details on Gauchospace.

Meanings: The Second Derivative



Point:

$$f''(x) > 0 \iff f'(x) \text{ is increasing}$$
$$\iff f(x) \text{ is concave up}$$

$$f''(x) < 0 \iff f'(x) \text{ is decreasing}$$
$$\iff f(x) \text{ is concave down}$$

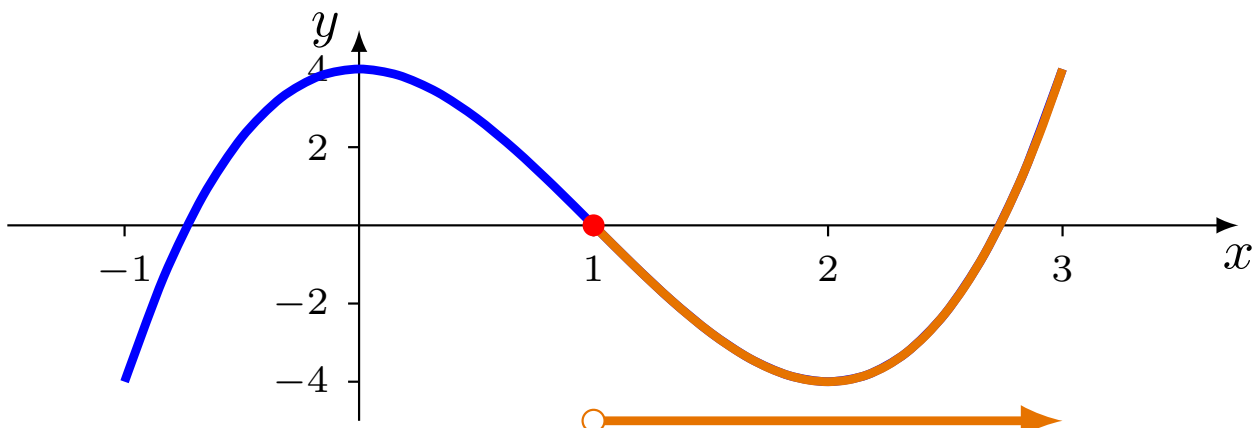
Concavity

$$f''(x) > 0 \iff f(x) \text{ is concave up}$$
$$f''(x) < 0 \iff f(x) \text{ is concave down}$$

(1) For which values of x is $f(x) = x^3 - 6x^2 + 3x + 2$ concave up?

A when $x = 0$ B when $x < 6$ C when $x > 6$
D when $x < 2$ E when $x > 2$ E

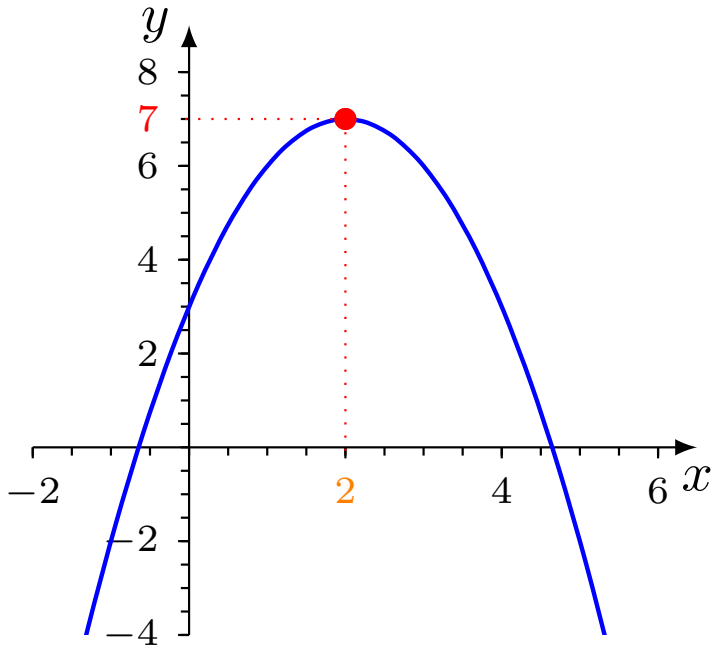
(2) Where is $f''(x) > 0$?



A when $x < 2$ B when $x > 2$ C when $x < 1$
D when $x > 1$ E when $-0.7 < x < 1$ D

§8.13: Max/Min problems

Often want to find the biggest, smallest, most, least, maximum, minimum of something.



Here's the graph of
$y = f(x) = -x^2 + 4x + 3$

The maximum value or just maximum of the function is 7.

The value of x which gives the maximum of $f(x)$ is $x = 2$

We write $f(2) = 7$.

For this example you can see this is the maximum because

$$f(x) = -x^2 + 4x + 3 = -(x-2)^2 + 7$$

$(x-2)^2$ is always positive except when $x = 2$
so the maximum must be at $x = 2$.

How To Find A Max / Min

(1) Find $f'(x)$

(2) Solve $f'(x) = 0$. This is the x value that gives the max / min.

(3) To find the maximum / minimum plug the value of x found in (2) back into $f(x)$.

Example: Use this method to find the x-value where maximum of the function $f(x) = 5x - e^{2x}$ occurs.

A $= 0$ B $= \ln(5)$ C $= 2\ln(5)$ D $= 2\ln(5/2)$ E $= \ln(5/2)/2$

Answer: E

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A $= 0$ B $= \ln(5)$ C $= 2\ln(5)$ D $= 2\ln(5/2)$ E $= \ln(5/2)/2$

$f'(x) = 5 - 2e^{2x}$

$5 - 2e^{2x} = 0$

$-2e^{2x} = -5$

$\ln(e^{2x}) = \frac{5}{2} = 2.5$

$2x = \ln(2.5)$

$x = \frac{1}{2}\ln(2.5)$

Word Problem #1

A ball is thrown into the air. After t seconds the height in meters above the ground of the ball is $h(t) = 40t - 10t^2$. How many meters high did the ball go?

A $= 2$ B $= 40 - 20t$ C $= 20$ D $= 40$ D

Word Problem #2

If an airline sells tickets at a price of $\$200 + 5x$ each the number of tickets it sells is $1000 - 20x$. What price should the tickets be if the airline wants to get the most money?

$$\text{A} = 5 \quad \text{B} = 25 \quad \text{C} = 175 \quad \text{D} = 200 \quad \text{E} = 225 \quad \text{E}$$

Word Problem #3

A fenced garden with an area of 100 m^2 will be made in the shape of a rectangle. It will be surrounded on all four sides by a fence. What length and width should be used so the least amount of fence is needed?

Approach:

(1) Express the total length of fence in terms of only one variable, either L = length of field, or W = width of field. This gives a formula for P = (total length of fence) involving, say, W.

(2) Find minimum by solving $\dfrac{dP}{dW} = 0$.

Students always find (1) the hardest part.
You have been prepared for this by word problems from chapter 3!

Word Problem #4

A fenced garden with an area of 1000 m^2 will be made in the shape of a rectangle. It will be surrounded on all four sides by a fence. Three sides are wood fence, and the remaining side is a brick wall.

- The wood fence costs \$5 per meter length.
- The brick wall costs \$20 per meter length.
- C = total cost of the fence and brick wall
- L = length of the brick wall
- W = width of the other side

(a) Find a formula for C in terms of only L.

A $= 2W + 2L$ B $= 2000L^{-1} + 2L$ C $= 25L + 10000L^{-1}$

D $= 20L + 10000WL^{-1}$ E $= 5L + 3000$ C

(b) What length of brick wall gives lowest cost?

A $= 20$ B $= 40$ C $= 50$ D $= 100$ E $= 25$ A

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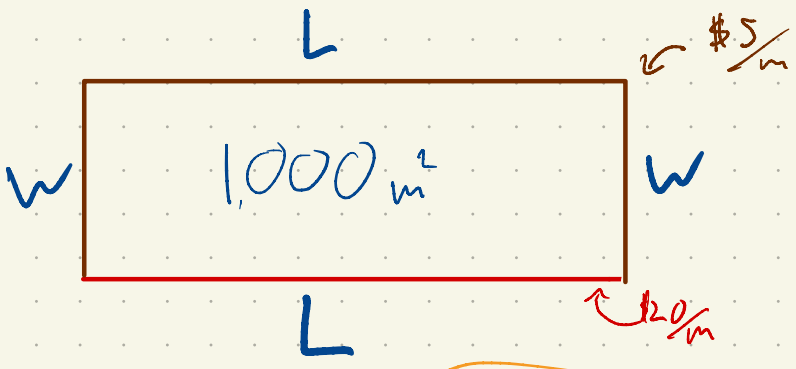
$$\text{A} = 2W + 2L \quad \text{B} = 2000L^{-1} + 2L \quad \text{C} = 25L + 10000L^{-1}$$
$$\text{D} = 20L + 10000WL^{-1} \quad \text{E} = 5L + 3000$$



$A = LW$

$1000 = LW$

$W = \frac{1000}{L}$

$C = \$5(2W + L) + \$20L$

$C = 5\left(2\left(\frac{1000}{L}\right) + L\right) + 20L$

$= 5\left(\frac{2000}{L} + L\right) + 20L$

$= \frac{10{,}000}{L} + 5L + 20L$

$= 10{,}000L^{-1} + 25L$

(b) What length of brick wall gives lowest cost?

A = 20 B = 40 C = 50 D = 100 E = 25

$C(L) = 10{,}000L^{-1} + 25L$

$C'(L) = -10{,}000L^{-2} + 25$

$-10{,}000L^{-2} + 25 = 0$

$-10{,}000L^{-2} = -25$

OR

$\frac{10000}{L}$

$\frac{1}{L^2} = \frac{25}{10000}$

$10000 = 25L^2$

$\frac{10000}{25} = L^2$

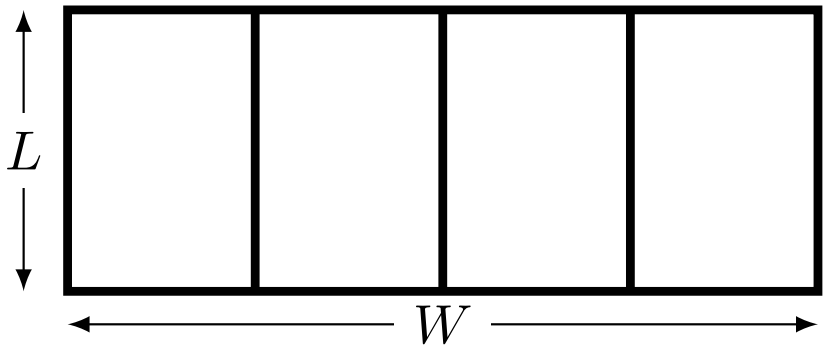
$(L^{-2})^{-1} = \left(\frac{25}{10{,}000}\right)^{-1}$

$\sqrt{L^2} = \sqrt{\frac{10{,}000}{25}}$

$L = \frac{100}{5} = \boxed{20}$

Word Problem #5

A rectangular field is surrounded by fence. It is divided into 4 equal



parts by 3 more dividing fences all parallel to one side of the field.

(a) What is the total length of all the fence needed?

$$\text{A } = 2L + 2W \quad \text{B } = LW \quad \text{C } = 5LW$$

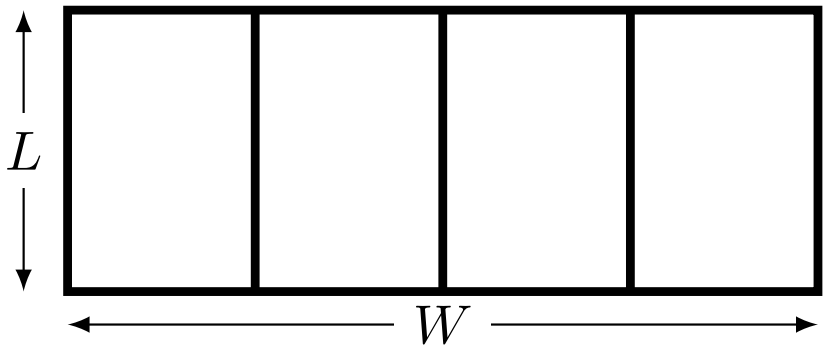
$$\text{D } = L + W \quad \text{E } = 5L + 2W \qquad \text{E}$$

(b) The field must have an area of 1000 m^2. Express W in terms of L.

$$\text{A } 1000 - L \quad \text{B } 1000L \quad \text{C } 1000/L \quad \text{D } 1000 + L \quad \text{C}$$

Word Problem #5 (cont'd)

A rectangular field is surrounded by fence. It is divided into 4 equal



parts by 3 more dividing fences all parallel to one side of the field.

(c) Express the total length of all the fence needed in terms of L.

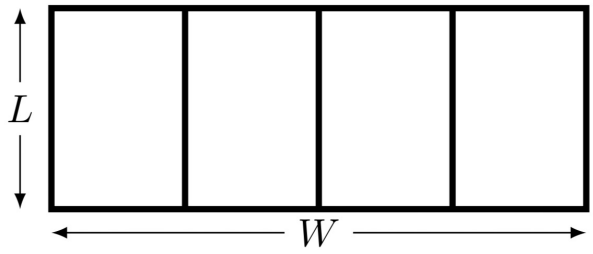
$$\text{A} = 5L + 1000 \quad \text{B} = 5L + 2000/L \quad \text{C} = 5L + 2/L \quad \text{B}$$

(d) What should L be so that the total length of fence used is a minimum?

$$\text{A} = 10 \quad \text{B} = 20 \quad \text{C} = 40 \quad \text{D} = 50 \quad \text{B}$$

Word Problem #5 (cont'd)

A rectangular field is surrounded by fence. It is divided into 4 equal



parts by 3 more dividing fences all parallel to one side of the field.

(c) Express the total length of all the fence needed in terms of L.

$\text{A} = 5L + 1000 \quad \text{B} = 5L + 2000/L \quad \text{C} = 5L + 2/L \quad$ B

(d) What should L be so that the total length of fence used is a minimum?

$\text{A} = 10 \quad \text{B} = 20 \quad \text{C} = 40 \quad \text{D} = 50$

$f(L) = 5L + 2000L^{-1}$

$f'(L) = 5 - 2000L^{-2}$

$5 - 2000L^{-2} = 0$

$2000L^{-2} = 5$

$L^{-2} = \frac{5}{2000}$

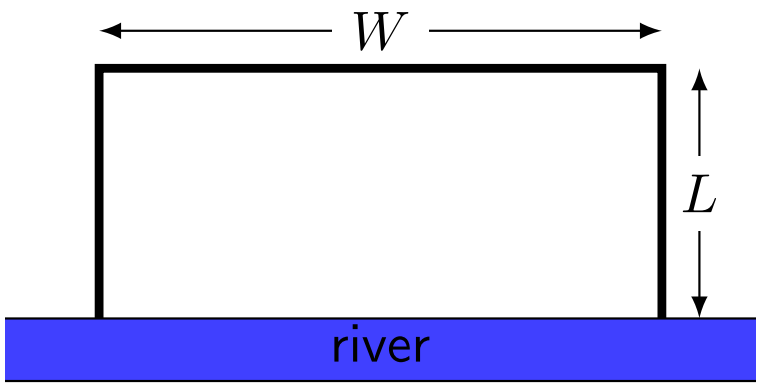
$L^2 = \frac{2000}{5} = 400$

$\boxed{L = 20}$

Word Problem #6

A rectangular field is surrounded on three sides by a fence and the fourth side runs along a perfectly straight river. What is the largest area field which can be so enclosed with 120 meters of fence?

A $= 1200$ m^2 B $= 1500$ m^2 C $= 1800$ m^2 D $= 1000$ m^2 C



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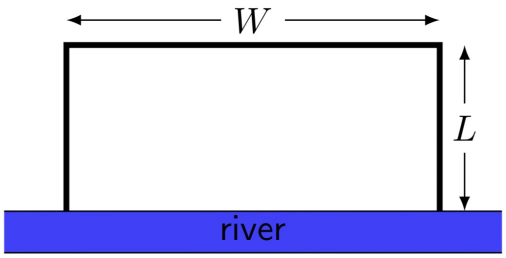

$120 = W + 2L$

$W = 120 - 2L$

$A = LW$

$A = L(120 - 2L)$

$A(L) = 120L - 2L^2$

$A'(L) = 120 - 4L$

Set $= 0$

$120 - 4L = 0$

$120 = 4L$

$30 = L$

Plug back in

$A(30) = 120(30) - 2(30)^2$

$= 3600 - 2(900)$

$= 3600 - 1800$

$= \boxed{1800}$

Word Problem #7

Tickets are going to be sold for a concert.

- If the price of each ticket is \$40, then $2,000$ tickets will be sold.
- For every \$1 the price is decreased, 100 more tickets will be sold.

(a) If the tickets are sold for $\$x$ each, how many will be sold?

$$\text{A} = 2000 - x \quad \text{B} = 2000 - 100x \quad \text{C} = 2000 + 100x$$

$$\text{D} = 6000 - 100x \quad \text{E} = 6000 + 100x \qquad \text{D}$$

(b) What is the total amount of money generated from selling tickets for $\$x$ each?

$$\text{A} = 6000x - 100x^2 \quad \text{B} = 2000x$$

$$\text{C} = 2000 - 40x^2 \quad \text{D} = 6000 - 100x \quad \text{A}$$

(c) What price should the tickets be to generate the most money from sales?

$$\text{A} = \$20 \quad \text{B} = \$22 \quad \text{C} = \$24 \quad \text{D} = \$30 \quad \text{E} = \$40 \quad \text{D}$$

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(a) If the tickets are sold for $\$x$ each, how many will be sold?

A $= 2000 - x$ B $= 2000 - 100x$ C $= 2000 + 100x$

D $= 6000 - 100x$ E $= 6000 + 100x$

This describes a line

$x = 40$, $y = 2{,}000$

$\Delta x = -1$, $\Delta y = +100$

$m = \frac{\Delta y}{\Delta x} = -100$

Point-Slope form

$(y - 2{,}000) = -100(x - 40)$

Simplify

$y = -100x + 4{,}000 + 2{,}000$

$y = 6{,}000 - 100x$

Word Problem #8

A farmer is growing wheat.

- On July 1, she has 1,000 bushels and this increases by 50 bushels per day.
- The price of a bushel on July 1 is \$10 and is dropping at a rate of 20 cents per day.
- She will harvest and sell on the same day.

How many days should she wait, assuming these trends continue?

A = 5 B = 10 C = 15 D = 20 E = other