Homework 2

1. Let $f : X \to Y$ be a continuous function, and consider the space

$$G_f = \{(x, f(x)) \mid x \in X\}$$

equipped with the subspace topology. Prove that the map $X \to G_f$ given by $x \mapsto (x, f(x))$ is a homeomorphism.

2. Prove that a map $F : X \to Y$ between metric spaces is continuous $\iff$ $f\left(\overline{A}\right) \subset \overline{f(A)}$ for all $A \subset X$. [Does your proof use the metric?]

3. Prove that if $\{A_\alpha\}_{\alpha \in \Gamma}$ are subsets of X, then $\overline{\bigcap_{\alpha \in \Gamma} A_\alpha} \subseteq \bigcap_{\alpha \in \Gamma} \overline{A}_\alpha$. Show equality need not hold.

4. Prove that $W \subset X \times Y$ is open with the product topology $\iff$ $\forall (x, y) \in W, \exists$ open subsets $U \subset X$, $V \subset Y$ such that $(x, y) \in U \times V \subset W$.

5. Prove that the topology on X is discrete $\iff$ the diagonal $\Delta = \{(x, y) \mid x \in X\}$ is open in $X \times X$.