5-19 More Derivatives

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HW20: Problem 8

$f(t) = 66 + 32 \cdot e^{-\frac{1}{24}t}$

$f'(t) = 0 + 32 \cdot \left(-\frac{1}{24}\right) e^{-\frac{1}{24}t}$

$f'(3) = -\frac{32}{24} e^{-\frac{1}{24} \cdot 3}$

$= -\frac{4}{3} e^{-\frac{3}{24}}$

The temperature in degrees Fahrenheit of a corpse t hours after death is $66 + 32e^{-t/24}$

(a) How quickly is the temperature decreasing after 5 hours?

degress Fahrenheit per hour

(b) What is the temperature of the surroundings of the corpse?

degrees Fahrenheit

(c) What was the temperature at the point of death?

degrees Fahrenheit

Question: Find $\frac{d}{dx}\left(4e^{3x} + 5x^3\right)$

A= $12e^{2x} + 15x^2$ B= $12e^{3x} + 15x^3$ C= $4e^{3x} + 15x^2$

D= $12e^{3x} + 15x^2$ E= Other D

$4e^{3x} \to 4 \cdot 3e^{3x}$

More Examples

$$\frac{d}{dx}\left(e^{kx}\right) = ke^{kx}$$

(1) $\dfrac{d}{dx}\left(\dfrac{3}{e^{2x}}\right) = ?$

A $= \frac{3}{2e^{2x}}$ B $= \frac{3}{2e^{x}}$ C $= \frac{6}{e^{2x}}$ D $= \frac{-6}{e^{2x}}$ D

(2) The number of grams of Einsteinium-253 after t days is $m(t) = 10e^{-t/30}$. How quickly is the mass changing (in grams per day) when $t = 0$?

A $= -1/30$ B $= -1/3$ C $= -10e^{-t/30}$ D $= -\frac{1}{3}\,e^{t/30}$

§8.12: The Second Derivative

Today: We can take the derivative of a function repeatedly!

Example: If $f(x) = x^3 - 3x + 2$, then

- $\frac{df}{dx} = f'(x) =$ $3x^2 - 3$ $\downarrow$ $6x$ $\downarrow$ 6 $\downarrow$ 0

position $\rightarrow$ velocity $\rightarrow$ acceleration $\rightarrow$ jerk

$\frac{d^2}{dx}(e^{3x}) = 9e^{3x}$

General idea: Differentiating the function n times gives us the nth derivative of f. It is written as

$$f'''\cdots'''(x) = f^{(n)}(x) = \frac{d^n f}{dx^n}.$$

$x^{\frac{1}{2}}$

$\frac{1}{2}\cdot x^{-\frac{1}{2}}$

$\frac{1}{2}\cdot(-\frac{1}{2})\cdot x^{-\frac{3}{2}}$

$= -\frac{1}{4}x^{-\frac{3}{2}}$

(1) What is the second derivative of $3x^2 - 5x + 7$?

A= 0 B= 7 C= 6 D= 3 E= -5 C

(2) $\dfrac{d^2}{dx^2}\left(x^5\right) = ?$

A= 20 B= $5x^4$ C= 0 D= $20x^4$ E= $20x^3$ E

(3) $\dfrac{d^2}{dx^2}\left(\sqrt{x}\right) = ?$

A= $\frac{1}{4}x^{-3/2}$ B= $\frac{-1}{4}x^{-1/2}$ C= $\frac{-1}{4}x^{-3/2}$ D= $\frac{1}{2}x^{-1/2}$ E= 0

(6) If $f(x) = x^3 - 4x^2 + 7x - 31$, then $f''(10) = ?$

A= 6 B= $3x^2 - 8x$ C= $6x$ D= 60 E= 52

$f(x) = x^3 - 4x^2 + 7x - 31$

$f'(x) = 3x^2 - 8x + 7$

$f''(x) = 6x - 8$ $f''(10) = 60 - 8 = 52$

A Rocket

A rocket is fired vertically upwards. The height after t seconds is $2t^3 + 5t^2$ meters.

Question: What is the acceleration ~~in m/sec^2~~ after t seconds?

A= $2t^3 + 5t^2$ B= $6t^2 + 10t$ C= $12t + 10$ D= 12 E= 0

$2t^3 + 5t^2$

$\downarrow$

$6t^2 + 10t$

$\downarrow$

$12t + 10$

Concavity

$f''(x) > 0$	$\iff$	$f(x)$ is concave up
$f''(x) < 0$	$\iff$	$f(x)$ is concave down

(1) For which values of x is $f(x) = x^3 - 6x^2 + 3x + 2$ concave up?

A when $x = 0$ B when $x < 6$ C when $x > 6$
D when $x < 2$ E when $x > 2$

$x^3 - 6x^2 + 3x + 2$

$\downarrow$

$3x^2 - 12x + 3$

$\downarrow$

$6x - 12$

$f''(x) = 6x - 12 > 0$

$6x > 12$

$x > 2$