1. (warm up, 5 points) Suppose you drop a 1 kg squirrel out of a helicopter 500m above the ground (sorry squirrel). We want to model its descent taking into account both the forces of gravity and air resistance. The most popular model for air resistance is that it is proportional to the velocity squared. Assume that the acceleration due to gravity is a constant -9.8m/s^2 If the constant of proportionality for our squirrel is $k = 0.4$ N/(m/s)^2, how long does it take the squirrel to reach the ground?

2. Ignoring other forces, and assuming that an objects' motion is in a straight line directly towards/away from the Earth, Newton's Law of Gravity says that if $y(t)$ is the distance of an object from the center of the Earth,

$$y'' = -\frac{k}{y^2}$$

Our brave squirrel is sitting in a coil-gun powered rocket, which is launched from the International Space Station (6,400 km from the center of the Earth), directly away from the Earth at a velocity of 9 km/s. The rocket's mass is such that in the equation above $k = 300{,}000 \text{ km}^3/\text{s}^2$.

a) **(5 points)** Use the substitution $v = y'$, $v\frac{dv}{dy} = y''$ to reduce this to a first-order ODE.

b) **(5 points)** What is the farthest away from the center of the Earth that the squirrel ever gets?

c) **(5 points)** How long does it take for the rocket to get to its highest point?

3 (5+5 points) This problem will guide you through the beginning of the proof of uniqueness of the solution of the first order IVP (for now don't worry about where ϕ is defined and the regularity, you can assume that it is defined everywhere and its derivative exists also everywhere and it is continuous).

1. Prove that ϕ satisfies the initial value problem:

$$\begin{cases} \phi'(x) = F(x, \phi(x)), \\ \phi(x_0) = y_0, \end{cases}$$

if and only if it satisfies the integral equation

$$\phi(x) = y_0 + \int_{x_0}^{x} F(t, \phi(t))dt.$$

Remember this problem has two parts. First assume that ϕ satisfies the IVP then show that it also satisfies the integral equation. Then assume that ϕ satisfies the integral equation and show that it also satisfies the IVP

HINT: USE FUNDAMENTAL THEOREM OF CALCULUS to prove either directions.