Welcome Back!
Differential Calculus

Instructor:

`schley@math.ucsb.edu`

Office Hours:

Suppose x and y are related variables. So as one changes, the other changes. We can ask:

How much does y change per unit change in x?

Answer: The derivative of y with respect to x tells us, and it depends on the current value of x!

If we write y as a function of x like this: $y = f(x)$, then the derivative is written as

$$\frac{\mathrm{d}y}{\mathrm{d}x} \quad \text{or} \quad \frac{\mathrm{d}f}{\mathrm{d}x} \quad \text{or} \quad f'(x)$$

It is the limit of "average rate of change" over shorter and shorter Δx:

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

also known as "instantaneous rate of change"

Standard Estimation Problem

Question: Approximate $\sqrt[3]{28}$.

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Better estimate: $\sqrt[3]{28} \approx 3.036589$, so the error in the tangent line approximation here is

$$\text{error} \approx 3.037037 - 3.036589 \approx .000448065$$

This is a percentage error of about .015%.

General Rule: (Review)

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Examples:

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A= $7x^7$ B= $6x^6$ C= $6x^7$ D= $7x^6$ E= 0

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Example: $n = 3$: Calculate the average rate of change of x^3 between x and $x + \Delta x$ then take limit as $\Delta x \to 0$.

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A similar calculation works for x^n for any n.

More Applications

3. What is the equation of the tangent line at $x = 1$ to the graph of $y = x^3 - x + 4$? The tangent line is $y = \ldots$?

A$= x + 3$ B$= 3x + 1$ C$= 2x - 2$ D$= 2x + 2$ E$= 6x - 2$

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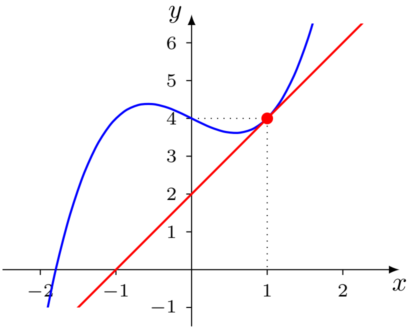

Another Example

4. The temperature in an oven after t minutes is $50 + t^3$ °F. How quickly is the temperature rising after 2 minutes?

A= 58   B= 3   C= 12   D= 50   E= 8

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$$\frac{d}{dx}(x\cdot x) = \frac{d}{dx}(x^2) = 2x$$

About Leibniz and the Product Rule

"It is completely natural to wonder if the derivative of a product is given by that (false) rule. Nobody is saying Leibniz thought it might be true for any extended amount of time. According to p. 254 of "The Historical Development of the Calculus" by C. H. Edwards, Leibniz wrote about his search for a product rule on November 11, 1675. He asked himself if $(uv) = uv$ and quickly dismissed it by the example you gave: $u = v = x$. He did not know a correct product rule at the time. By July 11, 1677 he had the product and quotient rules (see p. 255 of the book by Edwards)."
-Keith Conrad

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Question: $\frac{d}{dx}\left((x^2+1)(x^3+1)\right) = ?$

A$= 6x^3$ B$= 5x^4 + 3x^2 + 2x$ C$= x^5 + x^3 + x^2 + 1$ D$=$ Other

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Answer: $\boxed{\text{B}}$

Once upon a time. . .

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"When you work out derivatives ALWAYS write the $\frac{d}{dx}$ part so you write something like

$$\boxed{\frac{d}{dx}\left(3x^2+5x+2\right)=6x+5}$$

and you never-ever-ever write

$\boxed{3x^2+5x+2 \qquad 6x+5}$ or even worse $\boxed{3x^2+5x+2=6x+5.}$

Because if you don't do as I say I will become a sad math professor and you will repeat this class."

A Few Review Examples:

(1) If $f(x) = \sqrt{x}$, what is $f'(16)$?

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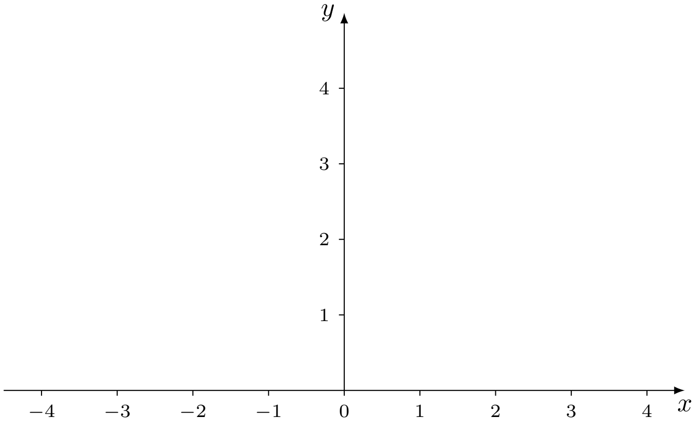
Yes:

$$\boxed{\frac{d}{dx}\left(e^x\right) = e^x.}$$

What's up with that?

The Derivative of $f(x) = a^x$



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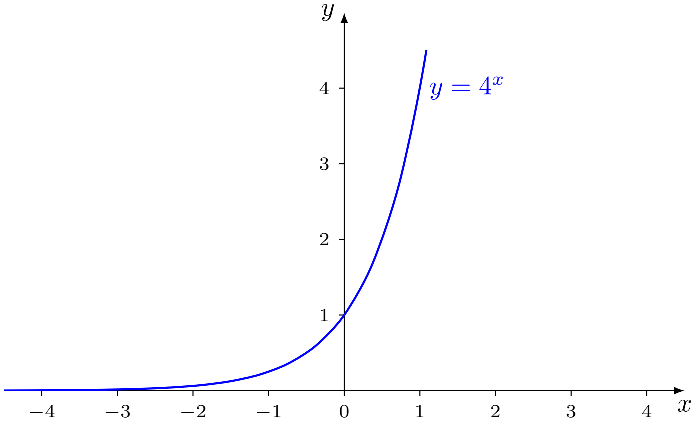

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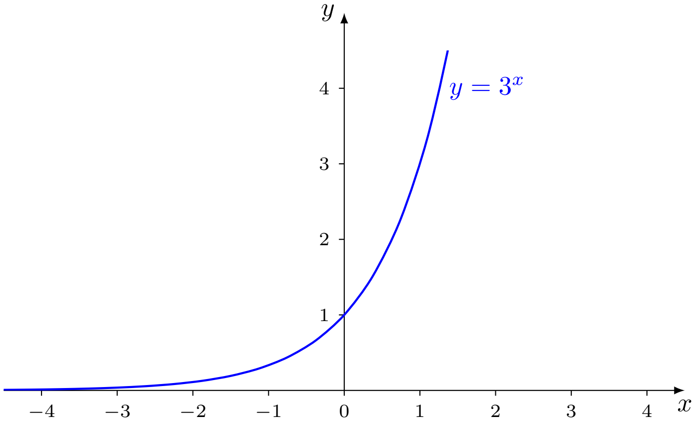

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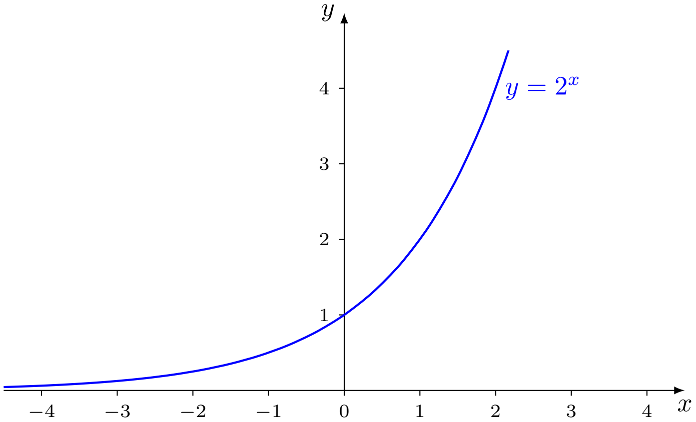

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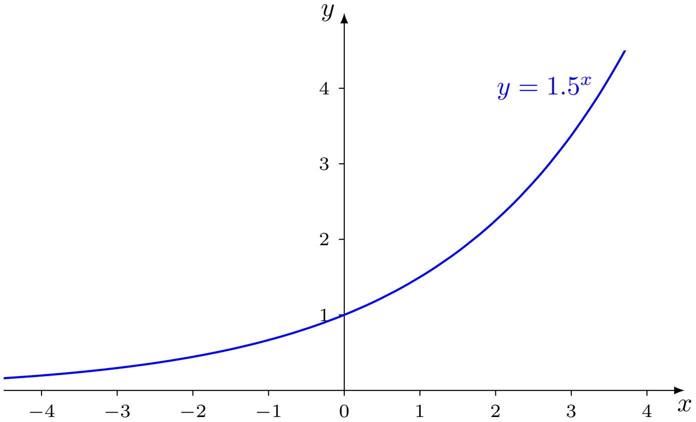

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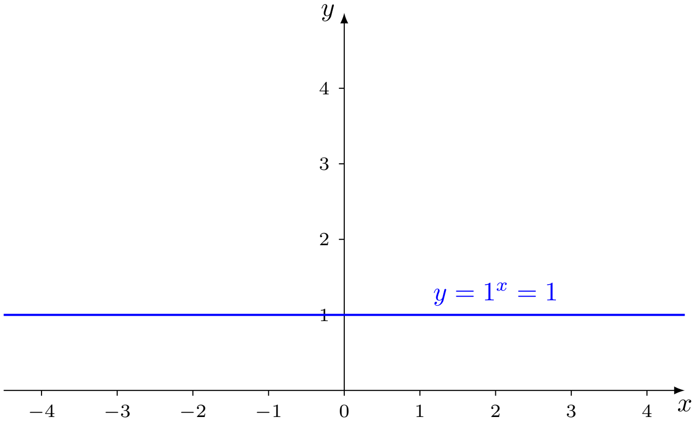

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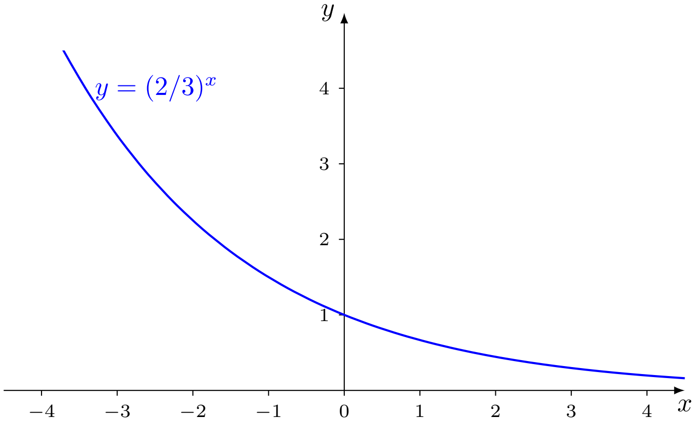

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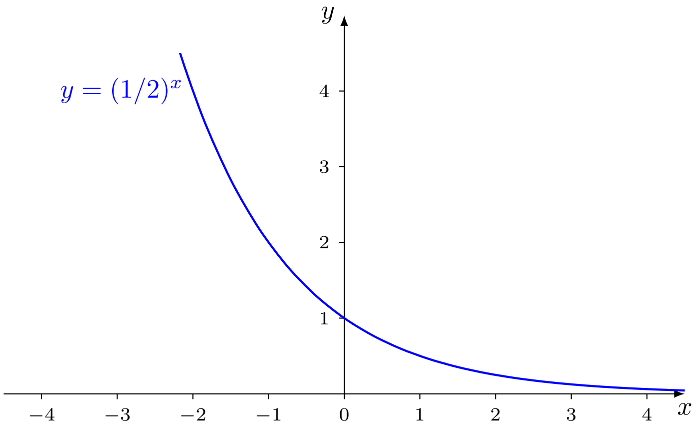

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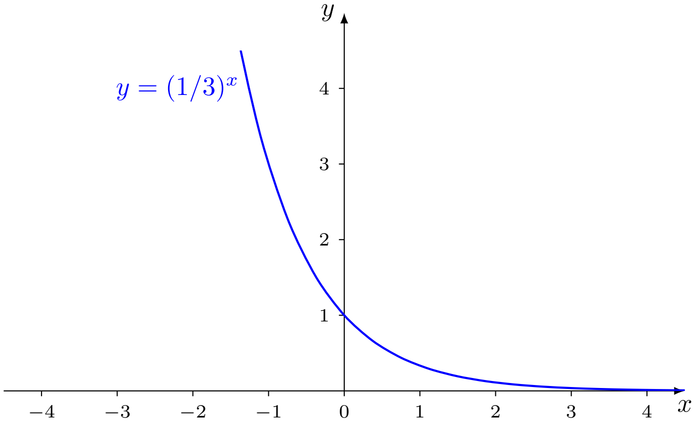

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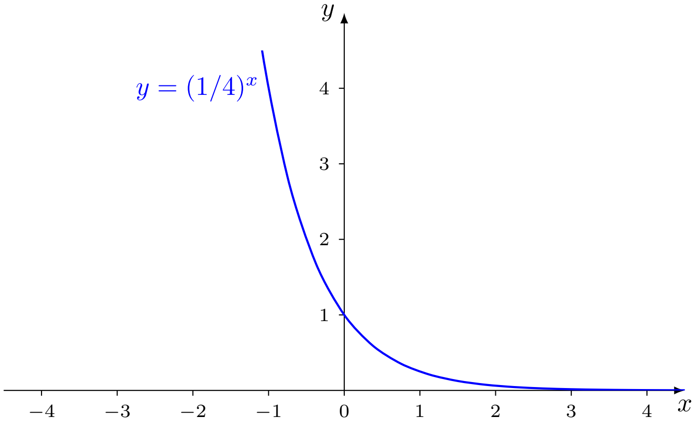

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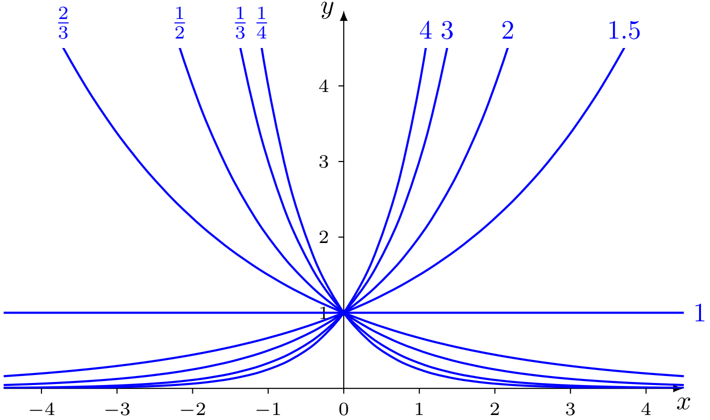

Question: Which "a" should we use?

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$$f'(x) \approx \frac{f(x+h) - f(x)}{h} = \frac{a^{x+h} - a^x}{h} = \frac{a^x(a^h - 1)}{h}$$

The Derivative of $f(x) = a^x$

The slope of the graph at $x = 0$ is

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This is a constant that depends on what a is.
Examples:

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Moral: The derivative of $f(x) = a^x$ is a multiple of itself!
Second Moral: That multiple is 1 when $a = 2.718\,281\,828\cdots = e$.

Factorials

$5! = 1 \times 2 \times 3 \times 4 \times 5$ is called 5 factorial and is the product of the whole numbers from 1 up to 5.

What is 5!?

A= 5 B= 20 C= 60 D= 120 E= 720

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Similarly $n!$ ("n factorial") is the product of all the whole numbers from 1 up to n.

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Factorials come up a lot in probability and statistics.

A Formula for e^x

It turns out that

$$e^x = 1 + x + \frac{x^2}{2} + \frac{x^3}{6} + \frac{x^4}{24} + \frac{x^5}{120} + \cdots + \frac{x^n}{n!} + \cdots$$

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A simple trick:
- The derivative of each term is the preceding one.
- The derivative of the first term is zero.

The Number e

The number $e = 2.718281828\cdots$ is a very important in math. It can be calculated to as much accuracy as needed by using more and more terms in this formula for e^x with $x = 1$ plugged in:

n	$1 + 1 + \frac{1}{2} + \cdots + \frac{1}{n!}$
1	2

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Key Facts about e and e^x

What you need to remember:

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Question: What is the equation of the tangent line to $y = e^x$ at $x = 0$?

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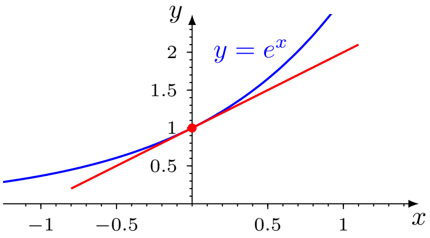

Differentiating $f(x) = e^{kx}$

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These are different functions.

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These are different functions. Both use exponents, sure, but in very different ways!

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Question: Find $\frac{d}{dx}\left(4e^{3x} + 5x^3\right)$

A= $12e^{2x} + 15x^2$ B= $12e^{3x} + 15x^3$ C= $4e^{3x} + 15x^2$
D= $12e^{3x} + 15x^2$ E= Other

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