Math 4B Summer 2020
Quiz #4

No calculators

PRINT NAME

PERM NUMBER

Put your answer in the box provided.

TA: ☐ Trevor ☐ Nick

Time: ☐ 4:30 ☐ 5:30 ☐ 6:30 ☐ 7:30

Variation of Parameters

Following is the formula for the variation of parameters method, which we will explore in today's quiz. I have suppressed the notation a bit, to make it easier to read and remember.

For an ODE of the form

$$y'' + p(t)y' + q(t)y = g(t)$$

with homogeneous solution

$$y_h = C_1 y_1 + C_2 y_2,$$

you can find a particular solution by

$$y_p = -y_1 \int \frac{y_2 g}{W} + y_2 \int \frac{y_1 g}{W}$$

where W is the Wronskian $W[y_1, y_2]$.

The general solution can be found by adding these together:

$$y_g = y_h + y_p.$$

Let's try a step-by-step example.

1. Find a general solution to the following differential equation.

$$y'' + 9y = 3\tan(3t)$$

(a) First we find y_h. To do this, find the general solution to $y'' + 9y = 0$.

$y_h =$

(b) Next we're going to use the formula $y_p = -y_1 \int \frac{y_2 g}{W} + y_2 \int \frac{y_1 g}{W}$, so let's compute the Wronskian $W[y_1, y_2]$ and write down everything we'll need to plug in:

$y_1 =$ $\quad$ $y_2 =$ $\quad$ $g =$ $\quad$ $W =$

(c) Now let's use the formula

$$y_p = -y_1 \int \frac{y_2 g}{W} + y_2 \int \frac{y_1 g}{W}.$$

By the way, there's no need to worry about $+C$ for the integrals in this formula.[1]

To make it a little easier to break up into steps, let's call $u_1 = -\int \frac{y_2 g}{W}$ and $u_2 = \int \frac{y_1 g}{W}$ so that

$$y_p = y_1 u_1 + y_2 u_2.$$

Find $u_1 = -\int \frac{y_2 g}{W}$.

[Hint: put everything in terms of cos and use the fact that $\int \frac{1}{\cos(x)} = \ln|\sec(x) + \tan(x)|$]

$u_1 =$

(d) Find $u_2 = \int \frac{y_1 g}{W}$.

$u_2 =$

[1] This is because when we want the most general solution, adding y_h does for us what $+C$ does for an indefinite integral.

(e) Now we write $y_p = y_1u_1 + y_2u_2$ and simplify.

$y_p =$

(f) To find the general solution, we just write $y_g = y_h + y_p$.

$y_g =$

2. Now try one on your own:

$$y'' - 2y' + y = \frac{e^t}{t^2 + 1}$$

[Hint: start by rewriting the formula from memory as much as possible, so you can work on memorizing it.]

$y_g =$

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