# Practice Problems 10

Math 4B, Spring 2017, Dr. Paul

Practice problems are for your own benefit. You won't turn them in or have them graded, but I have the expectation that you have done these when I write my tests. You can check answers with a TA, in Math Lab, or with the professor.

1. Go back to the pendulum problem from class. Let $\gamma = 2$, $k = 10$.

   (a) Confirm that the equilibria are of the types claimed by doing the calculations in greater detail using the Jacobian matrix.

   (b) Sketch the phase plane using the equilibrium types as your guide (you can compare your answer to the one in the textbook).

   (c) Use Euler's Method to graph $\theta(t)$ if the pendulum has initial position $\theta = 3\pi/4$ and initial velocity $\omega = 0$. Sketch this solution on the phase plane.

   (d) Use Euler's Method to graph $\theta(t)$ if the pendulum has initial position $\theta = 0$, and initial velocity $\omega = 3$. Sketch this solution on the phase plane.

   (e) Use Euler's Method to graph $\theta(t)$ if the pendulum has initial position $\theta = 0$, and initial velocity $\omega = 10$. Sketch this solution on the phase plane. Interpret the results, and in particular the difference between this part and the previous one.

2. Consider the system from class

$$\begin{aligned} x' &= 4x + 2y + 2x^2 - 3y^2 \\ y' &= 4x - 3y + 7xy \end{aligned}$$

   We showed that $(0, 0)$ was a saddle.The other equilibria are

$$(0.279, 1.065), \quad (0.933, -1.057), \quad (-2.354, -0.483)$$

   Classify each of these using the Jacobian matrix at each point.

3. Find and classify all equilibria for the system

$$\begin{aligned} x' &= x - x^2 - xy \\ y' &= 0.5y - 0.75xy - 0.25y^2 \end{aligned}$$