such that $f(x \times y) = 1$ then this contradicts $f(x \times y) = 0$, so f must be a constant function. Therefore $X \times Y$ is connected. □

Lemma If X & Y are homeomorphic & if X is connected, so is Y.

Let $f: X \to Y$ be the homeomorphism. Then let $g: Y \to \{0,1\}$ be a continuous function. Then $g \circ f$ is a continuous function $\Rightarrow g \circ f$ is constant. Since f is surjective $\Rightarrow g$ is constant. □