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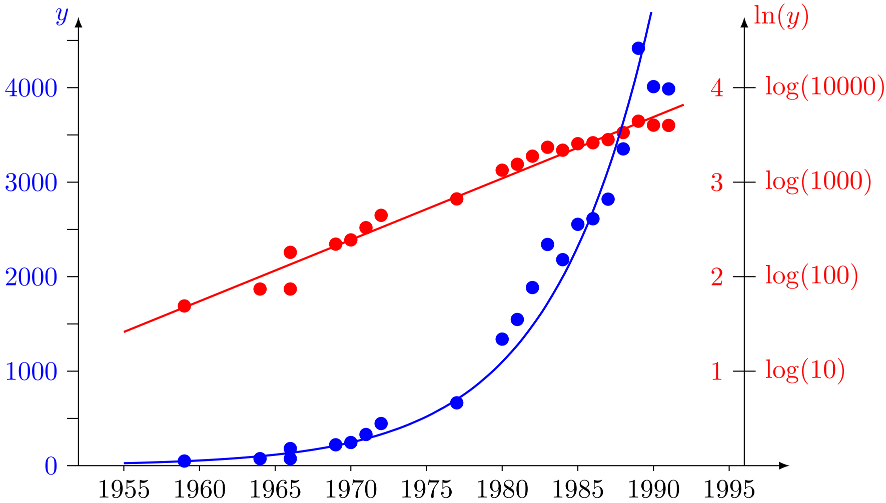


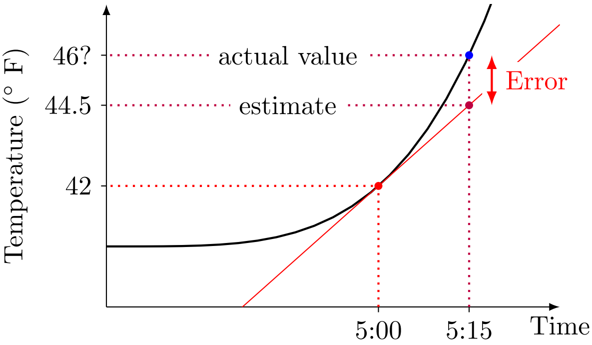
Elephant Seal Births in Channel Islands

§8.6: Tangent Line Approximation

Question: At 5am the temperature is 42° F and increasing at a rate of 10° F per hour. Which of the following do you think is closest to the temperature at 5:15am?

(A) 2.5° F (B) 52° F (C) 43.5° F (D) 44.5° F (E) 5.15° F

Answer: D



Continuing this example

Same set-up:

- $f(x)$ = temperature at time x hours after midnight
- $f(5) = 42$ (42° F at 5:00am)
- $f'(5) = 2$

(1) Find the equation of tangent line to $y = f(x)$ at $x = 5$.

(A) $y = 5x + 42$ (B) $y = 2x + 5$ (C) $y = 2(x - 5) + 42$
(D) $y - 5 = 2(x - 42)$ (E) $y - 42 = 2x - 5$ C

(2) Use this to predict the approximate temperature at 4am.

(A) 40 (B) 41 (C) 42 (D) 43 (E) 44 A

(3) The tangent line approximation is used to estimate the temperature at the following times. Which do you think is most accurate?

(A) 4am (B) 4:50am (C) 5:25am (D) 6am (E) midnight B

Tangent Line Approximation

To do a tangent line approximation:

(i) Find the equation of the tangent line.

(ii) Plug in the required value(s) into this equation.

Suppose $f(4) = 2$ and $f'(4) = 3$.

(a) The equation of the tangent line to $y = f(x)$ at $x = 4$ is $y =$?

(A) $4x - 14$ (B) $3x - 10$ (C) $2x - 6$ (D) $3x - 4$ (E) $2x - 5$ B

(b) Use this tangent line approximation to estimate $f(4.1)$.

(A) 2.3 (B) 1.7 (C) 2.6 (D) 1.4 (E) 2 A

(c) Use the tangent line approximation to estimate the value of x which gives $f(x) = 2.9$.

(A) 4.9 (B) 4.1 (C) 2.9 (D) 4.1 (E) 4.3 E

Standard Estimation Problem

Question: Approximate $\sqrt{26}$.

(A) 0.1 (B) 5.01 (C) 5.05 (D) 5.1 (E) 5.2 D

Hint: If $g(x) = \sqrt{x}$, then $g'(25) = 1/10$ and $g(25) = \sqrt{25} = 5$.

Better estimate: $\sqrt{26} \approx 5.09902$, so the error in the tangent line approximation here is

$$\text{error} \approx 5.1 - 5.09902 \approx 0.001$$

This is a percentage error of only 0.02%.

Another Example:

- $f(t)$ = number of grams of a chemical reagent after t seconds
- We're told $f(0) = 20$ and $f'(0) = -3$

Question: Roughly how many grams are there after t seconds?

(A) $4 - 3t$ (B) $20 - 3t$ (C) $20 - 4t$ (D) $20 + 4t$ (E) $32 - 3t$

Answer: $\boxed{\text{B}}$

Lake Cachuma (a linear approximation)

- Lake Cachuma was completed in 1950. *really completed 1953*
- It originally had a capacity of 205,000 acre feet (this is volume).
- In 2010 it has a capacity of approximately 190,000 acre-feet as a result of the accumulation of silt in the reservoir.
- $f(t)$ = capacity in acre-feet of Lake Cachuma t years after 1950.

(1) Write down a linear approximation from this information for $f(t)$.

(A) $205,000 - 15,000t$ (B) $190,000 + 250t$ (C) $205,000 - 250t$

(D) $190,000 - 250t$ (E) $190,000 - 125t$ C

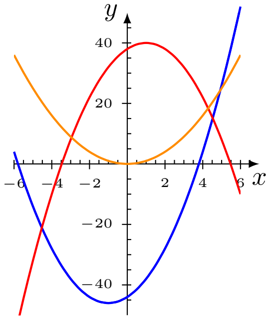
(2) Which of the following years is the best estimate for when 10% of its original capacity will have been lost due to silt?

(A) 2027 (B) 2032 (C) 2037 (D) 2042 (E) 2047 B

Sketching some simple graphs

It's useful to be able to sketch...

(1) Quadratics



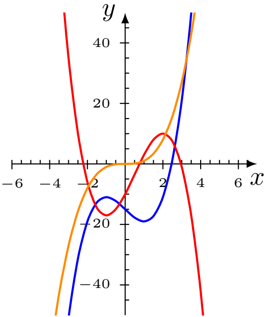
$y = 2x^2 + 4x - 44$
$y = -2x^2 + 4x + 38$

- $y = ax^2 + bx + c$
- Bowl-shaped:
  - ⋆ Opens up if $a > 0$
  - ⋆ Opens down if $a < 0$
- Model curve: $y = x^2$
  Shown here!

Sketching some simple graphs

It's useful to be able to sketch…

(2) Cubics



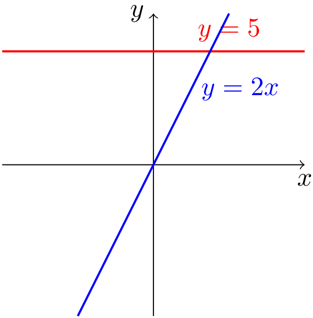
$y = 2x^3 - 6x - 15$

$y = -2x^3 + 3x^2 + 12x - 10$

- $y = ax^3 + bx^2 + cx + d$
- "S"-shaped:
  - ⋆ Goes to $+\infty$ if $a > 0$
  - ⋆ Goes to $-\infty$ if $a < 0$
- Model curve: $y = x^3$
  Shown here!

For a polynomial, the highest power of x dominates when x is big

The Derivatives of Simple Functions



The derivative of a constant is. . . ? zero because:

- derivative = rate of change
- constants don't change

- derivative = slope
- slope = 0

So $\frac{d}{dx}(5) = 0$

The derivative of a straight line is. . . ? its slope because

- derivative = slope

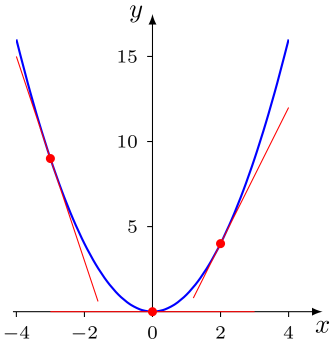
So $\frac{d}{dx}(2x) = 2$

Meaning of Derivatives

$$\frac{d}{dx}\left(x^2\right) = 2x$$

What this means

The slope of the graph of $y = x^2$ at $x = a$ is $2a$



at $x = -3$, slope is $2(-3) = -6$
at $x = 0$, slope is $2(0) = 0$
at $x = 2$, slope is $2(2) = 4$

derivative = rate of change = slope of graph = slope of tangent line

General Rule:

$$\frac{d}{dx}\left(x^{2}\right) = 2x$$
$$\frac{d}{dx}\left(x^{3}\right) = 3x^{2}$$
$$\frac{d}{dx}\left(x^{4}\right) = 4x^{3}$$

$$\boxed{\frac{d}{dx}\left(x^{n}\right) = nx^{n-1}}$$

The exponent comes out front. Then subtract one from exponent.

Examples:

(1) $\frac{d}{dx}\left(x^{7}\right) =$

(A) $7x^7$ (B) $6x^6$ (C) $6x^7$ (D) $7x^6$ (E) 0 D

(2) $\frac{d}{dx}\left(x^{-3}\right) =$

(A) $3x^{-2}$ (B) $-3x^{-2}$ (C) $-2x^{-4}$ (D) $-3x^{-4}$ D

What's the derivative of x^n?

$$\frac{d}{dx}\left(x^{n}\right) = nx^{n-1}$$

More Examples

$$\frac{d}{dx}\left(x^{n}\right) = nx^{n-1}$$

(3) $\frac{d}{dx}\left(x^{1/2}\right) =$

(A) $\frac{1}{2}x^{1/2}$ (B) $-\frac{1}{2}x^{-1/2}$ (C) $\frac{1}{2}x^{-1/2}$ C

Rule: ALWAYS rewrite the thing you want derivative of as x^n

(4) $\frac{d}{dx}\left(\frac{1}{x^3}\right) =$

(A) $\frac{1}{3x^2}$ (B) $-3x^{-2}$ (C) $-3x^{-4}$ C

(5) $\frac{d}{dx}\left(\sqrt{x}\right) =$

(A) $-\frac{1}{2}\sqrt{x}$ (B) $\frac{1}{2}x^{-1/2}$ (C) $-\frac{1}{2}x^{-1/2}$ B