Welcome To Math 34A! Differential Calculus

Instructor:

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Office Hours:

MTWR after class 2:00-3:00, and by appointment. Details on Gauchospace.

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Please do not distribute outside of this course.

Midterm 1: Next Tuesday in class

Bring:

- A pen or sharp pencil.
- A 3" $\times$ 5" notecard (both sides!).
- Student ID (so we can make sure it's you)

Don't bring:

- A calculator

Please Be Early!

See textbook for sample exam questions.

Warm-up

$\log_3(9) = ?$ means "How many times do we need to triple 1 to get 9?"

- $\log_3(9) = \boxed{2}$
- $\log_3(81) = \boxed{4}$
- $\log_3(1) = \boxed{0}$
- $\log_3(\frac{1}{3}) = \boxed{-1}$

Warm-up Part II

Let's try it with decupling!

- $\log_{10}(100) = \boxed{2}$
- $\log_{10}(1000) = \boxed{3}$
- $\log_{10}(1) = \boxed{0}$
- $\log_{10}(.0001) = \boxed{\text{-4}}$
- $\log_{10}(100000000000000000000000000000000000000000000000000000000000000000000000000000000000000000000000000000) = \boxed{100}$

Warm-up Part III

Closeness

- As x gets close to 0, $2+x$ gets close to... $\boxed{2}$
- As x gets close to 0, $5+2x$ gets close to... $\boxed{5}$
- As x gets close to 0, $3+x^2$ gets close to... $\boxed{3}$
- As x gets close to 3, $5x$ gets close to... $\boxed{15}$
- As x gets close to 2 and y gets close to 3, $\frac{x}{y}$ gets close to... $\boxed{\frac{2}{3}}$

§5.1: Error and Limit

Suppose the "real" answer is 10, but your approximate answer is 9.5

$$\text{error} = (\text{real answer}) - (\text{approximate answer})$$

In example error $= 10 - 9.5 = 0.5$

$$\%\ \text{error} = \left(\frac{\text{error}}{\text{real answer}}\right) \times 100\%$$

In other words it is the error expressed as a percentage of the real answer.
Often this is what matters.

1. You have \$50 in you pocket but YOU THINK you have only \$40. What is the percentage error?

A = 10% B = 20% C = 25% D = 40% E = 50% $\boxed{\text{B}}$

§5.1: Error and Limit

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1. You have \$50 in you pocket but YOU THINK you have only \$40. What is the percentage error?

A = 10% B = 20% C = 25% D = 40% E = 50%

error: $50 - 40 = \$10$

% error: $\frac{10}{50} = \frac{1}{5} \xrightarrow{\%} \frac{100}{5}\% = \boxed{20\%}$

Limits

Imagine you calculate more and more accurate approximations to a real answer that you don't know.

$$\begin{aligned} x_1 &= 1.3 \\ x_2 &= 1.33 \\ x_3 &= 1.333 \\ x_4 &= 1.3333 \\ &\vdots \\ &= \text{real answer???} \end{aligned}$$

These numbers get ever closer to $1.3333\cdots = 4/3$.
This is the real answer. The limit of this sequence is $4/3$:

$$\lim_{n\to\infty} x_n = 4/3$$

Read aloud as "The limit as n goes to infinity of x_n is $4/3$."

Guessing Limits

To work out (guess) a limit (when n goes to infinity) imagine plugging into the formula a REALLY BIG value for n like a thousand, or a million, or…

2. $\lim_{n\to\infty} \left(\frac{1}{n}\right) = ?$

$$A = \frac{1}{n} \qquad B = 0 \qquad C = 1 \qquad D = \frac{1}{\infty} \qquad E = \infty \qquad \boxed{B}$$

3. $\lim_{n\to\infty} \left(\frac{n}{n+3}\right) = ?$

$$A = 0 \qquad B = 1/3 \qquad C = 1 \qquad D = 1/4 \qquad E = \infty/(\infty+3). \qquad \boxed{C}$$

Guessing Limits

To work out (guess) a limit (when n goes to infinity) imagine plugging into the formula a REALLY BIG value for n like a thousand, or a million, or...

2. $\lim_{n\to\infty}\left(\frac{1}{n}\right) = ?$

$A = \frac{1}{n}$ $B = 0$ $C = 1$ $D = \frac{1}{\infty}$ $E = \infty$ $\boxed{B}$

3. $\lim_{n\to\infty}\left(\frac{n}{n+3}\right) = ?$

$A = 0$ $B = 1/3$ $C = 1$ $D = 1/4$ $E = \infty/(\infty+3)$.

$\lim_{n\to\infty}\frac{n}{n+3}$

$\frac{100}{103}$ oo <1

$\frac{100000}{100003}$

More Guessing Limits

4. $\lim_{n\to\infty}\left(\frac{2n+5}{9n+71}\right) = ?$

$$A = \frac{5}{71} \qquad B = \frac{2}{71} \qquad C = \frac{5}{9} \qquad D = \frac{2}{9} \qquad E = \frac{2\infty}{9\infty} \qquad \boxed{D}$$

For homework, you can use a calculator and plug in really big values for n then guess. For example if you plug in $n = 1000000$ and get the answer 16.0000361 you guess the limit is really 16.

For engineering, calculus students learn lots of tricks to work out limits. In this class we don't do that. Just UNDERSTAND the main idea.

More Guessing Limits

4. $\lim_{n\to\infty}\left(\frac{2n+5}{9n+71}\right) = ?$

$$\frac{2(10^{100}) + \cancel{5}}{9(10^{100}) + \cancel{71}} \approx \frac{2(10^{000})}{9(10^{000})} = \frac{2}{9}$$

More: Spot The Difference!

7. $\lim_{x\to 1}\left(\frac{x-1}{\underbrace{x^2-1}_{=(x-1)(x+1)}}\right) = \lim_{x\to 1}\left(\frac{\cancel{x-1}}{\cancel{(x-1)}(x+1)}\right) = \lim_{\substack{x\to 1\\ x\to\infty}}\left(\frac{1}{x+1}\right) = \frac{1}{2}$

small crazy small

9. $\lim_{x\to 0}\left(\frac{3x+x^2}{2x}\right) = ?$ $\quad \frac{3x+\cancel{x^2}}{2x} \approx \frac{3x}{2x} = \frac{3}{2}$

small

small

A = 0 B = $\frac{0}{0}$ C = $\frac{1}{0}$ D = $\frac{1}{2}$ E = $\frac{3}{2}$ $\boxed{\text{E}}$

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Even More Guessing Limits

5. $\lim_{n\to\infty}\left(\dfrac{2n+17}{5n+8}\right) = ?$

$$A = \frac{2}{5} \qquad B = \frac{17}{5} \qquad C = \frac{2}{8} \qquad D = \frac{17}{8} \qquad E = \frac{19}{13} \qquad \boxed{A}$$

6. $\lim_{n\to\infty}\left(3+\dfrac{1}{n}\right) = ?$

$$A = 1 \qquad B = 3 \qquad C = 0 \qquad D = \frac{1}{3} \qquad E = \infty \qquad \boxed{B}$$

More: Spot The Difference!

7. $\lim_{x \to 1} \left(\frac{x-1}{x^2-1} \right) = \frac{1}{2}$

8. $\lim_{x \to 1} \left(\frac{x+3}{x^2+1} \right) = ?$

A $= 3$ B $= 1$ C $= 4$ D $= 2$ E $= 0$ $\boxed{\text{D}}$

9. $\lim_{x \to 0} \left(\frac{3x+x^2}{2x} \right) = ?$

A $= 0$ B $= \frac{0}{0}$ C $= \frac{1}{0}$ D $= \frac{1}{2}$ E $= \frac{3}{2}$ $\boxed{\text{E}}$

More Guessing Limits

4. $\lim_{n\to\infty}\left(\dfrac{2n+5}{9n+71}\right) = ?$

$$\frac{2(10^{100})+5}{9(10^{100})+71} \approx \frac{2(10^{000})}{9(10^{000})} = \frac{2}{9}$$

More: Spot The Difference!

7. $\lim_{x\to 1}\left(\dfrac{x-1}{\underbrace{x^2-1}_{=(x-1)(x+1)}}\right) = \lim_{x\to 1}\left(\dfrac{\cancel{x-1}}{\cancel{(x-1)}(x+1)}\right) = \lim_{\substack{x\to 1\\ x\to\infty}}\left(\dfrac{1}{x+1}\right) = \dfrac{1}{2}$

small crazy small

9. $\lim_{x\to 0}\left(\dfrac{3x+x^2}{2x}\right) = ?$ $\quad \dfrac{3x+\cancel{x^2}}{2x} \approx \dfrac{3x}{2x} = \dfrac{3}{2}$

small small

A $= 0$ B $= \dfrac{0}{0}$ C $= \dfrac{1}{0}$ D $= \dfrac{1}{2}$ E $= \dfrac{3}{2}$ $\boxed{\text{E}}$

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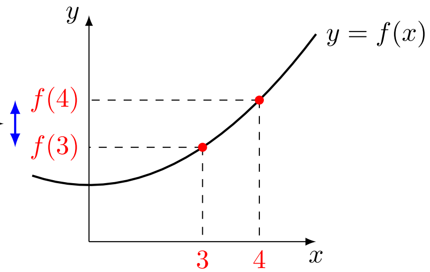
§5.2: Change in $f(x)$

$f(4) - f(3)$

$=$ change in $f(x)$
when x changes from 3 to 4



Example: $f(x) =$ stock value x years after 2010
Ex: $f(3) =$ stock value in 2013

$f(4) - f(3) =$?change in stock value from 2013 to 2014

Calculus is about change

The calculations involve limits.

10. What is the change in $f(x) = x^2$ between 2 and 3?

$A = 1$ $B = 4$ $C = 5$ $D = 6$ $E = 9$ $\boxed{\text{C}}$

11. What is the change in $f(x) = x^2$ between 2 and $2 + h$?

$A = 2$ $B = h^2 - 2$ $C = 4h$ $D = h^2$ $E = 4h + h^2$ $\boxed{\text{E}}$

Note: This <u>exact</u> example comes up when we do calculus.

Calculus is about change

The calculations involve limits.

10. What is the change in $f(x) = x^2$ between 2 and 3?

A = 1 B = 4 C = 5 D = 6 E = 9

Change means Secondone − first one

$$f(3) - f(2)$$
$$3^2 - 2^2$$
$$9 - 4$$
$$\boxed{5}$$

11. What is the change in $f(x) = x^2$ between 2 and $2 + h$?

Second − first

$$f(2+h) - f(2)$$
$$(2+h)^2 - 2^2$$
$$\cancel{4} + 2(2h) + h^2 - \cancel{4}$$
$$\boxed{4h + h^2}$$

ooo

$(a+b)^2 = a^2 + 2ab + b^2$

§5.3: Summation Notation

$$\sum_{n=1}^{7} n = 1 + 2 + 3 + 4 + 5 + 6 + 7$$

Read aloud: "The sum from n equals 1 up to 7 of n"

$$\sum_{n=1}^{4} n^2 = 1^2 + 2^2 + 3^2 + 4^2$$

$$\sum_{n=1}^{5} 2^n = 2^1 + 2^2 + 2^3 + 2^4 + 2^5$$

Σ is the Greek version of S

. . . as in Summation

. . . and the integral sign $\int$ (Math 34B)

Examples:

8. $$\sum_{k=100}^{150} (k^2 + k) = (100^2 + 100) + (101^2 + 101) \cdots + (150^2 + 150)$$

9. Summing entries in a table of data (or in a spreadsheet program)

$$\sum_{p=5}^{9} x_p = x_5 + x_6 + x_7 + x_8 + x_9$$

10. Summing values of a function

$$\sum_{i=-2}^{1} f(i) = f(-2) + f(-1) + f(0) + f(1)$$

Examples 2: Averages

The average of $5, 1, 4, 14$ is

$$\frac{5+1+4+14}{4}$$

Add up the numbers you have then divide by how many numbers you had.

Average of $x_1, x_2, \cdots, x_N$ is

$$\frac{1}{N} \sum_{i=1}^{N} x_i = \frac{x_1 + x_2 + \cdots + x_N}{N}.$$

Examples 3: Cool Sum Formulas

12. $$\left(\sum_{k=1}^{15} a_k\right) + \left(\sum_{k=16}^{35} a_k\right) = \sum_{k=1}^{35} a_k$$

To see why this works, just write it out!

$$(a_1 + \cdots + a_{15}) + (a_{16} + \cdots + a_{35}) = (a_1 + \cdots + a_{35})$$

13. $$\left(\sum_{k=1}^{50} f(k)\right) - \left(\sum_{k=20}^{50} f(k)\right) = \sum_{k=1}^{19} f(k)$$

This just says

$$(f(1) + \cdots + f(50)) - (f(20) + \cdots + f(50)) = (f(1) + \cdots + f(19))$$

And More Cool Sum Formulas

14. $$\left(\sum_{i=1}^{7} a_i\right) + \left(\sum_{i=1}^{7} b_i\right) = \sum_{i=1}^{7} (a_i + b_i)$$

This just says that

$$(a_1 + \cdots + a_7) + (b_1 + \cdots + b_7) = (a_1 + b_1) + \cdots + (a_7 + b_7)$$

15. $$\left(\sum_{i=1}^{100} p_i\right) - \left(\sum_{i=1}^{50} p_i\right) =$$

$$\text{A} = \sum_{i=50}^{100} p_i \qquad \text{B} = \sum_{i=1}^{50} p_i \qquad \text{C} = \sum_{i=1}^{150} p_i \qquad \text{D} = \sum_{i=51}^{100} p_i$$

Hint: Just write it out! $\boxed{\text{D}}$

$$(p_1 + \cdots + p_{100}) - (p_1 + \cdots + p_{50}) = (p_{51} + \cdots + p_{100})$$

That's it. Thanks for being here.