Math 450B
Homework 4
Dr. Fuller
Solutions

3. The linear transformations $(x,y) \mapsto 0$ and $(x,y) \mapsto y$ both satisfy the definition of the derivative of f on A.

4. Compute the partials: $\frac{\partial f}{\partial x}(0,0) = \lim_{t \to 0} \frac{\sqrt{|t \cdot 0|} - \sqrt{|0|}}{t} = 0$; similarly $\frac{\partial f}{\partial y}(0,0) = 0$. Assuming that f is differentiable, then necessarily $Df((0,0)) = 0$. Thus

$$\lim_{(x,y)\to(0,0)} \frac{\sqrt{|xy|}}{\sqrt{x^2+y^2}} = 0.$$

But this is false: let $\varepsilon = \frac{1}{2}$, then for any $\delta > 0$, we have $\|(\frac{\delta}{2}, \frac{\delta}{2})\| < \delta$, but $\frac{\sqrt{|\frac{\delta}{2}\frac{\delta}{2}|}}{\sqrt{\frac{\delta}{2}^2 + \frac{\delta}{2}^2}} = \frac{\sqrt{2}}{2} > \varepsilon$.

5. Let $\varepsilon > 0$, and pick $\delta = \varepsilon / M$. Note that $\|f(\mathbf{x})\| \le M\|\mathbf{x}\|^2$ implies that $f(\mathbf{0}) = 0$. Then with $Df(\mathbf{0}) = 0$ we get

$$\frac{\|f(\mathbf{x}) - f(\mathbf{0}) - Df(\mathbf{0})(\mathbf{x})\|}{\|\mathbf{x}\|} = \frac{\|f(\mathbf{x})\|}{\|\mathbf{x}\|} \le M\|\mathbf{x}\| < \varepsilon$$

for all $\|\mathbf{x}\| < \delta$.

6. $Dg(\mathbf{0}) = D(T+f)(\mathbf{0}) = DT(\mathbf{0}) + Df(\mathbf{0}) = T + 0 = T$. (The second equality uses problem 2, and the next-to-last one uses problem 5.)