Practice Problems 7

Math 4B,   Spring 2017,   Dr. Paul

Practice problems are for your own benefit. You won't turn them in or have them graded, but I have the expectation that you have done these when I write my tests. You can check answers with a TA, in Math Lab, or with the professor.

1. Consider the following system:

$$\frac{dx}{dt} = (y - x)(1 - x - y), \quad \frac{dy}{dt} = x(2 + y)$$

   (a) Find the critical points of the system

   (b) Under what circumstances is x increasing? Shade this region in the phase plane.

   (c) Under what circumstances is y increasing? Shade this region in the phase plane.

   (d) Give a rough sketch of the vector field in the phase plane, and sketch some solution curves.

2. Consider the system

$$\vec{x}'(t) = \begin{bmatrix} 1 & 1 \\ -2 & 4 \end{bmatrix} \vec{x}(t)$$

   (a) Find the general solution.

   (b) Solve the initial value problem with $x(0) = 2$, $y(0) = 4$.

3. Do the following for the system

$$\vec{x}' = \begin{bmatrix} 2 & 3 \\ 4 & 1 \end{bmatrix} \vec{x}$$

   (a) Characterize the circumstances where x is increasing/decreasing, and where y is increasing/decreasing.

   (b) At which points in the phase plane will the vectors be pointing directly towards the origin? Directly away from the origin?
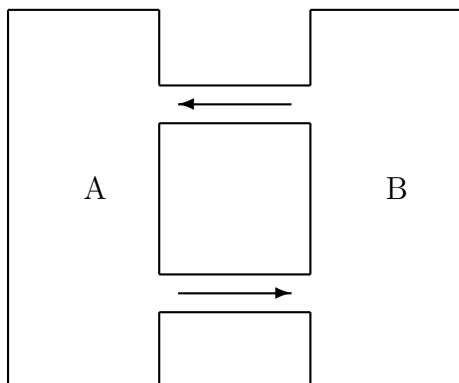
   (c) Sketch a phase portrait.

4. Find the general solutions for the following systems.

   (a) $x' = x + y, \quad y' = -2x + 4y$.

   (b) $\vec{x}' = \begin{bmatrix} -1 & 1 \\ 1 & -1 \end{bmatrix} \vec{x}$

   (c) $\begin{bmatrix} x' \\ y' \\ z' \end{bmatrix} = \begin{bmatrix} -2 & 1 & 1 \\ 1 & -2 & 1 \\ 1 & 1 & -2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix}$

5. Which of the systems from the previous problem models a closed system with two water tanks exchanging water?

Sketch its phase portrait and write a few sentences interpreting the "real-world" implications of the solutions.