Welcome Back!
Differential Calculus

Instructor:

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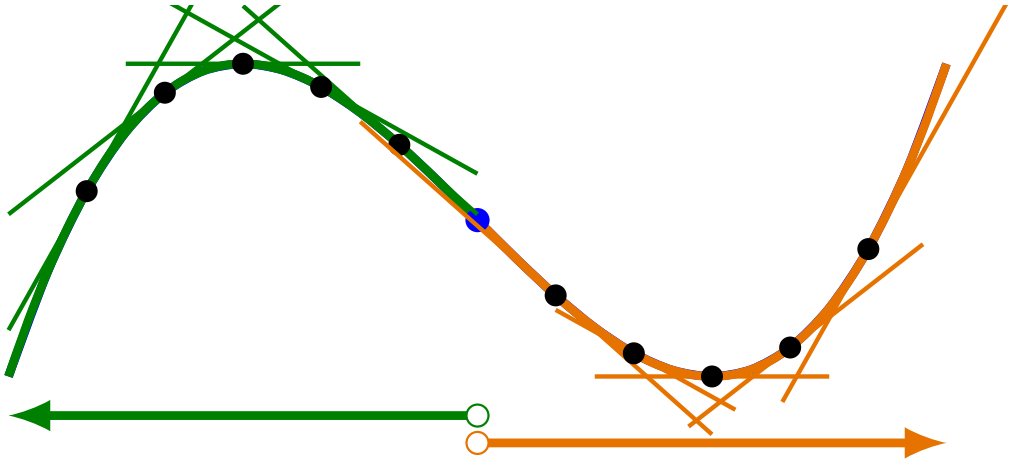
Office Hours:

Meanings: The Second Derivative



Point:

$$
\begin{aligned}
f''(x) > 0 &\iff f'(x) \text{ is increasing} \\
&\iff f(x) \text{ is concave up} \\
f''(x) < 0 &\iff f'(x) \text{ is decreasing} \\
&\iff f(x) \text{ is concave down}
\end{aligned}
$$

Concavity

$f''(x) > 0 \iff f(x)$ is concave up

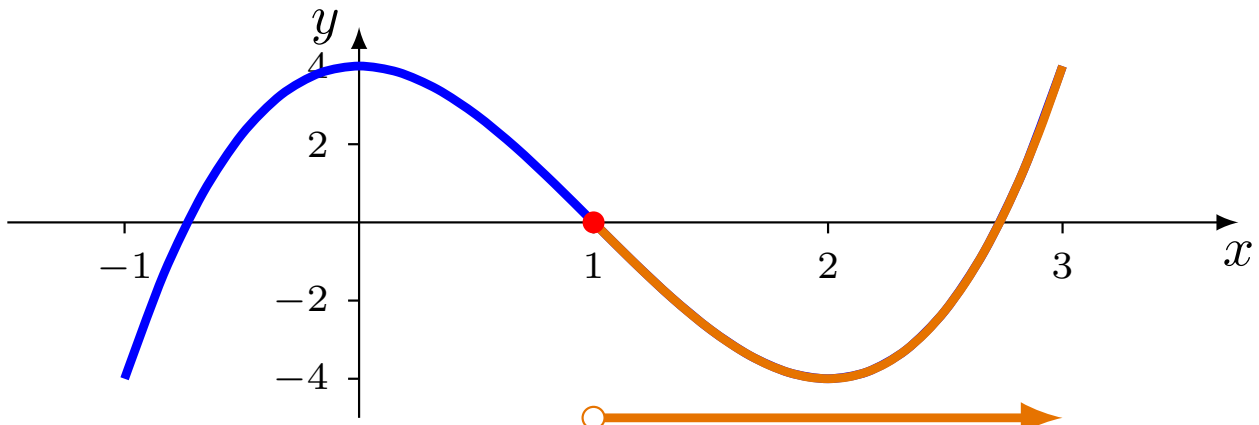
$f''(x) < 0 \iff f(x)$ is concave down

(1) For which values of x is $f(x) = x^3 - 6x^2 + 3x + 2$ concave up?

A when $x = 0$ B when $x < 6$ C when $x > 6$

D when $x < 2$ E when $x > 2$ E

(2) Where is $f''(x) > 0$?

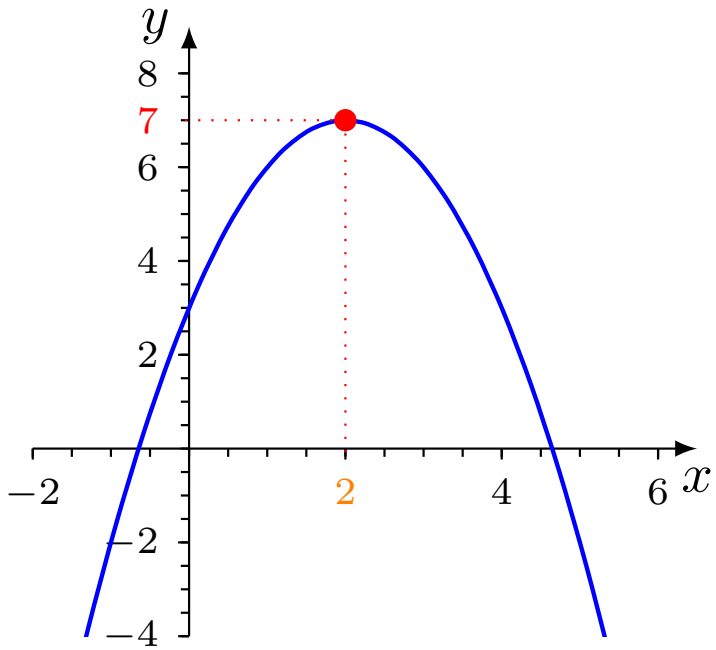


A when $x < 2$ B when $x > 2$ C when $x < 1$

D when $x > 1$ E when $-0.7 < x < 1$ D

§8.13: Max/Min problems

Often want to find the biggest, smallest, most, least, maximum, minimum of something.



Here's the graph of
$y = f(x) = -x^2 + 4x + 3$

The maximum value or just maximum of the function is 7.

The value of x which gives the maximum of $f(x)$ is $x = 2$

We write $f(2) = 7$.

For this example you can see this is the maximum because

$$f(x) = -x^2 + 4x + 3 = -(x - 2)^2 + 7$$

$(x - 2)^2$ is always positive except when $x = 2$
so the maximum must be at $x = 2$.

How To Find A Max / Min

(1) Find $f'(x)$

(2) Solve $f'(x) = 0$. This is the x value that gives the max / min.

(3) To find the maximum / minimum plug the value of x found in (2) back into $f(x)$.

Example: Use this method to find the x-value where maximum of the function $f(x) = 5x - e^{2x}$ occurs.

A $= 0$ B $= \ln(5)$ C $= 2\ln(5)$ D $= 2\ln(5/2)$ E $= \ln(5/2)/2$

Answer: E

Word Problem #1

A ball is thrown into the air. After t seconds the height in meters above the ground of the ball is $h(t) = 40t - 10t^2$. How many meters high did the ball go?

A $= 2$ B $= 40 - 20t$ C $= 20$ D $= 40$ D

Word Problem #2

If an airline sells tickets at a price of $\$200 + 5x$ each the number of tickets it sells is $1000 - 20x$. What price should the tickets be if the airline wants to get the most money?

$$\text{A} = 5 \quad \text{B} = 25 \quad \text{C} = 175 \quad \text{D} = 200 \quad \text{E} = 225 \quad \boxed{\text{E}}$$

Word Problem #3

A fenced garden with an area of 100 m^2 will be made in the shape of a rectangle. It will be surrounded on all four sides by a fence. What length and width should be used so the least amount of fence is needed?

Approach:

(1) Express the total length of fence in terms of only one variable, either L = length of field, or W = width of field. This gives a formula for P = (total length of fence) involving, say, W.

(2) Find minimum by solving $\dfrac{dP}{dW} = 0$.

Students always find (1) the hardest part.
You have been prepared for this by word problems from chapter 3!

Word Problem #4

A fenced garden with an area of 1000 m^2 will be made in the shape of a rectangle. It will be surrounded on all four sides by a fence. Three sides are wood fence, and the remaining side is a brick wall.

- The wood fence costs \$5 per meter length.
- The brick wall costs \$20 per meter length.
- C = total cost of the fence and brick wall
- L = length of the brick wall
- W = width of the other side

(a) Find a formula for C in terms of only L.

A $= 2W + 2L$ B $= 2000L^{-1} + 2L$ C $= 25L + 10000L^{-1}$

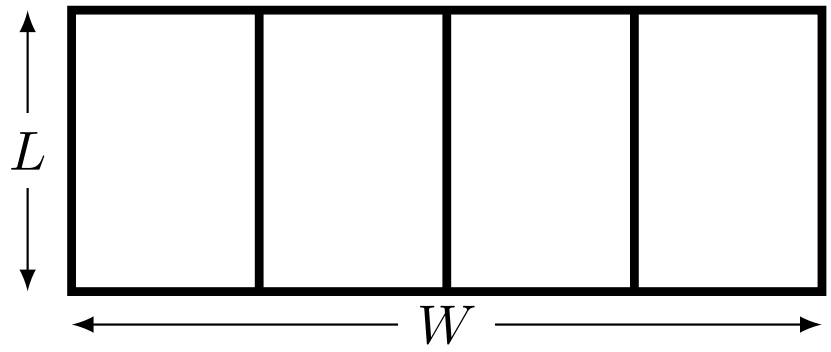
D $= 20L + 10000WL^{-1}$ E $= 5L + 3000$ C

(b) What length of brick wall gives lowest cost?

A $= 20$ B $= 40$ C $= 50$ D $= 100$ E $= 25$ A

Word Problem #5

A rectangular field is surrounded by fence. It is divided into 4 equal



parts by 3 more dividing fences all parallel to one side of the field.

(a) What is the total length of all the fence needed?

$$\text{A} = 2L + 2W \quad \text{B} = LW \quad \text{C} = 5LW$$

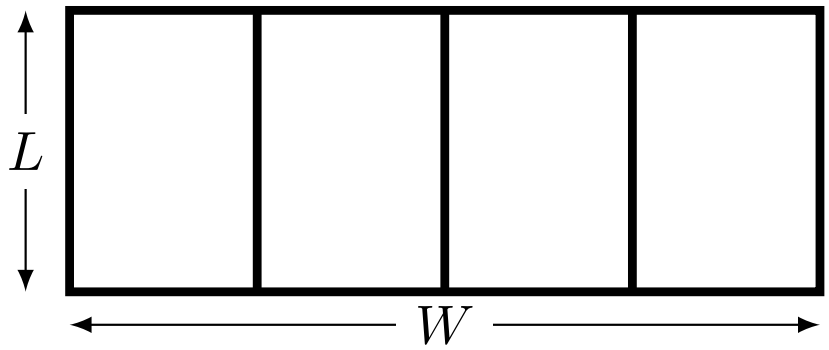
$$\text{D} = L + W \quad \text{E} = 5L + 2W \qquad \boxed{\text{E}}$$

(b) The field must have an area of 1000 m^2. Express W in terms of L.

$$\text{A } 1000 - L \quad \text{B } 1000L \quad \text{C } 1000/L \quad \text{D } 1000 + L \quad \boxed{\text{C}}$$

Word Problem #5 (cont'd)

A rectangular field is surrounded by fence. It is divided into 4 equal



parts by 3 more dividing fences all parallel to one side of the field.

(c) Express the total length of all the fence needed in terms of L.

$$\text{A} = 5L + 1000 \quad \text{B} = 5L + 2000/L \quad \text{C} = 5L + 2/L \quad \boxed{\text{B}}$$

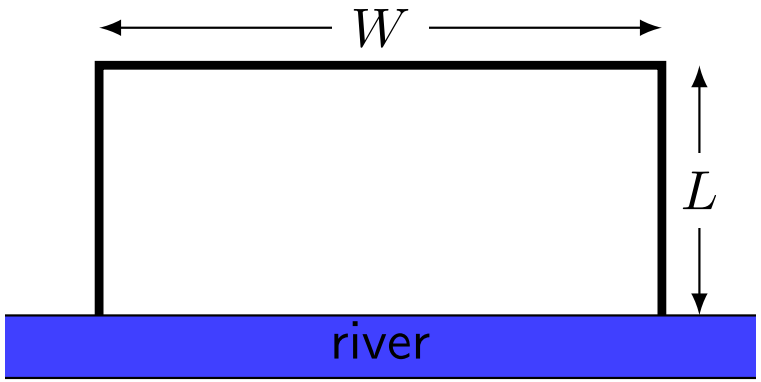
(d) What should L be so that the total length of fence used is a minimum?

$$\text{A} = 10 \quad \text{B} = 20 \quad \text{C} = 40 \quad \text{D} = 50 \quad \boxed{\text{B}}$$

Word Problem #6

A rectangular field is surrounded on three sides by a fence and the fourth side runs along a perfectly straight river. What is the largest area field which can be so enclosed with 120 meters of fence?

A $= 1200$ m^2 B $= 1500$ m^2 C $= 1800$ m^2 D $= 1000$ m^2 C



Word Problem #7

Tickets are going to be sold for a concert.

- If the price of each ticket is \$40, then $2,000$ tickets will be sold.
- For every \$1 the price is decreased, 100 more tickets will be sold.

(a) If the tickets are sold for $\$x$ each, how many will be sold?

$$\text{A} = 2000 - x \quad \text{B} = 2000 - 100x \quad \text{C} = 2000 + 100x$$

$$\text{D} = 6000 - 100x \quad \text{E} = 6000 + 100x \qquad \boxed{\text{D}}$$

(b) What is the total amount of money generated from selling tickets for $\$x$ each?

$$\text{A} = 6000x - 100x^2 \quad \text{B} = 2000x$$

$$\text{C} = 2000 - 40x^2 \quad \text{D} = 6000 - 100x \qquad \boxed{\text{A}}$$

(c) What price should the tickets be to generate the most money from sales?

$$\text{A} = \$20 \quad \text{B} = \$22 \quad \text{C} = \$24 \quad \text{D} = \$30 \quad \text{E} = \$40 \qquad \boxed{\text{D}}$$

Word Problem #8

A farmer is growing wheat.

- On July 1, she has $1,000$ bushels and this increases by 50 bushels per day.
- The price of a bushel on July 1 is \$10 and is dropping at a rate of 20 cents per day.
- She will harvest and sell on the same day.

How many days should she wait, assuming these trends continue?

A $= 5$ B $= 10$ C $= 15$ D $= 20$ E $=$ other □