Math 34A Winter 2020
Old Midterm 1 #3 Solutions

1. Solve for x in the equation $\dfrac{ax+7}{3x-4} = 5.$

Solution: We first multiply through by $(3x-4)$ to get $ax+7 = 5(3x-4)$. Distributing the 5 through, we get $ax+7 = 15x-20$. Adding 20 to both sides and subtracting ax from both sides, we get

$$ax + 7 + 20 - ax = 15x - 20 + 20 - ax \qquad \text{or} \qquad 27 = 15x - ax.$$

Factoring this, the equation becomes $(15-a)x = 27$. Dividing by the coefficient of x, we get $\boxed{x = 27/(15-a)}$.

2. Multiply out and simplify

$$(2a+5b)(3a-4b)+5ab.$$

Check your answer.

Solution: We distribute the multiplication and get

$$\begin{aligned}(2a+5b)(3a-4b)+5ab &= 2a\cdot(3a-4b)+5b\cdot(3a-4b)+5ab\\ &= 6a^2-8ab+15ab-20b^2+5ab.\end{aligned}$$

Now the three "ab" terms combine to give $\boxed{6a^2+12ab-20b^2}$.

We can check our answer by choosing specific values for a and b. If we pick $a=4$ and $b=3$, then

$$\begin{aligned}(2a+5b)(3a-4b)+5ab &= (2\cdot 4+5\cdot 3)(3\cdot 4-4\cdot 3)+5\cdot 4\cdot 3\\ &= (8+15)(12-12)+60 = 23\cdot 0+60 = 60.\end{aligned}$$

(Look! We tried to be clever and choose values for a and b that would make one of the terms in the product zero!) The simplified version is also

$$\begin{aligned}6a^2+12ab-20b^2 &= 6(4)^2+12\cdot 4\cdot 3-20(3)^2\\ &= 6\cdot 16+144-20\cdot 9 = 96+144-180 = 60.\end{aligned}$$

Thus the two expressions agree for these values of a and b. (Looking back, it probably would have been easier to pick numbers like $a=1$ and $b=0$.)

3. Substitute $x = (3 + 2/m)$ into $4m^2x - 3mx$. Simplify the result as much as possible.

Solution: We substitute $x = 3 + 2/m$ into $4m^2x - 3mx$ and get

$$\begin{aligned} 4m^2x - 3mx &= 4m^2(3 + 2/m) - 3m(3 + 2/m) \\ &= 12m^2 + 8m^2/m - 9m - 6m/m \\ &= 12m^2 + 8m - 9m - 6 \\ &= \boxed{12m^2 - m - 6}. \end{aligned}$$

Notice that we canceled one "m" in each term which was divided by m, then we combined the two $8m - 9m$ terms to get $-m$.

4. Solve for x and y in the simultaneous equations

$$3x + 2y = p \qquad\qquad 2x + 2y = 7.$$

Your answers will involve p only.

Solution: We solve for one of the variables in one equation, then plug this variable into the other equation. We choose to solve for y in the second equation, $2x+2y = 7$. This becomes $2y = 7-2x$ when we subtract $2x$ from both sides, then it becomes $y = (7 - 2x)/2$ when we divide by 2. Substituting into the first equation, we get

$$3x + 2\left(\frac{7 - 2x}{2}\right) = p \qquad \text{or} \qquad 3x + (7 - 2x) = p.$$

This becomes $x + 7 = p$, from which we get $x = p - 7$. Plugging this back into our expression $y = (7 - 2x)/2$, we get

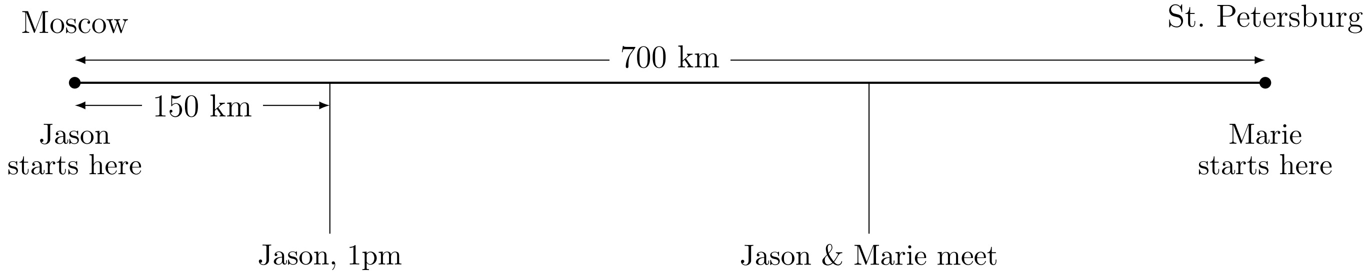
$$y = \frac{7 - 2x}{2} = \frac{7 - 2(p - 7)}{2} = \frac{7 - 2p + 14}{2} = \frac{21 - 2p}{2}.$$

(This could also be written $y = 21/2-p$ if we like.[1]) Thus the answers are $\boxed{x = p - 7}$ and $\boxed{y = (21 - 2p)/2}$.

[1] Notice! This is $\frac{21}{2} - p$, not (not *not* ***not***) $\frac{21}{2-p}$.

5. Jason leaves Moscow at noon driving to St. Petersburg on a road which is 700 km long. Marie leaves St. Petersburg at 1pm driving along the same road to Moscow. Marie's speed is 110 km/hr and Jason's speed is 150 km/hr.

For the solution, here's a little sketch of the situation:



(a) How many hours has *Marie* been driving when they meet? (Leave your answers as *fractions*).

Solution: Jason drives 150 km in the hour he drives before Marie starts. So at 1pm, Jason and Marie are $700 - 150 = 550$ km apart. The distance between them is decreasing at a speed of $110 + 150 = 260$ kmhr, so they will meet in

$$\frac{550 \text{ km}}{260 \text{ km/hr}} = \frac{55}{26} \text{ hours.}$$

(This is about 2 hours and 7 minutes.) Thus means Marie will be driving $\boxed{55/26 \text{ hours}}$ before meeting Jason.

(b) How many km apart are they 1 hour before they meet?

Solution: Since Jason and Marie are approaching each other at 260 km/hr, in the last hour before they meet they travel 260 km. This means that 1 hour before they meet, they are $\boxed{260 \text{ km}}$ apart.

(c) How many hours has *Jason* been driving when they are 200 km apart?

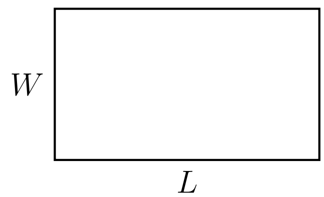
Solution: If we repeat part (a) with $700 - 200 = 500$ km, we'll find how long they're both driving before they're 200 km apart. At 1pm, they need to drive $500 - 150 = 350$ km, which takes

$$\frac{350 \text{ km}}{260 \text{ km/hr}} = \frac{35}{26} \text{ hours.}$$

(This is about 1 hour and 20.8 minutes.) Thus Jason (who has been driving since noon) must drive $1 + 35/26 = \boxed{61/26 \text{ hours}}$ before the two are 200 km apart.

6. A rectangle has length L and width W, perimeter $8X$ and area $3Y$.

To start the solution, let's summarize the situation. The perimeter of this rectangle is $2L+2W = 8X$ and the area is $LW = 3Y$. Here's a labeled picture with our two equations:



$$\begin{aligned} &(1) \quad 2L + 2W = 8X \\ &(2) \qquad\quad LW = 3Y \end{aligned}$$

(a) Express the length in terms of W and X.

Solution: Starting with equation (1), we subtract $2W$ from both sides to get

$$2L + 2W - 2W = 8X - 2W \qquad \text{or} \qquad 2L = 8X - 2W.$$

Now dividing both sides by 2 gives us an expression for length in terms of W and X: $\boxed{L = 4X - W}$.

(b) Give the length in terms of W and Y.

Solution: Starting with equation (2), we divide both sides by W to get $LW/W = 3Y/W$ or $\boxed{L = 3Y/W}$.

(c) Express W in terms of X and Y but not L. (Your answer should have a square root.)

Solution: From our two previous answers, we have two expressions for L. They must be the same (they're both L!), so $4X - W = 3Y/W$. Multiplying through by W, we get the equation

$$4XW - W^2 = 3Y \qquad \text{or} \qquad W^2 - 4XW + 3Y = 0.$$

Now the quadratic formula gives us the answer:

$$\begin{aligned} W &= \frac{4X \pm \sqrt{(4X)^2 - 4 \cdot 3Y}}{2} = \frac{4X \pm \sqrt{4 \cdot (4X^2 - 3Y)}}{2} = \frac{4X \pm 2\sqrt{4X^2 - 3Y}}{2} \\ &= \frac{2\left(2X \pm \sqrt{4X^2 - 3Y}\right)}{2} = \boxed{2X \pm \sqrt{4X^2 - 3Y}}. \end{aligned}$$

(There are two answers because one is W and the other is L.)