Project #1: Undoing the Chain Rule
Pre-Work

PW 1 Recall that the circle of radius 2 centered at $(1, 3)$ is described in the plane by the equation $(x-1)^2 + (y-3)^2 = 4$.

(a) This equation does **not** define a function. Why?

(b) However, the equation for a circle can be used to define several functions *implicitly*. What do you think this means?

(c) Use the chain rule to find the slope of a tangent line at a fixed point (x_0, y_0) of the graph of one of these implicit functions.

PW 2 The graph of the equation $y^2 = x^3 - x$ is a famous elliptic curve related to the proof of Fermat's Last Theorem[1].

(a) Give a formula for the slope of a tangent line to any point (x_0, y_0) of this curve. Where are the tangents vertical?

(b) Suppose a function $f(x)$ is implicitly defined by the curve $y^2 = x^3 - x$. What differential equation does $f(x)$ satisfy?

(c) Sketch the slope-field for the differential equation in (b). Plot the elliptic curve $y^2 = x^3 - x$ on the same plane. You can use the grid on the back of this page for your slope field.

(d) Does the differential equation you came up with in part (b) have any solutions that aren't defined implicitly by the curve $y^2 = x^3 - x$? If so, sketch one of these solutions on your slope-field. If not, explain.

[1] see http://en.wikipedia.org/wiki/Fermat

Use the dot grid below to sketch your slope field for PW 2 (c). Sketch a dash at each (x, y) gridpoint whose slope matches the value of y' for the corresponding values of (x, y).

You may find it helpful to make a table of x, y and y' values and take advantage of any symmetry you notice.