Final Exam - Math 3B Summer B 2020

Name:
PERM#:

Instructions: Initial each of the following lines after reading the corresponding exam instructions. Doing this is worth 2 points.

_____ You may use course notes and lecture videos, but no outside material.

_____ You may **not** use integral tables - you must show all work.

_____ **No calculators** or help from the Internet is allowed - please be honest about this!

_____ You must show **all** work to get credit. Answers without sufficient justification will not receive credit.

_____ Organize your work. Work that cannot be followed may not be graded.

_____ The exam was designed to take 90 minutes, however there is **no time limit** as long as you are finished before the due date.

_____ The exam will be turned in on Gradescope by **Friday**, September 11^{th} at 11:59 pm.

Sign below to agree that the work in this exam is your own work and that you have not received help from another student, friend, tutor, or other outside source.

Signature _____ **Date:** _____

Question 1: (20 pts) Determine if each of the following statements is True or False.

T / F (a) The length of the curve $y = x^3$, $0 \le x \le 1$ is

$$\ell = \int_0^1 \sqrt{1 + x^6}\, dx.$$

T / F (b) If $\int_{-\infty}^{\infty} f(x)\, dx$ is convergent, then $\int_0^{\infty} f(x)\, dx$ is convergent.

T / F (c) To evaluate the integral

$$\int \frac{1}{\sqrt{64 - \theta^2}}\, d\theta$$

an appropriate substitution is $x = 8 \sin \theta$.

T / F (d) If $\int_0^4 f(x)\, dx = 7$, then the average value of f on $[0, 4]$ is 28.

T / F (e) If f and g are continuous on $[a, b]$, then

$$\int_a^b [2f(x) + g(x)]\, dx = 2\int_a^b f(x)\, dx + 2\int_a^b g(x)\, dx$$

T / F (f) , If f has a discontinuity at $x = 0$, then $\int_{-1}^{1} f(x)\, dx$ does not exist.

T / F (g) If $f'(x)$ is continuous on $[1, 3]$, then $\int_1^3 f'(t)\, dt = f(3) - f(1)$.

T / F (h) If the curve $x = y + y^3$, $0 \le y \le 1$ is rotated about the y-axis, then the surface area of the resulting solid can be found using the integral

$$SA = \int_0^1 2\pi(y + y^3)\sqrt{2 + 6y^2 + 9y^4}\, dy$$

T / F (i) Using the definition of a definite integral it is true that

$$\int_{-1}^{3} (3 - 2\sin x)\, dx = \lim_{n\to\infty} \sum_{i=1}^{n} \frac{4}{n}\left(3 - 2\sin\left(-1 + \frac{4i}{n}\right)\right)$$

T / F (j) If $f(x) \le g(x)$ and $\int_0^{\infty} g(x)\, dx$ diverges, then $\int_0^{\infty} f(x)\, dx$ also diverges.

Question 2:

(a) (10 pts) Estimate the area under the graph of $f(x) = x^2 + 6$ on the interval $[-3, 3]$ using 3 approximating rectangles and the **left** end points. Sketch a picture.

(b) (10 pts) Write the definite integral

$$\int_{-2}^{3} \left(\sqrt[3]{x} - 3\right)\, dx$$

as the limit of a Riemann Sum.

Question 3:

(a) (10 pts) Use part 1 of the Fundamental Theorem of Calculus to find the **derivative** of the function

$$h(x) = \int_x^5 \frac{\sin(12t^8)}{\sqrt{5+8t^3}}\, dt$$

(b) (10 pts) Evaluate the following integral using u-substitution

$$\int_0^{\pi/4} \frac{7e^{\tan\theta}}{\cos^2\theta}\, d\theta$$

Question 4:

(a) (10 pts) Evaluate the following integral. Specify which method of integration is used.

$$\int \frac{5x + 34}{x^2 + x - 12}\, dx$$

(b) (10 pts) Evaluate the following integral. Specify which method of integration is used.

$$\int_0^{\pi/2} 8x \cos x\, dx$$

Question 5: For each of the following problems you must use **complete sentences** to justify your answer. You do **not** need to evaluate the integral.

(a) (4 pts) Determine which integration method can be used to evaluate the integral

$$\int \frac{1}{x^2 + x - 2}\, dx$$

(b) (4 pts) Determine which integration method can be used to evaluate the integral

$$\int \frac{2x + 1}{x^2 + x - 2}\, dx$$

(c) (4 pts) Determine which integration method can be used to evaluate the integral

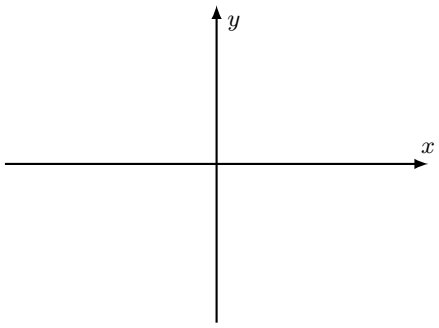
$$\int \frac{1}{x^2 + x + 2}\, dx.$$

(d) (4 pts) Determine which integration method can be used to evaluate the integral

$$\int (x^2 + x - 2) \ln x \, dx.$$

Question 6: Consider the region R bounded by $y = 4x - x^2$ and $y = x$.

(a) (4 pts) Graph $y = 4x - x^2$, and $y = x$, then identify the region bounded by the equations. Clearly label the graph.



(b) (8 pts) Set up, but do not evaluate, an integral for the volume of the solid obtained by rotating the region bounded by the given curves about the x-axis. Specify if you are using the disk, washer, or cylindrical shell method and explain why.

(c) (10 pts) Evaluate the volume equation from part (b).

Question 7: Consider the integral

$$\int_4^\infty \frac{1}{x^2 - 9}\, dx$$

(a) (10 pts) Explain, using **complete sentences**, why the integral is improper.

(b) (10 pts) Determine if the integral is convergent or divergent. If it is convergent, then evaluate the integral.

Question 8: (10 pts) Consider the following problem and solution:

Problem: Evaluate the following integral

$$\int_0^4 \frac{2}{x-3}\,dx$$

Solution: We will evaluate this using u-substitution. Let $u = x - 3$, then $du = dx$. The new lower bound is $u = 0 - 3 = -3$. The new upper bound is $u = 4 - 3 = 1$. Thus the integral can we rewritten as

$$\int_0^4 \frac{2}{x-3}\,dx = \int_{-3}^1 \frac{2}{u}\,du$$

This is something we can integrate. Therefore by part 2 of FTC we have that

$$\int_{-3}^1 \frac{2}{u}\,dx = 2\ln|u|\Big|_{-3}^1 = 2\ln|1| - 2\ln|-3| = 2\ln 3$$

Thus we conclude that

$$\int_0^4 \frac{2}{x-3}\,dx = 2\ln 3$$

The above solution is incorrect. Using **complete sentences** explain what mistakes were made? What should have been done instead?