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Proportionality Review

"y is proportional to x" or $y \propto x$ means

- When we double x, we double y
- When we triple x, we triple y
- When we halve x, we halve y
- $y = Kx$, where K is called the constant of proportionality.

Constant of Proportionality

Example: We are told

- Tax is proportional to income, and
- The tax on $\$1,000$ is $\$280$.

Express y = amount of tax paid in terms of x = the income. Then $y =$

(A) $1000x$ (B) $280x$ (C) $\dfrac{1,000}{280}x$

(D) $2.8x$ (E) $0.28x$ $\boxed{\text{E}}$

Question: What does the constant of proportionality $K = 0.28$ mean?

Answer: It is the tax on one dollar.

Example

For this question, we assume:

- The weight of an elephant is proportional to its height cubed, and
- An elephant 1 meter high weighs $1/3$ tons.

How many tons does an elephant h meters tall weigh?

(A) $h/3$ (B) h^3 (C) $h^3/3$ (D) $(h/3)^3$ (E) $(3h)^3$ C

Question: What does the constant of proportionality $K = 1/3$ mean?

Answer: It is the weight of 1 cubic meter of elephant.

More Complicated Examples

y is inversely proportional to x means $y \propto 1/x$

Example:

- I have \$300
- N = number of apples I can buy
- p = price per apple

Then N is inversely proportional to p: $N \propto 1/p$.

Question: What is the constant of proportionality?
Answer: It is \$300, the amount of money I have.

More Complicated Examples

z is jointly proportional to x and y means $z \propto x \cdot y$ (or $z = Kxy$)

Example:

- C = cost of a rectangular plot of land,
- ℓ = length (in meters) of plot, and
- w = width (in meters) of plot.

Then C is jointly proportional to ℓ and w: $C = K \cdot \ell \cdot w$.

Question: What does the constant of proportionality mean?

Answer: It is the cost of one square meter of land.

More Complicated Examples

Strength of Light

- P = strength of light (power per unit area)
  = amount of light on unit area
- R = distance to light source

Inverse Square Law: $P \propto 1/R^2$

Same idea for heat, gravity, sound, and many others...

Newton's Law of Gravity: $F \propto \dfrac{m_1 m_2}{r^2}$

Constant of proportionality: $G \approx 6.67 \times 10^{-11}\ \mathrm{m}^3/(\mathrm{kg}\cdot\mathrm{s}^2)$
(the Gravitational constant)

§5.1: Error and Limit

Suppose the "real" answer is 10, but your approximate answer is 9.5

$$\boxed{\text{error} = (\text{real answer}) - (\text{approximate answer})}$$

In example error $= 10 - 9.5 = 0.5$

$$\boxed{\%\ \text{error} = \left(\frac{\text{error}}{\text{real answer}}\right) \times 100\%}$$

In other words it is the error expressed as a percentage of the real answer.
Often this is what matters.

1. You have \$50 in you pocket but YOU THINK you have only \$40. What is the percentage error?

(A) 10% (B) 20% (C) 25% (D) 40% (E) 50% B

Limits

Imagine you calculate more and more accurate approximations to a real answer that you don't know.

$$x_1 = 1.3$$
$$x_2 = 1.33$$
$$x_3 = 1.333$$
$$x_4 = 1.3333$$
$$\vdots$$
$$= \text{real answer???}$$

These numbers get ever closer to $1.3333\cdots = 4/3$.
This is the real answer. The limit of this sequence is $4/3$:

$$\lim_{n\to\infty} x_n = 4/3$$

Read aloud as "The limit as n goes to infinity of x_n is $4/3$."

Guessing Limits

To work out (guess) a limit (when n goes to infinity) imagine plugging into the formula a REALLY BIG value for n like a thousand, or a million, or…

2. $\lim_{n\to\infty} \left(\frac{1}{n}\right) = ?$

(A) $\frac{1}{n}$ (B) 0 (C) 1 (D) $\frac{1}{\infty}$ (E) ∞ B

3. $\lim_{n\to\infty} \left(\frac{n}{n+3}\right) = ?$

(A) 0 (B) $1/3$ (C) 1 (D) $1/4$ (E) $\infty/(\infty+3)$ C

More Guessing Limits

4. $\lim_{n\to\infty}\left(\dfrac{2n+5}{9n+71}\right) = ?$

(A) $\dfrac{5}{71}$ (B) $\dfrac{2}{71}$ (C) $\dfrac{5}{9}$ (D) $\dfrac{2}{9}$ (E) $\dfrac{2\infty}{9\infty}$ D

For homework, you can use a calculator and plug in really big values for n then guess. For example if you plug in $n = 1000000$ and get the answer 16.0000361 you guess the limit is really 16.

For engineering, calculus students learn lots of tricks to work out limits. In this class we don't do that. Just UNDERSTAND the main idea.

Even More Guessing Limits

5. $\lim_{n\to\infty}\left(\dfrac{2n+17}{5n+8}\right) = ?$

(A) $\frac{2}{5}$ (B) $\frac{17}{5}$ (C) $\frac{2}{8}$ (D) $\frac{17}{8}$ (E) $\frac{19}{13}$ $\boxed{\text{A}}$

6. $\lim_{n\to\infty}\left(3+\dfrac{1}{n}\right) = ?$

(A) 1 (B) 3 (C) 0 (D) $\frac{1}{3}$ (E) ∞ $\boxed{\text{B}}$

More: Spot The Difference!

7. $\lim_{x \to 1} \left(\frac{x-1}{x^2-1} \right) = \frac{1}{2}$

8. $\lim_{x \to 1} \left(\frac{x+3}{x^2+1} \right) = ?$

(A) 3 (B) 1 (C) 4 (D) 2 (E) 0 D

9. $\lim_{x \to 0} \left(\frac{3x+x^2}{2x} \right) = ?$

(A) 0 (B) $\frac{0}{0}$ (C) $\frac{1}{0}$ (D) $\frac{1}{2}$ (E) $\frac{3}{2}$ E