Office Hours!

Instructor:

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Office Hours:

Mondays 2–3PM
Tuesdays 10:30–11:30AM
Thursdays 1–2PM
or by appointment

Office:

South Hall 6510

Homework Survey

Which homework problem were you totally stuck on and want to see this morning?

A Homework 14 #5 (The airplane ticket problem)

B Homework 14 #6 (The aquarium problem)

C Homework 14 #7 (The hummingbird problem)

D More than one of these

E None of these

Homework 14 #5

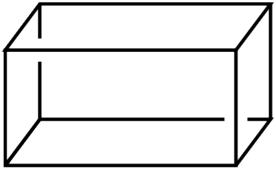
An airline sells all the tickets for a certain route at the same price.

- If it charges 200 dollars per ticket it sells $10,000$ tickets.
- For every 20 dollars the ticket price is reduced, an extra thousand tickets are sold. Thus if the tickets are sold for 180 dollars each then 11,000 tickets sell.
- It costs the airline 100 dollars to fly a person.

(a) Express the total profit P in terms of the number n of tickets sold.

(b) Express the total profit P in terms of the price p of one ticket.

Homework 14 #6



Frame shown thick

An aquarium with a square base has no top. There is a metal frame.

- Glass costs 5 dollars/m^2.
- The frame costs 2 dollars/m.
- The volume is to be 20 m^3.

Express the total cost C in terms of the height h in meters.

Hint: Work out the cost of the glass and frame separately.

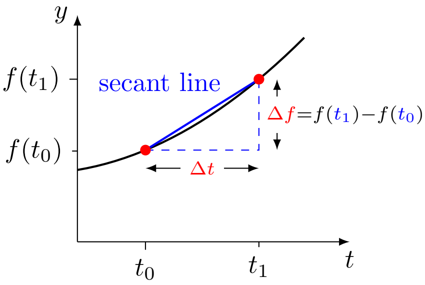
Homework 14 #7

A hummingbird needs 10 grams of sugar and 8 grams of protein each day.

- One honeysuckle flower provides 20 mg of sugar and 10 mg of protein.
- One nasturtium flower provides 10 mg of sugar and 10 mg of protein.
- It takes 20 seconds to feed from a nasturtium...
- ...and 10 seconds per honeysuckle.

How many minutes does it take to get exactly the food it needs?

Graphical Approach



Δf = change in f
Δt = change in t

Many ways to say same thing:

$$\left(\begin{array}{c}\text{average rate of}\\ \text{change of } f\end{array}\right) = \frac{\text{change in } f}{\text{change in } t}$$

$$= \frac{\Delta f}{\Delta t}$$

$$= \text{slope of secant line} = \frac{f(t_1) - f(t_0)}{t_1 - t_0}$$

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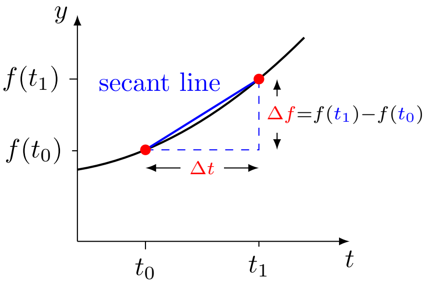

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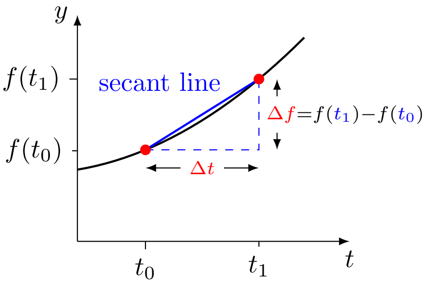

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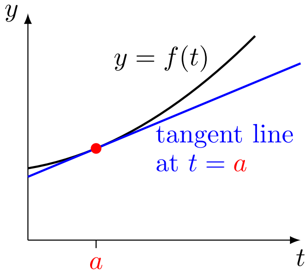
Idea: As t_1 moves closer to t_0 the secant line approaches the tangent line at t_0. This is the line with the same slope as the graph at t_0.

Understanding Derivatives

There are many ways to think about derivatives. You need to understand these to apply to problems.



slope of graph at a
= slope of tangent line
= instantaneous rate of change of f at a

$$= \left(\begin{array}{c} \text{limit of average rate of change} \\ \text{of } f \text{ over shorter and shorter} \\ \text{time intervals starting at } a \end{array} \right)$$

= limit of slopes of secant lines

$$= f'(a) \; = \; \left.\frac{df}{dt}\right|_{t=a}$$

Summary

- How fast something changes = rate of change
- Instantaneous rate of change is the limit of the average rate of change over shorter and shorter time spans. This gets around the $0/0$ problem.
- speed = rate of change of distance traveled.

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Our goal is that you understand the practical meaning of the derivative in various situations.

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$f(t) =$ temperature in $^\circ$ F at t hours after midnight

$f(7) = 48$ means the temperature at 7am was 48° F

$f'(7) = 3$ means at 7am the temperature was rising at a rate of 3° F/hr

$f'(9) = -5$ means at 9am the temperature was falling at a rate of 5° F/hr

or rising at a rate of -5° F/hr

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$g(t) =$ distance from origin in cm of hamster on x-axis after t seconds

$g(7) = 3$ means after 7 seconds hamster was 3 cm from origin

$g'(9) = -5$ means after 9 seconds our furry friend was running towards the origin at a speed of 5 cm/sec

Another Context

Suppose $f(t)$ = temperature of oven in °C after t minutes.

What do $f(3) = 20$ and $f'(3) = 15$ mean?

A After 20 minutes the oven was at 3° C and heating up at a rate of 15° C/min

B After 3 minutes oven temperature was 15° C and cooling down at a rate to 20° C/min

C The oven was heating up at rate of 3° C/min after 15 minutes and also after 20 minutes

D After 3 minutes the oven was at 20° C and heating up at a rate of 15° C/min

E None of the above

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- D After 3 minutes the oven was at 20° C and heating up at a rate of 15° C/min
- E None of the above

Answer: $\boxed{\text{D}}$

Yet Another Context

Now suppose $f(t)$ = the population of the ancient city of Lyrad in year t. We are told that $f(1550) = 1820$ and $f'(1650) = 1100$. Which of the following is true?

A In 1550, the population was 1820 and rising at a rate of 1100 people per year

B In 1650, the population was 1100 more than in 1550

C In 1650, Lyrad contained 1100 people

D In 1550, there were 1820 people in Lyrad, and by 1650 this had increased to 2920

E None of above

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E None of above

Answer: $\boxed{\text{E}}$

Context: Mathematics

Suppose $f(0) = 50$ and $f(10) = 70$. Which of the following is true?

A For all t between 0 and 10, the derivative is $f'(t) = 2$

B $f'(0) = 2$

C It is possible that $f'(0) = -8$

D It is impossible that $f'(0) = -8$

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Answer: $\boxed{\text{C}}$

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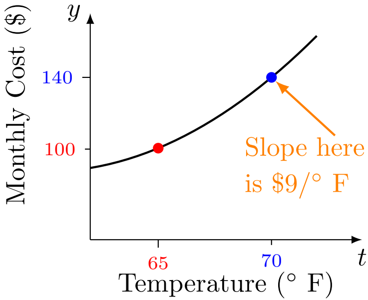
Answer: $\boxed{\text{C}}$

We'll see later that, for example, that $f(x) = x^2 - 8x + 50$ has $f(0) = 50$, $f(10) = 70$, and $f'(0) = -8$.

Notice: No Time



$f(x)$ = monthly cost of heating house to x° F

$f(70) = 140$ means it costs \$140 to heat the house for one month to a temperature of 70°F.

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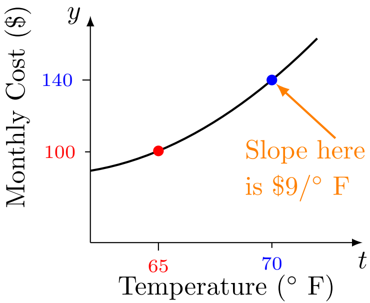

$f(x)$ = monthly cost of heating house to $x°$ F

$f(70) = 140$ means it costs \$140 to heat the house for one month to a temperature of 70°F.

$f'(70) = 9$ means rate at which cost increases as temperature changes is \$9 for each extra ° F.

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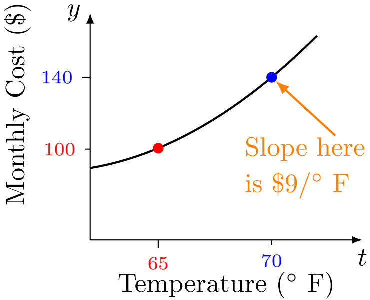

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In practical terms this means you pay an extra $9 during each month for each extra $1^o F$. If you turn it up two degrees you pay an extra $18 each month. Each extra degree of warmth costs an extra $9 each month. In economics this is called a marginal cost or marginal rate

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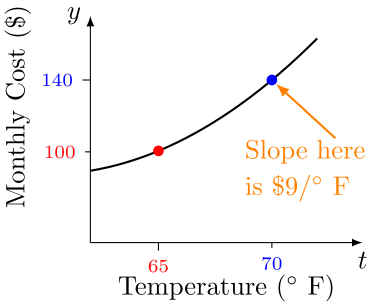

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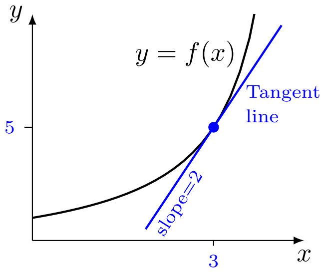
This is not exactly true:

average rate of change versus instantaneous rate of change.

In the following examples we will ignore this subtlety.

The Importance of Units



Told $f(3) = 5$ and $f'(3) = 2$

This means the slope of the tangent line to the graph $y = f(x)$ at $x = 3$ is 2.

The derivative is this slope, so...

$$\boxed{\text{The units of } \frac{dy}{dx} \text{ are } \frac{\text{units of y}}{\text{units of x}}}$$

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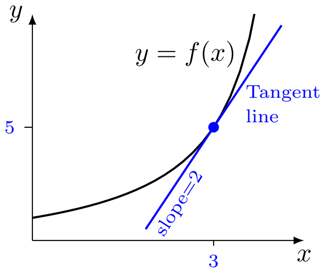

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Heating example: derivative units are $\$/^\circ$ F = dollars per degree F

Units help you understand the meaning of the derivative.

Get Pumped!

Adrenaline cause the heart to speed up.
x = number of mg (milligrams) of adrenaline in the blood.
$f(x)$ = number of beats per minute (bpm) of the heart with x mg of adrenaline in the blood.

What does $f'(5) = 2$ mean?

- A When there are 5 mg of adrenaline the heart beats at 2 pbm
- B When the amount of adrenaline is increased by 2 mg the heart speeds up by 5 bpm
- C When the heart beats at 5 bpm the adrenaline is increased by 2 mg
- D When there are 5 mg of adrenaline the heart speeds up by 2bpm
- E When there are 5 mg of adrenaline in the blood the heart speeds up by 2 bpm for each extra mg of adrenaline.

Hint: The units of $f'(5)$ are bpm per milligram of adrenaline

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