Welcome To Math 34A! Differential Calculus

Instructor:

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Chapter 7 (Logs)

§7.0 Logarithms and Exponentials

Applications:

- Chemistry: alkalinity and acidity, pH scale
- Finance: compound interest (get rich slow)
- Geology: Richter scale for earthquakes (did you feel the earth move too ?)
- Archeology: radio carbon dating (how old is that bone ?)
- Astronomy: stellar magnitude (brightness of stars)
- Sound: decibels (what did you say?, the music is too loud)
- Math: solving equations with exponents ...by performing an arithmetic operation to both sides (all of the above are examples of the use of this operation)

Chapter 7 (Logs)

Main Idea of Chapter 7:

> $\log(x)$ is how many times you multiply 1 by 10 to get x

Conclusion:

Before we do logs we should be really good at powers of 10.

Powers of Ten

1 meter ≈ 3 feet
1 centimeter $= 0.01$ meters $= 10^{-2}$ meters $\approx 1/2$ inch
1 kilometer $= 1,000$ meters $= 10^3$ meters $\approx 1/2$ mile

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Approximate distance (in meters), to nearest power of 10

10^7 meters	Size of Earth
10^9 meters	Distance to moon
10^{14} meters	Size of our solar system
10^{16} meters	One light-year
10^{21} meters	Size of the Milky Way galaxy
10^{27} meters	Size of the universe (about 93 billion light-years)
10^{80}	number of protons in the observable universe?
10^{100}	1 googol
10^{1000} meters	???

Exponential Basics

$$10^4 = 10 \times 10 \times 10 \times 10 \;=\; 10,000$$
$$= 4 \text{ lots of 10 multiplied together}$$
$$= 1 \text{ followed by } 4 \text{ zeroes}$$

$$10^x = \underbrace{10 \times 10 \times \cdots \times 10}_{x \text{ lots of } 10} = 1\,\underbrace{00000\cdots0}_{x \text{ zeros}}$$
$$= 1 \text{ followed by x zeroes}$$

Ex: $$10^2 \times 10^3 = (10 \times 10) \times (10 \times 10 \times 10)$$
$$= 10^{2+3} = 10^5.$$

$10^x \times 10^y = 10^{x+y}$ First Law of Exponents

Why? We can work it out!

Exponential Basics (cont'd)

$10^{x} \times 10^{y} = 10^{x+y}$ **First Law of Exponents**

Why? <u>We can work it out:</u>

(x lots of 10 multiplied together) $\times$ (y lots of 10 multiplied together)
$= (x + y)$ lots of 10 multiplied together

For now x and y are positive whole numbers.

More Exponentiation

$$\left(10^2\right)^3 = (10 \times 10)^3 = (10 \times 10) \times (10 \times 10) \times (10 \times 10) = 10^6$$

$$\boxed{\left(10^a\right)^b = 10^{ab} \qquad \text{Fourth Law of Exponents}}$$

Why? We can work it out:

$$10^a = \underbrace{10 \times 10 \times \cdots \times 10}_{a \text{ times}}$$

$$\left(10^a\right)^b = \underbrace{(10 \times \cdots \times 10) \times \cdots \times (10 \times \cdots \times 10)}_{b \text{ times}} = 10^{ab}.$$

Just count the zeros!

When the power is 0 or negative

What is 10^0?$= 1$ But why? We can work it out:

$$10^0 \times 10^1 = 10^{0+1}$$

$$\text{so} \quad 10^0 \times 10 = 10$$

$$\text{and therefore} \quad 10^0 = 10/10 = 1$$

Summary: we used the first law of exponents to figure out what 10^0 must be.

There is a second explanation in the book!

What is 10^{-2}?$= 1/100 = 0.01$ But why? We can work it out:

$$10^{-2} \times 10^2 = 10^{-2+2} = 10^0 = 1$$

$$\text{therefore} \qquad 10^{-2} = \frac{1}{10^2} \qquad \text{and} \qquad 10^{-a} = \frac{1}{10^a}$$

There is a second explanation in the book

The Five Laws of Exponents

(1) $10^a \times 10^b = 10^{a+b}$

(2) $10^0 = 1$

(3) $10^{-a} = 1/10^a$

(4) $\left(10^a\right)^b = 10^{ab}$

(5) $10^a/10^b = 10^{a-b}$

1. What is $10^3 \times 10^4$?

A$= 10^{12}$ B$= 10^7$ C$= 10^{34}$ D$= 10^0$ E$= 10^{-7}$ $\boxed{\text{B}}$

2. Find $10^3/10^4$

A$= 10^7$ B$= 10^1$ C$= 10^{-4}$ D$= 10^{-1}$ E$= 10^{-7}$ $\boxed{\text{D}}$

3. Find $\left(10^3\right)^4$.

A$= 10^7$ B$= 10^1$ C$= 10^{12}$ D$= 10^{-1}$ E$= 10^0$ $\boxed{\text{C}}$

The Five Laws of Exponents

(1) $10^a \times 10^b = 10^{a+b}$ (2) $10^0 = 1$

(3) $10^{-a} = 1/10^a$ (4) $\left(10^a\right)^b = 10^{ab}$

(5) $10^a/10^b = 10^{a-b}$

4. What is $\left(10^2 \times 10^3\right)^4$?

A= 10^8 B= 10^9 C= 10^{12} D= 10^{20} E= 10^{24} $\boxed{\text{D}}$

5. What is $(10^2 \times 10^6)/(10^2 \times 10^3)$?

A= 10^2 B= 10^3 C= 10^{-1} D= 10^7 E= 10^6 $\boxed{\text{B}}$

6. What is $\left(10^2/10^5\right)^{-2}$?

A= 10^{-6} B= 10^{-5} C= 10^6 D= 10^4 $E = 10^5$ $\boxed{\text{C}}$

§7.1 Fractional Exponents

We can work them out!

7. What is $10^{0.5} = 10^{1/2}$? Answer: $10^{0.5} = \sqrt{10} \approx 3.16288$

8. What is $10^{0.1} = 10^{1/10}$? Answer: $10^{0.1} = \sqrt[10]{10} \approx 1.258926$

9. Similarly: $10^{0.01} = \sqrt[100]{10} \approx 1.02329$

$$10^{0.001} = \sqrt[1000]{10} \approx 1.00231$$

10. What is $10^{0.27}$? Answer:

$$10^{0.27} = 10^{27/100} = \sqrt[100]{10^{27}} = \left(\sqrt[100]{10}\right)^{27} \approx 1.862$$

§7.2 Logarithms

$\log(y)$ is how many tens you multiply together to get y

$$10^{\log(y)} = y$$

$\log(10) = ?1$ because $10^1 = 10$

$\log(100) = ? = 2$ because $10^2 = 100$

$\log(1000) = ? = 3$ because $10^3 = 1000$

$\log(100,000) =$

A $= 2$ B $= 3$ C $= 4$ D $= 5$ E $= 6$ $\boxed{\text{D}}$

Still moving to Logarithms

log(y) is how may tens you multiply together to get y

$$10^{\log(y)} = y$$

$$\log(0.1) = ?-1 \qquad \text{because} \quad 10^{-1} = 1/10 = 0.1$$
$$\log(0.01) = ? = -2 \qquad \text{because} \quad 10^{-2} = 1/100 = 0.01$$
$$\log(10^x) = ? = x \qquad \text{duh?}$$

How confused are you?

A=not at all B=a bit C=a lot D= :'(

You try it:

$\log(100,000) = ?$

$$A = 2 \quad B = 3 \quad C = 4 \quad D = 5 \quad E = 6 \quad \boxed{D}$$

$\log(0.001) = ?$

$$A = 3 \quad B = 0 \quad C = 0.001 \quad D = -2 \quad E = -3 \quad \boxed{E}$$

$\log(100 \times 1000) = ?$

$$A = 6 \quad B = 5 \quad C = 3 \quad D = 9 \quad E = -5 \quad \boxed{B}$$

$\log(100/1000) = ?$

$$A = -1 \quad B = 0 \quad C = 1 \quad D = -3 \quad E = -5 \quad \boxed{A}$$

Key Fact Of Logs

First Law of Logs $\log(a\times b) = \log(a) + \log(b)$

This means logs convert multiplication into addition.

Example: $\log(100\times 1000) = \log(100) + \log(1000) = 2 + 3 = 5$

It is easy to understand why the first law works:

log(a) = (how many 10's you multiply to get a)
log(b) = (how many 10's you multiply to get b)

THEREFORE multiplying ALL these 10s gives $a\times b$

CONCLUDE $\log(a\times b)$ is this number of 10s: that is, $\log(a) + \log(b)$.

7.3 Using Log Tables

We are told: $\log(2) \approx 0.3$ (from table page 289)

$$\begin{aligned} \log(20) &= \log(10 \times 2) \\ &= \log(10) + \log(2) && \text{we know } \log(10) = 1 \\ &\approx 1 + 0.3 \\ &\approx 1.3 \end{aligned}$$

Use this method to find $\log(200)$

A$= 30$ B$= 3$ C$= 2.3$ D$= 30$ $\boxed{\text{C}}$

A few more

We are still told $\log(2) \approx 0.3$

Find $\log(0.002)$

$$\text{A}= -3.3 \quad \text{B}= -2.3 \quad \text{C}= -2.7 \quad \text{D}= -3.7 \quad \boxed{\text{C}}$$

Find $\log(2 \times 10^x)$

$$\text{A}= 2x \quad \text{B}= 2+x \quad \text{C}= .3x \quad \text{D}= 10x + \log(2) \quad \text{E}= x + .3 \ \boxed{\text{E}}$$

A Trick!

The graph and the table can both be used to find logs of numbers between 1 and 10.

To find the log of ANY number, we move the decimal point:

$$\boxed{\log(10^n \times x) = n + \log(x)}$$

Example:

$$\log(275.67) = \log(10^2 \times 2.7567) = 2 + \underbrace{\log(2.7567)}_{\text{look this up!}}$$

Its called the MOVING DECIMAL POINT TRICK because 2 is how many places you need to move the decimal point of 275.67 to obtain a number between 1 and 10.

Inverses!

logs are "opposite " of exponents (inverse function of antilog)
So every fact about exponents corresponds to a fact about logs:

	laws of exponents	corresponding law of logs
(1)	$10^a \times 10^b = 10^{a+b}$	$\log(xy) = \log(x) + \log(y)$
(2)	$10^0 = 1$	$\log(1) = 0$
(3)	$10^{-a} = 1/10^a$	$\log(1/x) = -\log(x)$
(4)	$(10^a)^p = 10^{ap}$	$\log(x^p) = p\log(x)$
(5)	$10^a/10^b = 10^{a-b}$	$\log(x/y) = \log(x) - \log(y)$

Example: $\log(x^a/y^b) =?$

$$A = a\log(x)/(b\log(y)) \qquad B = a\log(x) + b\log(y)$$
$$C = a\log(x) - b\log(y) \quad D = (a + \log(x)) - (b + \log(y)) \quad \boxed{\text{C}}$$

Rule (4): $\log(x^p) = p\log(x)$

Explanation of (4)

$\log(a \times a) = \log(a) + \log(a) = 2\log(a)$
$\log(a \times a \times a) = \log(a) + \log(a) + \log(a) = 3\log(a)$

In general: the number of tens you multiply to get x^p is p times as many tens as you multiply to get x.

What is $\log(\sqrt{x^7})$?

$A = 7 + \log(x)$ $B = (7/2) + \log(x)$ $C = 7/2$ $D = 7/2\log(x)$ $\boxed{\text{D}}$

Find x by solving $10^x = 5$.

A= 5 B= 0.5 C= $\log(5)$ D= $\log(0.5)$ E= $\log(5) - \log(10)$

$\boxed{\text{C}}$

That's it. Thanks for being here.