Student Name: _____

PERM: _____

Circle the section you ATTEND (if you are enrolled a different section, note which one):

Kyle: Tue:8am   Tue:4pm   Tue:7pm

David: Tue:5pm   Tue:6pm

Yihan: Mon:4pm   Mon:5pm   Mon:6pm   Mon:7pm

Tom: Tue:8am   Tue:4pm   Wed:8am

Matt: Tue:5pm   Tue:6pm   Tue:7pm

Math 4B, Midterm 1, Spring 2017
Version B

Instructions: Read the instructions for each question carefully. No calculators, cell phones, or other electronic devices are permitted. No notes or textbooks. Academic dishonesty will not be tolerated. Show your work, write legibly, and circle your answers.

Question	Points	Score
1	10	
2	10	
3	6	
4	8	
5	5	
Total:	39	

I understand UCSB's policies regarding academic dishonesty, and I certify that this test was taken with academic integrity.

Sign and date: _____

1. (10 points) Find the general solution to the ODE:

$$y' = y^2 \cos t$$

2. (10 points) Solve the initial value problem

$$y' = xe^{-\cos x} + y \sin x, \quad y(0) = 3$$

3. (6 points) In this problem, you need to set up a differential equation modeling the biomass M of trees in Tillamook State Forest in Oregon as a function of time t based on the following information.

- Natural tree growth causes biomass to increase at a constant rate.
- Logging permits are issued in such a way that allows biomass to be removed a rate proportional to the total biomass.

(a) Write down your differential equation and **one** complete sentence explaining it.

(b) Sketch the phase line for your differential equation.

(c) According to this model, what long-term prediction can you make about the biomass in the forest? Answer with **one** complete sentence.

4. (8 points) Answer the following questions about the ODE

$$y' = \sqrt{y} - 2e^t$$

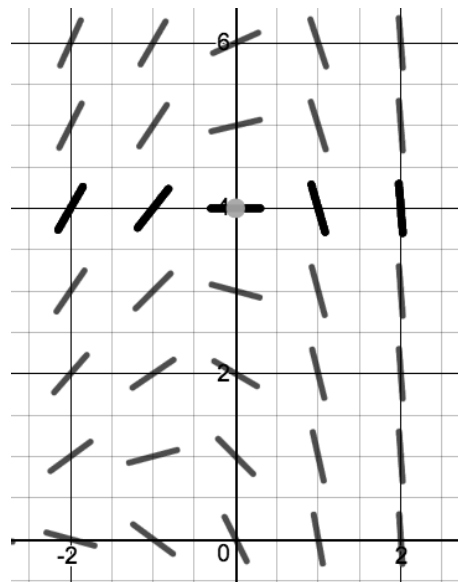(a) Below is an incomplete table in which Euler's method is being used. Fill in the three blanks.

t	y	y'
0	36	4
0.5	38	2.86
1	39.43	0.84
1.5	39.86	−0.66

(b) Suppose $y = f(t)$ is a solution to this differential equation, and that $f(t)$ has a local maximum at $t = 5$. What is the value of $f(t)$ at this maximum?
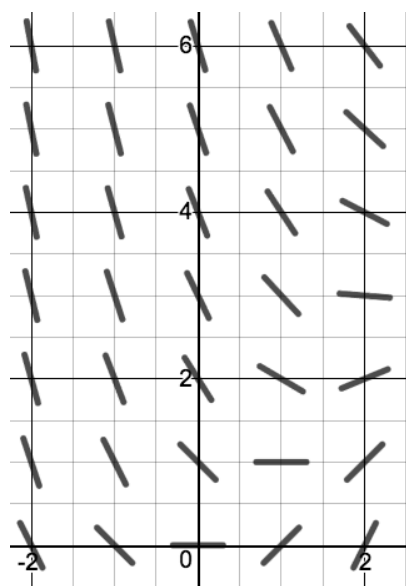
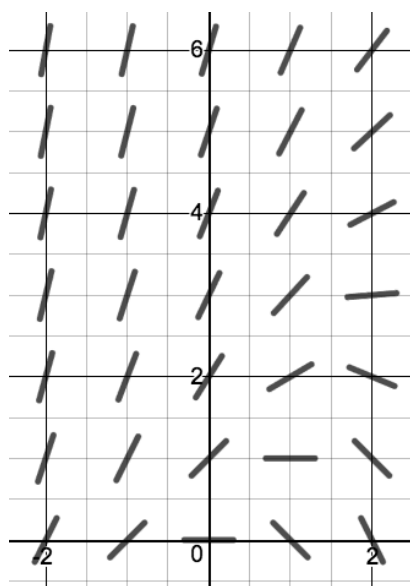(c) Circle the slope field that matches this ODE:
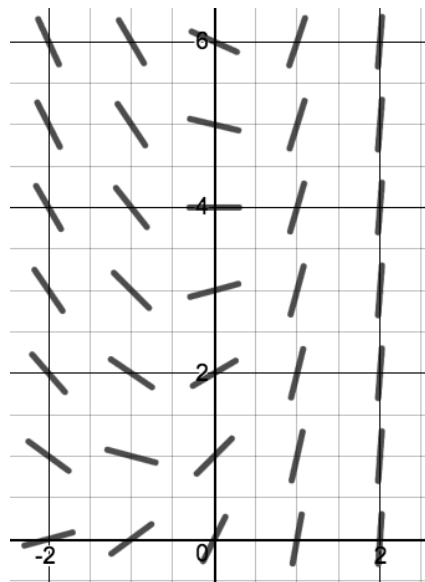
A

B

C

D

5. (5 points) DO ONE AND ONLY ONE of the following two problems. If you do both, we will just grade the first one. CLEARLY INDICATE which problem you are doing by CIRCLING that problem.

(a) Find the general solution to the following ODE by making the substitution $v = \dfrac{y}{x}$.

$$y' = \frac{xy - 3y^2}{x^2}$$

(b) Find the general solution to the following ODE. You should solve for $y(x)$ explicitly. Hint: think about exactness.

$$(2xy - \cos x) + (x^2 + 1)y' = 0$$

If you finish early, you must stay in your seat until the end. You should check your work, but if you are done, you can amuse yourself by coloring in these regular pentagonal tilings.