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Creating a Mathematical "B" Movie: The Effect of *b* on the Graph of a Quadratic

The suggestion to engage students in studying the effects on the graph of the quadratic function $y = ax^2 + bx + c$ that result from varying the parameters a, b, and c is certainly not a new idea. In this article, we first review a few of the specific suggestions that have occurred over the past fifty years. Then we propose an activity of our own that makes use of the new generation of handheld technologies.

A BRIEF HISTORY OF APPROACHES TO THIS PROBLEM

In *The Growth of Mathematical Ideas Grades K–12,* NCTM's Twenty-fourth Yearbook, Smith and Henderson (1959) described an algebra class that studied the effects on the graph of a quadratic function as the value of one of the parameters was changed while the other two were held constant. The students spent little time exploring the geometry of the resulting graphs, because they would have had to construct the graphs by hand, using paper and pencil. The focus of the investigation quickly moved to analyzing numerical tables that the students constructed in order to discover the relationships between the sum and product of the roots and the coefficients a, b, and c.

Curriculum and Evaluation Standards for School Mathematics (NCTM 1989), which appeared thirty years later, made the point that technology provides tools such as graphing utilities that can be used to study functions. This publication included the example of investigating the effects of varying the parameters a, b, c, and d on the graph of a quadratic function in the form $y = a(bx + c)^2 + d$. Although the power of using technology to support mathematics teaching was noted, specific details were lacking.

Professional Standards for Teaching Mathematics (NCTM 1991) then provided a classroom vignette in which students used graphing calculators to explore the effect on the graph when different values of the parameters a, b, and c were used. Here the focus was on classroom discourse, and, once again, specific details about how to use the technology to generate the desired results were not provided. Moreover, the students in the vignette explored the effects of changes in either a or c while the other two coefficients are held constant, but they did not consider changes in b while a and c are held constant.

Five years later, Edwards (1996) showed how to use graphing technology to systematically investigate the effects on the graph of a quadratic function when each of the parameters is varied

This department offers a forum where teachers can share innovative classroom activities and ideas related to teaching and learning mathematics through the use of technology. Ideas using all types of classroom technology are welcome. Send tips to the "Technology Tips" editors.

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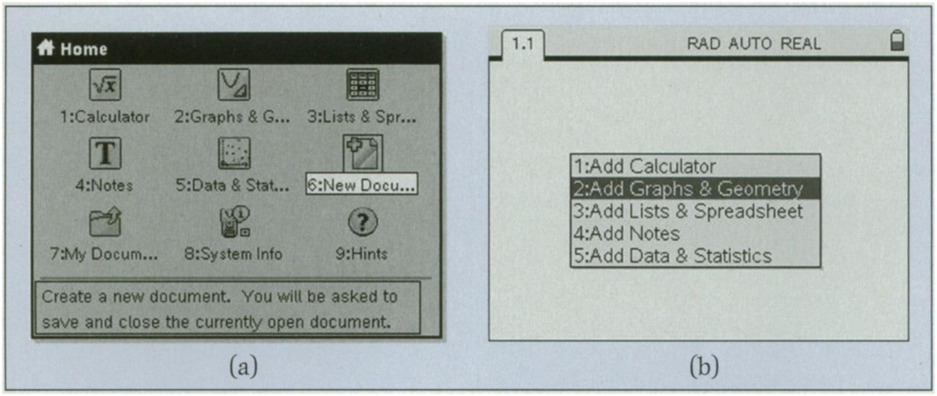

Fig. 1 Drawing a graph of $y = x^2 + b \cdot x + 2$ requires opening a new document (**a**) and choosing a graphic environment (**b**).

in turn while the other two are held constant. Where b is varied while a and c are held constant, he noted that the resulting locus can be analytically proven but leaves the details for the reader to work out.

We end our brief history of approaches to this problem by noting that *Principles and Standards for School Mathematics* (NCTM 2000) observed that explorations with $y = ax^2 + bx + c$ might lead to some interesting results and acknowledged the potential usefulness of computer algebra systems (CAS) in such investigations. Once again, few details were provided.

In today's world, it is difficult to ignore the advantages of using technology in mathematics classes. Indeed, one cannot help but sense the power of technology to enhance classroom investigations. NCTM has been making strong statements on behalf of the use of technology in mathematics education for the last two decades (NCTM 1989, 1991, 2000). Instructional technology is not static but is ever improving. In time, even good technology-based activities are worth revisiting with the use of newer technologies in mind.

New capabilities of the latest generation of technologies require such revisiting. Being able to manipulate a graphical representation dynamically and to observe the effects of such manipulation on the symbolic form is one of these new capabilities. In the activity that follows, we revisit one of Edwards's (1996) explorations and show how bringing the power of the new generation of handheld technology to bear on it can enrich the activity.

THE ACTIVITY

To begin, students need to create a new document on the TI-Nspire CAS by pressing the Home button on the calculator and then selecting 6: New Document (see **fig. 1a**). When prompted to add a new page, students should select 2: Add Graphs & Geometry (see **fig. 1b**). Now the Graphs & Geometry page is displayed, and students may enter $x^2 + b \cdot x + 2$ in the Entry Line. When they press enter to view the graph, nothing will happen, because b is not yet defined.

Creating a Slider

We suggest defining b on a slider to see its effects on the graph. To define b, students need to select menu and select 1: Actions and then select A: Insert Slider (see **fig. 2a**). Next, they need to replace the *v1* in the slider window by typing a b (see **fig. 2b**) and pressing enter. When they have done so, a graph will appear, because b now has a value (see **fig. 2c**).

The slider's default parameters include a minimum value of 0 and a maximum value of 10. To allow for some negative values of b, students should press ctrl and then menu 1: Settings and then adjust the five settings as needed so that they match those in **figure 2d**. Students can move from setting to setting by pressing the tab key, and they can change numerical values by typing in the new value.

Using the Slider to Change the Value of b

Before students manipulate the slider, we ask them to predict the behavior of the graph: "As you move the slider to the right [or left], what do you think will

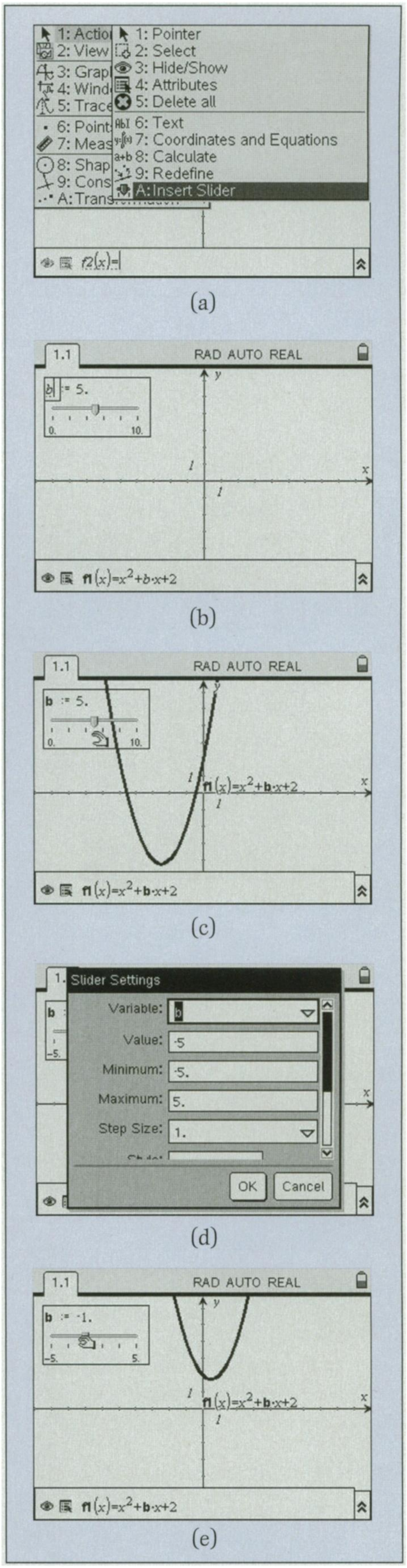

Fig. 2 A slider can be created with a few button pushes (**a**–**e**).

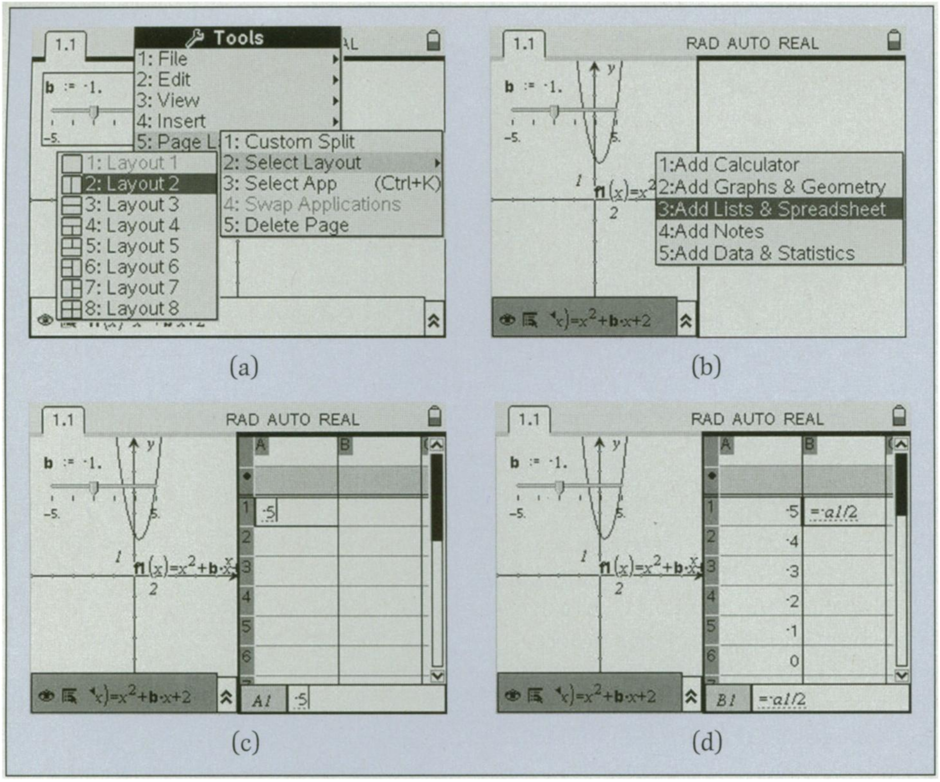

Fig. 3 Splitting the screen allows a spreadsheet to be seen alongside the graph (**a–d**).

happen to the graph?" To use the slider to change the value of b, students can point at the indicator on the slider, use the hand button to grab and hold it, and use the arrow key to move the indicator left or right (see **fig. 2e**). Now we ask them to compare their predictions with their observations: "Did your observations of how the graph moves match your predictions?"

Students should be encouraged to articulate what they are seeing. This is a good time to introduce the term *locus* into the discussion, because ultimately we will be studying the locus of the vertex of a "moving" parabola. At this point, some students may notice that the parabola appears to be moving along a path that is itself a parabola. The question then becomes, "Which parabola?"

Creating a Spreadsheet

Now it is time to organize information on the vertex of the parabola when b is changed. To do so efficiently, we suggest splitting the screen and creating a spreadsheet. To split the screen, students need to press the Wrench button (ctrl + Home) and choose options 5: Page Layout, then 2: Select Layout, and then 2: Layout 2 (see **fig. 3a**). To change windows, students need to press ctrl and then the tab key. Now, selecting menu will give them choices. They should select 3: Add Lists & Spreadsheet (see **fig. 3b**). An empty spreadsheet will appear with the cursor in the first row of column A. Now students can enter values of b in column A in steps of 1 from –5 to 5 by typing each value and pressing enter (see **fig. 3c**).

Students will also need to define formulas for columns B and C. They should go to the first cell of column B and type $= -a1/2$ (see **fig. 3d**). Similarly, in the first cell of column C, they should type $= b1^2 + a1 \cdot b1 + 2$. Now, we ask them to compare what they have just typed with the symbolic form of the quadratic function they have been investigating to see why defining columns B and C in such a way makes sense.

At this point, although three columns in the spreadsheet have been defined, only two are visible. To see all three columns at once, students will need to resize them. They should first place the cursor in column A, choose menu and then 1: Actions, 2: Resize, and 1: Resize Column Width (see **figs. 4a** and **4b**). Now they can use the left arrow to make column A narrower, pressing enter, until an appropriate width has been achieved. Students can then resize columns B and C by moving the cursor to the correct column and repeating the steps above (menu, 1: Actions, and so on) (see **fig. 4c**).

Finally, students need to fill down in columns B and C to see the coordinates of the vertex for each of the values of b that they entered in column A. They need to place the cursor in row 1 of column B, select menu, and choose 3: Data and then 3: Fill Down. Next, they choose the cells in column B that they would like to fill and press enter at the end (see **figs. 5a**,

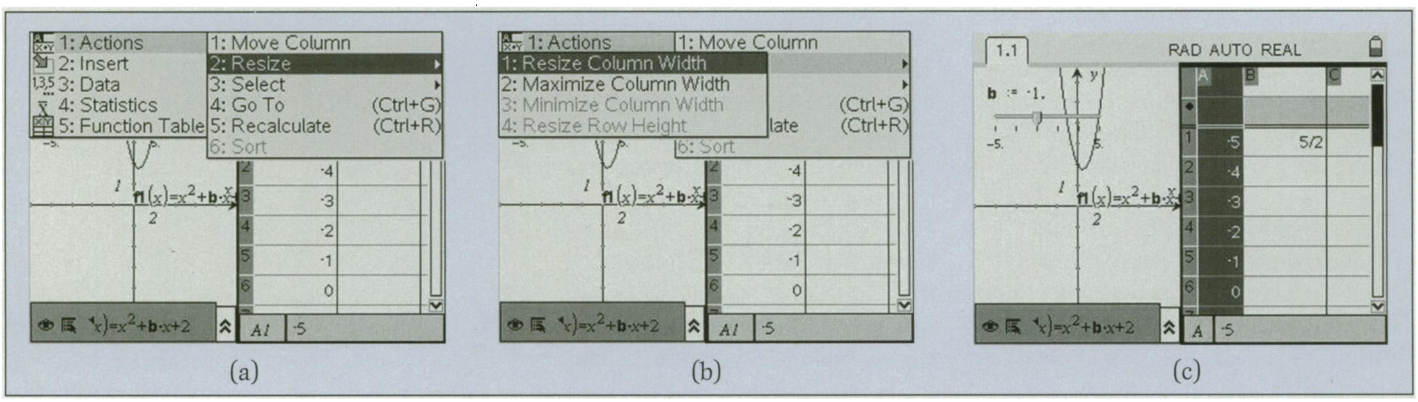

Fig. 4 Resizing the spreadsheet columns makes it easier to see the data (**a–c**).

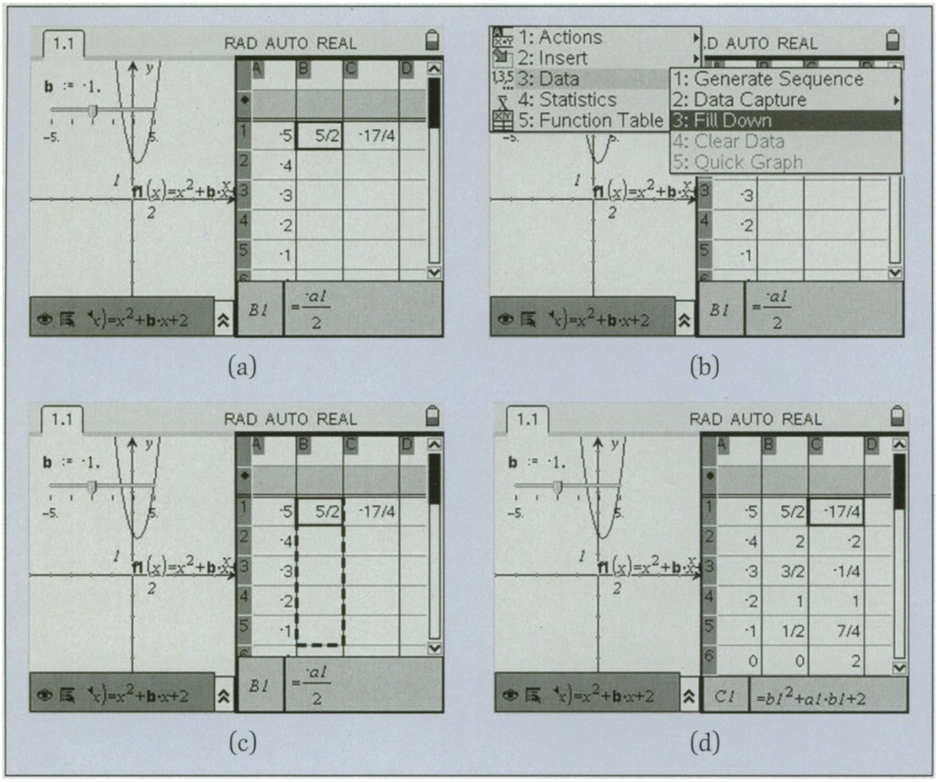

Fig. 5 Using the Fill Down option will complete the spreadsheet (**a–d**).

5b, and **5c**). The same steps should be completed for column C (see **fig. 5d**).

Creating a Scatter Plot

Students need to name columns B and C in order to plot them. They should go to the boxes next to the B and C labels by pressing the up arrow and type *p1* for column B and *p2* for column C and pressing enter after each entry (see **fig. 6a**). Now they need to go back to the graph window by pressing ctrl and the tab key, select menu, and choose 3: Graph Type and then 4: Scatter Plot (see **fig. 6b**). Pressing ctrl and the tab key gets them back to the graph window. Then they move forward with the next steps. The entry line will change and ask for the data for x and y. The options *p1* and *p2* will appear when students press enter. Choose *p1* for x, press enter, use the right arrow to go to the y box, press enter, choose *p2* for y, and press enter again. The scatter plot will be displayed (see **fig. 6c**).

Watching the Scatter Plot and the Moving Parabola

For a better view, students can hide the entry line by pressing ctrl and selecting menu while in the entry line and choosing 5: Hide Entry Line. They can also hide the other extra items on the screen by selecting menu and choosing 1: Actions and 3: Hide/Show. Finally, they can drag the indicator on the slider for b to watch the graph as it moves along the scatter plot (see **fig. 7**).

To facilitate student discovery, we ask questions such as, "Does the scatter plot suggest any particular sort of graph?" and "Which point of the moving parabola appears to coincide with the points on the scatter plot?" However, if given a bit of time to think, students sometimes do not need the foregoing prompts. The significant connection they must make is that the vertex of the moving parabola is tracing a curve that is also a parabola (Edwards 1996, p. 145).

Fitting a Curve to the Scatter Plot

Students can start a new page by pressing ctrl I, for insert. We then ask them to plot *p1* and *p2* by pressing Home and then 2: Graphs & Geometry and then menu, 3: Graph Type, and 4: Scatter Plot. Again, choose *p1* for x and *p2* for y. Pressing enter should produce the graph shown in **figure 8a**. After the scatter plot is graphed, we ask students to change the graph type by selecting

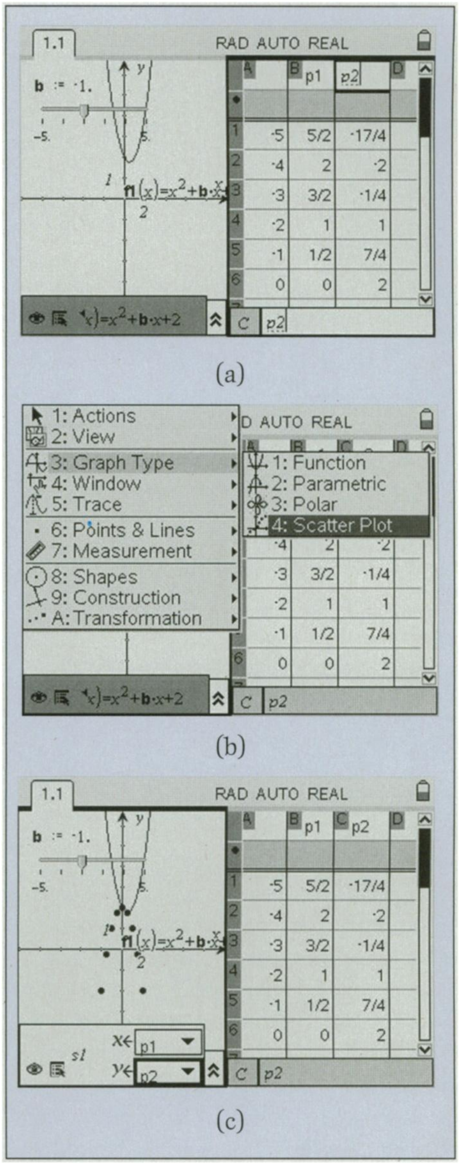

Fig. 6 The scatter plot shows the locus for the parabola's vertex (**a–c**).

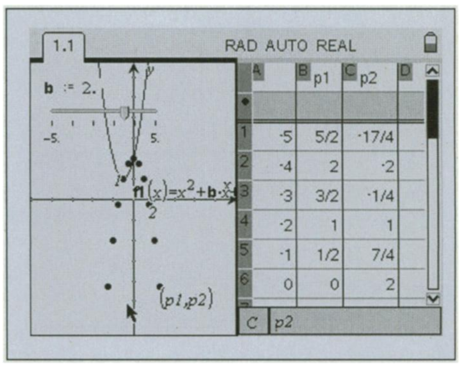

Fig. 7 The slider enables users to move the parabola from point to point on the scatter plot.

menu and choosing 3: Graph Type and 1: Function. Now they can draw $f(x) = x^2$ by typing the equation into the Entry Line (see **fig. 8b**) and hitting enter.

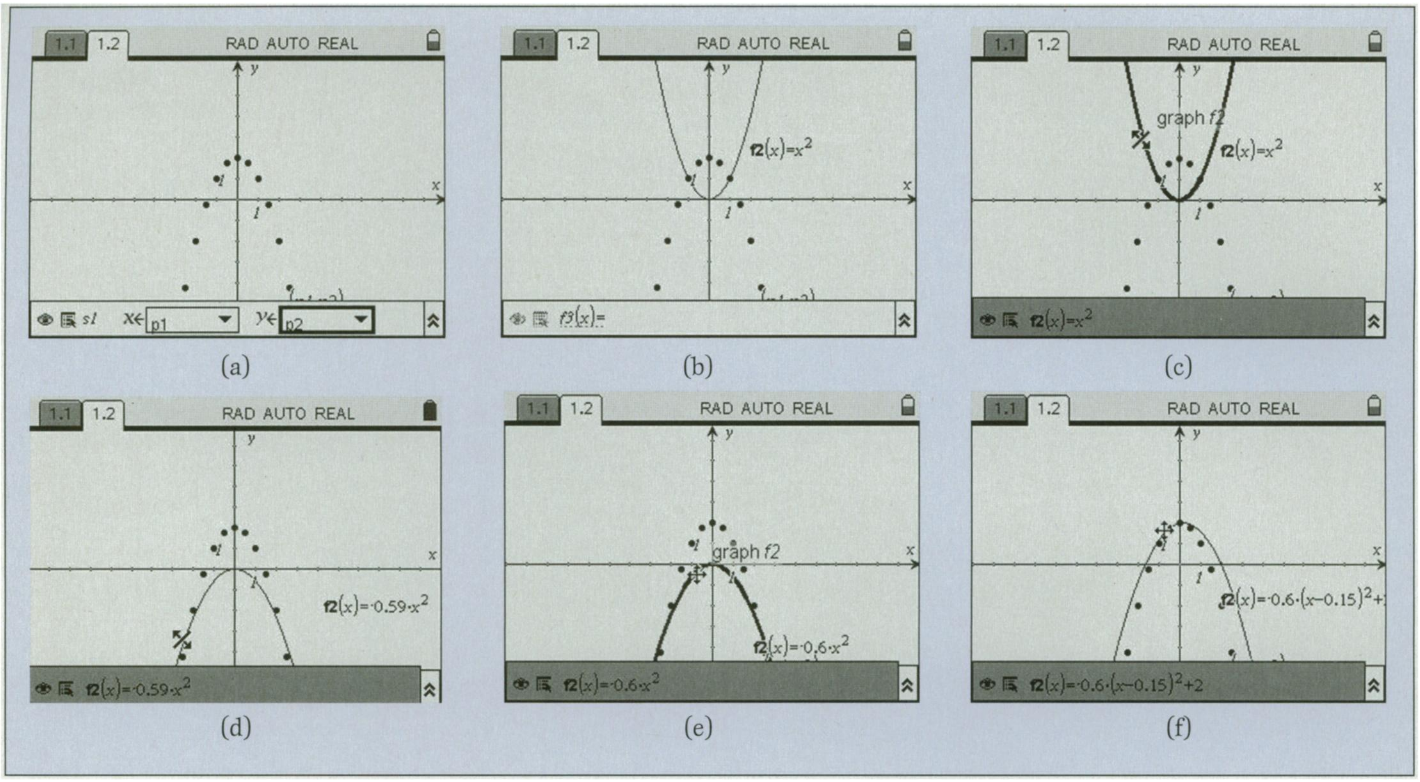

Fig. 8 Students can dynamically fit a curve to the scatter plot (**a–f**).

We ask students to try to match the data points on the scatter plot by dragging and pulling the graph of $y = x^2$. When they point the cursor to the graph, the cursor changes to a double-headed arrow (see **fig. 8c**). Students need to click and hold the hand button to bend the graph dynamically (see **fig. 8d**). To move up or down or to the left or right, they need to grab the graph at its vertex. Now the cursor changes to a four-headed arrow (see **fig. 8e**). Similarly, they can click and hold the hand button to drag the parabola by its vertex (see **fig. 8f**).

When students are satisfied that they have fitted a parabola to the points in the scatter plot, we ask them to compare the symbolic form of the resulting parabola, which should be close to $y = -x^2 + 2$, to the symbolic form with which we began the investigation, $y = x^2 + bx + 2$. When they do so, they are able to make the connection that the parabola they have fitted to the scatter plot of the vertices of $y = x^2 + bx + 2$ for various values of b has the coefficients $a = -1$, $b = 0$, and $c = 2$. After students work through a few more examples, we can elicit the generalization that the locus of the vertices of parabolas with fixed a and c and different values of b is the parabola whose equation is $y = -ax^2 + c$.

Using the CAS for a Symbolic Proof

Students need to start a new document by pressing the Home button on the calculator and then selecting 6: New Document (see **fig. 1a**). When prompted to add a new page, students should select 1: Add Calculator. Next, they will select menu and choose 1: Actions and 1: Define (see **fig. 9a**). Then they must type $y(x) = x^2 + b \cdot x + 2$ and then press enter (see **fig. 9b**). They also need to define t as $-b/2$ (see **fig. 9c**). This is a good time to discuss with students the role of t as defined by using the formula for the axis of symmetry. When they enter $y(t)$, they will see that it is $2 - b^2/4$ (see **fig. 9d**). We have not found it difficult to elicit from students the conjecture that $y(t) = 2 - t^2$. They can check their conjecture by typing $y(t) = 2 - t^2$, whereupon the calculator returns the message, True (see **fig. 9e**). A more general symbolic approach is also possible (see **fig. 9f**).

CONCLUDING THOUGHTS

Recent improvements to handheld technology make it possible for students to experience mathematical concepts in novel ways. Being able to collect data from a dynamic construction is one.

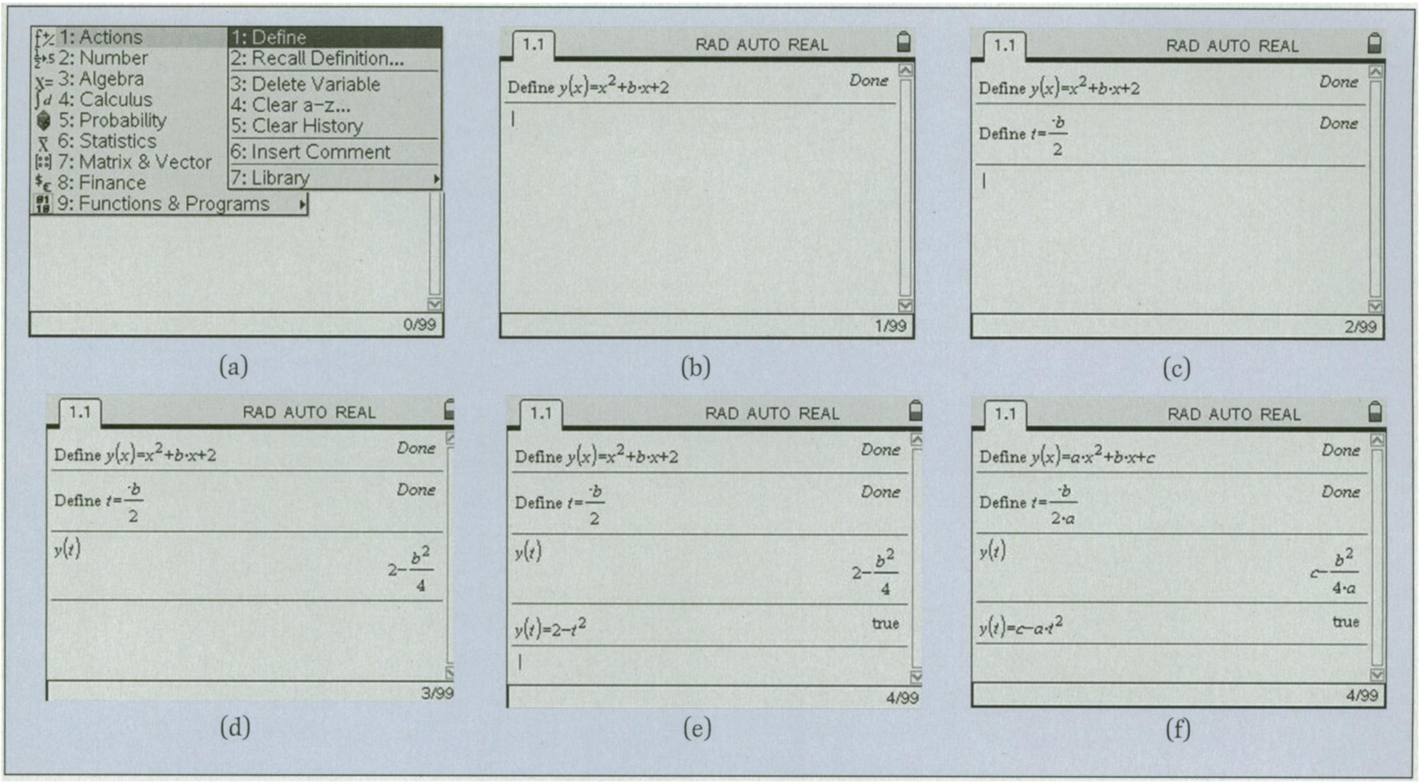

Fig. 9 A symbolic approach uses CAS (**a–f**).

Now, students have innovative ways to study patterns as well as visual and virtually tactile environments for creating and testing conjectures. Moreover, they can experience holding, moving, and bending the graph of a function to observe the effects of their actions on the function's algebraic form.

The ways of studying mathematics change, as do the mathematical topics themselves. The immediate feedback now possible allows mathematical concepts to spring to life in mathematics classrooms. Thus, some topics that previously have been ignored because of time limitations in the classroom or the limited capabilities of previous instructional technologies may now form the basis of interesting and productive student investigations.

REFERENCES

Edwards, Thomas G. "Exploring Quadratic Functions: From *a* to *c*." *Mathematics Teacher* 89, no. 2 (February 1996): 144–46.

National Council of Teachers of Mathematics (NCTM). *Curriculum and Evaluation Standards for School Mathematics.* Reston, VA: NCTM, 1989.

———. *Professional Standards for Teaching*

Mathematics. Reston, VA: NCTM, 1991.

———. *Principles and Standards for School Mathematics.* Reston, VA: NCTM, 2000.

Smith, Eugene P., and Kenneth B. Henderson. "Proof: Some Common Uses of the Term *Proof.*" In *The Growth of Mathematical Ideas Grades K–12,* Twenty-fourth Yearbook of the National Council of Teachers of Mathematics (NCTM), edited by Harold P. Fawcett and Phillip S. Jones et al., pp. 111–81. Washington, DC: NCTM, 1959.



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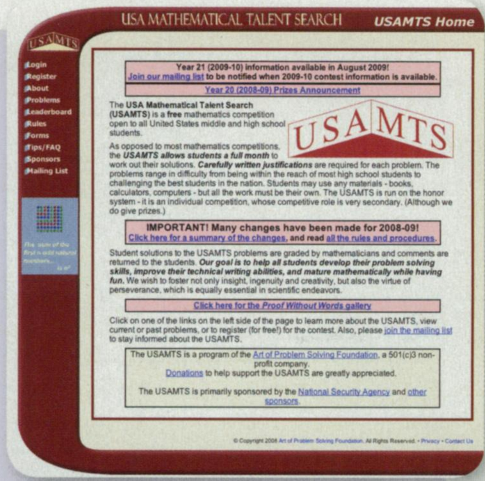
Surfing Note



The USA Mathematical Talent Search (USAMTS) Web site (www.usamts.org/), a program offered by the Art of Problem Solving Foundation (www.artofproblemsolving.org/), promotes a free mathematics contest that is open to high school and middle school students. The contest focuses not only on correct solutions but also on carefully written justifications. However, the plethora of problems and solutions that the Web site provides may be of benefit to teachers. In fact, under the Tips section is a nice discussion about how to find solutions and how to write solutions and proofs that may be of interest to teachers and students.

A sample problem follows:

> Two players are playing a game that starts with 2009 stones. The players take turns removing stones. A player may remove exactly 3, 4, or 7 stones on his or her turn, except that, if only 1 or 2 stones are remaining, then the player may remove them all. The player who removes the last stone wins. Determine, with proof, which player has a winning strategy, the first or the second player.