NAME(S): ____________________ TA (circle one): Elizabeth Christian

____________________ SECTION (circle one): 8AM 12PM 4PM 5PM 6PM 7PM

Project #4: Phase Portraits for Linear DE Systems
Solutions Page

Feedback[1]

quality of mathematical ideas (7 pts)	
clarity of communication (3 pts)	

Please write your group or individual solution on this page. Staple any additional work for your solutions on the back of this page to turn in during section on Wednesday, December 10th. If you cannot attend section, get your solutions to your TAs mailbox in SH 6623 by 4:00pm the day your section meets.

Problem 1 Consider the system

$$\vec{\mathbf{x}}' = \begin{bmatrix} 3 & -2 \\ -1 & 2 \end{bmatrix} \vec{\mathbf{x}}.$$

The matrix of the system has eigenvalues/vectors:

$$\lambda_1 = 1, \quad \vec{\mathbf{v}}_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix} \quad \text{and} \quad \lambda_2 = 4, \quad \vec{\mathbf{v}}_2 = \begin{bmatrix} -2 \\ 1 \end{bmatrix}$$

(a) Write down the general solution for the system.

(b) Is the equilibrium solution $\vec{\mathbf{0}}$ a source, sink or saddle? Explain.[2]

[1]Please see DP Evaluation Rubric handout, available on GauchoSpace, for more information on how DP solutions are evaluated.

[2]Please provide all of your explanations in complete sentences.

(c) Which eigenvector corresponds to the “fast” direction and which corresponds to the “slow” direction? Explain how you know and how this will affect solutions trajectories as they move away from the origin.

(d) Which of the phase portraits corresponds to this system?

Problem 2 Consider the system

$$\vec{\mathbf{x}}' = \begin{bmatrix} 1 & -1 \\ 5 & -1 \end{bmatrix} \vec{\mathbf{x}}.$$

The matrix of the system has eigenvalues/vectors:

$$\lambda_1 = 2i, \quad \vec{\mathbf{v}}_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix} + i \begin{bmatrix} 0 \\ 2 \end{bmatrix} \quad \text{and} \quad \lambda_2 = -2i, \quad \vec{\mathbf{v}}_2 = \begin{bmatrix} 1 \\ 1 \end{bmatrix} - i \begin{bmatrix} 0 \\ 2 \end{bmatrix}$$

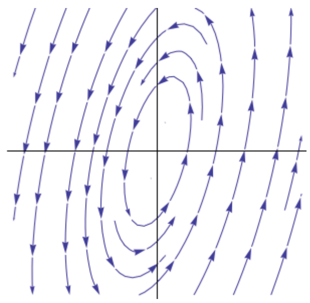
(a) Write down the general solution for the system.

(b) Are solution trajectories in the phase plane oscillating? Are they periodic? Explain how you can tell.

(c) Since the eigenvalues are complex, there should be some sort of rotation in the phase plane. Will this rotation be clockwise or counterclockwise? Explain how you can tell.

(d) Which of the phase portraits corresponds to this system?

Use the following phase portraits to answer part (d) of problems 1 and 2.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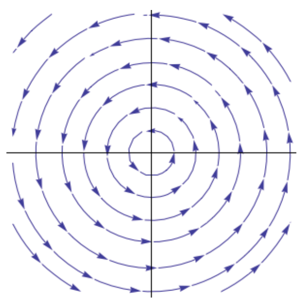
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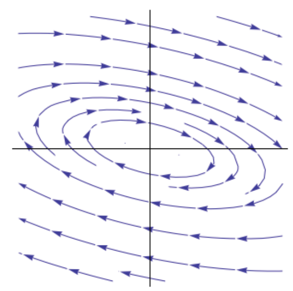
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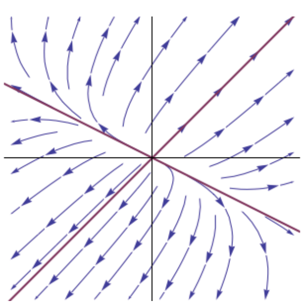
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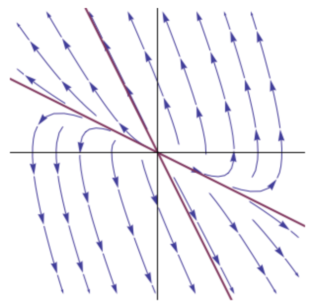
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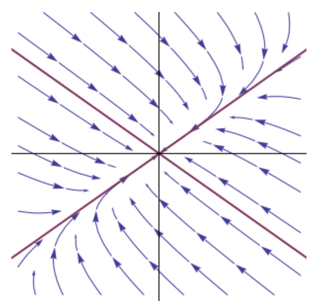
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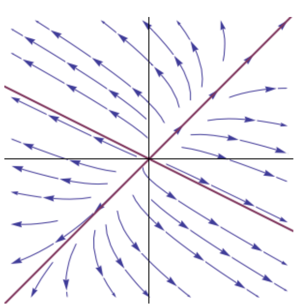
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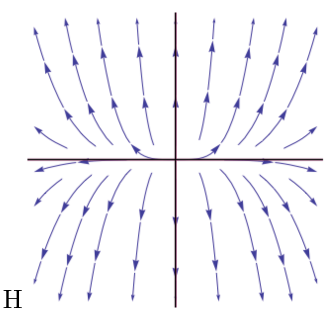
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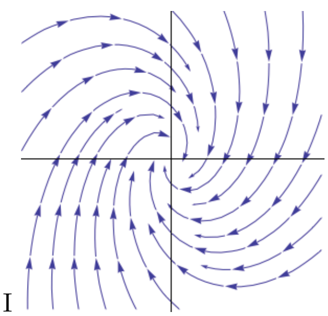
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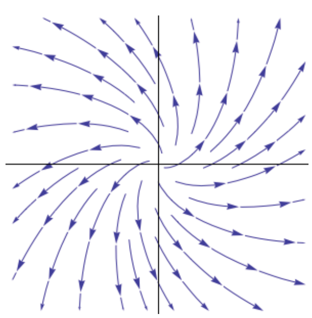
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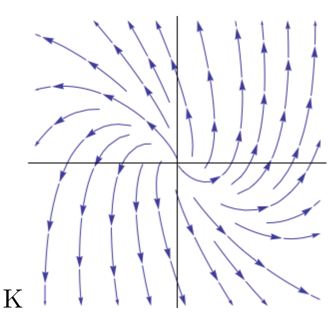
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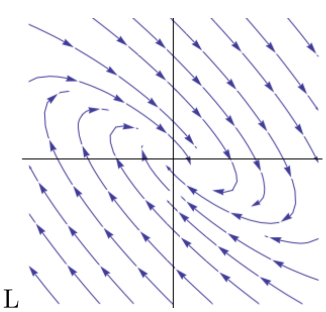
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