Office Hours!

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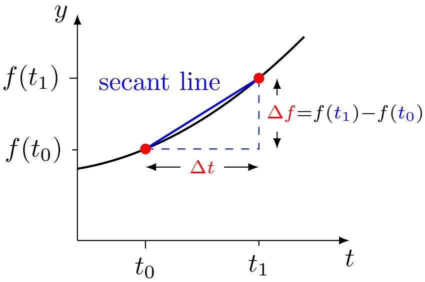
Graphical Approach



Δf = change in f
Δt = change in t

Many ways to say same thing:

$$\begin{pmatrix} \text{average rate of} \\ \text{change of } f \end{pmatrix} = \frac{\text{change in } f}{\text{change in } t}$$
$$= \frac{\Delta f}{\Delta t}$$
$$= \text{slope of secant line} = \frac{f(t_1) - f(t_0)}{t_1 - t_0}$$

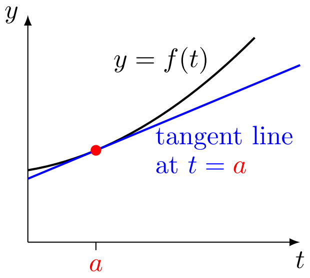
The derivative is defined to be

$$\lim_{\Delta t \to 0} \left(\frac{\Delta f}{\Delta t} \right) = \frac{df}{dt}$$

Idea: As t_1 moves closer to t_0 the secant line approaches the tangent line at t_0. This is the line with the same slope as the graph at t_0.

Understanding Derivatives

There are many ways to think about derivatives. You need to understand these to apply to problems.



slope of graph at a
= slope of tangent line
= instantaneous rate of change of f at a

$$= \left(\begin{array}{c} \text{limit of average rate of change} \\ \text{of } f \text{ over shorter and shorter} \\ \text{time intervals starting at } a \end{array} \right)$$

= limit of slopes of secant lines

$$= f'(a) \;=\; \left.\frac{df}{dt}\right|_{t=a}$$

Summary of Derivatives

One quantity, y, depends on another quantity x.
In other words y is a function of x so $y = f(x)$. Example: $y = 7x$

If you change x, then y changes.
Question: How quickly does y change as x changes?

Answer: The derivative tells you.

In our example, the derivative is 7. This tells you:

the output $= y$ of the function changes
7 times as fast
as the input $= x$ to the function.

If x is changed by 0.1 how much does y change by?

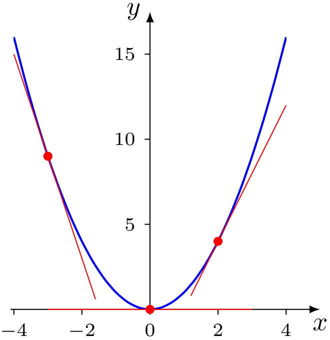
A= 7 B= 7.1 C= 0.7 D= 0.1/7 E= other $\boxed{\text{C}}$

Graphical Meaning

$$\frac{d}{dx}\left(x^2\right) = 2x$$

What this means

The slope of the graph of $y = x^2$ at $x = a$ is $2a$



at $x = -3$, slope is $2(-3) = -6$

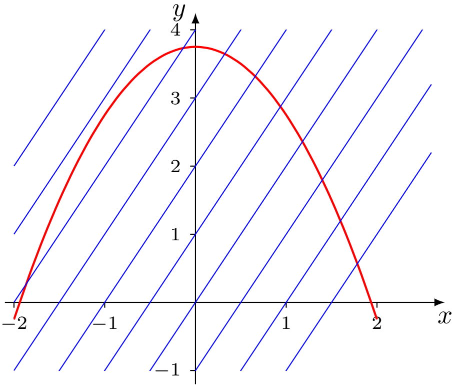
at $x = 0$, slope is $2(0) = 0$

at $x = 2$, slope is $2(2) = 4$

derivative = rate of change = slope of graph = slope of tangent line

Slope Question

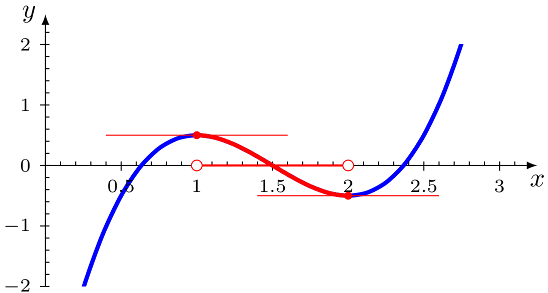
This graph shows $y = f(x)$ and lines parallel to $y = 2x$



Question: For which values of x is $f'(x) > 2$?

A $x < 1.2$ B $x < 0$ C $x < -1.5$ D $x < -1$ E $x < -0.5$ $\boxed{\text{D}}$

More Slope Questions



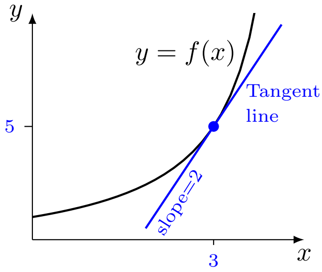
(1) For which values of x is $f'(x) = 0$?

A= none B= $\{0.63,\ 1.5, 2.38\}$ C= 1 D= $\{1, 2\}$ E= 2 $\boxed{\text{D}}$

(2) For which values of x is $f'(x) < 0$?

A $x < 0.63$ B $x < 1$ C $1 < x < 2$ D $1.5 < x < 2.38$ E none $\boxed{\text{C}}$

The Importance of Units



Told $f(3) = 5$ and $f'(3) = 2$

This means the slope of the tangent line to the graph $y = f(x)$ at $x = 3$ is 2.

The derivative is this slope, so...

$$\boxed{\text{The units of } \frac{dy}{dx} \text{ are } \frac{\text{units of y}}{\text{units of x}}}$$

Examples:
Heating: derivative units are $\$/^\circ$ F = dollars per degree F
Adrenaline: bpm/mg = beats per minute per mg of adrenaline.

Units help you understand the meaning of the derivative.

Interpretation of Derivatives I

Suppose $f(x)$ = the percentage of children who still get measles when x% of children are inoculated.

Question: Which of the following is a plausible value for $f'(40)$?

A$=0$ B$=2$ C$=50$ D$=-2$ E$=-50$ $\boxed{\text{D}}$

Question: If $f(40) = 20$ and $f'(40) = -2$, which must be true?

A when 20% of children are inoculated the percentage who gets measles decreases by 2%

B when 20% of children are inoculated then inoculating an extra 1% of children would reduce the number of measles cases by another 2%

C If the inoculation rate is 41% then 18% of children gets measles

D If the inoculation rate is 20% then 2% fewer cases of measles arise if an extra 1% of children can be inoculated

E none of the above

Answer: $\boxed{\text{C}}$

Interpretation of Derivatives II

Air temperature gets colder the higher you go.
$T(x)$ = air temperature in $^\circ C$ at a height x meters above sea level.
Question: Which of these is a plausible value for $T'(2000)$?

A$= -1$ B$= 1$ C$= 0$ D$= 1/200$ E$= -1/200$ $\boxed{\text{E}}$

Question: If $T(2000) = 10$ and $T'(2000) = -1/200$, which is most plausible?

- A the temperature at sea level is $16^o C$
- B the temperature 2400 meters above sea level is $8^o C$
- C the temperature 10 meters above sea level is $2000^o C$
- D 2000 meters above sea level the temperature is decreasing at a rate of $1/200^o C$ per minute.
- E none of these are plausible

Answer: $\boxed{\text{B}}$

Interpretation of Derivatives III

$x =$ money spent (in thousands of \$) in one month on advertising.

$f(x) =$ sales (in thousands of \$) in a month when x is spent on advertising.

Question: If $f(20) = 60$ and $f'(20) = 3$ which must be true?

A When the sales of the company are 20 thousand dollars in one month the amount spent on advertising is increasing at a rate of 3 thousand dollars per month

B When the company spends 20 thousand dollars per month on advertising the sales rise at a rate of 3 thousand dollars per month

C When the company spends 20 thousand dollars per month on advertising each extra dollar a month spent on advertising generates an extra 3 dollars of sales.

D When the company spends 3 thousand dollars per month on advertising the sales are increasing at a rate of 20 thousand dollars per month

E None of the above

Answer: $\boxed{\text{C}}$