Office Hours!

Instructor:

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Office Hours:

Mondays 2–3PM
Tuesdays 10:30–11:30AM
Thursdays 1–2PM
or by appointment

Office:

South Hall 6510

§7.9: Population Growth

Assume each generation of bunnies has 3 times as many bunnies as previous one. Initially have 100 bunnies. How many bunnies after n generations?

A$= 100 \times 3n$ B$= 100 + 3n$ C$= 100(1 + 3n)$
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After 3 generations have $100 \times 3 \times 3 \times 3$ bunnies
So…after n generations have

$$100 \times \underbrace{3 \times 3 \times \cdots \times 3}_{n \text{ times}} = 100 \times 3^n \text{ bunnies.}$$

More Bunnies

We saw that:

- if we start with 100 bunnies, and
- the bunny population triples every generation,

then we have 100×3^n bunnies after n generations.

1. How many generations until there are $10^7 = 10,000,000$ bunnies?

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$$\text{A} \approx 0.22 \qquad \text{B} \approx 4.52 \qquad \text{C} \approx 10.48$$
$$\text{D} \approx 1.67 \qquad \text{E} \approx 3,333 \qquad \boxed{\text{C}}$$

Flu Outbreak

2. At the start of an outbreak of H1N1 flu in a large herd of cattle, there were 5 infected individuals. The numbers doubles every 3 days. How many days until there are 80 infected cows?

A= $\log(16)/\log(2)$ B= $\log(16/2)$ C= $16/\log(2)$

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Doubling Time Formula

Suppose something doubles every K minutes*. If there is a mass of A at time $t = 0$, how much is there at time t minutes?

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Idea: t/K is number of doubling periods in t minutes.

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3. A disease spreads through a community. On March 1 there were 100 infected people. The number of people doubles in a 3 days. How many infected people are there t days after March 1?

A$= 2^t$ B$= 3 \times 2^{t/100}$ C$= 100 \times 2^t$ D$= 100 \times 2^{t/3}$

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How many days until there are $1,000$ infected people?

A$= \log(10)/\log(2)$ B$= 3\log(10)/\log(2)$ C$= 3\log(5)$
D$= 3(\log(10) - \log(2))$ E$= 3\log(20)$

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A More Complicated Example

$$\text{mass after t minutes} = A \times 2^{(t/K)}$$

where

- K is the doubling time, and
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4. A colony of mold is growing on a cheeseburger in the back of a dorm refrigerator. When discovered it has a mass of 10 mg. One week later it was found to have a mass of 30 mg. What is the doubling time measured in days?

A= $\log(2)/\log(3)$ B= $7\log(2)/\log(3)$ C= $7\log(2/3)$ D= $7\log(3/2)$

Hint: We know A and the mass t days after discovery (for some t).

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Solving $30 = 10 \times 2^{7/K}$ gives $\boxed{\text{B}}$

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In general: After n half-lives,

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6. An isotope has a half-life of 5 years.

(a) If we start with 70 grams, how many grams will be left after t years?

$$\text{A} = 70\left(\frac{1}{2}\right)^t \quad \text{B} = 5\left(\frac{1}{2}\right)^{70t} \quad \text{C} = 70\left(\frac{1}{2}\right)^{5t}$$

$$\text{D} = 70\left(\frac{1}{2}\right)^{t/5} \quad \text{E} = 0$$

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(b) How many years until 10 grams remain?

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7. (Radiocarbon Dating) A bone is found with 2% of the usual amount of carbon-14 in it. The half-life of carbon-14 is 5730 years. How old (in years) is the bone?

A= $5730 \log(.01)/\log(2)$ B= $5730 \log(50)/\log(2)$

C= 5730×50 D= wicked old

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Answer: $\boxed{\text{B}} \approx 32,000$ years

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