Welcome To Math 34A!
Differential Calculus

Instructor:

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South Hall 6431X (Grad Tower, 6th floor, blue side, first door on the right)

Office Hours:

MTWR after class 2:00-3:00, and by appointment. Details on Gauchospace.

Summary of Logs

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Each of these pairs of equalities says one thing!

General Compound Interest

If the interest rate is r%, then each year money multiplies by

$$m = 1 + \frac{r}{100}.$$

If you start with an initial amount A of money then after t years you have

$$A \times m^t = A \times \left(1 + \frac{r}{100}\right)^t$$

5. If you invest \$1000 at 14% interest, how much will you have 5 years later? (Guess!)

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How much is this?

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$$\$1,000 \times \left(1 + \frac{14}{100}\right)^5 = \$1,000 \times (1.14)^5.$$

How much is this? Smart way: 14% in 1 year ≈ 7% per year for 2.

§7.9: Population Growth

Assume each generation of bunnies has 3 times as many bunnies as previous one. Initially have 100 bunnies. How many bunnies after n generations?

A$= 100 \times 3n$ B$= 100 + 3n$ C$= 100(1 + 3n)$
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After 1 generation have 100×3 bunnies

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After 1 generation have 100×3 bunnies
After 2 generations have $100 \times 3 \times 3$ bunnies
After 3 generations have $100 \times 3 \times 3 \times 3$ bunnies
So. . .after n generations have

$$100 \times \underbrace{3 \times 3 \times \cdots \times 3}_{n \text{ times}} = 100 \times 3^n \text{ bunnies.}$$

More Bunnies

We saw that:

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1. How many generations until there are $10^7 = 10,000,000$ bunnies?

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$$\text{A} \approx 0.22 \qquad \text{B} \approx 4.52 \qquad \text{C} \approx 10.48$$
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Flu Outbreak

2. At the start of an outbreak of H1N1 flu in a large class of students, there were 5 infected individuals. The numbers doubles every 3 days. How many days until there are 80 infected students?

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4. A colony of mold is growing on a cheeseburger in the back of a dorm refrigerator. When discovered it has a mass of 10 mg. One week later it was found to have a mass of 30 mg. What is the doubling time measured in days?

A= $\log(2)/\log(3)$ B= $7\log(2)/\log(3)$ C= $7\log(2/3)$
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6. An isotope has a half-life of 5 years.

(a) If we start with 70 grams, how many grams will be left after t years?

$$\text{A} = 70\left(\frac{1}{2}\right)^{t} \quad \text{B} = 5\left(\frac{1}{2}\right)^{70t} \quad \text{C} = 70\left(\frac{1}{2}\right)^{5t}$$

$$\text{D} = 70\left(\frac{1}{2}\right)^{t/5} \quad \text{E} = 0$$

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(b) How many years until 10 grams remain?

$$A = 5(\log(7) - \log(2)) \quad B = \log(7)/\log(2) \quad C = 5\log(7/2)$$

$$D = 5\log(7)/\log(2) \quad E = \log(7)/(5\log(2))$$

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7. (Radiocarbon Dating) A bone is found with 2% of the usual amount of carbon-14 in it. The half-life of carbon-14 is 5730 years. How old (in years) is the bone?

A= $5730 \log(.01)/\log(2)$ B= $5730 \log(50)/\log(2)$

C= 5730×50 D= wicked old

[‡] Any time unit will work, not just years. Just be consistent!

Half-Life Formula

Suppose something has a half-life of K years[‡]. If there is a mass of A at time $t = 0$, how much is there at time t years?

$$\text{mass after } t \text{ years} = A \times \left(\tfrac{1}{2}\right)^{(t/K)}$$

Idea: t/K is number of half-lives in t years.

7. (Radiocarbon Dating) A bone is found with 2% of the usual amount of carbon-14 in it. The half-life of carbon-14 is 5730 years. How old (in years) is the bone?

A= $5730 \log(.01)/\log(2)$ B= $5730 \log(50)/\log(2)$

C= 5730×50 D= wicked old

Answer: $\boxed{\text{B}} \approx 32,000$ years

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$$\boxed{b^{\log_b(y)} = y} \qquad \boxed{\log_b\left(b^a\right) = a}$$

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Important bases:

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Teaser: e is special because the derivative of e^x is e^x whatever that means.

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...are about how quickly things change.

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This is all about *understanding* the world.

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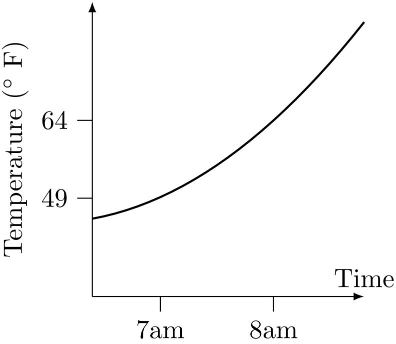

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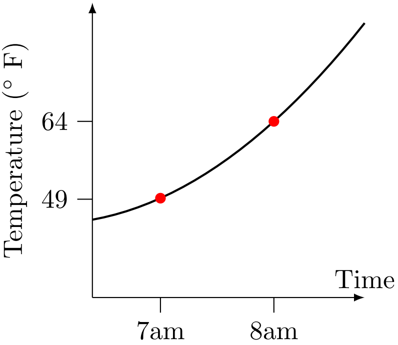
$= 64 - 49 = 15^\circ$ F



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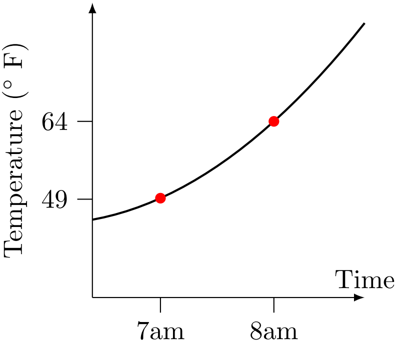

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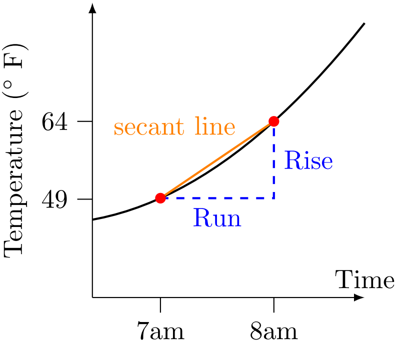

Continuing Example

Similarly,

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Suppose temperature at time t given by the formula $f(t) = t^2$.
Using a calculator one can find the average rate of change over shorter and shorter time spans Δt, starting at 7am:

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Answer: it is a limit!

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Mathematical solution: take the limit.

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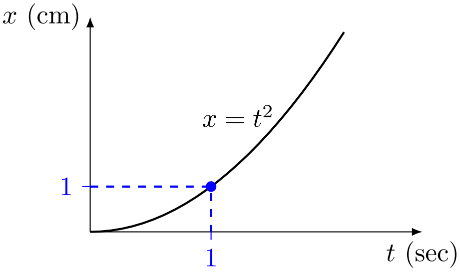
A hamster runs along the x-axis, so that after t seconds the hamster is t^2 cm from the origin. Our goal is to find the hamster's speed at time $t = 1$ sec.



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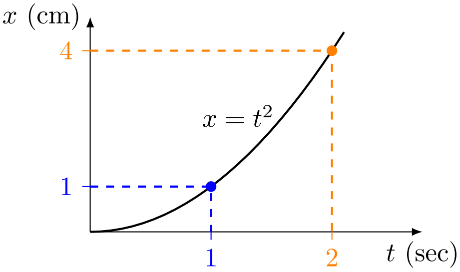

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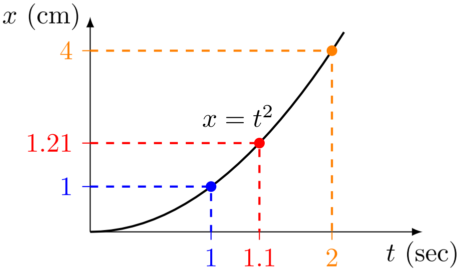

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Example Concluded

How do we work out the exact speed of the hamster after 1 second?

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Conclusion: At $t = 1$ sec, the exact speed of the hamster is 2 cm/sec.

Hamster Summary

Soon we will calculate that...

> the exact speed of the hamster after t seconds is $2t$ cm/sec.

Summary:

$f(t) = t^2$ = distance in cm of hamster from origin after t seconds

(a function that gives the distance the hamster has traveled at time t)

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Hamster Questions!

After t seconds, the hamster is $f(t) = t^2$ cm from origin.
(1) What is the exact speed (in cm/sec) of the hamster at $t = 2$?

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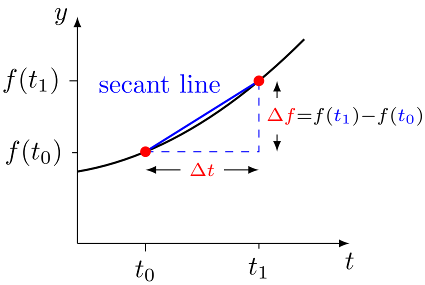
Does this make sense?

Graphical Approach



Δf = change in f
Δt = change in t

Many ways to say same thing:

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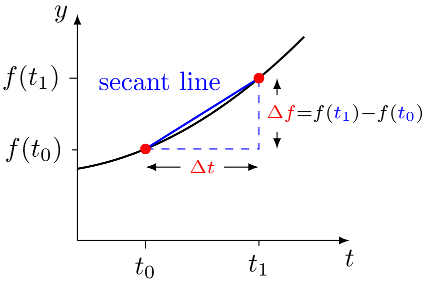

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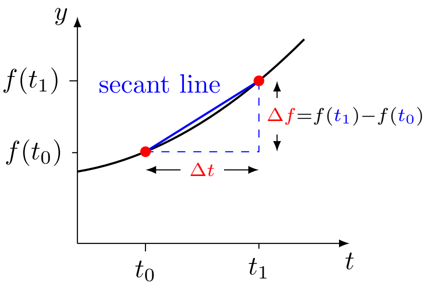

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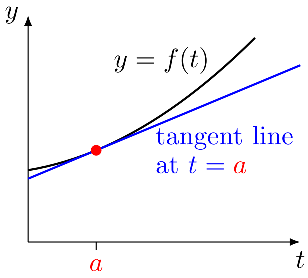
Idea: As t_1 moves closer to t_0 the secant line approaches the tangent line at t_0. This is the line with the same slope as the graph at t_0.

Understanding Derivatives

There are many ways to think about derivatives. You need to understand these to apply to problems.



slope of graph at a

= slope of tangent line

= instantaneous rate of change of f at a

$$= \left(\begin{array}{c} \text{limit of average rate of change} \\ \text{of } f \text{ over shorter and shorter} \\ \text{time intervals starting at } a \end{array} \right)$$

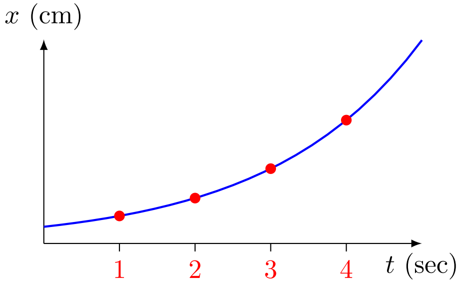
= limit of slopes of secant lines

$$= f'(a) = \left.\frac{df}{dt}\right|_{t=a}$$

Summary

- How fast something changes = rate of change
- Instantaneous rate of change is the limit of the average rate of change over shorter and shorter time spans. This gets around the $0/0$ problem.
- speed = rate of change of distance traveled.

Speed=Slope=Derivative



The graph shows the distance from the origin in cm after t seconds of a hamster. Which of the numbers below is the largest?

Hint: Speed is a slope!

A = speed of the hamster at $t = 1$

B = speed of the hamster at $t = 2$

C = speed of the hamster at $t = 3$

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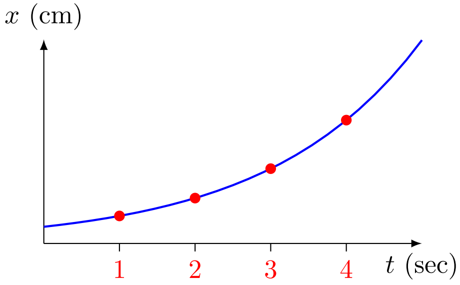

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Answer: $\boxed{\text{E}}$

Practical Meaning

Our goal is that you understand the practical meaning of the derivative in various situations.

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f(t) &= \text{temperature in }^\circ\text{ F at } t \text{ hours after midnight} \\
f(7) &= 48 \text{ means the temperature at 7am was } 48^\circ \text{ F} \\
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f'(9) &= -5 \text{ means at 9am the temperature was falling at a rate of } 5^\circ \text{ F/hr} \\
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g(t) &= \text{distance from origin in cm of hamster on } x\text{-axis after } t \text{ seconds}\\
g(7) &= 3 \text{ means after 7 seconds hamster was 3 cm from origin}\\
g'(9) &= -5 \text{ means after 9 seconds our furry friend was running towards}\\
&\qquad \text{the origin at a speed of 5 cm/sec}
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Another Context

Suppose $f(t)$ = temperature of oven in °C after t minutes.

What do $f(3) = 20$ and $f'(3) = 15$ mean?

- A After 20 minutes the oven was at 3° C and heating up at a rate of 15° C/min
- B After 3 minutes oven temperature was 15° C and cooling down at a rate to 20° C/min
- C The oven was heating up at rate of 3° C/min after 15 minutes and also after 20 minutes
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Answer: $\boxed{\text{D}}$

That's it. Thanks for being here.