# Welcome Back!
# Differential Calculus

## Instructor:

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## Office Hours:

T R 11-11:50, T 3:45-4:35 Details on Gauchospace.

# Warm-up

- $\log(x) = 5$ $\quad x = \boxed{10^5} = \boxed{100,000}$
- $10^x = 1,000,000$ $\quad x = \boxed{\log(10^6)} = \boxed{6}$
- $\log(\log(x)) = 2$ $\quad x = \boxed{10^{10^2}} = \boxed{10^{100}}$
- $10^x = 2$ $\quad x = \boxed{\log(2)} \approx \boxed{.3}$
- $10^x = 8700$ $\quad x = \boxed{\log(8700)} \approx ?$

  $8700 = 8.7 \cdot 10^3$ $\quad \log(8700) = \log(8.7) + 3$
- $10^{4x-5} = 7$ $\quad x = \boxed{(\log(7) + 5)/4} \approx \boxed{?}$ just leave it that way

# Logarithm Strategy

- $4^{2x+1} = 3$ $\qquad x = \boxed{(\log_4(3) - 1)/2} = (\frac{\log(3)}{\log(4)} - 1)/2$

In general,

$$\boxed{\log_b(x) = \frac{\log(x)}{\log(b)}}$$

# Midterm 2: One week from today

## Bring:

- A pen or sharp pencil.
- A 3" $\times$ 5" card with your notes.
- Student ID.

## Don't bring:

- A calculator

No bluebook or scratch paper necessary, just the above materials and hopefully a fresh, well-practiced you! Scratch paper will be provided.

# Midterm 2 Topics

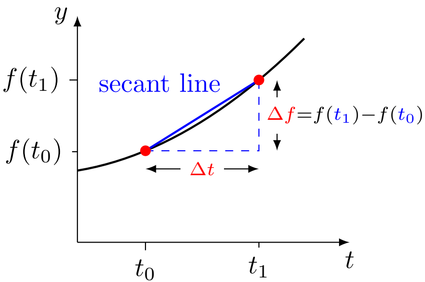
- All topics from Midterm 1
- Sums (like the example below, more examples on Gauchospace)

$$\sum_{n=1}^{4} 2^n - 1$$

- Advanced Logarithm Methods (the full chapter on logarithms in the book)
- Change and Average Rate of Change for a function or graph.
- Limits with $h$ (used to find exact speed, examples on the old midterm and extra problems)

If you struggled on Midterm 1 with algebra or word problems, you need to improve these skills immediately. **They are essential for success in this course**.

# Graphical Approach




$\Delta f$ = change in $f$
$\Delta t$ = change in $t$

Many ways to say same thing:

$$\left( \begin{array}{c} \text{average rate of} \\ \text{change of } f \end{array} \right) = \frac{\text{change in } f}{\text{change in } t}$$

$$= \frac{\Delta f}{\Delta t}$$

$$= \text{slope of secant line} = \frac{f(t_1) - f(t_0)}{t_1 - t_0}$$

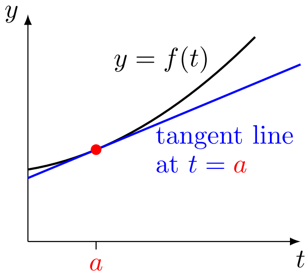
The derivative is defined to be

$$\lim_{\Delta t \to 0} \left( \frac{\Delta f}{\Delta t} \right) = \frac{df}{dt}$$

Idea: As $t_1$ moves closer to $t_0$ the secant line approaches the tangent line at $t_0$. This is the line with the same slope as the graph at $t_0$.

# Understanding Derivatives

There are many ways to think about derivatives. We need to understand how derivatives apply to problems.




slope of graph at a
$=$ slope of tangent line
$=$ instantaneous rate of change of $f$ at $a$

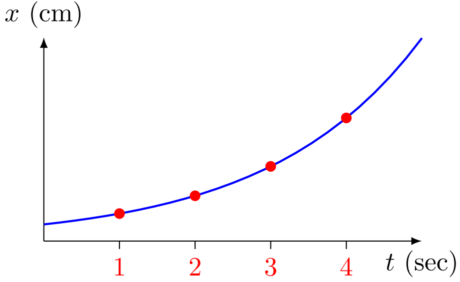
$$= \left( \begin{array}{c} \text{limit of average rate of change} \\ \text{of } f \text{ over shorter and shorter} \\ \text{time intervals starting at } a \end{array} \right)$$

$=$ limit of slopes of secant lines
$$= f'(a) \;=\; \left.\frac{df}{dt}\right|_{t=a}$$

# Summary

- How fast something changes = rate of change
- Instantaneous rate of change is the limit of the average rate of change over shorter and shorter time spans. This gets around the changing speed problem, and works a whole lot better that getting frustrated and trying 0/0.
- speed = rate of change of distance traveled.

# Speed=Slope=Derivative




The graph shows the distance from the origin in cm after $t$ seconds of a hamster. Which of the numbers below is the largest?

**Hint:** Speed is a slope!

A = speed of the hamster at $t = 1$

B = speed of the hamster at $t = 2$

C = speed of the hamster at $t = 3$

D = average speed of the hamster between $t = 2$ and $t = 3$

E = average speed of the hamster between $t = 3$ and $t = 4$

Answer: $\boxed{\text{E}}$

# Practical Meaning

Our goal is that you understand the practical meaning of the derivative in various situations.

$f(t) =$ temperature in ° F at $t$ hours after midnight

$f(7) = 48$ means the temperature at 7am was 48° F

$f'(7) = 3$ means at 7am the temperature was rising at a rate of 3° F/hr

$f'(9) = -5$ means at 9am the temperature was falling at a rate of 5° F/hr or rising at a rate of −5° F/hr

$g(t) =$ distance from origin in cm of hamster on $x$-axis after $t$ seconds

$g(7) = 3$ means after 7 seconds hamster was 3 cm from origin

$g'(9) = -5$ means after 9 seconds our furry friend was running towards the origin at a speed of 5 cm/sec

# Another Context

Suppose $f(t)$ = temperature of oven in °C after $t$ minutes.

What do $f(3) = 20$ and $f'(3) = 15$ mean?

- A After 20 minutes the oven was at 3° C and heating up at a rate of 15° C/min
- B After 3 minutes oven temperature was 15° C and cooling down at a rate to 20° C/min
- C The oven was heating up at rate of 3° C/min after 15 minutes and also after 20 minutes
- D After 3 minutes the oven was at 20° C and heating up at a rate of 15° C/min
- E None of the above

Answer: $\boxed{\text{D}}$

# Context: Population

Suppose $f(t)$ = the population of the ancient city of Lyrad in year $t$. We are told that $f(1550) = 1820$ and $f'(1650) = 1100$. Which of the following is true?

- A In 1550, the population was 1820 and rising at a rate of 1100 people per year
- B In 1650, the population was 1100 more than in 1550
- C In 1650, Lyrad contained 1100 people
- D In 1550, there were 1820 people in Lyrad, and by 1650 this had increased to 2920
- E None of above

Answer: $\boxed{\text{E}}$

# Context: Mathematics

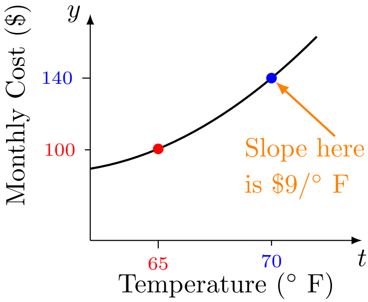
Suppose $f(0) = 50$ and $f(10) = 70$. Which of the following is true?

- A For all $t$ between 0 and 10, the derivative is $f'(t) = 2$
- B $f'(0) = 2$
- C It is possible that $f'(0) = -8$
- D It is impossible that $f'(0) = -8$
- E None of above

Answer: $\boxed{\text{C}}$

We'll see later that, for example, that $f(x) = x^2 - 8x + 50$ has $f(0) = 50$, $f(10) = 70$, and $f'(0) = -8$.

# It doesn't have to be about time!




$f(x)$ = monthly cost of heating house to $x°$ F

$f(70) = 140$ means it costs $140 to heat the house for one month to a temperature of 70°F.

$f'(70) = 9$ means rate at which cost increases as temperature changes is $9 for each extra ° F.

In practical terms this means you pay an extra $9 during each month for each extra $1^oF$ . If you turn it up two degrees you pay an extra $18 each month. Each extra degree of warmth costs an extra $9 each month. In economics this is called a marginal cost or marginal rate

This is not exactly true:

average rate of change versus instantaneous rate of change.

In the following examples we will ignore this subtlety.

# Get Pumped!

Adrenaline cause the heart to speed up.
$x$ = number of mg (milligrams) of adrenaline in the blood.
$f(x)$ = number of beats per minute (bpm) of the heart with $x$ mg of adrenaline in the blood.

What does $f'(5) = 2$ mean? Answer: $\boxed{\text{E}}$

- A When there are 5 mg of adrenaline the heart beats at 2 pbm
- B When the amount of adrenaline is increased by 2 mg the heart speeds up by 5 bpm
- C When the heart beats at 5 bpm the adrenaline is increased by 2 mg
- D When there are 5 mg of adrenaline the heart speeds up by 2bpm
- E When there are 5 mg of adrenaline in the blood the heart speeds up by 2 bpm for each extra mg of adrenaline.

**Hint:** The units of $f'(5)$ are bpm per milligram of adrenaline

# Summary of Derivatives

One quantity, $y$, depends on another quantity $x$.
In other words $y$ is a function of $x$ so $y = f(x)$. Example: $y = 7x$

If you change $x$, then $y$ changes.
Question: How quickly does $y$ change as $x$ changes?

Answer: The derivative tells you.

In our example, the derivative is 7. This tells you:

the output $= y$ of the function changes
7 times as fast
as the input $= x$ to the function.

If $x$ is changed by $0.1$ how much does $y$ change by?

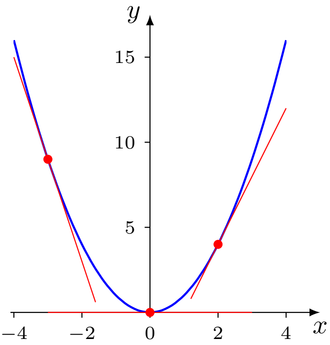
A= 7 B= 7.1 C= 0.7 D= 0.1/7 E= other $\boxed{\text{C}}$

# Graphical Meaning

$$\frac{d}{dx}\left(x^2\right) = 2x$$

What this means

The slope of the graph of $y = x^2$ at $x = a$ is $2a$

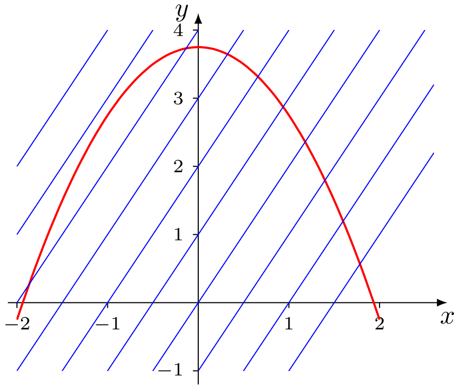



at $x = -3$, slope is $2(-3) = -6$
at $x = 0$, slope is $2(0) = 0$
at $x = 2$, slope is $2(2) = 4$

derivative = rate of change = slope of graph = slope of tangent line

# Slope Question

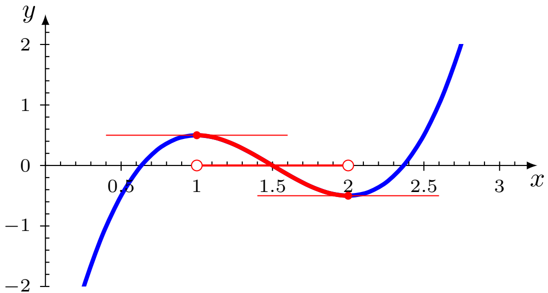
This graph shows $y = f(x)$ and lines parallel to $y = 2x$




Question: For which values of $x$ is $f'(x) > 2$?

A $x < 1.2$ B $x < 0$ C $x < -1.5$ D $x < -1$ E $x < -0.5$ $\boxed{\text{D}}$

# More Slope Questions




(1) For which values of $x$ is $f'(x) = 0$?

A= none  B= $\{0.63,\ 1.5, 2.38\}$  C= 1  D= $\{1, 2\}$  E= 2  $\boxed{\text{D}}$

(2) For which values of $x$ is $f'(x) < 0$?

A $x < 0.63$  B $x < 1$  C $1 < x < 2$  D $1.5 < x < 2.38$  E none  $\boxed{\text{C}}$

# That's it. Thanks for being here.