

Name:

Perm:

Math 34A Practice Midterm 1, Spring 2022

1. (2pts) Solve for x in the following equation.

$$\frac{4}{5(x-n)} - \frac{5}{4(x+n)} = 0$$

clear denominators

$$16(x+n) - 25(x-n) = 0$$

$$16\underline{x} + 16n - 25\underline{x} + 25n = 0 \rightarrow \text{get non-}x \text{ stuff on this side}$$

$$16x - 25x = -16n - 25n$$

$$-9x = -41n$$

$$x = \frac{41}{9}n$$

$$x = \boxed{\frac{41}{9}n}$$

2. (2pts) Multiply out and simplify.

$$(2x - 3y^{-1})(3x^{-1} + xy)$$

$$\begin{aligned} & \cancel{2x} \cdot \cancel{3x^{-1}} + 2x \cdot xy - 3y^{-1} \cdot \cancel{3x^{-1}} - \cancel{3y^{-1}} \cdot \cancel{xy} \\ &= 6 + 2x^2y - 9x^{-1}y^{-1} - 3x \end{aligned}$$

$$\boxed{6 + 2x^2y - 9x^{-1}y^{-1} - 3x}$$

3. (2pts) Substitute $x = ab + c$ into the expression below and simplify

$$2x^2 + 3cx - 1.$$

$$\begin{aligned} & 2(ab+c)^2 + 3c(ab+c) - 1 \\ &= 2(ab \cdot ab + 2ab \cdot c + c^2) + 3c \cdot ab + 3c \cdot c - 1 \\ &= 2a^2b^2 + \underline{4abc} + \underline{2c^2} + \underline{3abc} + \underline{3c^2} - 1 \\ &= 2a^2b^2 + 7abc + 5c^2 - 1 \end{aligned}$$

$$2a^2b^2 + 7abc + 5c^2 - 1$$

4. (2pts) Find the point of intersection of two lines. The first line passes through the points (0,0) and (2,3) while the second line passes through the points (0,5) and (4,0). Finding m_1, b_1 and m_2, b_2 for the equations below may help you find the answer.

$$m_1 = \frac{3}{2}, b_1 = 0$$

$$y = m_1x + b_1$$

$$y = m_2x + b_2$$

over 4, down 5

$$m_2 = -\frac{5}{4}$$

$$b_2 = 5$$

$$\left. \begin{aligned} y &= \frac{3}{2}x + 0 \\ y &= -\frac{5}{4}x + 5 \end{aligned} \right\} \begin{aligned} \frac{3}{2}x &= -\frac{5}{4}x + 5 \\ \cdot 4 \end{aligned}$$

$$6x = -5x + 20$$

$$11x = 20$$

$$x = \frac{20}{11}$$

$$\begin{aligned} y &= \frac{3}{2}x = \frac{3}{2} \cdot \frac{20}{11} \\ &= \frac{30}{11} \end{aligned}$$

$x =$

$$\frac{20}{11}$$

$y =$

$$\frac{30}{11}$$

5. (4pts) Jason started driving from Phoenix towards Isla Vista at noon at a speed of 75 mph. At 2pm Marie started driving from Isla Vista towards Phoenix at 100 mph. Jason and Marie met at 4pm. Meanwhile a bad guy follows Jason. He leaves Phoenix at 1pm and drives along the same route at 90 mph.

How many miles from Isla Vista was the bad guy when Jason and Marie met?

t = hours since noon.

Jason's distance: $75t$ ← started @ 2pm

Marie's distance: $100(t-2)$

At 4pm, their distances are 75.4 and 100(2) resp.

So Phoenix & IU are $300+200=500$ mi apart.

Bad guy's distance: $90(t-1)$

So he traveled $90(4-1)=90 \cdot 3=270$ mi by 4pm

He's $500-270=230$ mi from IU at 4pm.

230

miles

6. (4 points)

- (a) A cylinder with radius r and height h has a volume of $36\pi m^3$. Express the surface area in terms of the height h .

(The formula for the volume of a cylinder is $\pi r^2 h$, and the surface area formula is $2\pi r^2 + 2\pi r h$.)



$$A = 2\pi r^2 + 2\pi r h$$

$$36\pi = \pi r^2 h, \text{ so } r^2 = \frac{36\pi}{\pi h} = \frac{36}{h} \quad r = \sqrt{\frac{36}{h}}$$

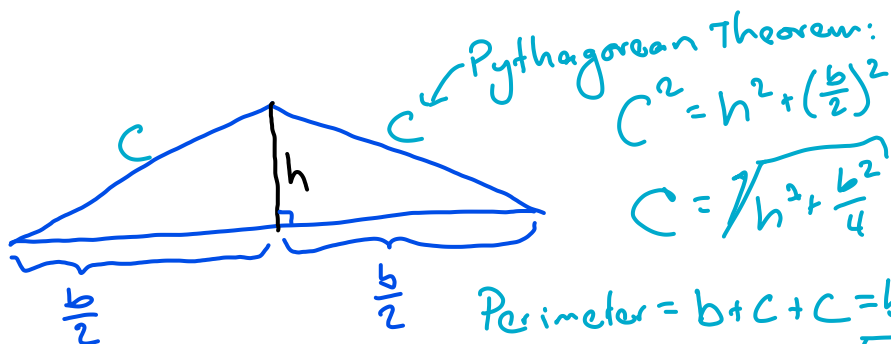
$$A = 2\pi\left(\frac{36}{h}\right) + 2\pi\sqrt{\frac{36}{h}} \cdot h \quad (\text{solve for } r)$$

$$= \frac{72\pi}{h} + 12\pi \cdot \sqrt{h}$$

Surface Area =

$$\frac{72\pi}{h} + 12\pi \cdot \sqrt{h}$$

- (b) The side of a roof is going to be built. Its shape is an isosceles triangle with base b and height h (figure below). The area of this triangle is $10m^2$. Express the perimeter in terms of the height.



$$10 = \frac{1}{2} b \cdot h, \text{ so } b = \frac{20}{h}.$$

(solve for b)

$$\begin{aligned} \text{Perimeter} &= b + c + c = b + 2c \\ &= \left(\frac{20}{h}\right) + 2\sqrt{h^2 + \left(\frac{20}{h} \cdot \frac{1}{2}\right)^2} \\ &= \left(\frac{20}{h}\right) + 2\sqrt{h^2 + \left(\frac{10}{h}\right)^2} \end{aligned}$$

Perimeter =

$$\frac{20}{h} + 2\sqrt{h^2 + \frac{100}{h^2}}$$

7. (3 points) What are the following limits?

(a) $\lim_{h \rightarrow 0} 10 - 7h$

← this part disappears

10

when $x \neq 2$,
 (b) $\lim_{x \rightarrow 2} \frac{x^2 - 4x + 4}{2x - 4}$ ← both go to zero as x gets close to 2 ∴
 Let's cancel!

$$\frac{x^2 - 4x + 4}{2x - 4} = \frac{(x-2)(x-2)}{2(x-2)}$$

$= \frac{x-2}{2}$ ← this approaches 0.
 ← this is always 2. $\frac{0}{2} = 0$ ✓

0

(c) $\lim_{n \rightarrow \infty} \frac{3n-2}{n+3}$ ← both get big.

$n = 1,000,000$
 $\frac{3,000,000 - 2}{1,000,000 + 3}$ is really close to 3.

3

8. (3 points) Compute the logarithms below.

(a) $\log_3(27)$

$27 = 1 \cdot 3 \cdot 3 \cdot 3$ or ... $27 = 3^{\textcircled{3}}$
 $\uparrow \uparrow \uparrow$
 1 2 3

3

(b) $\log_{10}(1,000,000)$

$1,000,000 = 1 \cdot 10 \cdot 10 \cdot 10 \cdot 10 \cdot 10 \cdot 10$ or ... $1,000,000 = 10^{\textcircled{6}}$
 $\uparrow \uparrow \uparrow \uparrow \uparrow \uparrow$
 6 10's

6

(c) $\log_2(\frac{1}{4})$

$\frac{1}{4} = 1 \div 2 \div 2$
 $\uparrow \uparrow$
 2 2's
 division means negative

-2