3-31 Algebra Review

Wednesday, January 5, 2022 2:29 PM

Left over from last time...

6. The perimeter of a rectangle is twice its area. Find a formula for the length of the rectangle in terms of its width.

Perimeter = 2 of each = $2\cdot l + 2\cdot w$. Area = product (of l & w) = $l\cdot w$

Perimeter = $2\cdot$ Area means $(2l+2w) = 2\cdot(l\cdot w)$

let's cancel the 2's... $\div 2$ $\div 2$

$\frac{2l+2w}{2} = \frac{2(l\cdot w)}{2}$

$l+w = l\cdot w$ now solve for l...

$l+w-l\cdot w = 0$

$l - lw = -w$

$l(1-w) = -w \rightarrow \boxed{l = \frac{-w}{1-w}}$

$\boxed{L = \frac{W}{W-1}}$ Same thing! (why?)

1. What is 3/4 as %? Using the trick: $\frac{3}{4}\cdot 100\% = \left(\frac{3}{4}\cdot 100\right)\% = 75\%$

Another way: $\frac{3}{4} = \frac{3\cdot 25}{4\cdot 25} = \frac{75}{100}$ This is the answer, because $\frac{75}{100} = 75\%$.

2. What is 20% of 30? One way: $20\% = \frac{20}{100} = \frac{1}{5}$. So 20% of $30 = \frac{1}{5}$ of 30. $\frac{1}{5}$ of 30 is $\boxed{6}$.

Another way: $20\%\cdot 30 = \frac{20}{100}\cdot 30 = \frac{20\cdot 30}{100} = \frac{600}{100} = \frac{6\cdot \cancel{100}}{\cancel{100}} = \frac{6}{1} = 6$.

3. Click A,B,C,D as you do these problems

(A) What is 20% of x? $20\% = \frac{20}{100} = \frac{1}{5} = .2$

So $20\%\cdot x = \frac{1}{5}\cdot x$. This could be $\boxed{.2x \text{ or } \frac{x}{5}}$

(B) What is 70% as a fraction?

$70\% = \frac{70}{100} = \boxed{\frac{7}{10}}$

(C) What is x% of 50? x% of 50 is also 50% of x.

$x\%\cdot 50 = \frac{x}{100}\cdot 50 = \frac{50x}{100} = \boxed{\frac{50}{100}\cdot x = \frac{1}{2}\cdot x.} = \frac{x}{2}$

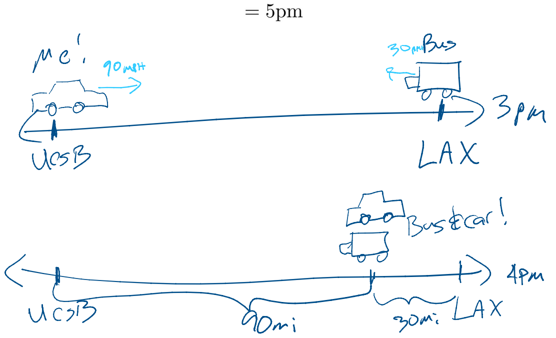
(D) What is $\frac{x}{x+1}$ as %? ?? Um...just apply the trick from before...

remember, $100\% = \frac{100}{100} = 1$

$\frac{x}{x+1} = \frac{x}{x+1}\cdot 100\% = \boxed{\frac{100x}{x+1}\%}$.

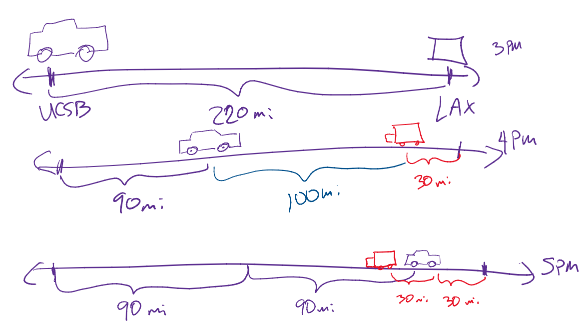
2. The Santa Barbara airbus leaves LAX at 3pm and drives to UCSB at an average speed of 30 mph. You leave UCSB at 3pm driving at 90 mph towards LAX. What time do you whiz past the airbus?

A = 1pm B = 3:30pm C = 4pm D = 4:45pm E = 5pm



What happens when you can't just use trial and error? One student asked about another distance: 220 miles between the bus and the reader. You can still solve the problem by diagramming! You can also solve using equations, if you like. There is an example of that below.

220 mi apart?



$\frac{100\text{mi}}{120\text{mi}}$ $\frac{100}{120}$ hour needed

$\frac{5}{6}$ hour

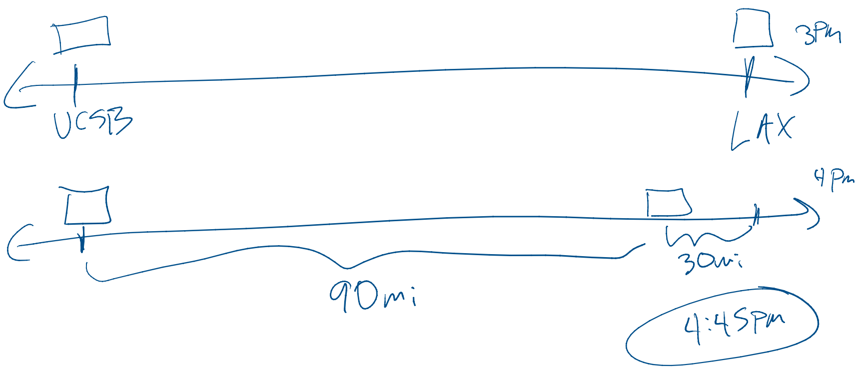
3. Same question/answers, but now you leave UCSB at 4pm

total distance covered $= 90(t-1) + 30t = 120$

$$90t + 30t \underbrace{-90}_{+90} = 120$$

$$120t = 210$$

$$t = \frac{210}{120} = \frac{21}{12} = \frac{7}{4} = 1\tfrac{3}{4}$$



At this point, 90 more miles need to be covered, and each hour we move 120 miles closer. So it should take 3/4 of an hour past 4PM to meet each other.

Another Word Problem

4. Two numbers add up to give 17 and their product is 60. What is the larger of the two numbers?

Method / Plan:

(i) ~~Name the two unknowns~~
(ii) create two equations
(iii) solve equations.

To solve: use one equation to eliminate one unknown from second equation, then factor the resulting quadratic.

A= I have answer B= working C=help

Another way to solve this problem: two numbers add to 17, which is odd, so one number has to be odd and the other has to be even. They also have to multiply to 60, so these two numbers are a pair, one odd and one even, that multiply to 60. 60 has a LOT of even divisors, so let's look at the odd ones: 1, 3, 5, 15.
1x60=60 -- that doesn't work because they add to 61.
3x20=60 -- also no, because they add to 23
5x12=60 -- success! 5 and 12 add to 17. We have the answer!

One way, volunteered by a student: This strategy involves a system of two equations, using substitution.

$$x + y = 17 \qquad y = \frac{60}{x}$$

$$xy = 60$$

$$x + \frac{60}{x} = 17$$

$$x^2 + 60 = 17x$$

$$x^2 - 17x + 60 = 0$$

$$x^2 - 5x - 12x + 60 = 0$$

$$x(x-5) - 12(x-5) = 0$$

now factor the (x-5) out of both terms

$$(x-12)(x-5) = 0$$

$$x - 5 = 0$$

or

$$x - 7 = 0$$

- My comment about factoring here: In order to factor the polynomial $x^2 - 17x + 60$, we have to find two numbers that add to 17 and multiply to 60 (this is to figure out how to split the 17 apart and continue factoring, as we do in red, or for another method you may have learned for factoring). My point was that in order to continue with this process, you end up solving the original problem. And if you already have two numbers that add to 17 and multiply to 60, then why not just use them for your answer?

5. A rectangle has perimeter 34 inches and area 60 square inches. What is the length of the shortest side?

A= I have answer B= working C=help

$A = l \cdot w = 60$

$P = 2l + 2w = 34$

$l \cdot w = 60$ $\quad$ $l + w = 17$

$(17-w) \cdot w = 60$ $\quad$ $l = 17 - w$

$$\boxed{0 = w^2 - 17w + 60}$$

6. Find $y = f(x)$, the function that gives $y\,°$F from $x\,°$C.

A $y = 9x/5$ B $y = 9(x{+}32)/5$ C $y = (9x/5){+}32$ D $y = (9x{+}32)/5$

$$y = \frac{x}{100} \cdot 180 + 32$$

Idea: You can plug in a power cord
Inverse is to unplug the cord.

Example: $f(x) = 3x - 2$ is a function of x

Start with $x \xrightarrow{\cdot 3} 3x \xrightarrow{-2} 3x-2$

$x \xleftarrow{\div 3} 3x \xleftarrow{+2} 3x-2$ f^{-1}

$f^{-1}(y) = \frac{y+2}{3}$

Try these, clicking as you go...

Click	$y = f(x)$	$x = f^{-1}(y)$
A	$y = 5x$	$x = y \div 5$
B	$y = x + 7$	$x = y - 7$
C	$y = 3x - 4$	$x = \frac{y+4}{3}$
D	$y = x^3$	$x = \sqrt[3]{y} = y^{1/3}$
E	$y = 2^x$	$x = \log_2(y)$

$\xrightarrow{\cdot 3}$ $\quad$ $\xrightarrow{-4}$

$\xleftarrow{\div 3}$ $\quad$ $\xleftarrow{+4}$

This table shows how to convert between Fahrenheit and Celsius.

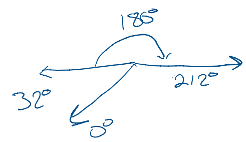
^oC	0	10	**20**	30	40	50	60	70	80	90	100
^oF	32	50	**68**	86	104	122	140	158	176	194	212

There is a function f that converts the temperature x in Celsius to the temperature y in Fahrenheit: $y = f(x)$

Example $f(20) = 68$ means $\mathbf{20^oC}$ is $\mathbf{68^oF}$

The inverse function $x = f^{-1}(y)$ converts Fahrenheit back into Celsius

20°C means 20% of the way from freezing to boiling.



$32^\circ + \underline{20\% \cdot 180^\circ}$

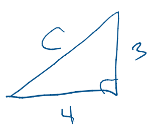
$20 \cdot \frac{180}{100} = 20 \cdot 1.8$

$\boxed{32 + 20 \cdot 1.8}$

$f(x) = 32 + x \cdot 1.8$

$f^{-1}(y) = \frac{y-32}{1.8}$

11. What is the length of the hypotenuse of a right triangle when the other two sides have length 3 and 4?

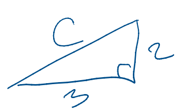


$c^2 = 3^2 + 4^2$, and $3^2 + 4^2 = 9 + 16 = 25$

So $c = 25$? No!

$c^2 = 25$. $c = 5$ ☺

12. Now lengths are 2 and 3. What's the hypotenuse?



$c^2 = 2^2 + 3^2 = 4 + 9 = 13$

$c^2 = 13$, so

$\boxed{c = \sqrt{13}.}$

13. Lengths $3x$ and $4x$. What's the hypotenuse?

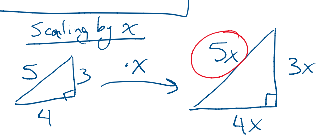
This one is close to # 11.

Using just algebra: $c^2 = a^2 + b^2 = (3x)^2 + (4x)^2 = 3^2 \cdot x^2 + 4^2 \cdot x^2 = (3^2 + 4^2) \cdot x^2$

So $c^2 = (9+16) \cdot x^2 = 25x^2$. $\boxed{c = \sqrt{25x^2} = 5x.}$

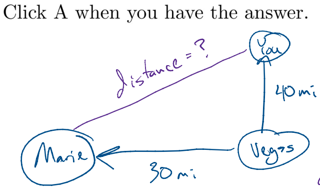
Using geometry: The 3,4,5 right triangle from before is scaled by x!

So scale 5 by x and get $5x$.

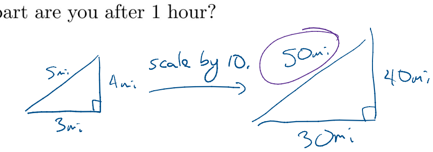


14. You and Marie are in Vegas. You drive north at 40 mph and Marie drives east at 30 mph. How far apart are you after 1 hour?

Click A when you have the answer.



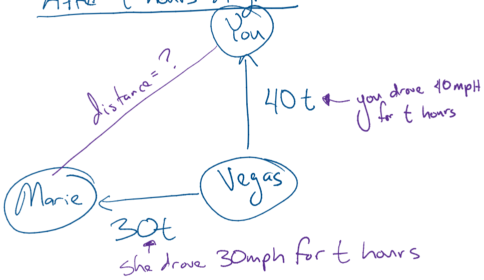
distance = 50mi



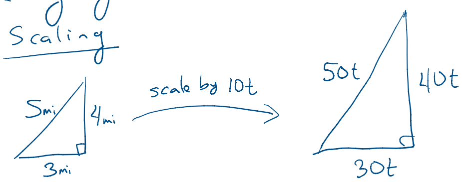
15. How many miles apart are you after t hours?

We're going to apply the same principle of "scaling" again.

After t hours diagram



Scaling



Dear 34A student,
If you're reading this between Thursday's class and you have a question about the problems above that I worked out after class, please say something at the beginning of class Tuesday. Also, if you think you would benefit from algebra practice, use the Pythagorean theorem to compute the distances above instead of the scaling idea. See you Tuesday.