Homework 1

Definition. Let Σ be a subset of (X, d). We say Σ is **bounded** if there exists $x_0 \in X$ and $0 < r < \infty$ such that $\Sigma \subset B_r(x_0)$.

1. Prove that Σ is bounded if and only if there exists $L > 0$ such that

$$d(x, x') \leq L$$

for any $x, x' \in \Sigma$.

2. Suppose Σ is bounded and $A \subset \Sigma$.

(a) Prove that A is bounded.

Definition. Define $\operatorname{diam}(\Sigma) = \sup\limits_{\delta, \delta' \in \Sigma} d(\delta, \delta')$.

(b) Prove that $\operatorname{diam}(A) \leq \operatorname{diam}(\Sigma)$.

3. Let (X, d) be a metric space, and let

$$d_p\big((x, y), (x', y')\big) = d(x, x') + d(y, y').$$

(a) Show that d_p is a metric on X^2.

(b) Prove that $d_p : X^2 \times X^2 \to (\mathbb{R}, \mathrm{MKM})$ is continuous.

4. Give examples to show that if $B_r(x) = B_s(y)$, it need not be true that $r = s$ or $x = y$.