

Name: *Solutions*

Perm:

Math 34A Midterm 1, Summer 2022

1. (2pts) Solve the system of equations below for x and y . Your answers should be in terms of a and b .

$$\begin{array}{l} 2x + 3y = a \\ x + y = b \end{array}$$

(1)
(2)

$x + y = b$ Start with eqn ②

$x = b - y$ Solve for x

$2x + 3y = a$ Plug into eqn ①

$2(b - y) + 3y = a$ Solve for y

$2b - 2y + 3y = a$

$2b + y = a$

$y = a - 2b$

$x + y = b$ Plug into eqn ①
 $x + a - 2b = b$
 $x = 3b - a$ Solve for x

$x = \boxed{3b - a}$

$y = \boxed{a - 2b}$

2. (2pts) Multiply out and simplify. Your answer should have no negative exponents in it.

$$\left(\frac{a^{-1}b}{xy}\right)^{-2} \cdot \frac{a^{-1}b}{\sqrt[4]{b^{-4}x^4y^{-8}}}$$

One way

$$\begin{aligned} & \left(\frac{a^{-1}b}{xy}\right)^{-2} \cdot \frac{a^{-1}b}{\sqrt[4]{b^{-4}x^4y^{-8}}} \\ &= \frac{(xy)^2}{(a^{-1}b)^2} \cdot \frac{a^{-1}b}{\sqrt[4]{b^{-4}x^4y^{-8}}} \\ &= \frac{ax^2y^2}{b(b^{-4}x^4y^{-8})^{1/4}} \\ &= \frac{ax^2y^2}{b \cancel{b^{-4}} \cancel{x^4} \cancel{y^{-8}}} \\ &= \boxed{axy^4} \end{aligned}$$

Another way

$$\begin{aligned} & (a^{-1}b x^{-1} y^{-1})^{-2} (a^{-1}b) (b^{-4} x^4 y^{-8})^{1/4} \\ &= a^2 b^2 x^2 y^2 a^{-1} b b x^{-1} y^2 \\ &= a^{2-1} \cdot b^{-2+1+1} \cdot x^{2-1} \cdot y^{2+2} \\ &= \boxed{axy^4} \end{aligned}$$

$\boxed{axy^4}$

3. (2pts) Substitute $x = a + b$ into the expression below and simplify completely. There should be no parentheses in your answer.

$$x(a^4 - a^3b + a^2b^2 - ab^3 + b^4)(x - 2b)$$

$$\underline{(a+b)}(a^4 - a^3b + a^2b^2 - ab^3 + b^4)\underline{(a+b-2b)}$$

multiply the smaller ones first

$$(a+b)(a+b-2b) \text{ combine like terms}$$

$$= (a+b)(a-b)$$

$$= (a^2 - b^2) \text{ difference of squares formula}$$

multiply by the big one

a^4	$-a^3b$	a^2b^2	$-ab^3$	b^4
a^6	$-a^5b$	a^4b^2	$-a^3b^3$	a^2b^4
$-b^2$	a^4b^2	a^3b^3	$-a^2b^4$	ab^5

$$= a^6 - a^5b + ab^5 - b^6$$

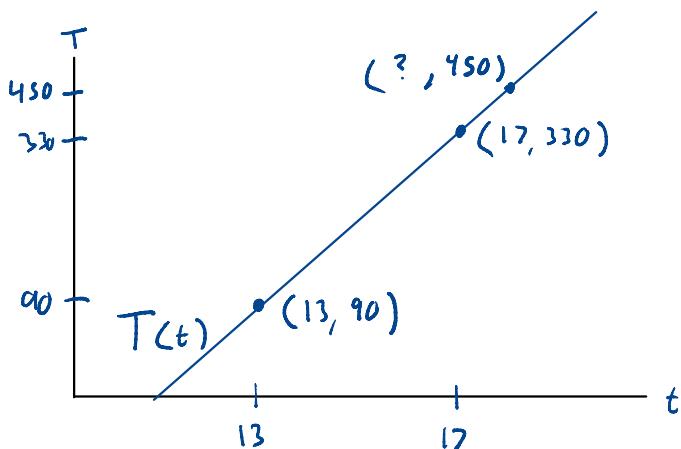
$$a^6 - a^5b + ab^5 - b^6$$

4. (4pts) Your classmate Eve has been studying for the 34A midterm so hard that they forgot to eat dinner. You want to make them a pizza as quickly as possible, and you set the oven to preheat to 450°. You notice that at exactly 8:13PM, the oven's temperature is 90°. You check back at exactly 8:17PM and the oven's temperature is now 330°. Using linear extrapolation, at what time do you estimate will the oven will be preheated to 450°?

Let t = minutes since 8:00 pm, let T = temperature.

Given: $T(13) = 90$, $T(17) = 330$

We're using linear extrapolation, so assume that the oven will heat heating w/ at the same rate:



Let's find an equation for $T(t)$:

$$\text{Slope} = \frac{\text{rise}}{\text{run}} = \frac{\Delta T}{\Delta t} = \frac{330 - 90}{17 - 13} = \frac{240}{4} = 60$$

Point-Slope form: $y - y_1 = m(x - x_1)$, or in our case, $T - T_1 = m(t - t_1)$

$$T - 90 = 60(t - 13)$$

Plug in $T = 450$ and solve:

$$450 - 90 = 60(t - 13)$$

$$360 = 60(t - 13)$$

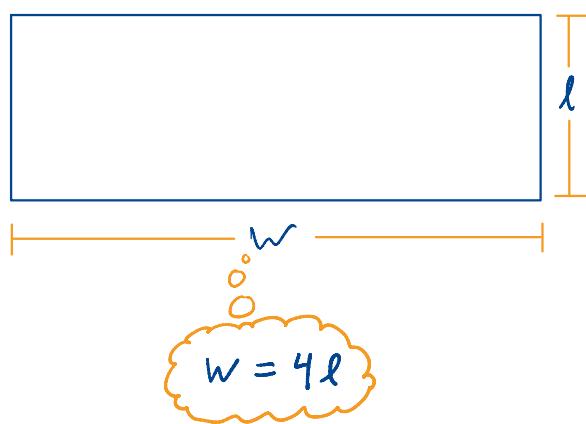
$$\frac{360}{60} = t - 13$$

$$6 = t - 13$$

$$t = 19$$

8:19 PM

5. (4pts) A city planner wants to build a park with a playground surrounded by a field, and to keep the kids safe she wants to build a fence around it. The field is to be four times as wide as it is long. Fencing purchases are \$350 for shipping plus \$33 per foot of fencing. Express the cost of fencing for the perimeter in terms of the length of the field. Simplify your answer.



Total cost = Shipping + $\frac{\text{Price}}{\text{foot}} \times \text{Perimeter}$

$$\begin{aligned} P &= 2l + 2w \\ &= 2l + 2(4l) \\ &= 2l + 8l \\ &= 10l \end{aligned}$$

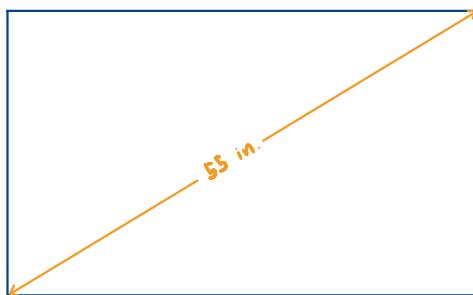
$$C = 350 + 33l (10)$$

$$C = 350 + 330l$$

Fencing Cost = \$

$350 + 330l$

6. (5 points) You are considering the purchase of a 55in TV (TV sizes are measured by the diagonal, not the length or width). You know that the aspect ratio of a screen is the ratio of the width to the height. However, the manufacturer will only disclose the height of the TV, not the width. Express the **aspect ratio** in terms of the height h of the TV.



Aspect ratio = $\frac{\text{width}}{\text{height}}$
Need w in terms of h

Pythagorean Theorem:

$$w^2 + h^2 = 55^2 \quad \text{solve for } w$$

$$w^2 = 55^2 - h^2$$

$$w = \sqrt{55^2 - h^2}$$

I think $\sqrt{55^2 - h^2}$ is simpler than $\sqrt{3025 - h^2}$, but they're both correct.

Plug in to aspect ratio:

$$\text{Aspect Ratio} = \frac{\sqrt{55^2 - h^2}}{h}$$

Aspect Ratio =

$$\frac{\sqrt{55^2 - h^2}}{h}$$

7. (3 points) What are the following limits?

$$(a) \lim_{x \rightarrow 6} 10x - 5$$

$$= 10(6) - 5$$

$$= 60 - 5$$

$$= 55$$

This looks like
mx + b, nothing weird
happens so we can just
Plug in.

$$55$$

$$(b) \lim_{x \rightarrow \infty} \frac{14x+4}{16x+3}$$

$$= \lim_{x \rightarrow \infty} \frac{14x}{16x}$$

$$= \frac{14}{16}$$

$$= \frac{7}{8}$$

x	f(x)
10	$\frac{144}{163}$
100	$\frac{1404}{1603}$
10000	$\frac{140004}{160003}$
1 googol	$\frac{14\text{googol}+4}{16\text{googol}+3}$

$$\frac{7}{8}$$

$$(c) \lim_{x \rightarrow \infty} \frac{10x^2+x}{-7x}$$

$$= \lim_{x \rightarrow \infty} \frac{x(10x+1)}{x(-7)}$$

$$= \lim_{x \rightarrow \infty} -\frac{10}{7}x - \frac{1}{7}$$

$$= -\infty$$

$\frac{\infty}{\infty}$... hmm let's
try to simplify

$$-\infty$$

8. (3 points) Compute the logarithms below.

$$(a) \log_2(8)$$

$$2^3 = 8$$

$$3$$

$$(b) \log_{10}(.01)$$

$$10^{-2} = .01$$

$$-2$$

$$(c) \log_5(125)$$

$$5^3 = 125$$

$$3$$