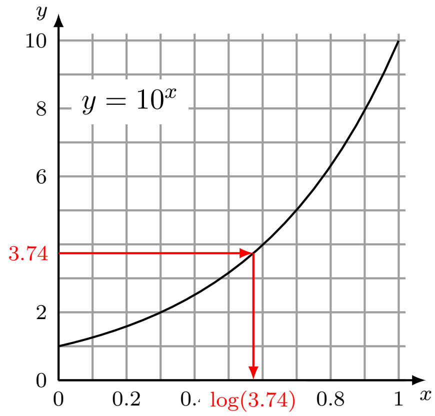
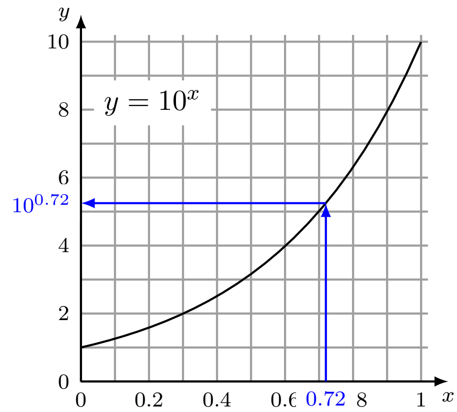
Math 34A Winter 2020
Old Midterm 2 #4 Solutions

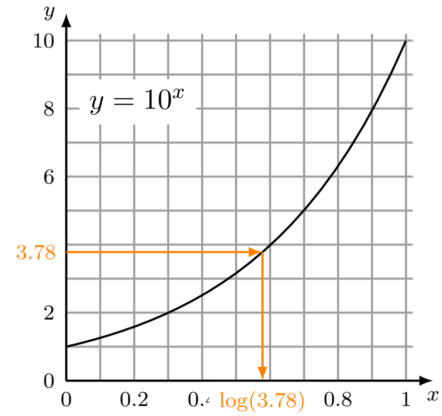
1. Here are the three graphs we'll use in solving these problems:



Part (a)



Part (b)



Part (c)

(a) Remember that we can use the move the decimal point trick:

$$\log(374) = \log(10^2 \times 3.74) = \log(10^2) + \log(3.74) = 2 + \log(3.74).$$

Now we can use the graph to find that $\log(3.74) \approx 0.57$, and so $\log(374) \approx \boxed{2.57}$. (Mathematica tells me that $\log(374) \approx 2.572\,871\,602\ldots$)

(b) The reverse version of the "move the decimal point trick" is what we need here:

$$10^{3.72} = 10^{3+0.72} = 10^3 \times 10^{0.72}.$$

We know that $10^3 = 1,000$, and we use the graph to find that $10^{0.72} \approx 5.25$. Thus $10^{3.72} \approx 1,000 \times 5.25 = \boxed{5{,}250}$. (Mathematica tells me that $10^{3.72} \approx 5248.074\,602\ldots$, so we're within 2 out of more than $5,200$.)

(c) First we use the rules of logarithms to write

$$\log(100 \times 378) = \log(100) + \log(378) = 2 + \log(378).$$

Now we again use the move the decimal point trick:

$$\log(378) = \log(10^2 \times 3.78) = \log(10^2) + \log(3.78) = 2 + \log(3.78).$$

Now we can use the graph to find that $\log(3.78) \approx 0.58$, and so $\log(378) \approx 2.58$. Thus

$$\log(100 \times 378) = \log(100) + \log(378) \approx 2 + 2.58 = \boxed{4.58}.$$

(Mathematica tells me that $\log(100 \times 378) \approx 4.577\,491\,799\,837\ldots$)

2. Let's start with this equation slightly simplified as

$$3 \times 6^{5x} = 12.$$

Now take the logarithm of both sides to get

$$\log(3 \times 6^{5x}) = \log(12).$$

We simplify this using more rules of logs:

$$\begin{aligned} \log(3) + \log(6^{5x}) &= \log(12) && \text{since } \log(xy) = \log(x) + \log(y) \\ \log(3) + 5x\log(6) &= \log(12) && \text{since } \log(a^p) = p\log(a). \end{aligned}$$

Now subtract $\log(3)$ from both sides, then divide by $5\log(6)$ to get

$$5x\log(6) = \log(12) - \log(3) \qquad \text{and then} \qquad x = \boxed{\frac{\log(12) - \log(3)}{5\log(6)}}.$$

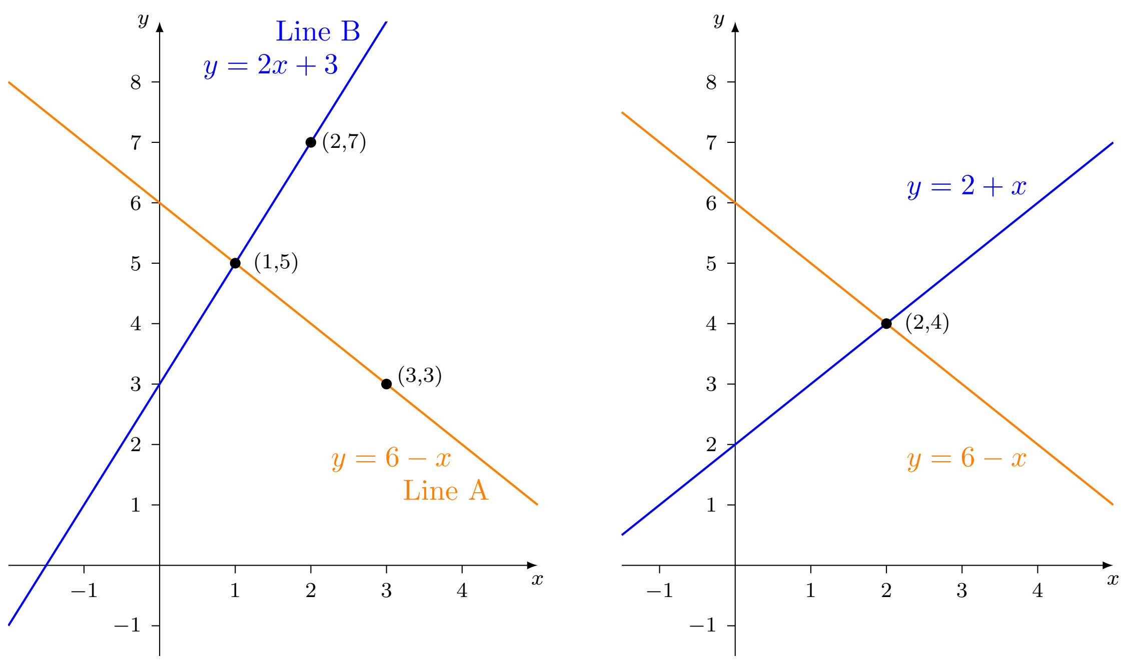
Since $\log(a) - \log(b) = \log(a/b)$, we can simplify the numerator to $\log(12) - \log(3) = \log(12/3) = \log(4)$. Thus we can write this as $x = \boxed{\dfrac{\log(4)}{5\log(6)}}$.

3. (a) The slope of Line A is

$$m = \frac{3-5}{3-1} = \frac{-2}{2} = -1.$$

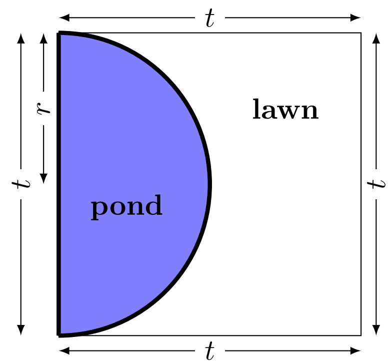
Thus Line A has equation $y = -x + b$ for some b. We find that value by plugging in either point $(x, y) = (1, 5)$ or $(3, 3)$. If we plug in the first point, we get $5 = -1 + b$, or $b = 6$. Thus Line A has equation $\boxed{y = -x + 6}$. Line A is shown on the left, below.

(b) Line B has slope 2, so it has equation $y = 2x + b$. We plug in $(x, y) = (2, 7)$ to find b: $7 = 2(2) + b$, or $b = 3$. Thus the equation of Line B is $\boxed{y = 2x + 3}$. Line B is shown with Line A on the left, below.



(c) The point of intersection of the lines $y = 2 + x$ and $y = 6 - x$ is where $2 + x = 6 - x$. Adding x and subtracting 2 from both sides, we get $2x = 4$. Dividing by 2 let's us see that $x = 2$. Plugging $x = 2$ into either line gives us $y = 4$. Thus the point of intersection is $(x, y) = \boxed{(2, 4)}$. (The lines and the point of intersection are shown above on the right.)

4. We reproduce the picture of the square garden here:



The length of each side is t, so the radius of the pond is $r = t/2$.

(a) The area of the pond is half the area of a circle of radius $r = t/2$, so $A = (1/2)\pi(t/2)^2 = \boxed{\pi t^2/8}$. (Here we've used the fact that the area of a circle is πr^2.)

(b) The perimeter of the pond is half the circumference of a circle of radius $r = t/2$ PLUS the straight side on the left (which has length t). Thus the perimeter of the pond is $P = (1/2)(2\pi(t/2)) + t = \boxed{\pi t/2 + t}$. (Here we've used the fact that the perimeter of a circle is $2\pi r$.)

(c) If the area of the square is 400, then since the area of the square is t^2, we get the length of each side is $t = 20$ (we just took the square root of $t^2 = 400$). Then from part (a), the area of the pond is $\pi(20)^2/8 = 50\pi$. The lawn is the part of the square *not* in the pond, so it has area $\boxed{400 - 50\pi}$.

5. (a) We're told that 90% of the 5 ounces of paint from Can A is red, and that 20% of the 5 ounces of paint from Can B is red. Thus the amount of red paint in the $5 + 5 = 10$ total ounces of the result is

$$90\% \times 5 + 20\% \times 5 = (0.90)(5) + (0.20)(5) = \boxed{5.5 \text{ ounces}}.$$

(b) This is very similar to part (a). We're told that 90% of the x ounces of paint from Can A is red, and that 20% of the $10 - x$ ounces of paint from Can B is red. Thus the amount of red paint in the $x + (10 - x) = 10$ total ounces of the result is

$$(0.90)(x) + (0.20)(10 - x) = 0.9x + 2 - 0.2x = \boxed{0.7x + 2 \text{ ounces}}.$$

Does this answer make sense? We can check three values of x pretty easily:

- When $x = 5$, this is simply part (a). Our answer was $0.7(5) + 2 = 3.5 + 2 = 5.5$ ounces, so this agrees.
- When $x = 10$, all the paint is from Can A, so we should get paint that is 90% red. Sure enough, plugging in $x = 10$ gives us a value of $(0.7)(10) + 2 = 9\ textounces$, so 9 of the 10 ounces in the result (or 90%) is red.
- Similarly, when $x = 0$, all the paint is from Can B, so we should get paint that is 20% red. We do, which I'll leave you to check.

(c) If the resulting 10 ounces is 70% red, then this means there is $70\% \times 10 = (0.7)(10) = 7$ ounces of red paint in the result. So we have to find the value of x in part (b) (remember, x is the amount of paint from Can A) so that

$$0.7x + 2 = 7 \text{ ounces}.$$

Solving, we get $\boxed{x = 50/7 \text{ ounces}}$ from Can A.

This answer is reasonable. When the split is 5 ounces from each can, we end up with 5.5 ounces of red paint in the result. To get 7 ounces, we'd need to increase the amount of paint from Can A (the can with a higher percent of red paint).