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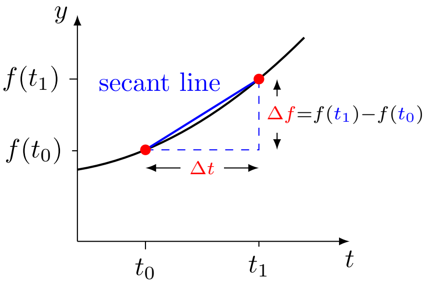
Graphical Approach



Δf = change in f
Δt = change in t

Many ways to say same thing:

$$\begin{pmatrix} \text{average rate of} \\ \text{change of } f \end{pmatrix} = \frac{\text{change in } f}{\text{change in } t}$$

$$= \frac{\Delta f}{\Delta t}$$

$$= \text{slope of secant line} = \frac{f(t_1) - f(t_0)}{t_1 - t_0}$$

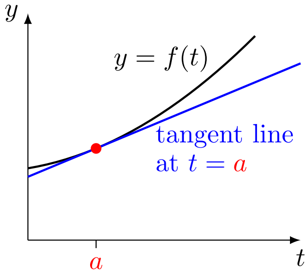
The derivative is defined to be

$$\lim_{\Delta t \to 0} \left(\frac{\Delta f}{\Delta t} \right) = \frac{df}{dt}$$

Idea: As t_1 moves closer to t_0 the secant line approaches the tangent line at t_0. This is the line with the same slope as the graph at t_0.

Understanding Derivatives

There are many ways to think about derivatives. You need to understand these to apply to problems.



slope of graph at a

= slope of tangent line

= instantaneous rate of change of f at a

$$= \left(\begin{array}{c} \text{limit of average rate of change} \\ \text{of } f \text{ over shorter and shorter} \\ \text{time intervals starting at } a \end{array} \right)$$

= limit of slopes of secant lines

$$= f'(a) \;=\; \left.\frac{df}{dt}\right|_{t=a}$$

Summary of Derivatives

One quantity, y, depends on another quantity x.
In other words y is a function of x so $y = f(x)$. Example: $y = 7x$

If you change x, then y changes.
Question: How quickly does y change as x changes?

Answer: The derivative tells you.

In our example, the derivative is 7. This tells you:

the output $= y$ of the function changes
7 times as fast
as the input $= x$ to the function.

If x is changed by 0.1 how much does y change by?

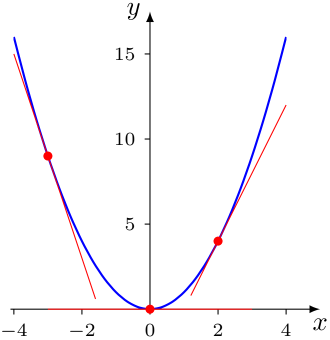
(A) 7 (B) 7.1 (C) 0.7 (D) 0.1/7 (E) other $\boxed{\text{C}}$

Graphical Meaning

$$\frac{d}{dx}\left(x^2\right) = 2x$$

What this means

The slope of the graph of $y = x^2$ at $x = a$ is $2a$

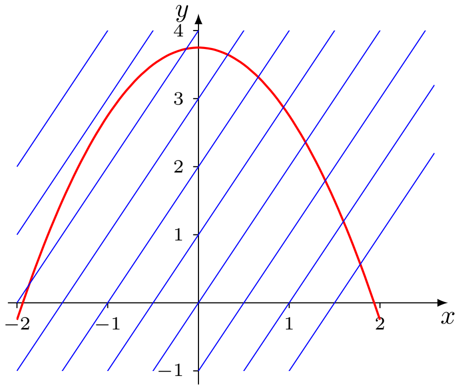


at $x = -3$, slope is $2(-3) = -6$
at $x = 0$, slope is $2(0) = 0$
at $x = 2$, slope is $2(2) = 4$

derivative = rate of change = slope of graph = slope of tangent line

Slope Question

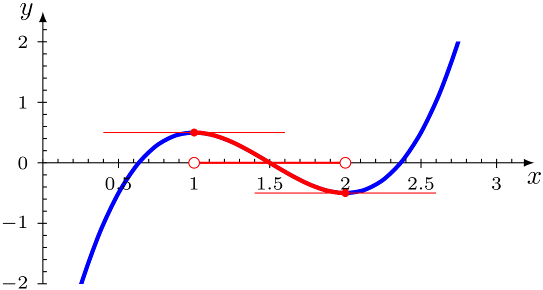
This graph shows $y = f(x)$ and lines parallel to $y = 2x$



Question: For which values of x is $f'(x) > 2$? Answer: D

(A) $x < 1.2$ (B) $x < 0$ (C) $x < -1.5$ (D) $x < -1$ (E) $x < -0.5$

More Slope Questions



(1) For which values of x is $f'(x) = 0$? Answer: D

(A) none (B) $\{0.63,\ 1.5, 2.38\}$ (C) 1 (D) $\{1, 2\}$ (E) 2

(2) For which values of x is $f'(x) < 0$? Answer: C

(A) $x < 0.63$ (B) $x < 1$ (C) $1 < x < 2$ (D) $1.5 < x < 2.38$ (E) none

Interpretation of Derivatives I

Suppose $f(x)$ = the percentage of children who still get measles when $x\%$ of children are inoculated.

Question: Which of the following is a plausible value for $f'(40)$?

(A) 0 (B) 2 (C) 50 (D) -2 (E) -50 D

Question: If $f(40) = 20$ and $f'(40) = -2$, which must be true?

(A) when 20% of children are inoculated the percentage who gets measles decreases by 2%

(B) when 20% of children are inoculated then inoculating an extra 1% of children would reduce the number of measles cases by another 2%

(C) If the inoculation rate is 41% then 18% of children gets measles

(D) If the inoculation rate is 20% then 2% fewer cases of measles arise if an extra 1% of children can be inoculated

(E) none of the above

Answer: (C)

Interpretation of Derivatives II

Air temperature gets colder the higher you go.
$T(x)$ = air temperature in $^\circ C$ at a height x meters above sea level.
Question: Which of these is a plausible value for $T'(2000)$?

(A) -1 (B) 1 (C) 0 (D) $1/200$ (E) $-1/200$ $\boxed{\text{E}}$

Question: If $T(2000) = 10$ and $T'(2000) = -1/200$, which is most plausible?

(A) the temperature at sea level is $16^o C$

(B) the temperature 2400 meters above sea level is $8^o C$

(C) the temperature 10 meters above sea level is $2000^o C$

(D) 2000 meters above sea level the temperature is decreasing at a rate of $1/200^o C$ per minute.

(E) none of these are plausible

Answer: $\boxed{\text{B}}$

Interpretation of Derivatives III

x = money spent (in thousands of \$) in one month on advertising.

$f(x)$ = sales (in thousands of \$) in a month when x is spent on advertising.

Question: If $f(20) = 60$ and $f'(20) = 3$ which must be true?

(A) When sales are $\$20,000$ in one month the amount spent on advertising is increasing at a rate of $\$3,000$ per month

(B) When the company spends $\$20,000$ per month on advertising the sales rise at a rate of $\$3,000$ per month

(C) When the company spends $\$20,000$ per month on advertising each extra dollar a month spent on advertising generates an extra $\$3$ of sales.

(D) When the company spends $\$3,000$ per month on advertising the sales are increasing at a rate of $\$20,000$ per month

(E) None of the above

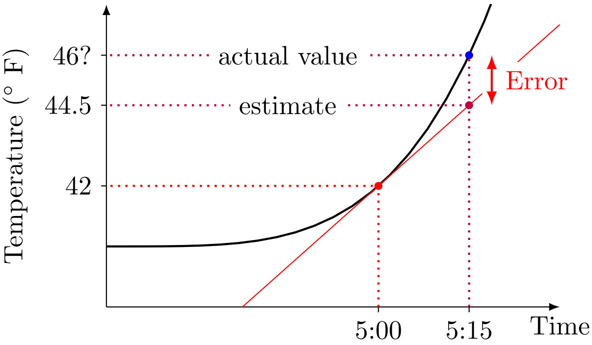
Answer: $\boxed{\text{C}}$

§8.6: Tangent Line Approximation

Question: At 5am the temperature is 42° F and increasing at a rate of 10° F per hour. Which of the following do you think is closest to the temperature at 5:15am?

(A) 2.5° F (B) 52° F (C) 43.5° F (D) 44.5° F (E) 5.15° F

Answer: D



Continuing this example

Same set-up:
- $f(x)$ = temperature at time x hours after midnight
- $f(5) = 42$ (42° F at 5:00am)
- $f'(5) = 2$

(1) Find the equation of tangent line to $y = f(x)$ at $x = 5$.

(A) $y = 5x + 42$ (B) $y = 2x + 5$ (C) $y = 2(x-5) + 42$
(D) $y - 5 = 2(x - 42)$ (E) $y - 42 = 2x - 5$ C

(2) Use this to predict the approximate temperature at 4am.

(A) 40 (B) 41 (C) 42 (D) 43 (E) 44 A

(3) The tangent line approximation is used to estimate the temperature at the following times. Which do you think is most accurate?

(A) 4am (B) 4:50am (C) 5:25am (D) 6am (E) midnight B