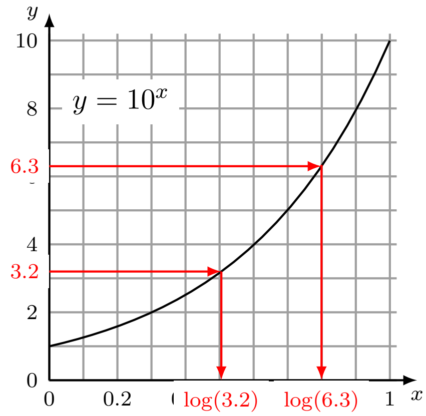
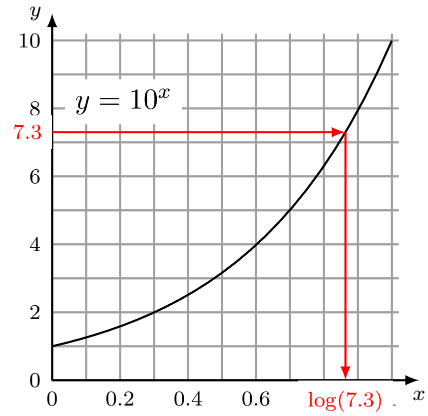
Math 34A Winter 2020
Old Midterm 3 #4 Solutions

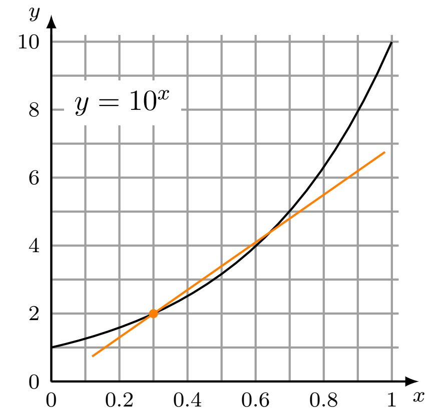
1. Here are the three graphs we'll use in solving these problems:



Part (a)



Part (b)



Part (c)

(a) Since $\log(6.3) \approx 0.80$ and $\log(3.2) \approx 0.51$ we get that $\log(6.3 \times 3.2) = \log(6.3) + \log(3.2) \approx 0.80 + 0.51 = \boxed{1.31}$.

(Mathematica tells me that $\log(6.3 \times 3.2) \approx 1.304\,490\,527\,773\ldots$, so we're close.)

(b) The solution to $10^x = 10/73$ is $x = \log(10/73) = \log(10) - \log(73)$. We use the "move the decimal point" trick to see that is what we need here:

$$\log(10/73) = \log(10) - \log(10 \times 7.3) = \log(10) - \Big(\log(10) + \log(7.3)\Big) = 1 - 1 - \log(7.3) = -\log(7.3).$$

We use the graph to find that $\log(7.3) \approx 0.86$, so $x = \log(10/73) \approx -\log(7.3) = \boxed{-0.86}$. (Mathematica tells me that $x = \log(10/73) \approx -0.863\,322\,860\ldots$, so we're pretty close.)

(c) We've drawn a line with slope 7 through the point at $x = 0.3$ on the third graph, above. This line is the secant through $x = 0.3$ and $x = c$ if $\boxed{c \approx 0.64}$.

2. We write down the answers without much commentary:

(a) $\dfrac{d}{dx}(5x^4 - 4x + 2) = 20x^3 - 4$

(b) $\dfrac{d^2}{dx^2}(2e^{5x} - 3x^2) = \dfrac{d}{dx}(10e^{5x} - 6x) = 50e^{5x} - 6$

(c) $\dfrac{d}{dx}(x^e + e^x + e^k) = ex^{e-1} + e^x + 0$. Notice that e^k is a constant (it doesn't change with x), so it has derivative zero.

3. The tangent line is the line through $(t, f(t)) = (7, f(7)) = (7, 100)$ with slope $f'(7) = -3$. Thus the line has equation

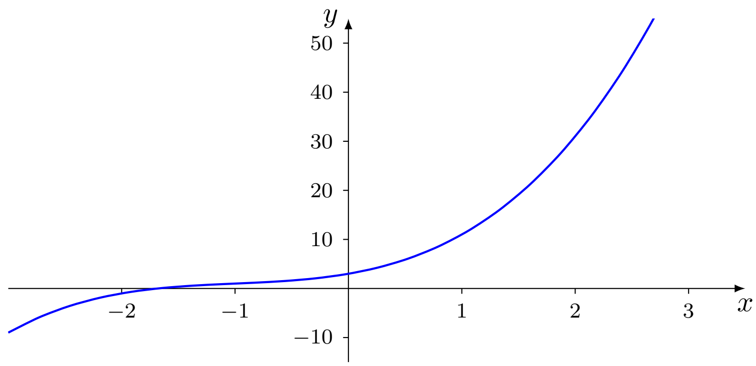
$$y - 100 = -3(t - 7) \qquad \text{or, equivalently,} \qquad y = -3t + 121.$$

The tangent line approximation is then $f(t) \approx -3t + 121$ for t near 7.

(a) The year 2020 is $t = 10$, and so the tangent line approximation estimates the depth of lake to be $f(10) \approx -3(10) + 121 = \boxed{\text{91 meters}}$.

(b) What is t when $f(t) \approx 70$? We solve $-3t + 121 = 70$, which gives $t = 17$. This is $\boxed{\text{the year 2027}}$.

4. Here is a picture of the graph of $y = x^3 + 3x^2 + 4x + 3$:



(a) The slope of the graph is the derivative, $f'(x)$. Since $f'(x) = 3x^2 + 6x + 4$, the slope of the graph at $x = -2$ is $3(-2)^2 + 6(-2) + 4 = \boxed{4}$.

(b) The tangent line at $x = -2$ has slope 4 (from part (a)) and passes through the point $(x, y) = (-2, f(-2)) = (-2, -1)$. Thus the equation of the tangent line is

$$y - (-1) = 4(x - (-2)) \qquad \text{or, equivalently} \qquad \boxed{y = 4x + 7}.$$

(c) The curve is concave up when $f''(x)$ is positive. Since $f''(x) = 6x + 6$, this is the case when $\boxed{x > -1}$ (or, equivalently, $\boxed{-1 < x < \infty}$).

(d) The slope is $f'(x) = 3x^2 + 6x + 4$, which is 4 when $3x^2 + 6x + 4 = 4$. This simplifies to $3x^2 + 6x = 0$ or, after factoring, $3x(x + 2) = 0$. Thus the solutions to this are $\boxed{x = 0 \text{ and } -2}$.

5. (a) The velocity of the rocket is $h'(t) = 20 - 10t$ m/s.

(b) The acceleration of the rocket is $h''(t) = -10$ m/s^2.

(c) The initial speed of the rocket is $h'(0)$. From part (a), this is 20 m/s.

(d) The velocity of the rocket is 15 m/s when $h'(t) = 15$. From part (a), this means when $20 - 10t = 15$. Solving, we get $t = 5/10 = \boxed{1/2 \text{ seconds}}$.

(e) The average speed of the rocket in the first two seconds is

$$\begin{aligned} \text{ave speed} &= \frac{\text{distance traveled in those 2 seconds}}{\text{2 seconds}} \\ &= \frac{h(2) - h(0)}{2} = \frac{420 - 400}{2} = \frac{\text{20 meters}}{\text{2 seconds}} \\ &= 10 \text{ m/s}. \end{aligned}$$