May 24 Optimization

Tuesday, May 24, 2022   3:07 PM

$$f(x) = 3e^{-2x} - 5e^{7x} + 15e^{x}$$

$$f'(x) = -6e^{-2x} - 35e^{7x} + 15e^{x}$$

$$f''(x) = 12e^{-2x} - 245e^{7x} + 15e^{x}$$

Candy bars   start at 0¢ for price 5¢ at a time
                    price = 5·x

$$\underline{\# \text{ sold} = 300 - 20x}$$

$$(300-20x)5x = 1500x - 100x^2$$

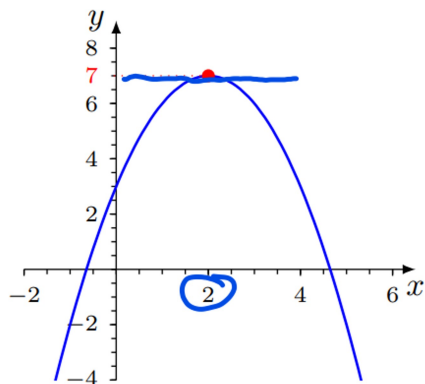$$\downarrow$$

$$1500 - 200x = 0$$

$$\boxed{x = 8}$$

$$200x = 1500$$

$$\boxed{x = \frac{1500}{200} = \frac{15}{2} = 7.5}$$

§8.13: Max/Min problems

Often want to find the biggest, smallest, most, least, maximum, minimum of something.

$-4 + 8 + 3$

Here's the graph of $y = f(x) = \boxed{-x^2 + 4x + 3}$

The __maximum value__ or just maximum of the function is 7.

$$f'(x) = -2x + 4 = 0$$

$$x = 2$$

1. Use this method to find the maximum of $f(x) = -x^2 + 8x + 5$.
The maximum value is. . .

   $A = 4$ $B = 5$ $C = -2x + 8$ $D = 21$ $E = 15$ $\boxed{D}$

16    $-x^2 - 4x + 12$ $f'(x) = -2x + 4$

2. Find the value of x which makes $f(x) = (2 - x)(x + 6)$ a maximum.

   $x = -2$

   $A = 16$ $B = 1$ $C = -1$ $D = 2$ $E = -2$

More Examples

$x^2 + 6x + 8 + 3 = x^2 + 6x + 11$

$\downarrow$

3. What is the minimum of $f(x) = (x + 2)(x + 4) + 3$?

   $(-1)(1)$ $f'(x) = 2x + 6 = 0$
   when
   $x = -3$

   $A = 0$ $B = 1$ $C = 2$ $D = 3$ $E = 4$

More Examples

3. What is the minimum of $f(x) = (x + 2)(x + 4) + 3$?

   $A = 0$ $B = 1$ $C = 2$ $D = 3$ $E = 4$

Answer: $\boxed{C}$

4. What is minimum of $f(x) = x^2 + 16x^{-2}$? $f'(x) = 2x - 32x^{-3} = 0$

   $A = 2$ $B = 4$ $C = 6$ $D = 8$ $E = 16$ $\dfrac{2x^4 - 32}{x^3} = \dfrac{0}{x^3}$

   $f(2) = 4 + \dfrac{16}{4} = 8$ $2x^4 - 32 = 0$
   $f(-2) = 4 + \dfrac{16}{4} = 8$ $x^4 = 16$
                                             $x = \pm 2$

More Examples

$\dfrac{2}{e} \cdot e^{2x}$

3. What is the minimum of $f(x) = (x+2)(x+4) + 3$?

$$A = 0 \quad B = 1 \quad C = 2 \quad D = 3 \quad E = 4$$

Answer: $\boxed{C}$

4. What is minimum of $f(x) = x^2 + 16x^{-2}$?

$$A = 2 \quad B = 4 \quad C = 6 \quad D = 8 \quad E = 16$$

Answer: $\boxed{D}$

$3x + 2 = 0$
$-e$
$3x = 2$
$e^{3x} = \ln(2)$
$3x = \ln(2)$

5. Find the value of x which makes $f(x) = -e^x - e^{-2x}$ a maximum.

$$A = 0 \quad B = \ln(2) \quad C = -\ln(2) \quad D = \ln(2)/3 \quad E = \ln(2)/3$$

$f'(x) = -e^x + 2e^{-2x} = 0$

$x = \dfrac{\ln(2)}{3}$

Word Problem #1

A ball is thrown into the air. After t seconds the height in meters above the ground of the ball is $h(t) = 40t - 10t^2$. How many meters high did the ball go?

$80 - 40$

$$A = 2 \quad B = 40 - 20t \quad C = 20 \quad D = 40$$

$h'(t) = 40 - 20t = 0$

$40 = 20t$

$2 = t$

Word Problem #2 $(\$225) \cdot 900$

If an airline sells tickets at a price of $\$200 + 5x$ each the number of tickets it sells is $1000 - 20x$. What price should the tickets be if the airline wants to get the most money?

$$A = 5 \quad B = 25 \quad C = 175 \quad D = 200 \quad E = 225$$

$(1000 - 20x)(200 + 5x) = 20 \cdot 5(50 - x)(40 + x)$

$= 100(50 - x)(40 + x)$

$(50 - x)(40 + x) = 2000 + 10x - x^2$

$100(2{,}000 + 10x - x^2)$

$\downarrow \frac{d}{dx}$

$f'(x) = 100(10 - 2x) = 0$

$10 = 2x$

$x = 5$

Word Problem #3

$$100 = \ell \cdot w \qquad w = \frac{100}{\ell}$$

A fenced garden with an area of 100 m^2 will be made in the shape of a rectangle. It will be surrounded on all four sides by a fence. What length and width should be used so the least amount of fence is needed?

$$P = 2\ell + 2w = 2\ell + 2\left(\frac{100}{\ell}\right) = 2\ell + 200\ell^{-1}$$

Approach:

(1) Express the total length of fence in terms of <u>only</u> one variable, either $L = $ length of field, or $W = $ width of field. This gives a formula for $P = $ (total length of fence) involving, say, W.

(2) Find minimum by solving $\dfrac{dP}{dW} = 0$.

$$P'(\ell) = 2 - \frac{200}{\ell^2} = 0$$

$$2\ell^2 = 200$$

$$\ell^2 = 100$$

$$\ell = 10$$

Students always find (1) the hardest part.
You have been prepared for this by word problems from chapter 3!