of the influence function. A key observation is that relatively crude approximations of the weights may still give realistic estimates for the overall error in the quantity of interest.

The current book is comprised of the notes from a series of lectures on this topic presented by the second author at ETH Zürich in 2002. The emphasis is very much on indicating the flexibility of the DWR approach through discussion of a range of applications and accompanying numerical examples. The basic approach is first illustrated in the context of simple linear elliptic PDEs and even ODEs where, perhaps surprisingly, the approach even offers new perspectives on the classical subject of error control for initial value problems. The ideas are then revisited in the more abstract setting of nonlinear variational problems. The remaining chapters are largely independent and illustrate the approach for PDE eigenvalue problems, optimal control and parameter estimation, parabolic and hyperbolic PDEs, and applications in structural and fluid mechanics.

The later chapters draw heavily on individual Ph.D. and Diploma theses produced in Rannacher's group, which gives some indication of the level of the presentation. In fact, very little in the way of prerequisites beyond basic knowledge of finite element approximation and familiarity with the application area is assumed. Most graduate students in engineering and physical sciences should be able to handle the material without excessive difficulty. The presentation is very much a tutorial approach promoting a hands-on experience, reinforced with practical exercises at the end of each chapter, aimed towards practitioners. The reader will not find much in the way of theoretical support in the book for the methodology proposed here, although there is a short chapter devoted to this, mainly because of the scarcity of mathematical justification in the literature. Much of the material presented in the book can be found in various survey articles by the second author, of which [1] is perhaps the most accessible. While the more seasoned practitioner will probably prefer the presentation in [1, 2], the present book provides a gentler introduction for the beginning graduate student or nonspecialist practitioner.

REFERENCES

[1] R. Becker and R. Rannacher, *An Optimal Control Approach to A Posteriori Error Estimation in Finite Element Methods*, Acta Numer. 10, Cambridge University Press, Cambridge, UK, 2001, pp. 1–102.

[2] M. B. Giles and E. Süli, *Adjoint Methods for PDEs: A Posteriori Error Analysis and Postprocessing by Duality*, Acta Numer. 11, Cambridge University Press, Cambridge, UK, 2002, pp. 145–236.

Mark Ainsworth
Strathclyde University

Mathematics in Population Biology. *By Horst R. Thieme.* Princeton University Press, Princeton, NJ, 2003. $49.50. xx+543 pp., softcover. ISBN 0-691-09291-5.

As the title suggests, the book *Mathematics in Population Biology* stresses the mathematics behind the models of population biology. For each model, the mathematical assumptions are clearly stated and the solution behavior rigorously verified in a theorem/proof format. The models studied in this book are deterministic, formulated primarily as either ordinary differential equations or as first-order partial differential equations. However, there is one chapter devoted to scalar difference equations. The chapters are not organized according to mathematical topic but according to biological model or biological principle. The first models discussed are for simple, single-species populations, but the models progress to more complex structured models. The book is divided into four parts. Part 1 covers single-species population growth models, Part 2 covers stage-structured models with demographics, and Part 3 covers infectious disease models. Part 4 is a toolbox of mathematical techniques and tools useful to Parts 1–3. Part 4 is divided into three appendices: Appendix A, Ordinary Differential Equations; Appendix B, Integration, Integral Equations, and Some Convex Analysis; and Appendix C, MAPLE Worksheets.

Part 1 contains 10 chapters, Chapters 2–11. Chapter 2 introduces population modeling using a general single-species population model, with births, deaths, immigration, and emigration. Solution behavior of the population model is shown to depend on the average intrinsic growth rate. The results in the first two chapters are useful in subsequent chapters when specific models are discussed. The classical population models, including logistic, Ricker, Beverton–Holt, von Bertalanffy, and a model with an Allee effect, are presented in Chapters 4–7. The mathematical formulations are justified based on biological principles. This is a nice feature not included in many other mathematical biology textbooks. For example, the Beverton–Holt functional form is derived from a resource-consumer model and from a model where adults cannibalize juveniles; the Ricker functional form is derived from cannibalism of juveniles by adults but where the juvenile stage is a fixed length. Nonautonomous models of population growth are discussed briefly in Chapter 8. However, nonautonomous models with time-periodic coefficients are discussed in other chapters as well. The discrete analogues of the classical models are discussed in Chapter 9 with a detailed analysis of more general first-order, scalar difference equations. The last two chapters in Part 1 present a formulation and an analysis of models for an aquatic population in a polluted environment and for a juvenile and adult stage-structured model. These two chapters use a variety of mathematical techniques from differential equations for which the mathematical theory is provided in Appendix A, e.g., Poincaré–Bendixson theory, Hopf bifurcation, stability theory, and persistence theory. These last two chapters provide the motivation for further study of stage-structured models in Part 2.

Part 2 contains five chapters, Chapters 12–16. Chapter 12 introduces the sojourn function of a stage, $\mathcal{F}$, and relates it to the duration of a stage. The sojourn function $\mathcal{F}(a)$ is the probability of still being in the stage a time units after having entered it. The sojourn function is then used to formulate age-structured models, where the population density is $u(t,a)$ at time t and age a. Two basic formulations are given, one based on Lotka's integral equation and a second one based on McKendrick's partial differential equation (also known as the McKendrick–von Foerster equation). Age-structured models with a birth rate depending linearly on the population density are studied in greater detail in Chapters 14 and 15. For example, the renewal equation and the basic reproduction ratio $\mathcal{R}_0$ are defined. Asymptotic results about solution behavior are verified for the case $u(t,a) = e^{st}v(a)$, referred to as balanced exponential growth. Continuous age-structured models with nonlinear birth rate are presented and analyzed in Chapter 16. Appendix B provides some mathematical background needed for Part 2 on Stieltjes integrals, Lebesgue–Stieltjes integrals, and Volterra's integral equation.

Part 3 contains eight chapters, Chapters 17–23. Chapter 17 begins with an interesting discussion about the impact infectious diseases have had on the human population. A glossary of important epidemiological terms is also included. The remaining chapters are divided into epidemic models and endemic models. Epidemic models are discussed in Chapters 18–20, where the disease is short-lived and population demography is not included. Endemic models are discussed in Chapters 21–23, where the disease is long-lived and the demography of the population is important. The first epidemic model discussed is the SIR model of Kermack and McKendrick. Then the SIR epidemic model is generalized to include contact rates that depend on the population size and variable infectivity. In Chapter 21, the SEIR endemic model is introduced. A simple case of the SEIR model is analyzed. It is assumed that there are no disease-related deaths and the total population size is constant. The basic replacement ratio $\mathcal{R}_0$ is defined and justified biologically. The notation $\mathcal{R}_0$ is the same notation used for the basic reproduction ratio in Part 2. But the two ratios are distinguished by their names, basic replacement ratio versus basic reproduction ratio, and by their interpretation. In the last two chapters in Part 3, multigroup models and age-structured epidemic models with vaccination dependent on age are studied.

The Perron–Frobenius theory for nonnegative matrices (summarized in Appendix A) is important in Chapter 23, where the basic replacement ratio is defined as the spectral radius of a next-generation matrix.

Most chapters conclude with bibliographic remarks. References are given to related work and to work that generalizes or extends the results in each chapter. In addition, there are exercises at the end of each chapter or at the end of each section within a chapter.

Mathematics in Population Biology provides a rigorous mathematical treatment of a wide variety of single-species population models, stage- and age-structured models, and epidemic models. The attention to mathematical detail is emphasized much more in this book than in many other popular mathematical biology textbooks [1, 2, 3, 4] and will be of particular interest to mathematicians. It is a good textbook for graduate students in mathematics interested in learning about model formulation and about mathematical techniques in population biology. The exercises and toolboxes in Part 4 make it attractive as a textbook. Selected topics from Parts 1–3 could be covered in a one-semester course with more advanced topics covered in a two-semester course. In addition, this book is an excellent reference for researchers interested in learning more about the mathematics behind the models of population biology.

REFERENCES

[1] F. Brauer and C. Castillo-Chávez, *Mathematical Models in Population Biology and Epidemiology*, Springer-Verlag, New York, 2001.

[2] L. Edelstein-Keshet, *Mathematical Models in Biology*, Random House/ Birkhäuser Math. Ser., New York, 1988.

[3] M. Kot, *Elements of Mathematical Ecology*, Cambridge University Press, Cambridge, UK, 2001.

[4] J. D. Murray, *Mathematical Biology*, 2nd ed., Springer-Verlag, Berlin, Heidelberg, New York, 1993.

Linda J. S. Allen
Texas Tech University

Elements of Number Theory. *By John Stillwell.* Springer-Verlag, New York, 2003. $49.95. xii+254 pp., hardcover. ISBN 0-387-95587-9.

Most mathematics departments in North America offer a course in elementary number theory. The content of this course has varied little over the years, consisting of an introduction to divisibility; the distinction between primes and composites; Euclid's algorithm for the greatest common divisor and the use of this to solve linear diophantine equations, congruences, and modular arithmetic; Fermat's little theorem; primitive roots; and quadratic reciprocity. All of this material was introduced in a systematic form by Gauss in his famous 1801 treatise *Disquisitiones Arithmeticae*. Indeed, many of the results that one proves in such a course were first proved correctly by Gauss in this book and it can still be read with profit by a well-motivated student. Gauss's proofs are often more concise and conceptual than those in many recent texts, although the proof of the law of quadratic reciprocity is much longer than some of his later proofs of this result.

In former years this course might have been populated by mathematics majors, students intending to go on to elementary or high school education, or by arts students seeking a mathematics credit in a course that doesn't require a background in calculus. However, since the advent of public key cryptography with its important applications of elementary number theoretic ideas, most universities have found that the course attracts many computer science students interested in such topics as the RSA cryptosystem and secure public key exchange.

The author of the text under review states that his text is not intended to be a standard number theory course, being based on two short courses he has offered at Monash University, one on elementary number theory and the other on ring theory with applications to algebraic number theory. However, as he points out, the first nine chapters of the text would be suitable for the standard elementary number theory course. The last three chapters on rings, ideals, and quadratic fields could serve as an introduction to algebraic number theory.