(e) If a function between Hausdorff spaces is continuous then the preimage of every compact set is compact.

This is false. Let $f: (\mathbb{R}, \mathcal{T}_1) \to (\mathbb{R}, \mathcal{T}_3)$ where $\mathcal{T}_1$ is the discrete topology & $\mathcal{T}_3$ is the standard topology & $f(x) = x$. Since $\mathcal{T}_1$ is the discrete topology, f is continuous. Further, both of these spaces are Hausdorff. However $[0,1]$ is compact in $\mathcal{T}_3$ but as we showed in part (a), not compact in $\mathcal{T}_1$. $\square$