Math 4B Summer 2020
Quiz #3

No calculators

PERM NUMBER

TA: ☐ Trevor ☒ Fabio

Time: ☒ 4:30 ☐ 5:30 ☐ 6:30 ☐ 7:30

Today we will be working in groups of 3. Discuss the following questions about exact equations with your group, and write down the answers your group comes up with. You can each write the same things, but **everyone in the group must turn in their own quiz.**

1. Why do you think exact differential equations are called "exact"? [There is a "right" answer here.]

The exact DE can be expressed exactly as the derivative of some function F.

$$\frac{d}{dx}[F] = 0$$

2. When you are deciding how to solve a given DE, what would give you the suspicion that it might be exact? [Is there a form they usually come in?]

I check if it can be written in the form

$$M(x,y) + N(x,y)y' = 0$$

3. If you suspect that a differential equation is probably exact, how can you know for sure?

An exact diff eq has the mixed partial equal to each other $f_{xy} = f_{yx}$.

In the form above, I check $M_y = N_x$

4. How do you solve an exact differential equation?

For an exact DE of form $M + Ny' = 0$

We have $\frac{d}{dx}[F] = C$

And $F_x = M$. So we integrate

$\int M = F$.

We eliminate any unknown $h(y)$ functions by taking the $\frac{\partial}{\partial y}$.

This results in an equation that is $= to$ N, since $F_y = N$

The end result is obtaining the function F which $= C$

5. Make up your own exact differential equation and solve it below. [Your answer here should be different from th other members of your group]

$M_y = 5xy^2 = N_x$ An exact DE has $f_{xy} = f_{yx}$ so I invent one.

$M = \int 5xy^2\,dy \qquad N = \int 5xy^2\,dx$

$M = 5x\frac{y^3}{3} + 0x \qquad = 5y^2\frac{x^2}{2} + 0y$

Our D.E. is $5x\frac{y^3}{3} + 5y^2\cdot\frac{x^2}{2}y' = 0$

$F = \int M\,dx$

$= \int \frac{5}{3}xy^3\,dx$

$F = \frac{5}{3}y^3\frac{x^2}{2} + h(y)$

$F_y = 5y^2\frac{x^2}{2} + h'(y) = N = 5y^2\frac{x^2}{2}$

$h' = 0$

$h = 0$ we choose $c = 0$

$\therefore F = \frac{5}{6}y^3x^2 = c$

6. Each member in turn, explain your solution to the rest of the group. While you are listening, write eacl differential equation and its solution below. [You don't need to copy all the work to solve, just listen to you groupmates' explanations.]

i.

Equation: $(2xy+3)dx + (x^2-1)dy = 0$

Solution: $x^2y + 3x - y = C$

ii.

Equation: $2xy + (x^2+1)y' = 0$

Solution: $x^2y + y = C$

iii.

Equation: $2y + (5y+2x)y' = 0$

Solution: $C = 2xy + \frac{5}{2}y^2$