Welcome To Math 34A! Differential Calculus

Instructor:

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Office Hours:

MTWR after class 2:00-3:00, and by appointment. Details on Gauchospace.

Warm-up

How many times do we need to triple 1 to get the following numbers?

- 9 $\boxed{2}$
- 81 $\boxed{4}$
- 1 $\boxed{0}$
- $\frac{1}{3}$ $\boxed{-1}$
- $\frac{1}{2}$??? ...something between -1 and 0.

Warm-up Part II

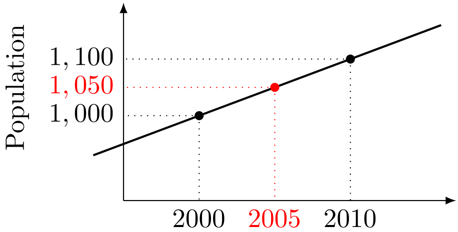
How many times do we need to decuple 1 (multiply 1 by 10) to get the following numbers?

- 100 $\boxed{2}$
- 1000 $\boxed{3}$
- 1 $\boxed{0}$
- .0001 $\boxed{-4}$
- A Googol $\boxed{100}$ A Googol is...
  10000000000000000000000000000000000000000000
  00000000000000000000000000000000000000000000
  0000000000000
  (The founders of Google spelled it wrong intentionally.)

Linear Interpolation

1. In 2000, a population was 1000. In 2010, it was 1100. What would you guess the population was in 2005?

A= 1005 B= 1020 C= 1050 D= 2050 E= 2010 $\boxed{\text{C}}$



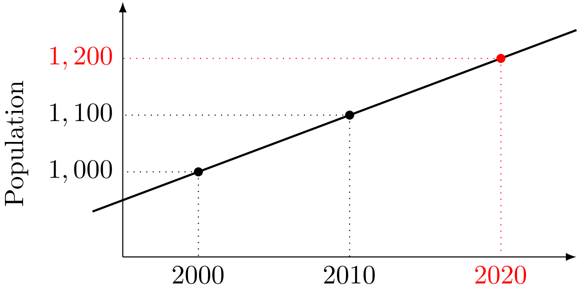
This is a guess based on the assumption that population grows at a constant rate.

"Constant rate" means that the graph of population is a straight line.

Linear Extrapolation

2. In 2000, a population was 1000. In 2010, it was 1100. What would you guess the population will be in 2020?

A= 1150 B= 1200 C= 1250 D= 2020 E=Other $\boxed{\text{B}}$



Again: a guess based on the assumption that population grows at a constant rate.

A Problem

You can't tell someone you just "guessed" the answer or just "drew a straight line". You need to make it sound more "scientific" so give it a complicated sounding name to impress people.

Linear Interpolation and Linear Extrapolation.

Linear means straight line
inter means between like intercity
extra means beyond like extraordinary

The idea is to assume the population (or whetever) grows at a constant rate.
Then use this to predict.

Method:
(1) Use given data to draw a straight line and find equation
$y = mx + b$
(2) Use the equation to make predictions.

3. In 2000, a population was 1000. In 2010, it was 1100. Let

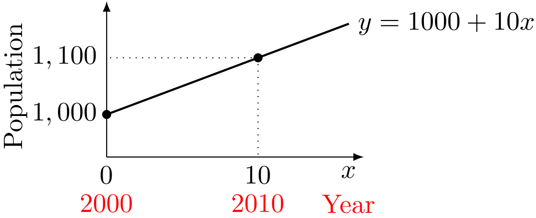
x = number of years after 2000 (Ex: $x = 3$ is the year 2003)

y = population in the year x

Find the equation of a line $y = mx + b$:

A: $2000 + 1000x$ B: $1000 + 2000x$ C: $1000 + 100x$ D: $1000 + 10x$

Answer: $\boxed{\text{D}}$



9. When will population be 1350?

A= 2015 B= 2025 C= 2035 D= 3350 E=Other $\boxed{\text{C}}$

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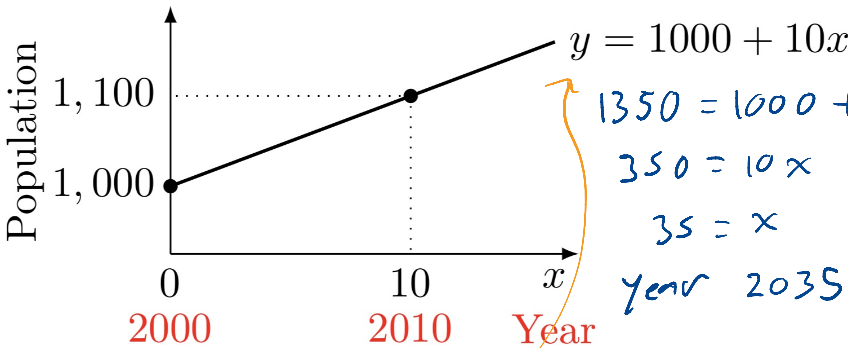

$1350 = 1000 + 10x$

$350 = 10x$

$35 = x$

year 2035

9. When will population be 1350?

$y = 1350$

A= 2015 B= 2025 C= 2035 D= 3350 E=Other

Another Example

5. The number of unemployed in LA on January 1, 2015 was $50,000$. After 100 days, it was $45,000$.

- Estimate the number of unemployed 300 days after Jan. 1.

A$= 40,000$ B$= 35,000$ C$= 30,000$ D$= 25,000$ E$= 300$ $\boxed{\text{B}}$

- Suppose $x =$ the number of days after January 1

  $y =$ number of unemployed people on day x.

  Then the equation of the line used for this linear extrapolation is $y =$

  A$= -100 + 50,000x$ B$= 50,000 - 100x$ C$= 45,000 - 100x$
  D$= 50,000 - 50x$ E$= 45,000 - 50x$ $\boxed{\text{D}}$

- How many days until unemployed reaches $30,000$?

  A$= 40$ B$= 140$ C$= 200$ D$= 300$ E$= 400$ $\boxed{\text{E}}$

Another Example

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- Suppose $x =$ the number of days after January 1
  $y =$ number of unemployed people on day x.

  Then the equation of the line used for this linear extrapolation is $y =$

  A= $-100 + 50,000x$ B= $50,000 - 100x$ C= $45,000 - 100x$
  D= $50,000 - 50x$ E= $45,000 - 50x$ $\boxed{\text{D}}$

- How many days until unemployed reaches 30,000? $y = 30,000$

  A= 40   B= 140   C= 200   D= 300   E= 400

$$30,000 = 50,000 - 50x$$

$$-20,00\cancel{0} = -5\cancel{0}x$$

$$\frac{2000}{5} = x$$

$$5\overline{)2000}\ \ 400$$

$$\boxed{400 = x}$$

Proportionality

Simple Idea: $y \propto x$ "y is proportional to x" means:

If you double x, then y doubles. Triple x then y triples. And so on.

Example: If you are paid by the hour then

$$\text{(amount you earn)} \propto \text{(number of hours you work)}$$

If you work for 10 hours, then you are paid \$50. How much are you paid if you work for 20 hours?

A= \$20 B= \$50 C= \$200 D= \$100 E=Other $\boxed{\text{D}}$

If you work for t hours, how much are you paid?

A= \$50 B= \$50t C= \$10t D= \$20t E= \$5t $\boxed{\text{E}}$

Because you are paid \$5/hour (or \$50 for 10 hours). The number "5" is called the constant of proportionality.

Proportionality Example:

Suppose $y \propto x$ and $y = 15$ when $x = 4$.

(a) What is y when $x = 8$?

A$= 15$ B$= 4$ C$= 8$ D$= 30$ E$= 60$ $\boxed{\text{D}}$

(b) What is y when $x = 12$?

A$= 15$ B$= 45$ C$= 30$ D$= 36$ E$= 12$ $\boxed{\text{B}}$

(c) What is x when $y = 150$?

A$= 14$ B$= 1500$ C$= 40$ D$= 450$ $\boxed{\text{C}}$

Constant of Proportionality

"y is proportional to x" means $y = Kx$, where K is called the constant of proportionality.

Example: We are told

- Tax is proportional to income, and
- The tax on $\$1,000$ is $\$280$.

Express $y =$ amount of tax paid in terms of $x =$ the income. Then $y =$

$$\text{A} = 1000x \quad \text{B} = 280x \quad \text{C} = \frac{1,000}{280}x$$
$$\text{D} = 2.8x \quad \text{E} = 0.28x \quad \boxed{\text{E}}$$

Question: What does the constant of proportionality $K = 0.28$ mean?

Answer: It is the tax on one dollar.

Constant of Proportionality

"y is proportional to x" means $y = Kx$, where K is called the constant of proportionality.

$y = Kx$

$280 = K(1000)$

$.28 = K$

$\boxed{y = .28x}$

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Express y = amount of tax paid in terms of x = the income. Then $y =$

$$\text{A} = 1000x \qquad \text{B} = 280x \qquad \text{C} = \frac{1,000}{280}x$$

$$\text{D} = 2.8x \qquad \text{E} = 0.28x$$

Example

For this question, we assume:

- The weight of an elephant is proportional to its height cubed, and
- An elephant 1 meter high weighs 1/3 tons.

How many tons does an elephant h meters tall weigh?

A= $h/3$ B= h^3 C= $h^3/3$ D= $(h/3)^3$ E= $(3h)^3$ $\boxed{\text{C}}$

Question: What does the constant of proportionality $K = 1/3$ mean?

Answer: It is the weight of 1 cubic meter of elephant.

Example

$$\frac{1}{1/3} = \frac{h}{w}$$

For this question, we assume:

- The weight of an elephant is proportional to its height cubed, and $w \propto h^3 \Leftrightarrow w = kh^3$
- An elephant 1 meter high weighs 1/3 tons. $k = \frac{1}{3}$

How many tons does an elephant h meters tall weigh?

A$= h/3$ B$= h^3$ $\boxed{\text{C}= h^3/3}$ D$= (h/3)^3$ E$= (3h)^3$

$$w = \frac{1}{3}h^3$$

More Complicated Examples

$$\boxed{y \text{ is inversely proportional to } x \text{ means } y \propto 1/x}$$

Example:

- I have \$300
- N = number of apples I can buy
- p = price per apple

Then N is inversely proportional to p: $N \propto 1/p$.

What is the constant of proportionality?

More Complicated Examples

$$\boxed{y \text{ is inversely proportional to } x \text{ means } y \propto 1/x}$$

Example:

k

$300 = pN$

- I have \$300
- N = number of apples I can buy
- p = price per apple

Then N is inversely proportional to p: $N \propto 1/p$. $N = \frac{k}{p}$

What is the constant of proportionality?

$k = 300$ $N = \frac{300}{p}$

equation

More Complicated Examples

Strength of Light

- $P =$strength of light (power per unit area)
  amount of light on unit area
- $R =$ distance to light source

Inverse Square Law: $P \propto 1/R^2$

Same idea for heat, gravity, sound, and many others...

Newton's Law of Gravity: $F \propto \frac{m_1 m_2}{r^2}$

Constant of proportionality: $G \approx 6.67 \times 10^{-11}\ \mathrm{m}^3/(\mathrm{kg}\,\mathrm{s}^2)$
(the Gravitational constant)

Review

1. Solve for x in the equation

$$\frac{3}{x+a} = \frac{a}{x+2}.$$

$$x = \frac{6-a^2}{a-3}$$

2. Multiply out and simplify. Check your answer.

$$(a-3b)(4a+2b)+6ab$$

$$= 4a^2 - 4ab - 6b^2$$

$=$

Review

1. Solve for x in the equation

$$\frac{3}{x+a} = \frac{a}{x+2}.$$

$$3(x+2) = a(x+a)$$

$$3x+6 = ax+a^2$$

$$-ax \quad -6 \quad -ax \quad -6$$

$$3x-ax = a^2-6$$

$$x(3-a) = a^2-6$$

$$\boxed{x = \frac{a^2-6}{3-a}}$$

? factor

$$\frac{(a-\sqrt{6})(a+\sqrt{6})}{-(a-3)}$$

2. Multiply out and simplify. Check your answer.

$$(a - 3b)(4a + 2b) + 6ab$$

$4a^2 + 2ab - 12ab - 6b^2 + 6ab$

$4a^2 - 4ab - 6b^2$

Review

3. Substitute $x = 3t - 4$ into

$$2x(x+1).$$

Simplify the result as much as possible.

$$= 6\left(3t^2 - 7t + 4\right)$$

(you don't <u>have</u> to pull out the 6)

4. Solve for x and y in the simultaneous equations

$$x + 2y = p \qquad x + y = 4.$$

$$x = 8 - p, \quad y = p - 4$$

Review

5. Marie leaves Santa Barbara at 10am, driving to Bakersfield on a route which is 150 miles long. Jason leaves Bakersfield at 11am driving the same route to Santa Barbara. Marie's speed is 40 miles/hr and Jason's speed is 60 miles/hr.

(a) How far apart are they at noon?

10 mi

(b) How far from Santa Barbara are they when they meet?

84 mi

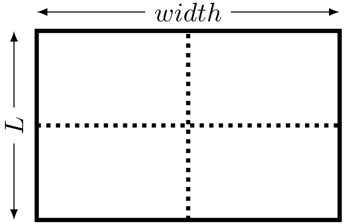
(c) How many hours has Jason been driving when they meet?
[leave your answers as fractions]

11/10 hours

Review

6. A farmer wants to partition a rectangular field into quarters, as shown. The total area of the field is 500 square meters. Suppose the length of the field is L meters.

(a) Express the width of the field in terms of L.(Answer: $500/L$)



(b) The outer boundary fence (on the perimeter of the field, shown solid) costs \$4 per meter, and the inside fence (shown dotted) costs \$3 per meter. Express the total cost of the fence needed in terms of L. (Answer: $\frac{11\left(L^2+500\right)}{L}$)

That's it. Thanks for being here.