Practice Problems 8

Math 4B, Spring 2017, Dr. Paul

Practice problems are for your own benefit. You won't turn them in or have them graded, but I have the expectation that you have done these when I write my tests. You can check answers with a TA, in Math Lab, or with the professor.

1. Consider a simplified Social Accounting Matrix in which we consider the flow of money among three institutional agents of the economy: Households (H), Firms (F), and Government (G). Each of these agents possesses a certain amount of wealth, h, f, and g respectively. The agents pay each other at the following yearly rates:

   - (H) pays (F) at a rate of 50% (of h) per year (consumer spending).
   - (F) pays (H) at a rate of 50% (of f) per year (wages).
   - (H) pays (G) at a rate of 5% (of h) per year (taxes).
   - (F) pays (G) at a rate of 10% (of f) per year (taxes).
   - (G) pays (H) at a rate of 100% (of g) per year (government wages and entitlements).
   - (G) pays (F) at a rate of 40% (of g) per year (government contracts).

   Answer the following.

   (a) Use the data above to write down a system of differential equations for h, f, and g.

   (b) Find the general solution (you can use a calculator).

   (c) Is/Are there equilibrium solution(s)?

   (d) In the U.S., $h = 82$, $f = 35$, and $g = 8$ (in trillions). How do you predict these numbers will change?

2. Find the eigenvectors and generalized eigenvectors of the matrix $\begin{bmatrix} 8 & -5 \\ 5 & -2 \end{bmatrix}$.

3. Sketch a phase portrait for $\vec{x}\,' = \begin{bmatrix} 8 & -5 \\ 5 & -2 \end{bmatrix} \vec{x}$. Make sure your portrait reflects where x and y are increasing or decreasing, and where vectors are pointing radially inward or outward.

4. Find the general solution to the system

$$\begin{bmatrix} x' \\ y' \\ z' \end{bmatrix} = \begin{bmatrix} 3 & 0 & -2 \\ 0 & 5 & 0 \\ 2 & 0 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

5. Solve the system $\vec{x}\,' = \begin{bmatrix} 3 & 5 \\ -1 & -1 \end{bmatrix} \vec{x}$ and sketch a phase portrait. Make sure your portrait reflects where x and y are increasing or decreasing, and where vectors are pointing radially inward or outward.

6. Find the general solution to the system

$$\begin{bmatrix} x' \\ y' \\ z' \end{bmatrix} = \begin{bmatrix} 3 & 0 & 0 \\ 0 & 3 & 1 \\ 0 & 0 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

7. Here are a few 3×3 or 4×4 systems to consider. Try finding the general solutions.

   (a) $\vec{x}\,' = \begin{bmatrix} 2 & 1 & 0 \\ 0 & 2 & 1 \\ 0 & 0 & 2 \end{bmatrix} \vec{x}$

   (b) $\vec{x}\,' = \begin{bmatrix} 1 & 2 & 0 & 0 \\ -2 & 1 & 0 & 0 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & -2 & 1 \end{bmatrix} \vec{x}$

   (c) $\vec{x}\,' = \begin{bmatrix} 1 & 2 & 1 & 0 \\ -2 & 1 & 0 & 1 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & -2 & 1 \end{bmatrix} \vec{x}$

   (d) $\vec{x}\,' = \begin{bmatrix} 5 & 1 & 0 & 0 \\ 0 & 5 & 0 & 0 \\ 0 & 0 & 5 & 1 \\ 0 & 0 & 0 & 5 \end{bmatrix} \vec{x}$