Project #3: First Order Linear Systems of DEs.
TA Guide Part 1

1. Print half as many pre-worksheets as their are students to promote collaboration. Have them get started on PW 1. If a quick group gets PW 1 well before the others, tell them to think about PW 2. Once about 2/3 of the room has (a) figured out, you can bring the class together and talk about the answer to that part. Give them a few more minutes with (b) and (c), then discuss those parts as a class before moving on to PW 2.

2. Once you've established what the matrix A is for part (b), ask them if they recall eigenvalues/vectors from 4A. The answer will be "no", so now you can go into an eigenvalue/vector refresher...maybe compute them for some other example matrix B, say something about what they mean for the mapping etc. Then have them compute the eigenstuff for part (b).

3. NOTE that this will be the first time they've seen a system of first order equations in this class. You can let them know that we'll start talking about these problems in lecture next Tuesday. For now, this is just warm-up.

PW 1 Let $\ddot{x}+3\dot{x}+2x=0$ be the equation of a damped vibrating spring with a unit mass, damping coefficient $b=3$ and spring constant $k=2$. We can convert this second order DE into a system of two first order DEs.

(a) If we make the substitution $y=\dot{x}$ we can find a system of two first order equations that describe the motion of the spring-block set-up. What two equations do you get?

(b) Find the general solution to the linear homogeneous DE $\ddot{x}+3\dot{x}+2x=0$.

(c) Now express your solution from part (a) in vector form, i.e. find an expression for the vector-valued function whose first entry is the position of the block and second entry is velocity of the block:

$$\begin{bmatrix} x(t) \\ y(t) \end{bmatrix} = \begin{bmatrix} ? \\ ? \end{bmatrix}$$

PW 2(a) Now convert your system into matrix form. You should get something like

$$\begin{bmatrix} \dot{x}(t) \\ \dot{y}(t) \end{bmatrix} = A \begin{bmatrix} x(t) \\ y(t) \end{bmatrix}.$$

What are the entries in your coefficient matrix A?

(b) Find the eigenvalues and eigenvectors of A.

(c) Can you express your vector-valued solution from part (b) in terms of the eigenvalues and eigenvectors you found in PW 1 part (c)?

(d) Will you always be able to do this for any second order linear DE with constant coefficients?