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Once upon a time. . .

There was a happy math professor and he told his happy students:

“When you work out derivatives ALWAYS write the $\frac{d}{dx}$ part so you write something like

$$\boxed{\frac{d}{dx}\left(3x^2+5x+2\right)=6x+5}$$

and you never-ever-ever write

$\boxed{3x^2+5x+2 \qquad 6x+5}$ or even worse $\boxed{3x^2+5x+2=6x+5.}$

Because if you don’t do as I say I will become a sad math professor and you will repeat this class.”

Exponential Functions (§8.8)

Is there a function $f(x)$ which equals its own derivative? That is, can you find a function $f(x)$ with

$$\boxed{f'(x) = f(x)?}$$

There are many many uses for it.

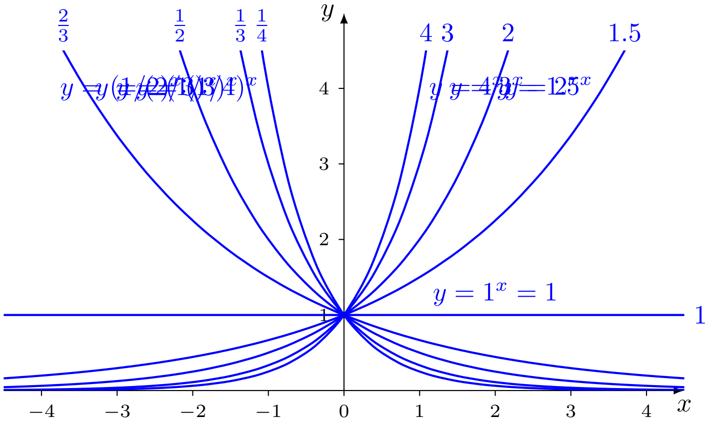
One boring answer: $f(x) = 0$. Is there another?

Yes:

$$\boxed{\frac{d}{dx}\left(e^x\right) = e^x.}$$

What's up with that?

The Derivative of $f(x) = a^x$



Question: Which "a" should we use?

The Derivative of $f(x) = a^x$

The slope of the graph at $x = 0$ is

$$f'(0) = \lim_{h\to 0} \frac{f(0+h) - f(0)}{h} = \lim_{h\to 0} \frac{a^h - a^0}{h} = \lim_{h\to 0} \frac{a^h - 1}{h}$$

This is a constant that depends on what a is.
Examples:

a	1	2	$2.718\cdots$	3	4
$f'(0)$	0	0.6931	1	1.0986	1.3863

More generally,

$$f'(x) \approx \frac{f(x+h) - f(x)}{h} = \frac{a^{x+h} - a^x}{h} = \frac{a^x(a^h - 1)}{h} = a^x \left(\frac{a^h - 1}{h}\right)$$

Moral: The derivative of $f(x) = a^x$ is a multiple of itself!
Second Moral: That multiple is 1 when $a = 2.718\,281\,828\cdots = e$.

Factorials

$5! = 1 \times 2 \times 3 \times 4 \times 5$ is called 5 factorial and is the product of the whole numbers from 1 up to 5.

What is 5!?

(A) 5 (B) 20 (C) 60 (D) 120 (E) 720 $\boxed{\text{D}}$

Why do we care? There are 5! orders in which to trim the nails on your left hand.

Similarly $n!$ ("n factorial") is the product of all the whole numbers from 1 up to n.

Question: What is $\dfrac{n!}{n}$?

(A) 1 (B) n (C) $(n-1)!$ (D) $(n+1)!$ $\boxed{\text{C}}$

Factorials come up a lot in probability and statistics.

A Formula for e^x

It turns out that

$$e^x = 1 + x + \frac{x^2}{2} + \frac{x^3}{6} + \frac{x^4}{24} + \frac{x^5}{120} + \cdots + \frac{x^n}{n!} + \cdots$$

How does it manage to equal it's own derivative?

$$\begin{aligned}
\frac{d}{dx}\left(e^x\right) &= \frac{d}{dx}\left(1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \frac{x^5}{5!} + \cdots\right) \\
&= 0 + 1 + \frac{2x}{2\times 1} + \frac{3x^2}{3\times 2\times 1} + \frac{4x^3}{4\times 3\times 2\times 1} + \frac{5x^4}{5\times 4\times 3\times 2\times 1} + \cdots \\
&= 0 + 1 + \frac{\cancel{2}x}{\cancel{2}\times 1} + \frac{\cancel{3}x^2}{\cancel{3}\times 2\times 1} + \frac{\cancel{4}x^3}{\cancel{4}\times 3\times 2\times 1} + \frac{\cancel{5}x^4}{\cancel{5}\times 4\times 3\times 2\times 1} + \cdots \\
&= 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \cdots \\
&= e^x
\end{aligned}$$

A simple trick:
- The derivative of each term is the preceding one.
- The derivative of the first term is zero.

The Number e

The number $e = 2.718281828\cdots$ is a very important in math. It can be calculated to as much accuracy as needed by using more and more terms in this formula for e^x with $x = 1$ plugged in:

n	$1 + 1 + \frac{1}{2} + \cdots + \frac{1}{n!}$
1	2
2	2.5
3	2.6666...
4	2.708333...
5	2.716666...
6	2.718055...
7	2.718253968...
8	2.718278770...
9	2.718281526...
10	2.718281801...
exact	2.7182818284590452354...

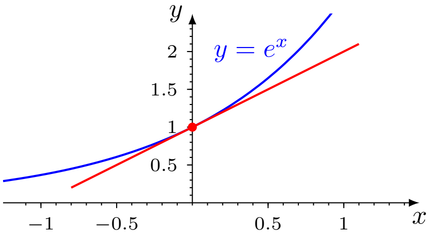
Key Facts about e and e^x

What you need to remember:

- $e^0 = 1$
- $\dfrac{d}{dx}\left(e^x\right) = e^x$

Question: What is the equation of the tangent line to $y = e^x$ at $x = 0$?

(A) $y = 1$ (B) $y = x$ (C) $y = x + 1$ (D) $y = ex + 1$ $\boxed{\text{C}}$



Differentiating $f(x) = e^{kx}$

$$\frac{d}{dx}\left(e^{kx}\right) = ke^{kx}$$

versus

$$\frac{d}{dx}\left(x^{n}\right) = nx^{n-1}$$

⚠ Do not get confused and write $\frac{d}{dx}\left(e^{kx}\right) = ke^{(k-1)x}$. ⚠

Question: Find $\frac{d}{dx}\left(4e^{3x} + 5x^3\right)$

(A) $12e^{2x} + 15x^2$ (B) $12e^{3x} + 15x^3$ (C) $4e^{3x} + 15x^2$

(D) $12e^{3x} + 15x^2$ (E) Other

$\boxed{\text{D}}$

Example

$$\frac{d}{dx}\left(e^{kx}\right) = ke^{kx}$$

The temperature (in $^\circ$ C) of a cup of coffee t hours after it is made is $f(t) = 50 + 40e^{-2t}$.

(a) What is the initial temperature when the coffee is made?

(A) 40 (B) 50 (C) 90 (D) 100 C

(b) How quickly is the coffee cooling down initially? This means how many degrees per hour is the temperature going down instantaneously at $t = 0$?

(A) 20 (B) 40 (C) 60 (D) 80 (E) 100 D

More Examples

$$\boxed{\frac{d}{dx}\left(e^{kx}\right) = ke^{kx}}$$

(1) $\dfrac{d}{dx}\left(\dfrac{3}{e^{2x}}\right) =?$

(A) $\frac{3}{2e^{2x}}$ (B) $\frac{3}{2e^{x}}$ (C) $\frac{6}{e^{2x}}$ (D) $\frac{-6}{e^{2x}}$ D

(2) The number of grams of Einsteinium-253 after t days is $m(t) = 10e^{-t/30}$. How quickly is the mass changing (in grams per day) when $t = 0$?

(A) $-1/30$ (B) $-1/3$ (C) $-10e^{-t/30}$ (D) $-\frac{1}{3}\,e^{t/30}$ B