Graded Problem 5

Math 4B, Spring 2017, Dr. Paul

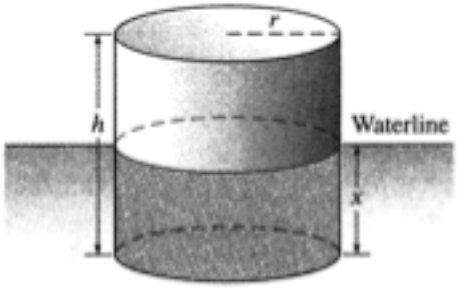
1. Consider the differential equation $y'' + y = 0$, whose general solution we already know how to find. Answer the following "boundary value" problems. Note that our Existence and Uniqueness Theorem does not necessarily apply in this context.

   (a) What solution(s) satisfy the conditions $y(0) = 4$, $y(\pi/6) = 3$?

   (b) What solution(s) satisfy the conditions $y(0) = 3$, $y(\pi) = 4$?

   (c) What solution(s) satisfy the conditions $y(0) = 0$, $y(2\pi) = 0$?

2. Consider a floating cylindrical buoy with radius r and height h of uniform density $\rho \leq 0.5\text{g/cm}^3$ (the density of water is 1 g/cm^3). The buoy is initially suspended at rest with its bottom at the top surface of the water and is released at $t = 0$. Thereafter it is acted upon by two forces: a downward gravitational force equal to its weight $mg = \pi r^2 hg\rho$, and an upward buoyancy force equal to the weight of the water it displaces $\pi r^2 xg$, where $x(t)$ is the depth of the bottom of the buoy beneath the surface



   at time t.

   (a) Set up a differential equation describing the motion of the buoy with respect to time. (Ignore water resistance).

   (b) Nonzero solutions to the differential equation above should display oscillatory behavior. What is the period of the oscillation (in terms of r, h, g, ρ)?

   (c) Find the solution given the constants $\rho = 0.5$ g/cm^3, $h = 200$ cm, $r = 10$ cm, and $g = 980$ cm/s^2.