Math 34A Winter 2020
Old Midterm 1 #4 Solutions

1. Solve for x in the equation $\dfrac{3}{x+a} = \dfrac{a}{x+2}$.

Solution: We multiply both terms by the denominators: $(x+a)$ and $(x+2)$. We get

$$\frac{3}{x+a}\cdot(x+a)(x+2) = \frac{a}{x+2}\cdot(x+a)(x+2) \qquad \text{and so} \qquad 3(x+2) = a(x+a).$$

Expanding these terms gives $3x+6 = ax+a^2$, and so $3x - ax = a^2 - 6$. Factoring the left-hand side yields $(3-a)x = a^2-6$, and so $\boxed{x = \dfrac{a^2-6}{3-a}}$. This is the same as $\boxed{x = \dfrac{6-a^2}{a-3}}$.

One way to check this is to pick a value for a. If we choose $a = 1$, then the first formula for x implies $x = \frac{1^2-6}{3-1} = -5/2$. Now let's see if $a = 1$ and $x = -5/2$ makes our original equality true:

$$\frac{3}{x+a} = \frac{3}{-5/2+1} = \frac{3}{-3/2} = -2 \qquad \text{and} \qquad \frac{a}{x+2} = \frac{1}{-5/2+2} = \frac{1}{-1/2} = -2.$$

Thus $\frac{3}{x+a} = \frac{a}{x+2}$ follows from $x = \frac{a^2-6}{3-a}$, at least when $a = 1$.

2. Multiply out and simplify

$$(a-3b)(4a+2b)+6ab.$$

Check your answer.

Solution: We distribute the first product and get

$$(a-3b)(4a+2b) = a\cdot 4a + a\cdot 2b - 3b\cdot 4a - 3b\cdot 2b = 4a^2 + 2ab - 12ab - 6b^2.$$

Thus

$$(a-3b)(4a+2b)+6ab = 4a^2+2ab-12ab-6b^2+6ab = \boxed{4a^2-4ab-6b^2}.$$

We can check this by picking values for a and b. If we say $a = 1$ and $b = 7$, then

$$(a-3b)(4a+2b)+6ab = (1-3\cdot7)(4\cdot1+2\cdot7)+6\cdot1\cdot7 = (1-21)(4+14)+42 = (-20)(18)+42 = -360+42 = -318$$

and

$$4a^2-4ab-6b^2 = 4(1)^2 - 4\cdot1\cdot7 - 6\cdot 7^2 = 4-28-6\cdot49 = -24-294 = -318.$$

(You, of course, could pick easier values for a and b.) Thus our simplification checks out for at least one value of a and b.

3. Substitute $x = 3t - 4$ into $2x(x+1)$. Simplify the result as much as possible.

Solution: We replace all the "x" with "$3t - 4$" and get

$$\begin{aligned} 2x(x+1) &= 2(3t-4)\big((3t-4)+1\big) = 2(3t-4)(3t-3) \\ &= 2\big(9t^2 - 12t - 9t + 12\big) = 2\big(9t^2 - 21t + 12\big) \\ &= \boxed{18t^2 - 42t + 24}. \end{aligned}$$

We can check this by picking a value of t. If we pick $t = 1$, then $x = 3(1) - 4 = -1$, so $2x(x+1) = 2(-1)(-1+1) = 0$. On the other hand,

$$18t^2 - 42t + 24 = 18(1)^2 - 42(1) + 24 = 18 - 42 + 24 = 0.$$

Thus our answer agrees with the original expression when $t = 1$.

4. Solve for x and y in the simultaneous equations

$$x + 2y = p \qquad\qquad x + y = 4.$$

Your answers will involve p only.

Solution: Solve for one of the variables in term of the others; we'll write $y = 4 - x$. Then plugging this into the other equation gives us

$$x + 2(4 - x) = p \qquad \text{or, simplifying,} \qquad 8 - x = p.$$

Thus $x = 8 - p$ and so, solving, $y = 4 - (8 - p) = p - 4$. That is, $\boxed{(x, y) = (8 - p, p - 4)}$.

5. Marie leaves Santa Barbara at 10am, driving to Bakersfield on a route which is 150 miles long. Jason leaves Bakersfield at 11am driving the same route to Santa Barbara. Marie's speed is 40 miles/hr and Jason's speed is 60 miles/hr.

(Leave your answers as *fractions*.)

(a) How far apart are they at noon?

Solution: At noon, Marie has been driving for 2 hours, so she has gone

$$\text{distance} = \text{rate} \times \text{time} = \left(40\ \frac{\text{miles}}{\text{hour}}\right)(2\text{ hours}) = 80\text{ miles.}$$

Similarly, Jason has been driving only 1 hour by noon, so he has gone

$$\text{distance} = \text{rate} \times \text{time} = \left(60\ \frac{\text{miles}}{\text{hour}}\right)(1\text{ hour}) = 60\text{ miles.}$$

Thus together they have gone $80 + 60 = 140$ miles. Since the route they are traveling is 150 miles, they are $\boxed{\text{10 miles apart}}$ at noon.

(b) How far from Santa Barbara are they when they meet?

Solution: They meet shortly after noon. Since they are only 10 miles apart at noon and they are traveling toward each other at $40 + 60 = 100$ miles/hour, it only takes another

$$\text{time} = \frac{\text{distance}}{\text{rate}} = \frac{10\text{ miles}}{100\text{ miles/hour}} = \frac{1}{10}\text{ hours}$$

for them to meet. In this 1/10 hour, Marie has gone an additional

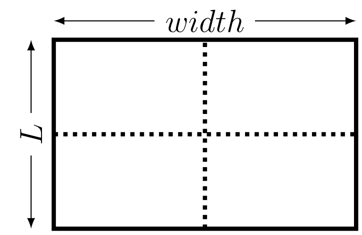
$$\text{distance} = \text{rate} \times \text{time} = \left(40\ \frac{\text{miles}}{\text{hour}}\right)\left(\frac{1}{10}\text{ hours}\right) = 4\text{ miles.}$$

Thus Marie has traveled a total of $\boxed{\text{84 miles from Santa Barbara}}$.

(c) How many hours has *Jason* been driving when they meet?

Solution: As we figured out in part (b), Jason drives one hour from 11am to noon, then an additional 1/10 hour. Thus Jason has been driving $1 + 1/10 = \boxed{\text{11/10 hours}}$ when they meet.

6. A farmer wants to partition a rectangular field into quarters, as shown.



The total area of the field is 500 square meters. Suppose the length of the field is L meters.

(a) Express the width of the field in terms of L.

Solution: The length of the field is L. We can relate L and w using the area. On the one hand, we're told the area is $A = 500$ square meters. On the other hand, we know the area of the rectangular field is $A = L \cdot w$. Thus $L \cdot w = 500$, so $\boxed{w = 500/L}$.

(b) The outer boundary fence (on the perimeter of the field, shown solid) costs \$4 per meter, and the inside fence (shown dotted) costs \$3 per meter. Express the total cost of the fence needed in terms of L.

Solution: The outer boundary fence has length equal to $2L + 2w$ meters and the interior fence have total length $L + w$ meters. Thus the cost of the outer boundary fence is

$$\left(\frac{\$4}{\text{meter}}\right)(2L + 2w \text{ meters}) = \$\,(8L + 8w)\ .$$

Similarly, the cost of the interior fence is

$$\left(\frac{\$3}{\text{meter}}\right)(L + w \text{ meters}) = \$\,(3L + 3w)\ .$$

Thus the total cost is $\$(11L + 11w)$. To write this in terms of only L, we replace w with $500/L$, so $11w = 11 \times 500/L = 5500/L$ and thus

$$\text{total cost} = \$\left(11L + \frac{5500}{L}\right)\ .$$