



Welcome Back!

Differential Calculus

Instructor:

Nathan Schley (*Sh+lye*), `schley@math.ucsb.edu`

South Hall 6701

Office Hours:

T R 11-11:50, T 3:45-4:35 Details on Gauchospace.

© 2022 Daryl Cooper, Peter M. Garfield, Ebrahim Ebrahim &
Nathan Schley

Please do not distribute outside of this course.

Today: Start Chapter 7 (Logs)

Applications:

- **Chemistry**: alkalinity and acidity, pH scale
- **Finance**: compound interest (get rich slow)
- **Geology**: Richter scale for earthquakes (did you feel the earth move too ?)
- **Archeology**: radio carbon dating (how old is that bone ?)
- **Astronomy**: stellar magnitude (brightness of stars)
- **Sound**: decibels (what did you say?, the music is too loud)
- **Math**: solving equations with exponents ...by performing an arithmetic operation to both sides (all of the above are examples of the use of this operation)

Today: Start Chapter 7 (Logs)

Main Idea of Chapter 7:

$\log(x)$ is how many times you multiply 1 by 10 to get x

Conclusion:

Before we do logs we should be really good at powers of 10.

Powers of Ten

1 meter \approx 3 feet

1 centimeter = 0.01 meters = 10^{-2} meters \approx 1/2 inch

1 kilometer = 1,000 meters = 10^3 meters \approx 1/2 mile

Approximate distance (in meters), to nearest power of 10

10^7 meters

Size of Earth

10^9 meters

Distance to moon

10^{14} meters

Size of our solar system

10^{16} meters

One light-year

10^{21} meters

Size of the Milky Way galaxy

10^{27} meters

Size of the universe (about 93 billion light-years)

10^{80}

number of protons in the observable universe?

10^{100}

1 googol

10^{1000} meters

???

Exponential Basics

$$\begin{aligned}
 10^4 &= 10 \times 10 \times 10 \times 10 = 10,000 \\
 &= 4 \text{ lots of } 10 \text{ multiplied together} \\
 &= 1 \text{ followed by } 4 \text{ zeroes}
 \end{aligned}$$

$$\begin{aligned}
 10^x &= \underbrace{10 \times 10 \times \cdots \times 10}_x \text{ lots of } 10 = 1 \underbrace{00000 \cdots 0}_x \text{ zeros} \\
 &= 1 \text{ followed by } x \text{ zeroes}
 \end{aligned}$$

Ex:

$$\begin{aligned}
 10^2 \times 10^3 &= (10 \times 10) \times (10 \times 10 \times 10) \\
 &= 10^{2+3} = 10^5.
 \end{aligned}$$

$$10^x \times 10^y = 10^{x+y} \quad \text{First Law of Exponents}$$

Why?

We can work it out!

Exponential Basics (cont'd)

$$10^x \times 10^y = 10^{x+y} \quad \text{First Law of Exponents}$$

Why? We can work it out:

$(x \text{ lots of } 10 \text{ multiplied together}) \times (y \text{ lots of } 10 \text{ multiplied together})$
 $= (x + y) \text{ lots of } 10 \text{ multiplied together}$

For now x and y are positive whole numbers.

More Exponentiation

$$\begin{aligned}
 (10^2)^3 &= (10 \times 10)^3 \\
 &= (10 \times 10) \times (10 \times 10) \times (10 \times 10) \\
 &= 10^6
 \end{aligned}$$

$$(10^a)^b = 10^{ab} \quad \text{Fourth Law of Exponents}$$

Why? We can work it out:

$$10^a = \underbrace{10 \times 10 \times \cdots \times 10}_{a \text{ times}}$$

$$\begin{aligned}
 (10^a)^b &= \underbrace{(10 \times \cdots \times 10) \times \cdots \times (10 \times \cdots \times 10)}_{b \text{ times}} \\
 &= 10^{ab}.
 \end{aligned}$$

Just count the zeros!

When the power is 0 or negative

What is 10^0 ? = 1 **But why?** We can work it out:

$$10^0 \times 10^1 = 10^{0+1}$$

$$\text{so } 10^0 \times 10 = 10$$

$$\text{and therefore } 10^0 = 10/10 = 1$$

Summary: we used the first law of exponents to figure out what 10^0 must be.

There is a second explanation in the book!

What is 10^{-2} ? = $1/100 = 0.01$ **But why?** We can work it out:

$$10^{-2} \times 10^2 = 10^{-2+2} = 10^0 = 1$$

$$\text{therefore } 10^{-2} = \frac{1}{10^2} \quad \text{and} \quad 10^{-a} = \frac{1}{10^a}$$

There is a second explanation in the book

The Five Laws of Exponents

$$(1) 10^a \times 10^b = 10^{a+b}$$

$$(2) 10^0 = 1$$

$$(3) 10^{-a} = 1/10^a$$

$$(4) (10^a)^b = 10^{ab}$$

$$(5) 10^a/10^b = 10^{a-b}$$

1. What is $10^3 \times 10^4$?

$$A = 10^{12} \quad B = 10^7 \quad C = 10^{34} \quad D = 10^0 \quad E = 10^{-7} \quad \boxed{B}$$

2. Find $10^3/10^4$

$$A = 10^7 \quad B = 10^1 \quad C = 10^{-4} \quad D = 10^{-1} \quad E = 10^{-7} \quad \boxed{D}$$

3. Find $(10^3)^4$.

$$A = 10^7 \quad B = 10^1 \quad C = 10^{12} \quad D = 10^{-1} \quad E = 10^0 \quad \boxed{C}$$

The Five Laws of Exponents

$$(1) 10^a \times 10^b = 10^{a+b}$$

$$(2) 10^0 = 1$$

$$(3) 10^{-a} = 1/10^a$$

$$(4) (10^a)^b = 10^{ab}$$

$$(5) 10^a/10^b = 10^{a-b}$$

4. What is $(10^2 \times 10^3)^4$?

$$A = 10^8 \quad B = 10^9 \quad C = 10^{12} \quad D = 10^{20} \quad E = 10^{24} \quad \boxed{D}$$

5. What is $(10^2 \times 10^6)/(10^2 \times 10^3)$?

$$A = 10^2 \quad B = 10^3 \quad C = 10^{-1} \quad D = 10^7 \quad E = 10^6 \quad \boxed{B}$$

6. What is $(10^2/10^5)^{-2}$?

$$A = 10^{-6} \quad B = 10^{-5} \quad C = 10^6 \quad D = 10^4 \quad E = 10^5 \quad \boxed{C}$$

Non-Integer Powers

We can work them out!

7. What is $10^{0.5} = 10^{1/2}$? **Answer:** $10^{0.5} = \sqrt{10} \approx 3.16288$

8. What is $10^{0.1} = 10^{1/10}$? **Answer:** $10^{0.1} = \sqrt[10]{10} \approx 1.258926$

9. Similarly: $10^{0.01} = \sqrt[100]{10} \approx 1.02329$

$$10^{0.001} = \sqrt[1000]{10} \approx 1.00231$$

10. What is $10^{0.27}$? **Answer:**

$$10^{0.27} = 10^{27/100} = \sqrt[100]{10^{27}} = \left(\sqrt[100]{10}\right)^{27} \approx 1.862$$

Moving to Logarithms

$\log(y)$ is how many tens you multiply together to get y

$$10^{\log(y)} = y$$

$$\log(10) = ? \text{ 1 } \quad \text{because } 10^{\text{1}} = 10$$

$$\log(100) = ? = \text{ 2 } \quad \text{because } 10^{\text{2}} = 100$$

$$\log(1000) = ? = \text{ 3 } \quad \text{because } 10^{\text{3}} = 1000$$

$$\log(100,000) =$$

$$A = 2 \quad B = 3 \quad C = 4 \quad D = 5 \quad E = 6 \quad \boxed{D}$$

Still moving to Logarithms

$\log(y)$ is how many tens you multiply together to get y

$$10^{\log(y)} = y$$

$$\log(0.1) = ? = -1 \quad \text{because} \quad 10^{-1} = 1/10 = 0.1$$

$$\log(0.01) = ? = -2 \quad \text{because} \quad 10^{-2} = 1/100 = 0.01$$

$$\log(10^x) = ? = x \quad \text{duh?}$$

How confused are you?

A=not at all B=a bit C=a lot D=:’(

You try it:

$$\log(100,000) = ?$$

$$A = 2 \quad B = 3 \quad C = 4 \quad D = 5 \quad E = 6 \quad \boxed{D}$$

$$\log(0.001) = ?$$

$$A = 3 \quad B = 0 \quad C = 0.001 \quad D = -2 \quad E = -3 \quad \boxed{E}$$

$$\log(100 \times 1000) = ?$$

$$A = 6 \quad B = 5 \quad C = 3 \quad D = 9 \quad E = -5 \quad \boxed{B}$$

$$\log(100/1000) = ?$$

$$A = -1 \quad B = 0 \quad C = 1 \quad D = -3 \quad E = -5 \quad \boxed{A}$$

Key Fact Of Logs

$$\text{First Law of Logs} \quad \log(a \times b) = \log(a) + \log(b)$$

This means logs convert **multiplication** into **addition**.

Example: $\log(100 \times 1000) = \log(100) + \log(1000) = 2 + 3 = 5$

It is easy to understand why the first law works:

$\log(a)$ = (how many 10's you multiply to get **a**)

$\log(b)$ = (how many 10's you multiply to get **b**)

THEREFORE multiplying ALL these 10s gives **$a \times b$**

CONCLUDE $\log(a \times b)$ is this number of 10s: that is, $\log(a) + \log(b)$.

Consequences of the Key Fact

We are told: $\log(2) \approx 0.3$ (from table page 289)

$$\begin{aligned}\log(20) &= \log(10 \times 2) \\ &= \log(10) + \log(2) && \text{we know } \log(10) = 1 \\ &\approx 1 + 0.3 \\ &\approx 1.3\end{aligned}$$

Use this method to find $\log(200)$

$$A = 30 \quad B = 3 \quad C = 2.3 \quad D = 30 \quad \boxed{C}$$

A few more

We are still told $\log(2) \approx 0.3$

Find $\log(0.002)$

$$A = -3.3 \quad B = -2.3 \quad C = -2.7 \quad D = -3.7 \quad \boxed{C}$$

Find $\log(2 \times 10^x)$

$$A = 2x \quad B = 2 + x \quad C = .3x \quad D = 10x + \log(2) \quad E = x + .3 \quad \boxed{E}$$

A Trick!

The graph and the table can both be used to find logs of numbers between 1 and 10.

To find the log of ANY number, we move the decimal point:

$$\log(10^n \times x) = n + \log(x)$$

Example:

$$\log(275.67) = \log(10^2 \times 2.7567) = 2 + \underbrace{\log(2.7567)}_{\text{look this up!}}$$

Its called the **MOVING DECIMAL POINT TRICK** because 2 is how many places you need to move the decimal point of 275.67 to obtain a number between 1 and 10.

Inverses!

logs are “**opposite**” of exponents (inverse function of antilog)

So every fact about exponents corresponds to a fact about logs:

	laws of exponents	corresponding law of logs
(1)	$10^a \times 10^b = 10^{a+b}$	$\log(xy) = \log(x) + \log(y)$
(2)	$10^0 = 1$	$\log(1) = 0$
(3)	$10^{-a} = 1/10^a$	$\log(1/x) = -\log(x)$
(4)	$(10^a)^p = 10^{ap}$	$\log(x^p) = p \log(x)$
(5)	$10^a / 10^b = 10^{a-b}$	$\log(x/y) = \log(x) - \log(y)$

Example: $\log(x^a/y^b) = ?$

$$\begin{aligned}
 A &= a \log(x) / (b \log(y)) & B &= a \log(x) + b \log(y) \\
 C &= a \log(x) - b \log(y) & D &= (a + \log(x)) - (b + \log(y)) \quad \boxed{C}
 \end{aligned}$$

Rule (4): $\log(x^p) = p \log(x)$

Explanation of (4)

$$\log(a \times a) = \log(a) + \log(a) = 2 \log(a)$$

$$\log(a \times a \times a) = \log(a) + \log(a) + \log(a) = 3 \log(a)$$

In general: the number of tens you multiply to get x^p is p times as many tens as you multiply to get x .

What is $\log(\sqrt{x^7})$?

$$A = 7 + \log(x) \quad B = (7/2) + \log(x) \quad C = 7/2 \quad D = 7/2 \log(x) \quad \boxed{D}$$

Find x by solving $10^x = 5$.

$$A = 5 \quad B = 0.5 \quad C = \log(5) \quad D = \log(0.5) \quad E = \log(5) - \log(10)$$

\boxed{C}

That's it. Thanks for being here.

