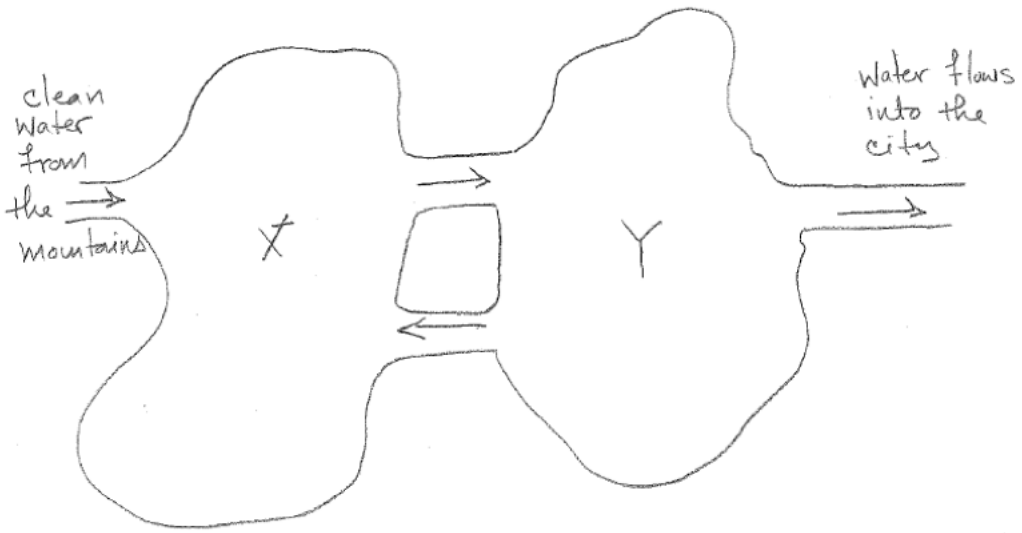
Graded Problem 7

Math 4B, Spring 2017, Dr. Paul

1. A city's water system is set up as shown.



Each of the two reservoirs holds 100 million gallons of water. Water flows into reservoir X from the mountains at a rate of 600,000 gallons, and out from reservoir Y to the city at the same rate. Water also circulates between the reservoirs: flowing from X to Y at a rate of 800,000 gallons per hour and from Y to X at a rate of 200,000 gallons per hour. Vector, the villian from *Despicable Me*, dumped 1,500 lb sodium hypochlorite into reservoir X.

(a) Find a system of differential equations which models the amounts of sodium hypochlorite in each reservoir as functions $x(t)$ and $y(t)$ of time.

(b) If $u(t) = 2x(t) + y(t)$, what is $u'(t)$ in terms of $u(t)$? Find the function $u(t)$.

(c) If $v(t) = 2x(t) - y(t)$, what is $v'(t)$ in terms of $v(t)$? Find the function $v(t)$.

(d) Using the functions $u(t)$ and $v(t)$ above, find the functions $x(t)$ and $y(t)$.

(e) Write a few sentences explaining how the process you used here relates to our eigenvector/alue method from class.

(f) How long does it take for the concentration in Y to get down to 3 lb/million gallons?