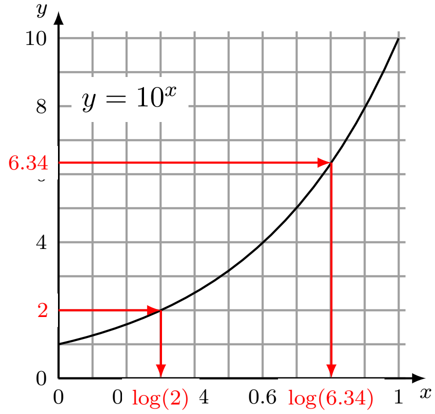
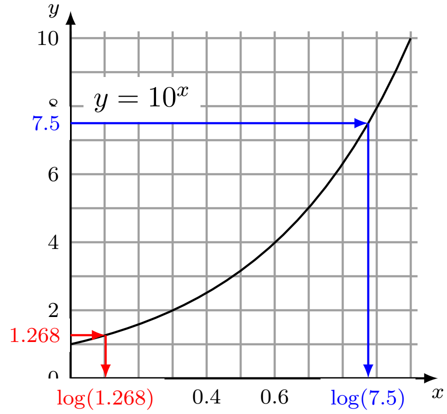
Math 34A Winter 2020
Old Midterm 3 #3 Solutions

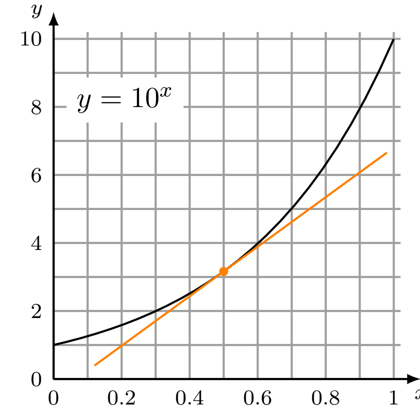
1. Here are the three graphs we'll use in solving these problems:



Part (a)



Parts (a) and (b)



Part (c)

(a) There are two natural ways to do this problem. One is to simply multiply $6.34 \times 2 = 12.68$, then compute the logarithm of this using the "move the decimal point" trick:

$$\log(6.34 \times 2) = \log(12.68) = \log(10^1 \times 1.268) = \log(10) + \log(1.268) = 1 + \log(1.268).$$

Now we can use the graph to find that $\log(1.268) \approx 0.10$, and so $\log(6.34 \times 2) \approx \boxed{1.10}$.

The second approach is to use the rules of logs to write

$$\log(6.34 \times 2) = \log(6.34) + \log(2),$$

then use the graph to find both $\log(6.34) \approx 0.80$ and $\log(2) \approx 0.30$. We then get $\log(6.34 \times 2) \approx \boxed{1.10}$, as before.

(Mathematica tells me that $\log(6.34 \times 2) \approx 1.103\,119\,25\ldots$)

(b) The solution to $10^x = 0.075$ is $x = \log(0.075)$. Again we use the "move the decimal point" trick to see that is what we need here:

$$\log(0.075) = \log(10^{-2} \times 7.5) = \log(10^{-2}) + \log(7.5) = -2 + \log(7.5).$$

We use the graph to find that $\log(7.5) \approx 0.88$, so $\log(0.075) \approx -2 + 0.88 = \boxed{-1.12}$. (Mathematica tells me that $\log(0.075) \approx -1.124\,938\,7\ldots$, so we're pretty close.)

(c) We've drawn the tangent line at $x = 0.5$ on the third graph, above. The slope of this line is about

$$m = \frac{6.1 - 1.0}{0.9 - 0.2} = \frac{5.1}{0.7} \approx \boxed{7.29}.$$

The actual slope of the tangent line to $y = 10^x$ at $x = 0.5$ is $m = \sqrt{10}\,\ln(2) \approx 7.281\,413\,400\ldots$, so as usual we're pretty close.

2. We write down the answers without much commentary:

(a) $\dfrac{d}{dx}(3x^6 - 2x^2 + 7) = 18x^5 - 4x$

(b) $\dfrac{d^2}{dx^2}(9x + 5e^{2x}) = \dfrac{d}{dx}(9 + 10e^{2x}) = 20e^{2x}$

(c) $\dfrac{d}{dx}(8\sqrt{x} + kx + k^2) = \dfrac{d}{dx}(8x^{1/2} + kx + k^2) = 8 \cdot \dfrac{1}{2}x^{-1/2} + k + 0 = 4x^{-1/2} + k$. This can also be written as $4/\sqrt{x} + k$.

3. The tangent line is the line through $(t, h(t)) = (0, h(0)) = (0, 9)$ with slope $h'(0) = -2$. Thus the line has equation

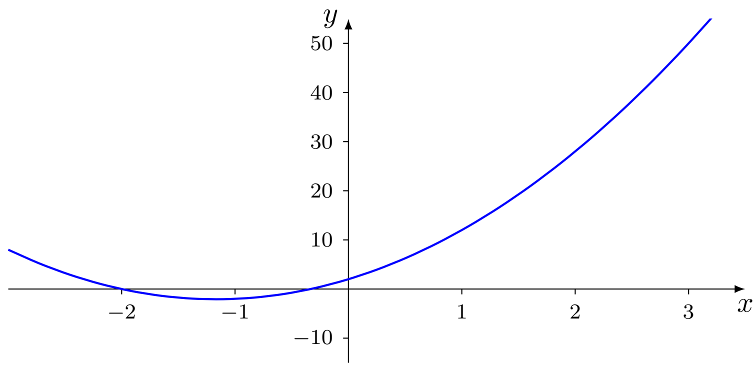
$$y - 9 = -2(t - 0) \qquad \text{or, equivalently,} \qquad y = -2t + 9.$$

(a) When $t = 2$, the depth of the water is $h(2) \approx -2(2) + 9 = 5$ meters. The volume of this water is therefore

$$\text{volume} = \text{Area of the base} \times \text{depth of the water} \approx (2 \text{ meters})^2 \times (5 \text{ meters}) = \boxed{20 \text{ m}^3}.$$

(b) This question can be re-phrased as: what is t when the depth is zero: $h(t) = 0$? We have estimated the depth as $h(t) \approx -2t + 9$, so this is zero when $-2t + 9 = 0$. Solving, we get $t = \boxed{4.5 \text{ hours}}$.

4. Here is a picture of the graph of $y = 3x^2 + 7x + 2$:



(a) The slope of the graph is the derivative, $\frac{dy}{dx}$. Since $\frac{dy}{dx} = 6x + 7$, the slope of the graph at $x = 1$ is $6(1) + 7 = \boxed{13}$.

(b) The tangent line at $x = 1$ has slope 13 (from part (a)) and passes through the point $(x, y) = (1, 3(1)^2 + 7(1) + 2) = (1, 12)$. Thus the equation of the tangent line is

$$y - 12 = 13(x - 1) \qquad \text{or, equivalently} \qquad \boxed{y = 13x - 1}.$$

(c) The slope is $\frac{dy}{dx} = 6x + 7$, so this is zero when $6x + 7 = 0$; that is, when $x = -7/6$. The y-coordinate at this point is

$$y = 3\left(-\frac{7}{6}\right)^2 + 7\left(-\frac{7}{6}\right) + 2 = \frac{49}{12} - \frac{49}{6} + 2 = \boxed{-\frac{25}{12}}.$$

(d) The slope is $\frac{dy}{dx} = 6x + 7$, which is 11 when $6x + 7 = 11$. The solution to this is $\boxed{x = 2/3}$.

5. (a) The velocity of the rocket is $h'(t) = -6t + 50$ m/s.

(b) The acceleration of the rocket is $h''(t) = -6$ m/s^2.

(c) The initial speed of the rocket is $h'(0)$. From part (a), this is 50 m/s.

(d) The speed of the rocket is zero when $h'(t) = 0$. From part (a), this means when $-6t + 50 = 0$. Solving, we get $t = 50/6 = \boxed{25/3 \text{ seconds}}$.

(e) The height of the rocket above the ground at time 2 seconds is $h(2) = 700 - 3(2)^2 + 50(2) = \boxed{788 \text{ meters}}$.