

# Welcome To Math 34A!

## Differential Calculus

### Instructor:

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### Office Hours:

MTWR after class 2:00-3:00, and by appointment. Details on Gauchospace.

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# Quiz Corrections

While doing well on the quizzes is important, what is more important to me is that you learn the material so that you can do well on the Midterm Exams and the Final. To that end, you can optionally do **Quiz Corrections** after your quiz is graded. To get credit back on your quizzes, please answer the following in your own **words**:

- What was the problem was asking you to do?
- What was the mistake(s) in your work?
- Correctly and completely rework the problem, explaining your steps as you go.
- We know that mistakes are simply an opportunity to learn; what did you learn from this mistake?

then **for each problem you correct, you will earn 50% of the missing points back on the corresponding Quiz**. This means a 50% can be corrected to a 75%, a 90% to a 95%, etc. Quiz corrections are due 1 week after the graded quiz is posted.

# Announcements

- Thursday is the last day to drop a class in Session A.
- I will have the exams graded ASAP (hopefully today or tomorrow morning).

# Counting and Our Logarithmic Perception of the World

Vsauce:

1,2,3,4,5,6,7,8,9,10,11,12,13,14,15,16,17,18,19,20,21,22,23,24,25,26,27,  
28,29,30,31,32,33,34,35...

<https://www.youtube.com/watch?v=Pxb5lSPLy9c>

# Warm-up

- $\log_9(9^4) = \boxed{4}$
- $\log_3(9^4) = \boxed{8}$
- $\log_{27}(27^5) = \boxed{5}$
- $\log_3(27^5) = \boxed{15}$
- $\log_5(25^{17}) = \boxed{34}$
- $\log_3(27^0) = \boxed{0}$
- $\log_8(2^{12}) = \boxed{4}$
- $\log_8(2) = \boxed{1/3}$

# Warm-up Part II

- $\log_{100}(100^7) = \boxed{7}$
- $\log_{10}(100^7) = \boxed{14}$
- $\log_{10}(1,000,000^{-4}) = \boxed{-24}$
- $\log_{10}(2) = \text{about } \boxed{.3}$     Still warm-up?
- $\log_{10}(2^{11}) = \text{about } .3 \cdot 11 = \boxed{3.3}$

Logs are “**opposite**” of exponentials (that’s why we sometimes call  $10^x$  as antilog). So every fact about exponents corresponds to a fact about logs:

laws of exponents	corresponding law of logs
(1) $10^a \times 10^b = 10^{a+b}$	$\log(xy) = \log(x) + \log(y)$
(2) $10^0 = 1$	$\log(1) = 0$
(3) $10^{-a} = 1/10^a$	$\log(1/x) = -\log(x)$
(4) $(10^a)^p = 10^{ap}$	$\log(x^p) = p \log(x)$
(5) $10^a/10^b = 10^{a-b}$	$\log(x/y) = \log(x) - \log(y)$

Example:  $\log(x^a/y^b) = ?$

$$A = a \log(x)/(b \log(y))$$

$$B = a \log(x) + b \log(y)$$

$$C = a \log(x) - b \log(y)$$

$$D = (a + \log(x)) - (b + \log(y))$$

C

# Rule (4): $\log(x^p) = p \log(x)$

Explanation of (4)

$$\log(a \times a) = \log(a) + \log(a) = 2 \log(a)$$

$$\log(a \times a \times a) = \log(a) + \log(a) + \log(a) = 3 \log(a)$$

In general: the number of tens you multiply to get  $x^p$  is  $p$  times as many tens as you multiply to get  $x$ .

What is  $\log\left(\sqrt{\frac{1}{x^7}}\right)$ ?

$$A = 7 - \log(x) \quad B = (7/2) - \log(x) \quad C = -7/2 \quad D = -(7/2) \log(x)$$

D



Find  $x$  by solving  $3^x = 5$ .

A  $\log(5)/\log(3)$

B  $\log(3)/\log(5)$

C  $\log(5)^3$

D  $\log(3) - \log(5)$

E  $\log(5) - \log(3)$

A

## §7.5: Using logs to multiply

First rule of logs:  $\log(a \times b) = \log(a) + \log(b)$

Example: Find  $2.7 \times 1.6$  using logs

### Method

- (i) Look up  $\log(2.7)$  and  $\log(1.6)$
- (ii) Add these
- (iii) Take the **antilog** of result from (ii)
- (iv) Think: Is the answer **reasonable** or did I goof up?

## §7.5: Using logs to multiply

First rule of logs:  $\log(a \times b) = \log(a) + \log(b)$

Example: Find  $2.7 \times 1.6$  using logs

Look how I write the answer.

- $\log(2.7 \times 1.6) = \log(2.7) + \log(1.6)$
- On the table we see that  $\log(2.7) \approx 0.43$  and  $\log(1.6) \approx 0.20$ , so  $\log(2.7 \times 1.6) \approx 0.43 + 0.20 = 0.63$
- Is this the answer? Heck No! It is the **log** of the answer
- $2.7 \times 1.6 \approx \text{antilog}(0.63) = 10^{0.63}$
- $10^{0.63} \approx 4.3$
- Is my answer 4.3 reasonable? Yes, about  $2 \times 2 = 4$ .

## §7.5: Using logs to divide

Remember Log Rule (5):  $\log(a \div b) = \log(a) - \log(b)$

**Example:** Use this rule to find  $38.2/1.77$

### Method

(i) Look up  $\log(3.82)$  and  $\log(1.77)$ , find  $\log(38.2)$

★ You can find  $\log(38.2)$  by adding 1 to  $\log(3.82)$  because 38.2 is 3.82 times one more power of 10.★

(ii) **Subtract!**

(iii) Take the **antilog** of result from (ii)

(iv) Think: Is the answer **reasonable** or did I goof up?

A = done

B = confused

## §7.5: Powers Using Logs

Or, exploiting Log Rule (4):  $\log(a^p) = p \log(a)$

Use this to find  $\sqrt{70}$ .

One Approach:

- (i) Use table and move decimal point trick to find  $\log(70)$   
★I will show the graph of the exponential function  $10^x$  and talk about the graph method next lecture.★
- (ii)  $\log(\sqrt{70}) = \log(70^{1/2}) = (1/2) \log(70)$
- (iii) Take the **antilog** of result from (ii)
- (iv) Think: Is the answer **reasonable** or did I goof up?

A= done    B= working    C= confused

Answer:  $\sqrt{70} \approx 8.32$ . Is that reasonable?

# Computer Applications

One kilobyte (1 **KB**) is  $2^{10}$ .

**Problem:** Calculate  $2^{10}$  using logs. **Hint:**  $\log(2) \approx 0.3$

$A \approx 3$     $B \approx 10.3$     $C \approx 30$     $D \approx 1000$     $E \approx 1100$

**D**

So:  $2^{10} \approx 10^3 = 1000$  (really  $2^{10} = 1024$ ).

**1KB** is really  $2^{10} = 1024 \approx 10^3$  (**K** is **Kilo** = thousand)

**1MB** is really  $2^{20} = (2^{10})^2 \approx (10^3)^2 = 10^6$  (**M** is **Mega** = million)

**1GB** is really  $2^{30} = (2^{10})^3 \approx (10^3)^3 = 10^9$  (**G** is **Giga** = billion)

**1TB** is really  $2^{40} = (2^{10})^4 \approx (10^3)^4 = 10^{12}$  (**T** is **Tera** = trillion)

Example: suppose on a certain island the population of rabbits doubles every generation. After 20 generations it multiplies by...  
 $2^{20} \approx 1$  million.

Powers of 2 are easy to do, even in your head. To work out  $2^n$  the **log** of the answer is approximately  $0.3n$ , so  $2^n$  is 1 followed by  $0.3n$  zeroes.

## §7.7: Solving Exponential Eq'ns

**1.** Find  $x$  by solving  $10^x = 5$ .

$$A = 5 \quad B = 0.5 \quad C = \log(5) \quad D = \log(0.5)$$

$$E = \log(5) - \log(10) \quad \boxed{C}$$

Look how I write the answer!

$$\log(10^x) = \log(5) \quad \text{Take logs of both sides}$$

$$x = \log(10^x) = \log(5) \quad \text{Using } \log(a^p) = p \log(a) \text{ and } \log(10) = 1$$

# Examples:

Use the Fourth Law:

$$\log(a^x) = x \log(a)$$

Slogan: Logs bring exponents down to ground level.

**2.** Solve  $3^x = 7$

$$A = \log(7/3) \quad B = \log(7) - \log(3) \quad C = \log(7) + \log(3)$$

$$D = \log(3)/\log(7) \quad E = \log(7)/\log(3) \quad \boxed{E}$$

Look how I write the answer:

$$\log(3^x) = \log(7)$$

$$x \log(3) = \log(3^x) = \log(7)$$

$$\text{So: } x = \log(7)/\log(3)$$

Take logs of both sides

Using  $\log(a^p) = p \log(a)$



# Examples:

Use the Fourth Law:

$$\log(a^x) = x \log(a)$$

Slogan: Logs bring exponents down to ground level.

**3.** Solve  $7^{x+2} = 30$ .

$$A = \frac{\log(30) - 2\log(7)}{\log(7)} \quad B = \frac{\log(30)}{\log(7)} - 2 \quad C = \frac{\log(30) - \log(49)}{\log(7)}$$

$$D = \frac{\log(30/49)}{\log(7)} \quad E \approx -0.25213$$

All are correct!

# Examples:

Use the Fourth Law:

$$\log(a^x) = x \log(a)$$

Slogan: Logs bring exponents down to ground level.

4. Solve  $7 \times 3^y = 2^{4y+3}$

$$A = \frac{3 \log(2) - \log(7)}{\log(3) - 4 \log(2)} \quad B = \frac{3 \log(2)}{7 \log(3)} \quad C = \frac{3 \log(2)}{7 \log(3) - 4 \log(2)}$$

$$D = \frac{7 \log(3) - 4 \log(2)}{3 \log(2)}$$

E = none of the above

A

# Compound Interest

At the end of each year a bank pays 7% interest into your account. Initially have \$10,000 in account. How much after 10 years?

**Think**  $10 \times 7\% = 70\%$  in 10 years, so have **\$17,000** but that is **wrong**.

After 1 year:  $\$10,000 \times 1.07 = \$10,700$

After 2 years:  $\$10,700 \times 1.07 = \$10,000 \times 1.07 \times 1.07 = \$11,449$

After 3 years:  $\$11,449 \times 1.07 = \$10,000 \times (1.07)^3 = \$12,250.40$

Each year **what you had before** is **multiplied** by 1.07. Thus **compound** interest.

So after 10 years have

$$\$10,000 \times \underbrace{1.07 \times 1.07 \times \cdots \times 1.07}_{10 \text{ times}} = 10,000 \times (1.07)^{10} \approx \boxed{\$20,000}$$

**Conclusion:** Money approximately doubles in 10 years!

So in 20 years multiplies by 4, in 30 years by 8,...

That's it. Thanks for being here.



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