A SIMULATION TO MODEL EXPONENTIAL GROWTH
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# SHARING TEACHING IDEAS

## A SIMULATION TO MODEL EXPONENTIAL GROWTH

Cancer cells, like other biological populations, can grow exponentially, at least for part of the time. This article describes a simulation that models exponential growth. Instead of cancer, teachers might choose a simulation that is based on the population growth of humans or bacteria.

### SUMMARY OF CANCER

Cancer typically grows from one bad cell, which reproduces itself. Students may remember from their biology class that this process of cell division is called *mitosis*. The two resulting cells, called *daughter cells,* reproduce, and so on. All the resulting cells are clones of the original cell. Although cancer cells can die, their tendency is to live and reproduce until they kill the patient.

### SUMMARY OF THE SIMULATION

Choose one person to be the founder of the population. Give that student a die, and ask him or her to toss the die over and over. Each toss represents a year. When a 3 appears, the student "reproduces," that is, the instructor chooses another person, who stands beside the founder and receives a die. On a signal from the teacher, representing a year, each person tosses a die. If either person gets a 3, he or she "reproduces" and the instructor selects another student to join the population. This population huddles in a circle, apart from the rest of the class.

Meanwhile, the teacher uses a transparency of an enlarged **figure 1** to plot points $(t, y)$. Here, $t$ is time in years (minutes, for bacteria) and $y$ is the corresponding number of cells or individuals in the population. Each point represents a paired observation. Such a diagram is called a *scatterplot*.

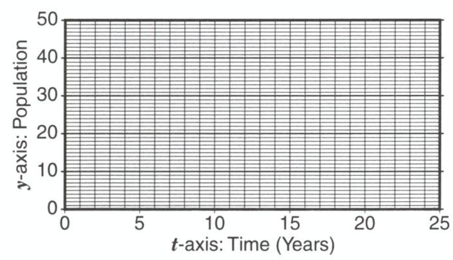


Fig. 1
Grid for scatterplot for cancer game

Continue playing this game and plotting points until all students have become cancer cells. With a class of about twenty-five students, the process takes less than twenty minutes. The game dramatizes how population increases with time. Explain that because the probability of getting a 3 is 1/6, the probability that an individual reproduces in a given year is also 1/6. Although reproduction is merely a probability for an individual, it is as "cer-



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tain as death and taxes" for a large population. The game models a population that is growing by 1/6 each year. The exponential function does not apply precisely in this game, because the growth is not continuous and the population is small. But an exponential function can approximate the growth; see the **appendix** for details.

**Figure 2** is a graph of this function. Lay the corresponding enlarged transparency on top of the enlarged transparency of **figure 1.** Students are often surprised at how closely it fits. **Figure 3** shows the results of one of these simulations, with the scatterplot and the overlaid exponential curve.

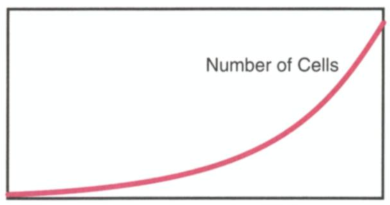


Fig. 2
$y = e^{0.1542t}$

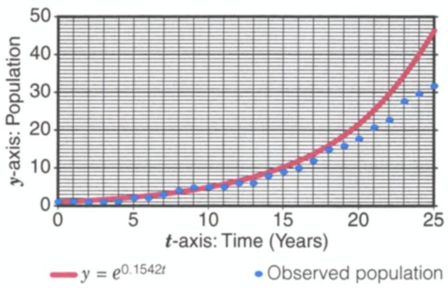


Fig. 3
Exponential curve superimposed on scatterplot

The teacher may wish to give each student an enlarged paper copy of **figure 1** and **figure 2.** Then students plot points on the paper **figure 1.** Ask students to lay one figure on top of the other and hold them up to a light.

## PLAYING THE GAME WITH LARGER OR SMALLER GROUPS

With a large group, use **figure 1** as long as possible, but it may be too small for the graph. The students themselves are a model of the number of cells. The population clusters in a circle, and everybody can see how it grows.

With a small group, one person can represent several cells by tossing several dice at once. The teacher still draws the graph. The teacher can ask a student to move objects, for example, blocks, to show the growth of the tumor.

## CONCLUSION

The main purpose of the game is to model exponential growth. Although the model is not precise, it does approximate such growth adequately for demonstration purposes.

## APPENDIX

In the game, each malignant cell has a probability of 1/6 of reproducing in one year. By the law of large numbers, a large population almost certainly increases by 1/6 in a year. So whatever the population, a year later it is 7/6 as large. The formula for exponential growth of a population is

$$y = Ne^{rt}.$$

In this equation,

- $y$ is the population size at a given time, $t$;
- $N$ is the population at time 0, when observation begins;
- $r$ is a positive parameter called the (instantaneous) growth rate; and
- $t$ is time in years.

At time $t + 1$, the population is

$$y = NE^{r(t+1)}.$$

Whatever the population at any time, a year later it is $e^r$ times as large. In this situation, the equation is

$$e^r = \frac{7}{6},$$

$$r = \ln \frac{7}{6}$$

$$\approx 0.1542.$$

In the game, the initial population is 1. So the desired exponential formula is

$$y = e^{0.1542t}.$$

Iovinelli (1997) describes a computer simulation of the logistic function. That function also describes the growth of a population. Appelbaum (forthcoming) gives further information on modeling cancerous growth.

***The game dramatizes how population increases with time***

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