Math 360
Section 1.4 Exercises

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In 22 and 24, describe all the elements in the cyclic subgroup of $GL(2,\mathbb{R})$ generated by the given 2×2 matrix.

22. $\begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix}$

    Answer: Let $A = \begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix}$. The elements of the subgroup are:

$$A = \begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix} \quad \text{and} \quad A^2 = I_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

24. $\begin{bmatrix} 3 & 0 \\ 0 & 2 \end{bmatrix}$

    Answer: Let $A = \begin{bmatrix} 3 & 0 \\ 0 & 2 \end{bmatrix}$. The elements of the subgroup are:

$$A = \begin{bmatrix} 3 & 0 \\ 0 & 2 \end{bmatrix} \quad \text{and} \quad A^2 = I_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

27. Find the order of the cyclic subgroup of $\mathbb{Z}_4$ generated by 3.

    Answer: The elements are $3, 2, 1, 0$, so the order is 4. (also, 3 and 4 are relatively prime, so the group cannot have less than 4 elements. I claim this intuitively; I expect we will prove it as a theorem in a later section, so I omit the proof here.)

36. Complete Table 1.4.25 (below) and use it for this problem.

    a. Complete the table to give a cyclic group $\mathbb{Z}_6$ of 6 elements (you need not prove the associative law.)

    Answer:

$\mathbb{Z}_6$:

+	0	1	2	3	4	5
0	0	1	2	3	4	5
1	1	2	3	4	5	0
2	2	3	4	5	0	1
3	3	4	5	0	1	2
4	4	5	0	1	2	3
5	5	0	1	2	3	4

    b. Compute the subgroups $\langle 0 \rangle$, $\langle 1 \rangle$, $\langle 2 \rangle$, $\langle 3 \rangle$, $\langle 4 \rangle$, and $\langle 5 \rangle$ of the group given in part (a).

    Answer: We will list the elements of the cyclic groups in order, by multiples of the generator.

$$\begin{array}{lll} \langle 0 \rangle = \{0\} & \langle 1 \rangle = \{1,2,3,4,5,0\} & \langle 2 \rangle = \{2,4,0\} \\ \langle 3 \rangle = \{3,0\} & \langle 4 \rangle = \{4,2,0\} & \langle 5 \rangle = \{5,4,3,2,1,0\} \end{array}$$

c. Which elements are generators for $\mathbb{Z}_6$?

Answer: 1 and 5.

d. $\mathbb{Z}_6 = \langle 1 \rangle = \langle 5 \rangle \begin{Bmatrix} \geq \langle 2 \rangle = \langle 4 \rangle \\ \geq \langle 3 \rangle \end{Bmatrix} \geq \langle 0 \rangle$