5. Prove that the product of two compact Hausdorff spaces is compact & Hausdorff.

Let X & Y be compact & Hausdorff. Consider $\{W_\alpha\}_{\alpha \in \Delta}$, an open cover of $X \times Y$. For each $W_\alpha \in$ $W_\alpha = \bigcup (U_\beta \times V_\beta)_\alpha$ where U_β is open in X & V_β is open in Y. Then $\{W_\alpha\} \subset \{(U_{\beta_\alpha} \times V_{\beta_\alpha})\}$ is an

open cover of $X \times Y$. Thus $\bigcup U_{\beta_\alpha} \times V_{\beta_\alpha} = \bigcup U_{\beta_\alpha} \times \bigcup V_{\beta\alpha}$

$\Rightarrow \{U_{\beta_\alpha}\}$ & $\{V_{\beta_\alpha}\}$ are open covers of X & Y.
Thus there a finite subcovers $\{U_{\beta_i}\}_{i=1}$ $\{V_j\}_{j=1}^{m}$.

Therefore $\{U_{\beta_i} \times V_i, V_j\}$ is an open cover of $X \times Y$.

Each of these was associated to some α so $\{W_i\}_{i=1}^{m+n}$

is an open cover of $X \times Y$.

Let $(x,y), (\bar{x},\bar{y}) \in X \times Y$. Then $\exists$ open sets U_x $U_{\bar{x}}$ of x & $\bar{x}$ with $U_x \cap U_{\bar{x}} = \emptyset$ & similarly $V_y, V_{\bar{y}}$. $\Rightarrow (U_x \times V_y) \cap (U_{\bar{x}} \times V_{\bar{y}}) = \emptyset$. $\square$