Welcome To Math 34A! Differential Calculus

Instructor:

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South Hall 6431X (Grad Tower, 6th floor, blue side, first door on the right)

Office Hours:

MTWR after class 2:00-3:00, and by appointment. Details on Gauchospace.

Videos

Vsauce: Logarithmic thinking:

https://www.youtube.com/watch?v=Pxb5lSPLy9c&t=1m53s

3Blue1Brown: Modeling Caronavirus Spread

https://youtu.be/Kas0tIxDvrg

Final Exam: Wednesday, July 17, 12:30-1:35PM

Bring:

- A pen or sharp pencil.
- 3" × 5" cards with your notes. **Three** notecards allowed.
  This is not a typo. The exam is 3 hours long.
- Student ID.

Don't bring:

- A calculator

Please Be Early!
Exam is cumulative, with its focus spread to all topics.
Please study the topics beyond just formulas and procedures. This approach will set you up the for success.

Final Exam: Wednesday, July 17, 12:30-1:35PM

Here are the topics we've covered:

- percentages, inverse functions, pythagorean theorem, units, areas and volumes, and lines.
- general topics from ch1 like word problems, substitution, and solving equations.
- limits, sums, change in a function, percent error, exponentiation, logarithms
- related types of word problems: like half-life and doubling time, etc.
- calculus! (i.e. derivatives)

Here's a more detailed list of topics relating to derivatives:

- calculus! (i.e. derivatives)
  - finding derivatives of expressions (mostly things made out of x^n and e^{kx})
  - understanding the definition of derivative (limit of average rate of change/slope of secant line)
  - linear approximation (finding and using eqn of tangent line, or just using derivative to get useful info somehow)
  - second derivative and concavity
  - optimization and the related word problems

Grade Breakdown

Homework 20%

Quizzes 10%

Midterm Exams 40% (Worth 20% each)

Final Exam 30%

iclicker - up to 3% extra credit

Let's refresh some Calculus

The derivative of y with respect to x tells us the rate of change, but it depends on the current value of x!

If we write y as a function of x like this: $y = f(x)$, then the derivative is written as

$$\frac{\mathrm{d}y}{\mathrm{d}x} \quad \text{or} \quad \frac{\mathrm{d}f}{\mathrm{d}x} \quad \text{or} \quad f'(x)$$

It is the limit of "average rate of change" over shorter and shorter Δx:

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

also known as "instantaneous rate of change"

Differentiation Power Moves

$$\boxed{\frac{d}{dx}\left(e^{kx}\right) = ke^{kx}}$$ and $$\boxed{\frac{d}{dx}\left(x^{n}\right) = nx^{n-1}}$$

Question: Find $\frac{d}{dx}\left(3x^2 + 4e^{-2x} - x^3\right)$

A= $12e^{2x} + 15x^2$ B= $12e^{3x} + 15x^3$ C= $4e^{3x} + 15x^2$
D= $6x + 8e^{-2x} - 15x^2$ E= Other E

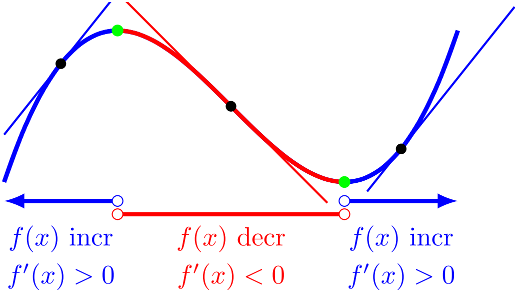
Tangent Line Approximation

To do a tangent line approximation:

(i) Find the equation of the tangent line.

(ii) Plug in the required value(s) into this equation.

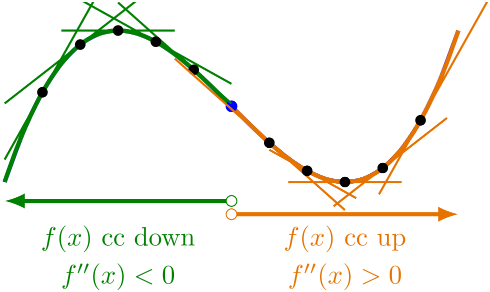
Meanings: The First Derivative



Point:

$$f'(x) > 0 \iff f(x) \text{ is increasing}$$
$$f'(x) < 0 \iff f(x) \text{ is decreasing}$$

Meanings: The Second Derivative



Point:

$$\begin{aligned} f''(x) > 0 &\iff f'(x) \text{ is increasing} \\ &\iff f(x) \text{ is concave up} \\[1ex] f''(x) < 0 &\iff f'(x) \text{ is decreasing} \\ &\iff f(x) \text{ is concave down} \end{aligned}$$

Concavity

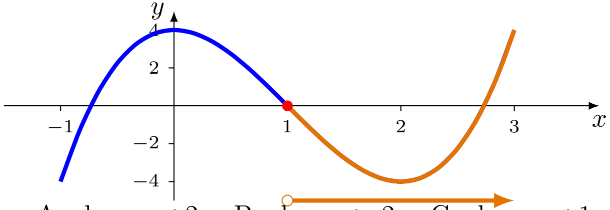
$f''(x) > 0 \iff f(x)$ is concave up

$f''(x) < 0 \iff f(x)$ is concave down

(1) For which values of x is $f(x) = x^3 - 6x^2 + 3x + 2$ concave up?

A when $x = 0$ B when $x < 6$ C when $x > 6$
D when $x < 2$ E when $x > 2$ E

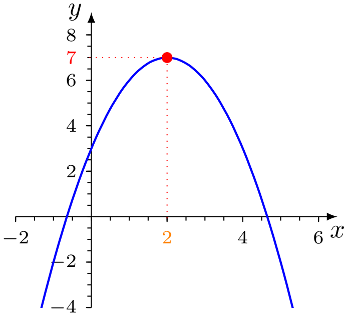
(2) Where is $f''(x) > 0$?



A when $x < 2$ B when $x > 2$ C when $x < 1$
D when $x > 1$ E when $-0.7 < x < 1$ D

§8.13: Max/Min problems

Often want to find the biggest, smallest, most, least, maximum, minimum of something.



Here's the graph of
$y = f(x) = -x^2 + 4x + 3$

The <u>maximum value</u> or just <u>maximum</u> of the function is 7.

The <u>value of x</u> which gives the maximum of $f(x)$ is $x = 2$

We write $f(2) = 7$.

For this example you can see this is the maximum because

$$f(x) = -x^2 + 4x + 3 = -(x-2)^2 + 7$$

$(x-2)^2$ is always positive except when $x = 2$
so the maximum must be at $x = 2$.

How To Find A Max / Min

(1) Find $f'(x)$

(2) Solve $f'(x) = 0$. This is the x value that gives the max / min.

(3) To find the maximum / minimum plug the value of x found in (2) back into $f(x)$.

Example: Use this method to find the x-value where maximum of the function $f(x) = 5x - e^{2x}$ occurs.

A $= 0$ B $= \ln(5)$ C $= 2\ln(5)$ D $= 2\ln(5/2)$ E $= \ln(5/2)/2$

Answer: E

Word Problem #8

A farmer is growing wheat.

- On July 1, she has $1,000$ bushels and this increases by 50 bushels per day.
- The price of a bushel on July 1 is \$10 and is dropping at a rate of 20 cents per day.
- She will harvest and sell on the same day.

How many days should she wait, assuming these trends continue?

$$A = 5 \quad B = 10 \quad C = 15 \quad D = 20 \quad E = \text{other} \quad C$$

Some Review Problems From Webwork

* An old midterm may be reviewed in class instead.
This midterm will be posted with worked out solutions.

Unit Conversion Problem

In this question, 5 miles equals 8 kilometers.

(a) In Canada some roads have a speed limit of 95 km/h. What is this limit in miles/hour?

(b) You are driving 75 mph in Nevada. How many miles do you travel in one minute?

You are driving on a two lane road that is 12 miles long. From the start, you have been stuck behind a car that is going 60 mph.

(c) How long (in minutes) will it take for you to reach the end of the road if you do not pass?

(d) How fast must you travel (in mph) to reduce this time by one minute if you pass right at the start?

Line Equation Problem

Find the equation of the line through (2,a) and (6,b).

y=

Linear Modeling Problem

The average sea-level in 1900 at London-bridge was 33 feet. In 1990 it was 33.08 feet. Use linear interpolation or extrapolation to find:

(a) What the average sea-level was in 1923.

feet

(b) In what year the average sea-level will be 40 feet.

Average Rate of Change Problem

For each of the values of h given, when x is increased from 4 to $4+h$, work out

$$\frac{\text{the change in } x^2}{h}$$

$h = 1$ ______ $h = .1$ ______

$h = .01$ ______ $h = .001$ ______

Log Properties Problem

- Rewrite $\log(xy)$.
- Rewrite $\log(x+y)$.
- Rewrite $\log(x-y)$.
- Rewrite $\log(x) - \log(y)$.
- Rewrite $\log(1/\sqrt{x})$.

Percent Word Problem

In the year 1900, in the country Acirema, there were 100 Lawyers and 6 million people. Every 10 years, the number of Lawyers doubles, and the population increases by 2 million. Let t be the number of years after 1900. Thus t=3 corresponds to 1903. Find the equation involving t whose solution tells you in which year 20 percent of the population are Lawyers. DO NOT SOLVE OR BEGIN TO SOLVE THIS EQUATION.

.2 =

Speed Estimation Problem

The table shows the position of a point on the x-axis during the time interval $0 \leq t \leq 1$ where x is measured in meters and t in seconds.

t	0	.2	.4	.6	.8	1
x	3	3.8	4.8	6.0	7.4	9.0

(a) Estimate the speed of the particle at t=0.9.

m/s

(b) When was the speed greatest?

During the interval starting at t=

seconds

(c) What was the average speed during the one second?

m/s

Exponential Decay Problem

The temperature in degrees Fahrenheit of a corpse t hours after death is $57 + 41e^{-t/24}$

(a) How quickly is the temperature decreasing after 2 hours?

______ degress Fahrenheit per hour

(b) What is the temperature of the surroundings of the corpse?

______ degrees Fahrenheit

(c) What was the temperature at the point of death?

______ degrees Fahrenheit

Please fill out evaluations for the course.
Your feedback is helpful!