MATH 3B

Discussion Worksheet - Thursday, May 24

Approximation of Integrals:

- Midpoint Rule:

- Trapezoidal Rule:

- Simpson's Rule:

- We say an integral *converges* if $\int_a^b f(x)\,dx =$

- We say an integral *diverges* otherwise.

- **Type I Improper Integrals**: Integrals on intervals of the form $(-\infty, b]$ or $[a, \infty)$

Examples:

- $\int_1^\infty \frac{1}{x}\,dx =$

- $\int_{-\infty} \frac{1}{\sqrt{3-x}}\,dx =$

- $\int_{-\infty}^\infty xe^{-x^2}\,dx =$

Type II Improper Integrals: Integrals of functions $f(x)$ on the interval $[a, b]$ where $f(x)$ is discontinuous or diverges at some $c \in [a, b]$

Examples:

- $\int_0^3 \frac{1}{\sqrt{3-x}}\,dx =$

- $\int_{-2}^3 \frac{1}{x^3}\,dx =$

- $\int_{-2}^2 \frac{x}{\sqrt{4-x^2}}\,dx =$

Arc Length: If f' is continuous on $[a, b]$, then the length of the curve

$$y = f(x), a \leqslant x \leqslant b \text{ is given by } L =$$

- Strategies:

Practice Problems:

Problem 1:

(HW 7) Compute the improper integrals below and write "D" if the integral diverges:

(a) $\displaystyle\int_0^1 \frac{1}{x^2}\,dx$

(b) $\displaystyle\int_0^1 \frac{1}{x}\,dx$

(c) $\displaystyle\int_0^1 \frac{1}{\sqrt{x}}\,dx$

(d) $\displaystyle\int_0^1 \ln(x)\,dx$

(e) $\displaystyle\int_{-1}^1 \frac{1}{\sqrt{1-x^2}}\,dx$

Problem 2:

(HW 8) For each of the improper integrals below, if the comparison test applies, write "converges" or "diverges" followed by the best function to use for comparison.

(a) $\displaystyle\int_1^\infty \frac{9+\sin(x)}{\sqrt{x-0.8}}\,dx$

(b) $\displaystyle\int_1^\infty \frac{\cos^2(x)}{x^2+4}\,dx$

(c) $\displaystyle\int_1^\infty \frac{x^2}{\sqrt{x^8+4}}\,dx$

(d) $\displaystyle\int_1^\infty \frac{e^{-x}}{x^2}\,dx$