Properties of Logarithms

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A note about Logarithms

The $\sqrt[3]{\ }$ undoes the 3 on top

Roots cancel/undo the exponent, $\sqrt[3]{8} = \sqrt[\cancel{3}]{2^{\cancel{3}}} = 2$

while logarithms cancel/undo the base.

The $\log_2$ undoes the 2 on the bottom.

$\log_2(8) = \cancel{\log_2}(\cancel{2}^3) = 3$

We need a different operation to undo the bottom (logarithms) than the top (roots) because in exponents the position matters. 2^3 3^2

these are not the same!

$\log_{10}(1{,}000) = \cancel{\log_{10}}(\cancel{10}^3) = 3$

$\log_5(25) = \cancel{\log_5}(\cancel{5}^2) = 2$

Properties of Logarithms

Exponent Properties		Log Properties
$10^a \cdot 10^b = 10^{a+b}$	$\longleftrightarrow$	$\log_{10}(a \cdot b) = \log_{10}(a) + \log_{10}(b)$
$10^a \div 10^b = 10^{a-b}$	$\longleftrightarrow$	$\log_{10}\left(\frac{a}{b}\right) = \log_{10}(a) - \log_{10}(b)$
$(10^a)^b = 10^{a \cdot b}$	$\longleftrightarrow$	$\log_{10}(a^b) = b \cdot \log_{10}(a)$ 3)

How to change base w/ a logarithm $\quad \log_b(a) = \dfrac{\log_{10}(a)}{\log_{10}(b)}$ 4)

Example

Exponent

1) $10^2 \cdot 10^5 = 100 \cdot 100{,}000 = 10{,}000{,}000 = 10^{2+5}$

Logarithm

$\underbrace{\log_{10}(100 \cdot 100{,}000)}_{7} \overset{?}{=} \underbrace{\log_{10}(100)}_{2} + \underbrace{\log_{10}(100{,}000)}_{5}$

Exponent

$(10^3)^7 = (1{,}000)^7 = 1{,}000 \cdot 1{,}000 \cdot \ldots \cdot 1{,}000 = 10^{21}$

Logarithm

$\underbrace{\log_{10}(1{,}000^7)}_{21} = 7 \cdot \log_{10}(1{,}000) = 7 \cdot 3 = 21$

(Just Logarithm)

$\underbrace{\log_{1{,}000}(1{,}000{,}000{,}000)}_{3} = \dfrac{\log_{10}(1{,}000{,}000{,}000)}{\log_{10}(1{,}000)} = \dfrac{9}{3} = 3$

Finding Change (Homework question)

$$\boxed{\begin{array}{l} f(x) = x^2 \\ x = 1 \end{array}}$$

$h = .001$

Find the change $\dfrac{f(x+h) - f(x)}{h}$

$$= \frac{(x+h)^2 - x^2}{h} = \frac{\cancel{x^2} + 2hx + h^2 - \cancel{x^2}}{h}$$

$$= \frac{2xh + h^2}{h} = \frac{\cancel{h}(2x+h)}{\cancel{h}} = \boxed{2x+h}$$

x: 2, h: .001

$2+h$ is the change

so 2.001.