Office Hours:

Instructor:

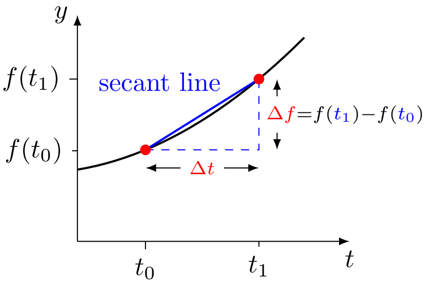
Peter M. Garfield	Mondays 11AM–12PM
`garfield@math.ucsb.edu`	Tuesdays 1:30–2:30PM
South Hall 6510	Wednesdays 1–2PM

TAs:

Trevor Klar	Wednesdays 2–3PM
`trevorklar@math.ucsb.edu`	South Hall 6431 X
Garo Sarajian	Mondays 1–2PM
`gsarajian@math.ucsb.edu`	South Hall 6431 F
Sam Sehayek	Wednesdays 3:30–4:30PM
`ssehayek@math.ucsb.edu`	South Hall 6432 P


Study Abroad in Winter/Spring 2020!
Applications will be due in May!!

Graphical Approach



Δf = change in f
Δt = change in t

Many ways to say same thing:

$$\left(\begin{array}{c}\text{average rate of}\\ \text{change of } f\end{array}\right) = \frac{\text{change in } f}{\text{change in } t}$$
$$= \frac{\Delta f}{\Delta t}$$
$$= \text{slope of secant line} = \frac{f(t_1) - f(t_0)}{t_1 - t_0}$$

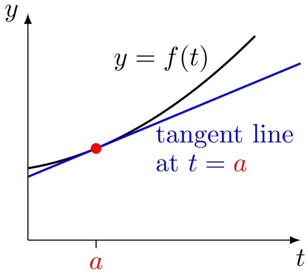
The derivative is defined to be

$$\lim_{\Delta t \to 0} \left(\frac{\Delta f}{\Delta t}\right) = \frac{df}{dt}$$

Idea: As t_1 moves closer to t_0 the secant line approaches the tangent line at t_0. This is the line with the same slope as the graph at t_0.

Understanding Derivatives

There are many ways to think about derivatives. You need to understand these to apply to problems.



slope of graph at a

= slope of tangent line

= instantaneous rate of change of f at a

$$= \left(\begin{array}{c} \text{limit of average rate of change} \\ \text{of } f \text{ over shorter and shorter} \\ \text{time intervals starting at } a \end{array} \right)$$

= limit of slopes of secant lines

$$= f'(a) \;=\; \left.\frac{df}{dt}\right|_{t=a}$$

Summary

- How fast something changes = rate of change
- Instantaneous rate of change is the limit of the average rate of change over shorter and shorter time spans. This gets around the $0/0$ problem.
- speed = rate of change of distance traveled.

Practical Meaning

Our goal is that you understand the practical meaning of the derivative in various situations.

$f(t) =$ temperature in $^\circ$ F at t hours after midnight

$f(7) = 48$ means the temperature at 7am was 48° F

$f'(7) = 3$ means at 7am the temperature was rising at a rate of 3° F/hr

$f'(9) = -5$ means at 9am the temperature was falling at a rate of 5° F/hr
or rising at a rate of -5° F/hr

$g(t) =$ distance from origin in cm of hamster on x-axis after t seconds

$g(7) = 3$ means after 7 seconds hamster was 3 cm from origin

$g'(9) = -5$ means after 9 seconds our furry friend was running towards the origin at a speed of 5 cm/sec

Another Context

Suppose $f(t)$ = temperature of oven in °C after t minutes.

What do $f(3) = 20$ and $f'(3) = 15$ mean?

(A) After 20 minutes the oven was at 3° C and heating up at a rate of 15° C/min

(B) After 3 minutes oven temperature was 15° C and cooling down at a rate to 20° C/min

(C) The oven was heating up at rate of 3° C/min after 15 minutes and also after 20 minutes

(D) After 3 minutes the oven was at 20° C and heating up at a rate of 15° C/min

(E) None of the above

Answer: $\boxed{\text{D}}$

Yet Another Context

Now suppose $f(t)$ = the population of the ancient city of Lyrad in year t. We are told that $f(1550) = 1820$ and $f'(1650) = 1100$. Which of the following is true?

(A) In 1550, the population was 1820 and rising at a rate of 1100 people per year

(B) In 1650, the population was 1100 more than in 1550

(C) In 1650, Lyrad contained 1100 people

(D) In 1550, there were 1820 people in Lyrad, and by 1650 this had increased to 2920

(E) None of above

Answer: $\boxed{\text{E}}$

Context: Mathematics

Suppose $f(0) = 50$ and $f(10) = 70$. Which of the following is true?

(A) For all t between 0 and 10, the derivative is $f'(t) = 2$

(B) $f'(0) = 2$

(C) It is possible that $f'(0) = -8$

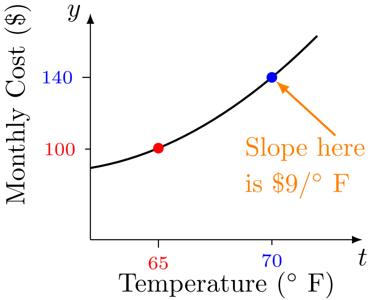
(D) It is impossible that $f'(0) = -8$

(E) None of above

Answer: $\boxed{\text{C}}$

We'll see later that, for example, that $f(x) = x^2 - 8x + 50$ has $f(0) = 50$, $f(10) = 70$, and $f'(0) = -8$.

Notice: No Time



$f(x)$ = monthly cost of heating house to x° F

$f(70) = 140$ means it costs $140 to heat the house for one month to a temperature of 70°F.

$f'(70) = 9$ means rate at which cost increases as temperature changes is $9 for each extra ° F.

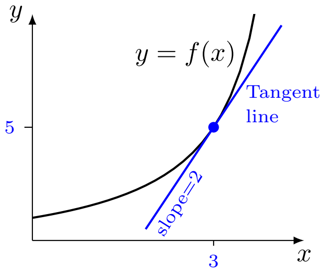
In practical terms this means you pay an extra $9 during each month for each extra 1^oF . If you turn it up two degrees you pay an extra $18 each month. Each extra degree of warmth costs an extra $9 each month. In economics this is called a marginal cost or marginal rate

This is not exactly true:

average rate of change versus instantaneous rate of change.

In the following examples we will ignore this subtlety.

The Importance of Units



Told $f(3) = 5$ and $f'(3) = 2$

This means the slope of the tangent line to the graph $y = f(x)$ at $x = 3$ is 2.

The derivative is this slope, so...

$$\boxed{\text{The units of } \frac{dy}{dx} \text{ are } \frac{\text{units of y}}{\text{units of x}}}$$

Heating example: derivative units are $\$/^\circ$ F = dollars per degree F

Units help you understand the meaning of the derivative.

Get Pumped!

$$x = \text{number of mg (milligrams) of adrenaline in the blood}$$
$$f(x) = \text{number of beats per minute (bpm) of the heart with } x \text{ mg of adrenaline in the blood.}$$

What does $f'(5) = 2$ mean? Answer: $\boxed{\text{E}}$

(A) When there are 5 mg of adrenaline the heart beats at 2 pbm

(B) When the amount of adrenaline is increased by 2 mg the heart speeds up by 5 bpm

(C) When the heart beats at 5 bpm the adrenaline is increased by 2 mg

(D) When there are 5 mg of adrenaline the heart speeds up by 2bpm

(E) When there are 5 mg of adrenaline in the blood the heart speeds up by 2 bpm for each extra mg of adrenaline.

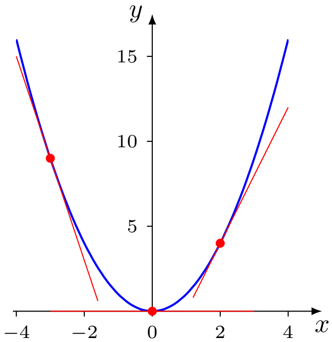
Hint: The units of $f'(5)$ are bpm per milligram of adrenaline

Graphical Meaning

$$\frac{d}{dx}\left(x^2\right) = 2x$$

What this means

The slope of the graph of $y = x^2$ at $x = a$ is $2a$

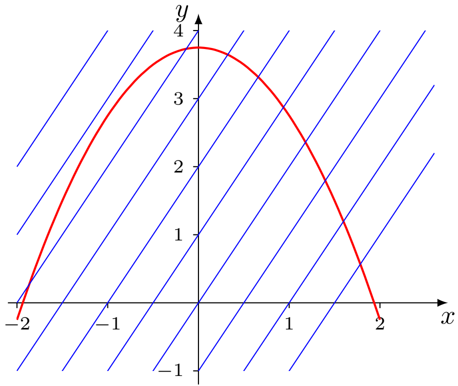


at $x = -3$, slope is $2(-3) = -6$
at $x = 0$, slope is $2(0) = 0$
at $x = 2$, slope is $2(2) = 4$

derivative = rate of change = slope of graph = slope of tangent line

Slope Question

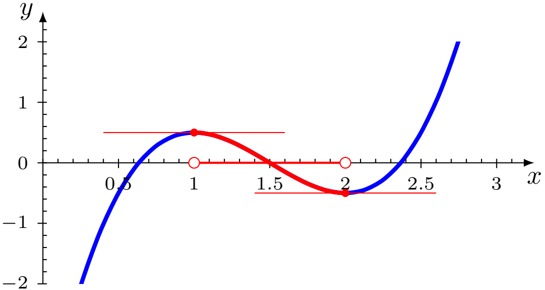
This graph shows $y = f(x)$ and lines parallel to $y = 2x$



Question: For which values of x is $f'(x) > 2$? Answer: D

(A) $x < 1.2$ (B) $x < 0$ (C) $x < -1.5$ (D) $x < -1$ (E) $x < -0.5$

More Slope Questions



(1) For which values of x is $f'(x) = 0$? Answer: D

(A) none (B) $\{0.63,\ 1.5, 2.38\}$ (C) 1 (D) $\{1, 2\}$ (E) 2

(2) For which values of x is $f'(x) < 0$? Answer: C

(A) $x < 0.63$ (B) $x < 1$ (C) $1 < x < 2$ (D) $1.5 < x < 2.38$ (E) none