

Welcome To Math 34A!

Differential Calculus

Instructor:

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South Hall 6431X (Grad Tower, 6th floor, blue side, first door on the right)

Office Hours:

MTWR after class 2:00-3:00, and by appointment. Details on Gauchospace.

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Chapter 7 (Logs)

§7.0 Logarithms and Exponentials

Applications:

- **Chemistry**: alkalinity and acidity, pH scale
- **Finance**: compound interest (get rich slow)
- **Geology**: Richter scale for earthquakes (did you feel the earth move too ?)
- **Archeology**: radio carbon dating (how old is that bone ?)
- **Astronomy**: stellar magnitude (brightness of stars)
- **Sound**: decibels (what did you say?, the music is too loud)

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- **Sound**: decibels (what did you say?, the music is too loud)
- **Math**: solving equations with exponents ...by performing an arithmetic operation to both sides (all of the above are examples of the use of this operation)

Chapter 7 (Logs)

Main Idea of Chapter 7:

$\log(x)$ is how many times you multiply 1 by 10 to get x

Conclusion:

Before we do logs we should be really good at powers of 10.

Powers of Ten

1 meter

1 centimeter = 0.01 meters = 10^{-2} meters

1 kilometer = 1,000 meters = 10^3 meters

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1 meter \approx 3 feet

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Approximate distance (in meters), to nearest power of 10

10^7 meters	Size of Earth
10^9 meters	Distance to moon

10^{14} meters	Size of our solar system
10^{16} meters	One light-year

10^{21} meters	Size of the Milky Way galaxy
10^{27} meters	Size of the universe (about 93 billion light-years)

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10^{80}	number of protons in the observable universe?

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10^{100}	1 googol

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10^{100}	1 googol
10^{1000} meters	???

Exponential Basics

$$\begin{aligned} 10^4 &= 10 \times 10 \times 10 \times 10 = 10,000 \\ &= 4 \text{ lots of } 10 \text{ multiplied together} \\ &= 1 \text{ followed by } 4 \text{ zeroes} \end{aligned}$$

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For now x and y are positive whole numbers.

More Exponentiation

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Why? We can work it out:

$$10^a = \underbrace{10 \times 10 \times \cdots \times 10}_{a \text{ times}}$$

$$\begin{aligned}(10^a)^b &= \underbrace{(10 \times \cdots \times 10) \times \cdots \times (10 \times \cdots \times 10)}_{b \text{ times}} \\ &= 10^{ab}.\end{aligned}$$

Just count the zeros!

When the power is 0 or negative

What is 10^0 ?

When the power is 0 or negative

What is $10^0 = 1$ But why?

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$$\text{so } 10^0 \times 10 = 10$$

$$\text{and therefore } 10^0 = 10/10 = 1$$

Summary: we used the first law of exponents to figure out what 10^0 must be.

There is a second explanation in the book!

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What is 10^{-2} ?

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The Five Laws of Exponents

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§7.1 Fractional Exponents

We can work them out!

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8. What is $10^{0.1} = 10^{1/10}$? **Answer:** $10^{0.1} = \sqrt[10]{10} \approx 1.258926$

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10. What is $10^{0.27}$? **Answer:**

$$10^{0.27} = 10^{27/100} = \sqrt[100]{10^{27}} = \left(\sqrt[100]{10}\right)^{27} \approx 1.862$$

§7.2 Logarithms

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$$\log(\textcolor{blue}{10}) = ?$$

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$$\begin{aligned}\log(10) &= 1 && \text{because } 10^1 = 10 \\ \log(100) &=?\end{aligned}$$

§7.2 Logarithms

$\log(y)$ is how many tens you multiply together to get y

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How confused are you?

A=not at all B=a bit C=a lot D=: '(

You try it:

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First Law of Logs $\log(a \times b) = \log(a) + \log(b)$

This means logs convert **multiplication** into **addition**.

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$\log(\textcolor{red}{a})$ = (how many 10's you multiply to get $\textcolor{red}{a}$)

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CONCLUDE $\log(a \times b)$ is this number of 10s: that is, $\log(a) + \log(b)$.

7.3 Using Log Tables

We are told: $\log(2) \approx 0.3$ (from table page 289)

$$\begin{aligned}\log(20) &= \log(10 \times 2) \\ &= \log(10) + \log(2) && \text{we know } \log(10) = 1 \\ &\approx 1 + 0.3 \\ &\approx 1.3\end{aligned}$$

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A few more

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Find $\log(0.002)$

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Find $\log(2 \times 10^x)$

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A Trick!

The graph and the table can both be used to find logs of numbers between 1 and 10.

To find the log of ANY number, we move the decimal point:

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Example:

$$\log(275.67) = \log(10^2 \times 2.7567) = 2 + \underbrace{\log(2.7567)}_{\text{look this up!}}$$

Its called the **MOVING DECIMAL POINT TRICK** because 2 is how many places you need to move the decimal point of 275.67 to obtain a number between 1 and 10.

Inverses!

logs are “**opposite**” of exponents (inverse function of antilog)

So every fact about exponents corresponds to a fact about logs:

	laws of exponents	corresponding law of logs
(1)	$10^a \times 10^b = 10^{a+b}$	$\log(xy) = \log(x) + \log(y)$
(2)	$10^0 = 1$	$\log(1) = 0$
(3)	$10^{-a} = 1/10^a$	$\log(1/x) = -\log(x)$
(4)	$(10^a)^p = 10^{ap}$	$\log(x^p) = p \log(x)$
(5)	$10^a/10^b = 10^{a-b}$	$\log(x/y) = \log(x) - \log(y)$

Example: $\log(x^a/y^b) = ?$

$$\begin{aligned} A &= a \log(x)/(b \log(y)) & B &= a \log(x) + b \log(y) \\ C &= a \log(x) - b \log(y) & D &= (a + \log(x)) - (b + \log(y)) \end{aligned}$$

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Rule (4): $\log(x^p) = p \log(x)$

Explanation of (4)

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In general: the number of tens you multiply to get x^p is p times as many tens as you multiply to get x .

What is $\log(\sqrt{x^7})$?

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That's it. Thanks for being here.

