

# Welcome Back!

# Differential Calculus

Instructor:

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Office Hours:

T R 11-11:50, T 3:45-4:35 Details on Gauchospace.

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Nathan Schley

Please do not distribute outside of this course.

# Warm-up

- $\log(x) = 5$

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•  $\log(x) = 5$

$$x = \boxed{10^5}$$

# Warm-up

- $\log(x) = 5$

$$x = \boxed{10^5} = \boxed{100,000}$$

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- $\log(x) = 5$        $x = \boxed{10^5} = \boxed{100,000}$
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# Warm-up

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- $10^x = 1,000,000$        $x = \boxed{\log(10^6)}$

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- $\log(x) = 5$        $x = \boxed{10^5} = \boxed{100,000}$
- $10^x = 1,000,000$        $x = \boxed{\log(10^6)} = \boxed{6}$

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- $\log(\log(x)) = 2$



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- $10^x = 8700$        $x = \boxed{\log(8700)} \approx ?$



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- $10^x = 8700$        $x = \boxed{\log(8700)} \approx ?$   
 $8700 = 8.7 \cdot 10^3$

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•  $10^{4x-5} = 7$

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•  $10^x = 8700$        $x = \boxed{\log(8700)} \approx ?$   
 $8700 = 8.7 \cdot 10^3$        $\log(8700) = \log(8.7) + 3$

•  $10^{4x-5} = 7$        $x = \boxed{(\log(7) + 5)/4}$

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- $10^x = 8700$        $x = \boxed{\log(8700)} \approx ?$   
 $8700 = 8.7 \cdot 10^3$        $\log(8700) = \log(8.7) + 3$
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- $10^x = 8700$        $x = \boxed{\log(8700)} \approx ?$   
 $8700 = 8.7 \cdot 10^3$        $\log(8700) = \log(8.7) + 3$
- $10^{4x-5} = 7$        $x = \boxed{(\log(7) + 5)/4} \approx \boxed{?}$  just leave it that way

# Logarithm Strategy

- $4^{2x+1} = 3$

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$$x = \boxed{(\log_4(3) - 1)/2}$$



# Logarithm Strategy

$$\bullet \quad 4^{2x+1} = 3 \qquad x = \boxed{(\log_4(3) - 1)/2} = (\frac{\log(3)}{\log(4)} - 1)/2$$

In general,

$$\boxed{\log_b(x) = \frac{\log(x)}{\log(b)}}$$

# Midterm 2: One week from today

## Bring:

- A pen or sharp pencil.
- A 3"  $\times$  5" card with your notes.
- Student ID.

## Don't bring:

- A calculator

No bluebook or scratch paper necessary, just the above materials and hopefully a fresh, well-practiced you! Scratch paper will be provided.

# Midterm 2 Topics

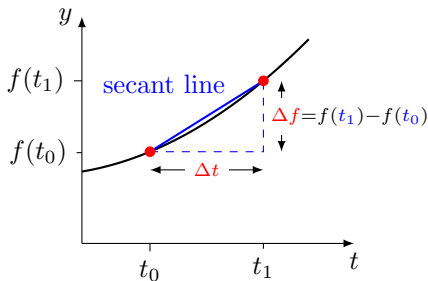
- All topics from Midterm 1
- Sums (like the example below, more examples on Gauchospace)

$$\sum_{n=1}^4 2^n - 1$$

- Advanced Logarithm Methods (the full chapter on logarithms in the book)
- Change and Average Rate of Change for a function or graph.
- Limits with  $h$  (used to find exact speed, examples on the old midterm and extra problems)

If you struggled on Midterm 1 with algebra or word problems, you need to improve these skills immediately. **They are essential for success in this course.**

# Graphical Approach



$\Delta f$  = change in  $f$

$\Delta t$  = change in  $t$

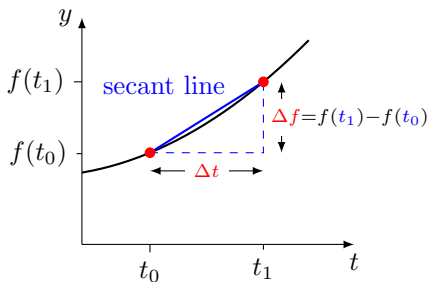
Many ways to say same thing:

$$\left( \begin{array}{c} \text{average rate of} \\ \text{change of } f \end{array} \right) = \frac{\text{change in } f}{\text{change in } t}$$

$$= \frac{\Delta f}{\Delta t}$$

$$= \text{slope of secant line} = \frac{f(t_1) - f(t_0)}{t_1 - t_0}$$

# Graphical Approach



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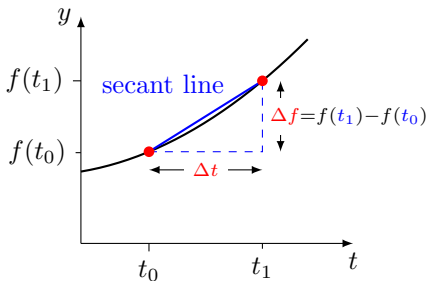
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The derivative is defined to be

$$\lim_{\Delta t \rightarrow 0} \left( \frac{\Delta f}{\Delta t} \right) = \frac{df}{dt}$$

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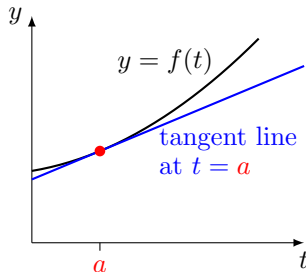
The derivative is defined to be

$$\lim_{\Delta t \rightarrow 0} \left( \frac{\Delta f}{\Delta t} \right) = \frac{df}{dt}$$

Idea: As  $t_1$  moves closer to  $t_0$  the secant line approaches the **tangent line** at  $t_0$ . This is the line with the **same slope** as the graph at  $t_0$ .

# Understanding Derivatives

There are many ways to **think** about derivatives. We **need** to understand how derivatives apply to problems.



slope of **graph** at **a**  
= slope of **tangent line**  
= **instantaneous rate of change** of  $f$  at **a**

=  $\left( \begin{array}{l} \text{limit of average rate of change} \\ \text{of } f \text{ over shorter and shorter} \\ \text{time intervals starting at } a \end{array} \right)$

= limit of slopes of secant lines

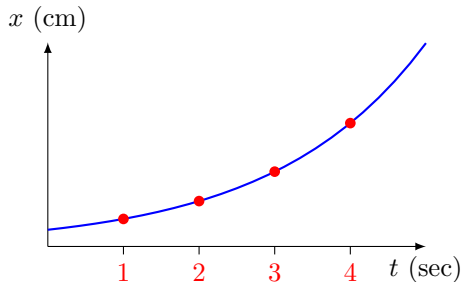
$$= f'(a) = \left. \frac{df}{dt} \right|_{t=a}$$

# Summary

- How fast something changes = **rate of change**
- **Instantaneous rate of change** is the **limit** of the average rate of change over shorter and shorter time spans. This gets around the changing speed problem, and works a whole lot better than getting frustrated and trying **0/0**.
- **speed** = rate of change of distance traveled.



# Speed=Slope=Derivative



The graph shows the distance from the origin in cm after  $t$  seconds of a hamster. Which of the numbers below is the largest?

**Hint:** Speed is a slope!

A = speed of the hamster at  $t = 1$

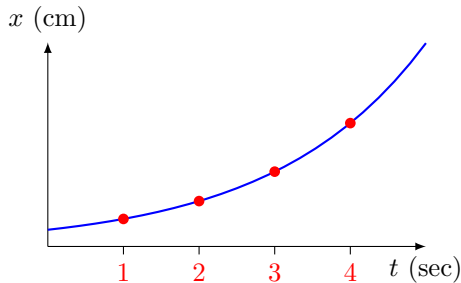
B = speed of the hamster at  $t = 2$

C = speed of the hamster at  $t = 3$

D = average speed of the hamster between  $t = 2$  and  $t = 3$

E = average speed of the hamster between  $t = 3$  and  $t = 4$

# Speed=Slope=Derivative



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Answer: E

# Practical Meaning

Our goal is that you understand the **practical meaning** of the derivative in various situations.

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$f(t)$  = temperature in  $^{\circ}$  F at  $t$  hours after midnight

$f(7)$  = 48 means the temperature at 7am was  $48^{\circ}$  F

$f'(7)$  = 3 means at 7am the temperature was rising at a rate of  $3^{\circ}$  F/hr

$f'(9)$  =  $-5$  means at 9am the temperature was **falling** at a rate of  $5^{\circ}$  F/hr  
or **rising** at a rate of  $-5^{\circ}$  F/hr

# Practical Meaning

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$f'(9)$  = -5 means at 9am the temperature was **falling** at a rate of 5 $^{\circ}$  F/hr  
or **rising** at a rate of -5 $^{\circ}$  F/hr

$g(t)$  = distance from origin in cm of hamster on  $x$ -axis after  $t$  seconds

$g(7)$  = 3 means after 7 seconds hamster was 3 cm from origin

$g'(9)$  = -5 means after 9 seconds our furry friend was running **towards**  
the origin at a speed of 5 cm/sec

# Another Context

Suppose  $f(t)$  = temperature of oven in  $^{\circ}\text{C}$  after  $t$  minutes.

What do  $f(3) = 20$  and  $f'(3) = 15$  mean?

- A After 20 minutes the oven was at  $3^{\circ}\text{C}$  and heating up at a rate of  $15^{\circ}\text{C/min}$
- B After 3 minutes oven temperature was  $15^{\circ}\text{C}$  and cooling down at a rate to  $20^{\circ}\text{C/min}$
- C The oven was heating up at rate of  $3^{\circ}\text{C/min}$  after 15 minutes and also after 20 minutes
- D After 3 minutes the oven was at  $20^{\circ}\text{C}$  and heating up at a rate of  $15^{\circ}\text{C/min}$
- E None of the above

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- E None of the above

Answer: D

# Context: Population

Suppose  $f(t)$  = the population of the ancient city of Lyrad in year  $t$ . We are told that  $f(1550) = 1820$  and  $f'(1650) = 1100$ . Which of the following is true?

- A In 1550, the population was 1820 and rising at a rate of 1100 people per year
- B In 1650, the population was 1100 more than in 1550
- C In 1650, Lyrad contained 1100 people
- D In 1550, there were 1820 people in Lyrad, and by 1650 this had increased to 2920
- E None of above



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- E None of above

Answer: E

# Context: Mathematics

Suppose  $f(0) = 50$  and  $f(10) = 70$ . Which of the following is true?

A For all  $t$  between 0 and 10, the derivative is  $f'(t) = 2$

B  $f'(0) = 2$

C It is possible that  $f'(0) = -8$

D It is impossible that  $f'(0) = -8$

E None of above

# Context: Mathematics

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Answer: C

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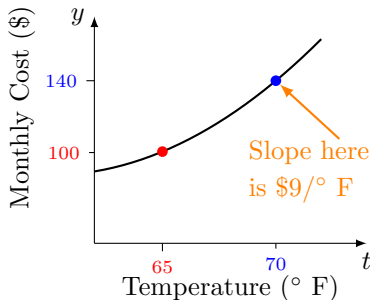
D It is impossible that  $f'(0) = -8$

E None of above

Answer: C

We'll see later that, for example, that  $f(x) = x^2 - 8x + 50$  has  $f(0) = 50$ ,  $f(10) = 70$ , and  $f'(0) = -8$ .

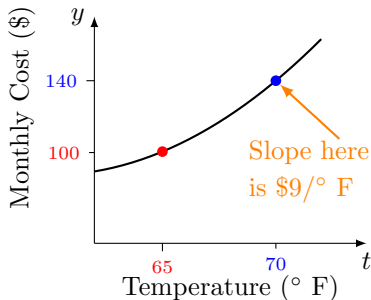
# It doesn't have to be about time!



$f(x)$  = monthly cost of heating house to  $x^{\circ}\text{F}$

$f(70) = 140$  means it costs \$140 to heat the house for one month to a temperature of  $70^{\circ}\text{F}$ .

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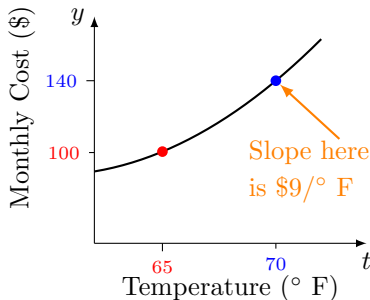


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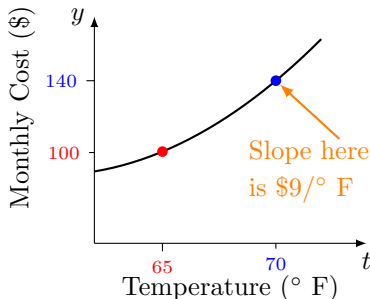
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In **practical** terms this means **you pay an extra \$9 during each month for each extra  $1^\circ$ F** . If you turn it up two degrees you pay an extra \$18 each month. **Each extra degree of warmth costs an extra \$9 each month**. In economics this is called a **marginal cost** or **marginal rate**

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This is not **exactly** true:

**average** rate of change versus **instantaneous** rate of change.

In the following examples we will ignore this subtlety.



# Get Pumped!

Adrenaline cause the heart to speed up.

$x$  = number of mg (milligrams) of adrenaline in the blood.

$f(x)$  = number of beats per minute (bpm) of the heart with  $x$  mg of adrenaline in the blood.

What does  $f'(5) = 2$  mean?

- A When there are 5 mg of adrenaline the heart beats at 2 pbm
- B When the amount of adrenaline is increased by 2 mg the heart speeds up by 5 bpm
- C When the heart beats at 5 bpm the adrenaline is increased by 2 mg
- D When there are 5 mg of adrenaline the heart speeds up by 2bpm
- E When there are 5 mg of adrenaline in the blood the heart speeds up by 2 bpm for each extra mg of adrenaline.

**Hint:** The units of  $f'(5)$  are bpm per milligram of adrenaline

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Answer: E

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- C When the heart beats at 5 bpm the adrenaline is increased by 2 mg
- D When there are 5 mg of adrenaline the heart speeds up by 2bpm
- E When there are 5 mg of adrenaline in the blood the heart speeds up by 2 bpm for each extra mg of adrenaline.

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# Summary of Derivatives

One quantity,  $y$ , depends on another quantity  $x$ .

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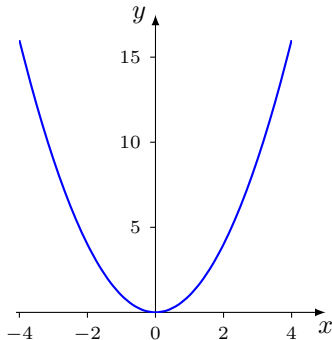


# Graphical Meaning

$$\frac{d}{dx}(x^2) = 2x$$

What this means

The slope of the graph  
of  $y = x^2$  at  $x = a$  is  $2a$

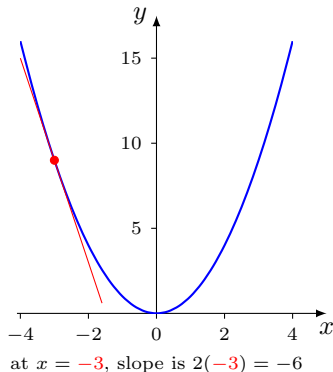


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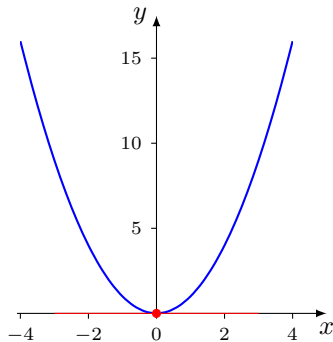


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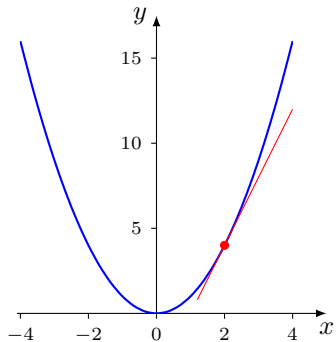
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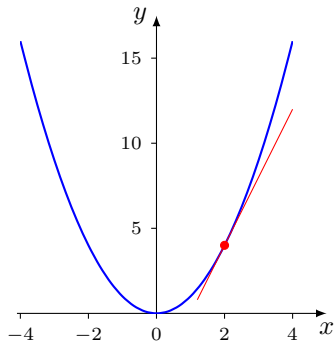
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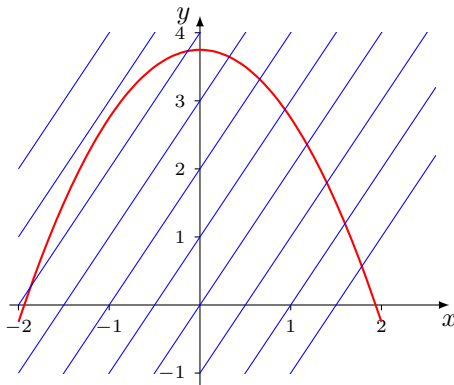


at  $x = 2$ , slope is  $2(2) = 4$

derivative = rate of change = slope of graph = slope of tangent line

# Slope Question

This graph shows  $y = f(x)$  and lines parallel to  $y = 2x$

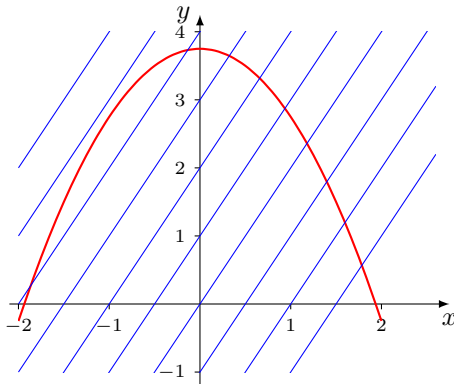


**Question:** For which values of  $x$  is  $f'(x) > 2$ ?

- A  $x < 1.2$     B  $x < 0$     C  $x < -1.5$     D  $x < -1$     E  $x < -0.5$

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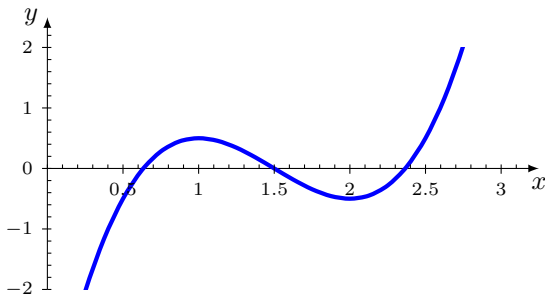


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# More Slope Questions

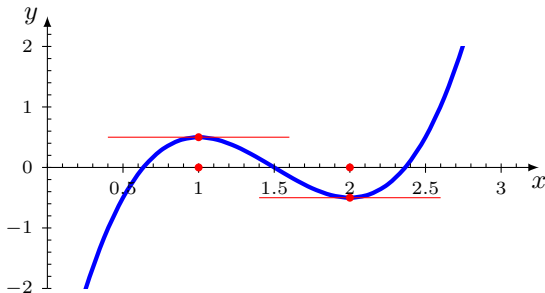


(1) For which values of  $x$  is  $f'(x) = 0$ ?

A= none    B=  $\{0.63, 1.5, 2.38\}$     C= 1    D=  $\{1, 2\}$     E= 2



# More Slope Questions



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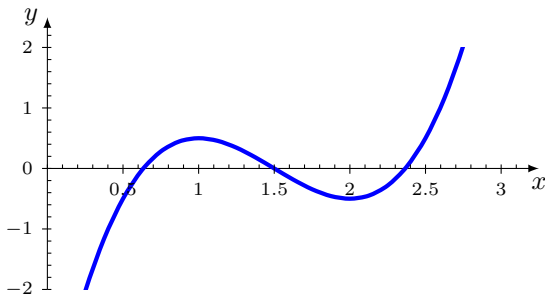
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# More Slope Questions



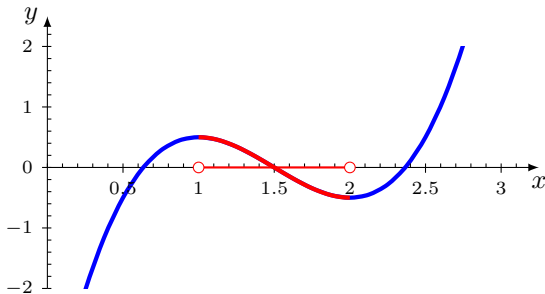
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(2) For which values of  $x$  is  $f'(x) < 0$ ?

A  $x < 0.63$     B  $x < 1$     C  $1 < x < 2$     D  $1.5 < x < 2.38$     E none

# More Slope Questions



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That's it. Thanks for being here.

