Office Hours!

Instructor:

Peter M. Garfield, garfield@math.ucsb.edu

Office Hours:

Mondays 2–3PM Tuesdays 10:30–11:30AM Thursdays 1–2PM or by appointment

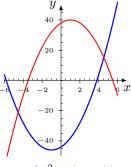
Office:

South Hall 6510

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It's useful to be able to sketch...

(1) Quadratics

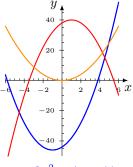


$$y = 2x^2 + 4x - 44$$
$$y = -2x^2 + 4x + 38$$

- Bowl-shaped:
 - \star Opens up if a>0
 - \star Opens down if a < 0
- Model curve: $y = x^2$

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(1) Quadratics

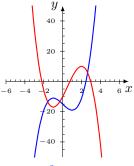


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- Bowl-shaped:
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(2) Cubics



$$y = 2x^3 - 6x - 15$$
$$y = -2x^3 + 3x^2 + 12x - 10$$

$$y = ax^3 + bx^2 + cx + d$$

• "S"-shaped:

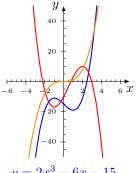
$$\star$$
 Goes to $+\infty$ if $a > 0$

★ Goes to
$$-\infty$$
 if $a < 0$

• Model curve:
$$y = x^3$$

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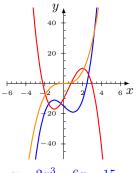
★ Goes to
$$-\infty$$
 if $a < 0$

• Model curve:
$$y = x^3$$

Shown here!

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(2) Cubics



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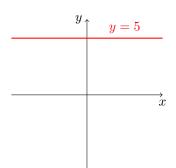
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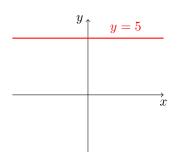
• Model curve:
$$y = x^3$$

Shown here!

For a polynomial, the highest power of x dominates when x is big

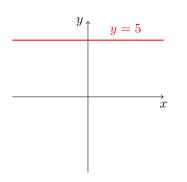
The derivative of a constant is...?





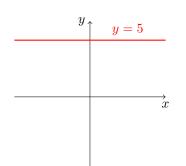
The derivative of a constant is zero because:

- derivative = rate of change
- constants don't change



The derivative of a constant is zero because:

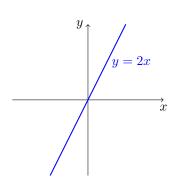
- derivative = rate of change
- constants don't change
- derivative = slope
- slope = 0



The derivative of a constant is zero because:

- derivative = rate of change
- constants don't change
- derivative = slope
- slope = 0

So
$$\frac{d}{dx}(5) = 0$$

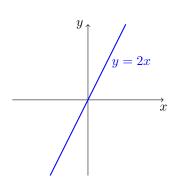


The derivative of a constant is zero because:

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The derivative of a straight line is...?



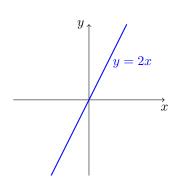
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The derivative of a straight line is its slope because

• derivative = slope



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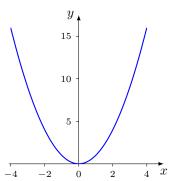
The derivative of a straight line is its slope because

• derivative = slope

So
$$\frac{d}{dx}(2x) = 2$$

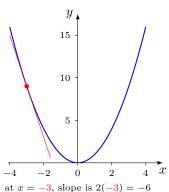
$$\frac{d}{dx}\left(x^2\right) = 2x$$

What this means



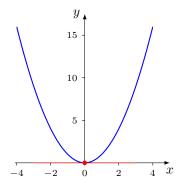
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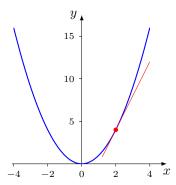
What this means



at
$$x = 0$$
, slope is $2(0) = 0$

$$\frac{d}{dx}\left(x^2\right) = 2x$$

What this means

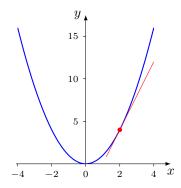


at
$$x = 2$$
, slope is $2(2) = 4$

$$\frac{d}{dx}\left(x^2\right) = 2x$$

What this means

The slope of the graph of $y = x^2$ at x = a is 2a



at
$$x = 2$$
, slope is $2(2) = 4$

derivative = rate of change = slope of graph = slope of tangent line

$$\frac{d}{dx}(x^2) = 2x$$
$$\frac{d}{dx}(x^3) = 3x^2$$
$$\frac{d}{dx}(x^4) = 4x^3$$

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$$\frac{d}{dx}\left(x^{\mathbf{n}}\right) = \mathbf{n}x^{\mathbf{n}-1}$$

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$$\frac{d}{dx}\left(x^{\mathbf{n}}\right) = \mathbf{n}x^{n-1}$$

$$\frac{1}{dx}\left(x^{7}\right) =$$

$$A = 7x^7$$
 $B = 6x^6$ $C = 6x^7$ $D = 7x^6$ $E = 0$

$$\frac{d}{dx}(x^2) = 2x$$

$$\frac{d}{dx}(x^3) = 3x^2$$

$$\frac{d}{dx}(x^4) = 4x^3$$

$$\frac{d}{dx}\left(x^{n}\right) = nx^{n-1}$$

$$\frac{d}{dx}\left(x^{7}\right) =$$

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$$2. \frac{d}{dx}\left(x^{-3}\right) =$$

$$A = 3x^{-2}$$
 $B = -3x^{-2}$ $C = -2x^{-4}$ $D = -3x^{-4}$

$$\frac{d}{dx}(x^2) = 2x$$

$$\frac{d}{dx}(x^3) = 3x^2$$

$$\frac{d}{dx}(x^4) = 4x^3$$

$$\frac{d}{dx}\left(x^{\mathbf{n}}\right) = \mathbf{n}x^{\mathbf{n}-1}$$

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$$\frac{d}{dx}\left(x^{n}\right) = nx^{n-1}$$

3.
$$\frac{d}{dx}(x^{1/2}) =$$

$$A = \frac{1}{2}x^{1/2}$$
 $B = -\frac{1}{2}x^{-1/2}$ $C = \frac{1}{2}x^{-1/2}$

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$$\frac{d}{dx}\left(\frac{1}{x^3}\right) =$$

$$A = \frac{1}{3x^2}$$
 $B = -3x^{-2}$ $C = -3x^{-4}$

$$\frac{d}{dx}\left(x^{\mathbf{n}}\right) = \mathbf{n}x^{\mathbf{n}-1}$$

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$$\frac{d}{dx}(\sqrt{x}) =$$

$$A = -\frac{1}{2}\sqrt{x}$$
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$$\frac{d}{dx}\left(x^{\mathbf{n}}\right) = \mathbf{n}x^{\mathbf{n}-1}$$

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$$\frac{d}{dx}\left(4x^5 + 7x^2 - 5x + 7\right) = 4(5)x^4 + 7(2)x^1 - 5 + 0$$

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Special cases

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Special cases

•
$$\frac{d}{dx}(7) = 0$$

6.
$$\frac{d}{dx}(3x^4+9x^3+7)=?$$

A= I have an answer

B= I am working on it

C = Help!

Polynomials

$$\frac{d}{dx}\left(4x^5 + 7x^2 - 5x + 7\right) = 4(5)x^4 + 7(2)x^1 - 5 + 0$$

Special cases

•
$$\frac{d}{dx}(7) = 0$$

6.
$$\frac{d}{dx}(3x^4+9x^3+7)=?$$

A = I have an answer

B = I am working on it

C = Help!

Answer: $12x^3 + 27x^2$

A = I got it B = I nearly got it

C=I want my mommy!

The derivative of $f(x) = x^2 + 3x + 1$ is $f'(x) = \frac{df}{dx} = 2x + 3$. This means:

• This is the slope of the graph $y = x^2 + 3x + 1$ at the point x

The derivative of $f(x) = x^2 + 3x + 1$ is $f'(x) = \frac{df}{dx} = 2x + 3$. This means:

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That f'(2) = 7 means:

• The slope of the graph y = f(x) at x = 2 is 7.

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- The slope of the tangent line to the graph at x = 2 is 7.
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- At x = 2 the output (value of f(x)) changes 7 times as fast as the input (value of (x)).

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- $\Delta f \approx 7\Delta x$ near x=2.
- $f(2 + \Delta x) \approx f(2) + 7\Delta x$.

7. What is the slope of the graph $y = 3x^2 - 7x + 5$ at x = 1?

$$A = -2$$
 $B = -1$ $C = 0$ $D = 1$ $E = 2$

7. What is the slope of the graph $y = 3x^2 - 7x + 5$ at x = 1?

$$A=-2$$
 $B=-1$ $C=0$ $D=1$ $E=2$ B

8. What is the instantaneous rate of change of $f(x) = x^3 - 2x + 3$ at x = 1?

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9. After t seconds a hamster on a skate board is $4t^2 + 2t$ cm from the origin on the x-axis. What is the exact speed of the hamster (in cm/sec) after 2 seconds?

$$A = 10$$
 $B = 16$ $C = 18$ $D = 20$ $E = 14$

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$$\frac{d}{dx}\left(x^{\mathbf{n}}\right) = \mathbf{n}x^{\mathbf{n}-1}$$

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$$\left(\begin{array}{c}
\text{average rate} \\
\text{of change between} \\
x \text{ and } x + h
\end{array}\right)$$

$$\frac{d}{dx}\left(x^{n}\right) = nx^{n-1}$$

$$\begin{pmatrix} \text{average rate} \\ \text{of change between} \\ x \text{ and } x + \frac{h}{h} \end{pmatrix} = \frac{(x+h)^3 - x^3}{(x+h) - x}$$

$$\frac{d}{dx}\left(x^{\mathbf{n}}\right) = \mathbf{n}x^{\mathbf{n}-1}$$

$$\begin{pmatrix} \text{average rate} \\ \text{of change between} \\ x \text{ and } x + \frac{h}{h} \end{pmatrix} = \frac{(x+h)^3 - x^3}{(x+h) - x}$$
$$= 3x^2 + 3xh + h^2$$

$$\frac{d}{dx}\left(x^{\mathbf{n}}\right) = nx^{n-1}$$

Example: n = 3: Calculate the average rate of change of x^3 between x and x + h then take limit as $h \to 0$.

$$\begin{pmatrix} \text{average rate} \\ \text{of change between} \\ x \text{ and } x + h \end{pmatrix} = \frac{(x+h)^3 - x^3}{(x+h) - x}$$
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Limit as $h \to 0$ is $3x^2$

$$\frac{d}{dx}\left(x^{n}\right) = nx^{n-1}$$

Example: n = 3: Calculate the average rate of change of x^3 between x and x + h then take limit as $h \to 0$.

$$\begin{pmatrix} \text{average rate} \\ \text{of change between} \\ x \text{ and } x + \frac{h}{h} \end{pmatrix} = \frac{(x+h)^3 - x^3}{(x+h) - x}$$
$$= 3x^2 + 3xh + h^2$$

Limit as $h \to 0$ is $3x^2$

A similar calculation works for x^n for any n.

More Applications

- 10. What is the equation of the tangent line at x = 1 to the graph of $y = x^3 x + 4$? The tangent line is $y = \dots$?
 - A = x + 3 B = 3x + 1 C = 2x 2 D = 2x + 2 E = 6x 2

More Applications

10. What is the equation of the tangent line at x = 1 to the graph of $y = x^3 - x + 4$? The tangent line is $y = \dots$?

$$A = x + 3$$
 $B = 3x + 1$ $C = 2x - 2$ $D = 2x + 2$ $E = 6x - 2$

Answer: D

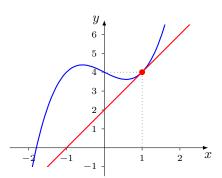
More Applications

10. What is the equation of the tangent line at x = 1 to the graph of $y = x^3 - x + 4$? The tangent line is $y = \dots$?

$$A = x + 3$$
 $B = 3x + 1$ $C = 2x - 2$ $D = 2x + 2$ $E = 6x - 2$

Answer: D

Here's a picture:



Another Example

11. The temperature in an oven after t minutes is $50 + t^3 \,^{\circ}$ F. How quickly is the temperature rising after 2 minutes?

$$A = 58$$
 $B = 3$ $C = 12$ $D = 50$ $E = 8$

Another Example

11. The temperature in an oven after t minutes is $50 + t^3$ ° F. How quickly is the temperature rising after 2 minutes?

$$A = 58$$
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Answer: C

A Warning!



$$\frac{d}{dx}\left(f(x)g(x)\right) \neq f'(x) \times g'(x) \qquad ?$$





$$\frac{d}{dx} (f(x)g(x)) \neq f'(x) \times g'(x)$$



Example:
$$5x^4 = \frac{d}{dx}(x^5) = \frac{d}{dx}(x^2 \cdot x^3) \neq (2x)(3x^2) = 6x^3$$

A Warning!



$$\frac{d}{dx} (f(x)g(x)) \neq f'(x) \times g'(x)$$



Example:
$$5x^4 = \frac{d}{dx}(x^5) = \frac{d}{dx}(x^2 \cdot x^3) \neq (2x)(3x^2) = 6x^3$$

Example: Find the derivative of (x+1)(2x+3)



$$\frac{d}{dx} (f(x)g(x)) \neq f'(x) \times g'(x) \qquad \text{ }$$



Example:
$$5x^4 = \frac{d}{dx}(x^5) = \frac{d}{dx}(x^2 \cdot x^3) \neq (2x)(3x^2) = 6x^3$$

Example: Find the derivative of (x+1)(2x+3)

12.
$$\frac{d}{dx}((x^2+1)(x^3+1)) = ?$$

$$A = 6x^3$$
 $B = 5x^4 + 3x^2 + 2x$ $C = x^5 + x^3 + x^2 + 1$ $D = Other$



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Answer: B

Once upon a time...

There was a happy math professor and he told his happy students:

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There was a happy math professor and he told his happy students:

"When you work out derivatives ALWAYS write the $\frac{d}{dx}$ part so you write something like

$$\frac{d}{dx}\left(3x^2 + 5x + 2\right) = 6x + 5$$

and you never-ever-ever write

$$3x^2 + 5x + 2$$
 $6x + 5$ or even worse

$$3x^2 + 5x + 2 = 6x + 5.$$

Because if you don't do as I say I will become a sad math professor and you will repeat this class."