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Exam 1 Results

Average: 12 out of 16 (75%)

Median: 13 out of 16 (81%)

If you scored **under 9** (or simply feel the need), please come talk to me or your TA to talk about how you can study and improve.

Remember: Logarithms

$\log(y)$ is how many tens you multiply together to get y

$$10^{\log(y)} = y$$

$$\log(\textcolor{blue}{100}) = ? = \textcolor{red}{2} \quad \text{because} \quad 10^{\textcolor{red}{2}} = \textcolor{blue}{100}$$

You Try It: $\log(\textcolor{blue}{100,000}) = ?$

(A) 2

(B) 3

(C) 4

(D) 5

(E) 6

D

A Few More:

$$\log(0.001) = ?$$

$A = 3$

$B = 0$

$C = 0.001$

$D = -2$

$E = -3$

E

$$\log(100 \times 1000) = ?$$

$A = 6$

$B = 5$

$C = 3$

$D = 9$

$E = -5$

B

$$\log(100/1000) = ?$$

$A = -1$

$B = 0$

$C = 1$

$D = -3$

$E = -5$

A

How confused are you ?

A = not at all

B = a bit

C = a lot

D = completely

How To Find Logarithms

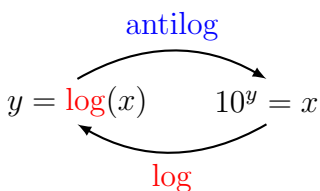
- (1) Use a calculator: efficient but not good for learning
- (2) Use the graph on page 290 of textbook
- (3) Use table of logarithms on page 289 of textbook

Our goal: use (2) and (3) to understand:

logs, functions and inverse functions.

Our main use of logs: solving certain kinds of equation.

Mistakes will follow unless you practice finding logs the old fashioned way.



log is the inverse function of
antilog

antilog is another name for
the 10-to-the-power function:

$$\text{antilog}(y) = 10^y.$$

| x | 0.00 | 0.01 | 0.02 | 0.03 | 0.04 | 0.05 | 0.06 | 0.07 | 0.08 | 0.09 |
|-----|--------|--------|--------|--------|--------|--------|--------|--------|--------|-------|
| 1.0 | 0.0000 | 0.0043 | 0.0086 | 0.0128 | 0.0170 | 0.0212 | 0.0253 | 0.0294 | 0.0334 | 0.037 |
| 1.1 | 0.0414 | 0.0453 | 0.0492 | 0.0531 | 0.0569 | 0.0607 | 0.0645 | 0.0682 | 0.0719 | 0.075 |
| 1.2 | 0.0792 | 0.0828 | 0.0864 | 0.0899 | 0.0934 | 0.0969 | 0.1004 | 0.1038 | 0.1072 | 0.110 |
| 1.3 | 0.1139 | 0.1173 | 0.1206 | 0.1239 | 0.1271 | 0.1303 | 0.1335 | 0.1367 | 0.1399 | 0.143 |
| 1.4 | 0.1461 | 0.1492 | 0.1523 | 0.1553 | 0.1584 | 0.1614 | 0.1644 | 0.1673 | 0.1703 | 0.173 |
| 1.5 | 0.1761 | 0.1790 | 0.1818 | 0.1847 | 0.1875 | 0.1903 | 0.1931 | 0.1959 | 0.1987 | 0.201 |
| 1.6 | 0.2041 | 0.2068 | 0.2095 | 0.2122 | 0.2148 | 0.2175 | 0.2201 | 0.2227 | 0.2253 | 0.227 |
| 1.7 | 0.2304 | 0.2330 | 0.2355 | 0.2380 | 0.2405 | 0.2430 | 0.2455 | 0.2480 | 0.2504 | 0.252 |
| 1.8 | 0.2553 | 0.2577 | 0.2601 | 0.2625 | 0.2648 | 0.2672 | 0.2695 | 0.2718 | 0.2742 | 0.276 |
| 1.9 | 0.2788 | 0.2810 | 0.2833 | 0.2856 | 0.2878 | 0.2900 | 0.2923 | 0.2945 | 0.2967 | 0.298 |
| 2.0 | 0.3010 | 0.3032 | 0.3054 | 0.3075 | 0.3096 | 0.3118 | 0.3139 | 0.3160 | 0.3181 | 0.320 |
| 2.1 | 0.3222 | 0.3243 | 0.3263 | 0.3284 | 0.3304 | 0.3324 | 0.3345 | 0.3365 | 0.3385 | 0.340 |
| 2.2 | 0.3424 | 0.3444 | 0.3464 | 0.3483 | 0.3502 | 0.3522 | 0.3541 | 0.3560 | 0.3579 | 0.359 |
| 2.3 | 0.3617 | 0.3636 | 0.3655 | 0.3674 | 0.3692 | 0.3711 | 0.3729 | 0.3747 | 0.3766 | 0.378 |
| 2.4 | 0.3802 | 0.3820 | 0.3838 | 0.3856 | 0.3874 | 0.3892 | 0.3909 | 0.3927 | 0.3945 | 0.396 |
| 2.5 | 0.3979 | 0.3997 | 0.4014 | 0.4031 | 0.4048 | 0.4065 | 0.4082 | 0.4099 | 0.4116 | 0.413 |
| 2.6 | 0.4150 | 0.4166 | 0.4183 | 0.4200 | 0.4216 | 0.4232 | 0.4249 | 0.4265 | 0.4281 | 0.429 |
| 2.7 | 0.4314 | 0.4330 | 0.4346 | 0.4362 | 0.4378 | 0.4393 | 0.4409 | 0.4425 | 0.4440 | 0.445 |
| 2.8 | 0.4472 | 0.4487 | 0.4502 | 0.4518 | 0.4533 | 0.4548 | 0.4564 | 0.4579 | 0.4594 | 0.460 |
| 2.9 | 0.4624 | 0.4639 | 0.4654 | 0.4669 | 0.4683 | 0.4698 | 0.4713 | 0.4728 | 0.4742 | 0.475 |
| 3.0 | 0.4771 | 0.4786 | 0.4800 | 0.4814 | 0.4829 | 0.4843 | 0.4857 | 0.4871 | 0.4886 | 0.490 |
| 3.1 | 0.4914 | 0.4928 | 0.4942 | 0.4955 | 0.4969 | 0.4983 | 0.4997 | 0.5011 | 0.5024 | 0.503 |
| 3.2 | 0.5051 | 0.5065 | 0.5079 | 0.5092 | 0.5105 | 0.5119 | 0.5132 | 0.5145 | 0.5159 | 0.517 |
| 3.3 | 0.5185 | 0.5198 | 0.5211 | 0.5224 | 0.5237 | 0.5250 | 0.5263 | 0.5276 | 0.5289 | 0.530 |
| 3.4 | 0.5315 | 0.5328 | 0.5340 | 0.5353 | 0.5366 | 0.5378 | 0.5391 | 0.5403 | 0.5416 | 0.542 |

Use table forwards to find logs: $\log(2.73) \approx 0.4362$

Use table backwards to find powers of 10: $10^{0.2923} \approx 1.96$

| x | 0.00 | 0.01 | 0.02 | 0.03 | 0.04 | 0.05 | 0.06 | 0.07 | 0.08 | 0.09 |
|-----|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|
| 1.0 | 0.0000 | 0.0043 | 0.0086 | 0.0128 | 0.0170 | 0.0212 | 0.0253 | 0.0294 | 0.0334 | 0.0376 |
| 1.1 | 0.0414 | 0.0453 | 0.0492 | 0.0531 | 0.0569 | 0.0607 | 0.0645 | 0.0682 | 0.0719 | 0.0757 |
| 1.2 | 0.0792 | 0.0828 | 0.0864 | 0.0899 | 0.0934 | 0.0969 | 0.1004 | 0.1038 | 0.1072 | 0.1107 |
| 1.3 | 0.1139 | 0.1173 | 0.1206 | 0.1239 | 0.1271 | 0.1303 | 0.1335 | 0.1367 | 0.1399 | 0.1431 |
| 1.4 | 0.1461 | 0.1492 | 0.1523 | 0.1553 | 0.1584 | 0.1614 | 0.1644 | 0.1673 | 0.1703 | 0.1733 |
| 1.5 | 0.1761 | 0.1790 | 0.1818 | 0.1847 | 0.1875 | 0.1903 | 0.1931 | 0.1959 | 0.1987 | 0.2015 |
| 1.6 | 0.2041 | 0.2068 | 0.2095 | 0.2122 | 0.2148 | 0.2175 | 0.2201 | 0.2227 | 0.2253 | 0.2279 |
| 1.7 | 0.2304 | 0.2330 | 0.2355 | 0.2380 | 0.2405 | 0.2430 | 0.2455 | 0.2480 | 0.2504 | 0.2529 |
| 1.8 | 0.2553 | 0.2577 | 0.2601 | 0.2625 | 0.2648 | 0.2672 | 0.2695 | 0.2718 | 0.2742 | 0.2766 |
| 1.9 | 0.2788 | 0.2810 | 0.2833 | 0.2856 | 0.2878 | 0.2900 | 0.2923 | 0.2945 | 0.2967 | 0.2989 |
| 2.0 | 0.3010 | 0.3032 | 0.3054 | 0.3075 | 0.3096 | 0.3118 | 0.3139 | 0.3160 | 0.3181 | 0.3202 |
| 2.1 | 0.3222 | 0.3243 | 0.3263 | 0.3284 | 0.3304 | 0.3324 | 0.3345 | 0.3365 | 0.3385 | 0.3405 |
| 2.2 | 0.3424 | 0.3444 | 0.3464 | 0.3483 | 0.3502 | 0.3522 | 0.3541 | 0.3560 | 0.3579 | 0.3599 |
| 2.3 | 0.3617 | 0.3636 | 0.3655 | 0.3674 | 0.3692 | 0.3711 | 0.3729 | 0.3747 | 0.3766 | 0.3784 |
| 2.4 | 0.3802 | 0.3820 | 0.3838 | 0.3856 | 0.3874 | 0.3892 | 0.3909 | 0.3927 | 0.3945 | 0.3963 |
| 2.5 | 0.3979 | 0.3997 | 0.4014 | 0.4031 | 0.4048 | 0.4065 | 0.4082 | 0.4099 | 0.4116 | 0.4133 |
| 2.6 | 0.4150 | 0.4166 | 0.4183 | 0.4200 | 0.4216 | 0.4232 | 0.4249 | 0.4265 | 0.4281 | 0.4298 |
| 2.7 | 0.4314 | 0.4330 | 0.4346 | 0.4362 | 0.4378 | 0.4393 | 0.4409 | 0.4425 | 0.4440 | 0.4456 |
| 2.8 | 0.4472 | 0.4487 | 0.4502 | 0.4518 | 0.4533 | 0.4548 | 0.4564 | 0.4579 | 0.4594 | 0.4609 |
| 2.9 | 0.4624 | 0.4639 | 0.4654 | 0.4669 | 0.4683 | 0.4698 | 0.4713 | 0.4728 | 0.4742 | 0.4757 |
| 3.0 | 0.4771 | 0.4786 | 0.4800 | 0.4814 | 0.4829 | 0.4843 | 0.4857 | 0.4871 | 0.4886 | 0.4900 |
| 3.1 | 0.4914 | 0.4928 | 0.4942 | 0.4955 | 0.4969 | 0.4983 | 0.4997 | 0.5011 | 0.5024 | 0.5038 |
| 3.2 | 0.5051 | 0.5065 | 0.5079 | 0.5092 | 0.5105 | 0.5119 | 0.5132 | 0.5145 | 0.5159 | 0.5172 |
| 3.3 | 0.5185 | 0.5198 | 0.5211 | 0.5224 | 0.5237 | 0.5250 | 0.5263 | 0.5276 | 0.5289 | 0.5302 |
| 3.4 | 0.5315 | 0.5328 | 0.5340 | 0.5353 | 0.5366 | 0.5378 | 0.5391 | 0.5403 | 0.5416 | 0.5429 |

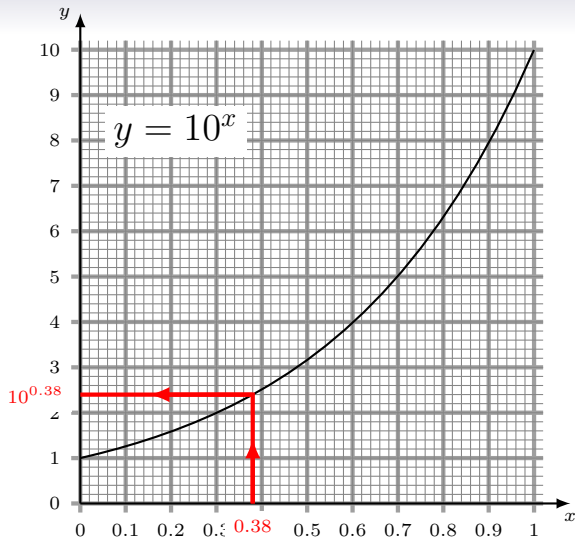
$\log(2.372)$ is

(A) ≈ 0.3729

(B) ≈ 0.3747

(C) ≈ 0.3766

B



Use the graph and a ruler to find $10^{0.38}$:

(A) ≈ 2.1

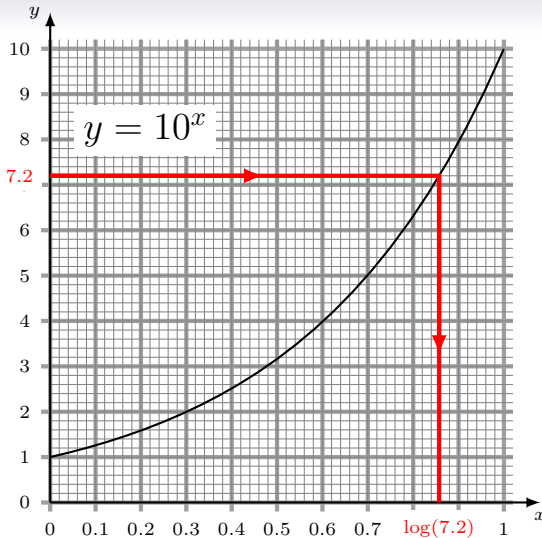
(B) ≈ 2.2

(C) ≈ 2.3

(D) ≈ 2.4

(E) ≈ 2.6

D



Use the graph backwards and a ruler to find $\log(7.2)$:

- (A) ≈ 0.81 (B) ≈ 0.82 (C) ≈ 0.83 (D) ≈ 0.84 (E) ≈ 0.86 **E**

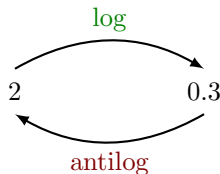
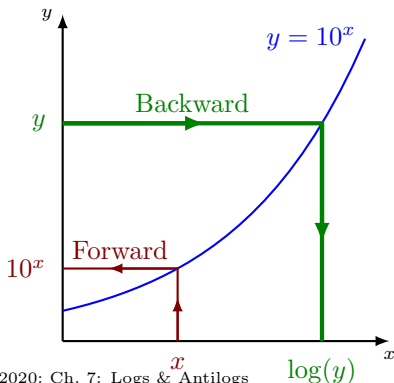
Do you have questions about using the graph of $y = 10^x$?

(A) lots (B) a few (C) one (D) none (E) move on already!

Do you have questions about using the table of logs?

(A) lots (B) a few (C) one (D) none (E) move on already!

Using the Graph **Forward** and **Backward**



Summary

- $\log(y)$ is how many tens you multiply to get y
- \log is the **inverse function** to **antilog**.
- So: method to find $x = \log(y)$ is the **opposite** of method to find $y = 10^x$

Use the graph $y = 10^x$ **forwards** to find 10^x . This means to find $10^{0.38}$ **start at $x = 0.38$ on x -axis**. Using graph as intended.

Use graph **backwards** to find $\log(y)$. This means to find $\log(7.2)$ **start at $y = 7.2$ on the y -axis**.

Same deal with log tables. Use the tables **forwards** to find $\log(2.73)$. This means **find 2.73 in the left column/top row** and the answer $\log(2.73) = 0.4362$ is in the middle of the table.

Use the table **backwards** to find $10^{0.2923}$. This means **hunt through the middle of the table** until you find 0.2932 then $10^{0.2932} = 1.96$ is in the left column/top row.

Key Fact Of Logs

First Law of Logs $\log(a \times b) = \log(a) + \log(b)$

This means logs convert **multiplication** into **addition**.

Example: $\log(100 \times 1000) = \log(100) + \log(1000) = 2 + 3 = 5$

It is easy to understand why the first law works:

$\log(a)$ = (how many 10's you multiply to get **a**)

$\log(b)$ = (how many 10's you multiply to get **b**)

THEREFORE multiplying ALL these 10s gives **$a \times b$**

CONCLUDE $\log(a \times b)$ is this number of 10s: that is, $\log(a) + \log(b)$.

Does this make sense to you?

(A) completely

(B) mostly

(C) a glimmer

(D) no!

Consequences of the Key Fact

We are told: $\log(2) \approx 0.3$ (from table)

$$\begin{aligned}
 \log(20) &= \log(10 \times 2) \\
 &= \log(10) + \log(2) && \text{we know } \log(10) = 1 \\
 &\approx 1 + 0.3 \\
 &\approx 1.3
 \end{aligned}$$

Use this method to find $\log(200)$

(A) 30

(B) 3

(C) 2.3

(D) 30

C

A few more

We are still told $\log(2) \approx 0.3$

Find $\log(0.002)$

(A) -3.3

(B) -2.3

(C) -2.7

(D) -3.7

(E) -3.0

Find $\log(2 \times 10^x)$

(A) $2x$

(B) $2 + x$

(C) $x \log(2)$

(D) $10x + \log(2)$

(E) $x + \log(2)$

Answer: **(E)**

A Trick!

The graph and the table can both be used to find logs of numbers between 1 and 10.

To find the log of ANY number, we move the decimal point:

$$\log(10^n \times x) = n + \log(x)$$

Example:

$$\log(275.67) = \log(10^2 \times 2.7567) = 2 + \underbrace{\log(2.7567)}_{\text{look this up!}}$$

It's called the **MOVING DECIMAL POINT TRICK** because 2 is how many places you need to move the decimal point of 275.67 to obtain a number between 1 and 10.

You Try It!

Use the log tables to find $\log(5.73)$

(A) I have done it (B) I am confused (C) I have no log table

$\log(5.73) \approx 0.7582$ Did you get this?

(A) YES! (B) No

What is $\log(57.3) \approx$?

(A) 7.582 (B) $10 + 0.7582$ (C) $1 + 0.7582$ (D) Other C