Math 34A Winter 2020
Old Midterm 1 #2 Solutions

1. Solve for x in the equation $\dfrac{a}{5} = \dfrac{3x+a}{x+2}$.

Solution: If we multiply this equation by 5 and $x+2$, we get

$$\frac{a}{5} \cdot 5(x+2) = \frac{3x+a}{x+2} \cdot 5(x+2) \qquad \text{or} \qquad a(x+2) = (3x+a)(5).$$

Expanding both sides, we get $ax + 2a = 15x + 5a$. Subtracting $15x$ and $2a$ from both sides gives us

$$ax + 2a - 15x - 2a = 15x + 5a - 15x - 2a \qquad \text{or} \qquad (a-15)x = 3a.$$

Dividing both sides by $(a-15)$ gives us $\boxed{x = 3a/(a-15)}$.

We could check this by choosing a value for a and seeing if the value of x we get satisfies $a/5 = (3x + a)/(x+2)$. We'll pick $a = 5$ (so $5/5 = 1$), from which we get $x = 3a/(a-15) = 3(5)/(5-15) = -1.5$. Then

$$\frac{3x+a}{x+2} = \frac{3(-1.5)+5}{-1.5+2} = \frac{-4.5+5}{0.5} = \frac{0.5}{0.5} = 1,$$

the same as $a/5 = 5/5 = 1$. Thus our answer checks out for at least one choice of a.

2. Put

$$\frac{5}{a-4} + \frac{3}{a+4}$$

over a common denominator, multiply out, and simplify.

Solution: To put these two fractions over a common denominator, we multiply the first one by $(a+4)/(a+4)$ and the second by $(a-4)/(a-4)$ (both of which are really 1):

$$\begin{aligned} \frac{5}{a-4} + \frac{3}{a+4} &= \frac{5}{a-4} \cdot \frac{a+4}{a+4} + \frac{3}{a+4} \cdot \frac{a-4}{a-4} \\ &= \frac{5(a+4) + 3(a-4)}{(a+4)(a-4)}. \end{aligned}$$

The numerator (the top) simplifies to

$$5(a+4) + 3(a-4) = 5a + 20 + 3a - 12 = 8a + 8$$

and the denominator (the bottom) is just $(a+4)(a-4) = a^2 - 16$. Thus $\dfrac{5}{a-4} + \dfrac{3}{a+4} = \boxed{\dfrac{8a+8}{a^2-16}}$.

3. Substitute $x = 2t - 3$ into $x(x+5)$. Simplify the result as much as possible.

Solution: We substitute in and get

$$x(x+5) = (2t-3)(2t-3+5) = (2t-3)(2t+2).$$

Multiplying this out gives

$$\begin{aligned}(2t-3)(2t+2) &= 2t(2t+2) - 3(2t+2)\\ &= 4t^2 + 4t - 6t - 6\\ &= 4t^2 - 2t - 6.\end{aligned}$$

Thus $x(x+5) = \boxed{4t^2 - 2t - 6}$ when $x = 2t - 3$.

We can check this by choosing a particular value of t. If we pick $t = 1$, then $x = 2t-3$ is $x = 2(1)-3 = 2-3 = -1$. Thus $x(x+5) = (-1)(-1+5) = -4$, while $4t^2 - 2t - 6 = 4(1)^2 - 2(1) - 6 = 4-2-6 = -4$. Thus this checks out, for one choice of t anyway.

4. Solve for x and y in the simultaneous equations

$$2x + y = p \qquad\qquad x - y = 5.$$

Your answers will involve p only.

Solution: To solve for x and y, we'll solve for one variable in one equation and plug it into the other equation. We could fairly easily solve for y in either equation, but instead we'll solve for x in the second equation: $x = 5 + y$. Plugging this into the first equation to get

$$2(5+y) + y = p \qquad \text{or} \qquad 10 + 3y = p.$$

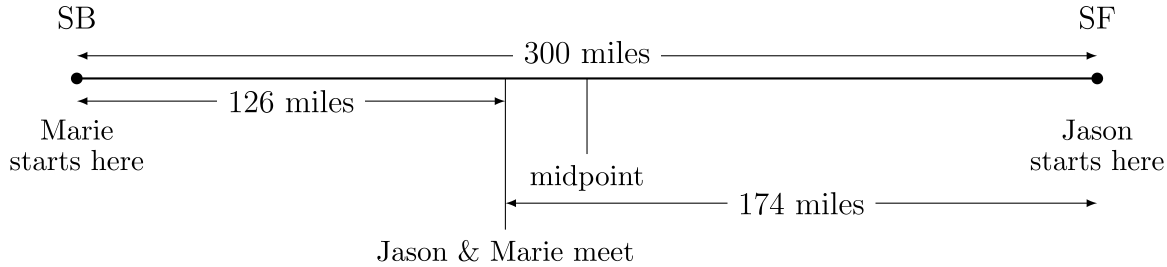
Solving, we find that $3y = p - 10$, and so $y = (p-10)/3$. Plugging this back into the equation $x = 5 + y$, we get

$$x = 5 + y = 5 + \frac{p-10}{3} = \frac{15 + (p-10)}{3} = \frac{p+5}{3}.$$

Thus the solutions are $\boxed{x = (p+5)/3}$ and $\boxed{y = (p-10)/3}$.

We can check this by simply plugging in and checking that $2x + y = p$ and $x - y = 5$. We're going to do something simpler, and simply check this for one value of p. We'll take $p = 1$, so $x = 2$ and $y = -3$. Then $2x + y = 2(2) + (-3) = 1$, which equals p. Similarly, $x - y = 2 - (-3) = 5$, as desired. Thus the solution works for at least one value of p.

5. Marie left Santa Barbara at 4am driving at 42 mph along a route that is 300 miles long to San Francisco. Jason left San Francisco at the same time driving along the same route at 58 mph. Here's a little sketch of the situation:



(a) What time do they meet?

Solution: Marie and Jason are approaching each other at $42 + 58 = 100$ mph, so together they cover the 300 miles in

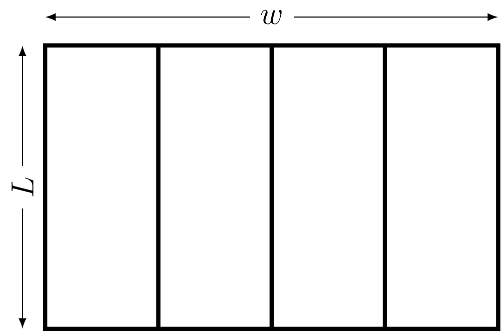
$$\frac{300 \text{ miles}}{100 \text{ miles/hr}} = 3 \text{ hours.}$$

This means they meet at 7am (three hours after 4am).

(b) How far from the midpoint of the route are they when they meet?

Solution: The midpoint is 150 miles from either end (either Santa Barbara or San Francisco). In the three hours of driving, Marie has traveled (42 mph)(3 hrs) = 126 miles. Similarly, Jason has gone (58 mph)(3 hrs) = 174 miles. From either of these we can see that their meeting place is 24 miles from the midpoint (either $150 - 126 = 24$ miles or $174 - 150 = 24$ miles).

6. A farmer wants to make a rectangular field with a total area of 400 square meters. It is surrounded by a fence.



It is divided into 4 equal areas as shown. The width of the field is w meters.

(a) Express the length of the field in terms of w.

Solution: The picture above has been tweaked so that the length now is labeled "L" (rather than "length"). We can relate L and w using the area. On the one hand, we're told the area is $A = 400$ square meters. On the other hand, we know the area of the rectangular field is $A = L \cdot w$. Thus $L \cdot w = 400$, so $\boxed{L = 400/w}$.

(b) Express the total number of meters of fence needed in terms of w.

Solution: The total length of fence needed (from counting lines!) is $2w+5L$. Using $L = 400/w$, we find the total length of fence needed is $2w + 5 \cdot 400/w$ or $\boxed{2w + 2000/w}$.