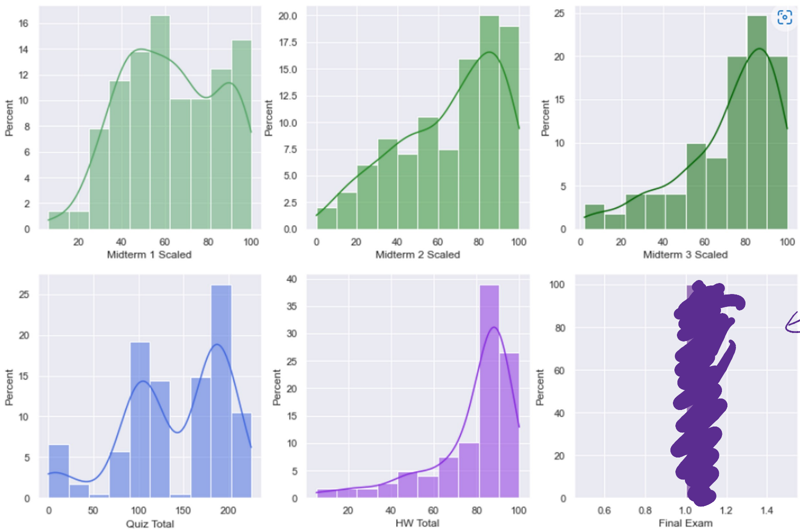
May 31 Optimization Problems

Sunday, May 29, 2022 3:10 PM



Placeholder for now

Grade Breakdown

Homework 20%

Quizzes 10%

Midterm Exams 40% (Worth 20% each)

Final Exam 30%

iclicker - up to 3% extra credit

A 3 Variable Optimization Problem???

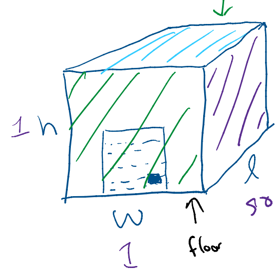
You are asked to design a shipping container with a square entrance wall. It needs to be able to store $50m^3$ of stuff. Your job is to find the length, width and height that will minimize the material cost. If these dimensions are reasonable, your company will move forward with production. Here is the cost information:
The **walls** and **roof** will only cost $50 per square meter.
The **floor** needs to be heavily reinforced, so the material cost $200 per cubic meter. This is a daunting problem, so you break it up into pieces.

- If the entrance is 1×1, then what is the materials cost?
- If the entrance is 5×5, then what is the materials cost? $\ell = 2m$
- If the entrance is 10×10, then what is the materials cost?
- Find the cost for any width w. $\boxed{h = w,\ \ell = \frac{50}{w^2}}$
- Believe it or not, you're almost done! How can you find the dimensions that would yield the lowest cost? Are these dimensions feasible? What is the minimum cost and how could that be used for planning?

solve for ℓ $\quad \ell = \frac{50}{w^2}$

$$\boxed{A = \ell \cdot w \cdot h = 50}$$

$\ell = 50$



front & back cost \$50 each

left & right cost \$50² each

same for the top

bottom costs \$200·50

$$\text{Cost}: 2 \cdot \$50 \cdot \left(\frac{50}{w^2} \cdot w\right) + 2 \cdot \$50 \cdot w^2 + \$50 \cdot \left(\frac{50}{w^2} \cdot w\right) + \$200 \cdot \left(\frac{50}{w^2} \cdot w\right)$$

$$\to 100w^2 + \frac{50}{w}(100 + 50 + 200)$$

$$= 100w^2 + \frac{50 \cdot 350}{w}$$

$\downarrow \frac{d}{dw}$

$$200w - 50 \cdot 350 w^{-2} = 0$$

$$200w^3 - 50 \cdot 350 = 0$$

$$w^3 = \frac{50 \cdot 350}{200} = \frac{5 \cdot 35}{2}$$

$$\sqrt[3]{175}$$

$A = \ell \cdot w = 1{,}000$

means $w = \frac{1{,}000}{\ell}$

Word Problem #4

Word Problem #4

A fenced garden with an area of 1000 m^2 will be made in the shape of a rectangle. It will be surrounded on all four sides by a fence. Three sides are wood fence, and the remaining side is a brick wall.

- The wood fence costs \$5 per meter length.
- The brick wall costs \$20 per meter length.
- C = total cost of the fence and brick wall
- L = length of the brick wall
- W = width of the other side

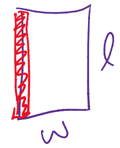
(a) Find a formula for C in terms of only L.

A $= 2W + 2L$ B $= 2000L^{-1} + 2L$ C $= 25L + 10000L^{-1}$
D $= 20L + 10000WL^{-1}$ E $= 5L + 3000$ $\boxed{}$

(b) What length of brick wall gives lowest cost?

A $= 20$ B $= 40$ C $= 50$ D $= 100$ E $= 25$ $\boxed{A}$

$A = \ell \cdot w = 1{,}000$
means $w = \frac{1{,}000}{\ell}$



$5 \cdot 2w$
$= 10 \cdot \frac{1000}{\ell}$
$= \frac{10{,}000}{\ell}$

$Cost = \underbrace{5 \cdot (2w)}_{\text{2 fences}} + \underbrace{5 \cdot \ell}_{\text{fence}} + \underbrace{20 \cdot \ell}_{\text{brick}}$

$\boxed{Cost = \frac{10{,}000}{\ell} + 25\ell}$

$\downarrow \frac{d}{d\ell}$

$-10{,}000 \cdot \ell^{-2} + 25 = 0$

$25\ell^2 = 10{,}000$

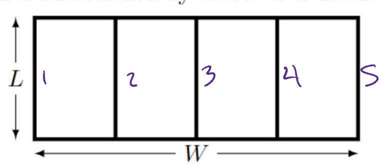
$\boxed{5^2 \cdot \ell^2 = 100^2 \quad 5\ell = 100 \quad \ell = 20}$

$w^3 = \frac{50 \cdot 350}{200} = \frac{5 \cdot}{2}$

$w = 7\sqrt[3]{\frac{175}{2}}$

Word Problem #5

A rectangular field is surrounded by fence. It is divided into 4 equal



parts by 3 more dividing fences all parallel to one side of the field.

$5L + 2W$

(a) What is the total length of all the fence needed?

A $= 2L + 2W$ ~~B $= LW$~~ ~~C $= 5LW$~~
D $= L + W$ E $= 5L + 2W$ $\boxed{E}$

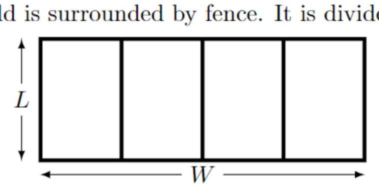
(b) The field must have an area of 1000 m^2. Express W in terms of L.

A $1000 - L$ B $1000L$ C $1000/L$ D $1000 + L$ $\boxed{}$

$Area = L \cdot W = 1{,}000$
$W = \frac{1{,}000}{L}$

Word Problem #5 (cont'd)

A rectangular field is surrounded by fence. It is divided into 4 equal



parts by 3 more dividing fences all parallel to one side of the field.

(c) Express the total length of all the fence needed in terms of L.

A $= 5L + 1000$ B $= 5L + 2000/L$ C $= 5L + 2/L$ $\boxed{B}$

(d) What should L be so that the total length of fence used is a minimum?

A $= 10$ B $= 20$ C $= 40$ D $= 50$ $\boxed{B}$

$5L + 2W = 5L + 2\left(\frac{1000}{L}\right)$

$\downarrow \frac{d}{dL}$

$5 - 2000 \cdot L^{-2} = 0$

$5L^2 = 2000$

$L^2 = 400 = 20^2 \qquad L = 20$

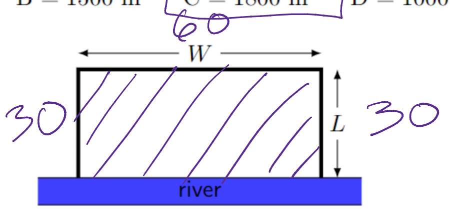
Word Problem #6

$Fencing = 120 = 2L + W$

Word Problem #6

A rectangular field is surrounded on three sides by a fence and the fourth side runs along a perfectly straight river. What is the largest area field which can be so enclosed with 120 meters of fence?

A = 1200 m^2 B = 1500 m^2 C = 1800 m^2 D = 1000 m^2 $\boxed{\text{C}}$



$\text{Fencing} = 120 = 2L + W$

$W = 120 - 2L$

$\text{Area} = L \cdot W = L(120-2L)$

$= 120L - 2L^2$

$\downarrow \frac{d}{dL}$

$120 - 4L = 0$

$4L = 120$

$L = 30$

$\boxed{30 \cdot 60 = 1800}$

Word Problem #7

Tickets are going to be sold for a concert.

- If the price of each ticket is \$40, then 2,000 tickets will be sold.
- For every \$1 the price is decreased, 100 more tickets will be sold.

(a) If the tickets are sold for $\$x$ each, how many will be sold?

A = $2000 - x$ B = $2000 - 100x$ C = $2000 + 100x$

D = $6000 - 100x$ E = $6000 + 100x$ $\boxed{\text{D}}$

(b) What is the total amount of money generated from selling tickets for $\$x$ each?

A = $6000x - 100x^2$ B = $2000x$

C = $2000 - 40x^2$ D = $6000 - 100x$ $\boxed{\text{A}}$

(c) What price should the tickets be to generate the most money from sales?

A = \$20   B = \$22   C = \$24   D = \$30   E = \$40 $\boxed{\text{D}}$

Word Problem #8

A farmer is growing wheat.

- On July 1, she has 1,000 bushels and this increases by 50 bushels per day.
- The price of a bushel on July 1 is \$10 and is dropping at a rate of 20 cents per day.
- She will harvest and sell on the same day.

How many days should she wait, assuming these trends continue?

A = 5   B = 10   C = 15   D = 20   E = other   ☐

HW23: Problem 2

(1 point) Cooper_oml/Cooper_oml.v2/Cooper_8/Cooper_8_13_9.pg

This set is **visible to students.**

Cooper 8.13.9

A manufacturer wishes to make tee-shirts for the band Indigo Girls. They sell for 14 dollars each. She must deliver all the tee-shirts in 20 days time. The manufacturer will first set up some machines. Once all machines are set up she will turn them on. Each machine takes one day to set up. A machine produces 200 tee-shirts in one day. All the machines must be set before any of them are turned on.

(a) Express the amount of money she will receive for the shirts in terms of the number of machines m she decides to set up.

\$ ________

plug in 10.

$\text{money} = 14 \cdot (4000m - 200m^2)$

number of t-shirts made $200 \cdot m$ each day, so

$200 \cdot m(20 - m) = 200(20m - m^2)$

$= 4000m - 200m^2$

(a) Express the amount of money she will receive for the shirts in terms of the number of machines m she decides to set up.

$

(b) Use calculus to find out how many machines she should set up to obtain the most money.

$m =$

$200 \cdot m$ each day

$200 \cdot m(20-m) = 200(20m-m^2)$

$= 4000m - 200m^2$

$\frac{d}{dm}$

$4000 - 400m = 0$

$10 - m = 0$ $m = 10$

HW23: Problem 6

(1 point) Cooper_oml/Cooper_oml.v2/Cooper_8/Cooper_8_13_15.pg

This set is **visible to students.**

Cooper 8.13.15

On a certain island there are two populations of deer. After t years the numbers of deer in the two populations are $p(t) = 200e^t$ and $q(t) = 3000e^{-t}$. When is the total population smallest?

t=

$200e^t - 3{,}000e^{-t} = 0$

$200e^{2t} = 3000$

$2e^{2t} = 30$

$e^{2t} = 15$

$2t = \ln(15)$

$\boxed{t = \frac{\ln(15)}{2}}$