Math 4B **Name:** ______________________

Summer Session A

Final Exam

30 July 2020

---

1. For each nonhomogeneous linear equation below, determine what the guess solution y_p would be for the method of undetermined coefficients.

   (a) $y'' - 4y' + 13y = 80e^{-3t}$

   (b) $y'' - 6y' + 9y = (t+1)e^{3t}$

   (c) $y'' + 4y = \sin(2t) + t^3 - 1$

   (d) $4y'' - 4y' + y = 16e^{t/2}$

2. Find the general solution using the method of undetermined coefficients:

   (a) $y'' - 2y' - 3y = 3e^{2t}$

   (b) $y'' - 5y' + 6y = 4t^2 + 3t + 1$

3. Find the solution of the initial value problem:

$$y'' + 9y = \cos t, \quad y(0) = 0, \quad y'(0) = 1.$$

4. Find the general solution using variation of parameters:

$$y'' - y' - 2y = 2e^{-t}$$

5. Let $\vec{x_1}(t) = \langle ae^{k_1 t}, be^{k_2 t}\rangle$ and $\vec{x_2}(t) = \langle ce^{k_1 t}, de^{k_2 t}\rangle$

   (a) Compute $W[\vec{x_1}, \vec{x_2}](t)$

   (b) What conditions on t make $\vec{x_1}$ and $\vec{x_2}$ linearly dependent?

   (c) What conditions on a, b, c, d make $\vec{x_1}$ and $\vec{x_2}$ linearly dependent?

6. The real 2×2 matrix A has eigenvector $\vec{v_1} = \langle -1, 2+3i\rangle$ for eigenvalue $r_1 = 1 + 3i$ and eigenvector $\vec{v_2} = \langle -1, 2-3i\rangle$ for eigenvalue $r_2 = 1 - 3i$. Given this information, write down the general solution to the system $\vec{x}' = A\vec{x}$.

7. Find the general solution:

   (a)

$$\vec{x}' = \begin{pmatrix} 1 & -1 \\ 1 & 3 \end{pmatrix} \vec{x}.$$

   (b)

$$\vec{x}' = \begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix} \vec{x}.$$

8. Consider the third order equation

$$2t^3y' - 2y''' = -3t^4.$$

   (a) Write the equation above as an equivalent system of first order differential equations. Use $x_1 = y, x_2 = y'$, and $x_3 = y''$.
   (b) Express your system of equations in matrix-vector form: $\vec{x}' = A(t)\vec{x} + \vec{g}(t)$

9. Let

$$A = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 & 0 \\ 0 & 0 & 3 & 0 & 0 \\ 0 & 0 & 0 & 4 & 0 \\ 0 & 0 & 0 & 0 & 5 \end{pmatrix}$$

   (a) Find the general solution to the sysem of equations $\vec{x}' = A\vec{x}$
   (b) Solve the initial value problem $x_1(0) = x_2(0) = \cdots = x_5(0) = 5$.