Welcome To Math 34A!
Differential Calculus

Instructor:

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MTWR after class 2:00-3:00, and by appointment. Details on Gauchospace.

A nice thing about derivatives...

$$\frac{d}{dx}(a \cdot f(x) + b \cdot g(x)) = a\frac{d}{dx}f(x) + b\frac{d}{dx}g(x)$$
$$= a \cdot f'(x) + b \cdot g'(x)$$

For example...

$$\begin{aligned} \tfrac{d}{dx}(3x^2 + 5x) &= 3\tfrac{d}{dx}x^2 + 5\tfrac{d}{dx}x \\ &= 3(2x) + 5(1) \\ &= 6x + 5 \end{aligned}$$

A Warning! (Again)

⚠ $$\frac{d}{dx}\left(f(x)g(x)\right) \neq f'(x) \times g'(x)$$ ⚠

Example: What is the derivative of $(x^3+1)(2x^2-3x+5)$?

Question: Find $\frac{d}{dx}\left((x^3+1)(2x^2-3x+5)\right)$.

$$\begin{aligned} \text{A} &= 10x^4 - 8x^3 + 10x^2 + 12x - 3 \\ \text{B} &= 10x^4 - 12x^3 + 15x^2 + 4x + 5 \\ \text{C} &= 10x^4 - 12x^3 + 15x^2 + 4x - 3 \\ \text{D} &= \text{Other} \end{aligned}$$

Hint: $2x^5 - 3x^4 + 5x^3 + 2x^2 - 3x + 6$

Answer: C

Differentiating $f(x) = e^{kx}$

$$\frac{d}{dx}\left(e^{kx}\right) = ke^{kx}$$

versus

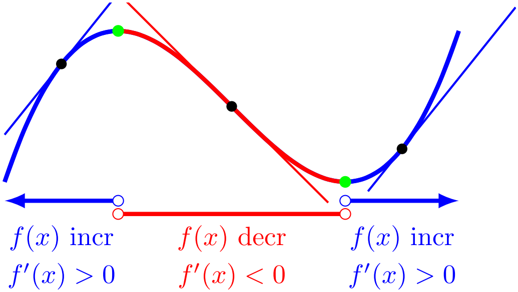
$$\frac{d}{dx}\left(x^{n}\right) = nx^{n-1}$$

These are not polynomials. $\frac{d}{dx}\left(e^{kx}\right) \neq ke^{(k-1)x}$.

Question: Find $\frac{d}{dx}\left(4e^{3x} + 5x^3\right)$

A$= 12e^{2x} + 15x^2$ B$= 12e^{3x} + 15x^3$ C$= 4e^{3x} + 15x^2$
D$= 12e^{3x} + 15x^2$ E$=$ Other D

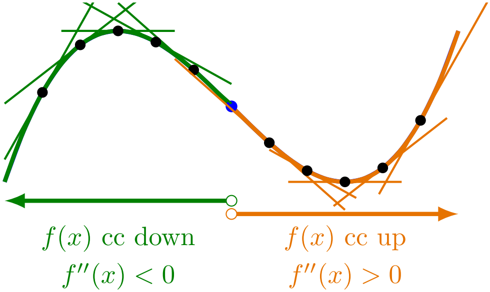
Meanings: The First Derivative



Point:

$$f'(x) > 0 \iff f(x) \text{ is increasing}$$
$$f'(x) < 0 \iff f(x) \text{ is decreasing}$$

Meanings: The Second Derivative



Point:

$$\begin{aligned} f''(x) > 0 &\iff f'(x) \text{ is increasing} \\ &\iff f(x) \text{ is concave up} \\ f''(x) < 0 &\iff f'(x) \text{ is decreasing} \\ &\iff f(x) \text{ is concave down} \end{aligned}$$

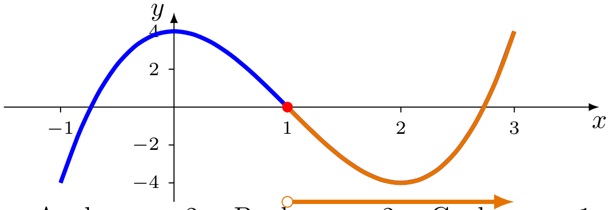
Concavity

$$f''(x) > 0 \iff f(x) \text{ is concave up}$$
$$f''(x) < 0 \iff f(x) \text{ is concave down}$$

(1) For which values of x is $f(x) = x^3 - 6x^2 + 3x + 2$ concave up?

A when $x = 0$ B when $x < 6$ C when $x > 6$
D when $x < 2$ E when $x > 2$ E

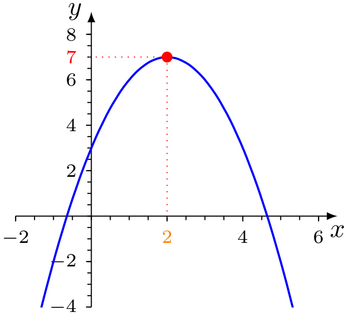
(2) Where is $f''(x) > 0$?



A when $x < 2$ B when $x > 2$ C when $x < 1$
D when $x > 1$ E when $-0.7 < x < 1$ D

§8.13: Max/Min problems

Often want to find the biggest, smallest, most, least, maximum, minimum of something.



Here's the graph of
$y = f(x) = -x^2 + 4x + 3$

The maximum value or just maximum of the function is 7.

The value of x which gives the maximum of $f(x)$ is $x = 2$

We write $f(2) = 7$.

For this example you can see this is the maximum because

$$f(x) = -x^2 + 4x + 3 = -(x-2)^2 + 7$$

$(x-2)^2$ is always positive except when $x = 2$
so the maximum must be at $x = 2$. But there is an easier way.

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§8.13: Max/Min problems

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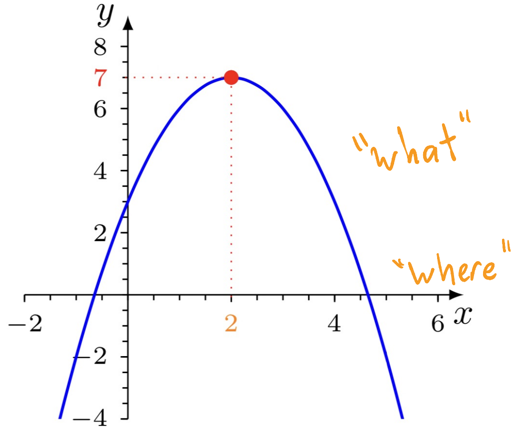

Here's the graph of $y = f(x) = -x^2 + 4x + 3$

The maximum value or just maximum of the function is 7.

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We write $f(2) = 7$.

How To Find A Maximum

(1) At the highest point, it's not going up or down. So find $f'(x)$ to look for the flat part.

(2) Solve $f'(x) = 0$ for x. The x value that gives the max must be one of these! (Usually there is just one.)

(3) To find the maximum for $f(x)$, use the x-value you just found...because it gives you the maximum!

1. Use this method to find the maximum of $f(x) = -x^2 + 8x + 5$. The maximum value is...

$\mathrm{A} = 4 \quad \mathrm{B} = 5 \quad \mathrm{C} = -2x + 8 \quad \mathrm{D} = 21 \quad \mathrm{E} = 15 \quad \mathrm{D}$

2. Find the value of x which makes $f(x) = (2 - x)(x + 6)$ a maximum.

$\mathrm{A} = 16 \quad \mathrm{B} = 1 \quad \mathrm{C} = -1 \quad \mathrm{D} = 2 \quad \mathrm{E} = -2 \quad \mathrm{E}$

How To Find A Maximum

(1) At the highest point, it's not going up or down. So find $f'(x)$ to look for the flat part.

(2) Solve $f'(x) = 0$ for x. The x value that gives the max must be one of these! (Usually there is just one.)

(3) To find the maximum for $f(x)$, use the x-value you just found...because it gives you the maximum!

1. Use this method to find the maximum of $f(x) = -x^2 + 8x + 5$. The maximum value is...

A $= 4$ B $= 5$ C $= -2x + 8$ D $= 21$ E $= 15$

$f'(x) = -2x + 8$

Set equal to zero

$-2x + 8 = 0 \Rightarrow 8 = 2x \Rightarrow x = 4$

Plug in to find what the max value is

$f(4) = -(16) + 8(4) + 5$

$= 21$

2. Find the value of x which makes $f(x) = (2-x)(x+6)$ a maximum. Set $f'(x) = 0$

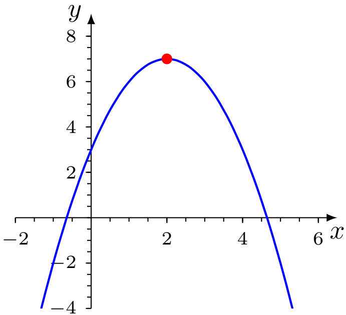
A $= 16$ B $= 1$ C $= -1$ D $= 2$ E $= -2$

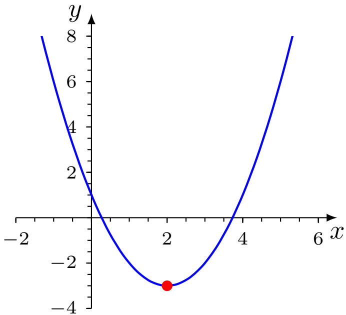
Can't do $\frac{d}{dx}[(2-x)(x+6)]$

$f(x) = (2-x)(x+6) = -x^2 - 4x + 12$

$f'(x) = -2x - 4 = 0 \Rightarrow x = -2$

How To Find A Minimum?





What this technique actually does is find both maxima and minima

In Math 34A a problem will have either a maximum or a minimum, but not both. So the technique will find what you want. In Math 34B you discover how to do problems which have both a maximum and a minimum and find out which is which.

More Examples

3. What is the minimum of $f(x) = (x+2)(x+4)+3$?

$$A = 0 \quad B = 1 \quad C = 2 \quad D = 3 \quad E = 4$$

Answer: C

4. What is minimum of $f(x) = x^2 + 16x^{-2}$?

$$A = 2 \quad B = 4 \quad C = 6 \quad D = 8 \quad E = 16$$

Answer: D

5. Find the value of x which makes $f(x) = -e^x - e^{-2x}$ a maximum.

$$A = 0 \quad B = \ln(2) \quad C = -\ln(2) \quad D = \ln(2)/3 \quad E = \ln(2)/3$$

Answer: E

More Examples

can't do $\frac{d}{dx}$ here

3. What is the minimum of $f(x) = (x+2)(x+4)+3$?

A = 0 B = 1 C = 2 D = 3 E = 4

$f(x) = (x+2)(x+4)+3 = x^2+6x+11$

$f'(x) = 2x+6$

find x-value, set $=0$

$2x+6=0$

$2x=-6$

$x=-3$

To find what the max is, plug in.

$f(-3)=(-1)(1)+3 = \boxed{2}$

4. What is minimum of $f(x) = x^2 + 16x^{-2}$?

A = 2 B = 4 C = 6 D = 8 E = 16

$f'(x) = 2x - 32x^{-3}$

set $=0$

$2x-32x^{-3}=0$ mult. by x^3 on both sides

$2x^4-32=0$

$2x^4=32$

$x^4=16$ take $\sqrt{\ }$

$x^2=4$ take $\sqrt{\ }$ again

$x=\pm 2$

Plug in

$f(2) = 4 + \frac{\cancel{16}^{\,4}}{\cancel{4}} = \boxed{8}$

5. Find the value of x which makes $f(x) = -e^x - e^{-2x}$ a maximum.

A = 0 B = ln(2) C = − ln(2) D = ln(2)/3 E = ln(2)/3

$f'(x) = -e^x + 2e^{-2x} = 0$

$-e^x + \frac{2}{e^{2x}} = 0$ clear the fraction, mult. by e^{2x}

$-e^{3x}+2=0$

$\ln[e^{3x}]=3x$ $\quad\circ\circ\circ\; e^{3x}=2$ take ln of both sides

$$3x = \ln 2$$

$$\boxed{x = \frac{1}{3}\ln 2}$$

Word Problem #1

A ball is thrown into the air. After t seconds the height in meters above the ground of the ball is $h(t) = 40t - 10t^2$. How many meters high did the ball go?

$$\text{A} = 2 \quad \text{B} = 40 - 20t \quad \text{C} = 20 \quad \text{D} = 40 \quad \text{D}$$

Word Problem #2

If an airline sells tickets at a price of $\$200 + 5x$ each the number of tickets it sells is $1000 - 20x$. What price should the tickets be if the airline wants to get the most money?

$$\mathrm{A} = 5 \quad \mathrm{B} = 25 \quad \mathrm{C} = 175 \quad \mathrm{D} = 200 \quad \mathrm{E} = 225 \quad \mathrm{E}$$

Word Problem #1

A ball is thrown into the air. After t seconds the height in meters above the ground of the ball is $h(t) = 40t - 10t^2$. How many meters high did the ball go? maximize $h(t)$

A = 2 B = $40 - 20t$ C = 20 D = 40

find $h'(t) = 0$

$$h'(t) = 40 - 20t = 0 \quad \Rightarrow \quad t = 2$$

Plug in

$$h(2) = 80 - 10(4) = \boxed{40}$$

Word Problem #2

If an airline sells tickets at a price of $\underbrace{\$200 + 5x}_{\text{price}}$ each the number of tickets it sells is $\underbrace{1000 - 20x}_{n}$. What price should the tickets be if the airline wants to get the most money?

A = 5 B = 25 C = 175 D = 200 E = 225

Revenue = Price × number of tix

$$R(x) = (200 + 5x)(1000 - 20x) = 200{,}000 - 4{,}000x + 5{,}000x - 100x^2$$
$$= 200{,}000 + 1{,}000x - 100x^2$$

$$R'(x) = 1{,}000 - 200x = 0 \quad \Rightarrow \quad x = 5$$

$$p(x) = 200 + 5x$$
$$p(5) = 200 + 25 = \boxed{\$225}$$

Word Problem #3

A fenced garden with an area of 100 m^2 will be made in the shape of a rectangle. It will be surrounded on all four sides by a fence. What length and width should be used so the least amount of fence is needed?

Approach:

(1) Express the total length of fence in terms of only one variable, either L = length of field, or W = width of field. This gives a formula for P = (total length of fence) involving, say, W.

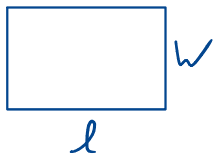
(2) Find minimum by solving $\dfrac{dP}{dW} = 0$.

Students always find (1) the hardest part.
You have been prepared for this by word problems from chapter 3!

Word Problem #3

A fenced garden with an area of 100 m^2 will be made in the shape of a rectangle. It will be surrounded on all four sides by a fence. What length and width should be used so the least amount of fence is needed?

minimize Perimeter



Area $= l \cdot w$

$100 = lw$

$\frac{100}{l} = w$

$P(l,w) = 2l + 2w$

$P(l) = 2l + 2\left(\frac{100}{l}\right)$

$= 2l + 200l^{-1}$

Optimize: Take $P'(l)$

$P'(l) = 2 - 200l^{-2}$

set $=0$

$l^2\left(2 - \frac{200}{l^2}\right) = (0)\,l^2$

$2l^2 - 200 = 0$

$2l^2 = 200$

$l^2 = 100$

$\boxed{l = 10}$

$100 = lw$

$\boxed{w = 10}$

interpret

$P'(1) = -198$