Math 550
Homework 5

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1. Calculate the differential of each of the following.

   (a) $\omega = e^{xy}\,dx$
   Answer. $d\omega = (ye^{xy}\,dx + xe^{xy}\,dy) \wedge dx$

   $$= -xe^{xy}\,dx \wedge dy$$

   (b) $\omega = x_1 x_2\,dx_3 \wedge dx_4$
   Answer. $d\omega = (x_2\,dx_1 + x_1\,dx_2) \wedge dx_3 \wedge dx_4$

   $$= x_2\,dx_1 \wedge dx_3 \wedge dx_4 + x_1\,dx_2 \wedge dx_3 \wedge dx_4$$

   (c) $\omega = f(x,y)\,dx + g(x,y)\,dy$
   Answer. $d\omega = (\partial f_x\,dx + \partial f_y\,dy) \wedge dx + (\partial g_x\,dx + \partial g_y\,dy) \wedge dy = \partial g_x\,dx \wedge dy - \partial f_y\,dx \wedge dy$

   $$= (\partial g_x - \partial f_y)\,dx \wedge dy$$

   (d) $\omega = f(x,y,z)\,dy \wedge dz - g(x,y,z)\,dx \wedge dz + h(x,y,z)\,dx \wedge dy$
   Answer. $d\omega = (\partial f_x + \partial g_y + \partial h_z)\,dx \wedge dy \wedge dz$. (A lot of computation can be skipped, leaving only partials and sign changes to compute mentally.)