Welcome To Math 34A! Differential Calculus

Instructor:

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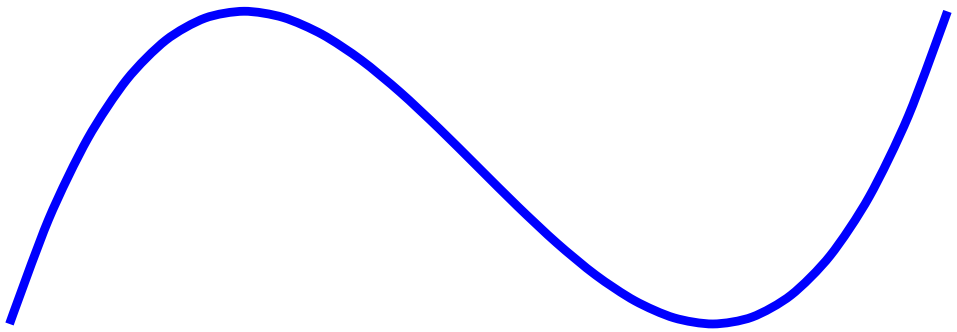
South Hall 6431X (Grad Tower, 6th floor, blue side, first door on the right)

Office Hours:

MTWR after class 2:00-3:00, and by appointment. Details on Gauchospace.

Meanings: The Second Derivative



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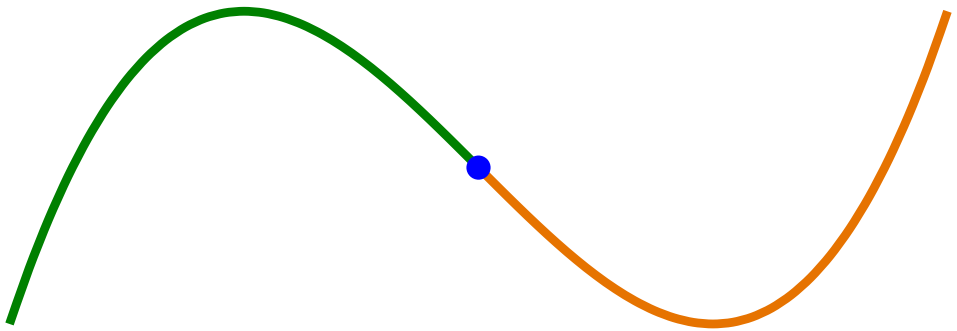

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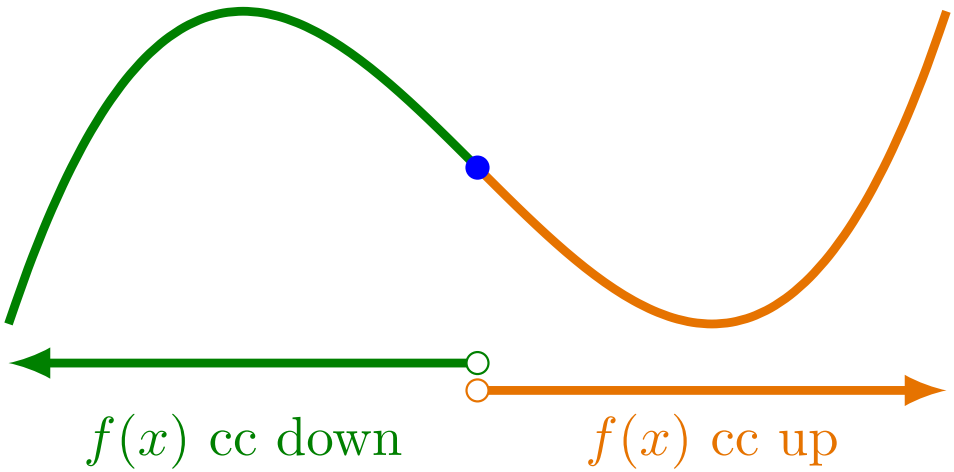

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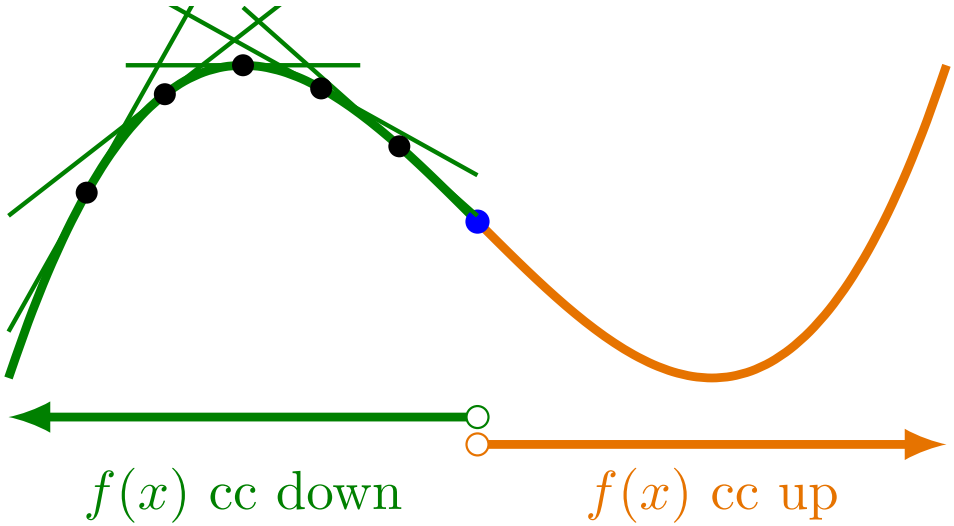

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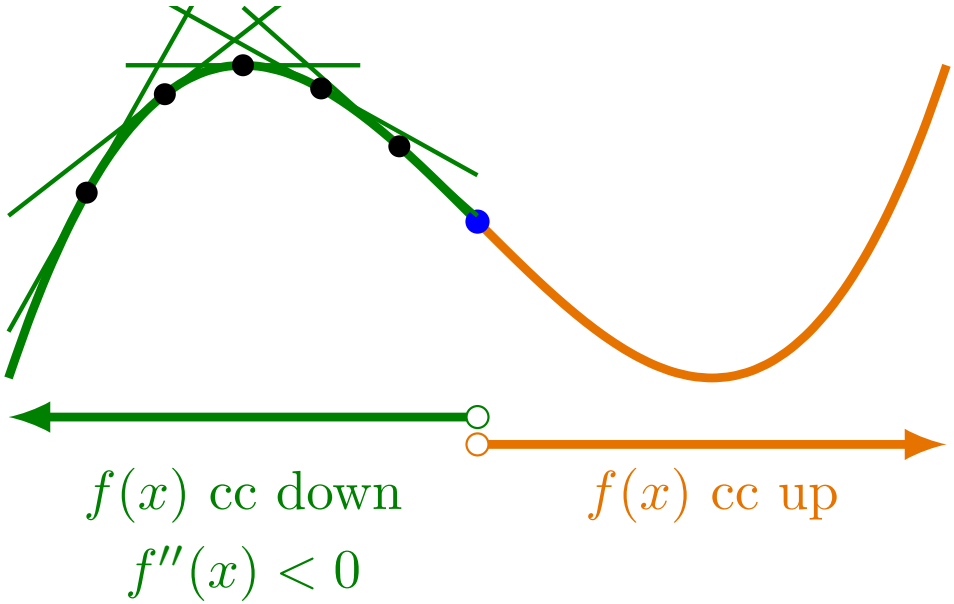

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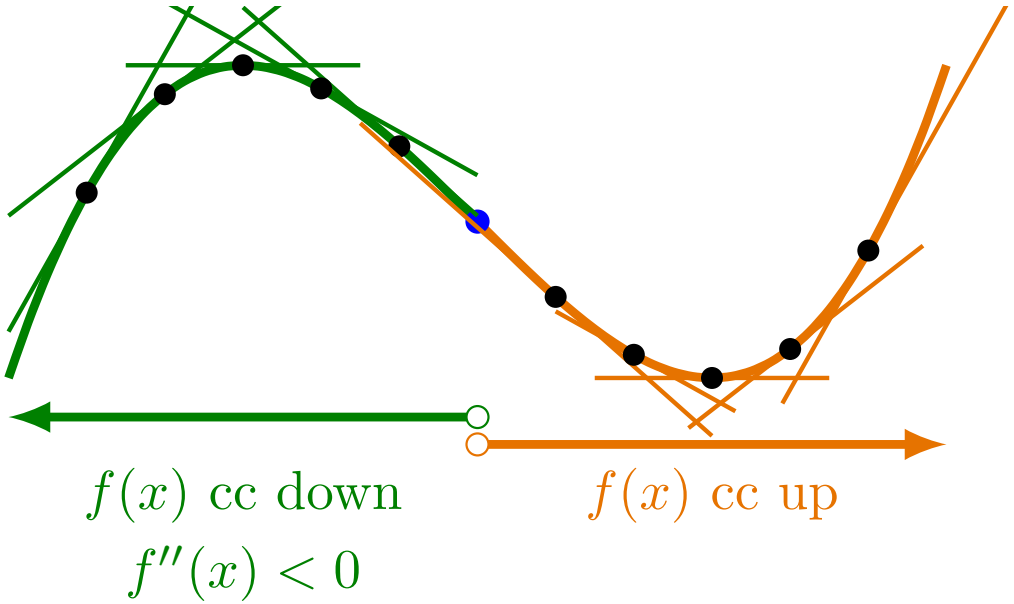

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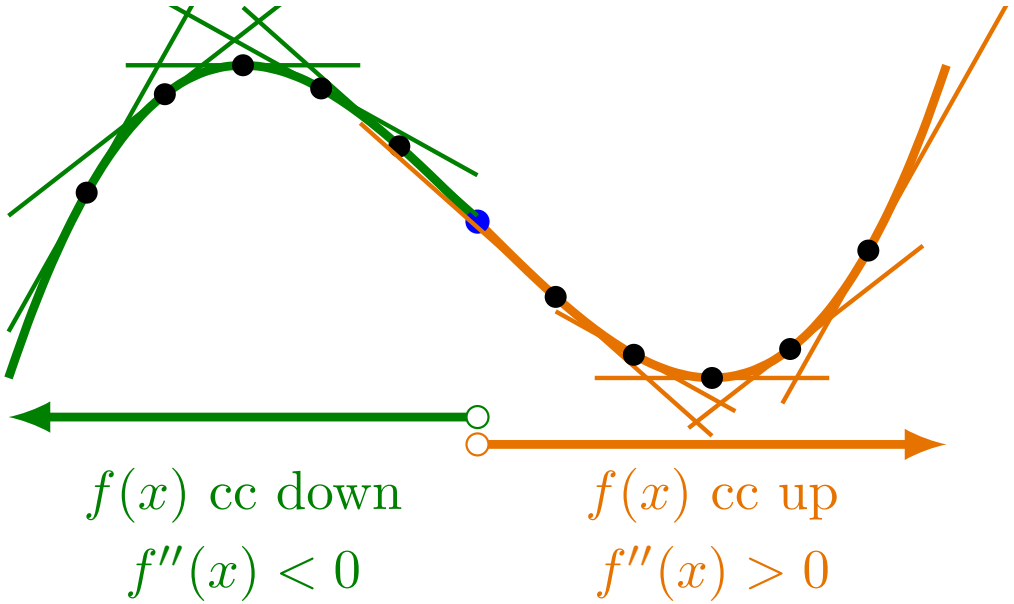

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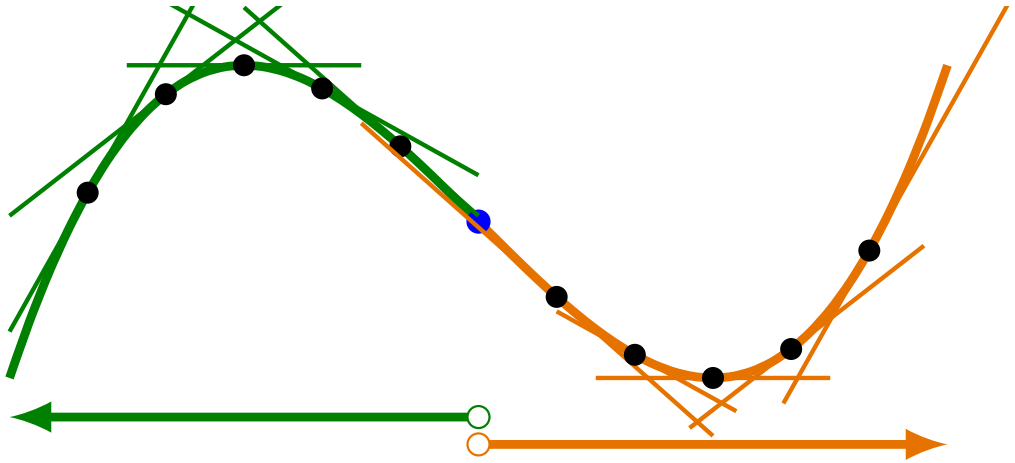

Point:

$$
\begin{aligned}
f''(x) > 0 &\iff f'(x) \text{ is increasing} \\
&\iff f(x) \text{ is concave up} \\
f''(x) < 0 &\iff f'(x) \text{ is decreasing} \\
&\iff f(x) \text{ is concave down}
\end{aligned}
$$

Concavity

$$f''(x) > 0 \iff f(x) \text{ is concave up}$$
$$f''(x) < 0 \iff f(x) \text{ is concave down}$$

(1) For which values of x is $f(x) = x^3 - 6x^2 + 3x + 2$ concave up?

A when $x = 0$ B when $x < 6$ C when $x > 6$
D when $x < 2$ E when $x > 2$

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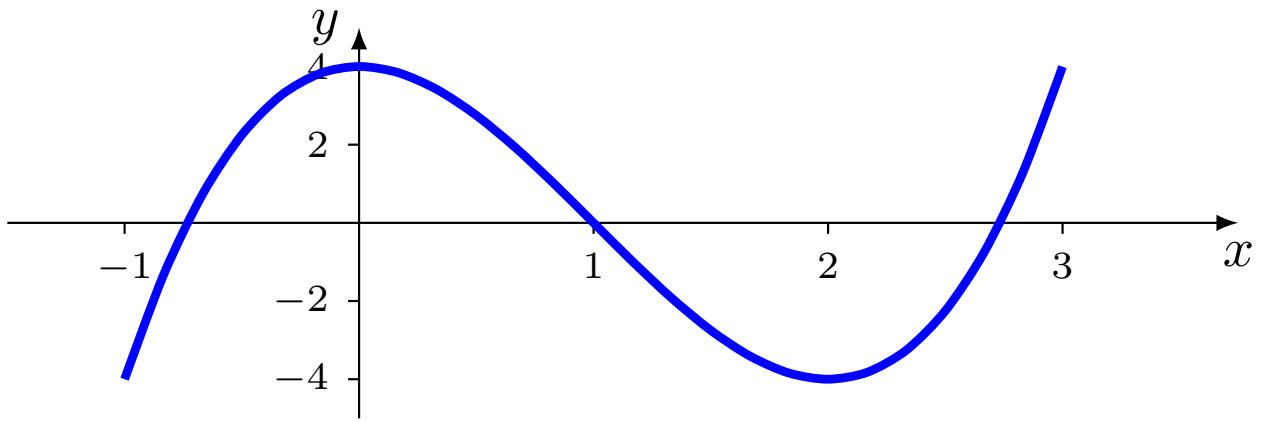
(2) Where is $f''(x) > 0$?



A when $x < 2$ B when $x > 2$ C when $x < 1$
D when $x > 1$ E when $-0.7 < x < 1$

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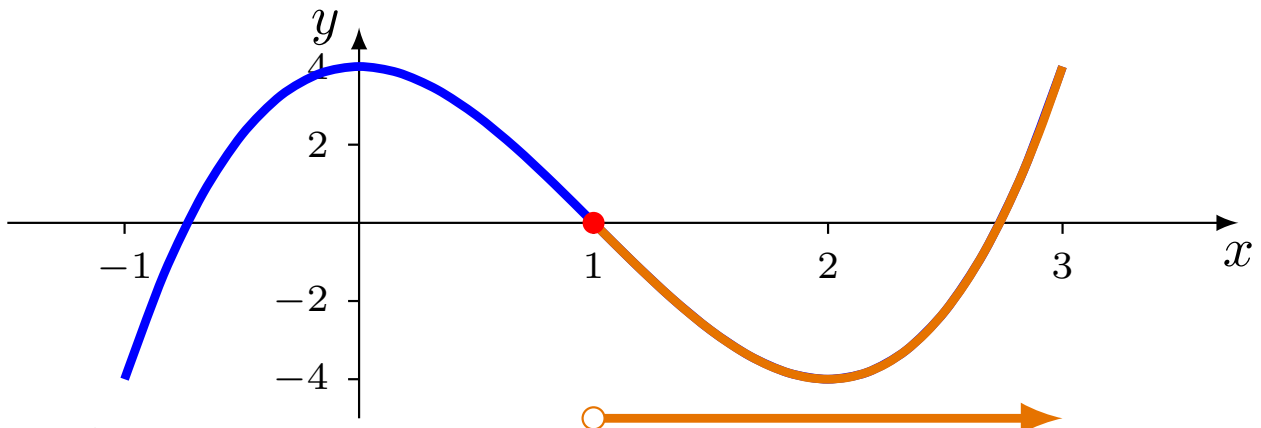

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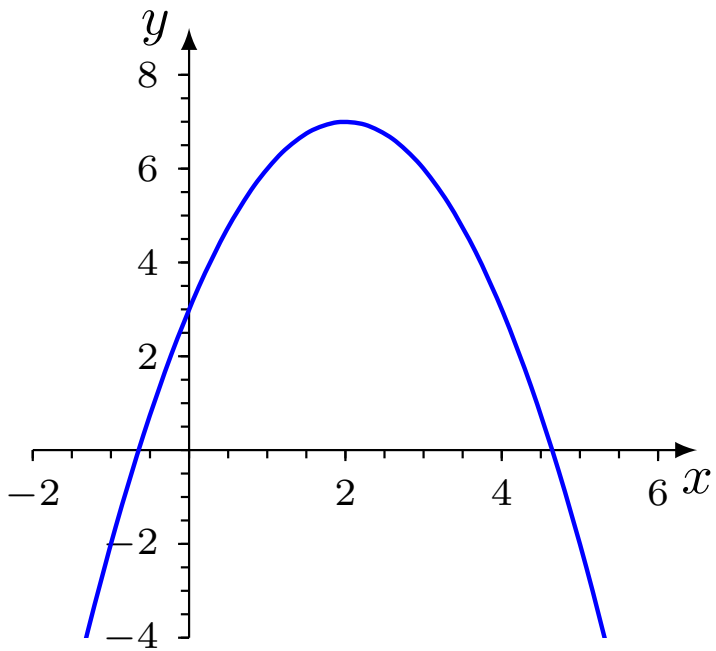
§8.13: Max/Min problems

Often want to find the biggest, smallest, most, least, maximum, minimum of something.



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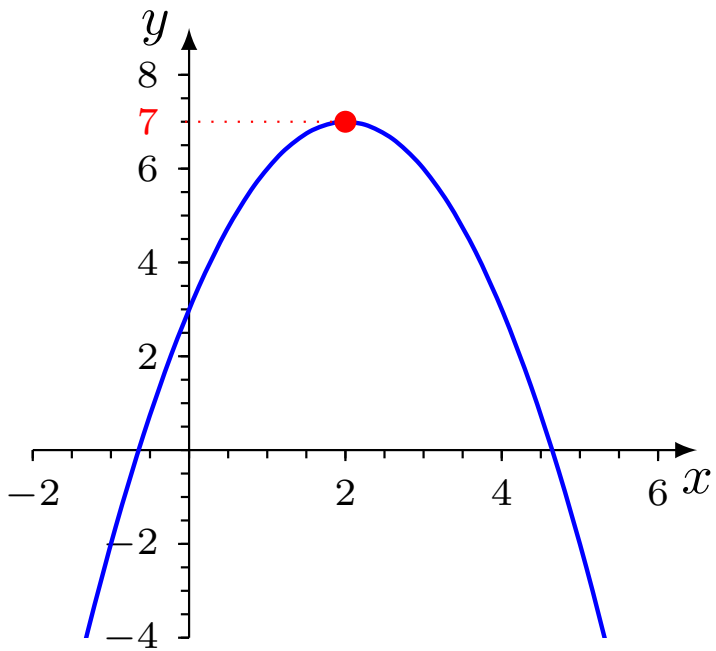

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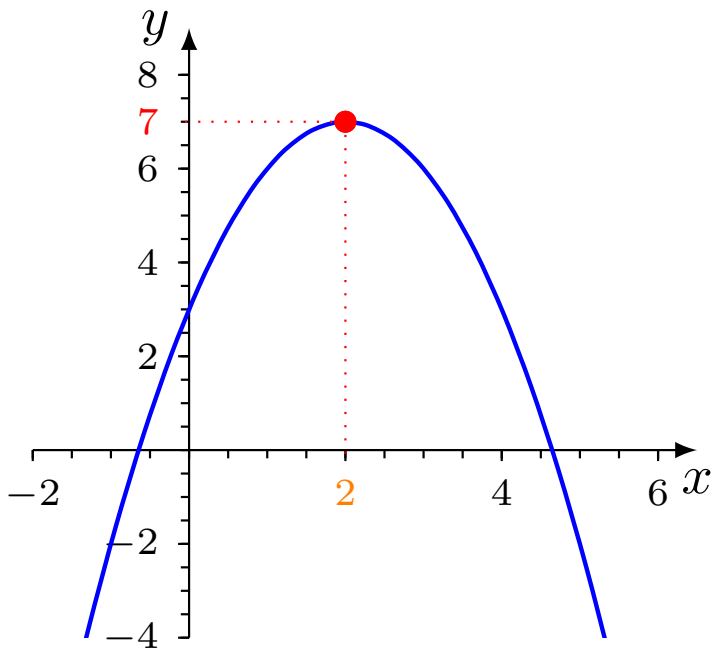

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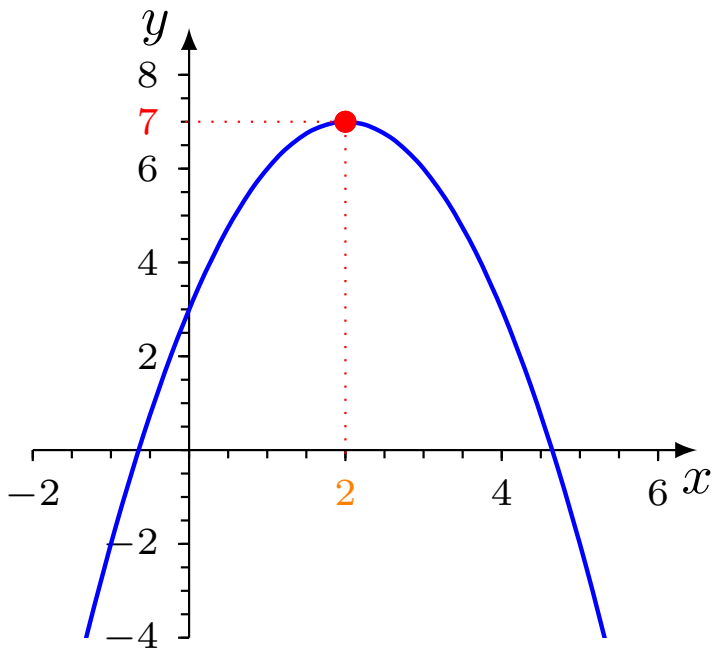

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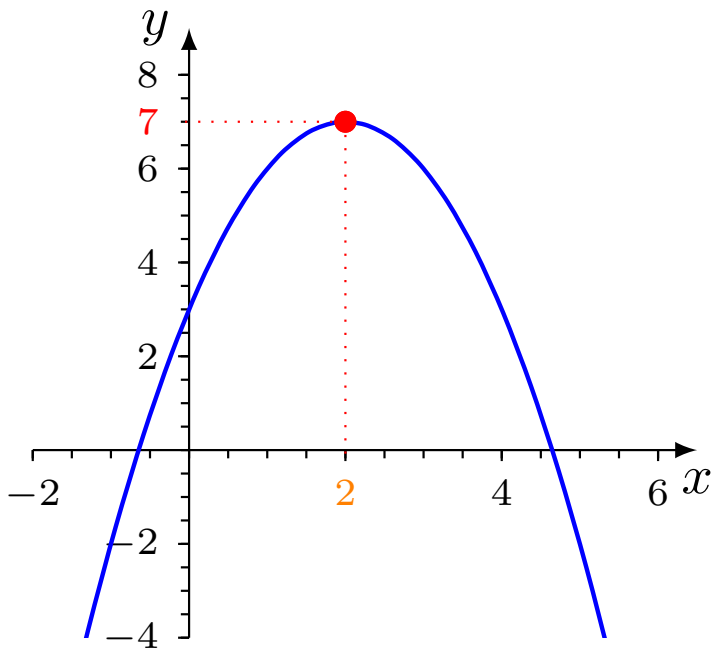

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For this example you can see this is the maximum because

$$f(x) = -x^2 + 4x + 3 = -(x - 2)^2 + 7$$

$(x - 2)^2$ is always positive except when $x = 2$
so the maximum must be at $x = 2$.

How To Find A Max / Min

(1) Find $f'(x)$

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Example: Use this method to find the x-value where maximum of the function $f(x) = 5x - e^{2x}$ occurs.

A $= 0$ B $= \ln(5)$ C $= 2\ln(5)$ D $= 2\ln(5/2)$ E $= \ln(5/2)/2$

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Answer: E

Word Problem #1

A ball is thrown into the air. After t seconds the height in meters above the ground of the ball is $h(t) = 40t - 10t^2$. How many meters high did the ball go?

$$\text{A} = 2 \quad \text{B} = 40 - 20t \quad \text{C} = 20 \quad \text{D} = 40$$

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Word Problem #2

If an airline sells tickets at a price of $\$200 + 5x$ each the number of tickets it sells is $1000 - 20x$. What price should the tickets be if the airline wants to get the most money?

$$\text{A} = 5 \quad \text{B} = 25 \quad \text{C} = 175 \quad \text{D} = 200 \quad \text{E} = 225$$

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Word Problem #3

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Students always find (1) the hardest part.
You have been prepared for this by word problems from chapter 3!

Word Problem #4

A fenced garden with an area of 1000 m^2 will be made in the shape of a rectangle. It will be surrounded on all four sides by a fence. Three sides are wood fence, and the remaining side is a brick wall.

- The wood fence costs \$5 per meter length.
- The brick wall costs \$20 per meter length.

Word Problem #4

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- The wood fence costs \$5 per meter length.
- The brick wall costs \$20 per meter length.
- C = total cost of the fence and brick wall
- L = length of the brick wall
- W = width of the other side

(a) Find a formula for C in terms of only L.

$$\text{A} = 2W + 2L \quad \text{B} = 2000L^{-1} + 2L \quad \text{C} = 25L + 10000L^{-1}$$

$$\text{D} = 20L + 10000WL^{-1} \quad \text{E} = 5L + 3000$$

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(b) What length of brick wall gives lowest cost?

A $= 20$ B $= 40$ C $= 50$ D $= 100$ E $= 25$

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Word Problem #5

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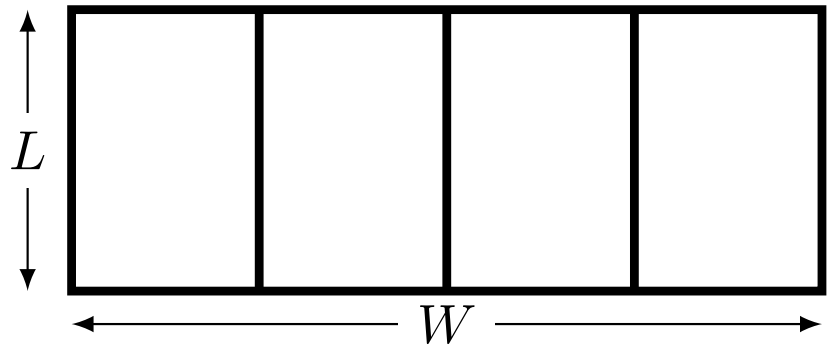

parts by 3 more dividing fences all parallel to one side of the field.

(a) What is the total length of all the fence needed?

$$\text{A} = 2L + 2W \qquad \text{B} = LW \qquad \text{C} = 5LW$$

$$\text{D} = L + W \qquad \text{E} = 5L + 2W$$

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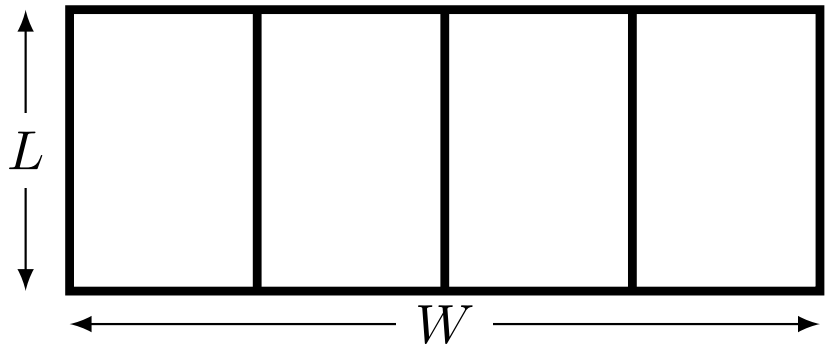

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(b) The field must have an area of 1000 m^2. Express W in terms of L.

$$\text{A } 1000 - L \quad \text{B } 1000L \quad \text{C } 1000/L \quad \text{D } 1000 + L$$

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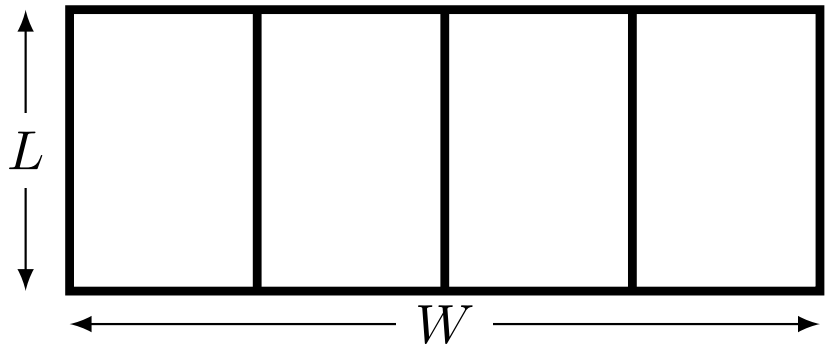

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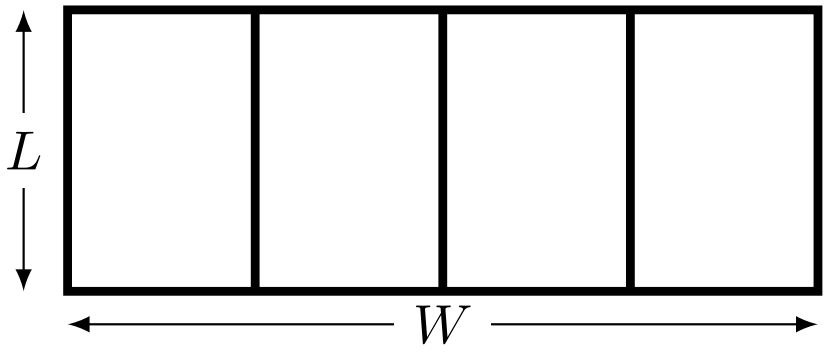

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(c) Express the total length of all the fence needed in terms of L.

$$\text{A} = 5L + 1000 \qquad \text{B} = 5L + 2000/L \qquad \text{C} = 5L + 2/L$$

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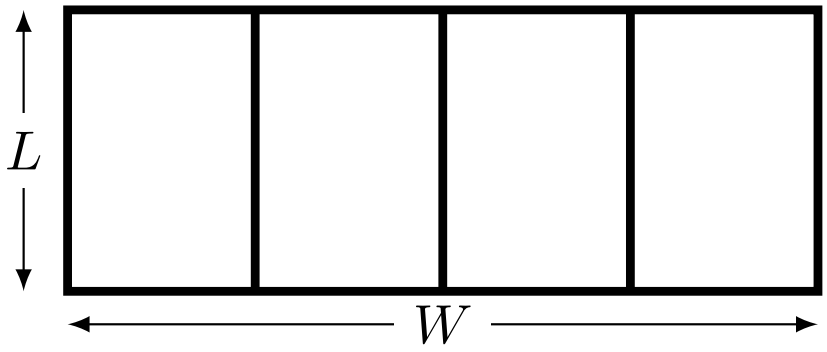

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(d) What should L be so that the total length of fence used is a minimum?

$$\text{A} = 10 \quad \text{B} = 20 \quad \text{C} = 40 \quad \text{D} = 50$$

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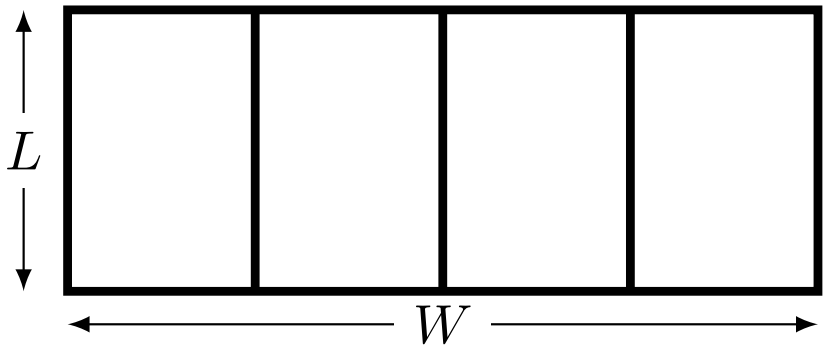

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Word Problem #6

A rectangular field is surrounded on three sides by a fence and the fourth side runs along a perfectly straight river. What is the largest area field which can be so enclosed with 120 meters of fence?

A $= 1200$ m^2 B $= 1500$ m^2 C $= 1800$ m^2 D $= 1000$ m^2

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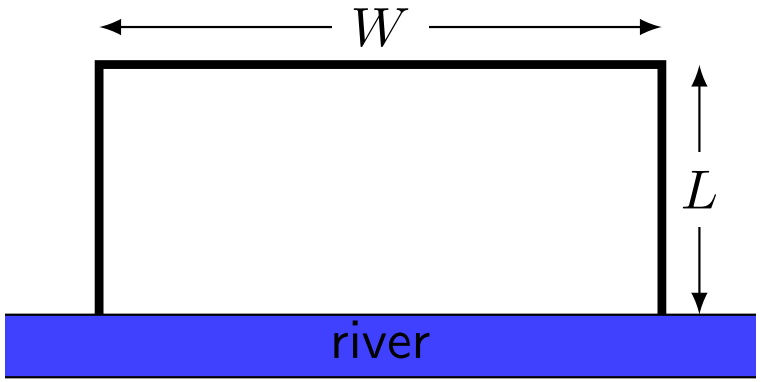

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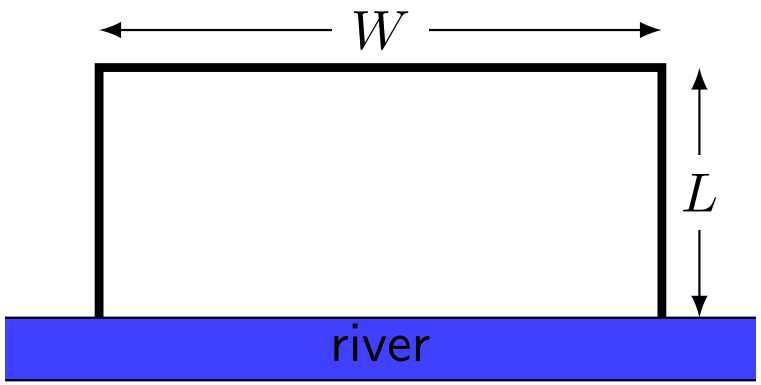

Word Problem #7

Tickets are going to be sold for a concert.

- If the price of each ticket is \$40, then $2,000$ tickets will be sold.
- For every \$1 the price is decreased, 100 more tickets will be sold.

(a) If the tickets are sold for $\$x$ each, how many will be sold?

$$\text{A} = 2000 - x \qquad \text{B} = 2000 - 100x \qquad \text{C} = 2000 + 100x$$

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$$\text{D} = 6000 - 100x \qquad \text{E} = 6000 + 100x \qquad \text{D}$$

(b) What is the total amount of money generated from selling tickets for $\$x$ each?

$$\text{A} = 6000x - 100x^2 \qquad \text{B} = 2000x$$

$$\text{C} = 2000 - 40x^2 \qquad \text{D} = 6000 - 100x$$

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$$\text{C} = 2000 - 40x^2 \quad \text{D} = 6000 - 100x \quad \text{A}$$

(c) What price should the tickets be to generate the most money from sales?

$$\text{A} = \$20 \quad \text{B} = \$22 \quad \text{C} = \$24 \quad \text{D} = \$30 \quad \text{E} = \$40$$

Word Problem #7

Tickets are going to be sold for a concert.

- If the price of each ticket is \$40, then 2,000 tickets will be sold.
- For every \$1 the price is decreased, 100 more tickets will be sold.

(a) If the tickets are sold for $\$x$ each, how many will be sold?

$$\text{A} = 2000 - x \quad \text{B} = 2000 - 100x \quad \text{C} = 2000 + 100x$$

$$\text{D} = 6000 - 100x \quad \text{E} = 6000 + 100x \qquad \text{D}$$

(b) What is the total amount of money generated from selling tickets for $\$x$ each?

$$\text{A} = 6000x - 100x^2 \quad \text{B} = 2000x$$

$$\text{C} = 2000 - 40x^2 \quad \text{D} = 6000 - 100x \quad \text{A}$$

(c) What price should the tickets be to generate the most money from sales?

$$\text{A} = \$20 \quad \text{B} = \$22 \quad \text{C} = \$24 \quad \text{D} = \$30 \quad \text{E} = \$40 \quad \text{D}$$

Word Problem #8

A farmer is growing wheat.

- On July 1, she has 1,000 bushels and this increases by 50 bushels per day.
- The price of a bushel on July 1 is \$10 and is dropping at a rate of 20 cents per day.
- She will harvest and sell on the same day.

How many days should she wait, assuming these trends continue?

$$\text{A} = 5 \quad \text{B} = 10 \quad \text{C} = 15 \quad \text{D} = 20 \quad \text{E} = \text{other}$$

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