Project #3: First Order Linear Systems of DEs.
Pre-Work

PW 1 Let $\ddot{x} + 3\dot{x} + 2x = 0$ be the equation of a damped vibrating spring with a unit mass, damping coefficient $b = 3$ and spring constant $k = 2$. We can convert this second order DE into a system of two first order DEs.

(a) If we make the substitution $y = \dot{x}$ we can find a system of two first order equations that describe the motion of the spring-block set-up. What two equations do you get?

(b) Now convert your system into matrix form. You should get something like

$$\begin{bmatrix} x'(t) \\ y'(t) \end{bmatrix} = A \begin{bmatrix} x(t) \\ y(t) \end{bmatrix}.$$

What are the entries in your coefficient matrix A?

(c) Find the eigenvalues and eigenvectors of A.

PW 2 (a) Find the general solution to the linear homogeneous DE $\ddot{x} + 3\dot{x} + 2x = 0$.

(b) Now express your solution from part (a) in vector form, i.e. find an expression for the vector-valued function whose first entry is the position of the block and second entry is velocity of the block:

$$\begin{bmatrix} x(t) \\ y(t) \end{bmatrix} = \begin{bmatrix} ? \\ ? \end{bmatrix}$$

(c) Can you express your vector-valued solution from part (b) in terms of the eigenvalues and eigenvectors you found in PW 1 part (c)?

(d) Will you always be able to do this for any second order linear DE with constant coefficients?