Office Hours!

Instructor:

Peter M. Garfield, `garfield@math.ucsb.edu`

Office Hours:

Mondays 2–3PM
Tuesdays 10:30–11:30AM
Thursdays 1–2PM
or by appointment

Office:

South Hall 6510

Exam 1: Wednesday in class

Bring:

- A blue book
- A 3” × 5” card (both sides!) with your notes.
- A pen / pencil
- An ID

Don’t bring:

- A calculator

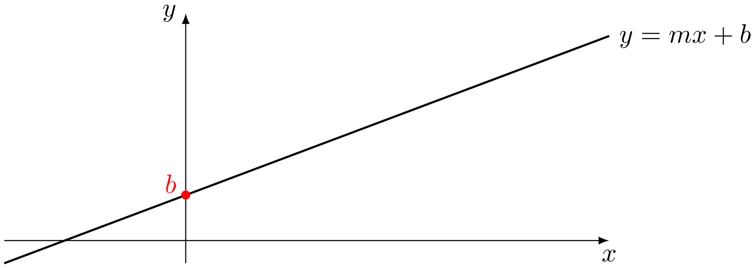
Please Be Early!

See Gauchospace and textbook for sample exams.

Straight Lines (§6.1)

The Slope Intercept Form

The **slope intercept** equation of a straight line is $\boxed{y = mx + b}$.



m = the **slope**. CRUCIAL for calculus.

b = where the line crosses the y-axis (the "y-intercept").

WHY? Because when you plug in $x = 0$, you get $y = b$.

Example

1. Find the equation of the line $\boxed{y = mx + b}$ through the points $(1, 3)$ and $(7, 5)$.

Plan: Find m, then find b.

(a) What is m?

A$= 1$ B$= 3$ C$= 5$ D$= 1/3$ E$= 2$ $\boxed{\text{D}}$

So $y = \frac{1}{3}x + b$. What is b? Plug in either point!

(b) What do you get for b?

A$= 1/3$ B$= 4/3$ C$= 7/3$ D$= 8/3$ E$= 10/3$ $\boxed{\text{D}}$

(c) Can we check?

You Try It

2. A line has slope 1/2 and goes through the point $(2, 5)$. What is the y-coordinate of the point on this line where $x = 6$?

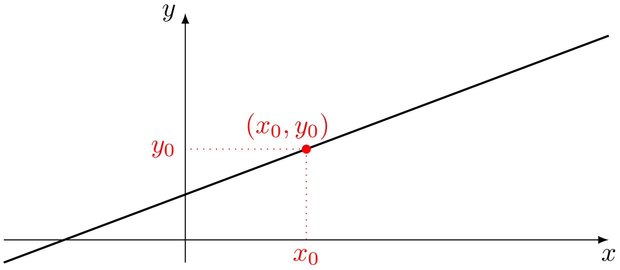
A$= 3$ B$= 4$ C$= 5$ D$= 6$ E$= 7$ $\boxed{\text{E}}$

Plan: 1. Find equation of the line.
2. Plug in $x = 6$ to find y.

Another Equation of a Line

The Point-Slope Form

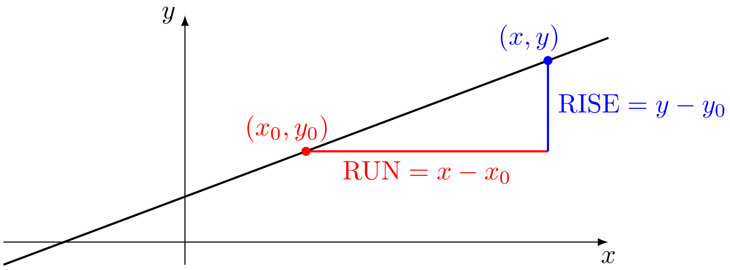
The **point slope** equation of a straight line is $\boxed{y = y_0 + m(x - x_0)}$.



m = the **slope**. Still CRUCIAL for calculus.

(x_0, y_0) = any point on the line.

Why Does This Work?

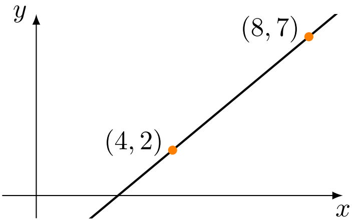


(x, y) lies on the line exactly when

$$\frac{y - y_0}{x - x_0} = m$$
$$y - y_0 = m(x - x_0)$$
$$y = y_0 + m(x - x_0)$$

Examples

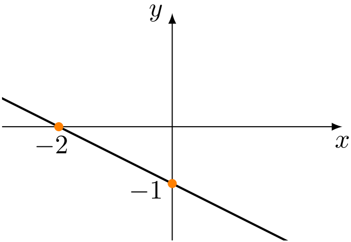
3. Find the equations of these lines (whose slopes we've already found):



$$m = 5/4$$

$$y - 2 = \frac{5}{4}(x - 4)$$

$$y = \frac{5}{4}x - 3$$



$$m = -1/2$$

$$y - (-1) = -\frac{1}{2}(x - 0)$$

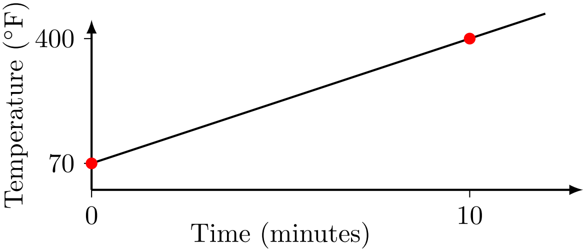
$$y = -\frac{1}{2}x - 1$$

And. . . ?

Yes, but what's this got to do with calculus?

Derivatives are about **rate of change** and that is what **slope** is!

Example: This graph shows the temperature in an oven as it heats up:



4. How quickly (in °F/min) is the oven heating up?

A= 70 B= 10 C= 40 D= 33 E=Other $\boxed{\text{D}}$

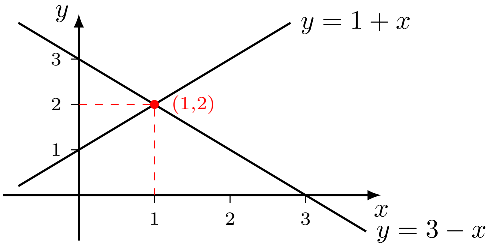
Moral: Rate of increase = **slope**

One More Example

5. Where does the line $y = 1 + x$ cross the line $y = 3 - x$? Find both the x and y coordinates of the crossing point.

Plan:

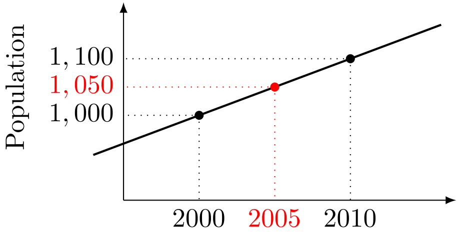
1. Draw a picture! showing two straight lines crossing.
2. Solve the two simultaneous equations
3. THINK why this gives the answer!



Linear Interpolation

6. In 2000, a population was 1000. In 2010, it was 1100. What would you guess the population was in 2005?

A= 1005 B= 1020 C= 1050 D= 2050 E= 2010 $\boxed{\text{C}}$

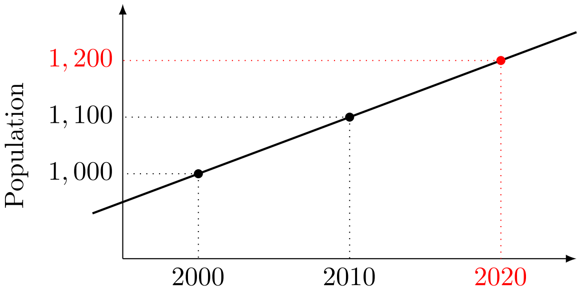


This is a guess based on the assumption that population grows at a constant rate.
"Constant rate" means that the graph of population is a straight line.

Linear Extrapolation

7. In 2000, a population was 1000. In 2010, it was 1100. What would you guess the population will be in 2020?

A= 1150 B= 1200 C= 1250 D= 2020 E=Other $\boxed{\text{B}}$



Again: a guess based on the assumption that population grows at a constant rate.

A Problem

You can't tell someone you just "guessed" the answer or just "drew a straight line". You need to make it sound more "scientific" so give it a complicated sounding name to impress people.

Linear Interpolation and Linear Extrapolation.

Linear means straight line
inter means between like intercity
extra means beyond like extraordinary

The idea is to assume the population (or whetever) grows at a constant rate.
Then use this to predict.

Method:
(1) Use given data to draw a straight line and find equation
$y = mx + b$
(2) Use the equation to make predictions.

Wiktionary: interpolo (Latin)

Latin [edit]

Etymology [edit]

From *inter-* + *poliō* + *-ō*.

Pronunciation [edit]

- (*Classical*) IPA(key): /inˈter.po.loː/, [ɪnˈtɛr.pɔ.ɫoː]

Verb [edit]

interpolō (*present infinitive* **interpolāre**, *perfect active* **interpolāvī**, *supine* **interpolātum**); *first conjugation*

1. I give a new form, shape, or appearance
2. I polish, furbish, dress up
3. (*of writing*) I alter, falsify, insert text

Linear Extrapolation

8. In 2000, a population was 1000. In 2010, it was 1100. Let

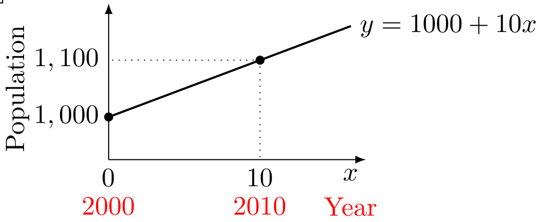
x = number of years after 2000 (Ex: $x = 3$ is the year 2003)

y = population in the year x

Find the equation of a line $y = mx + b$:

A: $2000 + 1000x$ B: $1000 + 2000x$ C: $1000 + 100x$ D: $1000 + 10x$

Answer: $\boxed{\text{D}}$



9. When will population be 1350?

A= 2015 B= 2025 C= 2035 D= 3350 E=Other $\boxed{\text{C}}$

Another Example

10. The number of unemployed in LA on January 1, 2015 was $50,000$. After 100 days, it was $45,000$.

(a) Estimate the number of unemployed 300 days after Jan. 1.

A$= 40,000$ B$= 35,000$ C$= 30,000$ D$= 25,000$ E$= 300$ $\boxed{\text{B}}$

(b) Suppose $x =$ the number of days after January 1

$y =$ number of unemployed people on day x.

Then the equation of the line used for this linear extrapolation is $y =$

A$= -100 + 50,000x$ B$= 50,000 - 100x$ C$= 45,000 - 100x$
D$= 50,000 - 50x$ E$= 45,000 - 50x$ $\boxed{\text{D}}$

(c) How many days until unemployed reaches $30,000$?

A$= 40$ B$= 140$ C$= 200$ D$= 300$ E$= 400$ $\boxed{\text{E}}$

Proportionality

Simple Idea: $y \propto x$ "y is proportional to x" means:

> If you double x, then y doubles. Triple x then y triples. And so on.

Example: If you are paid by the hour then

$$\text{(amount you earn)} \propto \text{(number of hours you work)}$$

If you work for 10 hours, then you are paid \$50. How much are you paid if you work for 20 hours?

A= \$20 B= \$50 C= \$200 D= \$100 E=Other $\boxed{\text{D}}$

If you work for t hours, how much are you paid?

A= \$50 B= \$50t C= \$10t D= \$20t E= \$5t $\boxed{\text{E}}$

Because you are paid \$5/hour (or \$50 for 10 hours). The number "5" is called the constant of proportionality.

Proportionality Example:

Suppose $y \propto x$ and $y = 15$ when $x = 4$.

(a) What is y when $x = 8$?

A$= 15$ B$= 4$ C$= 8$ D$= 30$ E$= 60$ $\boxed{\text{D}}$

(b) What is y when $x = 12$?

A$= 15$ B$= 45$ C$= 30$ D$= 36$ E$= 12$ $\boxed{\text{B}}$

(c) What is x when $y = 150$?

A$= 14$ B$= 1500$ C$= 40$ D$= 450$ $\boxed{\text{C}}$

Constant of Proportionality

"y is proportional to x" means $y = Kx$, where K is called the constant of proportionality.

Example: We are told

- Tax is proportional to income, and
- The tax on \$1,000 is \$280.

Express y = amount of tax paid in terms of x = the income. Then $y =$

$$\text{A}= 1000x \qquad \text{B}= 280x \qquad \text{C}= \frac{1,000}{280}x$$

$$\text{D}= 2.8x \qquad \text{E}= 0.28x \qquad \boxed{\text{E}}$$

Question: What does the constant of proportionality $K = 0.28$ mean?

Answer: It is the tax on one dollar.