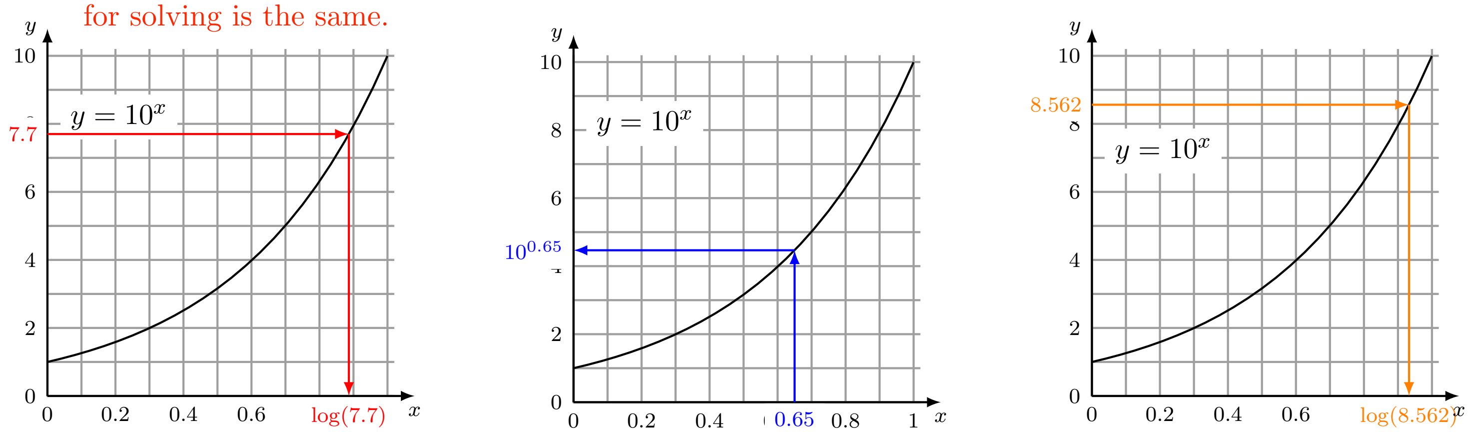
Math 34A Winter 2020
Old Midterm 2 #1 Solutions

1. Here are the three graphs we'll use in solving these problems:

Note to Trevor's class: we used a log table instead of a graph of 10^x, but ignoring that, the strategy for solving is the same.



Part (a) Part (b) Part (c)

(a) Remember that there is no way to simplify $\log(a+b)$, so instead we just add $35+42=77$ to realize we just need to find $\log(77)$. We can use the move the decimal point trick:

$$\log(77) = \log(10 \times 7.7) = \log(10) + \log(7.7) = 1 + \log(7.7).$$

Now we can use the graph to find that $\log(7.7) \approx 0.89$, and so $\log(77) \approx \boxed{1.89}$. (Mathematica tells me that $\log(77) \approx 1.886\,490\,725\ldots$, so we're pretty close.)

(b) The reverse version of the "move the decimal point trick" is what we need here:

$$10^{3.65} = 10^{3+0.65} = 10^3 \times 10^{0.65}.$$

We know that $10^3 = 1{,}000$, and we use the graph to find that $10^{0.65} \approx 4.47$. Thus $10^{3.65} \approx 1{,}000 \times 4.47 = \boxed{4{,}470}$. (Mathematica tells me that $10^{3.65} \approx 4{,}466.835\,921\,509\,631\ldots$, so we're within 4 out of more than $4{,}400$.)

(c) First we use the rules of logarithms and the move the decimal point trick to write

$$\log(\sqrt{8562}) = \frac{1}{2}\log(8562) = \frac{1}{2}\log(10^3 \times 8.562) = \frac{1}{2}\left(3 + \log(8.562)\right).$$

Now we can use the graph to find that $\log(8.562) \approx 0.93$. Thus

$$\log(\sqrt{8562}) = \frac{1}{2}\left(3 + \log(8.562)\right) \approx \frac{1}{2}(3 + 0.93) = \frac{3.93}{2} \approx \boxed{1.96}.$$

(Mathematica tells me that $\log(\sqrt{8562}) \approx 1.966\,287\,61\ldots$)

2. Let's start with this equation slightly simplified as

$$7^{4x+1} = 5.$$

Now take the logarithm of both sides to get

$$\log(7^{4x+1}) = \log(5).$$

We simplify this using rules of logs to

$$(4x+1)\log(7) = \log(5) \qquad \text{since } \log(a^p) = p\log(a).$$

Now distribute the product on the left to get

$$4x\log(7) + \log(7) = \log(5).$$

Now subtract $\log(7)$ from both sides, then divide by $4\log(7)$ to get

$$4x\log(7) = \log(5) - \log(7) \qquad \text{and then} \qquad x = \boxed{\frac{\log(5) - \log(7)}{4\log(7)}}.$$

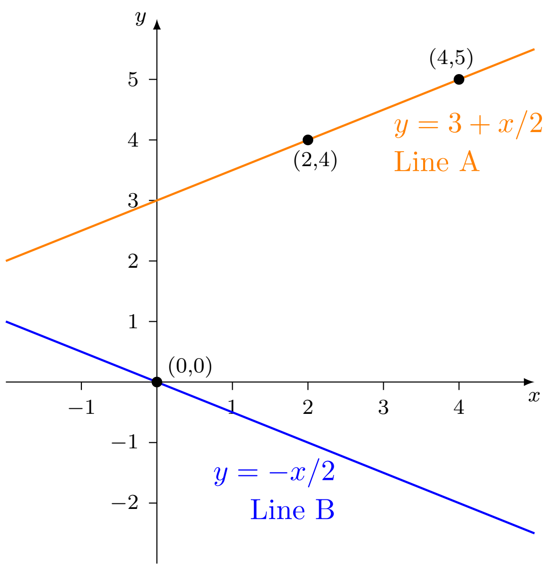
Since $\log(a) - \log(b) = \log(a/b)$, we can simplify the numerator to $\log(5) - \log(7) = \log(5/7)$. Thus we can write this as $x = \boxed{\frac{\log(5/7)}{4\log(7)}}$.

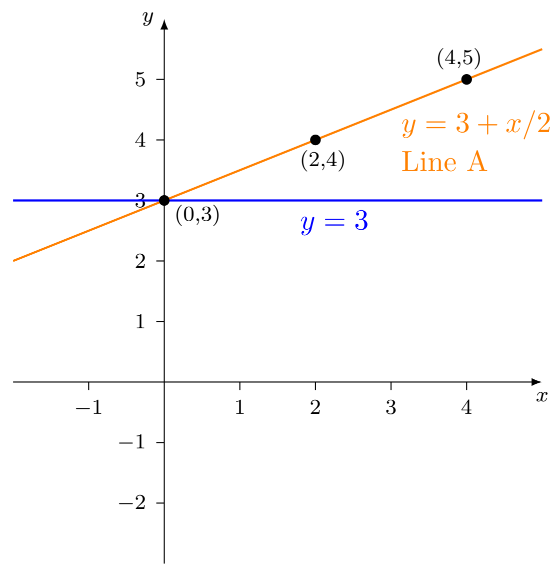
3. (a) The slope of Line A is

$$m = \frac{5-4}{4-2} = \frac{1}{2}.$$

Thus Line A has equation $y = \frac{1}{2}x + b$ for some b. We find that value by plugging in either point $(x, y) = (2, 4)$ or $(4, 5)$. If we plug in the first point, we get $4 = \frac{1}{2}(2) + b$, or $b = 3$. Thus Line A has equation $\boxed{y = \frac{1}{2}x + 3}$ or $\boxed{y = 3 + x/2}$. Line A is shown on the left, below.

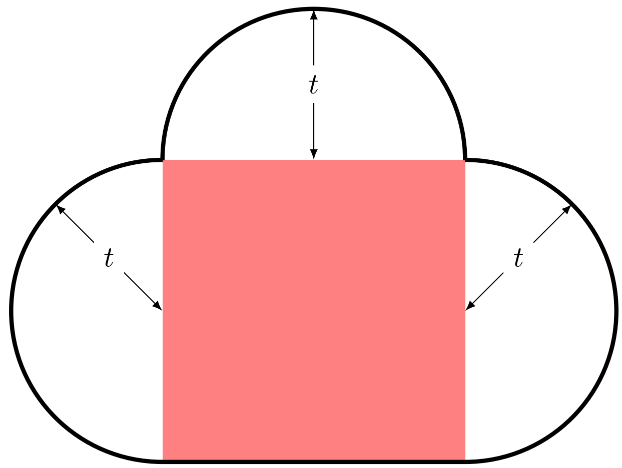
(b) Line B has slope minus the slope of Line A, or $-\frac{1}{2}$, so it has equation $y = -\frac{1}{2}x + b$. We plug in $(x, y) = (0, 0)$ to find b: $0 = -\frac{1}{2}(0) + b$, or $b = 0$. Thus the equation of Line B is $\boxed{y = -x/2}$. Line B is shown with Line A on the left, below.





(c) The point of intersection of Line A and the line $y - 3 = 0$ is where $y = 3$ (this is just the second line, slightly re-arranged). Solving $3 = 3 + x/2$ (since Line A is $y = 3 + x/2$), we get $x = 0$. Thus the point of intersection is $(x, y) = \boxed{(0, 3)}$. (The lines and the point of intersection are shown above on the right.)

4. We reproduce the picture of the garden here, with an added-in notation showing that the radius of the circles is t:



The radius of each semicircle is t, so the length of each side of the square is $2t$.

(a) The area of the garden is the area of three semicircles of radius r plus the area of a square of side length $2r$. The area of a semicircle is half the area of a circle; that is, a semicircle of radius $r = t$ has area $\pi r^2/2 = \pi t^2/2$. Thus the area of the garden is

$$A = 3\left(\pi t^2/2\right) + (2t)^2 \qquad \text{or} \qquad \boxed{A = 3\pi t^2/2 + 4t^2}.$$

(b) The perimeter of the garden is the perimeter of three semicircles of radius t plus the length of one side of the square (and we know the side length is $2t$). The perimeter of a semicircle is half the perimeter of a circle; that is, a semicircle of radius $r = t$ has perimeter $2\pi r/2 = \pi r = \pi t$. Thus the perimeter of the garden is $P = \boxed{3\pi t + 2t}$.

(c) If the area of the square is 100, then since the area of the square is $(2t)^2 = 4t^2$, we get the radius of each semicircle is $t = 5$ (we just took the square root of $t^2 = 100/4 = 25$). Then from part (b), the perimeter of the garden is $3\pi(5) + 2(5) = \boxed{15\pi + 10}$.

5. (a) I've poured half of the 6 liters of red paint into can B, so can B now contains 9 liters of blue paint and 3 liters of red paint. What is the percentage of paint that is red in can B? It's simply

$$\frac{\text{amount of red paint in can B}}{\text{total amount of paint in can B}} \times 100\% = \frac{3 \text{ liters}}{9 + 3 \text{ liters}} \times 100\% = \frac{3}{12} \times 100\% = 25\%.$$

That is, can B is now $\boxed{25\%}$ red.

(b) There were 12 liters in can B after my mixing experiment in part (a). Half of this is 6 liters, which I now add to the 3 liters of red paint remaining in can A. Thus there are $6 + 3 = \boxed{9 \text{ liters}}$ of paint now in can A.

(c) So how much red paint is now in can A? The 3 liters from can A were 100% red, and the 6 liters coming from can B were 25% red (as we found in part (a)). Thus the amount of red paint in can A is

$$100\%\big(3 \text{ liters}\big) + 25\%\big(6 \text{ liters}\big) = (1)(3) + (0.25)(6) \text{ liters} = \boxed{4.5 \text{ liters}}.$$

Sums

1. Find the following sum:

$$\sum_{n=1}^{6}(n+1)(n+2)$$

$2\cdot 3+3\cdot 4+4\cdot 5+5\cdot 6+6\cdot 7+7\cdot 8$
$= 6+12+20+30+42+56$

$$\begin{array}{r} 56 \\ 42 \\ 30 \\ 20 \\ 12 \\ +\ 6 \\ \hline 166 \end{array}$$

$\boxed{166}$

2. Find the following sum:

$$\sum_{m=2}^{4}\frac{m^2}{1-m}.$$

$$\frac{2^2}{-1}+\frac{3^2}{-2}+\frac{4^2}{-3}$$

$$= -\frac{4}{1}\cdot\frac{6}{6}-\frac{9}{2}\cdot\frac{3}{3}-\frac{16}{3}\cdot\frac{2}{2} = -\frac{24}{6}-\frac{27}{6}-\frac{32}{6} = \boxed{-\frac{83}{6}}$$

$$\begin{array}{r} 24 \\ 27 \\ +\ 32 \\ \hline 83 \end{array}$$

Limits

3. $\lim\limits_{x\to\infty} 4-\frac{1}{x}$ $\quad \frac{1}{x}\to 0$ $\quad \boxed{4}$

4. $\lim\limits_{h\to 0} \frac{4h-4h^2}{h} = \lim\limits_{h\to 0}\frac{\cancel{h}(4-h)}{\cancel{h}} = \boxed{4}$

5. $\lim\limits_{h\to 0} \frac{147h+21h^2+h^3}{h}$ $\quad$ $h\to 0$, so $21h+h^2\to 0$

$$= \lim_{h\to 0}\frac{\cancel{h}(147+21h+h^2)}{\cancel{h}}$$

$$= \boxed{147}$$

Average Speed

6. Find the average speed of a race car over the time period from 2 seconds to 3 seconds if $f(t)=t^3$ is the distance from the starting line t seconds after the start.

$$\text{Avg speed} = \frac{\Delta f}{\Delta t} = \frac{(3)^2-(2)^2}{3-2} = \frac{9-4}{1} = \boxed{5 \text{ units per second}}$$