

Welcome To Math 34A!

Differential Calculus

Instructor:

Trevor Klar, trevorklar@math.ucsb.edu

South Hall 6431X (Grad Tower, 6th floor, blue side, first door on the right)

Office Hours:

MTWR after class 2:00-3:00, and by appointment. Details on Gauchospace.

© 2017-22 Daryl Cooper, Peter Garfield, Ebrahim Ebrahim, Nathan Schley, and Trevor Klar
Please do not distribute outside of this course.

A nice thing about derivatives...

$$\begin{aligned}\frac{d}{dx}(a \cdot f(x) + b \cdot g(x)) &= a \frac{d}{dx} f(x) + b \frac{d}{dx} g(x) \\ &= a \cdot f'(x) + b \cdot g'(x)\end{aligned}$$

For example...

$$\begin{aligned}\frac{d}{dx}(3x^2 + 5x) &= 3 \frac{d}{dx} x^2 + 5 \frac{d}{dx} x \\ &= 3(2x) + 5(1) \\ &= 6x + 5\end{aligned}$$

A Warning!



$$\frac{d}{dx} (f(x)g(x)) \neq f'(x) \times g'(x)$$



Example: $5x^4 = \frac{d}{dx} (x^5) = \frac{d}{dx} (x^2 \cdot x^3) \neq (2x)(3x^2) = 6x^3$

Example: Find the derivative of $(x+1)(2x+3)$

Question: $\frac{d}{dx} ((x^2+1)(x^3+1)) = ?$

A = $6x^3$ B = $5x^4 + 3x^2 + 2x$ C = $x^5 + x^3 + x^2 + 1$ D = Other

Answer: B

Review Examples:

(1) What is the x -coordinate of the point on the graph of $y = 4x^2 - 3x + 7$ where the graph has slope 13?

A = 0 B = 1 C = 2 D = 3 E = 4 C

(2) A circle is expanding so that after R seconds it has radius R cm. What is the rate of increase of area inside the circle after 2 seconds?

A = 4π B = $2\pi R^2$ C = 2 D = $2\pi R$ E = πR^2 A

Differentiating $f(x) = e^{kx}$

$$\frac{d}{dx} (e^{kx}) = ke^{kx}$$

versus

$$\frac{d}{dx} (x^n) = nx^{n-1}$$

Do not get confused and write $\frac{d}{dx} (e^{kx}) = ke^{(k-1)x}$.

Question: Find $\frac{d}{dx} (4e^{3x} + 5x^3)$

$$A = 12e^{2x} + 15x^2$$

$$B = 12e^{3x} + 15x^3$$

$$C = 4e^{3x} + 15x^2$$

$$D = 12e^{3x} + 15x^2$$

$$E = \text{Other}$$

$$D$$

Example

$$\frac{d}{dx} (e^{kx}) = k e^{kx}$$

The temperature (in °C) of a cup of coffee t hours after it is made is $f(t) = 50 + 40e^{-2t}$.

(a) What is the **initial** temperature when the coffee is made?

A = 40 B = 50 C = 90 D = 100 C

(b) How quickly is the coffee **cooling down** initially? This means how many degrees per hour is the temperature **going down** instantaneously at $t = 0$?

A = 20 B = 40 C = 60 D = 80 E = 100 D

More Examples

$$\frac{d}{dx} (e^{kx}) = ke^{kx}$$

(1) $\frac{d}{dx} \left(\frac{3}{e^{2x}} \right) = ?$

A = $\frac{3}{2e^{2x}}$ B = $\frac{3}{2e^x}$ C = $\frac{6}{e^{2x}}$ D = $\frac{-6}{e^{2x}}$ D

(2) The number of grams of [Einsteinium-253](#) after t days is $m(t) = 10e^{-t/30}$. How quickly is the mass changing (in grams per day) when $t = 0$?

A = $-1/30$ B = $-1/3$ C = $-10e^{-t/30}$ D = $-\frac{1}{3}e^{t/30}$ B

§8.12: The Second Derivative

Today: We can take the derivative of a function repeatedly!

Example: If $f(x) = x^3 - 3x + 2$, then

- $\frac{df}{dx} = f'(x) = 3x^2 - 3$
- The **second derivative** of $f(x)$ is $\frac{d}{dx} \left(\frac{df}{dx} \right) = f''(x) = 6x$.
This is written $f''(x)$ or $\frac{d^2 f}{dx^2}$.
- The **third derivative** of $f(x)$ is $\frac{d}{dx} \left(\frac{d^2 f}{dx^2} \right) = f'''(x) = 6$.
This is written $f'''(x)$ or $\frac{d^3 f}{dx^3}$.
- **Keep Going!** The **fourth derivative** is $\frac{d^4 f}{dx^4} = f''''(x) = 0$.
- The fun ends here, for this $f(x)$ all **higher derivatives** are zero.

Examples

General idea: Differentiating the function n times gives us the n th derivative of f . It is written as

$$f^{\prime\prime\prime\prime\prime\prime}(x) = f^{(n)}(x) = \frac{d^n f}{dx^n}.$$

(1) What is the second derivative of $3x^2 - 5x + 7$?

$$A=0 \quad B=7 \quad C=6 \quad D=3 \quad E=-5 \quad C$$

(2) $\frac{d^2}{dx^2}(x^5) = ?$

$$A=20 \quad B=5x^4 \quad C=0 \quad D=20x^4 \quad E=20x^3 \quad E$$

(3) $\frac{d^2}{dx^2}(\sqrt{x}) = ?$

$$A=\frac{1}{4}x^{-3/2} \quad B=\frac{-1}{4}x^{-1/2} \quad C=\frac{-1}{4}x^{-3/2} \quad D=\frac{1}{2}x^{-1/2} \quad E=0 \quad C$$

More Examples

(4) $\frac{d^2}{dt^2}(e^{3t}) = ?$

A = e^{3t} B = $3e^{2t}$ C = $9e^{3t}$ D = $3e^{3t}$ E = $9e^t$ C

(5) Find $f'''(x)$ when $f(x) = x^3$.

A = $6x^2$ B = 0 C = $3x$ D = $3x^2$ E = 6 E

(6) If $f(x) = x^3 - 4x^2 + 7x - 31$, then $f''(10) = ?$

A = 6 B = $3x^2 - 8x$ C = $6x$ D = 60 E = 52 E

Example: Acceleration

The **acceleration** due to gravity is

$$32 \text{ feet per second per second} = 32 \text{ ft/sec}^2.$$

This means:

every second you fall,
your speed increases by $32 \text{ ft/sec} \approx 22 \text{ mph}$.

acceleration = rate of change of **velocity** = derivative of **velocity**.

velocity = rate of change of **distance** = derivative of **distance**.

Therefore

acceleration = second derivative of **distance**

Example: Height of ball is $h(t) = 20t - 5t^2$ meters after t seconds.

(a) **Velocity** of ball after t seconds is $h'(t) = 20 - 10t \text{ m/sec}$

(b) **Acceleration** of ball after t seconds is $h''(t) = -10 \text{ m/sec}^2$

It's not the speed that kills

Suppose you hit a brick wall at 60 mph.

Question: What is your (sudden!) acceleration?

$$\left(\begin{array}{c} \text{Average rate of} \\ \text{change of velocity} \\ \text{in stopping} \end{array} \right) = \frac{\Delta \text{ velocity}}{\Delta \text{ time}} = \frac{-60 \text{ mph}}{1/10 \text{ sec}}$$
$$\approx \frac{-88 \text{ ft/sec}}{1/10 \text{ sec}} = -880 \text{ ft/sec}^2.$$

Since 1 gravity = 32 ft/sec², this is about

$$880 \text{ ft/sec}^2 = (880 \text{ ft/sec}^2) \times \frac{1 \text{ gravity}}{32 \text{ ft/sec}^2} \approx 28 \text{ "g"}.$$

The force at which the brick wall pushes you is **28** times your weight.

If you weigh 110 pounds, this force is about **3000 pounds = 1.5 tons**.

A Rocket

A rocket is fired vertically upwards. The height after t seconds is $2t^3 + 5t^2$ meters.

Question: What is the acceleration in m/sec^2 after t seconds?

$$A = 2t^3 + 5t^2 \quad B = 6t^2 + 10t \quad C = 12t + 10 \quad D = 12 \quad E = 0 \quad C$$

Idea:

- $h(t)$ = height in meters at time t seconds
- $h'(t)$ = velocity in m/sec at time t seconds
- $h''(t)$ = acceleration in m/sec^2 at time t seconds

More Questions:

- (a) What can we say about $f(t)$ if $f'(t) = 0$ for **all** t ?
- (b) What can we say about $f(t)$ if $f''(t) = 0$ for **all** t ?

Application 2: Concavity

$$\frac{df}{dx} = \text{rate of change of } f(x)$$

$$\text{and so } \frac{d^2f}{dx^2} = \frac{d}{dx} \left(\frac{df}{dx} \right) = \text{rate of change of } \frac{df}{dx}$$

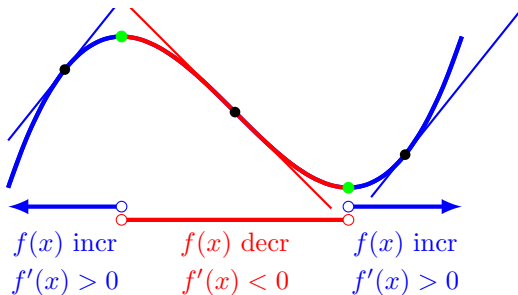
Conclusion:

The second derivative tells you how quickly the **rate of change** is changing.

Uses of second derivative:

- We've seen: **acceleration** is the rate of change of velocity
So: **acceleration** is the second derivative of distance traveled.
- Is the graph **concave up** or **concave down**?
- Are things **changing for better or worse**?

Meanings: The First Derivative

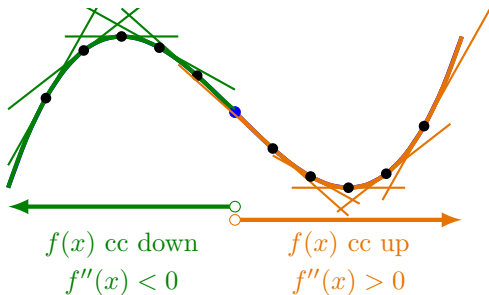


Point:

$$f'(x) > 0 \iff f(x) \text{ is increasing}$$

$$f'(x) < 0 \iff f(x) \text{ is decreasing}$$

Meanings: The Second Derivative



Point:

$$f''(x) > 0 \iff f'(x) \text{ is increasing}$$

$$\iff f(x) \text{ is concave up}$$

$$f''(x) < 0 \iff f'(x) \text{ is decreasing}$$

$$\iff f(x) \text{ is concave down}$$

Concavity

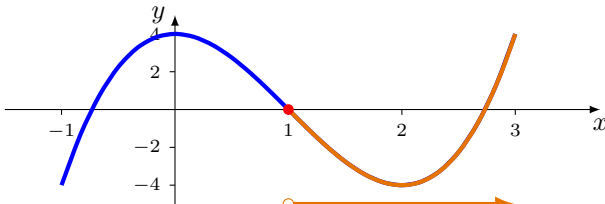
$$f''(x) > 0 \iff f(x) \text{ is concave up}$$

$$f''(x) < 0 \iff f(x) \text{ is concave down}$$

(1) For which values of x is $f(x) = x^3 - 6x^2 + 3x + 2$ concave up?

A when $x = 0$ B when $x < 6$ C when $x > 6$
D when $x < 2$ E when $x > 2$ E

(2) Where is $f''(x) > 0$?



A when $x < 2$ B when $x > 2$ C when $x < 1$
D when $x > 1$ E when $-0.7 < x < 1$ D