

Welcome To Math 34A!

Differential Calculus

Instructor:

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South Hall 6431X (Grad Tower, 6th floor, blue side, first door on the right)

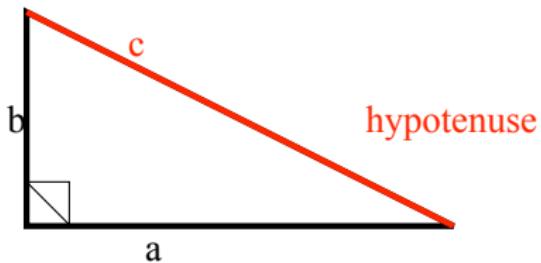
Office Hours:

MTWR after class 2:00-3:00, and by appointment. Details on Gauchospace.

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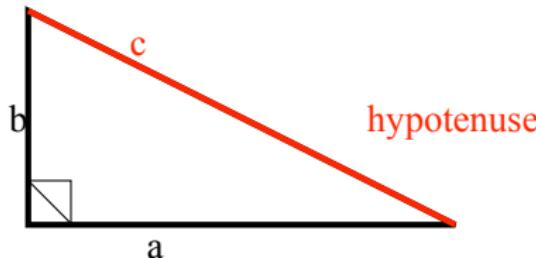
§ 1.7: Pythagoras' Theorem



hypotenuse

$$a^2 + b^2 = c^2$$

§ 1.7: Pythagoras' Theorem



$$a^2 + b^2 = c^2$$

11. What is the length of the hypotenuse of a right triangle when the other two sides have length 3 and 4 ?

A = 3 B = 4 C = 6 D = 25 E = none of these

12. Now lengths are 2 and 3 . What's the hypotenuse?

A = $\sqrt{5}$ B = $\sqrt{13}$ C = 13 D = 5

13. Lengths 3 x and 4x . What's the hypotenuse?

A = 5 + x B = 5 x 2 C = 25 x D = 5x



Pythagorean Theorem Applications

This is **very useful** to calculate how far apart two things are.

14. You and Marie are in Vegas. You drive north at 40 mph and
Marie drives east at 30 mph. How far apart are you after 1 hour?

Click A when you have the answer.

15. How many miles apart are you after t hours?

$$A = 50t \quad B = 50 + t \quad C = 50t^2 \quad D = 2500t^2$$

a

11. What is the length of the hypotenuse of a right triangle when the other two sides have length 3 and 4?

$$A = 3 \quad B = 4 \quad C = 6 \quad D = 25 \quad E = \text{none of these}$$

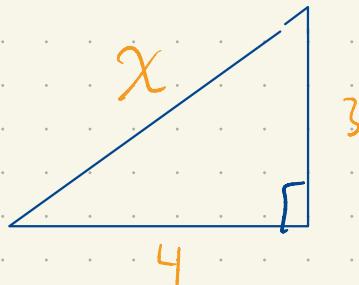
12. Now lengths are 2 and 3. What's the hypotenuse?

$$A = \sqrt{5} \quad B = \sqrt{13} \quad C = 13 \quad D = 5$$

13. Lengths $3x$ and $4x$. What's the hypotenuse?

$$A = 5 + x \quad B = 5x^2 \quad C = 25x \quad D = 5x$$

(11)



$$a^2 + b^2 = c^2$$

$$3^2 + 4^2 = x^2$$

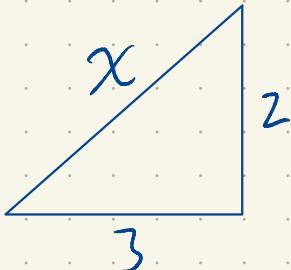
$$9 + 16 = x^2$$

$$25 = x^2$$

$$\sqrt{25} = \sqrt{x^2}$$

$$5 = x$$

(12)



$$2^2 + 3^2 = x^2$$

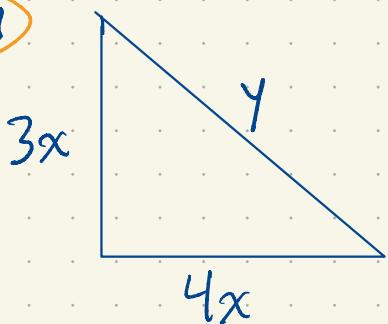
$$4 + 9 = x^2$$

$$13 = x^2$$

$$\sqrt{13} = x$$



(13)



Fractions

$$(3x)^2 + (4x)^2 = y^2$$

$$9x^2 + 16x^2 = y^2$$

$$25x^2 = y^2$$

$$\sqrt{25x^2} = y$$

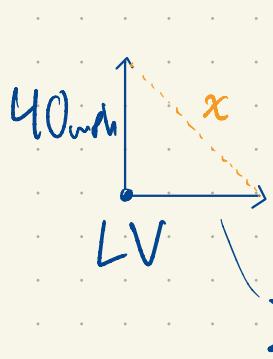
$$\sqrt{25} \sqrt{x^2} = y$$

$$5x = y$$

14. You and Marie are in Vegas. You drive north at 40 mph and Marie drives east at 30 mph. How far apart are you after 1 hour?
Click A when you have the answer.

15. How many miles apart are you after t hours?

$$A = 50t \quad B = 50 + t \quad C = 50t^2 \quad D = 2500t^2$$



$$x^2 = 40^2 + 30^2$$

$$x^2 = 1600 + 900$$

$$x^2 = 2500$$

$$x = \sqrt{2500} = \sqrt{25} \sqrt{100} = (5)(10)$$

$$= \boxed{50}$$

Note :

$$\sqrt{1600 + 900} \neq \sqrt{1600} + \sqrt{900}$$

$$\sqrt{1600 \times 900} = \sqrt{1600} \sqrt{900}$$

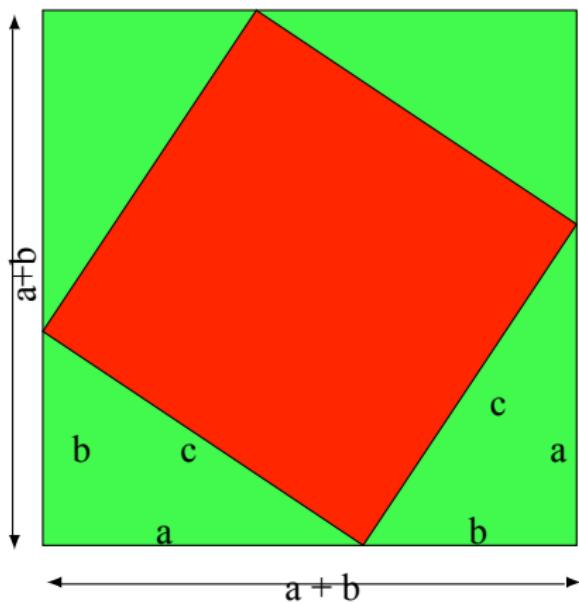
A word problem

1. The vertical mast of a yacht is 40 feet high. A rope runs in a straight line from the top to a pulley 30 feet horizontally from the base of the mast. How many feet long is the rope?

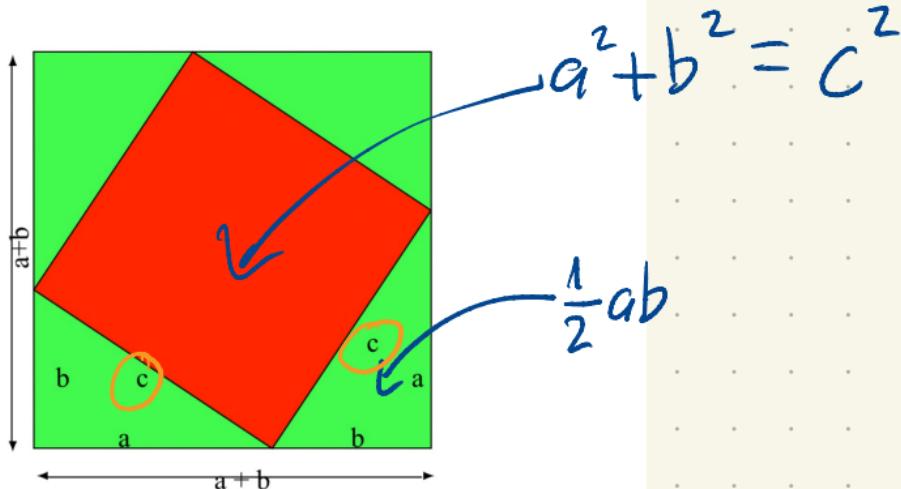
Hint: Draw a picture!

$$A = 30 \quad B = 40 \quad C = 50 \quad D = 60 \quad E = 70$$

Why Pythagorean Theorem works



Why Pythagorean Theorem works



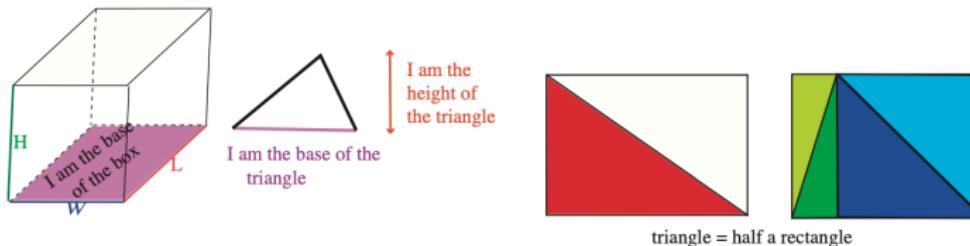
$$A = (a+b)^2 = a^2 + b^2 + 2ab$$

$$4 \left(\frac{1}{2} ab \right)$$

§ 4.2: Area and Volume

You need to know:

- Area of a rectangle = length \times width
- Area of a circle = πR^2 (R = radius)
- Circumference of a circle = $2\pi R$
- Area of a triangle = half base \times height = $\frac{1}{2}bh$
- volume of rectangular box = (length \times width) \times height
 $=$ (area of **base**) \times height



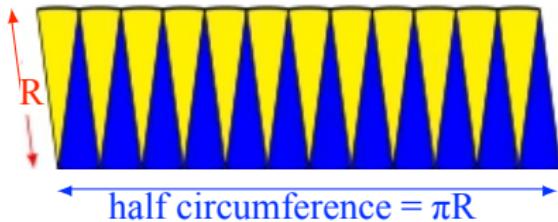
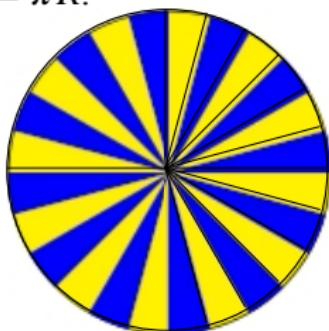
What is the (circumference of a circle) divided by the diameter?

$$A = R \quad B = 2\pi \quad C = \pi \quad D = \text{the what now?}$$

The definition of π is

$$\pi = \frac{\text{circumference of circle}}{\text{diameter}} = \frac{C}{2R},$$

$$\text{so } \frac{C}{2} = \pi R.$$



$$\text{Thus Area} = (R)(\pi R) = \pi R^2$$

Applications

2. A rectangular parking lot is to be made in the shape of a rectangle. It will have an area of 2000 square meters. Express the length of the parking lot in terms of the W = width .

$$A = (2000 - 2W) / 2 \quad B = 2000 / W \quad C = 2000 - W$$

D = Other

3. The parking lot will be surrounded by a fence. Express the total length of the fence in terms of W .

$$A = 2000 + 2W \quad B = L + W \quad C = 4000W^{-1} + 2W$$

4. The fence costs \$7 per meter. Express the total cost of all the fence in terms of W .

$$A = 7 \times 2000 \quad B = 7 \times 4000W^{-1} + 2W \\ C = 28000W^{-1} + 14W$$

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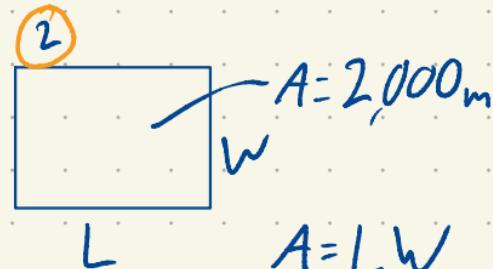
4. The fence costs \$7 per meter. Express the total cost of all the fence in terms of W .

$$A = 7 \times 2000 \quad B = 7 \times 4000W^{-1} + 2W$$
$$C = 28000W^{-1} + 14W$$

3) $P = 2L + 2W$

$$= 2\left(\frac{2000}{W}\right) + 2W$$

$$= \frac{4000}{W} + 2W$$



$$2000 = LW$$

$$L = \frac{2000}{W}$$

4) $\text{cost} = \$7/\text{meter}$

Applications II

5. A rectangular poster is to have a total area of 500 cm^2 . There is an empty margin where nothing is printed 6 cm wide at the top and 4 cm wide along the sides and bottom. The rest is the printed area.

Hint: Draw a picture! Name your unknowns!

- Express printed area in terms of width W and height H of the poster.

$$A = HW \quad B = (H - 8)(W - 8) \quad C = \text{Other}$$

- Express the area of the printed part in terms of the width W of the poster.

$$A = \text{got it!} \quad B = \text{working on it} \quad C = \text{help}$$

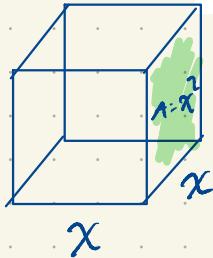
Hint: Express H in terms of W .

3.2.41 Express the **total surface area** of a cube in terms of its **volume** V .

Draw a picture! Name the unknowns!

3.2.41 Express the total surface area of a cube in terms of its volume

Draw a picture! Name the unknowns!



x

$$SA = 6x^2$$

$$V = x \cdot x \cdot x = x^3$$

use this to sub. for x

$$V = x^3$$

$$\sqrt[3]{V} = x$$

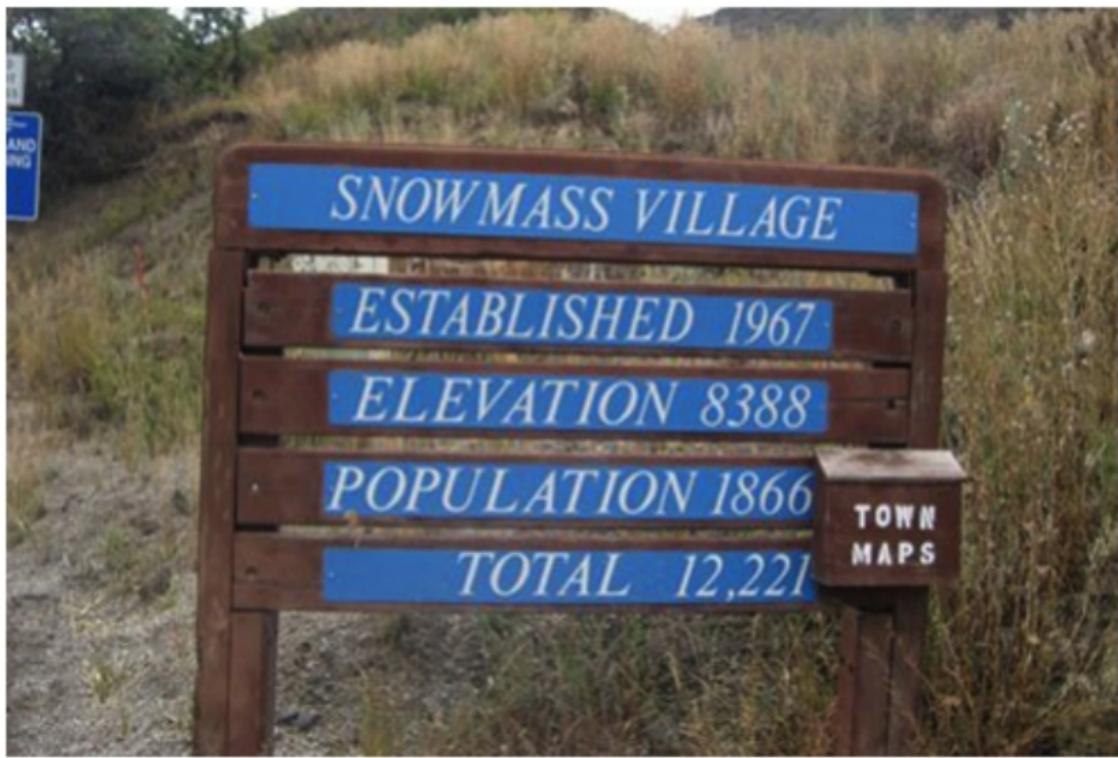
Sub

$$SA = 6(\sqrt[3]{V})^2$$

$$= 6(V^{2/3})$$

$$SA = 6V^{2/3}$$

Units



Units

Rule: Only add or subtract things measured in same units

- 3 meters + 7 inches is NOT 10 of anything
- 2 days + 5 hours = 7
- 3 nickels + 2 dimes = 5

BUT! You can multiply or divide things in different units:

$$\text{average speed} = (\text{distance gone}) / (\text{time taken})$$

$$(50 \text{ miles}) / (1 \text{ hour}) = 50 (\text{ miles / hours}) = 50 \text{ miles per hour} = 50 \text{ m p h}$$

You must multiply or divide the units too !

miles divided by hours is miles per hour

When a problem has **mixed units** like **miles and feet** or **years and seconds** decide what units you will use (like **miles** and **seconds**) and convert everything into those units, or

SUFFER

Units conversions

6. How fast does your hair grow... in mph?

$$A = 10^{-3} \quad B = 10^{-4} \quad C = 10^{-5} \quad D = 10^{-6} \quad E = 10^{-8}$$

???

I don't know either.

7. How fast does your hair grow... in cm / month?

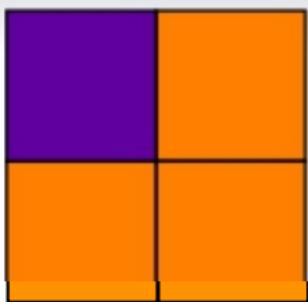
$$A = \text{faster} \quad B = 10 \quad C = 1 \quad D = 1 / 10 \quad E = \text{slower}$$

C

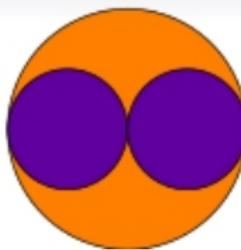
Conversions:

$$2.54 \text{ cm} = 1 \text{ inch} \quad 12 \text{ inches} = 1 \text{ foot} \quad 5280 \text{ feet} = 1 \text{ mile}$$

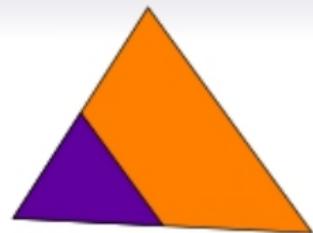
$$30 \text{ days} = 1 \text{ month} \quad 24 \text{ hours} = 1 \text{ day}$$



The **large square** is 2 times the base of the **small square**. It has $2 \times 2 = 4$ times the area.



The **large circle** is 2 times the size of the **small circle**. It has 4 times the area.



The **large triangle** is 2 times the size of the **small triangle**. It has 4 times the area.

When you **double** the size of a shape the **area** is multiplied by **4**
If you make a shape **3** times larger the area is **9** times as much
x times larger gives **x 2** times as much area

area grows as **the square** of the linear dimensions

When you **double** the size of a solid object,
the volume is **8** times as much

What is going on?

An area has **two dimensions** : **length** and **width**.
Both of these get doubled so area is doubled **twice** so multiplied by 2^2

A solid object has **three dimensions** : **length, width** and **height**.
Each dimension is doubled so volume doubled **three times** : multiplied by 2^3

Make a solid object **x** times bigger, volume is **x³** times as much.

volume grows as **the cube** of the linear dimensions

Conclusion Volume and area grow at **different rates**

As you make an object bigger the volume gets bigger faster (**cubing**)
than the area (only **squaring**). Opposite effect when you make it
smaller: volume gets smaller faster than area.

Consequences!

Many important consequences read section 4.4

Why do babies get cold faster than adults?

Why can an ant pick up something weighing 10 times its own weight?

Why are humans 60 feet tall **mathematically impossible** ?

Why can't you build a jumbo jet twice as big?

Why are my lungs crinkly?

A planet made of rock behaves like a liquid

Why can a fly walk on the ceiling, but I cant?

Why is water so dangerous to an insect but not gravity?

Paraphrasing **J.B.S.Haldane** Falling down a thousand yard mine shaft

A mouse walks away

A rat is killed

A man is broken

A horse splashes

8. An oil leak!

- Oil is leaking from an oil tanker at the rate of 4000 liters per hour.
 - 8 liters of oil spread out over 10 square meters of ocean surface.
 - A **SQUARE** oil slick forms.
-
- Express the length, X , of one side of the square oil slick as a function of the time t (in hours) the tank has been leaking.
 - After **how many hours** will the oil slick be a square with side length 2 kilometers?

PLAN:

- (i) How many liters of oil on ocean after t hours?
- (ii) How much area does this oil cover?

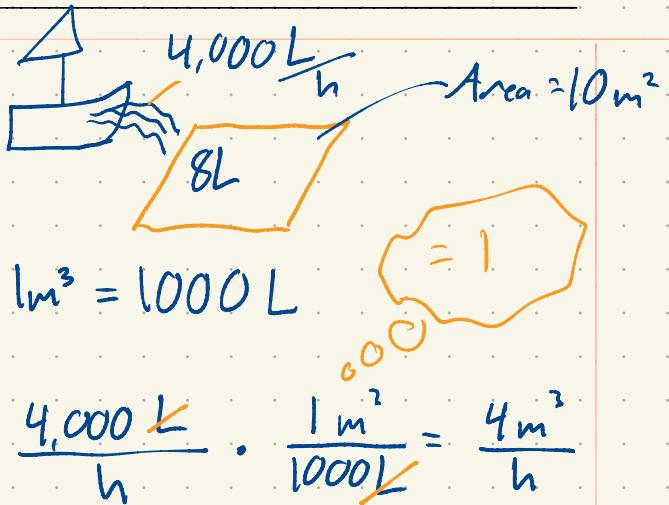
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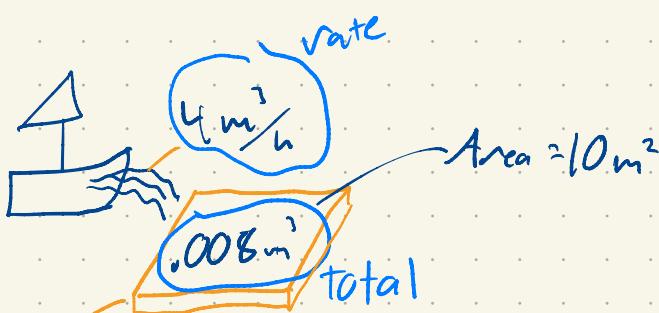
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- How much area does this oil cover?



$$8L \cdot \frac{1 \text{ m}^3}{1000 \text{ L}} = .008 \text{ m}^3$$



$$\text{Total (m}^3\text{)} \div \text{rate (m}^3/\text{h}) = \text{time (h)}$$

$$.008 \text{ m}^3 \cdot \frac{1 \text{ h}}{4 \text{ m}^3} = \frac{.008}{4} \text{ h} = .002 \text{ h} = \text{time}$$

$$\text{height: } .008 \text{ m}^3 \cdot \frac{1}{10 \text{ m}^2} = .0008 \text{ m} = \text{thickness}$$

$$\text{Area} = \frac{4(t) \text{ m}^3}{.0008 \text{ m}}$$

$$\text{Side} = \sqrt{A_{\text{m}^2}} = \sqrt{\frac{4t}{.0008}} \text{ m}$$

$$= \sqrt{\frac{t}{.0002}} \text{ m}$$

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PLAN:

- (i) How many liters of oil on ocean after t hours?
- (ii) How much area does this oil cover?

Answer: $t = 800$ hours

Office Hours:

Cooper 1.3.10

$$\left(\frac{x}{100} \cdot 9 + \frac{y}{100} \cdot 3 \right)$$

Express $x\%$ of 9 plus $y\%$ of 3 as a percentage of 11.

.109

$$\frac{9x}{100} + \frac{3y}{100}$$

$$= \frac{9x+3y}{100}$$

Online Math Lab resources for this problem:

- Order of Operations
- Polynomials
- Percents
- Fractions
- Exponents

5 as a % of 11?

↳ what fraction

$$\frac{5}{11}$$

has a percent

mult. by 100 %

$$\frac{5}{11} \cdot 100 \% = \frac{500}{11} \%$$

$\frac{9x+3y}{100}$ as a % of 11?

↳ what fraction

$$\frac{9x+3y}{100} \cdot \frac{1}{11} = \frac{9x+3y}{1100}$$

↳ as a percent

$$\left(\frac{9x+3y}{1100} \right) \cdot 100 \%$$

$$\frac{(9x+3y)(100)}{1100} \%$$

$$= \boxed{\frac{9x+3y}{11} \%}$$

HW01: Problem 11

(1 point)

Cooper 1.5.34

Make the following substitutions for x in the expression

$$\sqrt{x} + \frac{1}{x} + 3x^2$$

- (a) $x = 2$ (b) $x = a$ (c) $x = c^2$ (d) $x = a + b$ (e) $x = y + y^{-1}$

(a) $\sqrt{x} + \frac{1}{x} + 3x^2$

$$\sqrt{2} + \frac{1}{2} + 3(2)^2$$

$$\sqrt{2} + \frac{1}{2} + 12$$

$$\sqrt{2} + 12.5$$

$\boxed{\sqrt{2} + 12.5}$

(b) $\sqrt{x} + \frac{1}{x} + 3x^2$

$$\boxed{\sqrt{a} + \frac{1}{a} + 3a^2}$$

$$y \cdot y^{-1} = y^{1-1} = y^0 = 1$$

$$yy^{-1} = \frac{y}{1-y} = \frac{y}{y} = 1$$

(c) $\sqrt{x} + \frac{1}{x} + 3x^2$

$$\boxed{\sqrt{c^2} + \frac{1}{c^2} + 3(c^2)^2}$$

| | | |
|----------|----------|----------|
| y | y^{-1} | |
| y | y^2 | 1 |
| y^{-1} | 1 | y^{-2} |

$$= y^2 + 2 + y^{-2}$$

(d) $\sqrt{x} + \frac{1}{x} + 3x^2$

$$\boxed{\sqrt{a+b} + \frac{1}{a+b} + 3(a+b)^2}$$

First
Outside
Inside
Last

$$(a+b)(a+b)$$

$$= 3(a^2 + 2ab + b^2)$$

$$\sqrt{a+b} + \frac{1}{a+b} + 3(a^2 + 2ab + b^2)$$

not simpler

$$c + \frac{1}{c^2} + 3c^4$$

$$c + 1/c^2 + 3c^4$$

$$c + c^{-2} + 3c^4$$

$$c^{-2} = \frac{1}{c^2}$$

(e) $\sqrt{x} + \frac{1}{x} + 3x^2$

$$\boxed{\sqrt{y+y^{-1}} + \frac{1}{y+y^{-1}} + 3(y+y^{-1})^2}$$

$$\frac{1}{ab^{-1}} = \frac{b}{a}$$

$$\frac{1}{a+b^{-1}} \neq \frac{b}{a}$$

$$(x^a)^b = x^{ab}$$

$$(x^{-1})^2 = x^{-2}$$

$$\boxed{\sqrt{y+y^{-1}} + \frac{1}{y+y^{-1}} + 3(y+y^{-1})^2}$$

Exercise

9. When you substitute $x = y + 3$ into $x^2 - 6x + 8$ you get...

A = $y^2 - 6y - 1$ B = $y^2 + 35$ C = $y^2 - 6y + 35$ D = $y^2 - 1$

Exercise

9. When you substitute $x = y + 3$ into $x^2 - 6x + 8$ you get...

A = $y^2 - 6y - 1$ B = $y^2 + 35$ C = $y^2 - 6y + 35$ D = $y^2 - 1$

Answer: D

10. Can you check your answer to the previous question?

Hint: What are the expressions when $y = 1$?

What is x when $y = 1$?

When $y = 1$, $x = 4$ so $x^2 - 6x + 8 = 4^2 - 6(4) + 8 = 0$.

The other expressions are...

Exercise

9. When you substitute $x = y + 3$ into $x^2 - 6x + 8$ you get...

A = $y^2 - 6y - 1$ B = $y^2 + 35$ C = $y^2 - 6y + 35$ D = $y^2 - 1$

Answer: D

10. Can you check your answer to the previous question?

Hint: What are the expressions when $y = 1$?

What is x when $y = 1$?

When $y = 1$, $x = 4$ so $x^2 - 6x + 8 = 4^2 - 6(4) + 8 = 0$.

The other expressions are...

$$A = y^2 - 6y - 1 = -6 \qquad B = y^2 + 35 = 36$$

$$C = y^2 - 6y + 35 = 30 \qquad D = y^2 - 1 = 0$$



That's it. Thanks for being here.

