

Math 34A - Final Exam

1. Short Answers

1.1. Problem 1. Solve for x :

$$(x + 1)(x + 2) = (x + 3)^2$$

Scratch Work

Answer: $x =$

1.2. Problem 2. Solve for x :

$$12x^2 + 23x - 6 = 2x^2 - 1$$

Scratch Work

Answer: $x =$

1.3. Problem 3. Solve for x :

$$-2^{x+1} - 2^{x+3} + 2^{2x+4} + 1 = 0$$

Scratch Work

Answer: $x =$

1.4. Problem 4. Solve for x :

$$3 \cdot 7^{2x+1} = 42$$

If there are any logs in your answer, leave them - do not round to a decimal.

Scratch Work

Answer: $x =$

1.5. Problem 5. Simplify the following expression as much as possible:

$$\frac{(x-2)^2 - (2x-1)^2 - 3}{x}$$

Scratch Work

Answer:

1.6. Problem 6. Compute the derivative of $f(x) = (2x - 1)^2 + 4x$.

Scratch Work

Answer: $f'(x) =$

1.7. Problem 7. Let $f(x) = (2x + 3)(7 - 2x)$. What is the largest value that f can take on?

Scratch Work

Answer: $f(x) =$

1.8. Problem 8. Solve the system of equations:

$$2x - y = 20$$

$$7x - 12y = 2$$

Scratch Work

Answer: $(x, y) =$

1.9. Problem 9. Let $g(x) = x^3 - x^2 - x + 1$. Find the equation of the tangent line to $y = g(x)$ at $x = 1$. Write your answer in $y = mx + b$ form.

Scratch Work

Answer: $y =$

2. Top Chef: Calzones

2.1. Units. It's impossible to find a good calzone in IV, so we're going to make one. To start, we need to mix some flour and water to make dough. Our recipe calls for 6 tablespoons of water for each cup of flour.

- “One cup” is a unit of volume. It is equal to 236588 (millimeters cubed.)
- “One tablespoon” is also a unit of volume. There are 16 tablespoons in a cup.

These are fun facts, but you will not need them to answer the questions. You will, however, need the following:

- One cup of flour weighs 120 grams.
 - One cup of water weighs 280 grams.
- (1) How many **grams of flour** do we need if we want to use exactly one cup of water? Round your answer to the nearest decimal place (e.g. if the answer is 23424.2342341354565 grams, just write 23424.2 grams)
 - (2) We have exactly 420 grams of flour on hand. How many **cups** of water will we need? Round your answer to the nearest decimal place (e.g. if the answer is 23424.2342341354565 cups, just write 23424.2. cups)
 - (3) How much does the dough weigh if we use 420 grams of flour? Give your answer in grams. Round your answer to the nearest decimal place (e.g. if the answer is 23424.2342341354565 grams, just write 23424.2 grams)

2.1.1. *Scratch Work.*

2.1.2. *Answers.*

- (1)
- (2)
- (3)

2.2. The dough rises. we've mixed our flour and water, and also added some yeast to make our calzone dough. Yeast is alive! It makes the dough rise if you just let it sit. To start, we have 5 cups of dough.

- It takes 25 minutes for the dough to double in volume.
 - We need to let the dough rise until it has tripled in volume.
- (1) Write down a formula for the volume of the dough t minutes after we started.
 - (2) What is the volume of the dough 15 minutes after we start? Write your answer in cups. Round your answer to the nearest decimal - e.g. if you compute 1.23456, write down 1.2 cups.
 - (3) How many minutes should the dough sit to reach the target volume?

2.2.1. *Scratch Work.*

2.2.2. *Answers.*

(1)

(2)

(3)

2.3. Selling the calzones. We're now going to sell our calzones. To make a single calzone, we need to spend \$ 3 on ingredients.

If the price of a calzone is set at \$ 12 dollars, we will sell 150 calzones. If we increase the price by a dollar, we will sell 6 fewer calzones.

- (1) If the price is set at d dollars, how many calzones will we sell? (Your answer should be a function involving d).
- (2) What is the total revenue if we sell at d dollars per calzone?
- (3) What is the profit if we sell at d dollars per calzone?
- (4) How many much should we charge to maximize profit?

2.3.1. *Scratch Work.*



2.3.2. *Answers.*

- (1)
- (2)
- (3)

3. Geometry

3.1. Linear meets quadratic. Let $q(x)$ be the quadratic polynomial:

$$q(x) = (x - 2)^2 - (2x + 1)^2$$

- The graph of $q(x)$ looks like an upside-down U-shape.
- The graph of $q(x)$ contains the points $(1, -8)$ and $(-3, 0)$.

(See Figure 1.)

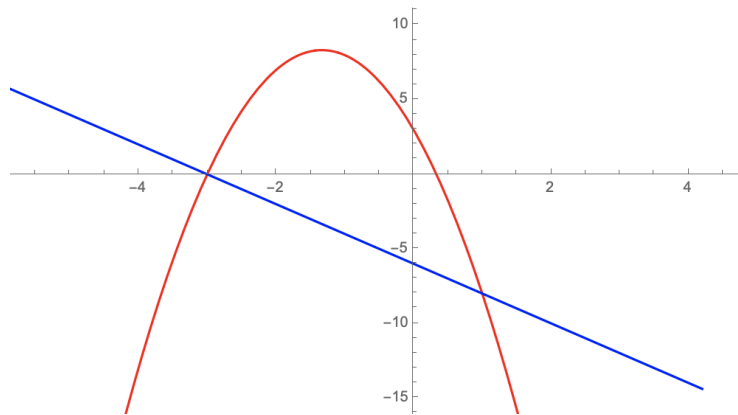


FIGURE 1. Problem 3.1

- (1) For which value of x is $q(x)$ as big as possible?
- (2) Find the equation of the line which is tangent to $q(x)$ at the point that makes $q(x)$ as big as possible.
- (3) Find the equation of the line that passes through $(1, -8)$ and $(-3, 0)$.
- (4) Compute the intersection point of those two lines.

3.1.1. *Scratch Work.*

3.1.2. *Answers.*

- (1)
- (2)
- (3)
- (4)

3.2. Cubic. Let f be the cubic:

$$f(x) = 2x^3 - x^2 - 8x + 4$$

(See Figure 2.)

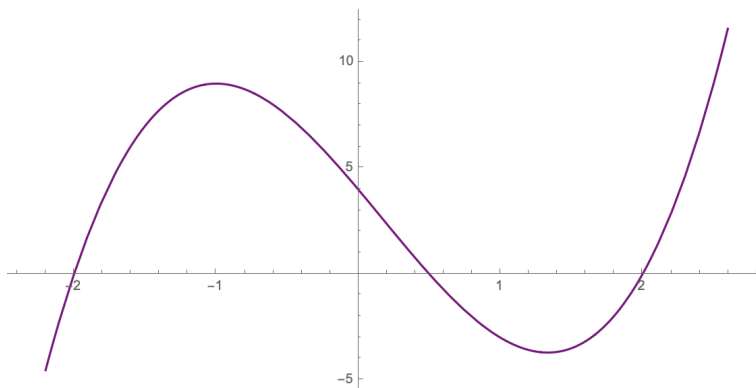


FIGURE 2. Problem 3.2

- (1) Compute the derivative of f .
- (2) Find the solutions to $f'(x) = 0$.
- (3) Find the coordinates of the local max.

3.2.1. *Scratch Work.*



3.2.2. *Answers.*

- (1)
- (2)
- (3)

3.3. Disks and Squares. We're going to grill in the middle of a square park of side length 120 m. The aroma of the food forms a disk around us, and the radius of that disk is increasing at a rate of 4 meters per minute.

- (1) After t minutes, what is the area of the disk where you can smell what we're cooking?
- (2) How quickly is the area of the disk growing after 12 minutes?
- (3) Extra Credit: How long will it take until everybody in the park can smell the food?

3.3.1. *Scratch Work.*

3.3.2. *Answers.*

- (1)
- (2)
- (3)