First day agenda

- Introductions
- Syllabus review
- High-level overview of course concepts
- Important examples (and why we care about UQ)

New instructor

Pros	Cons
	- I'm nervous.... - I might not know everything....

New instructor

Pros	Cons
- I will put SO much effort into this class - Seriously, I will do everything to make this course a **useful** and **positive** experience for you - You have more agency. Hate the HW? Bored in lecture? Your feedback can directly influence the class	- I'm nervous.... - I might not know everything....

Course websites

- Github: https://github.com/lalyman/cme-270/
  - Syllabus*
  - HW assignments*
  - Lecture notes (posted after class)*
  - Jupyter notebooks

  *These items will also be posted on Canvas, but they will *first* be updated on Github

- Canvas: https://canvas.stanford.edu/courses/144967
  - Class announcements
  - HW submission (optional; can submit in class or by email)
  - Discussions
  - Returned work

Syllabus

- Review the content here: https://github.com/lalyman/cme-270/

Coding constraints

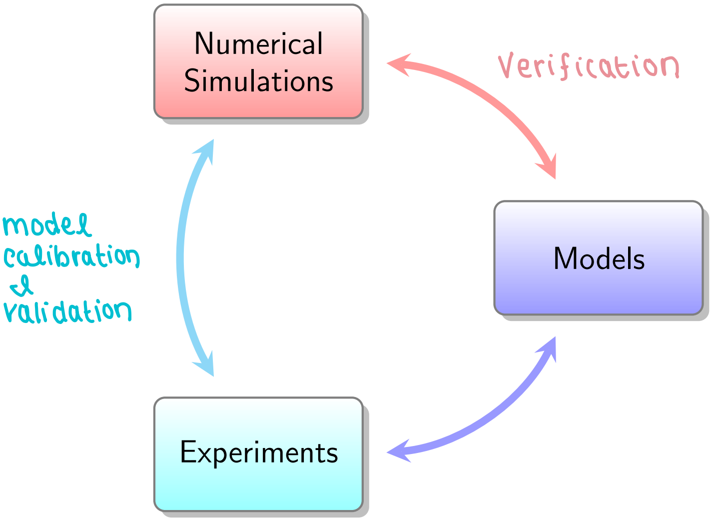
Programming

- The "official" format I will use is Python via Jupyter notebooks
- No hardcore reason for this choice; it looks pretty and it's nice to execute cells individually; (also, open source is a bonus)

What language can I use for HW assignments and/or the final project?

- Go wild! I can figure it out
- Just know if you need support with debugging, I might not be *as* help if I'm less familiar with the language

Predictive science



Hurricane Ida

- Image source:
  https://www.bbc.com/news/world-us-canada-58378788

Hurricane prediction

- Broadly speaking, we *know* the physics
- Modeled by the Navier–Stokes equations + some thermodynamic assumptions; the solution to these equations gives you whatever you're interested in as a function of space and time

velocity

pressure

temperature

$$
\begin{aligned}
\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho v) &= 0 \\
\frac{\partial v}{\partial t} &= -v \cdot \nabla v - \frac{1}{\rho}\nabla p - g\hat{k} - 2\Omega \times v \\
\rho c_V \frac{\partial T}{\partial t} + p\nabla \cdot v &= -\nabla \cdot F + \nabla \cdot (k \nabla T) + \rho q(T, p, \rho) \\
\frac{\partial m_j}{\partial t} &= \cdots \\
&\vdots
\end{aligned}
$$

density

- So what's the problem?

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\end{aligned}
$$

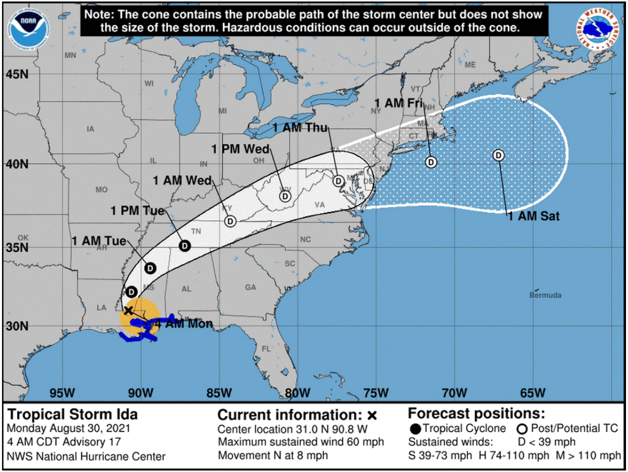
- So what's the problem? There is **uncertainty in the initial conditions**

But why can't you figure out the initial conditions....

1. ...with weather stations?
   - Can't place weather stations everywhere
   - Biased to North America
2. ...with satellite imagery?
   - Best resolution is 4 x 4 km ($\sim 2.49 \times 2.49$ miles)

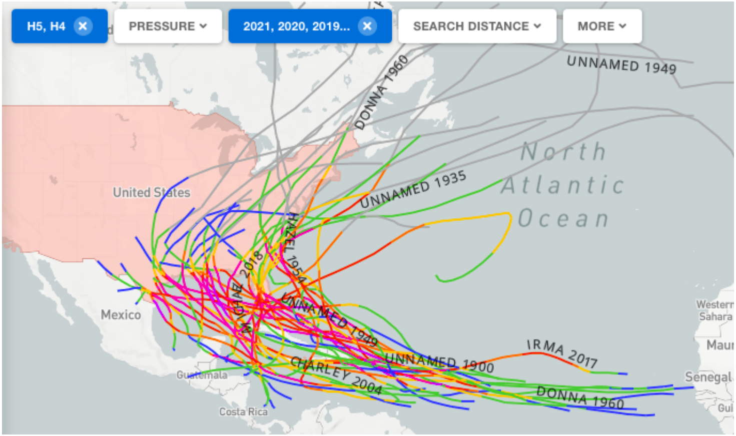
- The best we can do is make informed *guesses* about initial conditions
- Solution: sample different possible initial conditions (guesses); get different possible hurricane paths based on these samples
- The envelope of predicted paths is the "cone of uncertainty"; gives you the probable trajectories of the storm center, based on the samples you were able to take
- In general, this is an example of an **uncertainty propagation problem**

Cone of Uncertainty



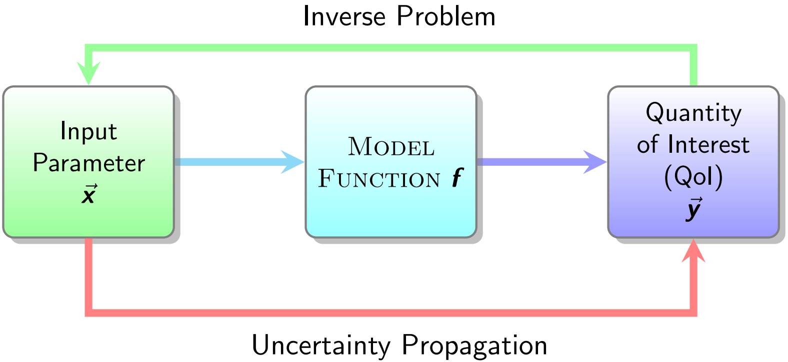
- Tropical storm Ida (August 30, 2021)
- Image source: here

Hurricane paths



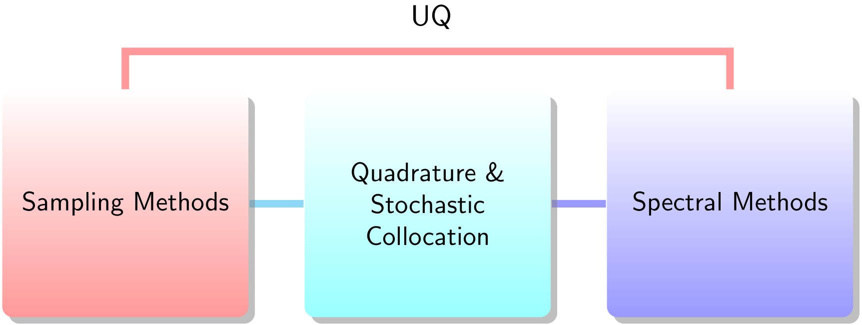
- Category 4 & 5 hurricane tracking data from 1900 - 2021 (68 total)
- https://oceanservice.noaa.gov/news/historical-hurricanes/

Uncertainty Propagation

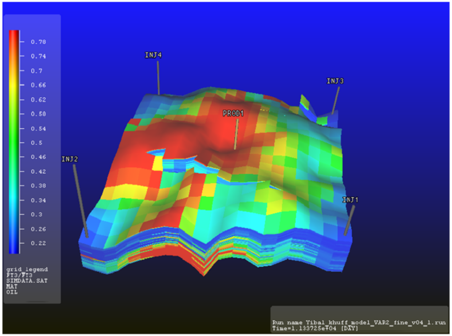


- Put a distribution on the (random variable) input parameter $\boldsymbol{x}$
- $\vec{\boldsymbol{y}} \approx \boldsymbol{f}(\vec{\boldsymbol{x}})$
- The uncertainty *propagates* through your model $\boldsymbol{f}$
- Determine the resulting distribution on $\vec{\boldsymbol{y}}$ or some statistics of interest (mean, variance, etc.) for $\vec{\boldsymbol{y}}$

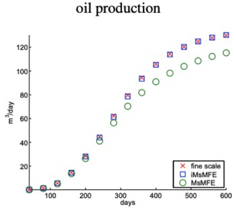
Three pillars of UQ



Subsurface geology & hydrology



simulation



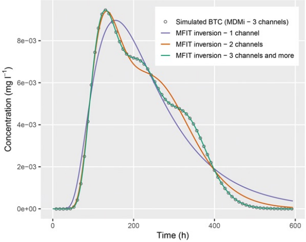


Image sources:

- Multiscale Method for Simulating Two-and Three-Phase Flow in Porous Media (M. Pal et al, 2013)
- Multi-Flow Inversion of Tracer Breakthrough Curves in Fractured and Karst Aquifers (J. Bodin, 2020)

Verification versus validation

The oft-cited AIAA (American Institute of Aeronautics and Astronautics) Guide defines these terms as follows.

- **Verification**: The process of determining that a model implementation accurately represents the developer's conceptual description of the model and the solution to the model.
- **Validation**: The process of determining the degree to which a model is an accurate representation of the *real world* from the perspective of the intended uses of the model.

More colloquial/heuristic approach

- **Verification** tries to answer....

  Are we solving the equations correctly?

- **Validation** tries to answer....

  Are we solving the correct equations?

Epistemic vs aleatoric uncertainty

Aleatoric uncertainty

- Also called **irreducible**, **statistical**, or **stochastic** uncertainty
- Randomness inherent to a to a problem or system that cannot be reduced simply by taking more measurements
- In principle, this uncertainty *cannot* be reduced by additional physical or experimental knowledge

Epistemic uncertainty

- Also called **systemic** or **reducible** uncertainty
- Stems from simplifying model assumptions, missing physics, or a basic lack of knowledge
- We can reduce this uncertainty (for a price)

This categories are *heuristic*. The definitions are not rigorous, there is overlap, & the distinctions can become philosophical....

Examples (aleatory or epistemic?)

1. Uncertainty in a physical law

$$\underbrace{\rho(\overset{\text{state}}{q})}_{\substack{\text{probability of occupying}\\ \text{a given state } q\\ \text{(positions of all atoms)}}} \propto \exp\left(-\frac{\overbrace{V(q)}^{\text{empirical potential}}}{k_E T}\right)$$

Boltzmann constant

temperature

Examples (aleatory or epistemic?)

1. Uncertainty in a physical law

$$\rho(q) \propto \exp\left(-\frac{V(q)}{k_E T}\right)$$

   - Categorize as **epistemic**
   - There is a fixed potential $V(q)$; we just don't know what it is....

2. Subsurface examples
   - The composition of the porous medium (soil) might be unknown, but at a given time and location, it is overwhelmingly fixed
   - This indicates an **epistemic** uncertainty
   - We could know the soil composition if we could just dig it up and examine it everywhere; our understanding of it *would* improve with more and/or better measurements

Is any uncertainty truly aleatory?

- Somewhat philosophical question...
- Is nature random?

- **Ex. of aleatory uncertainty**. Background thermal radiation
  - Penzias & Wilson (Bell Labs researchers): no matter where they pointed a special antenna, it always picked up some microwave radio frequencies; they realized it was thermal radiation left over from the Big Bang
  - TV fuzz "looks" pretty random
  - *Arguably* epistemic; *some* might say if we knew the precise initial conditions of the Big Bang, we would be able to predict this

TV fuzz