DATA 558: Homework 1

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Exercise 1

In this section, I will compute the derivatives of the following functions with respect to the parameter $\beta \in \mathbb{R}$.

1. $f(\beta) = \frac{1}{2n} \sum_{i=1}^{n} (y_1 - x_i\beta)^2$

$$\begin{aligned} \frac{d}{d\beta} f(\beta) &= -\frac{2}{2n} \sum_{i=1}^{n} x_i(y_i - x_i\beta) \\ &= -\frac{1}{2n} \sum_{i=1}^{n} x_i y_i - x_i^2 \beta \end{aligned}$$

2. $f(\beta) = \frac{e^{x\beta}}{e^{x\beta}+1}$

$$\begin{aligned} \frac{d}{d\beta} f(\beta) &= \frac{d}{d\beta} \left(e^{x\beta}(e^{x\beta}+1)^{-1} \right) \\ &= \frac{xe^{x\beta}}{e^{x\beta}+1} - \frac{e^{x\beta} x e^{x\beta}}{(e^{x\beta}+1)^2} \\ &= \frac{xe^{x\beta}}{e^{x\beta}+1} - \frac{xe^{2x\beta}}{(e^{x\beta}+1)^2} \end{aligned}$$

3. $f(\beta) = \log(\sum_{i=1}^{n} e^{x_i\beta - 1})$

$$\begin{aligned} \frac{d}{d\beta} f(\beta) &= \frac{1}{\sum_{i=1}^{n} e^{x_i\beta - 1}} \cdot \sum_{i=1}^{n} x_i e^{x_i\beta - 1} \\ &= \sum_{i=1}^{n} x_i \end{aligned}$$

4. $f(\beta) = |y - x\beta|$

$$f(\beta) = \begin{cases} y - x\beta & \beta \leq \frac{y}{x} \\ -(y - x\beta) & \beta > \frac{y}{x} \end{cases}$$

$$\frac{d}{d\beta} f(\beta) = \begin{cases} -x & \beta \leq \frac{y}{x} \\ x & \beta > \frac{y}{x} \end{cases}$$