Infinity Tutors

Calculus II

Topic: Parametric Equations

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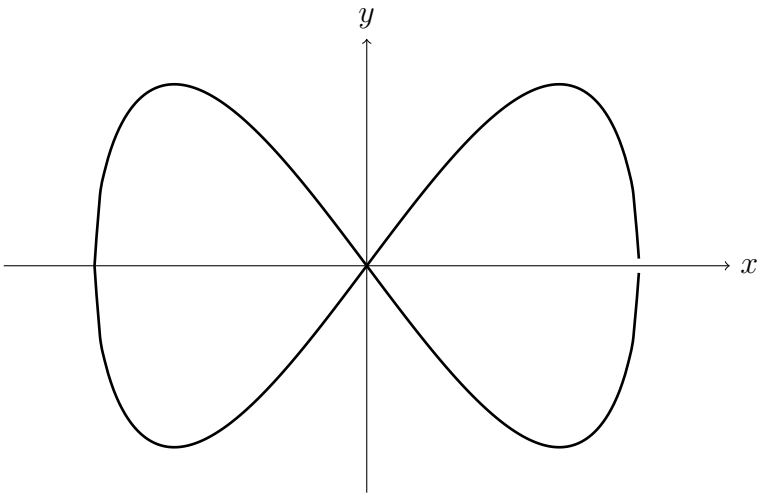
Parametric Equations Exercise

1. By revolving the curve $x = r\cos t$, $y = r\sin t$ for $0 \leq t \leq \pi$ about the x-axis, show that the surface area of a sphere of radius r is $4\pi r^2$.

2. The figure below shows the curve C, known as the lemniscate of Bernoulli, defined by the parametric equations:

$$x = 3\sin\theta, \quad y = 2\sin(2\theta) \quad (0 \leq \theta \leq \pi)$$

The curve is symmetrical about both the x- and y-axes.



   (a) Show that a Cartesian equation of C is:

$$18y^2 = 16x^2(9 - x^2)$$

   (b) Find the area of one loop of C.

3. Find the equation of the tangent to the curve at the point corresponding to the given value of the parameter $x = t^3 + 1, \quad y = t^4 + 1 : t = -1$

4. Find the parametric equation for the path of a particle that moves counter clockwise halfway around the circle $(x-2)^2 + y^2 = 4$, from the top to the bottom.

5. Find the area bounded by loop of the curve with parametric equation $x = t^2, \quad y = t^3 - 3t$

6. Find the parametric equation of the following

   (a) $\dfrac{ax^2}{a^2} + \dfrac{y^2}{a^2} = 1$

   (b) $\dfrac{x^2}{a^2} - \dfrac{y^2}{a^2} = 1$

   (c) $x^2 + y^2 = 4$

7. For each of the following determine $\dfrac{dy}{dx}$ and $\dfrac{d^2y}{dx^2}$

   (a) $x = \sin t, \quad y = \cos t$

   (b) $x = 3t^2 + 1, \quad y = t^3 - 2t^2$

   (c) $x = 3t^2 + 4t, \quad y = \sin 2t$