Exercises on Hyperbolic Functions

Trevour Jim

December 4, 2024

Instructions

Answer all the questions. Show all necessary steps and calculations.

Exercise 1: Proving Hyperbolic Identities

1. Use the definitions of sinh x and cosh x in terms of exponential functions to prove that;

   (a) $\cosh x + \sinh x = e^x$

   (b) $\cosh 2x = 2\cosh^2 x - 1$

   (c) $\cosh 2x = 1 + 2\sinh^2 x$

   (d) $\cosh^2 x + \sinh^2 x = \cosh 2x$

   (e) $\sinh(x + y) = \sinh(x)\cosh(y) + \cosh(x)\sinh(y)$

   (f) $\cosh(x + y) = \cosh(x)\cosh(y) + \sinh(x)\sinh(y)$

Exercise 2: Osborn's Rule

1. Given the following trigonometric formulae, use Osborn's rule to write down the corresponding hyperbolic function formulae;

   (a) $\sin A - \sin B = 2\cos\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right)$

   (b) $\sin 3A = 3\sin A - 4\sin^3 A$

   (c) $\cos^2\theta + \sin^2\theta = 1$

   (d) $\sin 2A = 2\sin A\cos A$

   (e) $\cos 2A = 1 - 2\sin^2 A$

Exercise 2: Differentiation of Hyperbolic Functions

1. Differentiate the following:

   (a) $y = \cosh 3x$

(b) $y = \sinh^2 x$

(c) $y = \sinh(e^x)$

(d) $y = \tanh(\sinh x)$

(e) $y = x \sinh x$

(f) $y = \sinh^3 4x$

(g) $y = \sqrt{(\cosh 4x)}$

(h) $y = e^x \cosh x$

(i) $y = \sinh^{-1}(3x)$

(j) $y = \cosh^{-1}(2x + 4)$

(k) $y = x \sinh(x) + \cosh(x)$

(l) $y = \cosh^2(x) - \sinh^2(x)$

2. Prove that:

(a) $\frac{d}{dx}(\sinh(x)) = \cosh(x)$

(b) $\frac{d}{dx}(\cosh(x)) = \sinh(x)$

Exercise 3: Integration Involving Hyperbolic Functions

1. Evaluate:

(a) $\int \sinh(4x)\, dx$

(b) $\int \cosh(4x + 6)\, dx$

(c) $\int x \sinh(x)\, dx$

(d) $\int \cosh^2(x)\, dx$

(e) $\int e^{2x} \sin 4x\, dx$

(f) $\int x^2 \cosh x^3 + 4\, dx$

(g) $\int \cosh 2x \sinh 3x\, dx$

(h) $\int \sinh^4 x\, dx$

(i) $\int \cosh^3 7x\, dx$

Exercise 4: Differentiation of Inverse Hyperbolic Functions

1. Differentiate

(a) $\cosh^{-1}(2x + 4)$

(b) $\sinh^{-1}(\sqrt{x})$

(c) $\tanh^{-1}(3x + 1)$

(d) $x^2 \sinh^{-1} 2x$

(e) $\cosh^{-1}\left(\frac{1}{x}\right)$

(f) $\sinh^{-1}(\cosh 2x)$

Exercise 5: Solving Equations Involving Hyperbolic Functions

1. Solve the following equations, leaving your answers as natural logarithms:

   (a) $4\sinh(x) + 3\cosh(x) = 7$

   (b) $2\cosh^2(x) - 5\sinh(x) = 3$

   (c) $3\sinh(x) - \cosh(x) = 1$

2. Prove that $\cosh^2(x) + \sinh^2(x) = \cosh(2x)$.

Exercise 6: Use of Hyperbolic Functions In Integration

NOTE: The following integrals can be used as hints.

(a) $\displaystyle\int \frac{1}{\sqrt{(ax)^2 + b^2)}}\,dx = \sinh^{-1}\left(\frac{ax}{b}\right) + C$

(b) $\displaystyle\int \frac{1}{\sqrt{((ax)^2 - b^2)}}\,dx = \cosh^{-1}\left(\frac{ax}{b}\right) + C$

1. Evaluate the following integrals

   (a) $\displaystyle\int \frac{1}{\sqrt{(x^2+1)}}\,dx$

   (b) $\displaystyle\int \frac{1}{\sqrt{(x^2-4)}}\,dx$

   (c) $\displaystyle\int \frac{1}{\sqrt{(4x^2-9)}}\,dx$

   (d) $\displaystyle\int \frac{1}{\sqrt{(16x^2+25)}}\,dx$

   (e) $\displaystyle\int \frac{1}{\sqrt{(x^2+6x+8)}}\,dx$

   (f) $\displaystyle\int \frac{1}{\sqrt{(x^2+2x5)}}\,dx$

   (g) $\displaystyle\int \frac{1}{\sqrt{(4x^2+-8x-5)}}\,dx$

END OF QUESTIONS