Simple DC circuits

Main effects of electric current

There are three main effects of an electric current:

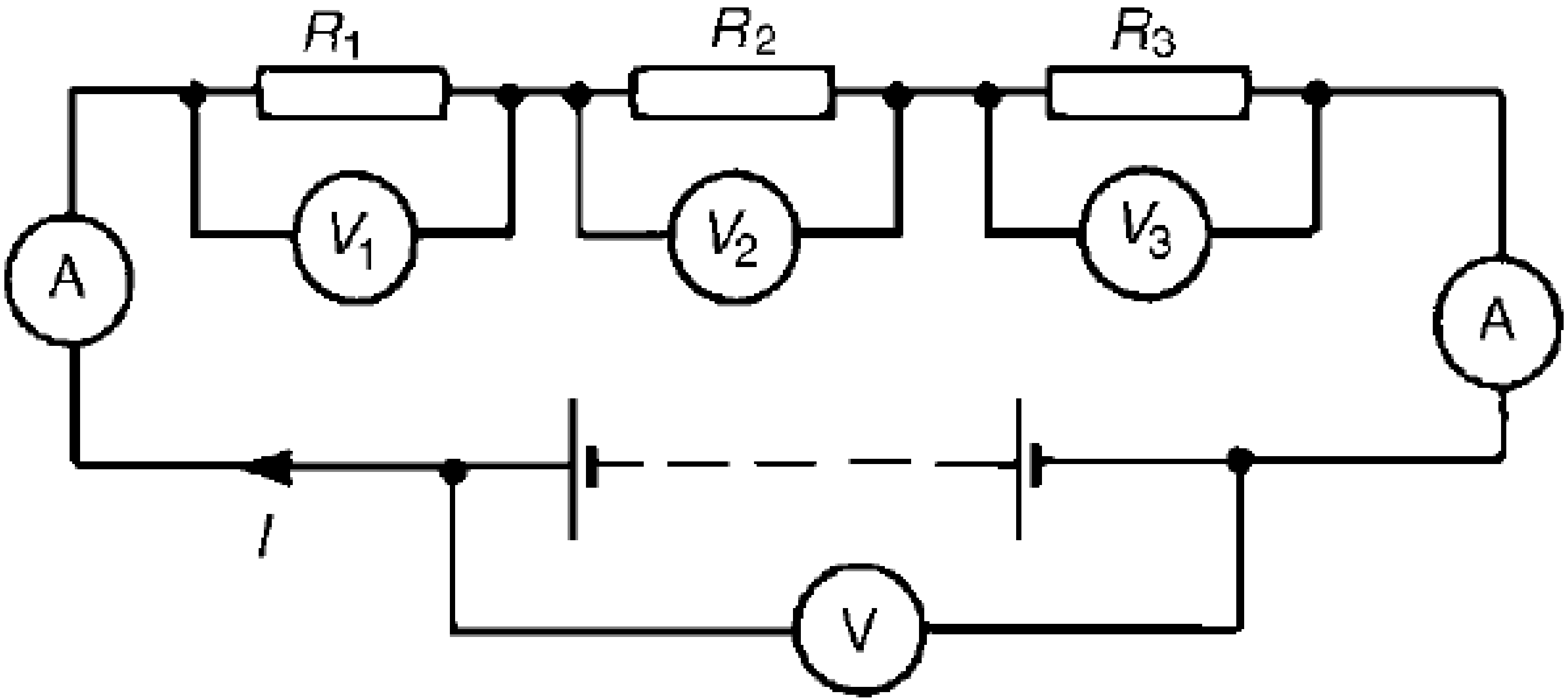
1. **Magnetic effect**: bells, relays, motors, generators, transformers, telephones, car-ignition and lifting magnets
2. C**hemical effect** :primary and secondary cells and electroplating
3. H**eating effect:** cookers, water heaters, electric fires, irons, furnaces

Simple DC circuits

1. Series circuit
2. Parallel circuit

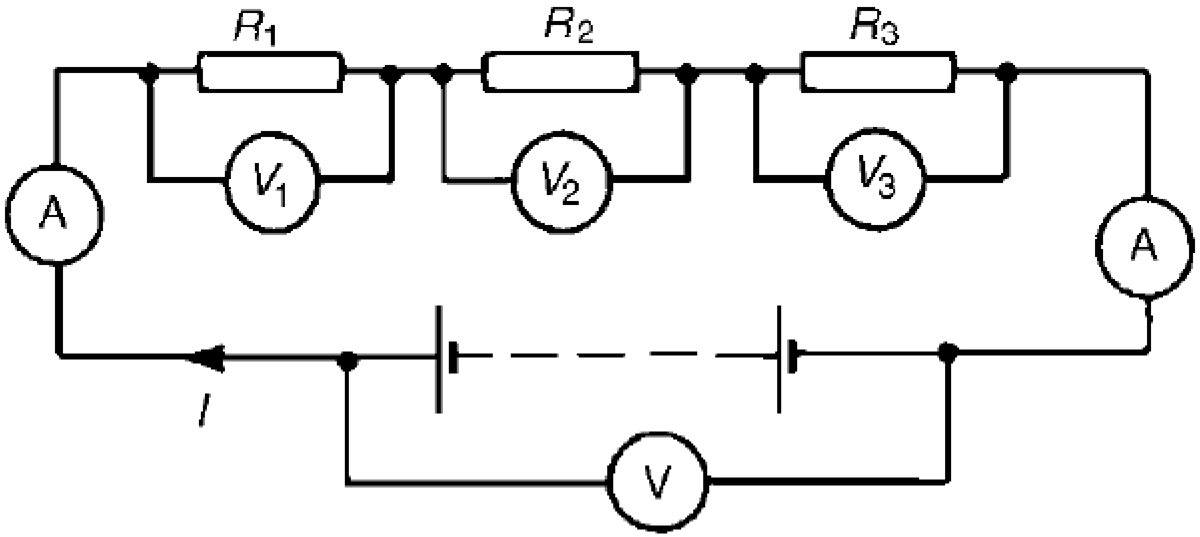
Series circuits

- Series circuits diagram



Series circuits

- Resistors R_1, R_2 and R_3 connected end to end.
- The current **I** is the same in all parts of the circuit
- the sum of the voltages V_1, V_2 and V_3 is equal to the total applied voltage, V, i.e. $\boldsymbol{V = V_1 + V_2 + V_3}$

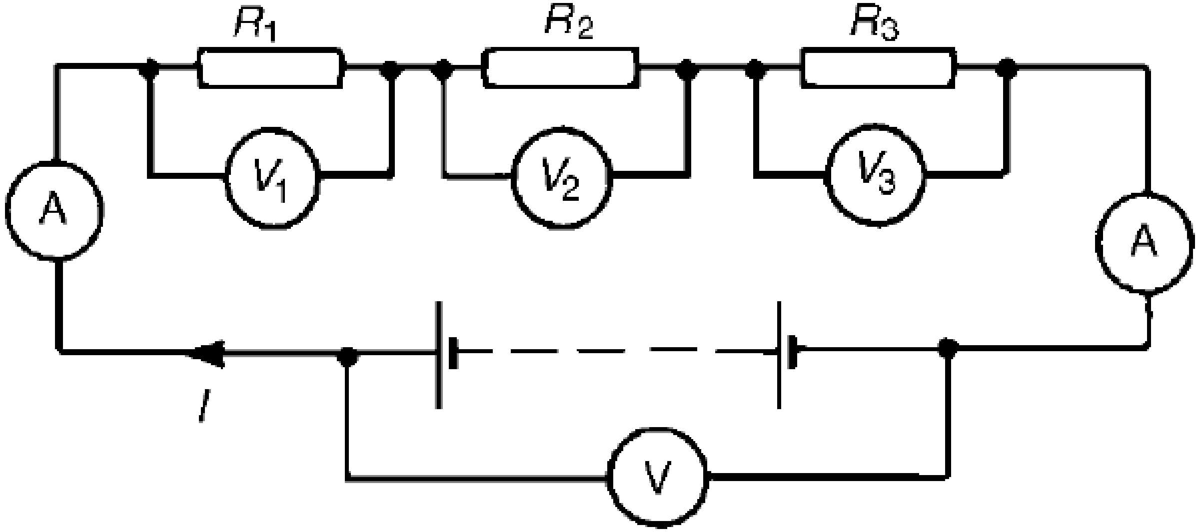


Series circuits

From Ohm's law:

- $V_1 = IR_1$, $V_2 = IR_2$, $V_3 = IR_3$ and $V = IR$
  where R is the total circuit resistance.
- Since $V = V_1 + V_2 + V_3$
- then $IR = IR_1 + IR_2 + IR_3$
- Dividing throughout by I gives:

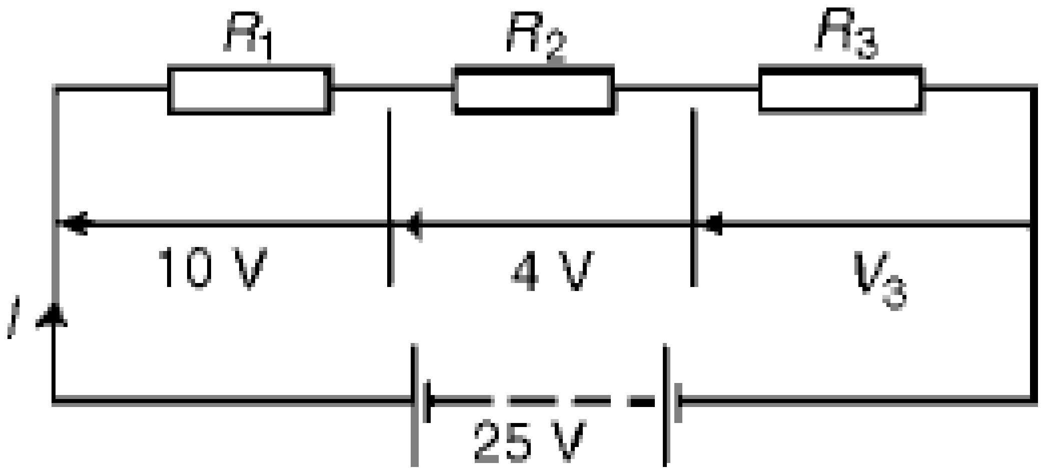
$$\boldsymbol{R = R_1 + R_2 + R_3}$$



Series circuits

Example:

For the circuit shown in Figure below, determine the p.d. across resistor R_3. If the total resistance of the circuit is 100, determine the current flowing through resistor R_1. Find also the value of resistor R_2.



Series circuits

Example:

P.d. across R_3, $V_3 = 25 - 10 - 4 = \mathbf{11\ V}$

Current $I = \frac{V}{R} = \frac{25}{100} = \mathbf{0.25\ A}$, which is the current flowing in each resistor

Resistance $R_2 = \frac{V_2}{I} = \frac{4}{0.25} = \mathbf{16\ \Omega}$

Series circuits

Four resistors are connected in series with $R_1=82\Omega$,$R_2=45\Omega$,$R_3=23\Omega$ and $R_4=50\Omega$.The supply voltage is 50 volts.(i)draw a well labelled circuit diagram(ii)determine

(a)The total circuit resistance

(b)The supply current

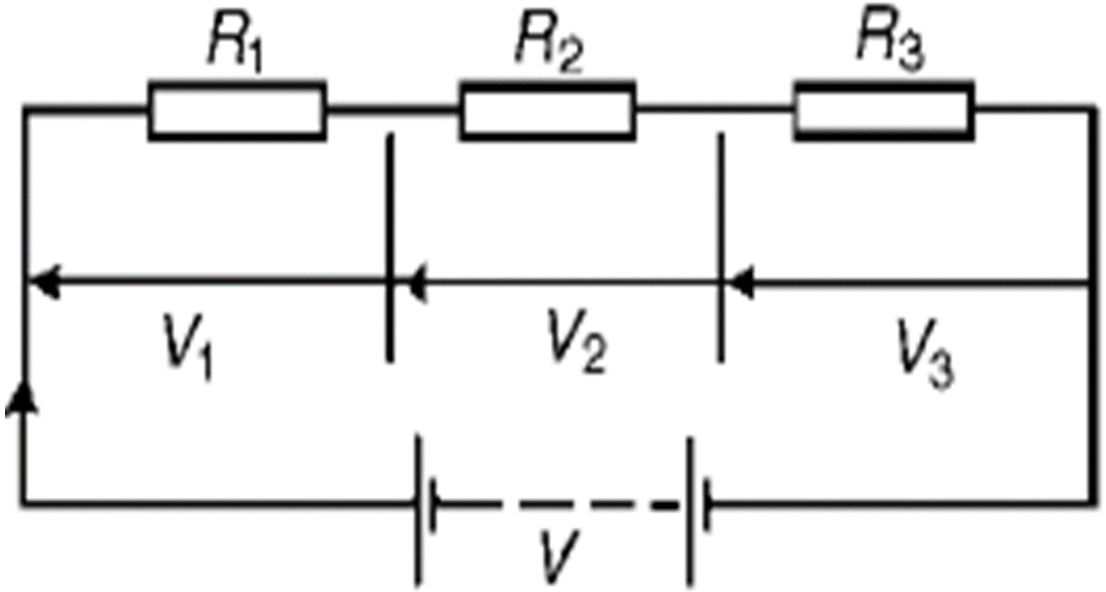
(c)Voltage across each resistor

(d) The power dissipated in R_2

(e)Energy consumed by the circuit

Series circuits

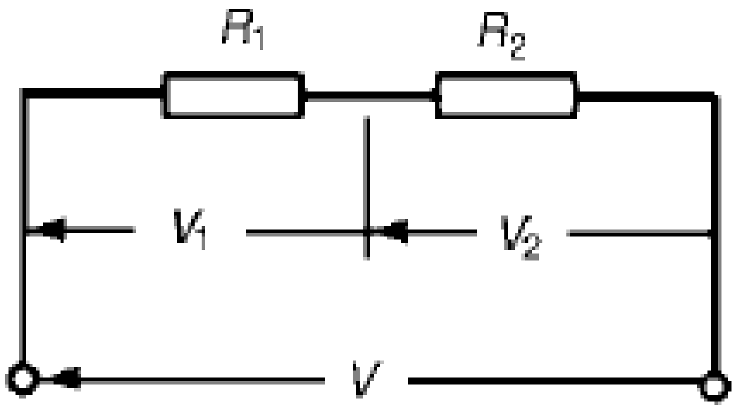
- For the circuits shown in the figure, given that V1=5 Volts ,V2=2 Volts and V3=6 Volts and supply current =4 amps. Determine (a)Battery voltage (b)Total circuit resistance(c)Values of resistance of resistors R1,R2 and R3.



Series circuits

Potential divider

- The ratio of the voltages depends on the ratio of the resistances.
- Potential (voltage) divider.



$$V_1 = \left(\frac{R_1}{R_1 + R_2}\right)\mathbf{V}$$

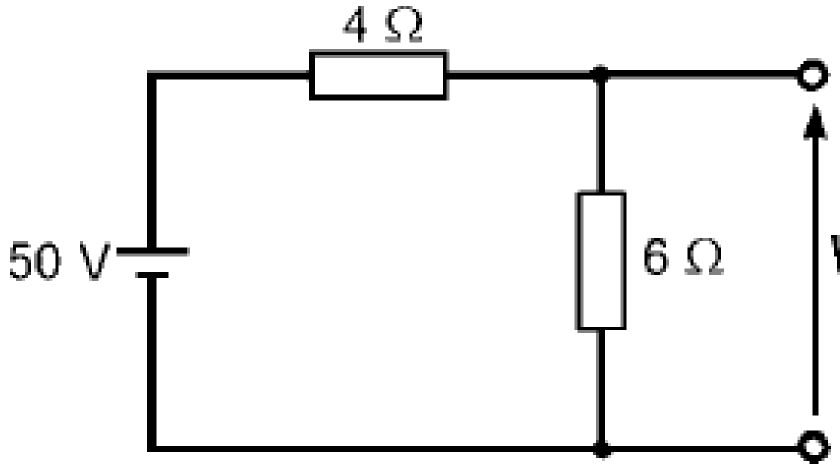
$$V_2 = \left(\frac{R_2}{R_1 + R_2}\right)\mathbf{V}$$

Series circuits

Potential divider

Example:

Determine the value of voltage *V* shown in Figure below

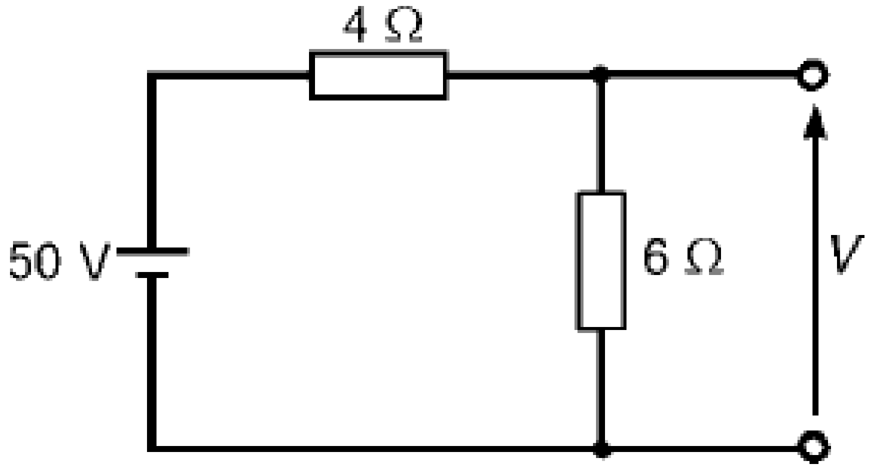


Series circuits

Potential divider

Example:

Determine the value of voltage *V* shown in Figure below



$$V = \left(\frac{6}{6+4}\right)(50) = \mathbf{30\ V}$$

Parallel circuits

- the sum of the currents I_1, I_2 and I_3 is equal to the total circuit current, I, i.e. $I = I_1 + I_{2\ Y}\ I_3$, and the source p.d., V volts, is the same across each of the resistors.
- From Ohm's law:

$$I_1 = \frac{V}{R_1},\ I_2 = \frac{V}{R_2},\ I_3 = \frac{V}{R_3} \text{ and } I = \frac{V}{R}$$

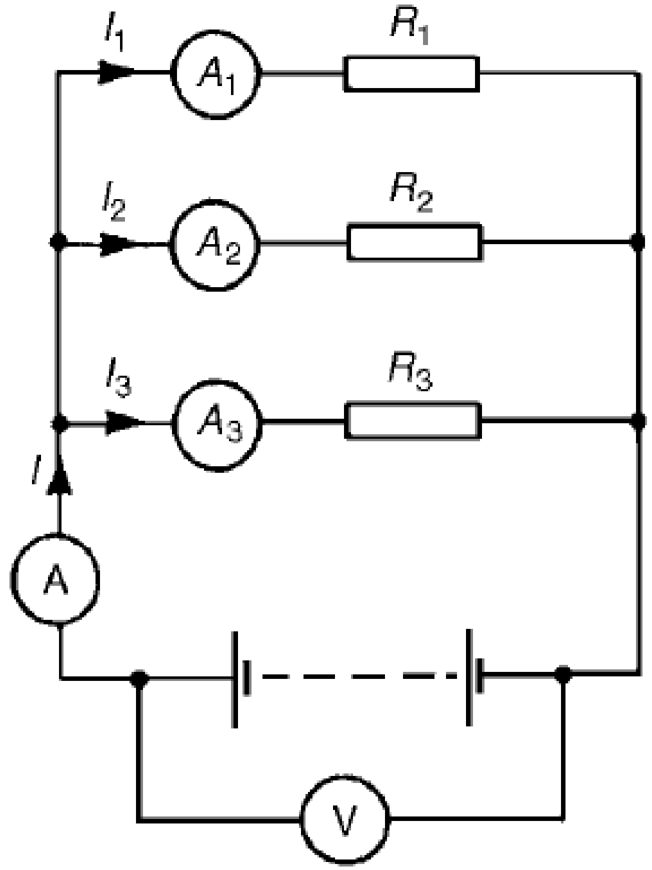
where R is the total circuit resistance.

- Since $I = I_1 + I_2 + I_3$

$$\text{then, } \frac{V}{R} = \frac{V}{R_1} + \frac{V}{R_2} + \frac{V}{R_3}$$

- Dividing throughout by V gives:

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$



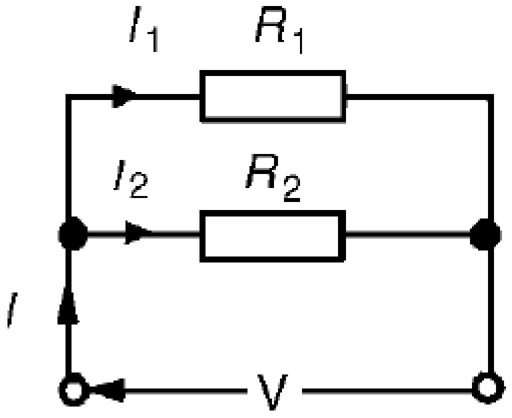
Two Resistors in parallel

For only two resistors in parallel:

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} = \frac{R_2 + R_1}{R_1 R_2}$$

Hence $\boxed{\boldsymbol{R = \frac{R_1 R_2}{R_1 + R_2}}}$ $\left(\text{i.e. } \frac{\text{product}}{\text{sum}}\right)$

Current division



- For the circuit the total circuit resistance, R_T is given by:

$$R_T = \frac{R_1 R_2}{R_1 + R_2}$$

and $V = IR_T = I\left(\frac{R_1 R_2}{R_1 + R_2}\right)$

$$\text{Current } I_1 = \frac{V}{R_1} = \frac{I}{R_1}\left(\frac{R_1 R_2}{R_1 + R_2}\right) = \left(\frac{R_2}{R_1 + R_2}\right)(I)$$

$$\text{current } I_2 = \frac{V}{R_2} = \frac{I}{R_2}\left(\frac{R_1 R_2}{R_1 + R_2}\right) = \left(\frac{R_1}{R_1 + R_2}\right)(I)$$

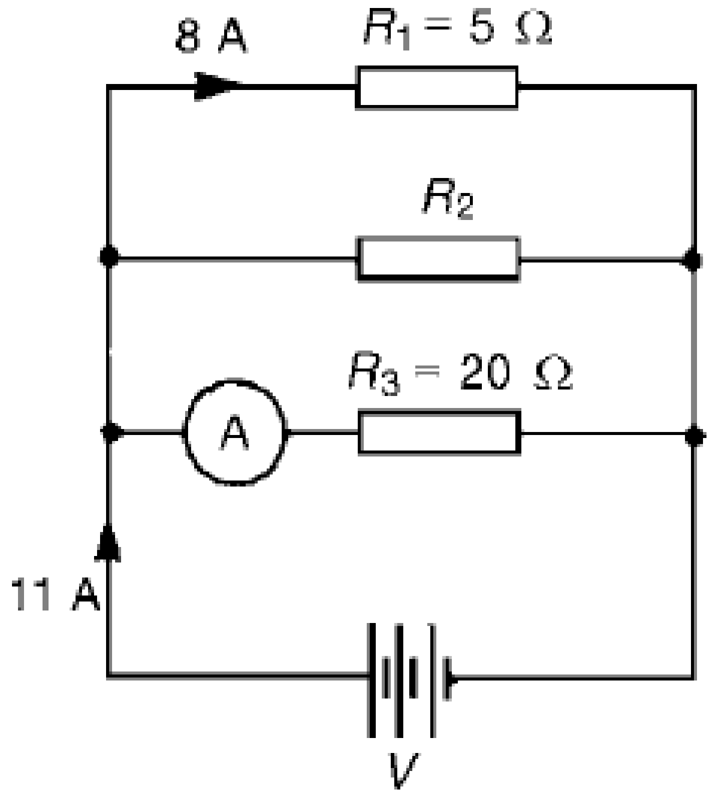
$$I_1 = \left(\frac{R_2}{R_1 + R_2}\right)(I)$$

$$I_2 = \left(\frac{R_1}{R_1 + R_2}\right)(I)$$

Parallel circuit

Example:

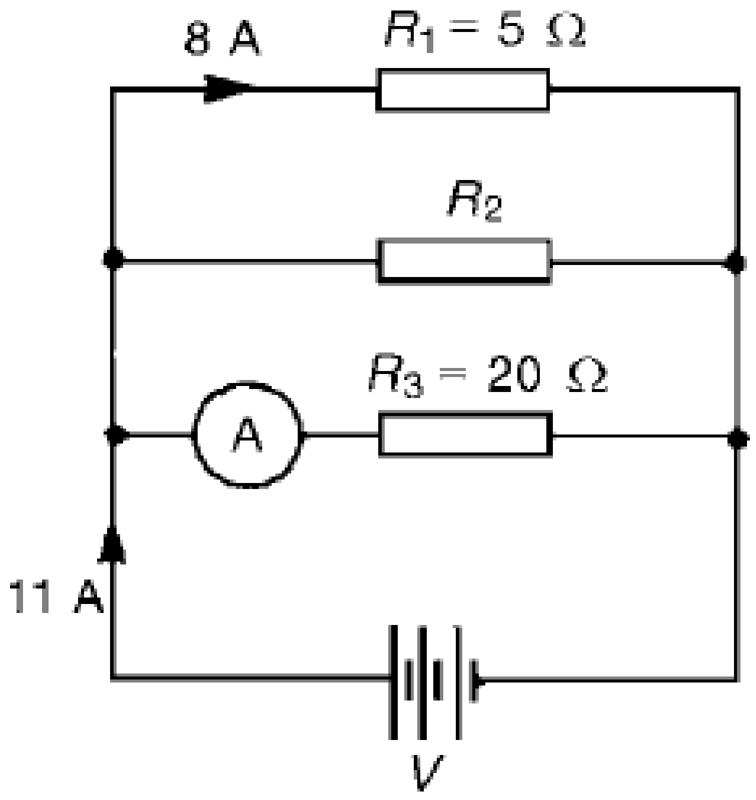
For the circuit shown in Figure below determine (a) the reading on the ammeter, and (b) the value of resistor R_2.



Parallel circuit

Example:

For the circuit shown in Figure below determine (a) the reading on the ammeter, and (b) the value of resistor R_2.



P.d. across R_1 is the same as the supply voltage V.

Hence supply voltage, $V = 8 \times 5 = 40\,\text{V}$

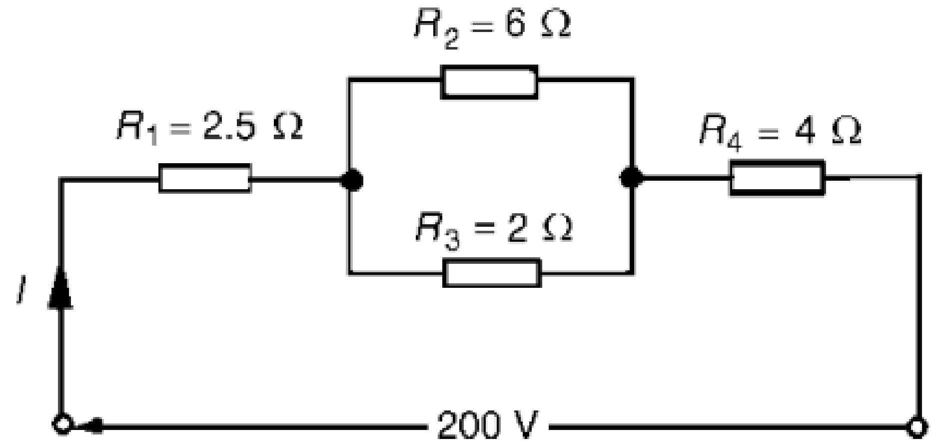
(a) Reading on ammeter, $I = \dfrac{V}{R_3} = \dfrac{40}{20} = \mathbf{2\,A}$

(b) Current flowing through $R_2 = 11 - 8 - 2 = 1\,\text{A}$

Hence, $R_2 = \dfrac{V}{I_2} = \dfrac{40}{1} = \mathbf{40\,\Omega}$

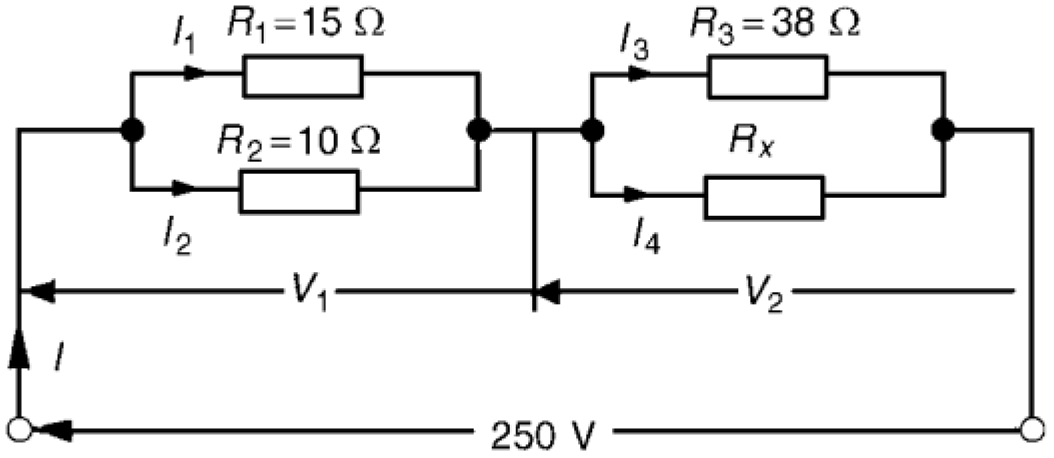
Example

For the series-parallel arrangement shown in Figure , find (a) the supply current, (b) the current flowing through each resistor and (c) the p.d. across each resistor.



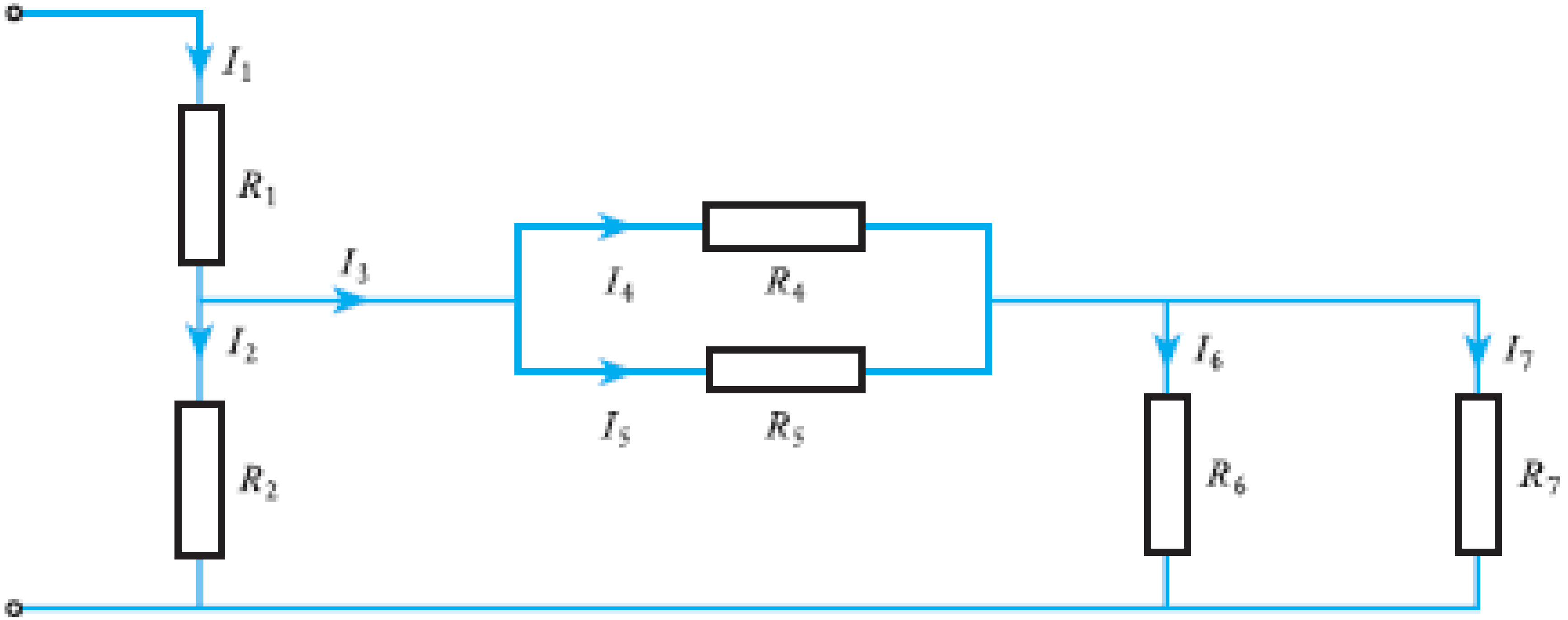
Example

For the circuit shown in Figure calculate (a) the value of resistor Rx such that the total power dissipated in the circuit is 2.5 kW, and (b) the current flowing in each of the four resistors.



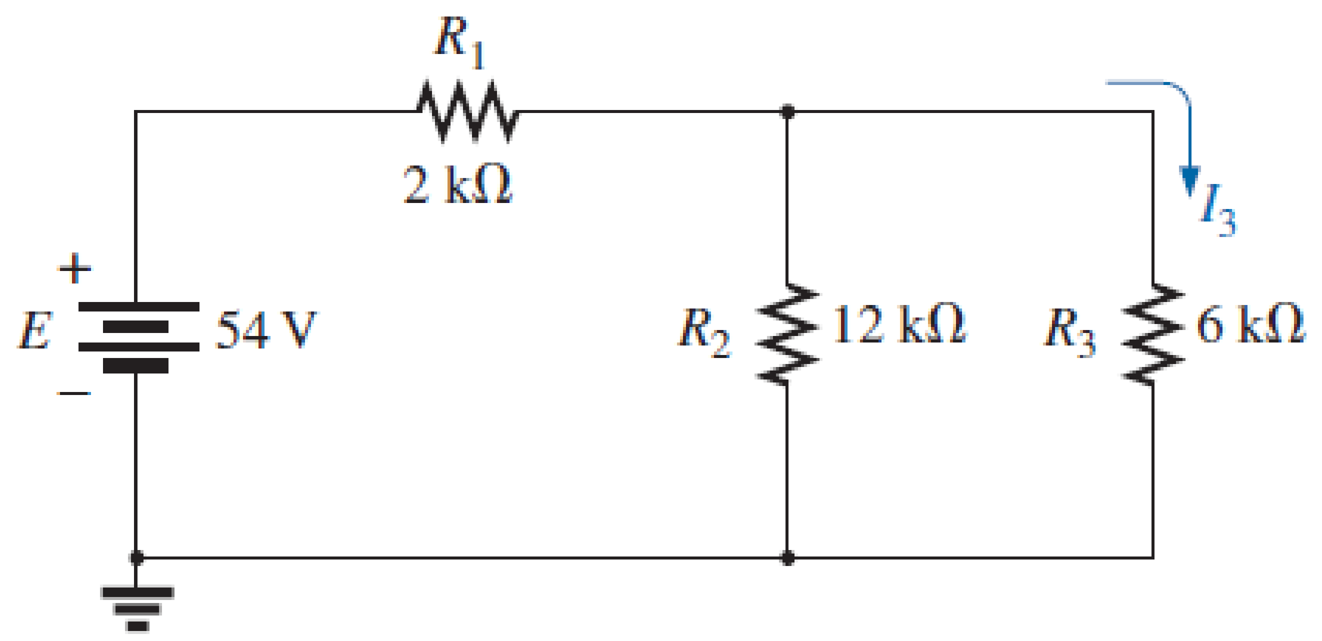
Series-parallel networks

- R_4 is in parallel with R_5 and R_6 is in parallel with R_7.
- The network comprising R_4 and R_5 is in series with the network comprising R_6 and R_7.
- R_2 is in parallel with the network comprising R_4, R_5, R_6 and R_7 and R_1 in series with that combination



Example of series-parallel

- Find current I_3 for the series-parallel network



Example of series-parallel

- R_2 and R_3 are in parallel, their total resistance is

$$R' = R_2 \parallel R_3 = \frac{R_2 R_3}{R_2 + R_3} = \frac{(12\text{ k}\Omega)(6\text{ k}\Omega)}{12\text{ k}\Omega + 6\text{ k}\Omega} = 4\text{ k}\Omega$$

Resistors R_1 and R' are then in series, resulting in a total resistance of:

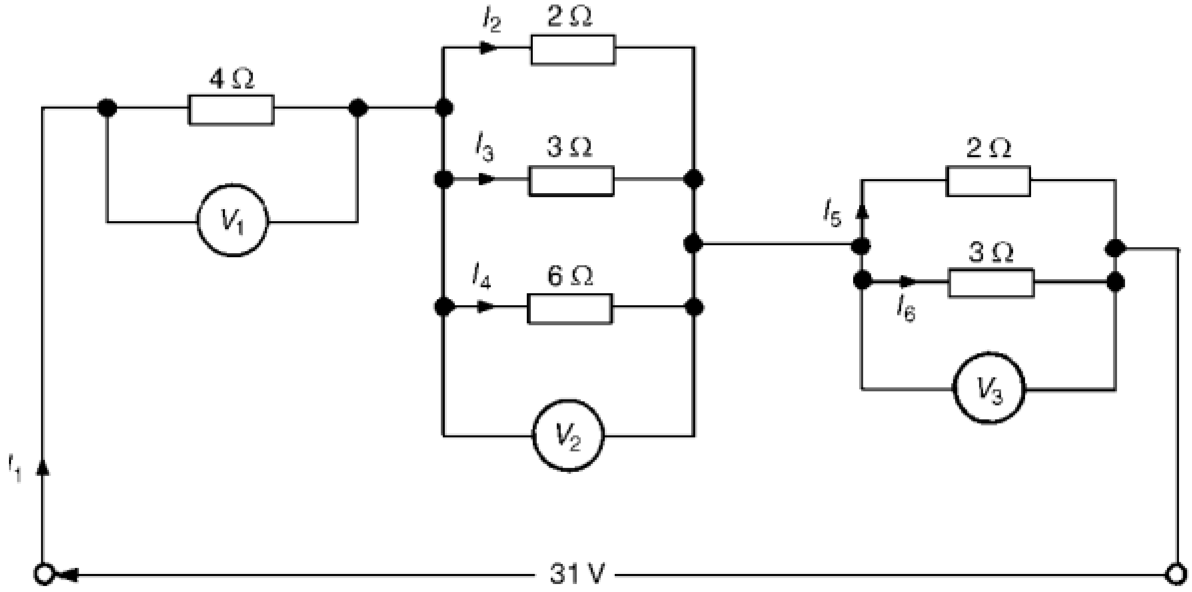
$$R_T = R_1 + R' = 2\text{ k}\Omega + 4\text{ k}\Omega = 6\text{ k}\Omega$$

- The source current is then determined using Ohm's law: $$I_s = \frac{E}{R_T} = \frac{54\text{ V}}{6\text{ k}\Omega} = 9\text{ mA}$$
- since R_1 and R' are in series, they have the same current I_s: $$I_1 = I_s = 9\text{ mA}$$
- I_1 is the total current entering the parallel combination of R_2 and R_3. Applying the current divider rule :

$$I_3 = \left(\frac{R_2}{R_2 + R_3}\right) I_1 = \left(\frac{12\text{ k}\Omega}{12\text{ k}\Omega + 6\text{ k}\Omega}\right) 9\text{ mA} = \mathbf{6\text{ mA}}$$

EXAMPLE

- Determine the currents and voltages indicated in the circuit shown in Figure



EXAMPLE

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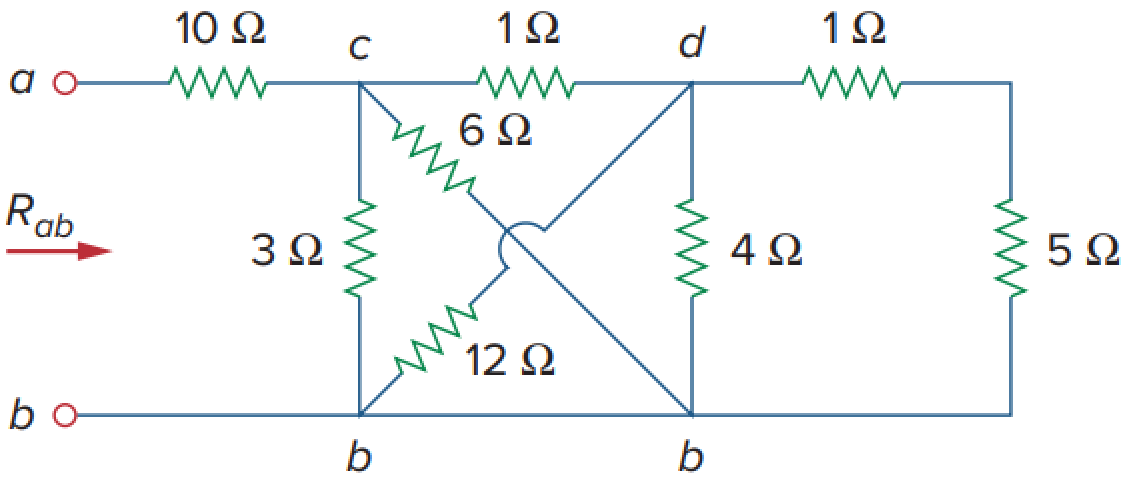
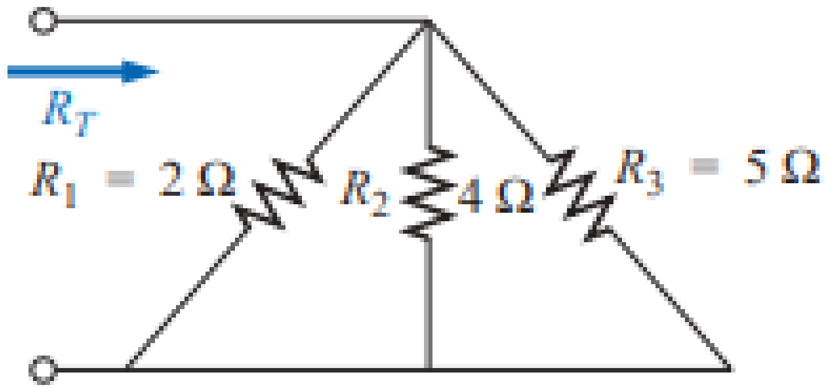
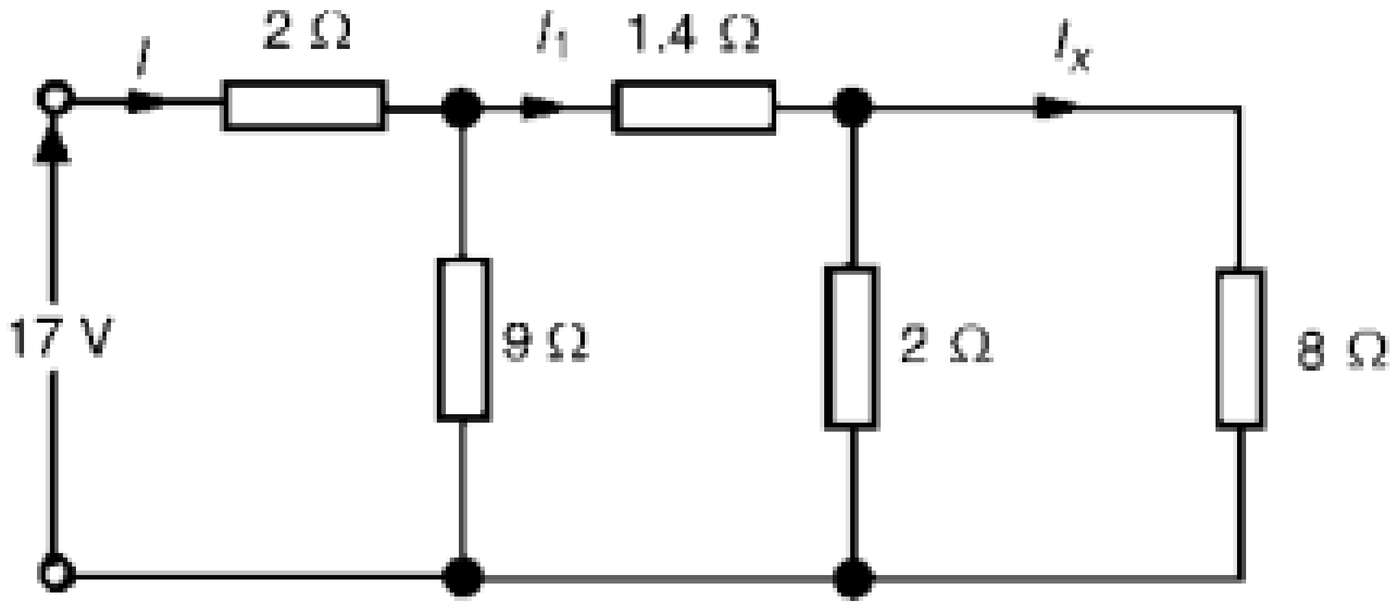


EXAMPLE



EXAMPLE

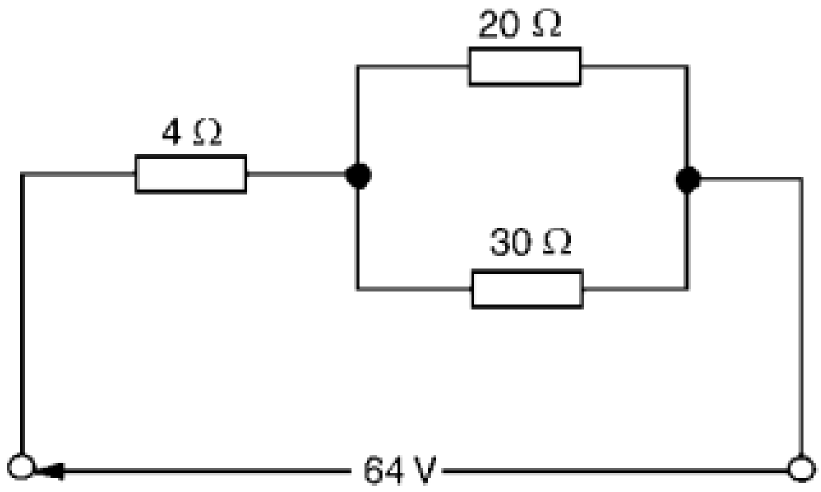
- For the arrangement shown in Figure , find the current Ix



EXAMPLE

(a) Calculate the current flowing in the 30 Ω resistor shown in Figure

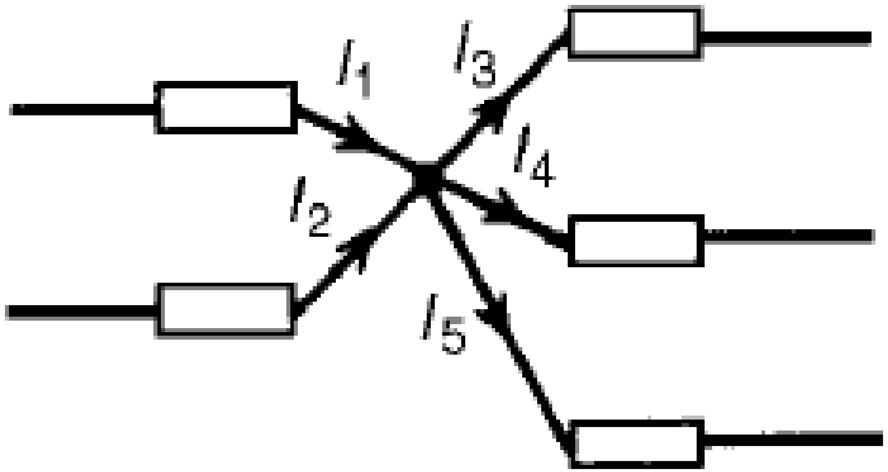
(b) What additional value of resistance would have to be placed in parallel with the 20 Ω and 30 Ω resistors to change the supply current to 8 A, the supply voltage remaining constant.



Kirchhoff's (First) Current Law (KCL)

At any junction in an electric circuit the total current flowing towards that junction is equal to the total current flowing away from the junction,

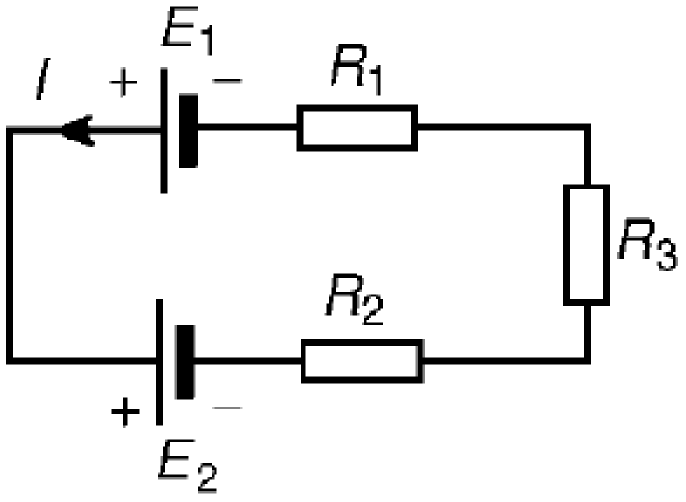
i.e. ΣI = 0



$$I_1 + I_2 = I_3 + I_4 + I_5 \quad \text{or} \quad I_1 + I_2 - I_3 - I_4 - I_5 = 0$$

Kirchhoff's (Second) Voltage Law

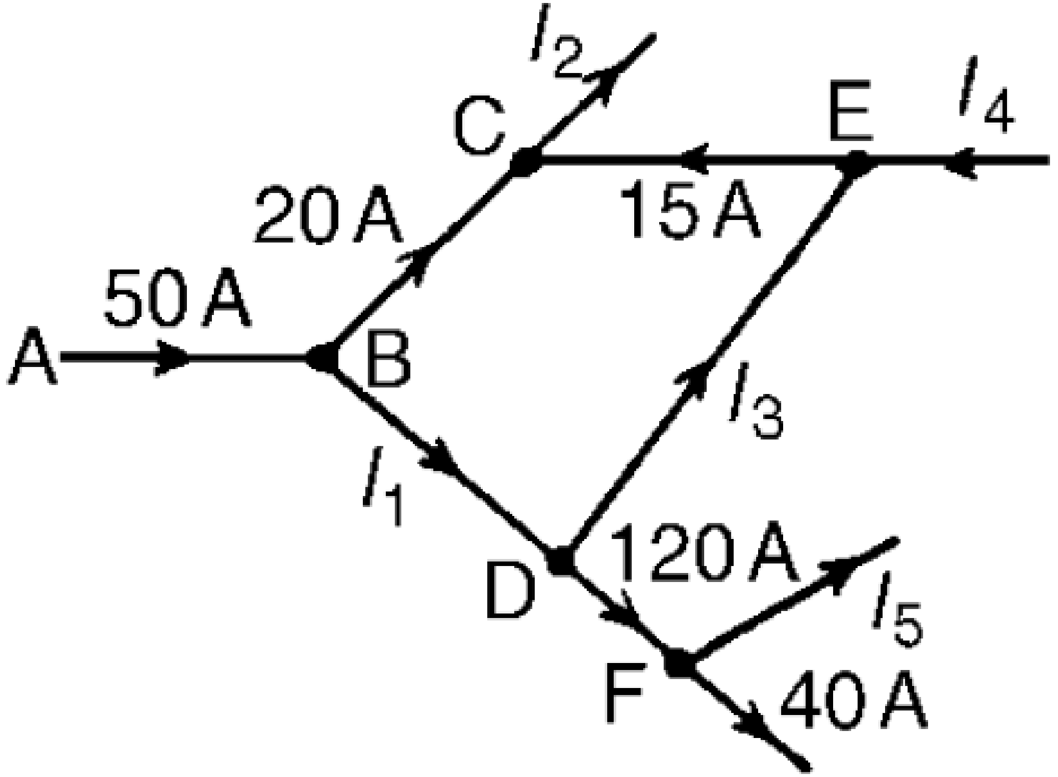
In any closed loop in a network, the algebraic sum of the voltage drops (i.e. products of current and resistance) taken around the loop is equal to the resultant e.m.f. acting in that loop.



$$E_1 - E_2 = IR_1 + IR_2 + IR_3$$

KCL Example

Find the marked unknown currents in the figure.



KCL Example

Applying Kirchhoff's current law:

For junction B: $50 = 20 + I_1$. Hence $\boldsymbol{I_1 = 30\ \mathrm{A}}$

For junction C: $20 + 15 = I_2$. Hence $\boldsymbol{I_2 = 35\ \mathrm{A}}$

For junction D: $I_1 = I_3 + 120$

i.e. $30 = I_3 + 120$. Hence $\boldsymbol{I_3 = -90\ \mathrm{A}}$

(i.e. in the opposite direction to that shown in Figure

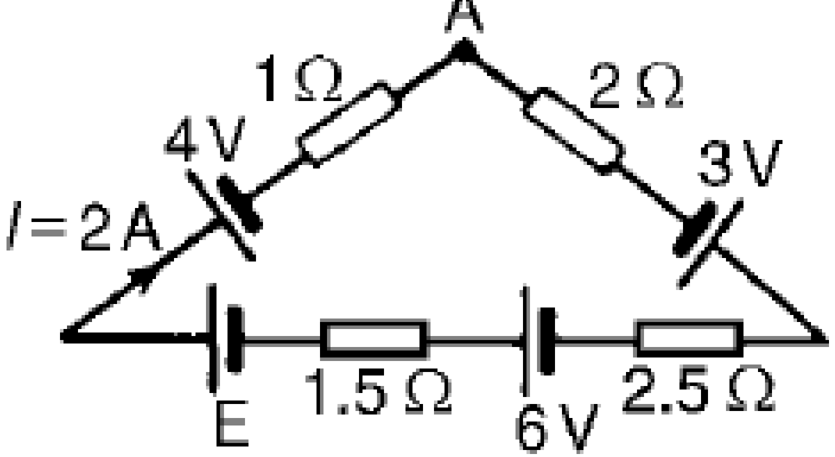
For junction E: $I_4 + I_3 = 15$

i.e. $I_4 = 15 - (-90)$. Hence $\boldsymbol{I_4 = 105\ \mathrm{A}}$

For junction F: $120 = I_5 + 40$. Hence $\boldsymbol{I_5 = 80\ \mathrm{A}}$

KVL Example

Determine the value of e.m.f. E in the Figure.



Applying Kirchhoff's voltage law and moving clockwise around the loop of starting at point A:

$$3 + 6 + E - 4 = (I)(2) + (I)(2.5) + (I)(1.5) + (I)(1)$$

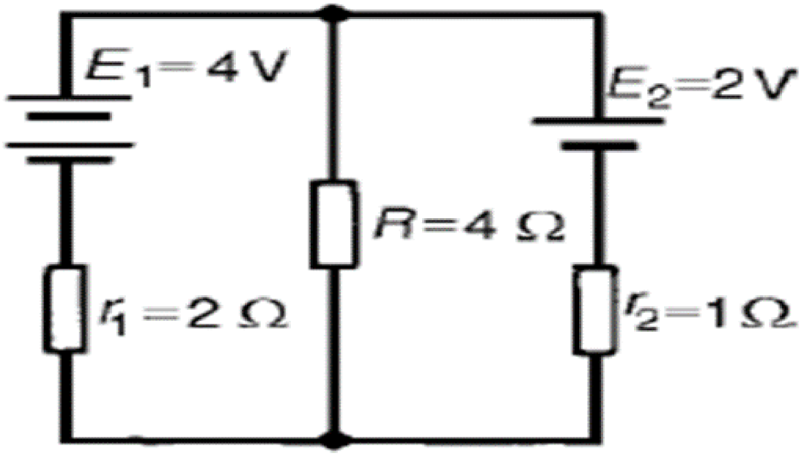
$$= I(2 + 2.5 + 1.5 + 1)$$

i.e. $\quad 5 + E = 2(7), \text{ since } I = 2 \text{ A}$

Hence $\quad E = 14 - 5 = \mathbf{9\ V}$

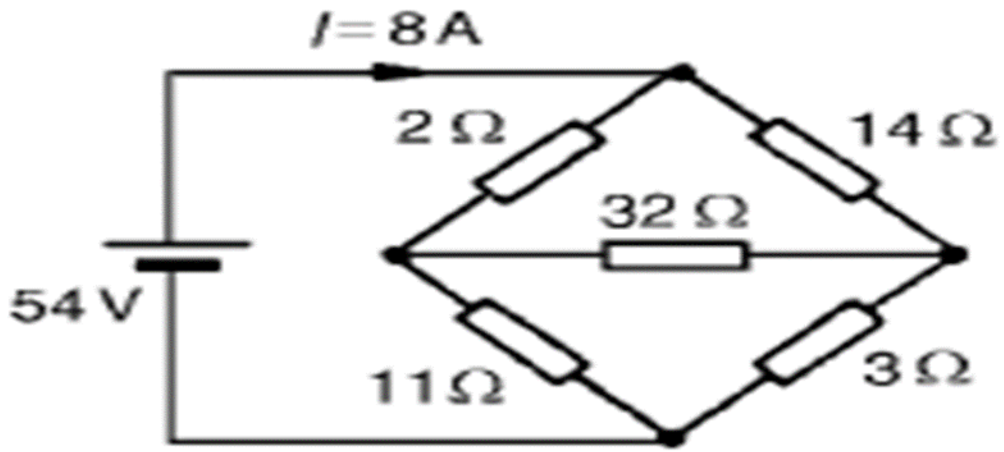
KVL Example

- Determine the current flowing in each branch using Kirchhoff's current and voltage laws



KVL Example

- For the bridge network shown in the figure below, determine the current in each of the resistors



Resistance Variation

The resistance of an electrical conductor depends on the following 4 factors,

(a) the length of the conductor,

(b) the cross-sectional area of the conductor,

(c) the type of material and

(d) the temperature of the material.

Resistivity

- Resistance, R, is directly proportional to length, *l*, of a conductor, i.e. R α *l*.
- Resistance, R, is inversely proportional to cross-sectional area, a, of a conductor, i.e. R α 1/a.
- The constant of proportionality in this relationship is the type of material used and is known as the resistivity of the material and is given the symbol *ρ* (Greek rho). Thus,

$$R = \frac{\rho l}{a} \text{ ohms}$$

- *ρ* is measured in ohm-metres (Ωm)

Typical values of resistivity

Material	ρ (Ω m) at 0 °C
Aluminium	2.7×10^{-8}
Brass	7.2×10^{-8}
Copper	1.59×10^{-8}
Eureka	49.00×10^{-8}
Manganin	42.00×10^{-8}
Carbon	6500.00×10^{-8}
Tungsten	5.35×10^{-8}
Zinc	5.37×10^{-8}

Problem 4. Calculate the resistance of a 2 km length of aluminium overhead power cable if the cross-sectional area of the cable is 100 mm^2. Take the resistivity of aluminium to be 0.03×10^{-6} Ωm

Example

A coil is wound from a 10 m length of copper wire having a cross-sectional area of 1.0 mm^2. Calculate the resistance of the coil. The resistivity of copper is 1.59 x 10^{-8}

$$R = \rho \frac{l}{A} = \frac{1.59 \times 10^{-8} \times 10}{1 \times 10^{-6}} = 0.159\ \Omega$$

Temperature coefficient of resistance

- As the temperature of a material increases, most conductors increase in resistance, insulators decrease in resistance, whilst the resistance of some special alloys remain almost constant.
- The temperature coefficient of resistance of a material is the increase in the resistance of a 1 Ω resistor of that material when it is subjected to a rise of temperature of 1°C.
- The symbol used for the temperature coefficient of resistance is ***α*** (Greek alpha)
- The units are usually expressed as ‘per °C’.

Temperature coefficient of resistance

- If the resistance of a material at 0°C is known the resistance at any

other temperature can be determined from:

$$R_\theta = R_0(1 + \alpha_0\theta)$$

where

R_0 = resistance at 0°C

R_θ = resistance at temperature θ °C

α = temperature coefficient of resistance at 0°C

- If a material having a resistance R_0 at 0 °C, taken as the standard temperature, has a resistance R_1 at θ_1 and R_2 at θ_2, and if α_0 is the temperature coefficient of resistance at 0 °C, then

$$\frac{R_1}{R_2} = \frac{(1 + \alpha_0\theta_1)}{(1 + \alpha_0\theta_2)}$$

Temperature coefficient of resistance

- If the resistance of a material at room temperature (approximately 20°C), R_{20}, and the temperature coefficient of resistance at 20°C, α_{20}, are known then the resistance R_θ at temperature θ °C is given by:

$$R_\theta = R_{20}[1 + \alpha_{20}(\theta - 20)]$$

Typical temperature coefficients of resistance referred to 0 °C

- Some materials such as carbon have a negative temperature coefficient of resistance,
- i.e. their resistances fall with increase in temperature.

Material	α_0(/°C) at 0 °C
Aluminium	0.003 81
Copper	0.004 28
Silver	0.004 08
Nickel	0.006 18
Tin	0.004 4
Zinc	0.003 85
Carbon	–0.000 48
Manganin	0.000 02
Constantan	0
Eureka	0.000 01
Brass	0.001

Example

A coil of copper wire has a resistance of 100 Ω when its temperature is 0°C. Determine its resistance at 70°C if the temperature coefficient of resistance of copper at 0°C is 0.0043/°C

$$\text{Resistance } R_\theta = R_0(1 + \alpha_0\theta)$$

$$\text{Hence resistance at } 70^\circ\text{C, } R_{70} = 100[1 + (0.0043)(70)]$$

$$= 100[1 + 0.301] = 100(1.301)$$

$$= \mathbf{130.1\ \Omega}$$

Example

A coil of copper wire has a resistance of 10 at 20°C. If the temperature coefficient of resistance of copper at 20°C is 0.004/°C determine the resistance of the coil when the temperature rises to 100°C

$$\text{Resistance at } \theta^\circ\text{C}, \; R = R_{20}[1 + \alpha_{20}(\theta - 20)]$$

$$\text{Hence resistance at } 100^\circ\text{C}, \; R_{100} = 10[1 + (0.004)(100 - 20)]$$

$$= \mathbf{13.2\ \Omega}$$

Homework

When a potential difference of 10 V is applied to a coil of copper wire

of mean temperature 20 °C, a current of 1.0 A flows in the coil. After

some time the current falls to 0.95 A yet the supply voltage remains unaltered. Determine the mean temperature of the coil given that

the temperature coefficient of resistance of copper is 4.28×10^{-3}/°C

referred to 0 °C.