CAPACITORS

Introduction to Capacitors

The capacitor is a widely used electrical component:

- **Storing energy** eg in power supplies and camera flash circuit
- **Timing** – for example with a 555 timer IC controlling the charging and discharging;
- **Smoothing** – eg in a power supply;
- **Coupling** – eg between stages of an audio system and a loudspeaker;
- **Filtering** – eg in the tone control of an audio system;
- **Tuning** – eg in a radio system;
- **Decoupling** in logic circuits for prevent spikes and ripple on the supply lines

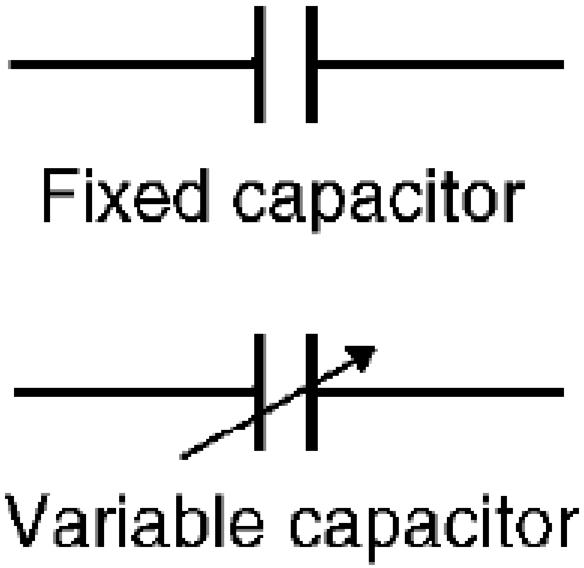
Capacitors

- A capacitor is a device that has the ability to store a quantity of static electricity.
- In its simplest form a capacitor consists of two plates which are separated by an insulating material known as a **dielectric**.
- The **charge *Q*** stored in a capacitor is given by:

$$Q = I \times t \text{ coulombs}$$

- where *I* is the current in amperes and *t* the time

in seconds.



Capacitance

- It is the property of a capacitor to store an electric charge when its plates are at different potentials.
- The unit of capacitance is termed the farad (F) which may be defined as the capacitance of a capacitor between the plates of which there appears a potential difference of 1 volt when it is charged by 1 coulomb of electricity.
- Hence:

$$\frac{\text{Charge [coulombs]}}{\text{Applied p.d. [volts]}} = \text{capacitance [farads]}$$

$$C = \frac{Q}{V}$$

Example

A direct current of 4 A flows into a previously uncharged 20 μF capacitor for 3 ms. Determine the p.d. between the plates.

$I = 4\ \mathrm{A};\ C = 20\ \mu\mathrm{F} = 20 \times 10^{-6}\ \mathrm{F};\ t = 3\ \mathrm{ms} = 3 \times 10^{-3}\ \mathrm{s}$

$Q = It = 4 \times 3 \times 10^{-3}\ \mathrm{C}$

$$V = \frac{Q}{C} = \frac{4 \times 3 \times 10^{-3}}{20 \times 10^{-6}} = \frac{12 \times 10^{6}}{20 \times 10^{3}} = 0.6 \times 10^{3} = 600\ \mathrm{V}$$

Example

A 5 µF capacitor is charged so that the p.d. between its plates is 800 V. Calculate how long the capacitor can provide an average discharge current of 2 mA.

$$C = 5\ \mu\text{F} = 5 \times 10^{-6}\ \text{F};\ V = 800\ \text{V};\ \ I = 2\ \text{mA} = 2 \times 10^{-3}\ \text{A}$$

$$Q = CV = 5 \times 10^{-6} \times 800 = 4 \times 10^{-3}\ \text{C}$$

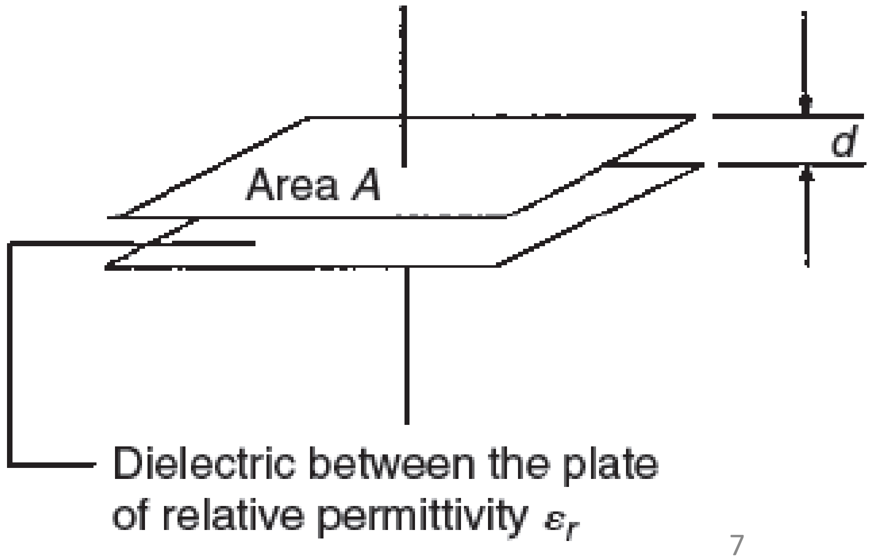
$$\text{Also, } Q = It. \text{ Thus, } t = \frac{Q}{I} = \frac{4 \times 10^{-3}}{2 \times 10^{-3}} = 2\ \text{s}$$

Parallel plate capacitor

- For a parallel plate capacitor, capacitance *C* is proportional to the area *A* of a plate, inversely proportional to the plate spacing *d* (i.e. the dielectric thickness) and depends on the nature of the dielectric:

$$\textbf{Capacitance } C = \frac{\varepsilon_0 \varepsilon_r A}{d} \text{ farads}$$

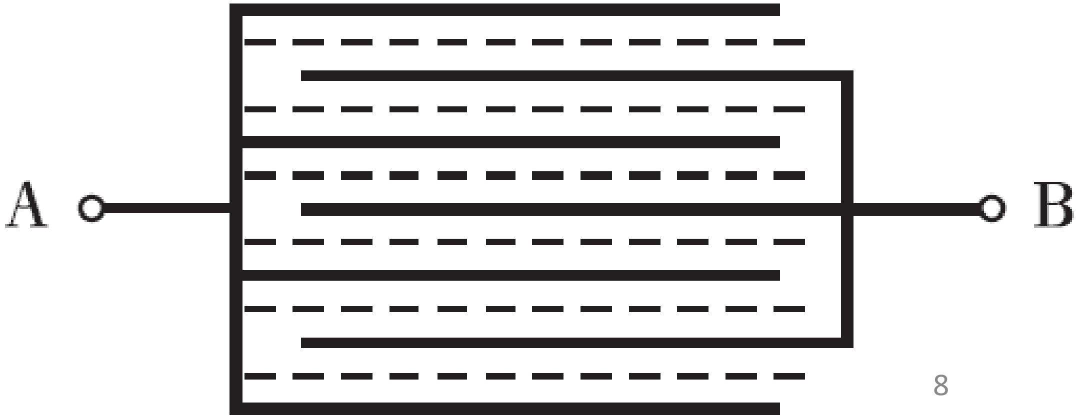
- where ε_0 = 8.85×10−12 F/m (constant)
- ε_r = relative permittivity
- *A* = area of one of the plates in m2 and
- *d* = thickness of dielectric in m.



Capacitance of a multi-plate capacitor

- The capacitance can be increased by interleaving several plates.
- In the figure, 7 plates are shown, forming 6 capacitors with a capacitance 6 times that of one pair of plates.
- If such an arrangement has n plates then capacitance $C \propto (n-1)$.

$$\therefore \qquad \text{Capacitance} = \frac{\epsilon_0 \epsilon_r (n-1) A}{d} \quad \text{farads}$$



Example

(a) A ceramic capacitor has an effective plate area of 4cm^2 separated by 0.1mm of ceramic of relative permittivity 100. Calculate the capacitance of the capacitor in picofarads.

$$\text{Capacitance } C = \frac{\varepsilon_0 \varepsilon_r A}{d} \text{ farads}$$

$$= \frac{8.85 \times 10^{-12} \times 100 \times 4 \times 10^{-4}}{0.1 \times 10^{-3}} \text{F}$$

$$= \frac{8.85 \times 4}{10^{10}} \text{F} = \frac{8.85 \times 4 \times 10^{12}}{10^{10}} \text{pF}$$

$$= \mathbf{3540\ pF}$$

(b) If the capacitor in part (a) is given a charge of 1.2μC, what will be the p.d. between the plates?

$$Q = CV \text{ thus } V = \frac{Q}{C} = \frac{1.2 \times 10^{-6}}{3540 \times 10^{-12}}$$

$$V = \mathbf{339\ V}$$

Example

A capacitor is made with 7 metal plates and separated by sheets of mica having a thickness of 0.3 mm and a relative permittivity of 6. The area of one side of each plate is 500 cm2. Calculate the capacitance in microfarads.

$$\therefore \qquad \text{Capacitance} = \frac{\epsilon_0 \epsilon_r (n-1) A}{d} \quad \text{farads}$$

$$\therefore \qquad C = \frac{8.85 \times 10^{-12} \times 6 \times 6 \times 0.05}{0.0003} = 0.0531 \times 10^{-6}\ \text{F}$$

$$= \mathbf{0.053\ \mu F}$$

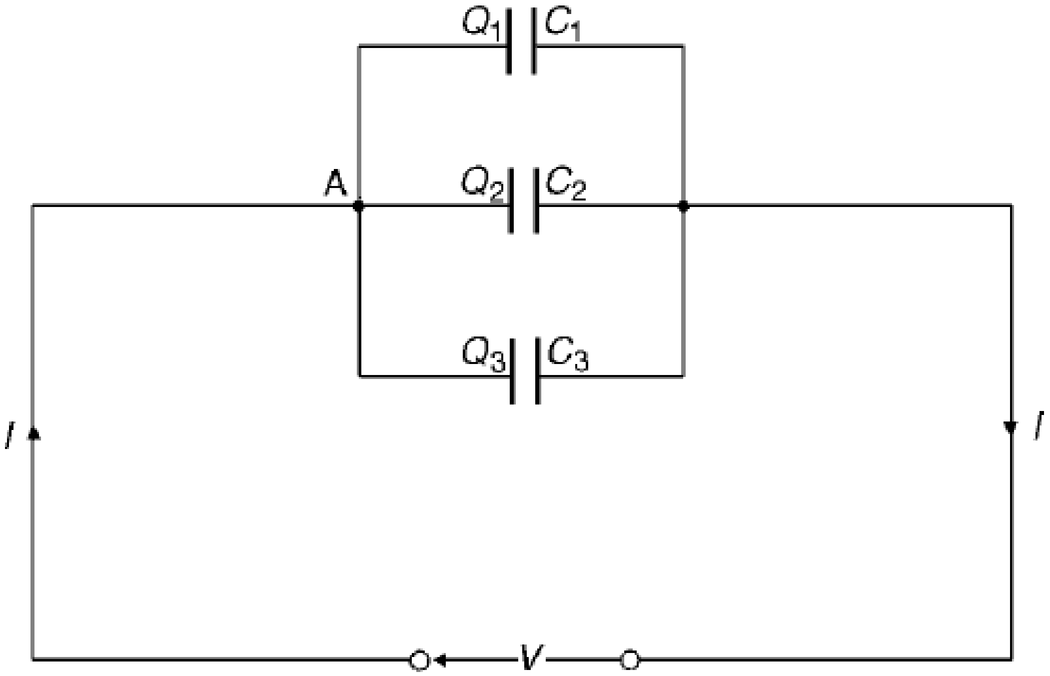
Capacitors connected in parallel

- Since the current divides, the total charge (**Q_T= I x t)** is also divided between the three capacitors.

$$Q_T = Q_1 + Q_2 + Q_3$$

- But $Q_T = CV$, $Q_1 = C_1V$, $Q_2 = C_2V$ and $Q_3 = C_3V$

where C is the total equivalent circuit Capacitance:



$$CV = C_1V + C_2V + C_3V$$

$$C = C_1 + C_2 + C_3$$

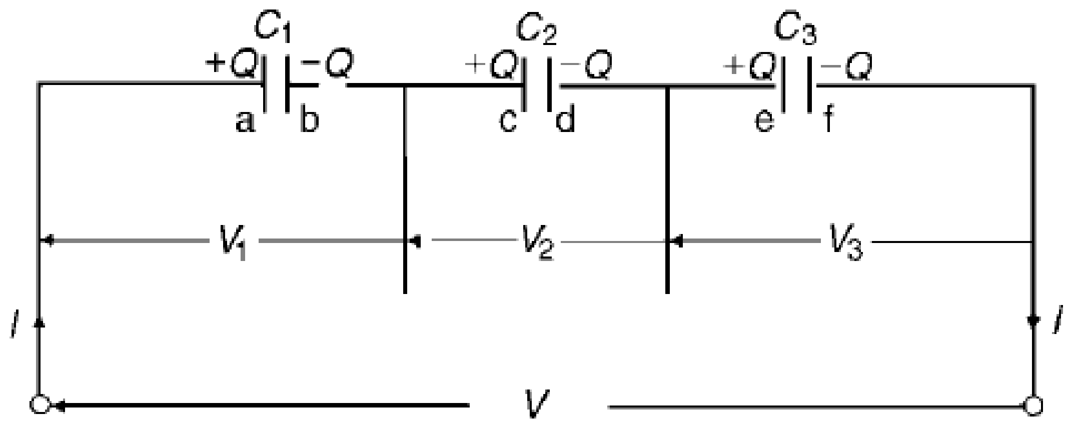
- Note that this formula is similar to that used for **resistors** connected in **series**

Capacitors connected in series

- Let the p.d. across the individual capacitors be V_1, V_2 and V_3 and the charge on each capacitor is found to be the same, *Q* coulombs.
- In a series circuit: $V = V_1 + V_2 + V_3$ Since $V = \frac{Q}{C}$ then $\frac{Q}{C} = \frac{Q}{C_1} + \frac{Q}{C_2} + \frac{Q}{C_3}$
- where C is the total equivalent circuit capacitance, thus $\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$
- Note that this formula is similar to that used for resistors connected in parallel
- For the special case of two capacitors

in series: $\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} = \frac{C_2 + C_1}{C_1\, C_2}$

Hence $\boxed{C = \frac{C_1 C_2}{C_1 + C_2}}$ $\left(\text{i.e. } \frac{\text{product}}{\text{sum}}\right)$



Example

- Calculate the equivalent capacitance of two capacitors of 6 μF and 4 μF connected (a) in parallel and (b) in series

(a) In parallel, equivalent capacitance ***C*** = C1 + C2 = 6 μF +4 μF =**10 μF**

(b) In series, equivalent capacitance C is given by:

$$C = \frac{C_1 C_2}{C_1 + C_2}$$

$$C = \frac{6 \times 4}{6 + 4} = \frac{24}{10} = \mathbf{2.4\ \mu F}$$

Example

Capacitances of 1 μF, 3 μF, 5 μF and 6 μF are connected in parallel to a direct voltage supply of 100 V. Determine

(a) the equivalent circuit capacitance,

$$C = C_1 + C_2 + C_3 + C_4$$

$$C = 1 + 3 + 5 + 6 = \mathbf{15\ \mu F}$$

(b) the total charge

$$\text{Total charge } Q_T = CV$$

$$Q_T = 15 \times 10^{-6} \times 100 = 1.5 \times 10^{-3}\ C = \mathbf{1.5\ mC}$$

(c) the charge on each capacitor

$$Q_1 = C_1V = 1 \times 10^{-6} \times 100$$

$$= \mathbf{0.1\ mC}$$

$$Q_2 = C_2V = 3 \times 10^{-6} \times 100$$

$$= \mathbf{0.3\ mC}$$

$$Q_3 = C_3V = 5 \times 10^{-6} \times 100$$

$$= \mathbf{0.5\ mC}$$

$$Q_4 = C_4V = 6 \times 10^{-6} \times 100$$

$$= \mathbf{0.6\ mC}$$

Check

$$Q_T = Q_1 + Q_2 + Q_3 + Q_4$$

$$= 1.5\ \text{mC}$$

Example

Capacitances of 3 μF, 6 μF and 12 μF are connected in series across a 350 V supply. Calculate (a) the equivalent circuit capacitance,

$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$$

i.e. $$\frac{1}{C} = \frac{1}{3} + \frac{1}{6} + \frac{1}{12} = \frac{4+2+1}{12} = \frac{7}{12}$$

$$\mathbf{C = \frac{12}{7} = 1\frac{5}{7}\ \mu F}$$

(b) the charge on each capacitor:

$$Q_T = CV,$$

$$Q_T = \frac{12}{7} \times 10^{-6} \times 350 = 600\ \mu\text{C or } 0.6\ \text{mC}$$

Since the capacitors are in series 0.6 mC is the charge on each.

(c) the pd across each capacitor.

$$V_1 = \frac{Q}{C_1} = \frac{0.6 \times 10^{-3}}{3 \times 10^{-6}}$$

$$= \mathbf{200\ V}$$

$$V_2 = \frac{Q}{C_2} = \frac{0.6 \times 10^{-3}}{6 \times 10^{-6}}$$

$$= \mathbf{100\ V}$$

$$V_3 = \frac{Q}{C_3} = \frac{0.6 \times 10^{-3}}{12 \times 10^{-6}}$$

$$= \mathbf{50\ V}$$

Dielectric strength

The maximum amount of field strength that a dielectric can withstand is called the dielectric strength of the material.

$$E_m = \frac{V_m}{d}$$

Energy stored

- The energy, W, stored by a capacitor is given by

$$W = \frac{1}{2} CV^2 \text{ joules}$$

Energy stored

Example: Determine the energy stored in a 3μF capacitor when charged to 400V. (b) Find also the average power developed if this energy is dissipated in a time of 10 μs.

(a) **Energy stored** $W = \frac{1}{2}CV^2$ joules

$$= \frac{1}{2} \times 3 \times 10^{-6} \times 400^2$$

$$= \frac{3}{2} \times 16 \times 10^{-2}$$

$$= \mathbf{0.24\ J}$$

(b) **Power** $= \dfrac{\text{Energy}}{\text{time}} = \dfrac{0.24}{10 \times 10^{-6}}\ \text{W} = \mathbf{24\ kW}$

Discharging a Capacitor

- When a capacitor has been disconnected from the supply it may still be charged and it may retain this charge for some considerable time.
- Thus precautions must be taken to ensure that the capacitor is automatically discharged after the supply is switched off.
- This is done by connecting a high-value resistor across the capacitor terminals.