

## Principles of Communication HW #4

Exercise 14.2: Consider transmitting a signal with values chosen from the six level alphabet  $\pm 1, \pm 3, \pm 5$

- a) Suppose that all 6 symbols are equally likely. Identify  $N$ ,  $x_i$ , and  $p(x_i)$ , and calculate the information  $I(x_i)$  associated with each:

$$N=6 \quad \begin{matrix} +1 & -1 & +3 & -3 & +5 & -5 \end{matrix}$$

$$p(x_1) = p(x_2) = p(x_3) = p(x_4) = p(x_5) = p(x_6)$$

$$= p(x_i) = \frac{1}{6}$$

$$I(x_i) = \log_2 \left( \frac{1}{(1/6)} \right) = \log_2(6) = 2.585 \approx 3 \text{ bits}$$

b)

$x_1 = +1$	$p(x_1) = 1/4$
$x_2 = -1$	$+ p(x_2) = 1/4$
$x_3 = +3$	$- p(x_3) = 1/8$
$x_4 = -3$	$- p(x_4) = 1/8$
$x_5 = +5$	$+ p(x_5) = 1/4$
$x_6 = -5$	$- p(x_6) = 0$
	<hr/>
	100%

Probability of  
 $-5 = 0$

The information conveyed by each of the symbols:

$$I(x_1 \& x_2) = 2 \text{ bits}$$

$$I(x_3 \& x_4) = 3 \text{ bits}$$

$$I(x_5) = 2 \text{ bits}$$

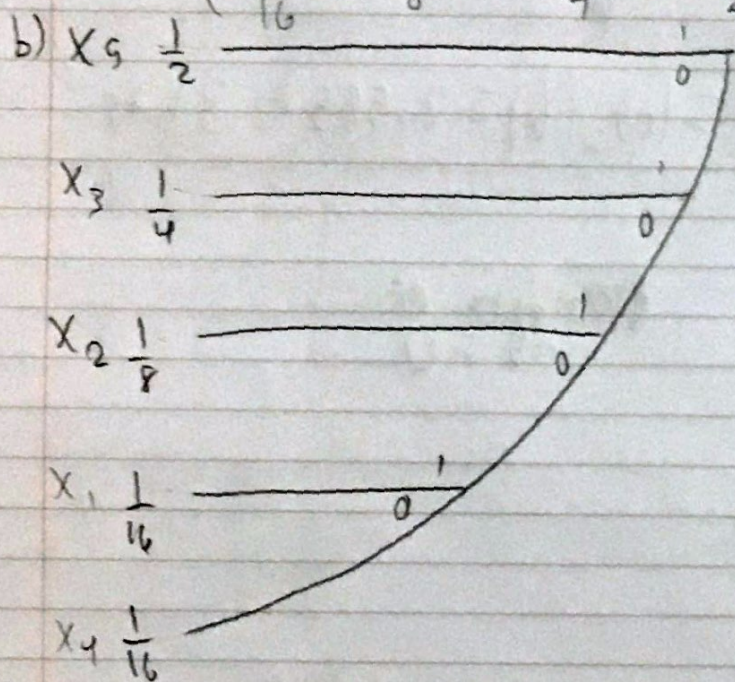
$$I(x_6) = \text{DNE}$$



Exercise 14.16:  $N=5$   $P(x_1) = \frac{1}{16}$   $P(x_2) = \frac{1}{8}$   $P(x_3) = \frac{1}{4}$

$$P(x_4) = \frac{1}{16} \quad P(x_5) = \frac{1}{2}$$

$$a) H(X) = - \left( \frac{1}{16} \log\left(\frac{1}{16}\right) + \frac{1}{8} \log\left(\frac{1}{8}\right) + \frac{1}{4} \log\left(\frac{1}{4}\right) + \frac{1}{2} \log\left(\frac{1}{2}\right) \right) \\ = - \left( \frac{-4}{16} + \frac{-3}{8} + \frac{-2}{4} + \frac{-1}{2} \right) = 1.875$$



c)

$$\begin{aligned} x_5 &= 1 \\ x_3 &= 01 \\ x_2 &= 001 \\ x_1 &= 0001 \\ x_4 &= 0000 \end{aligned}$$

$$x_1, x_4, x_2, x_2, x_3, x_3, x_3, x_3 = 30 / 16 = 1.875$$

$$d) \frac{1.875}{1.875} = 1 = \text{efficiency}$$

e) naive: 3 bits per symbol

$$\text{efficiency} = \frac{1.875}{3} = 0.625$$