Finite State Automata and Search

Doing NLP with FST

Trevor Sullivan

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University of Arizona

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Introduction to FSM

Nomenclature

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1. **Finite State Acceptor** (usually what is meant by Finite State Automaton)

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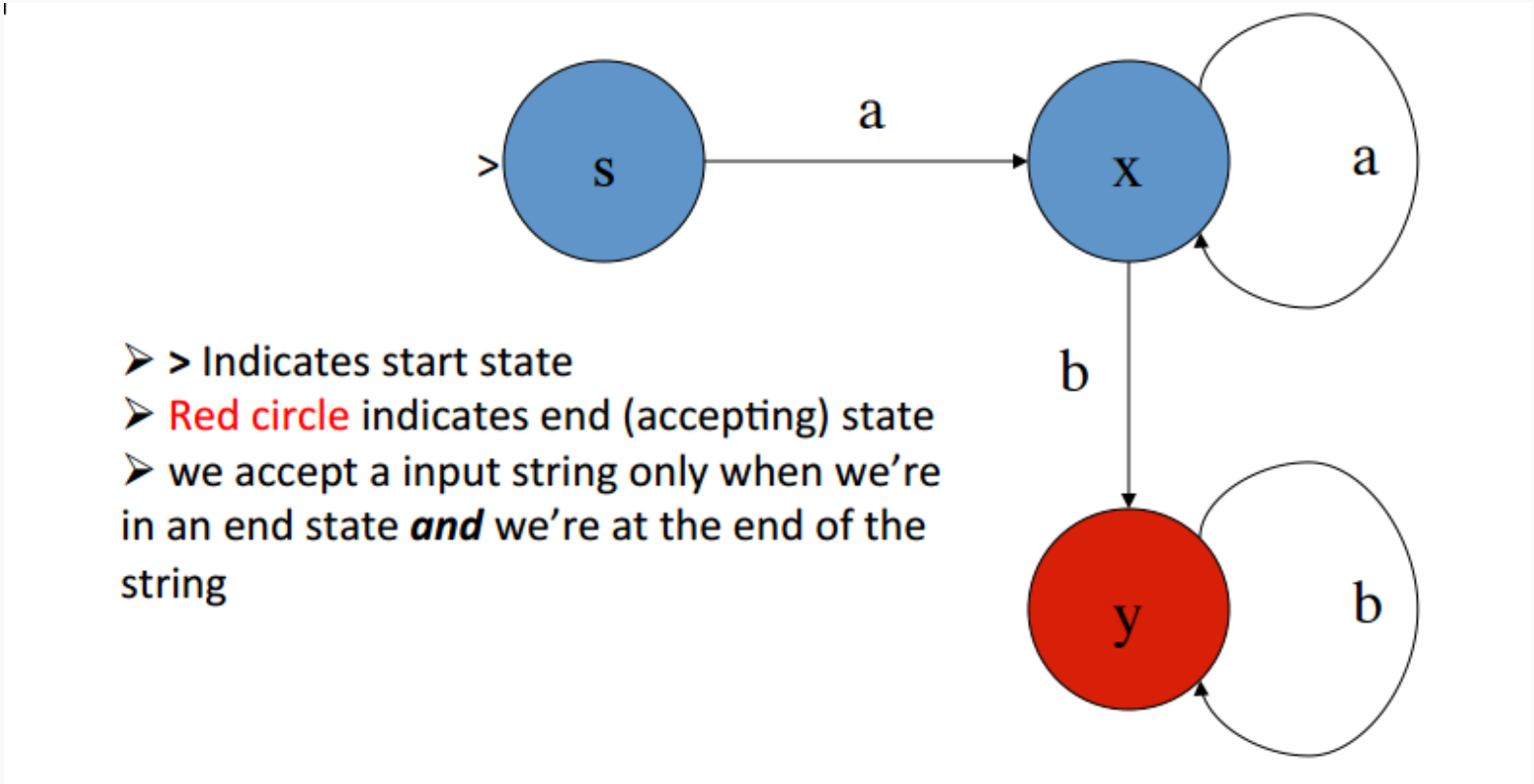
Two types of FSM:

1. **Finite State Acceptor** (usually what is meant by Finite State Automaton)
2. **Finite State Transducer**

What is a FSM?

What is a FSM?

A directed graph



- > Indicates start state
- Red circle indicates end (accepting) state
- we accept a input string only when we're in an end state ***and*** we're at the end of the string

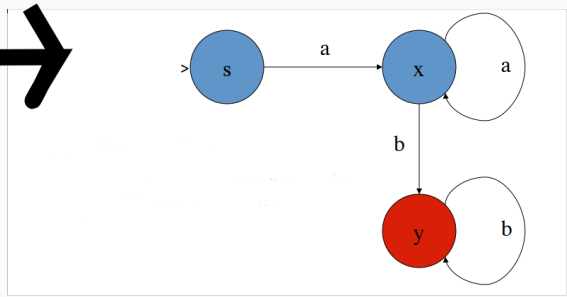
$L = \{a^+ b^+\}$

What is a FSM?

REGEX

{a+b+}

FSA



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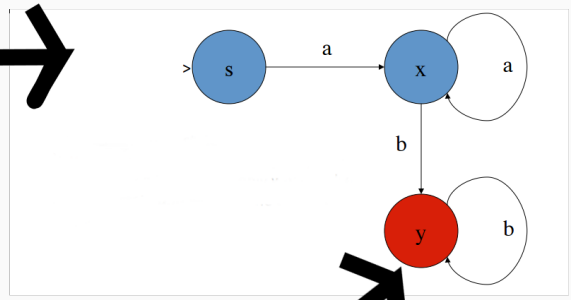
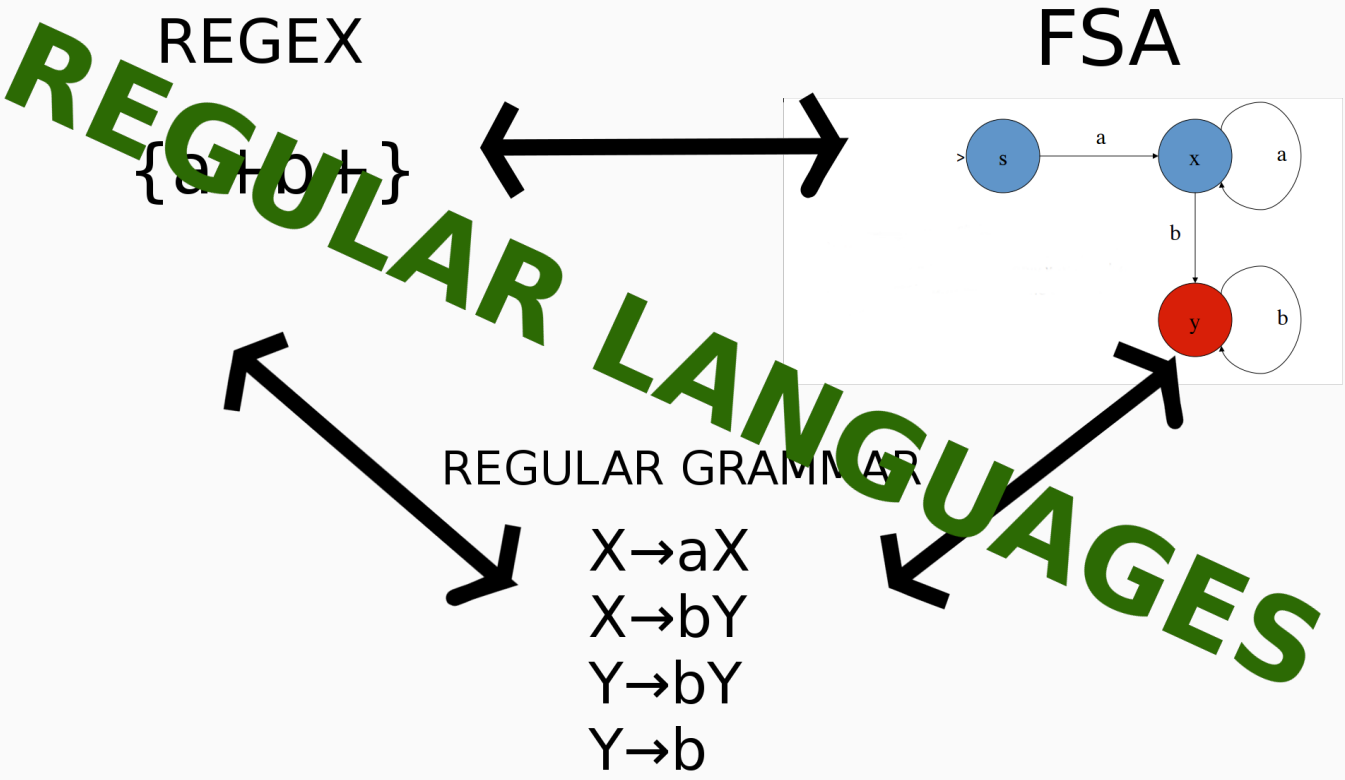
FSA



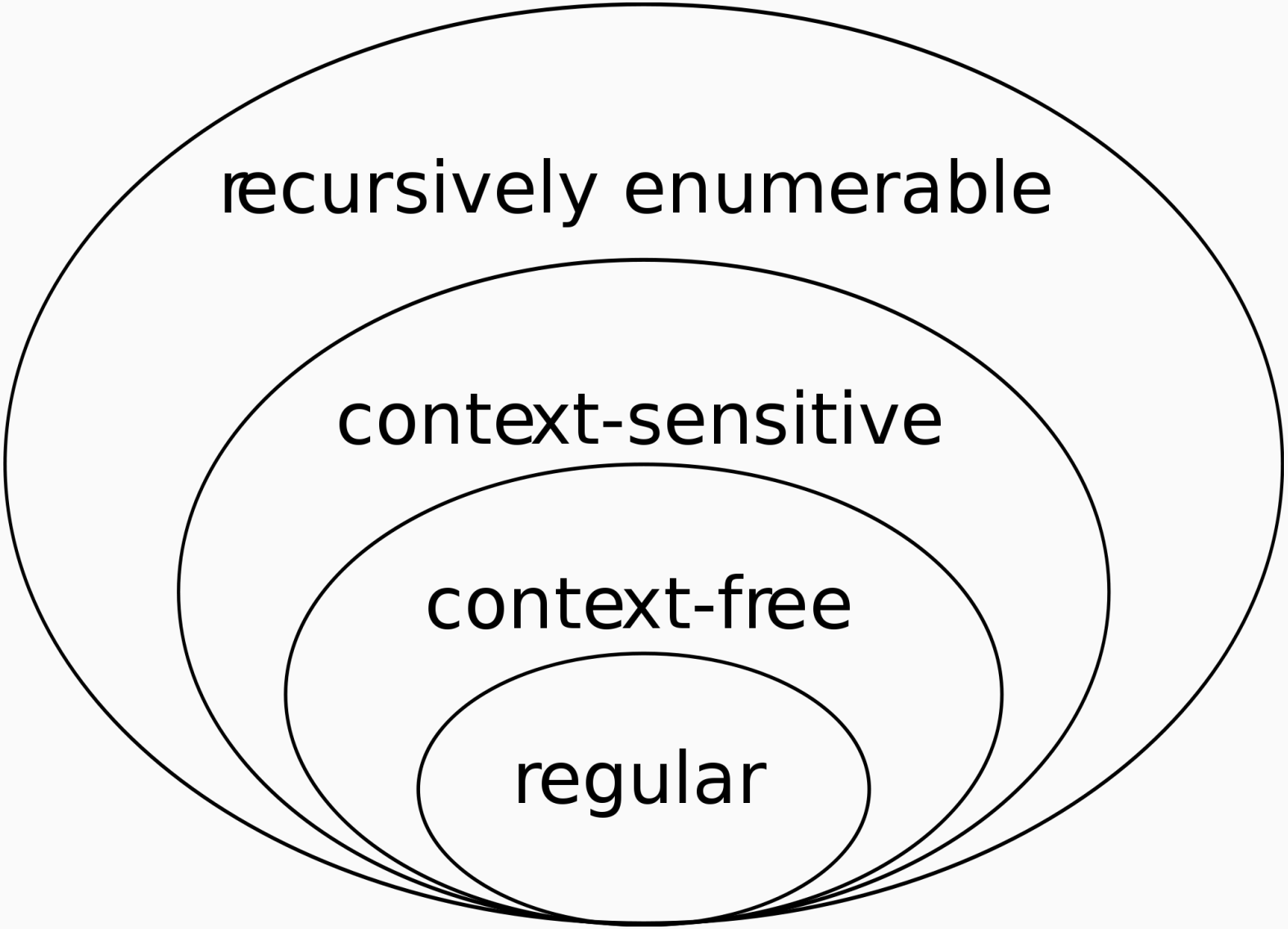
REGULAR GRAMMAR

X→aX
X→bY
Y→bY
Y→b

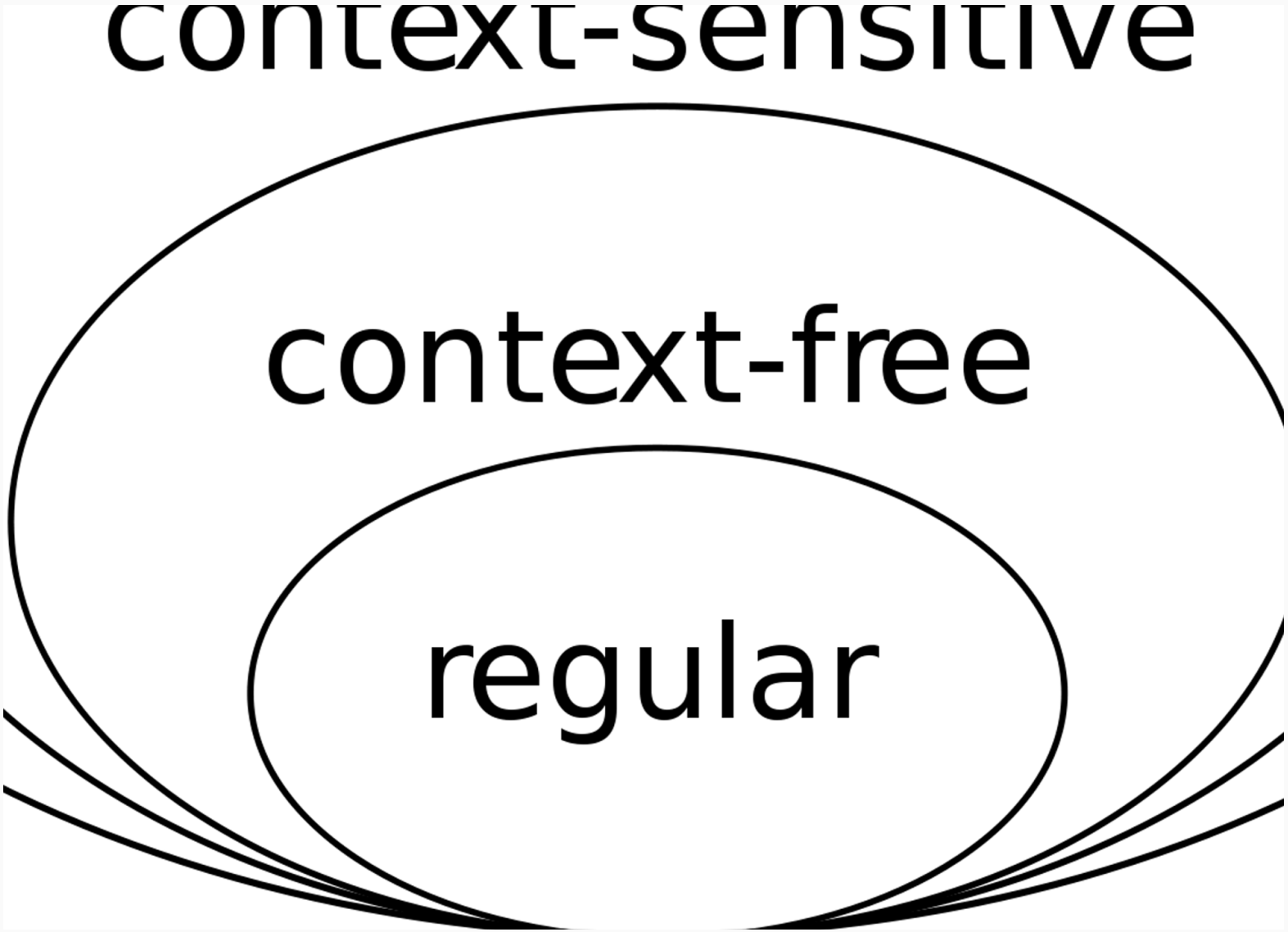
What is a FSM?


REGEX
{a+b+}
FSA
s
x
y
a
a
b
b
REGULAR GRAMMAR
X→aX
X→bY
Y→bY
Y→b
REGULAR LANGUAGES

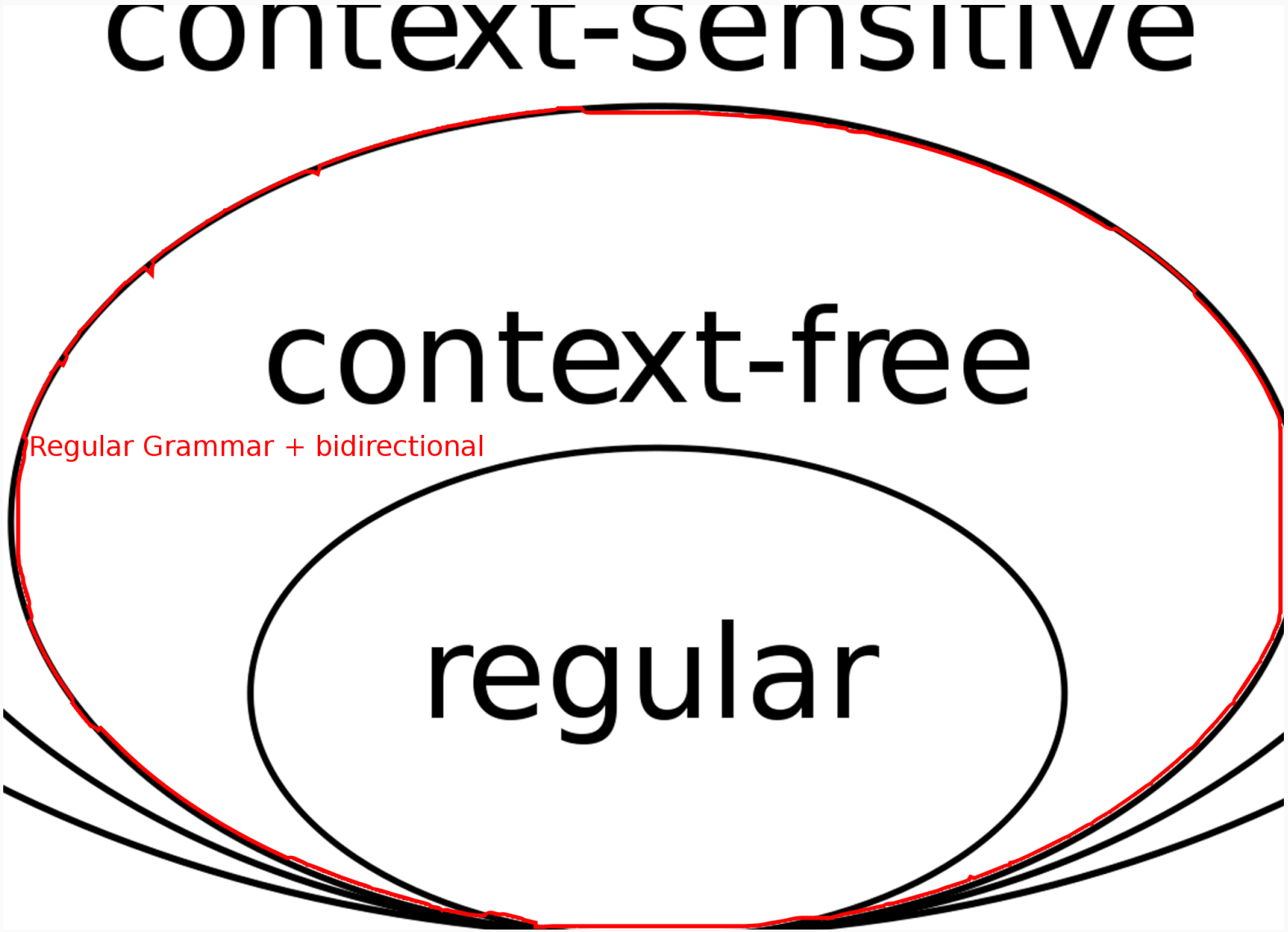
Chomsky Hierarchy



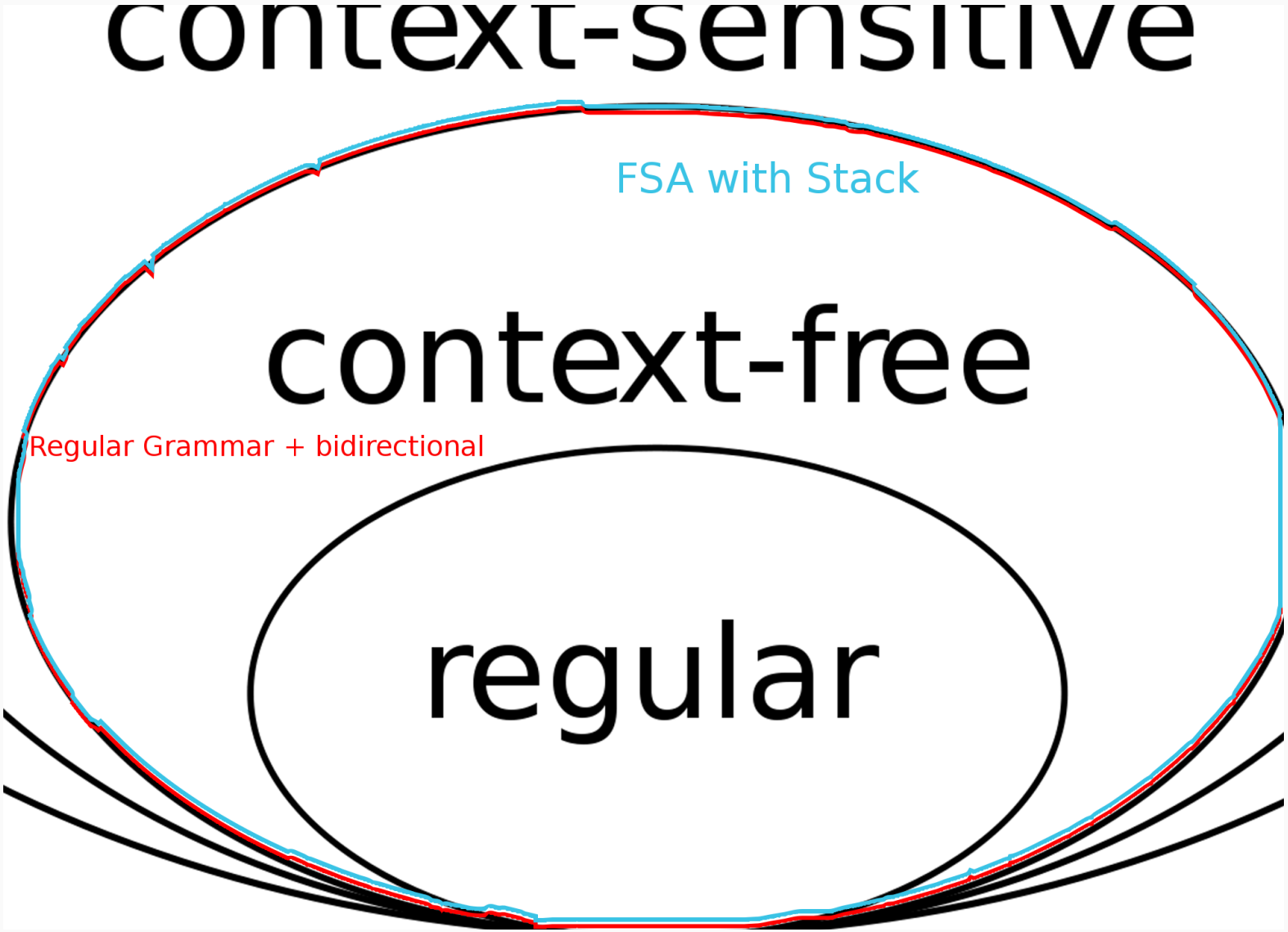
Chomsky Hierarchy



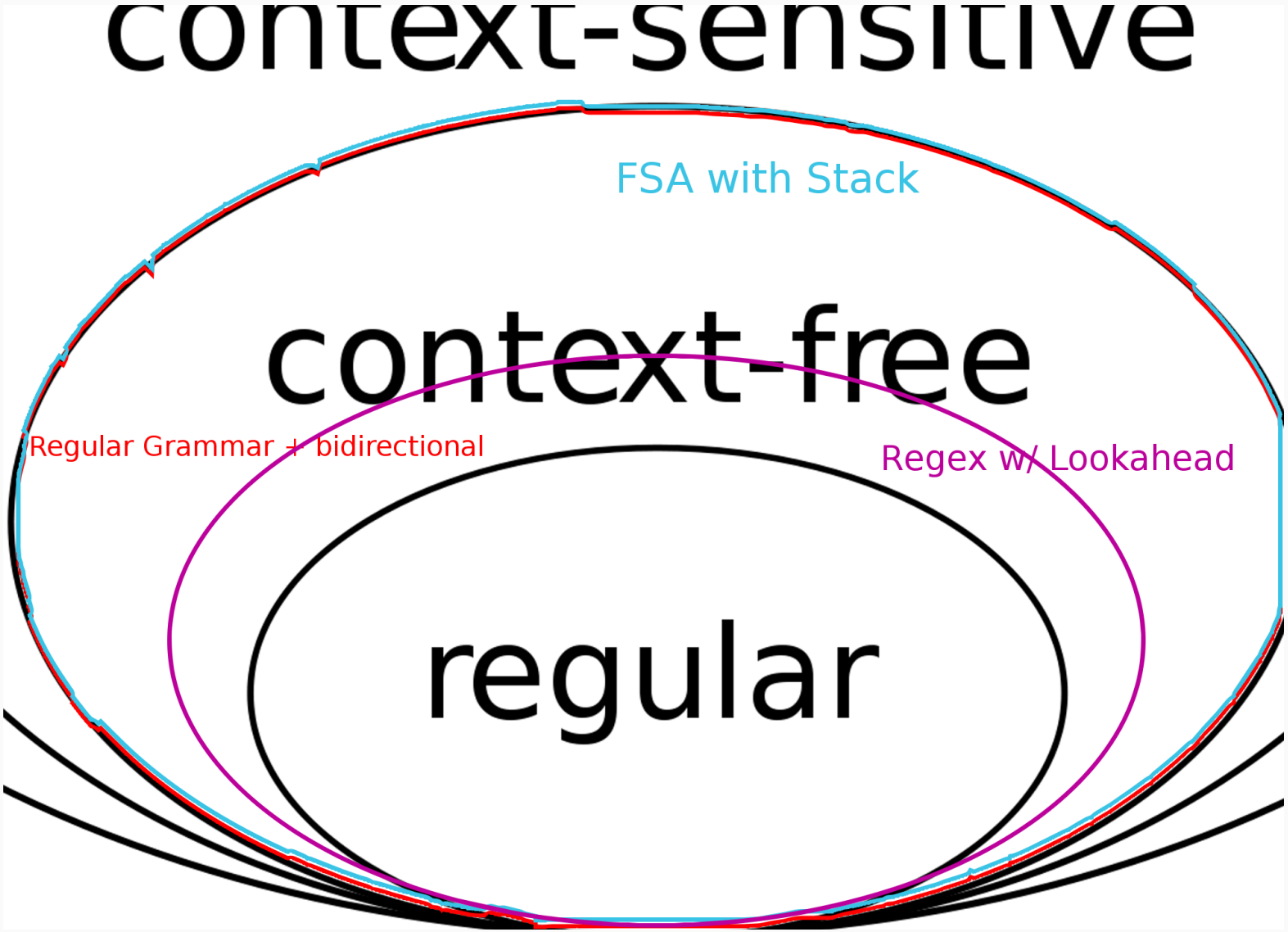
Chomsky Hierarchy



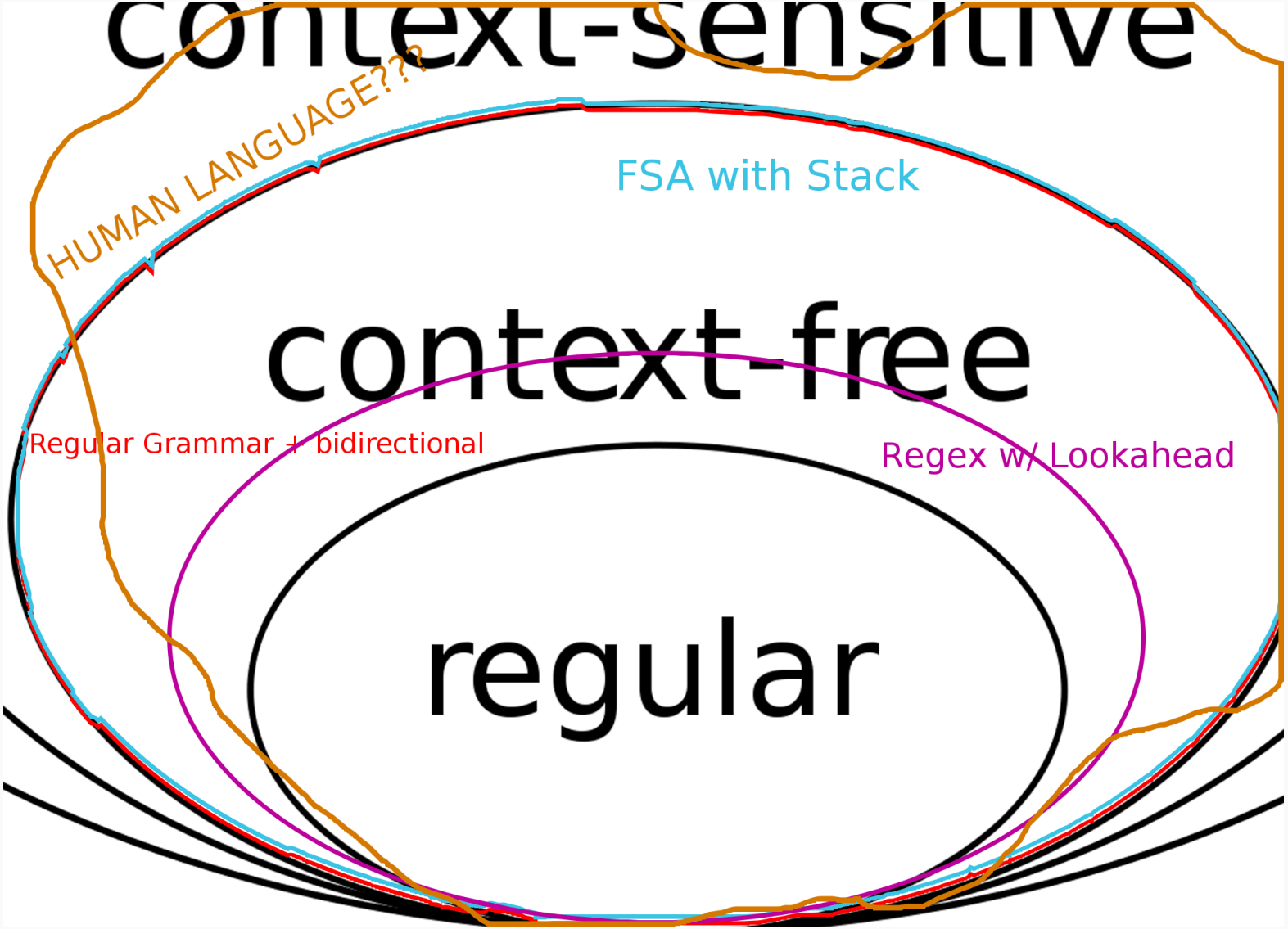
Chomsky Hierarchy



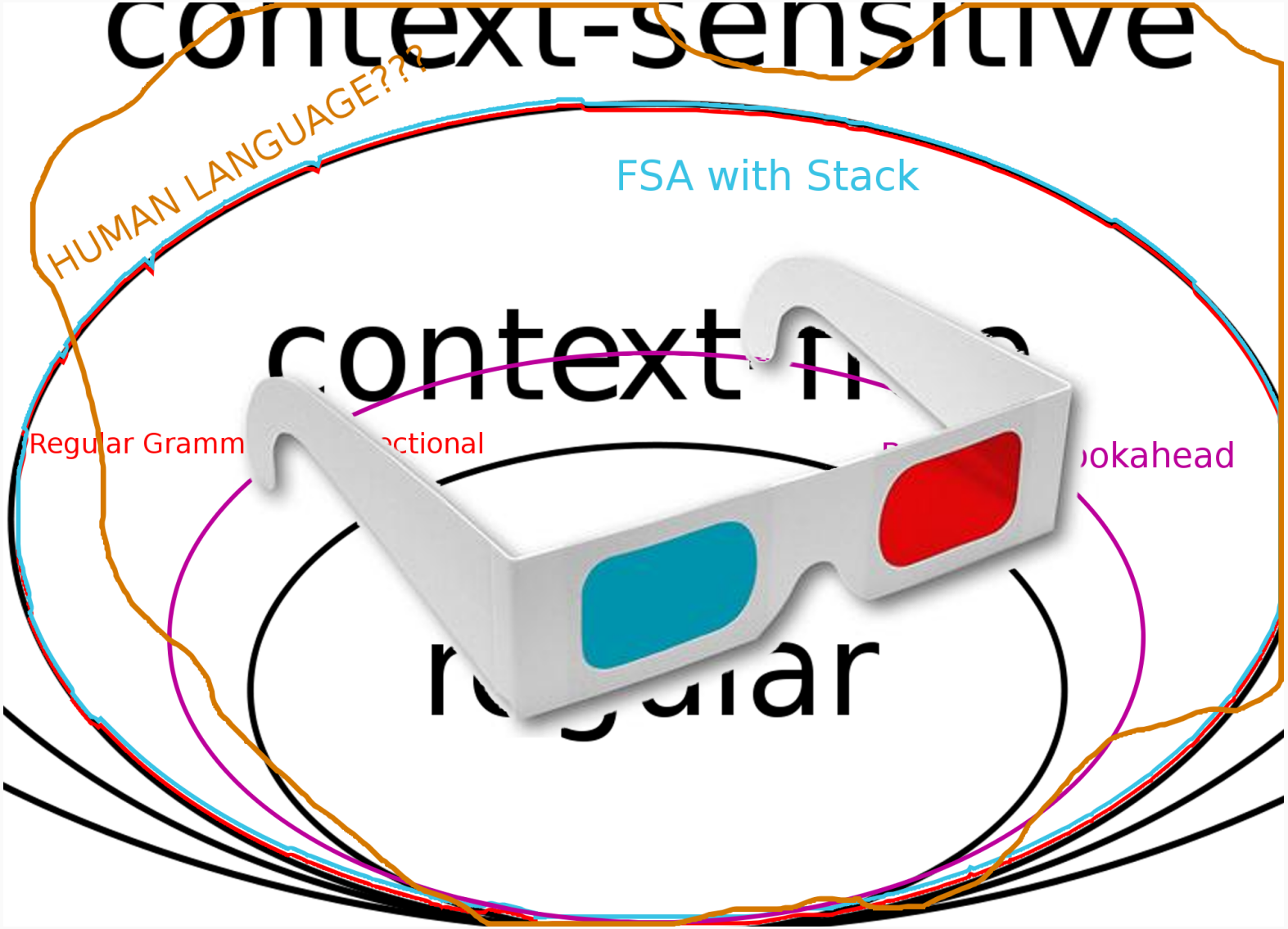
Chomsky Hierarchy



Chomsky Hierarchy



Chomsky Hierarchy



FST

A finite state machine that turns one regular language into another regular language.

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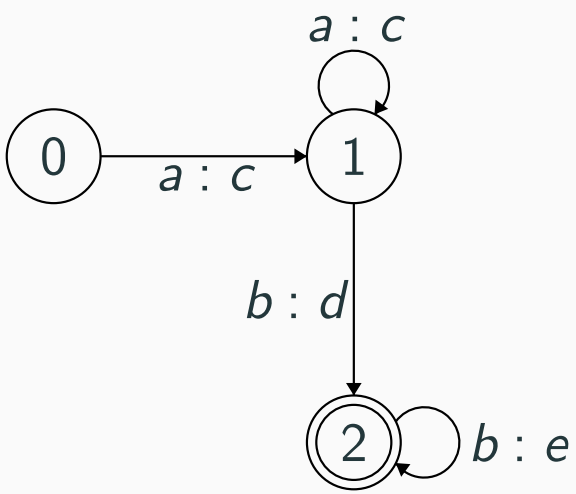

FST

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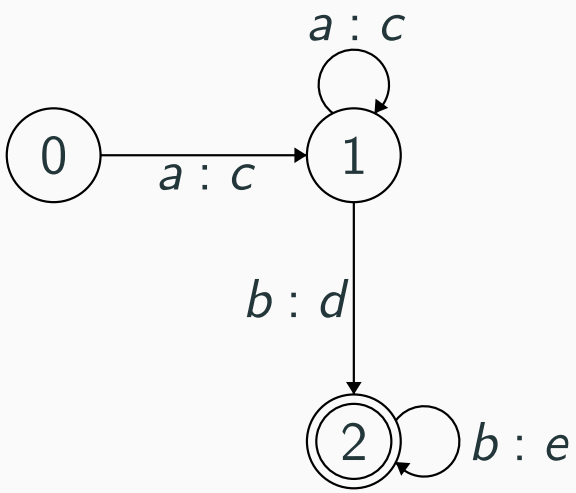

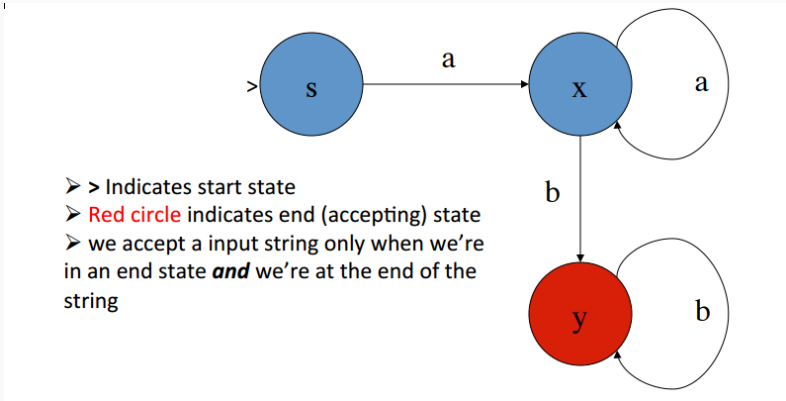

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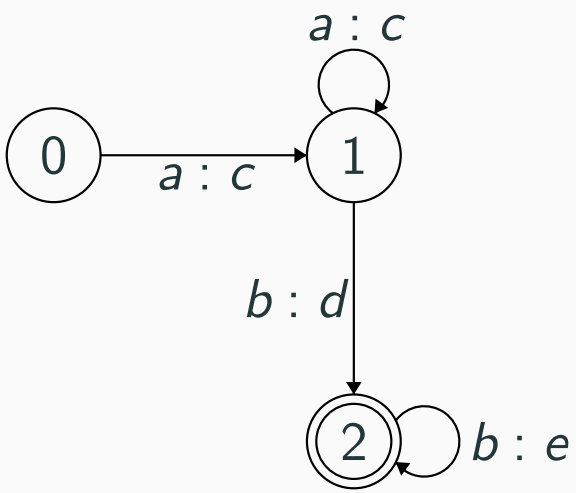

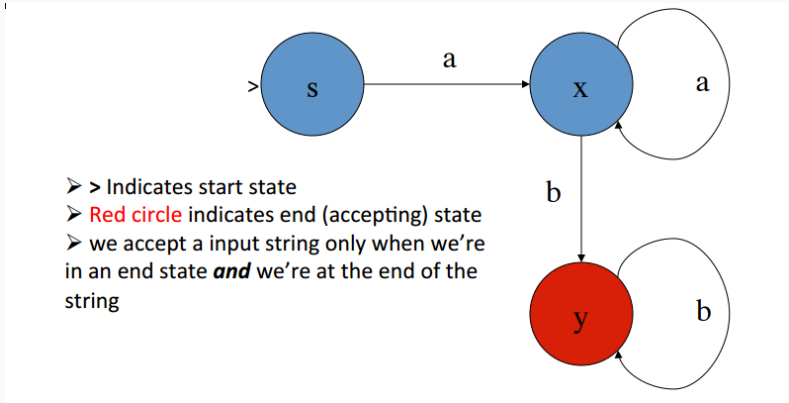

FST that transforms $\{a + b+\}$ to $\{c + de*\}$

Formal Definition

Formal Definition

$\langle \Sigma, S, s_0, \delta, F \rangle$

Σ alphabet

S states

$s_0 \in S$ start state

δ transition function
$\delta : S, \Sigma \to S$
(Can be partial or total)

$\mathbf{F} \subset S$ final states
(can be $\emptyset$)

Uses in Computational Linguistics

Morphology!

There are dictionaries of derived forms of words, but these can never be fully complete.

Cortois and Laporte (1991) demonstrated that most terms found missing from LADL[1]s DELAS[2] by comparing against natural language text are derived forms that were missed.

[1] Labratorie d'Automatique Documentaire et Linguistique
[2] Dictionairie Electronique du LADL S

SDICOS, the morphological transducer

Problem:

Present in DICOS	Missing from DICOS
activier	activable
	activabilite
	suractivation
	suractivable

Attempts to add these to the dictionary inevitably miss many and dramatically increase the dictionary size and reduce search speed without much benefit.

Solution: FST

Rather than storing a word list, make a transducer that changes an input word to output dictionary information.

Solution: FST

- inculperont $\Longleftrightarrow$ inculper, V+f3p
- aimerant $\Longleftrightarrow$ aimer, V+f3p
- chevaux $\Longleftrightarrow$ cheval, N+mp
- carnavals $\Longleftrightarrow$ carnaval, N+mp
- pommes $\Longleftrightarrow$ pomme, N+fp
- pommes $\Longleftrightarrow$ pommer, V+P2s
- pommes $\Longleftrightarrow$ pommer, V+S2s

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(disambiguation problem: we don't care)

SDICOF

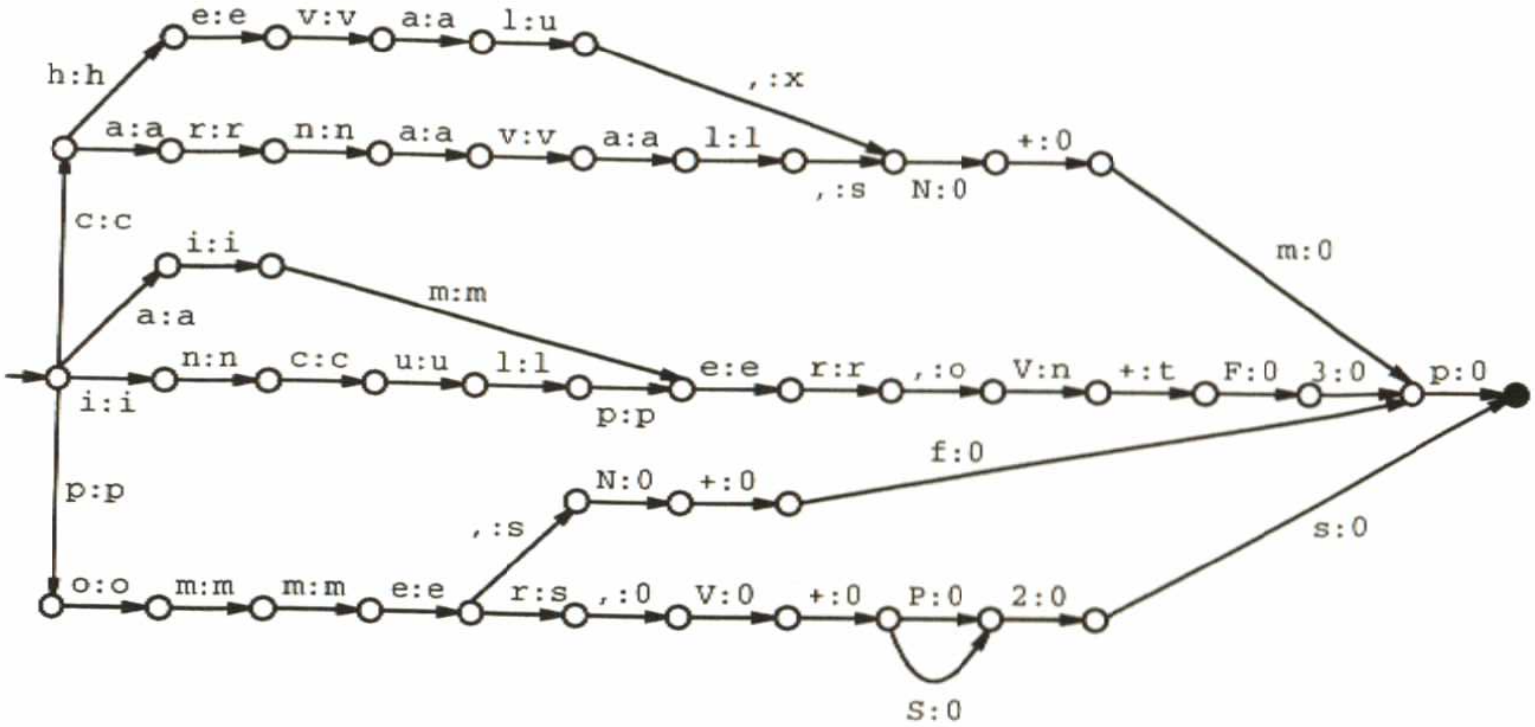

Figure 2.4: A sample of the DICOF transducer

Performance

	NUMBER OF ENTRIES	SIZE (Ko)	SPEED (words/second)
DICOF TRANSDUCER	612,848	948	1,100
NEW DICOF TRANSDUCER	1,301,976	1,014	1,100

Issues

Not very convenient because you cannot link a derivitive in a text to the root of it's derivational tree.

We need a system that given a derivitive returns the trees that contain it.

New FST

- C1 + activier + Vable.32RA/S $\Longleftrightarrow$ coactivable
- I3 + C1 + activier + Vable.32RA/S $\Longleftrightarrow$ incoactivable
- O1 + activier + Vable.32RA/S $\Longleftrightarrow$ coactivable
- I3 + O1 + activier + Vable32RA/S $\Longleftrightarrow$ incoactivable

New FST

- C1 + activier + Vable.32RA/S $\iff$ coactivable
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- O1 + activier + Vable.32RA/S $\iff$ coactivable
- I3 + O1 + activier + Vable32RA/S $\iff$ incoactivable

(C = symmetry of subjects, O = symmetry of objects, eg

"IBM and Microsoft codeveloped OS/2" vs "The alarm and the lock are always coactivated")

SDICOS

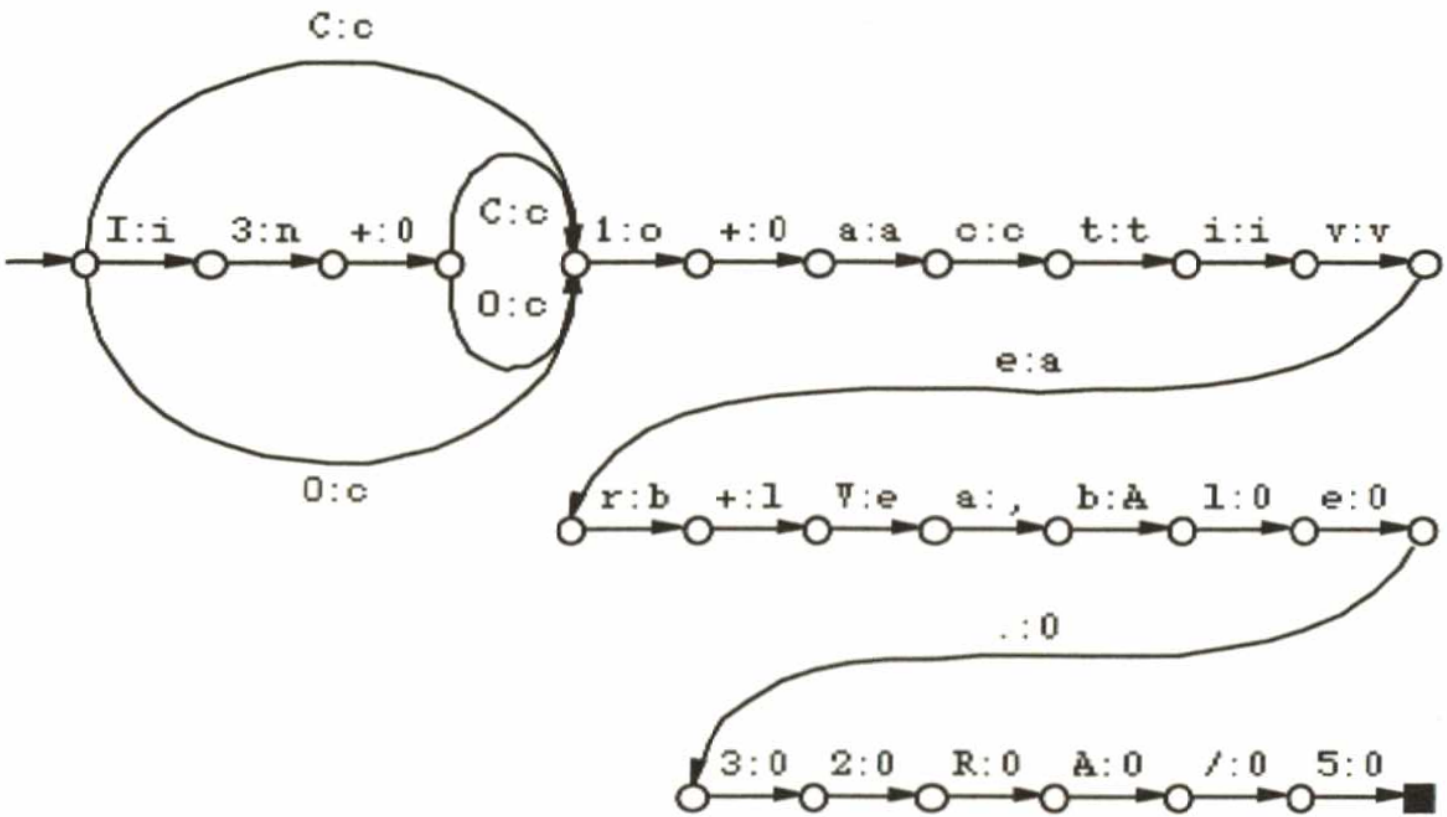

Figure 2.5: SDICOS, the Structured DICOS transducer

SDICOS

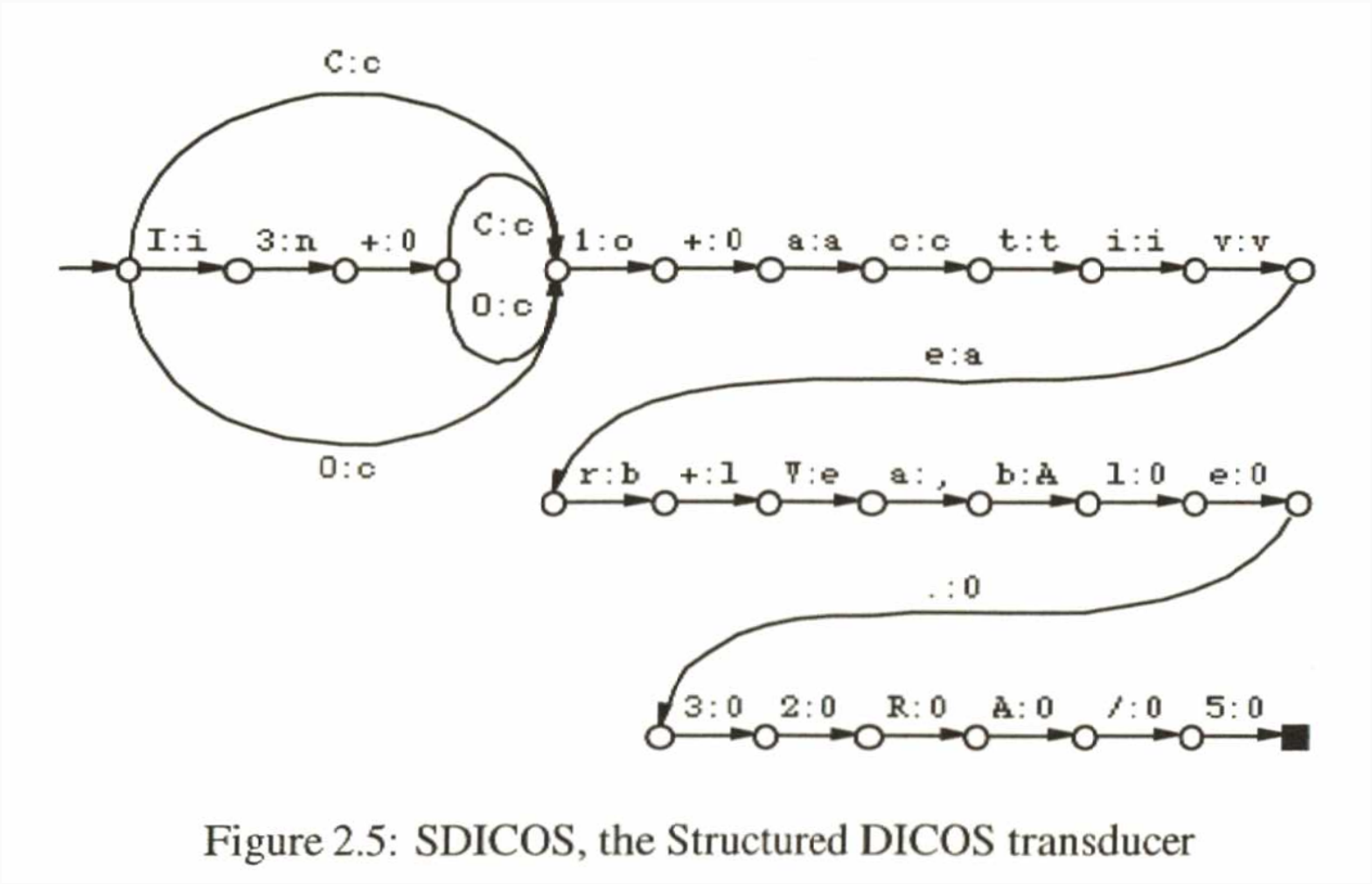

Figure 2.5: SDICOS, the Structured DICOS transducer

Not as fast (519 w/s), significantly smaller memory imprint (97Kb)

More Generic

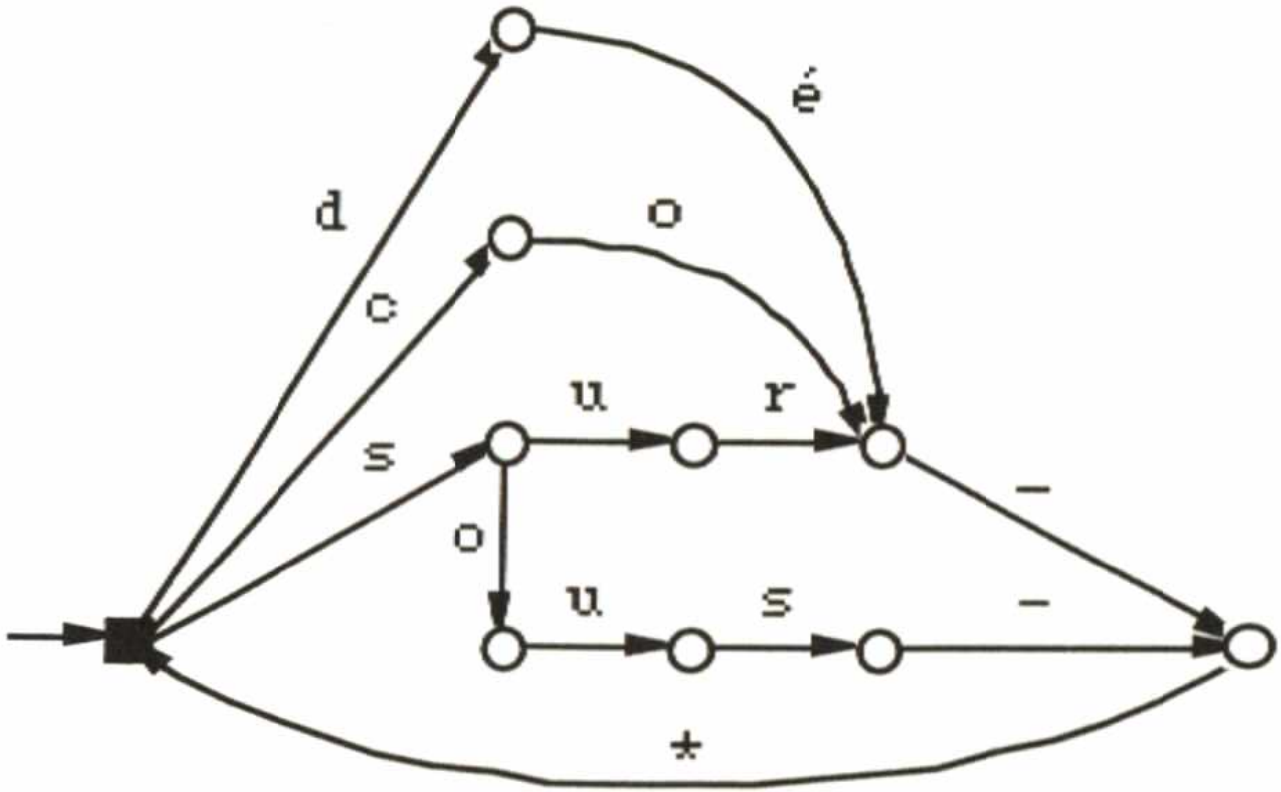

Figure 2.6: A sample of PREF(A)

More Generic

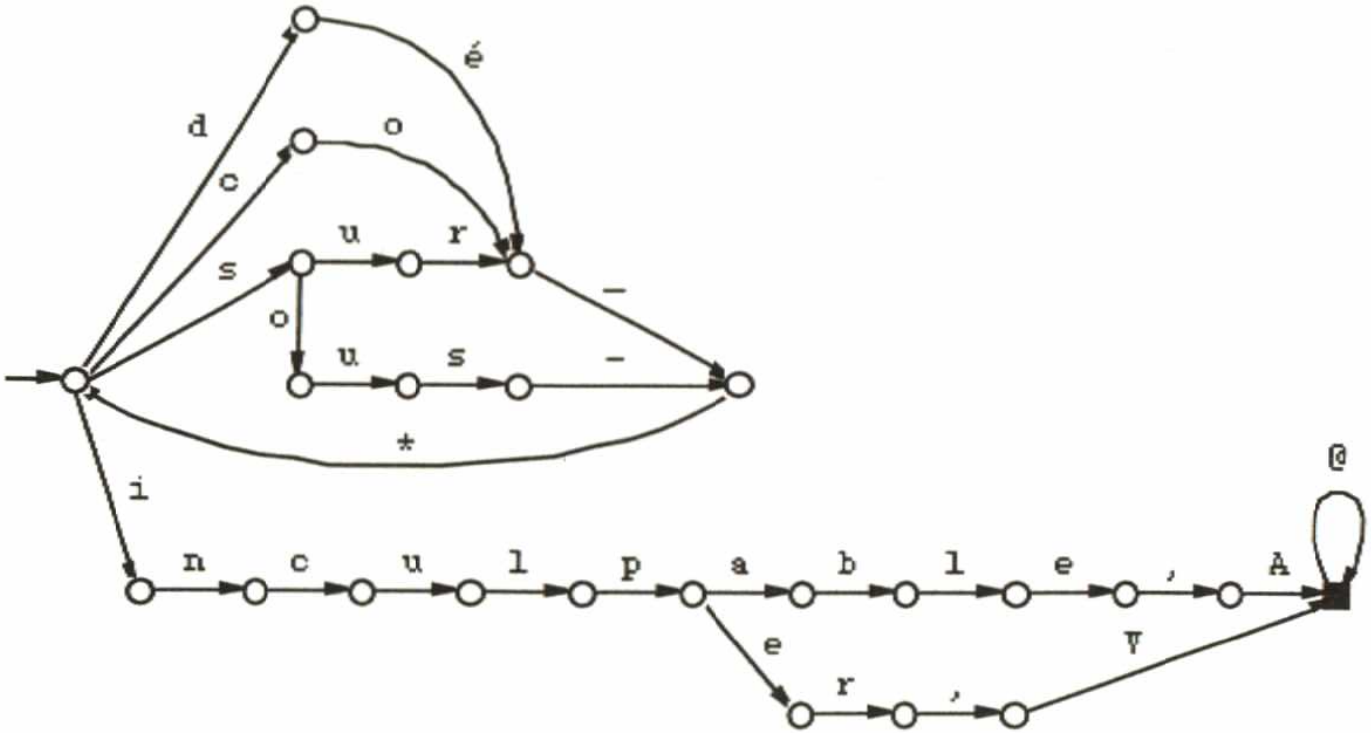

Figure 2.7: A sample of PREF(A) • DICOS(A)

More Generic

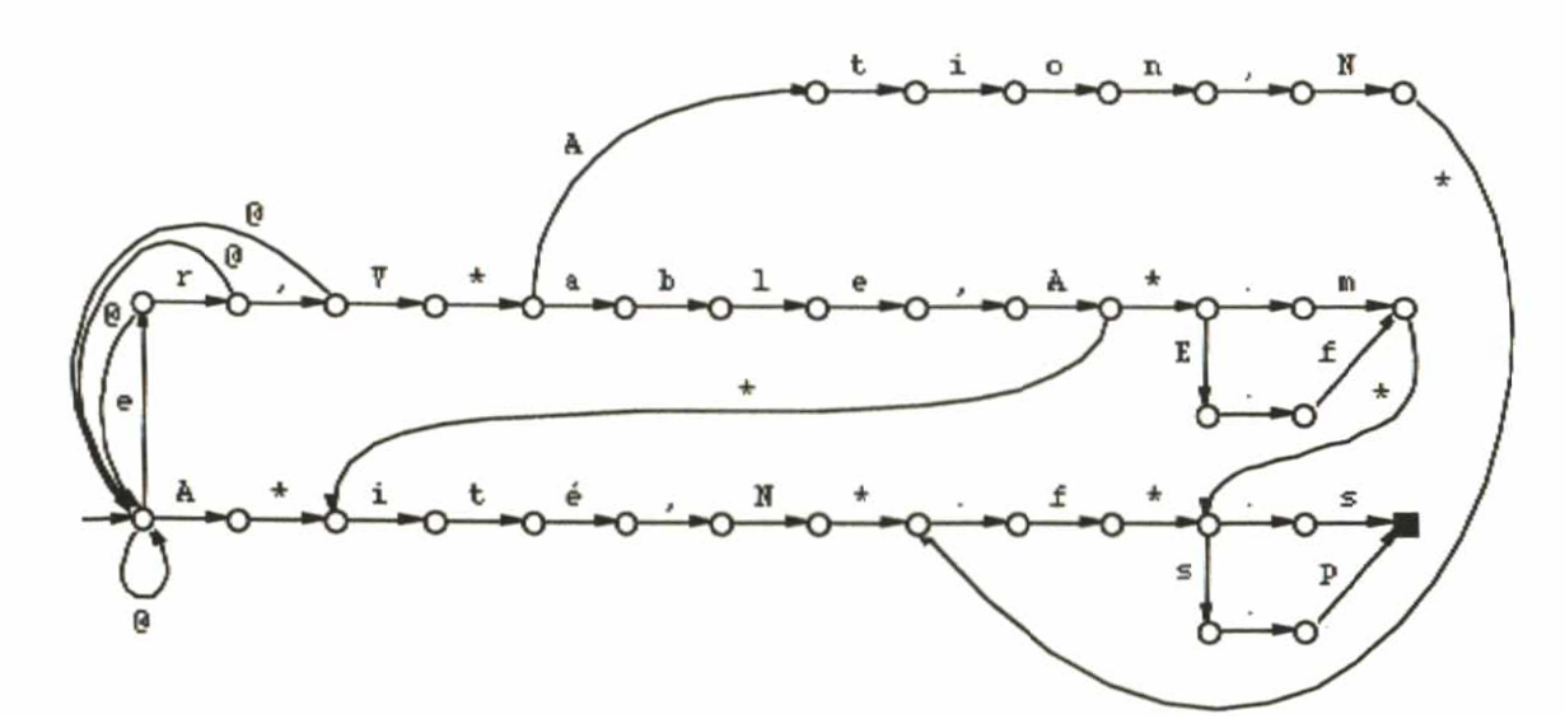

Figure 2.8: A sample of SUF(A)

More Generic

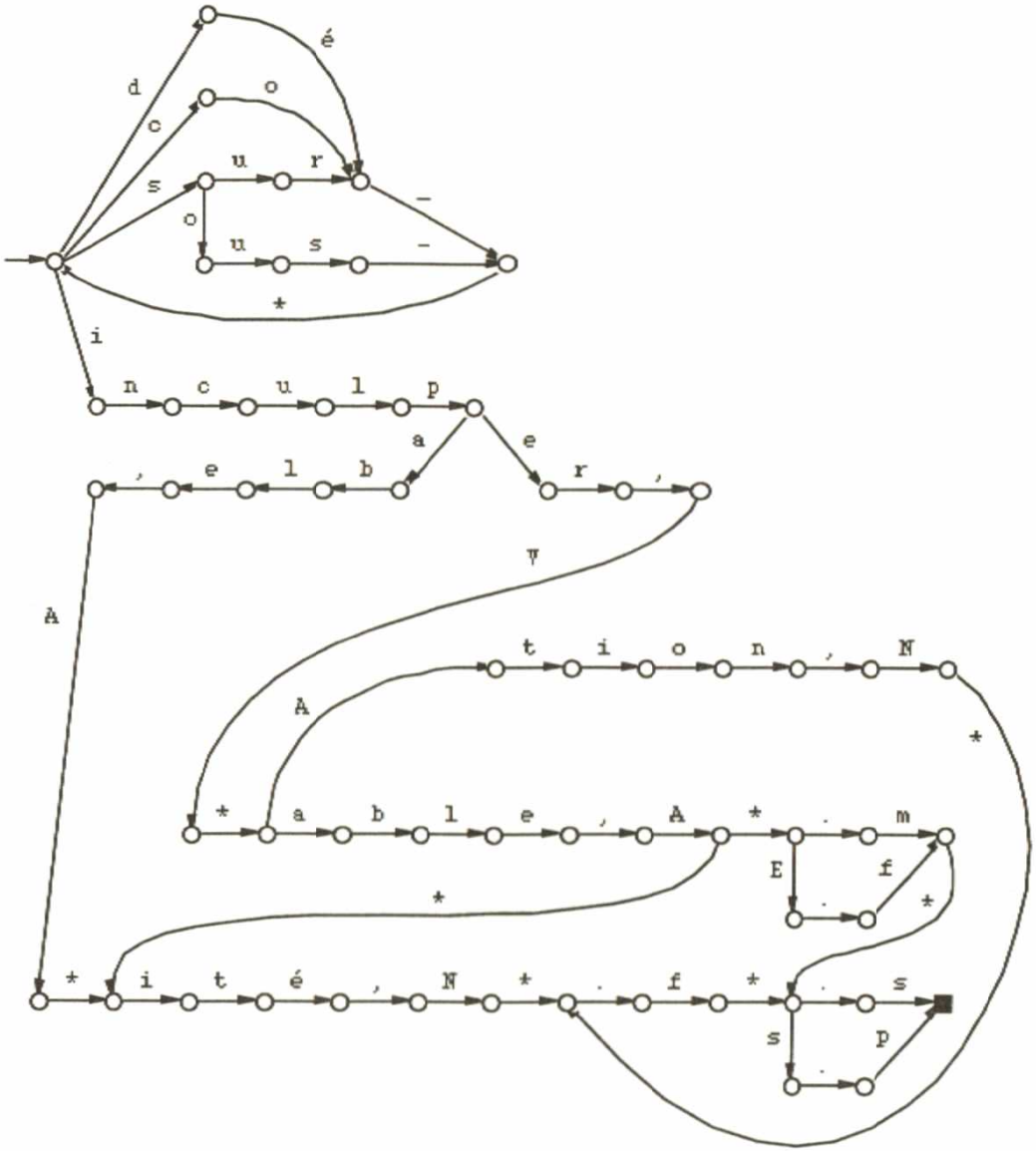

Figure 2.9: A sample of (PREF(A) • DICOS(A)) ∩ SUF(A)

Parsing?!

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The Earley algorithm converts a deterministic CFG into a FSA by using dot notation (LR(0), LR(1))

This technique involves using FSMs to model grammars by combining many grammar rules into finite states.

Advantages

- Compact

Reconceptualize parsing as a string transformation.

[3] Roche 1993

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Reconceptualize parsing as a string transformation.

Parser: $\Sigma_w^* \implies 2^{(\Sigma_g^* \cdot \Sigma_w^* \cdot \Sigma_g^*)}$

"Bob left this morning" $\iff$ "(S(N Bob N)(V left (N this morning N)V)S)"

[3] Roche 1993

CFG

N *thinks that* S	S
N *kept* N	S
John	N
Peter	N
the book	N

Table 8.1: syntactic dictionary for *John thinks that Peter kept the book*

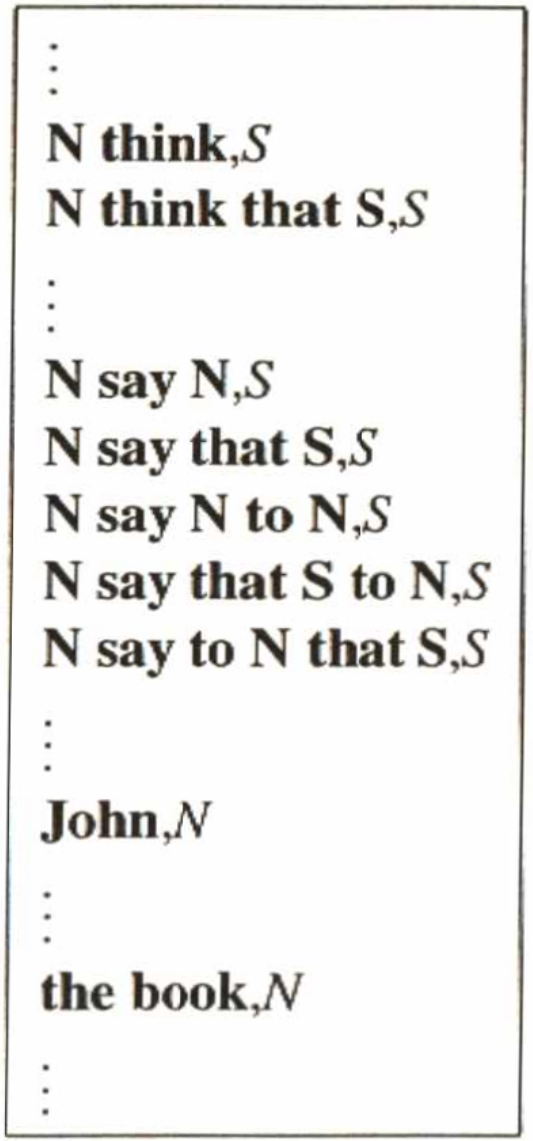


Figure 8.1: Sample of the Syntactic Dictionary

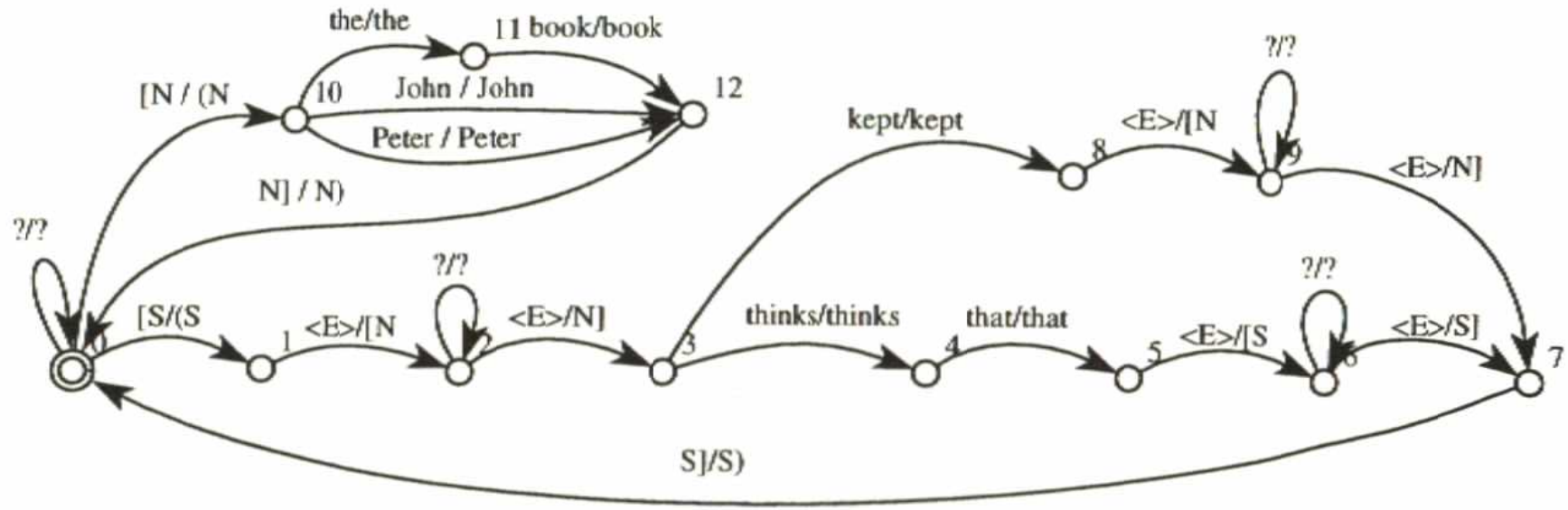


Figure 8.2: Transducer T_{dic_1} representing the syntactic dictionary dic_1

IFSP

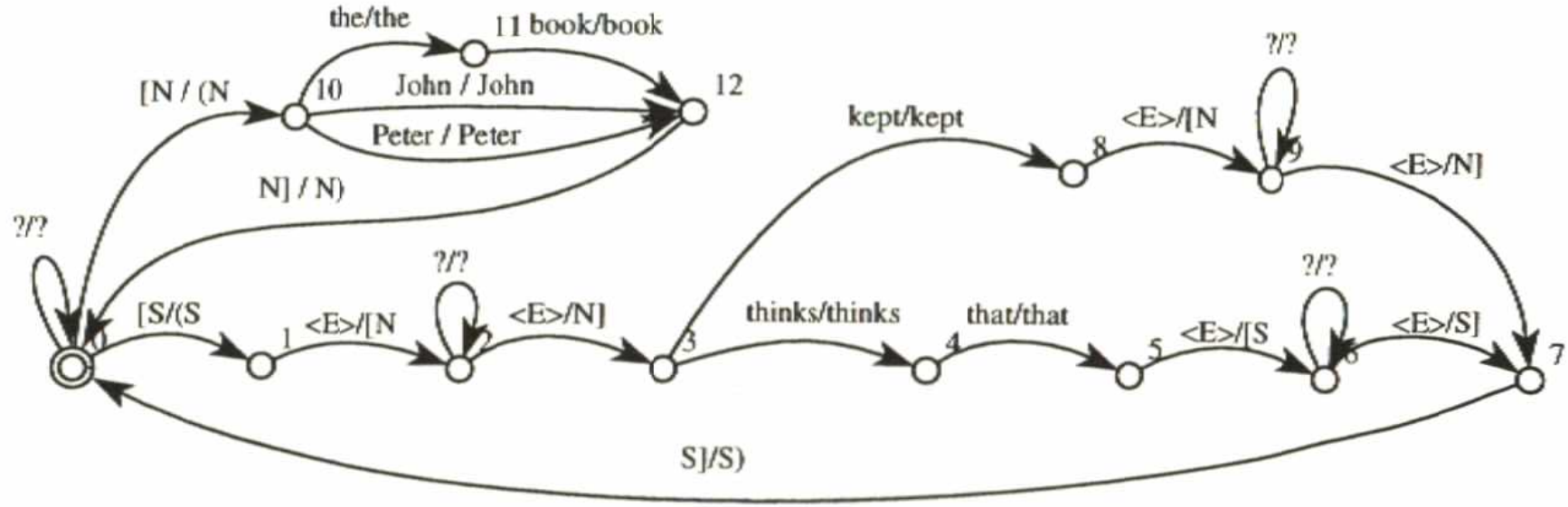


Figure 8.2: Transducer T_{dic_1} representing the syntactic dictionary dic_1

[S Mike thinks that Robert kept the book S]

IFSP

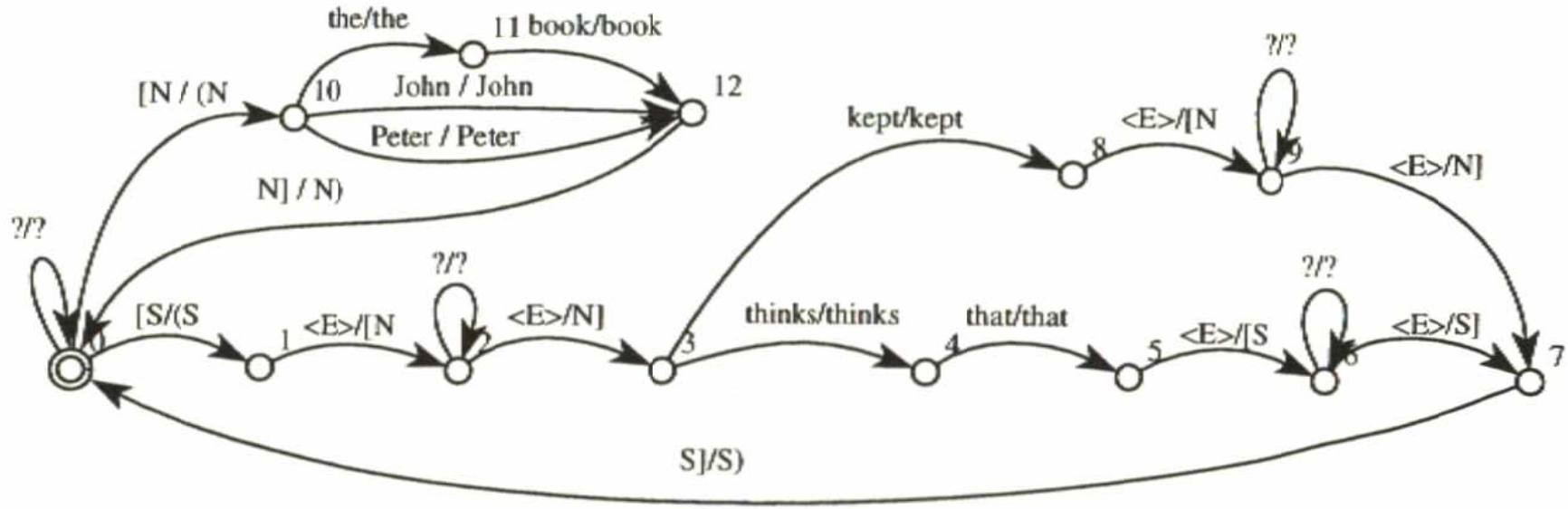
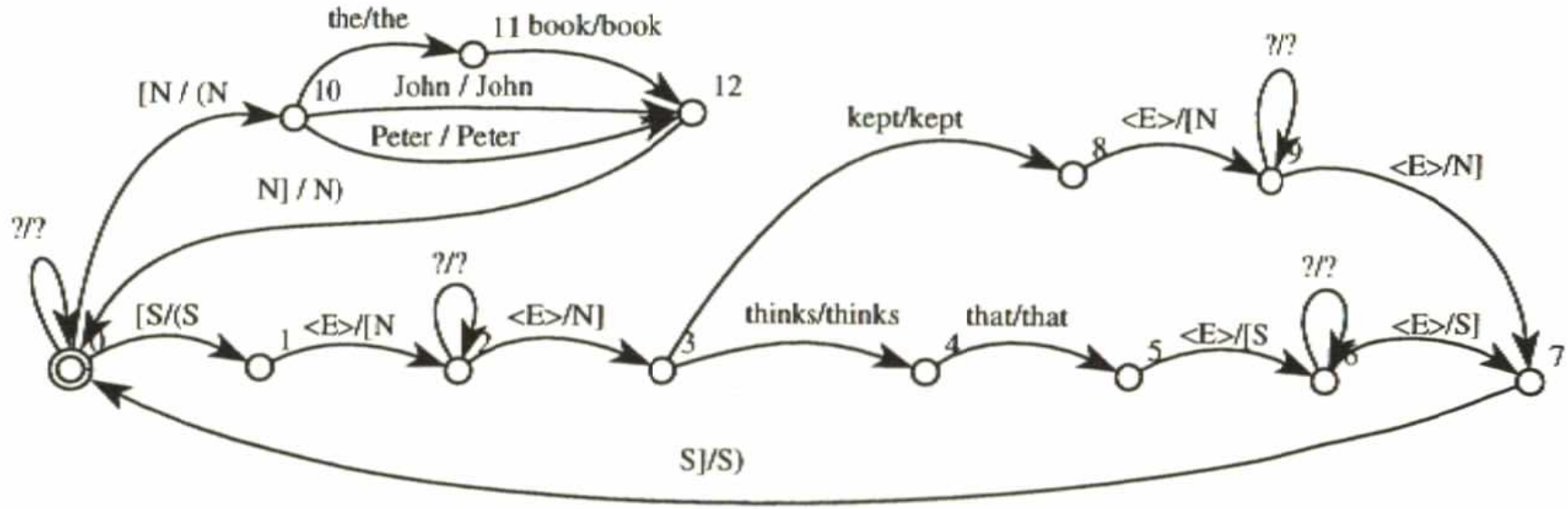


Figure 8.2: Transducer T_{dic_1} representing the syntactic dictionary dic_1

(S [N Mike thinks that Robert N] kept [S the book S]S)

(S [N Mike N] thinks that [S Robert kept the book S]S)

IFSP



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(S [N Mike N] thinks that [S Robert kept the book S]S)

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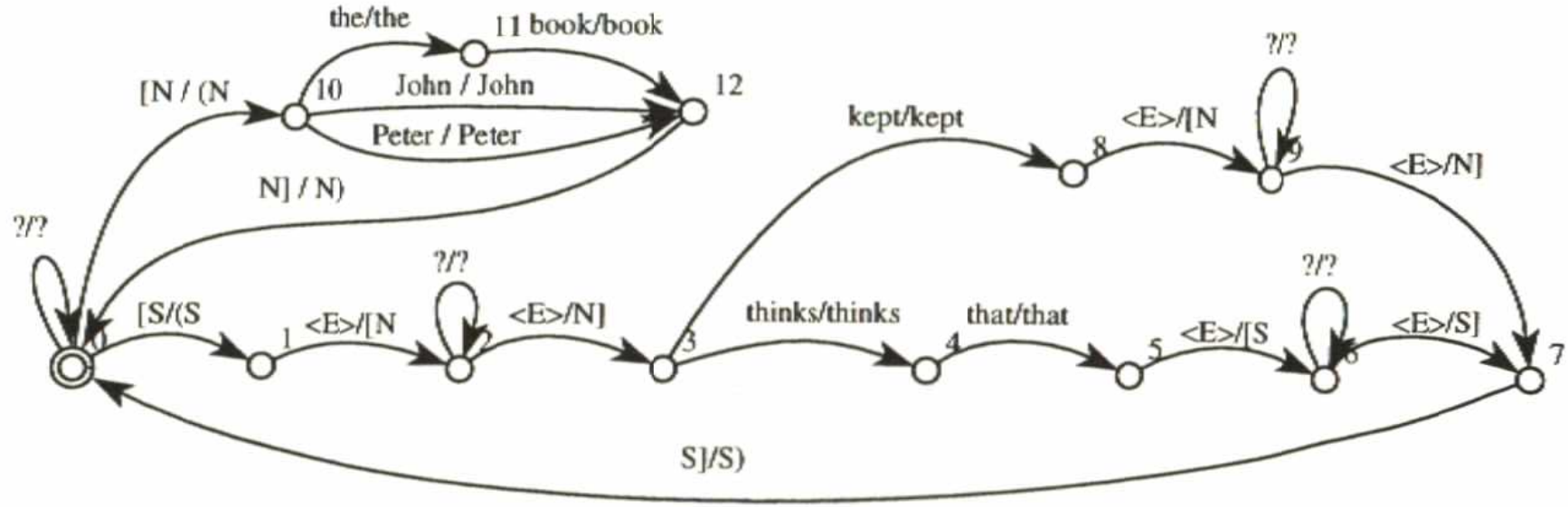

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(S (N Mike N) thinks that (S [N Robert N] kept [N the book N]S)S)

IFSP

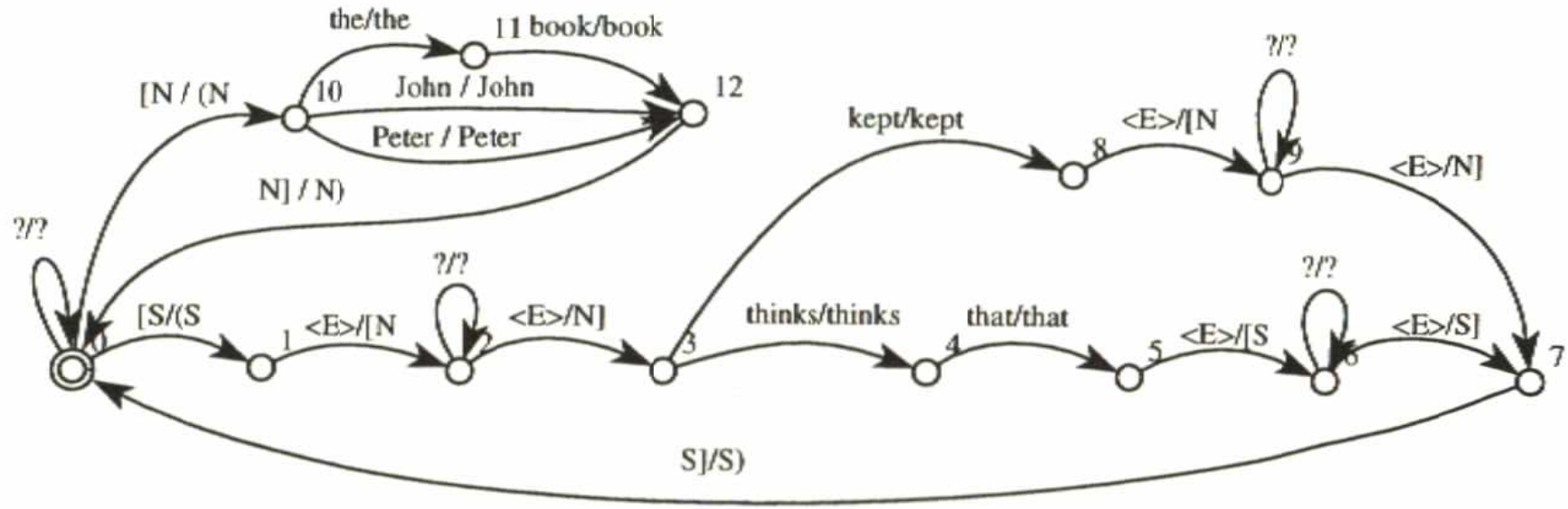


Figure 8.2: Transducer T_{dic_1} representing the syntactic dictionary dic_1

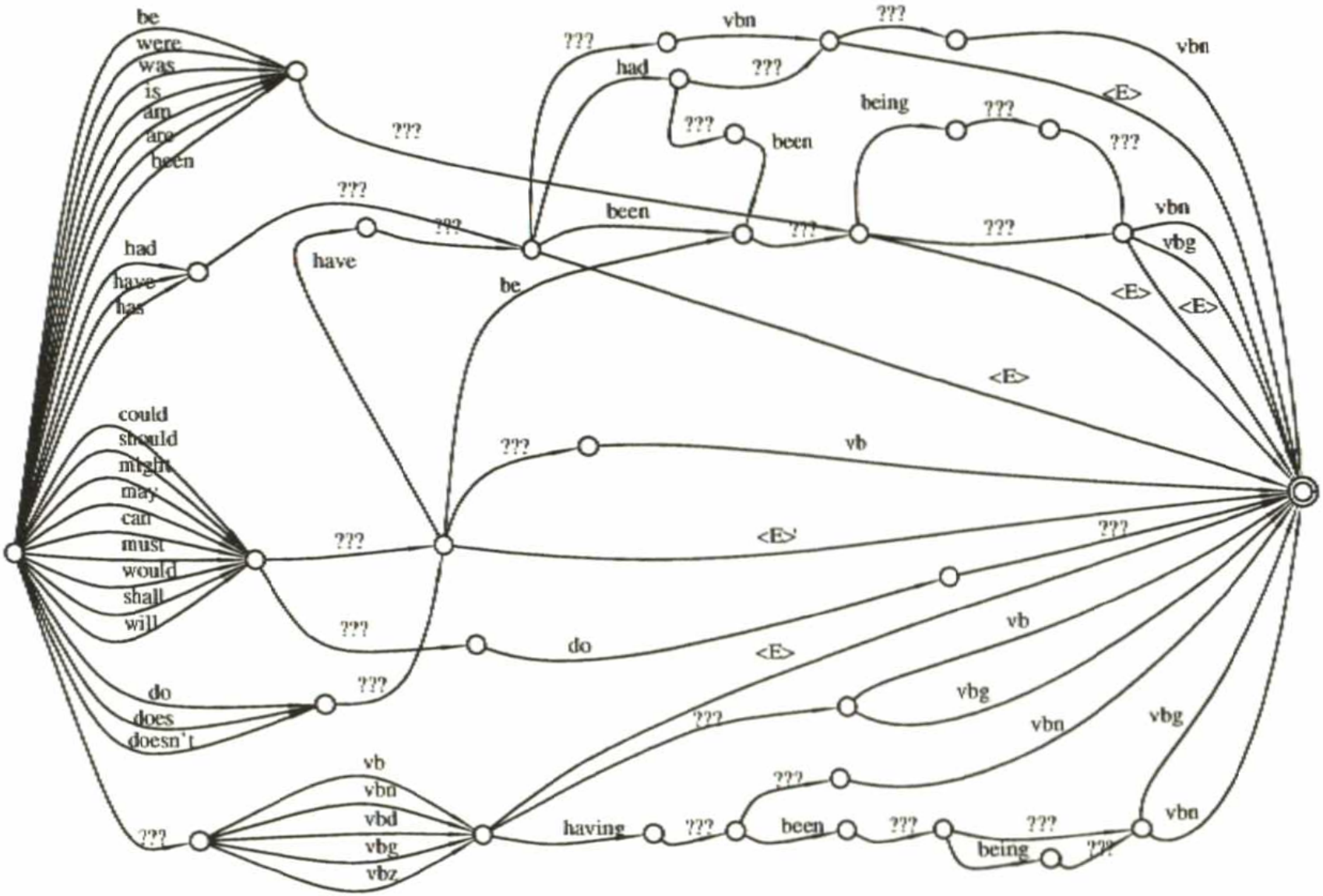
(S (N Mike N) thinks that (S (N Robert N) kept (N the book N)S)S)

Algorithm

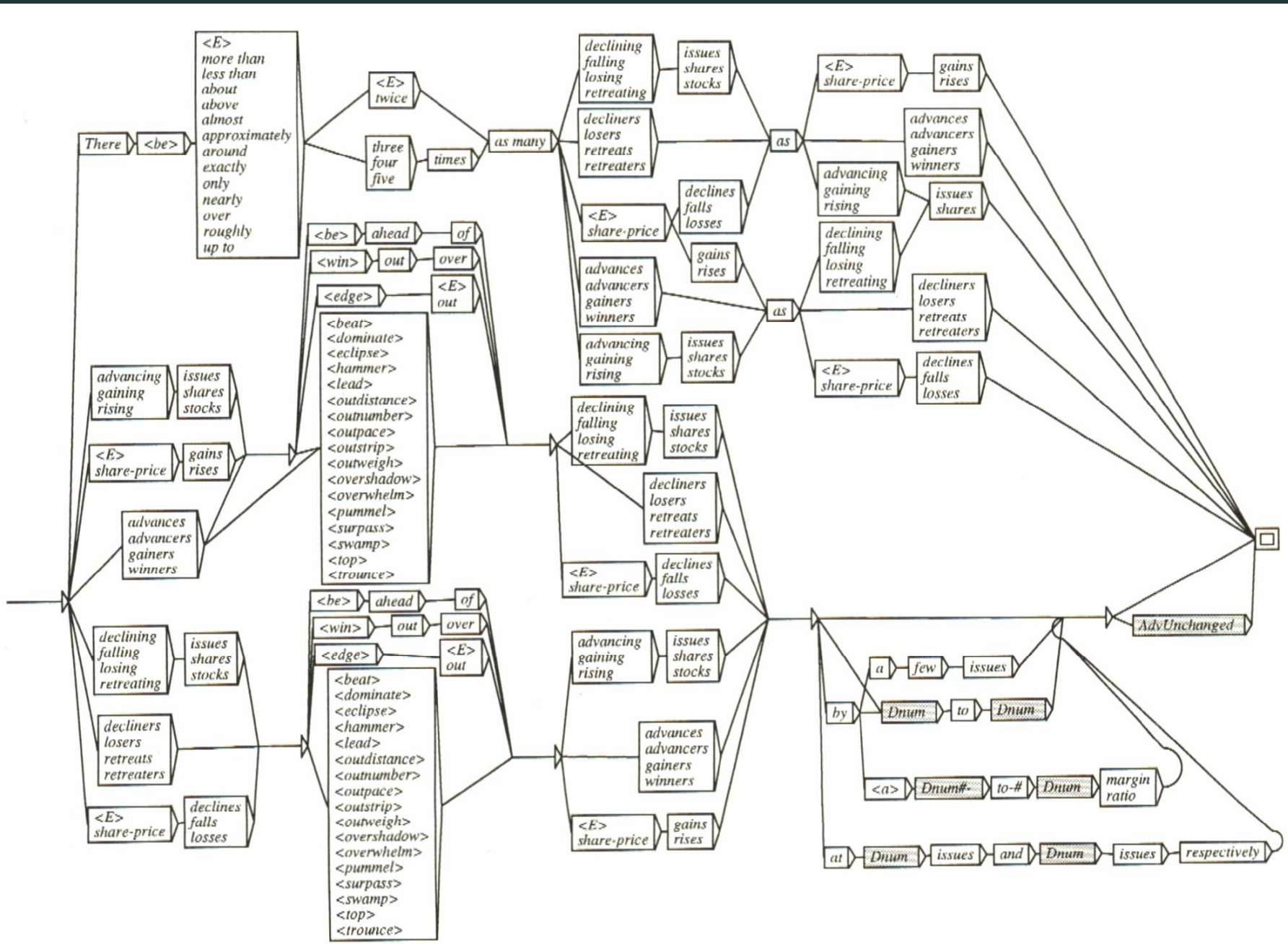
```
TransducerParse(T, sentence):
  sent1 = sentence

  while(sent2 = Apply_Transducer(T, sent1) != sent1):
    sent1 = sent2

  return sent1
```

note: I haven't worked out the math, but I'm pretty sure this algorithm actually has the full power of a turing machine and can describe any Chomsky type 0 language.

Auxilliary Verb Complex



MORE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falling
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issues
respectively

Great, but can I use it to do real work?

Uses in NLP (search)

Search

We have some data indexed with words (ie, natural language), and we want to find the correct stuff based on a query word.

Search

We have some data indexed with words (ie, natural language), and we want to find the correct stuff based on a query word.

How do??

Trie

Trie

- Directed (Deteriministic)

Trie

- Directed (Deteriministic)
- Acyclic

Trie

- Directed (Deteriministic)
- Acyclic
- Word

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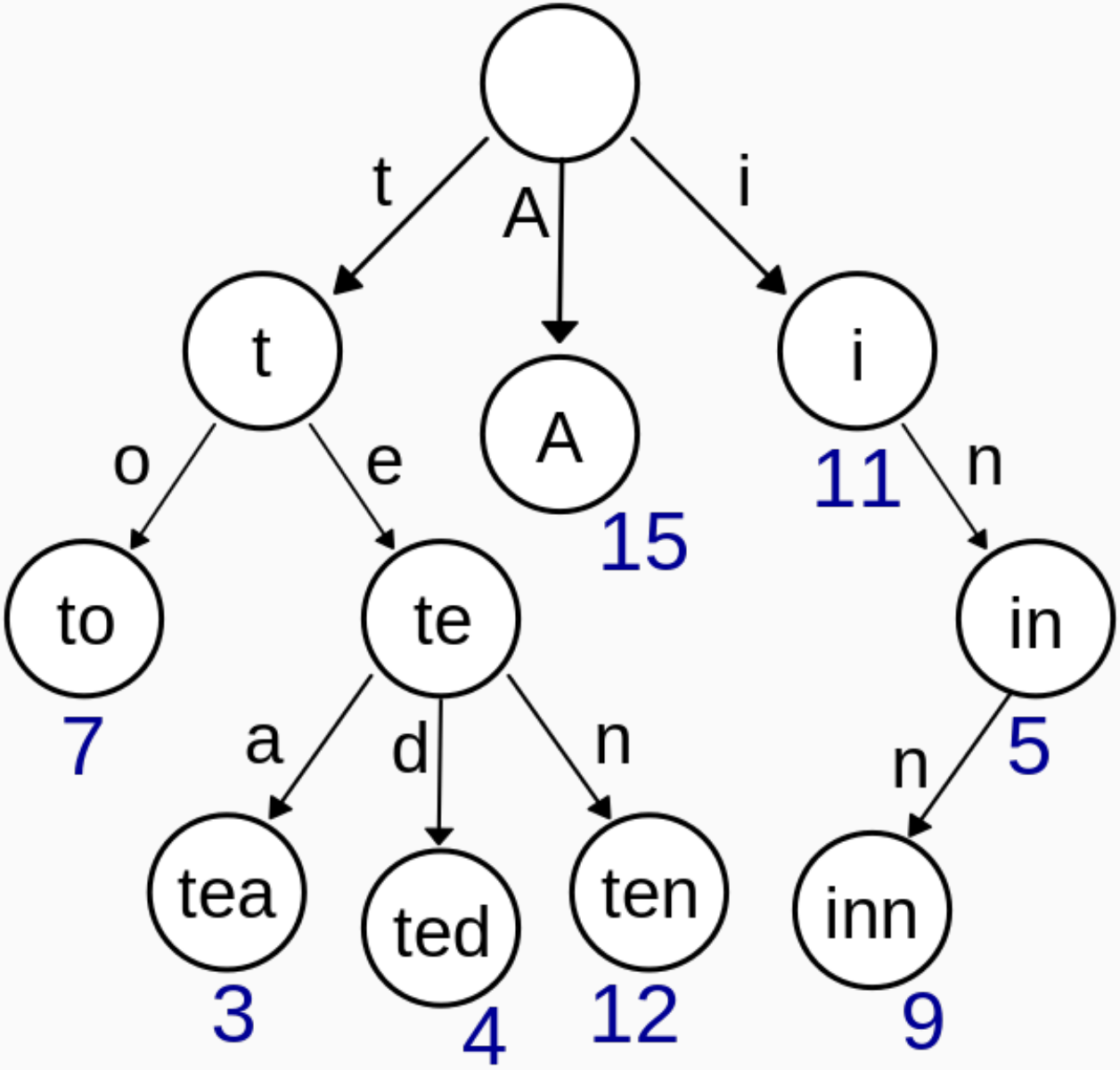
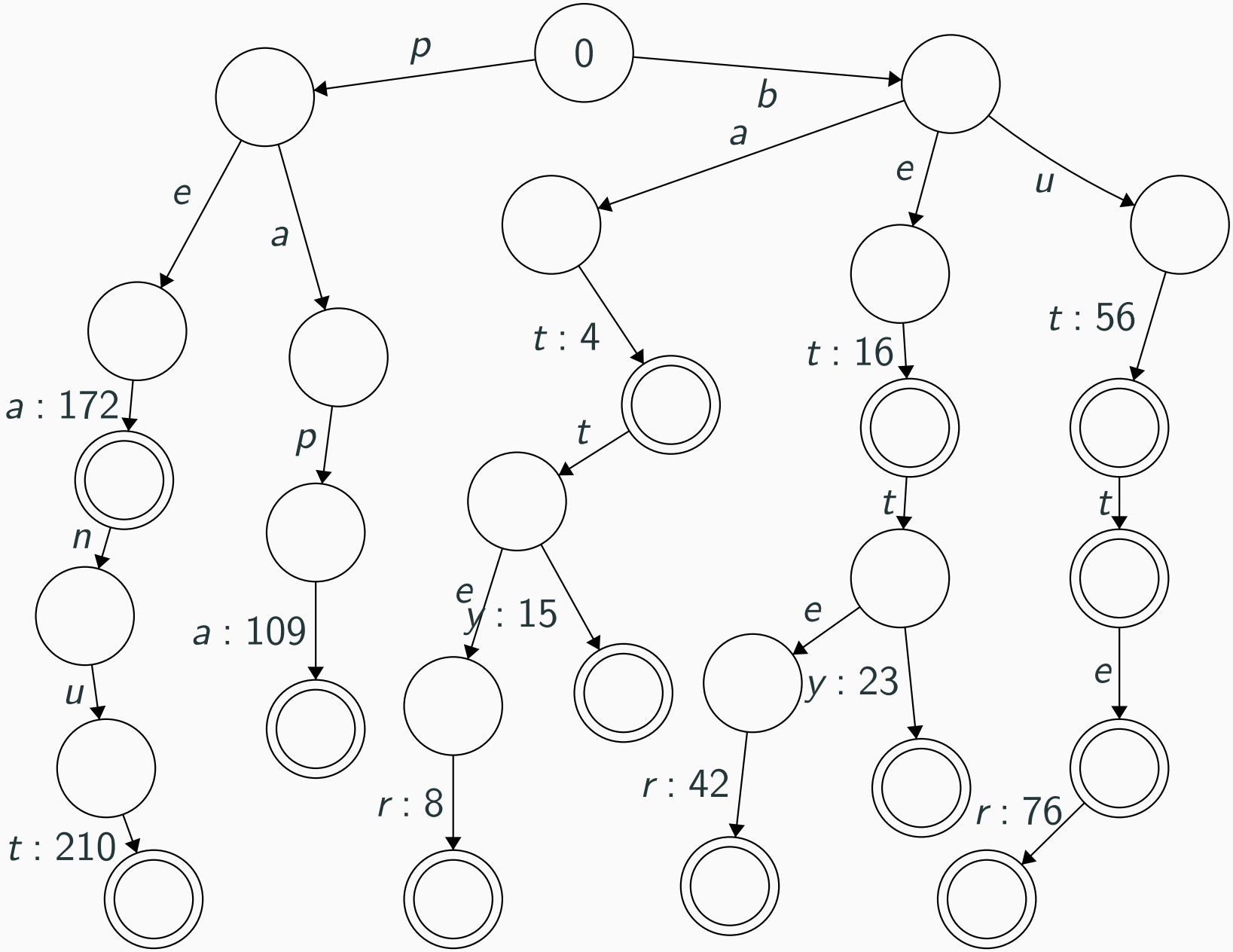
Trie/DAWG

A finite state transducer that contains a dictionary by storing words as paths through the FSM

Trie/DAWG



Trie/DAWG


0
p
b
e
a
a
e
u
t : 4
t : 16
t : 56
a : 172
p
t
t
t
n
e
y : 15
e
e
a : 109
y : 23
u
r : 8
r : 42
r : 76
t : 210

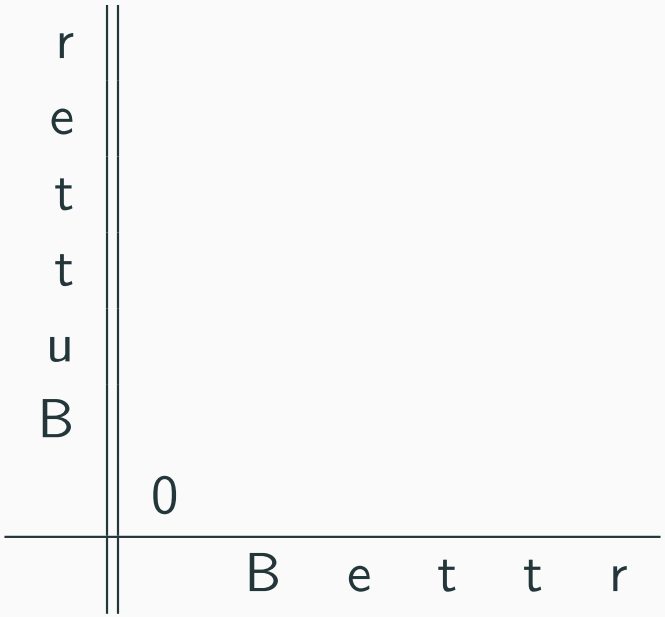
Cool, so what?

Cool, so what?

If only there was a way to take steps through the trie that are close to but not exactly the same as the query...

Levenshtein

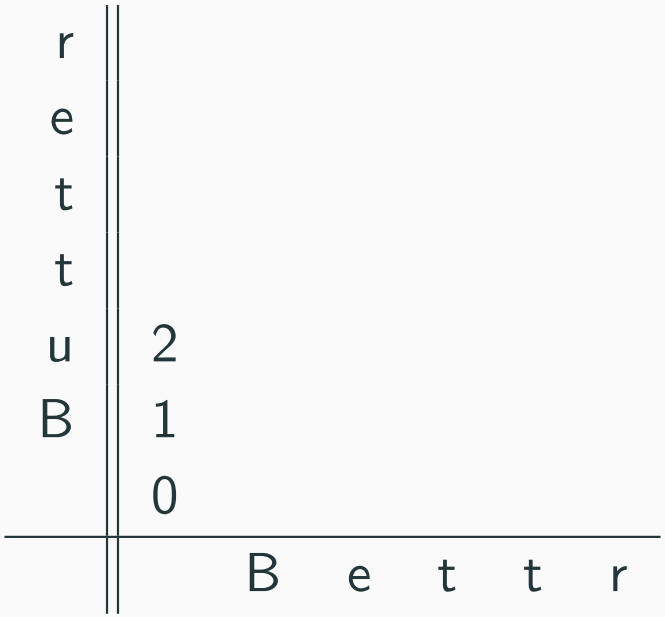
Levenshtein



Levenshtein

r						
e						
t						
t						
u						
B	1					
	0					
		B	e	t	t	r

Levenshtein



Levenshtein

r	6					
e	5					
t	4					
t	3					
u	2					
B	1					
	0					
		B	e	t	t	r

Levenshtein

r	6	5				
e	5	4				
t	4	3				
t	3	2				
u	2	1				
B	1	0				
	0	1				
		B	e	t	t	r

Levenshtein

r	6	5	4			
e	5	4	3			
t	4	3	3			
t	3	2	2			
u	2	1	1			
B	1	0	1			
	0	1	2			
		B	e	t	t	r

Levenshtein

r	6	5	4	3		
e	5	4	3	2		
t	4	3	3	1		
t	3	2	2	1		
u	2	1	1	2		
B	1	0	1	2		
	0	1	2	3		
		B	e	t	t	r

Levenshtein

r	6	5	4	3	3	
e	5	4	3	2	2	
t	4	3	3	1	1	
t	3	2	2	1	1	
u	2	1	1	2	3	
B	1	0	1	2	3	
	0	1	2	3	4	
		B	e	t	t	r

Levenshtein

r	6	5	4	3	3	2
e	5	4	3	2	2	2
t	4	3	3	1	1	2
t	3	2	2	1	1	2
u	2	1	1	2	3	4
B	1	0	1	2	3	4
	0	1	2	3	4	5
		B	e	t	t	r

Levenshtein distance: 2

Levenshtein

r	6	5	4	3	3	2
e	5	4	3	2	2	2
t	4	3	3	1	1	2
t	3	2	2	1	1	2
u	2	1	1	2	3	4
B	1	0	1	2	3	4
	0	1	2	3	4	5
		B	e	t	t	r

Levenshtein distance: 2

Way too expensive to do for every string

Levenshtein

r	6	5	4	3	3	2
e	5	4	3	2	2	2
t	4	3	3	1	1	2
t	3	2	2	1	1	2
u	2	1	1	2	3	4
B	1	0	1	2	3	4
	0	1	2	3	4	5
		B	e	t	t	r

Levenshtein distance: 2

Way too expensive to do for every string

but...

Levenshtein Automata

Levenshtein Automata

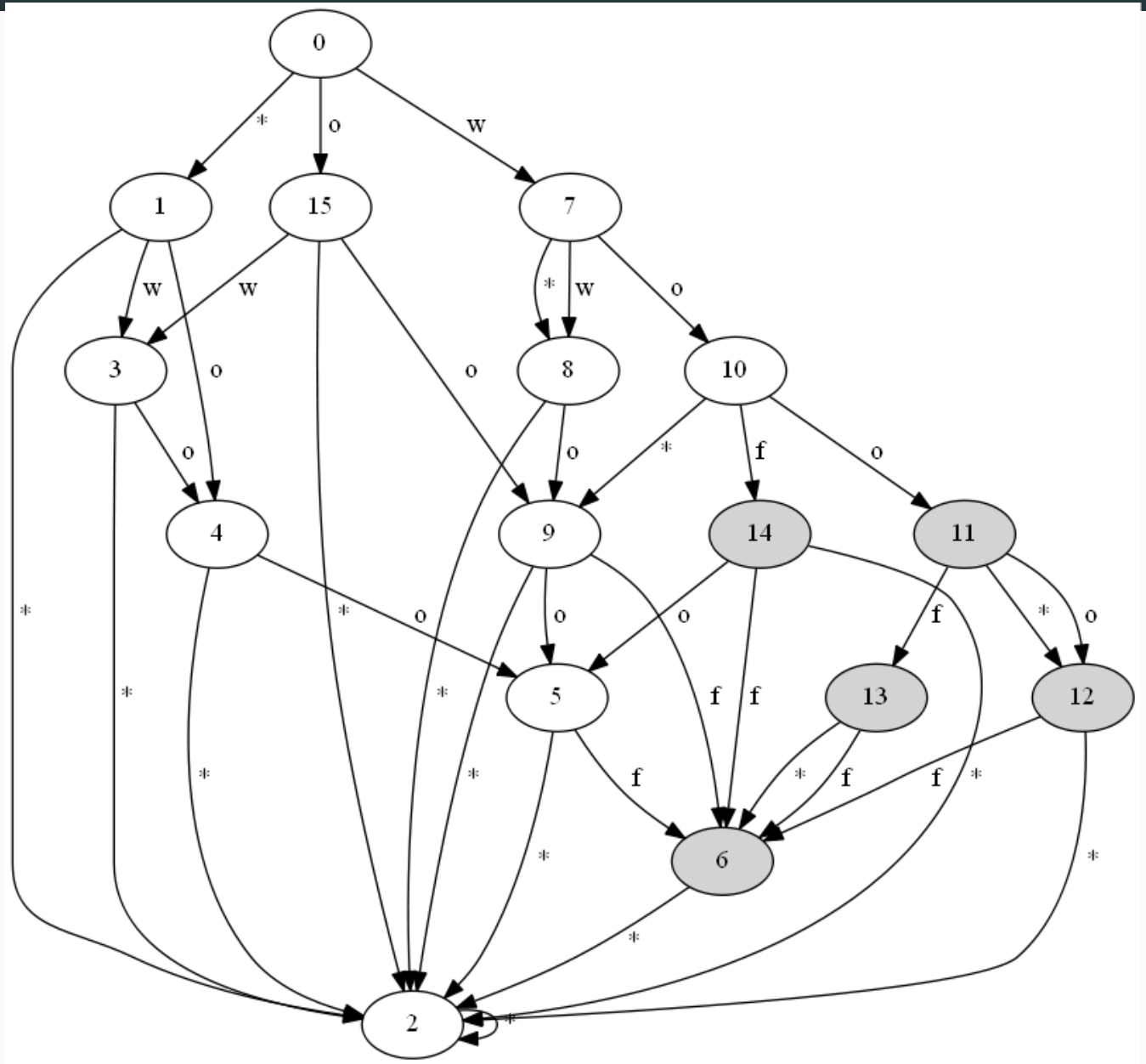
r	6	5	4	3	3	2
e	5	4	3	2	2	2
t	4	3	3	1	1	2
t	3	2	2	1	1	2
u	2	1	1	2	3	4
B	1	0	1	2	3	4
	0	1	2	3	4	5
		B	e	t	t	r

Mihov and Schultz (2002)

Python implementation

```
class LevenshteinAutomaton:
  def __init__(self, string, n):
      self.string = string
      self.max_edits = n
  def start(self):
      return range(len(self.string)+1)
  def step(self, state, c):
      new_state = [state[0]+1]
      for i in range(len(state)-1):
          cost = 0 if self.string[i] == c else 1
          new_state.append(min(new_state[i]+1, state[i]+cost,
            state[i+1]+1))
      return [min(x,self.max_edits+1) for x in new_state]
  def is_match(self, state):
      return state[-1] <= self.max_edits
  def can_match(self, state):
      return min(state) <= self.max_edits
```

Levenshtein Automaton (credit to Jules Jacobs)



Properties

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Numbers of states is linearly bounded by the length of the query string

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builds in O(n) time

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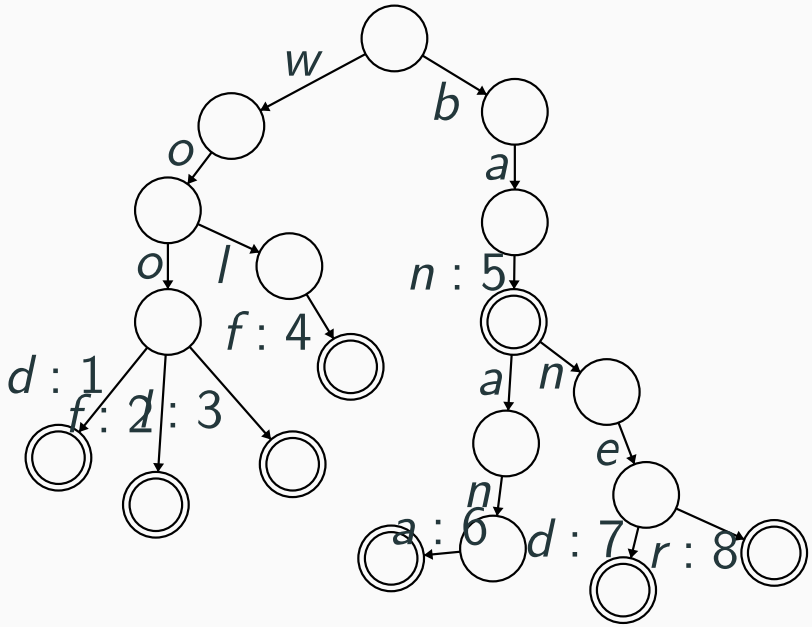
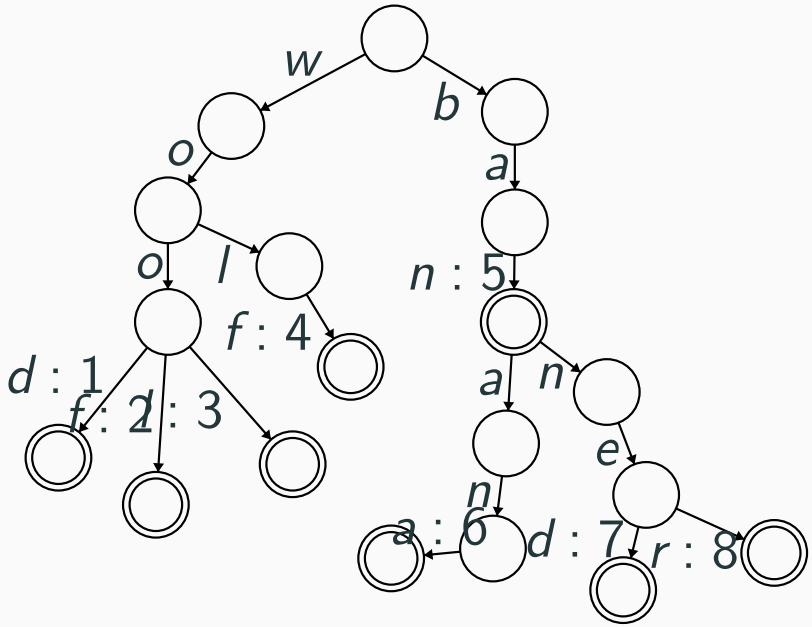
builds in O(n) time

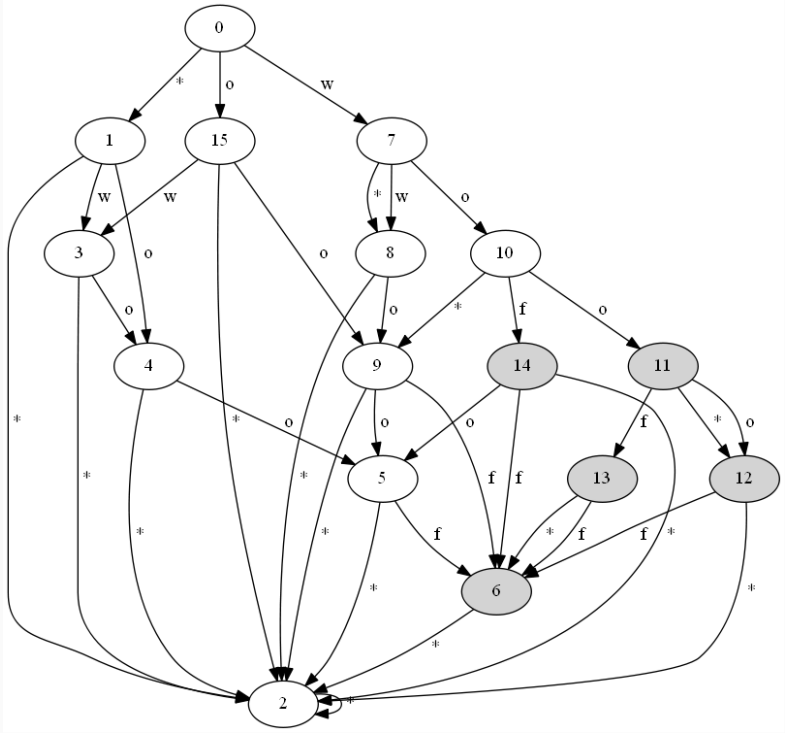
(can cut that down to O(δ) by using sparse vectors)

Use



Use





Wrap-up

Summary

We can do better.

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What if we don't have the whole search term from the user to make a Levenshtein Automaton with?

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What if we don't have the whole search term from the user to make a Levenshtein Automaton with?

What if it's only a prefix?

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We can do better.

What if we don't have the whole search term from the user to make a Levenshtein Automaton with?

What if it's only a prefix?

There still isn't a super efficient way to do fuzzy prefix searching.

Questions?

Tools and resources

Apache Lucene full stack FOSS search solution

Marisa_trie Python trie, shares interface with DAWG

DAWG another Python trie

Roche and Schabes 1997 primary source on FS comp ling

Mihov and Schultz 2002 levenshtein automata

References I