# EOM

October 8, 2021

# 1 6DOF Equations of Motion

Acronym	Meaning
$\overline{\text{CG}}$	Center of Gravity
DAVE-ML	Dynamic Aerospace Vehicle Exchange Markup Language
EOM	Equations of Motion
FRD	Forward Right Down
IC	Initial Condition
LLA	Latitude, Longitude and Altitude
MathML	Mathematical Markup Language
NED	North, East, Down
PCPF	Planet Centered, Planet Fixed (aka, ECEF on Earth)
SI	International System of Units

The goal of the software is to implement the EOM for DAVE-ML models in Python.

The software implements flat earth and the oblate rotating planet EOM as derived in Stevens and Lewis. Stevens and Lewis derives the equations of motion; therefore they are not described here.

All internal calculations are in SI.

To upgrade jupyter: pip3 install -upgrade jupyterlab

## 1.1 Greek Alphabet

alpha $\alpha A$	beta $\beta B$	gamma $\gamma \Gamma$
delta $\delta \Delta$	epsilon $\epsilon E$	zeta $\zeta Z$
eta $\eta H$	theta $\theta \Theta$	iota $\iota I$
kappa $\kappa K$	lambda $\lambda \Lambda$	mu $\mu M$
nu $\nu N$	xi $\xi \Xi$	omicron $oO$
pi $\pi \Pi$	rho $\rho P$	sigma $\sigma \Sigma$
tau $\tau T$	upsilon $v \Upsilon$	phi $\phi \Phi$
chi $\chiX$	psi $\psi \Psi$	omega $\omega\Omega$

### 1.2 Unit Test Class

A base class is created for doing unit testing. Other classes derive from this class.

```
[1]: import logging

class ppUnitTest:
    FailCount = 0
    ClassName = " "

    def TestValue(self, actualValue, testValue, label, tol):
        if abs(actualValue - testValue) > tol:
            self.FailCount += 1
            labelStr = "[" + self.ClassName + "]" + label
            actStr = labelStr + ": Test failed. Expected: {};".

    →format(actualValue)
        calStr = " Calculated: {}".format(testValue)
        logging.error(actStr+calStr)
```

```
class ppUnitTestCheck(ppUnitTest):
    def UnitTest(self):
        self.ClassName = "UnitCheck"
        # Same test, different tolerances
        self.TestValue(9.9, 10.1, "Should Pass", 1.0)
        self.TestValue(9.9, 10.1, "Should Fail", 1e-1)

        print("Number of ppConvert failed tests: ", self.FailCount)

unitTestCheck = ppUnitTestCheck()
unitTestCheck.UnitTest()
```

ERROR:root:[UnitCheck]Should Fail: Test failed. Expected: 9.9; Calculated: 10.1 Number of ppConvert failed tests: 1

### 1.3 Conversion Class

Create a class for doing common conversions.

unit	abbreviation
second	s
minute	min
inch	inch
foot	ft
meter	m
nautical mile	nmi
statute mile	$\operatorname{smi}$
kilometer	$\mathrm{km}$
centimeter	$\mathrm{cm}$
millimeter	mm
pound force	lbf
Newton	N

unit	abbreviation
kilogram force	kgf
kilogram	kg
pound mass	lbm
slug	slug
degree	$\deg$
radian	rad
knot (nmi/hr)	kt
nondimensional	$\operatorname{nd}$

```
[3]: import math
     class ppConvert(ppUnitTest):
         KnotToFps = 1.6878097112860893
         FpsToKnot = (1.0 / KnotToFps)
         MinToSec = 60.0
         FeetToMeter = 0.3048
         MeterToFeet = (1.0 / FeetToMeter)
         NmToFeet = 6076.115485564304
         FeetToNm = (1.0 / NmToFeet)
         SqMeterToSqFeet = (MeterToFeet*MeterToFeet)
         SqFeetToSqMeter = 1.0 / SqMeterToSqFeet
         PoundToNewton = 4.4482216152605
         NewtonToPound = 1.0 / PoundToNewton
         SlugToKg = 14.593902937
         KgToSlug = 1.0 / SlugToKg
         Slugft2ToKgm2 = 1.3558179618926
         Kgm2ToSlugft2 = 1.0 / Slugft2ToKgm2
         DegToRad = math.radians(1.0)
         RadToDeg = math.degrees(1.0)
         MpsToKt = 1.94384
         EnglishToSI = {
             "lbf": PoundToNewton,
             "slug": SlugToKg,
             "slugft2": Slugft2ToKgm2,
             "ft": FeetToMeter,
             "ft_s": FeetToMeter,
             "ft2": SqFeetToSqMeter,
             "deg": DegToRad,
             "deg_s": DegToRad,
             "km": 1000.0,
             "km_s": 1000.0,
             "nd": 1
         }
```

```
SiToEnglish = {
    "m": MeterToFeet,
    "m2": SqMeterToSqFeet,
    "rad": RadToDeg,
    "rad_s": RadToDeg,
    "m_s": MpsToKt,
    "n": NewtonToPound,
    "kg": KgToSlug,
    "kgm2": Kgm2ToSlugft2,
    "s": 1
}
def LogWarn(self, inUnit):
   warnStr = units + " not recognized in ppConvert. No conversion done."
    logging.warning(warnStr)
# Convert English units to SI
def SetIC(self, inIC):
   print("======= SetIC =======")
    icData = {}
    for key,value in inIC.items():
        units = value[1].lower()
        factor = 1
        if units in self.EnglishToSI:
            factor = self.EnglishToSI[units]
        elif units not in self.SiToEnglish:
            self.LogWarn(units)
        icData[key] = factor * value[0]
   return icData
def ToSI(self, value, inUnits):
   units = inUnits.lower()
   factor = 1
    if units in self.EnglishToSI:
        factor = self.EnglishToSI[units]
    elif units not in self.SiToEnglish:
        self.LogWarn(inUnits)
    return value*factor
# Convert SI to English units
def ToEnglish(self, value, inUnits):
   units = inUnits.lower()
   factor = 1
    if units in self.SiToEnglish:
        factor = self.SiToEnglish[units]
    elif units in self.EnglishToSI:
        self.LogWarn(inUnits)
```

```
convertedValue = []
       for v in value:
           convertedValue.append(v*factor)
       return convertedValue
   def UnitTest(self):
       self.ClassName = "ppConvert"
       checkData = (
           (
                123.0,
                              72.876*self.KnotToFps, "KnotToFps", 1e-3),
                 78.8,
                               133.0*self.FpsToKnot, "FpsToKnot", 1e-3),
           (
           (
                300.0,
                                    5.0*self.MinToSec, "MinToSec", 1e-12),
           ( 395.9352,
                         1299.0*self.FeetToMeter, "FeetToMeter", 1e-4),
              1299.0, 395.9352*self.MeterToFeet, "MeterToFeet", 1e-4),
                                  653.0*self.NmToFeet, "NmToFeet", 0.1),
           (3967703.4,
                653.0,
                            3967703.4*self.FeetToNm, "FeetToNm", 0.1),
           (10.763910,
                                 self.SqMeterToSqFeet, "SqMeterToSqFeet", __
\rightarrow1e-6).
           (
                 13.0,
                         2.92252*self.PoundToNewton, "PoundToNewton", 1e-4),
                 1.0, 0.73756215*self.Slugft2ToKgm2, "Slugft2ToKgm2", 1e-7),
           (
              54.864,
                               self.FeetToMeter*180.0, "FeetToMeters", 1e-4),
           (1742.12598,
                               self.MeterToFeet*531.0, "MetersToFeet", 1e-5),
           ( 178.5596, self.SqFeetToSqMeter*1922.0, "SqFeetToSqMeters", 1e-4),
           ( 4412.64, self.PoundToNewton*992.0, "PoundsToNewtons", 0.01),
           (21.6930872, self.Slugft2ToKgm2*16.0, "SlugFt2ToKgM2", 1e-6),
           (161.0256, self.ToSI(36.2,"lbf"), "ToSI lbf->N", 1e-3),
           (161.0256, self.ToSI(36.2,"LBf"), "ToSI lbf->N", 1e-3),
           (105.7538001, self.ToSI(78.0, "slugft2"), "ToSI slugf2->kgm2", 1e-6),
           (531.0, self.ToSI(1742.12598,"ft"), "ToSI f->m", 1e-5),
           (9.7536, self.ToSI(32.0, "ft_s"), "ToSI fps-mps", 1e-4),
           (140.6552, self.ToSI(1514,"ft2"), "ToSI f2-m2", 1e-4),
           (math.pi, self.ToSI(180.0, "deg"), "ToSI deg->rad", 1e-6),
           (0.25*math.pi, self.ToSI(45.0,"deg_s"), "ToSI dps->rps", 1e-6),
           (93200, self.ToSI(93.2,"km"), "ToSI km->m", 1e-6),
           (4221, self.ToSI(4.221,"km_s"), "ToSI km_s->m_s", 1e-6)
       )
       for d in checkData:
           self.TestValue( d[0], d[1], d[2], d[3] )
       self.ClassName = "ppConvert IC"
       icTest = {
           "newtonTest": [36.2, "lbf"],
           "inertiaTest": [78.0, "slugft2"],
           "feetTest": [1742.12598, "ft"],
           "fpsTest": [32.0, "ft_s"],
           "ft2Test": [1514, "ft2"],
```

```
"degTest": [180, "deg"],
    "dpsTest": [45.0, "deg_s"],
    "kmTest": [93.2, "km"],
    "kpsTest": [4.221, "km_s"]
}
icData = self.SetIC(icTest)
self.TestValue(161.0256, icData["newtonTest"], "lbf->N", 1e-3)
self.TestValue(105.7538001, icData["inertiaTest"], "slugf2->kgm2", 1e-6)
self.TestValue(531.0, icData["feetTest"], "f->m", 1e-5)
self.TestValue(9.7536, icData["fpsTest"], "fps-mps", 1e-4)
self.TestValue(140.6552, icData["ft2Test"], "f2-m2", 1e-4)
self.TestValue(math.pi, icData["degTest"], "deg->rad", 1e-6)
self.TestValue(0.25*math.pi, icData["dpsTest"], "dps->rps", 1e-6)
self.TestValue(93200, icData["kmTest"], "km->m", 1e-6)
self.TestValue(4221, icData["kpsTest"], "km_s->m_s", 1e-6)
#TODO: add ToEnglish unit tests
print("Number of ppConvert failed tests: ", self.FailCount)
```

Create an instance of the conversion class to be used globally.

```
[4]: gvConvert = ppConvert()
gvConvert.UnitTest()
```

### 1.4 DAVE Model Parser

Parse a DAVE-ML model and store as a Python class.

In XML, you must escape:

```
" with "
< with &lt;
& with &amp;</pre>
```

```
[5]: import xml.etree.ElementTree as ET
import math

# this function puts the namespace prefix on DAVE-ML tags
def Tag(name):
    daveNs = "{http://daveml.org/2010/DAVEML}"
    return (daveNs + name)

class ppVarDefStruct:
    name = None
    varID = None
```

```
units = None
   axisSystem = None
   sign
              = None
              = None
   alias
   symbol
              = None
   hasInitialValue = False
   initialValue = 0
   hasMath = False
   code = compile("1", "<string>", "eval")
   codeText = None
   isInput
             = True
   isOutput = False
   isStdAIAA = False
   isState = False
   isStateDeriv = False
class ppBpDefStruct:
   name = None
   bpID = None
   units = None
   bpVals = []
   def Clear(self):
       self.bpVals.clear()
class ppGtDefStruct:
   name = None
   gtID = None
   units = None
   bpRef = []
   dataTableStr = None
   dataTable = []
   def Clear(self):
       self.bpRef.clear()
       self.dataTable.clear()
class ppFunctionStruct:
   name = None
   fdName = None
   gtID = None
   numBreakPts = 0
   dependentVarID = None
   independentVarRef = []
   bpVals = []
```

```
dataTable = []
    def Clear(self):
        self.independentVarRef.clear()
        self.bpVals.clear()
        self.dataTable.clear()
    def Evaluate(self, data):
        index = 0
        if self.numBreakPts == 1:
            inValue = data[self.independentVarRef[0].varID]
            for v in self.bpVals[0]:
                if v <= inValue:</pre>
                    #print("i: ", i, "v: ", v, "inValue: ", inValue)
                    i += 1
            index = i - 1
        elif self.numBreakPts == 2:
            x = data[self.independentVarRef[1].varID]
            y = data[self.independentVarRef[0].varID]
            #print("::: iv x: ", self.independentVarRef[1].varID)
            #print("::: iv y: ", self.independentVarRef[0].varID)
            jmax = len(self.bpVals[1])
            i = 0
            for a in self.bpVals[1]:
                if a <= x:
                    #print("i: ", i, "a: ", a, "x: ", x)
                    i += 1
            i -= 1
            j = 0
            for b in self.bpVals[0]:
                if b <= y:</pre>
                    #print("j: ", j, "b: ", b, "y: ", y)
                    j += 1
            j -= 1
            index = j*jmax + i
            #print("i: ", i, " j: ", j, " jmax: ", jmax, " index: ", index)
        data[self.dependentVarID] = self.dataTable[index]
        return
class ppFunctionVarStruct:
    varID = None
```

```
min = 0
    max = 0
    extrapolate = "neither"
    interpolate = "linear"
class ppAeroModel:
   11 11 11
    A class hold the DAVE-ML aerodynamic data
    Attributes
    _____
    Data : dictionary
        a key-value pair containing aero data
    VarIdMap : dictionary
        a key-value pair for varID elements
    Methods
    _____
    Clear()
        delete all values in the model
    Update()
        update the values of the aero data
    Data = \{\}
    NameToId = {}
    IdToName = {}
    VarDef = []
    BpDef = []
    GtDef = []
    FunctionDef = []
    def Clear(self):
        self.Data.clear()
        self.NameToId.clear()
        self.IdToName.clear()
        self.VarDef.clear()
        for b in self.BpDef:
            b.Clear()
        self.BpDef.clear()
        for g in self.GtDef:
            g.Clear()
        self.GtDef.clear()
        for f in self.FunctionDef:
            f.Clear()
        self.FunctionDef.clear()
```

```
def HasName(self, inName):
       return inName in self.NameToId
  def DataFromName(self, inName):
       outVarID = self.NameToId[inName]
       return self.Data[outVarID]
  def Set(self, inName, inValue = 0):
       if not (inName in self.NameToId):
           infoStr = inName + " not in DAVE model. Value set to " +_
→str(inValue)
           logging.info(infoStr)
           self.NameToId[inName] = inName
           self.Data[inName] = inValue
       else:
           self.Data[self.NameToId[inName]] = inValue
  def PreProcess(self, printDebugData):
       # Change variable names in equations to self.Data[] dictionary
       for v in self.VarDef:
           # function variables are not inputs
           for f in self.FunctionDef:
               if v.varID == f.dependentVarID:
                   v.isInput = False
           if v.hasMath:
               newText = v.codeText.replace("{", "self.Data[\"")
               newText = newText.replace("}", "\"]")
               v.code = compile(newText, "<string>", "eval")
               if printDebugData:
                   print(" <<", v.varID, ">>")
                   print(" [raw] -> ", v.codeText)
                   print(" [python] -> ", newText)
       print("++++ MODEL INPUTS AND OUTPUTS +++++")
       for v in self.VarDef:
           if v.isInput:
               print("++> Input: ", v.varID)
           if v.isOutput:
               print("++> Output: ", v.varID)
       print("+++++++++++++++++++++++++++++")
       # connect the gridded tables with break points to functions
       for f in self.FunctionDef:
           for gt in self.GtDef:
               if f.gtID == gt.name:
```

```
f.dataTable = gt.dataTable
                  if printDebugData:
                      print("----> depVar: ", f.dependentVarID)
                      print("---> f.dataTable: ", f.dataTable)
                      print("---> bpRef: ", gt.bpRef)
                      print("---> gt.name: ", gt.name)
                      print("---> f.gtID: ", f.gtID)
                  bpv = []
                  for bpr in gt.bpRef:
                      for bp in self.BpDef:
                         if bp.bpID == bpr:
                             bpv.append(bp.bpVals)
                             if printDebugData:
                                 print("---> f.bp name: ", bpr)
                                 print("---> f.bpVals: ", bp.bpVals)
                                 print("---> bpv: ", bpv)
                  f.bpVals = bpv
   def Update(self):
       # Update all the functions
       for f in self.FunctionDef:
           f.Evaluate(self.Data)
       # Evaluate the MATH-ML equations
       for v in self.VarDef:
           if v.hasMath:
              self.Data[v.varID] = eval(v.code)
gvAeroModel = ppAeroModel()
def FileHeader(e):
   print("Model: ", e.get('name'))
   for fhTag in e:
       if fhTag.tag == Tag("creationDate"):
           print("creation date: ", fhTag.get('date'))
       if fhTag.tag == Tag("fileVersion"):
           print("file version: ", fhTag.text)
   def VariableDef(e, printDebugData):
   varDefStruct = ppVarDefStruct();
   varDefStruct.name = e.get('name')
   varDefStruct.varID = e.get('varID')
   varDefStruct.units = e.get('units')
```

```
varDefStruct.axisSystem = e.get('axisSystem')
varDefStruct.sign = e.get('sign')
varDefStruct.alias = e.get('alias')
varDefStruct.symbol = e.get('symbol')
varDefStruct.initialValue = e.get('initialValue')
value = 0
if varDefStruct.initialValue != None:
    value = varDefStruct.initialValue
    varDefStruct.hasInitialValue = True
   varDefStruct.isInput = False
gvAeroModel.Data[varDefStruct.varID] = float(value)
gvAeroModel.NameToId[varDefStruct.name] = varDefStruct.varID
gvAeroModel.IdToName[varDefStruct.varID] = varDefStruct.name
for label in e:
    if label.tag == Tag("isStdAIAA"):
        varDefStruct.isStdAIAA = True
    if label.tag == Tag("isOutput"):
        varDefStruct.isOutput = True
        varDefStruct.isInput = False
    if label.tag == Tag("isState"):
        varDefStruct.isState = True
    if label.tag == Tag("isStateDeriv"):
        varDefStruct.isStateDeriv = True
    if label.tag == Tag("calculation"):
        varDefStruct.hasMath = True
        varDefStruct.isInput = False
        for pl in label:
            if pl.tag == Tag("python"):
                varDefStruct.codeText = pl.text
# TODO: add MathML
gvAeroModel.VarDef.append(varDefStruct)
if printDebugData:
   print("-variableDef-")
    print(" varDefStruct.name: ", varDefStruct.name)
   print(" varDefStruct.varID: ", varDefStruct.varID)
   print(" varDefStruct.units: ", varDefStruct.units)
    print(" varDefStruct.axisSystem: ", varDefStruct.axisSystem)
   print(" varDefStruct.sign: ", varDefStruct.sign)
    print(" varDefStruct.alias: ", varDefStruct.alias)
   print(" varDefStruct.symbol: ", varDefStruct.symbol)
    print(" varDefStruct.hasInitialValue: ", varDefStruct.hasInitialValue)
```

```
print(" varDefStruct.initialValue: ", varDefStruct.initialValue)
       print(" varDefStruct.isStdAIAA: ", varDefStruct.isStdAIAA)
       print(" varDefStruct.isOutput: ", varDefStruct.isOutput)
        print(" varDefStruct.hasMath: ", varDefStruct.hasMath)
        print(" varDefStruct.codeText: ", varDefStruct.codeText)
def BreakpointDef(e, printDebugData):
   bpDefStruct = ppBpDefStruct()
   bpDefStruct.name = e.get('name')
    bpDefStruct.bpID = e.get('bpID')
   bpDefStruct.units = e.get('units')
   for label in e:
        if label.tag == Tag("bpVals"):
            bpList = []
            for i in label.text.split(','):
                bpList.append( float(i) )
            bpDefStruct.bpVals = bpList
   gvAeroModel.BpDef.append(bpDefStruct)
   if printDebugData:
       print("-bpDefStruct-")
       print(" bpDefStruct.name: ", bpDefStruct.name)
       print(" bpDefStruct.bpID: ", bpDefStruct.bpID)
       print(" bpDefStruct.units: ", bpDefStruct.units)
       print(" bpDefStruct.bpVals: ", bpDefStruct.bpVals)
def GriddedTableDef(e, printDebugData):
   gtDefStruct = ppGtDefStruct()
   gtDefStruct.name = e.get('name')
   gtDefStruct.gtID = e.get('gtID')
   gtDefStruct.units = e.get('units')
   gtDefStruct.bpRef.clear()
   bpr = []
   for label in e:
        if label.tag == Tag("breakpointRefs"):
            for refs in label:
                if refs.tag == Tag("bpRef"):
                    bpr.append( refs.get('bpID') )
        if label.tag == Tag("dataTable"):
            gtDefStruct.dataTableStr = label.text
   gtDefStruct.bpRef = bpr
   gtDefStruct.dataTable.clear()
   dt = []
    for i in gtDefStruct.dataTableStr.split(','):
```

```
dt.append( float(i) )
   gtDefStruct.dataTable = dt
   gvAeroModel.GtDef.append(gtDefStruct)
    if printDebugData:
       print("-gtDefStruct-")
       print(" gtDefStruct.name: ", gtDefStruct.name)
       print(" gtDefStruct.gtID: ", gtDefStruct.gtID)
       print(" gtDefStruct.units: ", gtDefStruct.units)
       print(" gtDefStruct.bpRef: ", gtDefStruct.bpRef)
       print(" gtDefStruct.dataTableStr: ", gtDefStruct.dataTableStr)
       print(" gtDefStruct.dataTable: ", gtDefStruct.dataTable)
# TODO: add ungridded table parsing
def UngriddedTableDef(e):
   logging.info("-UngriddedTableDef-: COMING SOON")
def Function(e, printDebugData):
   funDefStruct = ppFunctionStruct()
   funDefStruct.name = e.get('name')
   funDefStruct.independentVarRef.clear()
   iVar = []
   for label in e:
        if label.tag == Tag("independentVarRef"):
            indVar = ppFunctionVarStruct()
            indVar.varID = label.get('varID')
            indVar.min = float( label.get('min') )
            indVar.max = float( label.get('max') )
            indVar.extrapolate = label.get('extrapolate')
            indVar.interpolate = label.get('interpolate')
            iVar.append(indVar)
        if label.tag == Tag("dependentVarRef"):
            funDefStruct.dependentVarID = label.get('varID')
        if label.tag == Tag("functionDefn"):
            funDefStruct.fdName = label.get('name')
            for tVar in label:
                if tVar.tag == Tag("griddedTableRef"):
                    funDefStruct.gtID = tVar.get('gtID')
                if tVar.tag == Tag("griddedTable"):
                    funDefStruct.gtID = tVar.get('name')
                    GriddedTableDef(tVar, printDebugData)
   funDefStruct.independentVarRef = iVar
   funDefStruct.numBreakPts = len(funDefStruct.independentVarRef)
    gvAeroModel.FunctionDef.append(funDefStruct)
```

```
if printDebugData:
        print("-functionStruct-")
       print(" funDefStruct.name: ", funDefStruct.name)
       print(" funDefStruct.fdName: ", funDefStruct.name)
        print(" funDefStruct.gtID: ", funDefStruct.gtID)
        print(" funDefStruct.numBreakPts: ", funDefStruct.numBreakPts)
       print(" funDefStruct.dependentVarID: ", funDefStruct.dependentVarID)
        for iv in funDefStruct.independentVarRef:
            print(" independentVarRef.varID: ", iv.varID)
            print(" independentVarRef.min: ", iv.min)
            print(" independentVarRef.max: ", iv.max)
            print(" independentVarRef.extrapolate: ", iv.extrapolate)
            print(" independentVarRef.interpolate: ", iv.interpolate)
class ppSignalStruct:
   signalType = None
    signalName = None
    signalUnits = None
   varID = None
    signalID = None
   signalValue = 0
   tol = 1e-6
class ppCheckData:
   name = []
   signal = []
   numSignals = []
   def Clear(self):
       self.name.clear()
        self.signal.clear()
        self.numSignals.clear()
gvCheckData = ppCheckData()
def ppPrint(str1, str2, printDebugData):
   if printDebugData:
       print(str1, str2)
def CheckData(e, printDebugData):
   pd = printDebugData
   ppPrint("-checkData-", "", pd)
   for ssTag in e:
        if ssTag.tag == Tag("staticShot"):
            ppPrint("staticShot: ", ssTag.get('name'), pd)
            gvCheckData.name.append(ssTag.get('name'))
            numSignals = 0
```

```
for signalType in ssTag:
                for signal in signalType:
                    if signal.tag == Tag("signal"):
                        localSignal = ppSignalStruct()
                        ppPrint(" signal type: ", signalType.tag, pd)
                        localSignal.signalType = signalType.tag
                        numSignals += 1
                        for oneSignal in signal:
                            if oneSignal.tag == Tag("signalName"):
                                localSignal.signalName = oneSignal.text
                                ppPrint(" signal name: ", localSignal.
 ⇒signalName, pd)
                            if oneSignal.tag == Tag("signalID"):
                                localSignal.signalID = oneSignal.text
                                ppPrint(" signal ID: ", localSignal.signalID, __
→pd)
                            if oneSignal.tag == Tag("varID"):
                                localSignal.varID = oneSignal.text
                                ppPrint(" signal varID: ", localSignal.varID, u
→pd)
                            if oneSignal.tag == Tag("signalUnits"):
                                localSignal.signalUnits = oneSignal.text
                                ppPrint(" signal units: ", localSignal.
→signalUnits, pd)
                            if oneSignal.tag == Tag("signalValue"):
                                localSignal.signalValue = float(oneSignal.text)
                                ppPrint(" signal value: ", localSignal.
→signalValue, pd)
                            if oneSignal.tag == Tag("tol"):
                                localSignal.tol = oneSignal.text
                                ppPrint(" signal tol: ", localSignal.tol, pd)
                        sigStr = "{} signal #: {}".format(ssTag.get('name'),__
→numSignals)
                        ppPrint(" [ localSignal append ] -> ", sigStr, pd)
                        gvCheckData.signal.append(localSignal)
            gvCheckData.numSignals.append(numSignals)
            numStr = "{} signals in ".format(numSignals)
            ppPrint(numStr, ssTag.get('name'), pd)
def ppLoadDml(dmlFile, printDebugData=True):
    """Pass in DAVE-ML model format file"""
    # TODO add a quiet mode to not print out all loading data
    gvAeroModel.Clear()
    gvCheckData.Clear()
```

```
root = ET.parse(dmlFile).getroot()
if root.tag == Tag("DAVEfunc"):
    for daveFcn in root:
        if daveFcn.tag == Tag("fileHeader"):
            FileHeader(daveFcn)
        if daveFcn.tag == Tag("variableDef"):
            VariableDef(daveFcn, printDebugData)
        if daveFcn.tag == Tag("breakpointDef"):
            BreakpointDef(daveFcn, printDebugData)
        if daveFcn.tag == Tag("griddedTableDef"):
            GriddedTableDef(daveFcn, printDebugData)
        if daveFcn.tag == Tag("ungriddedTableDef"):
            UngriddedTableDef(daveFcn)
        if daveFcn.tag == Tag("function"):
            Function(daveFcn, printDebugData)
        if daveFcn.tag == Tag("checkData"):
            CheckData(daveFcn, printDebugData)
if printDebugData:
    print("\n--- PreProcess Equations and Functions ---\n")
gvAeroModel.PreProcess(printDebugData)
if printDebugData:
    print("Number of check cases: ", len(gvCheckData.name))
   print("\n--Variables defined in model--\n")
    for i in gvAeroModel.VarDef:
        print(i.name)
print("\n---- DAVE-ML MODEL PARSE COMPLETE ----")
```

#### 1.5 Check Model Function

Function to run the check cases.

```
[6]: import logging

def CheckModel():
    print("\n---- CheckModel ----\n")
    print("numSignals: ", gvCheckData.numSignals)
    i = 0
    shotCount = 0
    for ss in gvCheckData.name:
        prevSignalType = Tag("checkInputs")
```

```
for si in range(gvCheckData.numSignals[shotCount]):
           signal = gvCheckData.signal[i]
           name = signal.varID if signal.signalName == None else signal.
⇒signalName
           i += 1
           if signal.signalType == Tag("checkInputs"):
               gvAeroModel.Data[signal.varID] = float(signal.signalValue)
           if signal.signalType != prevSignalType:
               gvAeroModel.Update()
           if signal.signalType == Tag("internalValues"):
               modelValue = gvAeroModel.Data[signal.varID]
               checkValue = signal.signalValue
               if abs(modelValue - checkValue) > float(signal.tol):
                   errStr = "internal: {} -> [{}] Calculated {}, Expected {}".
→format(ss, name, modelValue, checkValue)
                   logging.error(errStr)
           if signal.signalType == Tag("checkOutputs"):
               modelValue = gvAeroModel.Data[signal.varID]
               checkValue = signal.signalValue
               if abs(modelValue - checkValue) > float(signal.tol):
                   errStr = "output: {} -> [{}] Calculated {}, Expected {}".
→format(ss, name, modelValue, checkValue)
                   logging.error(errStr)
           prevSignalType = signal.signalType
       shotCount += 1
   print("\n---- END CheckModel ----\n")
```

#### 1.5.1 1-D Gridded Table

Load and Check the DAVE-ML model. Start with some test models.

```
[7]: ppLoadDml('models/tests/oneD_table.dml')
CheckModel()

*********************************

Model: One Dimensional Table Test
creation date: 2021-04-26
```

\*\*\*\*\*\*\*\*\*\*\*\*

-variableDefvarDefStruct.name: angleOfAttack

file version: \$Revision: 100 \$

```
varDefStruct.varID:
                     alpha
 varDefStruct.units:
                      deg
 varDefStruct.axisSystem:
                          None
 varDefStruct.sign: None
varDefStruct.alias: None
 varDefStruct.symbol: None
 varDefStruct.hasInitialValue: False
 varDefStruct.initialValue: None
 varDefStruct.isStdAIAA: False
 varDefStruct.isOutput: False
 varDefStruct.hasMath: False
 varDefStruct.codeText: None
-variableDef-
 varDefStruct.name:
varDefStruct.varID:
varDefStruct.units: nd
 varDefStruct.axisSystem:
                          None
 varDefStruct.sign: None
 varDefStruct.alias: None
 varDefStruct.symbol: None
 varDefStruct.hasInitialValue: False
 varDefStruct.initialValue: None
 varDefStruct.isStdAIAA: False
 varDefStruct.isOutput: False
varDefStruct.hasMath: False
 varDefStruct.codeText: None
-bpDefStruct-
 bpDefStruct.name:
                   alpha
 bpDefStruct.bpID:
                   ALPHA1
 bpDefStruct.units: deg
 bpDefStruct.bpVals: [-10.0, -5.0, 0.0, 5.0, 10.0, 15.0, 20.0, 25.0, 30.0,
35.0, 40.0, 45.0]
-gtDefStruct-
 gtDefStruct.name: Cnp_table
gtDefStruct.gtID: None
 gtDefStruct.units: None
 gtDefStruct.bpRef:
                    ['ALPHA1']
 gtDefStruct.dataTableStr:
             .061, .052, .052, -.012, -.013, -.024, .050, .150, .130, .158,
.240, .150
 gtDefStruct.dataTable: [0.061, 0.052, 0.052, -0.012, -0.013, -0.024, 0.05,
0.15, 0.13, 0.158, 0.24, 0.15]
-functionStruct-
 funDefStruct.name:
                    Cnp
 funDefStruct.fdName: Cnp
funDefStruct.gtID: Cnp_table
```

```
funDefStruct.numBreakPts: 1
 funDefStruct.dependentVarID:
 independentVarRef.varID: alpha
 independentVarRef.min: -10.0
 independentVarRef.max: 45.0
 independentVarRef.extrapolate:
 independentVarRef.interpolate:
-checkData-
staticShot: AOA 5 deg
 signal type: {http://daveml.org/2010/DAVEML}checkInputs
 signal name: angleOfAttack 5 deg
 signal varID: alpha
 signal value: 5.0
 [localSignal append] -> AOA 5 deg signal #: 1
 signal type: {http://daveml.org/2010/DAVEML}checkOutputs
 signal name: Cnp at 5 deg
 signal varID: cnp
 signal value: -0.012
 signal tol: 0.000001
 [localSignal append] -> AOA 5 deg signal #: 2
2 signals in AOA 5 deg
staticShot: AOA 10 deg
 signal type: {http://daveml.org/2010/DAVEML}checkInputs
signal name: angleOfAttack 10 deg
signal varID: alpha
 signal value: 10.0
 [localSignal append] -> AOA 10 deg signal #: 1
 signal type: {http://daveml.org/2010/DAVEML}checkOutputs
 signal name: Cnp at 10 deg
 signal varID: cnp
 signal value: -0.013
 signal tol: 0.000001
 [localSignal append] -> AOA 10 deg signal #: 2
2 signals in AOA 10 deg
staticShot: AOA 29 deg
 signal type: {http://daveml.org/2010/DAVEML}checkInputs
 signal name: AOA at 29 deg
 signal varID: alpha
signal value: 29.0
 [localSignal append] -> AOA 29 deg signal #: 1
 signal type: {http://daveml.org/2010/DAVEML}checkOutputs
 signal name: Cnp at 29 deg AOA
 signal varID: cnp
 signal value: 0.15
signal tol: 0.000001
 [localSignal append] -> AOA 29 deg signal #: 2
2 signals in AOA 29 deg
```

```
--- PreProcess Equations and Functions ---
   +++++ MODEL INPUTS AND OUTPUTS +++++
   ++> Input: alpha
   ----> depVar: cnp
   ----> f.dataTable: [0.061, 0.052, 0.052, -0.012, -0.013, -0.024, 0.05, 0.15,
   0.13, 0.158, 0.24, 0.15]
   ----> bpRef: ['ALPHA1']
   ----> gt.name: Cnp_table
   ----> f.gtID: Cnp_table
   ----> f.bp name: ALPHA1
   ----> f.bpVals: [-10.0, -5.0, 0.0, 5.0, 10.0, 15.0, 20.0, 25.0, 30.0, 35.0,
   40.0, 45.0]
   ----> bpv: [[-10.0, -5.0, 0.0, 5.0, 10.0, 15.0, 20.0, 25.0, 30.0, 35.0, 40.0,
   45.0]]
   Number of check cases: 3
   --Variables defined in model--
   angleOfAttack
   Cnp
   ---- DAVE-ML MODEL PARSE COMPLETE ----
   ---- CheckModel -----
   numSignals: [2, 2, 2]
   ---- END CheckModel -----
   Do a short test to check of the function cnp which is a function of alpha.
[8]: gvAeroModel.Data["alpha"] = 29.0
    gvAeroModel.Update()
    print("f cnp: ", gvAeroModel.Data["cnp"])
   f cnp: 0.15
   1.5.2 2-D Gridded Table
[9]: ppLoadDml('models/tests/twoD_table.dml')
    CheckModel()
   ************
   Model: 2D gridded table
   file version: $Revision: 108 $
    ************
```

```
-variableDef-
varDefStruct.name: angleOfAttack
varDefStruct.varID: ALPHA
varDefStruct.units: deg
varDefStruct.axisSystem:
varDefStruct.sign: None
varDefStruct.alias: None
varDefStruct.symbol: #x3B1
varDefStruct.hasInitialValue: False
varDefStruct.initialValue: None
varDefStruct.isStdAIAA: False
varDefStruct.isOutput: False
varDefStruct.hasMath: False
varDefStruct.codeText: None
-variableDef-
varDefStruct.name: Mach
varDefStruct.varID: MACH
varDefStruct.units: ND
varDefStruct.axisSystem:
                          None
varDefStruct.sign: None
varDefStruct.alias: None
varDefStruct.symbol: M
varDefStruct.hasInitialValue: False
varDefStruct.initialValue: None
varDefStruct.isStdAIAA: False
varDefStruct.isOutput: False
varDefStruct.hasMath: False
varDefStruct.codeText: None
-variableDef-
varDefStruct.name: coefficientOfLift
varDefStruct.varID: CL
varDefStruct.units: ND
varDefStruct.axisSystem:
                          None
varDefStruct.sign: None
varDefStruct.alias: None
varDefStruct.symbol: CL
varDefStruct.hasInitialValue: False
varDefStruct.initialValue: None
varDefStruct.isStdAIAA: False
varDefStruct.isOutput: False
varDefStruct.hasMath: False
varDefStruct.codeText: None
-bpDefStruct-
bpDefStruct.name: alpha
```

bpDefStruct.bpVals: [-4.0, 0.0, 4.0, 8.0, 12.0, 16.0]

bpDefStruct.bpID: ALPHA1
bpDefStruct.units: deg

```
-bpDefStruct-
 bpDefStruct.name:
                    mach
bpDefStruct.bpID: MACH1
 bpDefStruct.units: ND
 bpDefStruct.bpVals: [0.0, 0.4, 0.8, 0.9, 0.95, 0.99, 1.0, 1.01, 1.05, 1.2]
-gtDefStruct-
 gtDefStruct.name:
                    CL TABLE
 gtDefStruct.gtID:
 gtDefStruct.units: None
 gtDefStruct.bpRef: ['MACH1', 'ALPHA1']
 gtDefStruct.dataTableStr:
      9.5013e-01, 6.1543e-01, 5.7891e-02, 1.5274e-02, 8.3812e-01, 1.9343e-01,
      2.3114e-01, 7.9194e-01, 3.5287e-01, 7.4679e-01, 1.9640e-02, 6.8222e-01,
      6.0684e-01, 9.2181e-01, 8.1317e-01, 4.4510e-01, 6.8128e-01, 3.0276e-01,
      4.8598e-01, 7.3821e-01, 9.8613e-03, 9.3181e-01, 3.7948e-01, 5.4167e-01,
      8.9130e-01, 1.7627e-01, 1.3889e-01, 4.6599e-01, 8.3180e-01, 1.5087e-01,
      7.6210e-01, 4.0571e-01, 2.0277e-01, 4.1865e-01, 5.0281e-01, 6.9790e-01,
      4.5647e-01, 9.3547e-01, 1.9872e-01, 8.4622e-01, 7.0947e-01, 3.7837e-01,
      1.8504e-02, 9.1690e-01, 6.0379e-01, 5.2515e-01, 4.2889e-01, 8.6001e-01,
      8.2141e-01, 4.1027e-01, 2.7219e-01, 2.0265e-01, 3.0462e-01, 8.5366e-01,
      4.4470e-01, 8.9365e-01, 1.9881e-01, 6.7214e-01, 1.8965e-01, 5.9356e-01
 gtDefStruct.dataTable: [0.95013, 0.61543, 0.057891, 0.015274, 0.83812,
0.19343, 0.23114, 0.79194, 0.35287, 0.74679, 0.01964, 0.68222, 0.60684, 0.92181,
0.81317, 0.4451, 0.68128, 0.30276, 0.48598, 0.73821, 0.0098613, 0.93181,
0.37948, 0.54167, 0.8913, 0.17627, 0.13889, 0.46599, 0.8318, 0.15087, 0.7621,
0.40571, 0.20277, 0.41865, 0.50281, 0.6979, 0.45647, 0.93547, 0.19872, 0.84622,
0.70947, 0.37837, 0.018504, 0.9169, 0.60379, 0.52515, 0.42889, 0.86001, 0.82141,
0.41027, 0.27219, 0.20265, 0.30462, 0.85366, 0.4447, 0.89365, 0.19881, 0.67214,
0.18965, 0.59356]
-functionStruct-
 funDefStruct.name: Basic CL
 funDefStruct.fdName: Basic CL
 funDefStruct.gtID: CL TABLE
 funDefStruct.numBreakPts: 2
 funDefStruct.dependentVarID:
 independentVarRef.varID: MACH
 independentVarRef.min: 0.3
 independentVarRef.max: 0.95
 independentVarRef.extrapolate:
 independentVarRef.interpolate:
 independentVarRef.varID: ALPHA
 independentVarRef.min: 0.0
 independentVarRef.max: 15.0
 independentVarRef.extrapolate:
                                 both
 independentVarRef.interpolate:
                                 None
```

```
-checkData-
staticShot: AOA 4 deg; Mach 0.9
 signal type: {http://daveml.org/2010/DAVEML}checkInputs
 signal name: AOA
signal varID: ALPHA
 signal value: 4.0
 [localSignal append] -> AOA 4 deg; Mach 0.9 signal #: 1
 signal type: {http://daveml.org/2010/DAVEML}checkInputs
signal name: MACH number
 signal varID: MACH
 signal value: 0.9
 [localSignal append] -> AOA 4 deg; Mach 0.9 signal #: 2
 signal type: {http://daveml.org/2010/DAVEML}checkOutputs
 signal name: CL
 signal varID: CL
 signal value: 0.0098613
 signal tol: 0.000001
 [localSignal append] -> AOA 4 deg; Mach 0.9 signal #: 3
3 signals in AOA 4 deg; Mach 0.9
--- PreProcess Equations and Functions ---
+++++ MODEL INPUTS AND OUTPUTS +++++
++> Input: ALPHA
++> Input: MACH
----> depVar: CL
---> f.dataTable: [0.95013, 0.61543, 0.057891, 0.015274, 0.83812, 0.19343,
0.23114, 0.79194, 0.35287, 0.74679, 0.01964, 0.68222, 0.60684, 0.92181, 0.81317,
0.4451, 0.68128, 0.30276, 0.48598, 0.73821, 0.0098613, 0.93181, 0.37948,
0.54167, 0.8913, 0.17627, 0.13889, 0.46599, 0.8318, 0.15087, 0.7621, 0.40571,
0.20277, 0.41865, 0.50281, 0.6979, 0.45647, 0.93547, 0.19872, 0.84622, 0.70947,
0.37837, 0.018504, 0.9169, 0.60379, 0.52515, 0.42889, 0.86001, 0.82141, 0.41027,
0.27219, 0.20265, 0.30462, 0.85366, 0.4447, 0.89365, 0.19881, 0.67214, 0.18965,
0.59356]
----> bpRef: ['MACH1', 'ALPHA1']
----> gt.name: CL TABLE
----> f.gtID: CL_TABLE
----> f.bp name: MACH1
----> f.bpVals: [0.0, 0.4, 0.8, 0.9, 0.95, 0.99, 1.0, 1.01, 1.05, 1.2]
---> bpv: [[0.0, 0.4, 0.8, 0.9, 0.95, 0.99, 1.0, 1.01, 1.05, 1.2]]
----> f.bp name: ALPHA1
----> f.bpVals: [-4.0, 0.0, 4.0, 8.0, 12.0, 16.0]
----> bpv: [[0.0, 0.4, 0.8, 0.9, 0.95, 0.99, 1.0, 1.01, 1.05, 1.2], [-4.0, 0.0,
4.0, 8.0, 12.0, 16.0]]
Number of check cases: 1
--Variables defined in model--
```

```
angleOfAttack
Mach
coefficientOfLift
---- DAVE-ML MODEL PARSE COMPLETE ----
---- CheckModel ----
numSignals: [3]
---- END CheckModel -----
```

#### 1.5.3 Various DAVE-ML Tests

```
[10]: ppLoadDml('models/tests/test.dml')
CheckModel()
```

\*\*\*\*\*\*\*\*\*\*\*\*

Model: Test File

creation date: 2009-06-01
file version: \$Revision: 1 \$

\*\*\*\*\*\*\*\*\*\*\*\*

-variableDef-

varDefStruct.name: rtd
varDefStruct.varID: rtd
varDefStruct.units: deg\_rad
varDefStruct.axisSystem: None

varDefStruct.sign: None
varDefStruct.alias: None
varDefStruct.symbol: None

varDefStruct.hasInitialValue: True varDefStruct.initialValue: 57.29577951

varDefStruct.isStdAIAA: False
varDefStruct.isOutput: False
varDefStruct.hasMath: False
varDefStruct.codeText: None

-variableDef-

varDefStruct.name: angleOfAttack

varDefStruct.varID: alpha
varDefStruct.units: nd

varDefStruct.axisSystem: None

varDefStruct.sign: None
varDefStruct.alias: None
varDefStruct.symbol: None

varDefStruct.hasInitialValue: False

varDefStruct.initialValue: None
varDefStruct.isStdAIAA: False
varDefStruct.isOutput: False
varDefStruct.hasMath: False
varDefStruct.codeText: None

-variableDef-

varDefStruct.name: beta
varDefStruct.varID: beta
varDefStruct.units: deg

varDefStruct.axisSystem: None

varDefStruct.sign: None
varDefStruct.alias: None
varDefStruct.symbol: None

varDefStruct.hasInitialValue: True
varDefStruct.initialValue: 5.0
varDefStruct.isStdAIAA: False
varDefStruct.isOutput: False
varDefStruct.hasMath: False
varDefStruct.codeText: None

-variableDef-

varDefStruct.name: Avariable

varDefStruct.varID: a
varDefStruct.units: nd

varDefStruct.axisSystem: None

varDefStruct.sign: None
varDefStruct.alias: None
varDefStruct.symbol: None

varDefStruct.hasInitialValue: True
varDefStruct.initialValue: 2.0
varDefStruct.isStdAIAA: False
varDefStruct.isOutput: False
varDefStruct.hasMath: False
varDefStruct.codeText: None

-variableDef-

varDefStruct.name: Bvariable

varDefStruct.varID: b
varDefStruct.units: nd

varDefStruct.axisSystem: None

varDefStruct.sign: None
varDefStruct.alias: None
varDefStruct.symbol: None

varDefStruct.hasInitialValue: False
varDefStruct.initialValue: None
varDefStruct.isStdAIAA: False
varDefStruct.isOutput: False
varDefStruct.hasMath: False
varDefStruct.codeText: None

-variableDef-

varDefStruct.name: cnp
varDefStruct.varID: cnp
varDefStruct.units: nd

varDefStruct.axisSystem: None

varDefStruct.sign: None
varDefStruct.alias: None
varDefStruct.symbol: None

varDefStruct.hasInitialValue: False
varDefStruct.initialValue: None
varDefStruct.isStdAIAA: False
varDefStruct.isOutput: False
varDefStruct.hasMath: False
varDefStruct.codeText: None

-variableDef-

varDefStruct.name: dtr
varDefStruct.varID: dtr
varDefStruct.units: r\_d

varDefStruct.axisSystem: None

varDefStruct.sign: None
varDefStruct.alias: None
varDefStruct.symbol: None

varDefStruct.hasInitialValue: False
varDefStruct.initialValue: None
varDefStruct.isStdAIAA: False
varDefStruct.isOutput: False
varDefStruct.hasMath: True

varDefStruct.codeText: 3.14159265/180.0

-variableDef-

varDefStruct.name: alpr
varDefStruct.varID: ALPR
varDefStruct.units: rad

varDefStruct.axisSystem: None

varDefStruct.sign: None
varDefStruct.alias: None
varDefStruct.symbol: None

varDefStruct.hasInitialValue: False
varDefStruct.initialValue: None
varDefStruct.isStdAIAA: False
varDefStruct.isOutput: False
varDefStruct.hasMath: True

varDefStruct.codeText: {alpha} \* {dtr}

-variableDef-

varDefStruct.name: dummy
varDefStruct.varID: dummy
varDefStruct.units: nd

varDefStruct.axisSystem: None

varDefStruct.sign: None
varDefStruct.alias: None

```
varDefStruct.symbol: None
varDefStruct.hasInitialValue: False
varDefStruct.initialValue: None
varDefStruct.isStdAIAA: False
varDefStruct.isOutput: False
varDefStruct.hasMath: True
varDefStruct.codeText: 10 * (25 - 15) + 8/2 + 9 * 9 + 3 + 2
-variableDef-
varDefStruct.name: Xvar
varDefStruct.varID: x
varDefStruct.units: nd
varDefStruct.axisSystem:
                          None
varDefStruct.sign: None
varDefStruct.alias: None
varDefStruct.symbol: None
varDefStruct.hasInitialValue: False
varDefStruct.initialValue: None
varDefStruct.isStdAIAA: False
varDefStruct.isOutput: False
varDefStruct.hasMath: True
varDefStruct.codeText: 5.0 ** {a}
-variableDef-
varDefStruct.name: y
varDefStruct.varID: y
varDefStruct.units: nd
varDefStruct.axisSystem:
                          None
varDefStruct.sign: None
varDefStruct.alias: None
varDefStruct.symbol: None
varDefStruct.hasInitialValue: False
varDefStruct.initialValue: None
varDefStruct.isStdAIAA: False
varDefStruct.isOutput: False
varDefStruct.hasMath: True
varDefStruct.codeText: {cnp} + 0.0
-variableDef-
varDefStruct.name: Cz1
varDefStruct.varID: cz1
varDefStruct.units: nd
varDefStruct.axisSystem:
                          None
varDefStruct.sign: down
varDefStruct.alias: None
varDefStruct.symbol: None
varDefStruct.hasInitialValue: False
varDefStruct.initialValue: None
varDefStruct.isStdAIAA: False
varDefStruct.isOutput: False
varDefStruct.hasMath: True
```

```
varDefStruct.codeText: {czt}*(1.-({beta}/{rtd})**2)-0.19*{del}
-variableDef-
varDefStruct.name: cnt
varDefStruct.varID: cnt
varDefStruct.units: nd
varDefStruct.axisSystem: None
varDefStruct.sign: nose right
varDefStruct.alias: None
varDefStruct.symbol: None
varDefStruct.hasInitialValue: False
varDefStruct.initialValue: None
varDefStruct.isStdAIAA: False
varDefStruct.isOutput: False
varDefStruct.hasMath: True
varDefStruct.codeText: -10 if {beta} < 0 else 10</pre>
-variableDef-
varDefStruct.name: ALP_UNLIM
varDefStruct.varID: ALP_UNLIM
varDefStruct.units: deg
varDefStruct.axisSystem:
                          None
varDefStruct.sign: None
varDefStruct.alias: None
varDefStruct.symbol: None
varDefStruct.hasInitialValue: False
varDefStruct.initialValue: None
varDefStruct.isStdAIAA: False
varDefStruct.isOutput: False
varDefStruct.hasMath: False
varDefStruct.codeText: None
-variableDef-
varDefStruct.name: ALP_MAX_LIM
varDefStruct.varID: ALP_MAX_LIM
varDefStruct.units: deg
varDefStruct.axisSystem:
varDefStruct.sign: None
varDefStruct.alias: None
varDefStruct.symbol: None
varDefStruct.hasInitialValue: True
varDefStruct.initialValue: 5.0
varDefStruct.isStdAIAA: False
varDefStruct.isOutput: False
varDefStruct.hasMath: False
varDefStruct.codeText: None
-variableDef-
varDefStruct.name: Limited_angle_of_attack
varDefStruct.varID: ALP
varDefStruct.units: deg
varDefStruct.axisSystem:
                          None
```

```
varDefStruct.sign: anu
 varDefStruct.alias: None
varDefStruct.symbol: None
 varDefStruct.hasInitialValue: False
varDefStruct.initialValue: None
 varDefStruct.isStdAIAA: False
 varDefStruct.isOutput: False
varDefStruct.hasMath: True
 varDefStruct.codeText: -2.0 if {ALP_UNLIM} < -2 else {ALP_MAX_LIM} if
{ALP_UNLIM} > {ALP_MAX_LIM} else {ALP_UNLIM}
-variableDef-
varDefStruct.name: sinTest
 varDefStruct.varID: sinTest
varDefStruct.units: nd
varDefStruct.axisSystem:
 varDefStruct.sign: none
varDefStruct.alias: None
 varDefStruct.symbol: None
varDefStruct.hasInitialValue: False
 varDefStruct.initialValue: None
 varDefStruct.isStdAIAA: False
 varDefStruct.isOutput: False
 varDefStruct.hasMath: True
 varDefStruct.codeText: math.sin(0.5235987756)
-bpDefStruct-
 bpDefStruct.name:
                   alpha
 bpDefStruct.bpID:
                   ALPHA1
 bpDefStruct.units: deg
 bpDefStruct.bpVals: [-10.0, -5.0, 0.0, 5.0, 10.0, 15.0, 20.0, 25.0, 30.0,
35.0, 40.0, 45.0]
-gtDefStruct-
gtDefStruct.name:
                   Cnp_table
gtDefStruct.gtID:
 gtDefStruct.units: None
 gtDefStruct.bpRef:
                    ['ALPHA1']
 gtDefStruct.dataTableStr:
                             .061, .052, .052, -.012, -.013, -.024, .050, .150,
.130, .158, .240,
        .150
 gtDefStruct.dataTable: [0.061, 0.052, 0.052, -0.012, -0.013, -0.024, 0.05,
0.15, 0.13, 0.158, 0.24, 0.15]
-functionStruct-
 funDefStruct.name:
 funDefStruct.fdName: Cnp
 funDefStruct.gtID: Cnp_table
 funDefStruct.numBreakPts:
 funDefStruct.dependentVarID: cnp
 independentVarRef.varID: alpha
 independentVarRef.min: -10.0
```

```
independentVarRef.max: 45.0
independentVarRef.extrapolate:
                               neither
independentVarRef.interpolate:
-checkData-
staticShot: Nominal
signal type: {http://daveml.org/2010/DAVEML}checkInputs
signal name: Bvariable
signal varID: b
signal value: -2.5
 [localSignal append] -> Nominal signal #: 1
signal type: {http://daveml.org/2010/DAVEML}checkInputs
signal name: angleOfAttack
signal varID: alpha
signal value: 5.0
 [localSignal append] -> Nominal signal #: 2
signal type: {http://daveml.org/2010/DAVEML}checkInputs
signal name: ALP_UNLIM
signal varID: ALP_UNLIM
signal value: -1.0
 [localSignal append] -> Nominal signal #: 3
signal type: {http://daveml.org/2010/DAVEML}checkInputs
signal name: angleOfSideslip
signal varID: beta
signal value: 5.0
 [localSignal append] -> Nominal signal #: 4
signal type: {http://daveml.org/2010/DAVEML}checkInputs
signal name: elevatorDeflection
signal varID: del
signal units: d
signal value: 0.0
 [localSignal append] -> Nominal signal #: 5
signal type: {http://daveml.org/2010/DAVEML}checkInputs
signal name: czt
signal varID: czt
signal units: d
signal value: -49.5
[localSignal append] -> Nominal signal #: 6
signal type: {http://daveml.org/2010/DAVEML}internalValues
signal name: Radian to degree
signal varID: rtd
signal value: 57.29577951
 [localSignal append] -> Nominal signal #: 7
signal type:
              {http://daveml.org/2010/DAVEML}internalValues
signal name: Degree to radian
signal varID: dtr
signal value: 0.01745329252
 [localSignal append] -> Nominal signal #: 8
signal type: {http://daveml.org/2010/DAVEML}internalValues
```

```
signal name: a
signal varID: a
signal value: 2.0
[localSignal append] -> Nominal signal #: 9
signal type: {http://daveml.org/2010/DAVEML}internalValues
signal name: b
signal varID: b
signal value: -2.5
[localSignal append] -> Nominal signal #: 10
signal type: {http://daveml.org/2010/DAVEML}internalValues
signal name: beta
signal varID: beta
signal value: 5.0
[localSignal append] -> Nominal signal #: 11
signal type: {http://daveml.org/2010/DAVEML}internalValues
signal name: ALP_UNLIM
signal varID: ALP_UNLIM
signal value: -1.0
[localSignal append] -> Nominal signal #: 12
signal type: {http://daveml.org/2010/DAVEML}internalValues
signal name: sinTest
signal varID: sinTest
signal value: 0.5
[localSignal append] -> Nominal signal #: 13
signal type: {http://daveml.org/2010/DAVEML}internalValues
signal name: ALPR
signal varID: ALPR
signal value: 0.0872665
[localSignal append] -> Nominal signal #: 14
signal type: {http://daveml.org/2010/DAVEML}checkOutputs
signal name: dummy
signal varID: dummy
signal value: 190.0
signal tol: 0.000001
[localSignal append] -> Nominal signal #: 15
signal type: {http://daveml.org/2010/DAVEML}checkOutputs
signal name: Xvar
signal varID: x
signal value: 25.0
signal tol: 0.000001
[localSignal append] -> Nominal signal #: 16
signal type: {http://daveml.org/2010/DAVEML}checkOutputs
signal name: cz1
signal varID: cz1
signal value: -49.123036
signal tol: 0.000001
[localSignal append] -> Nominal signal #: 17
signal type: {http://daveml.org/2010/DAVEML}checkOutputs
```

```
signal name: y
 signal varID: y
 signal value: -0.012
 signal tol: 0.000001
 [localSignal append] -> Nominal signal #: 18
 signal type: {http://daveml.org/2010/DAVEML}checkOutputs
 signal name: cnt
 signal varID: cnt
 signal value: 10.0
 signal tol: 0.000001
 [localSignal append] -> Nominal signal #: 19
 signal type: {http://daveml.org/2010/DAVEML}checkOutputs
 signal name: Limited_angle_of_attack
 signal varID: ALP
 signal value: -1.0
 signal tol: 0.000001
 [localSignal append] -> Nominal signal #: 20
 signal type: {http://daveml.org/2010/DAVEML}checkOutputs
 signal name: Cnp
 signal varID: cnp
 signal value: -0.012
 signal tol: 0.000001
 [localSignal append] -> Nominal signal #: 21
21 signals in Nominal
--- PreProcess Equations and Functions ---
 << dtr >>
  [raw]-> 3.14159265/180.0
  [python] -> 3.14159265/180.0
 << ALPR >>
  [raw]-> {alpha} * {dtr}
  [python]-> self.Data["alpha"] * self.Data["dtr"]
 << dummy >>
  [raw] \rightarrow 10 * (25 - 15) + 8/2 + 9 * 9 + 3 + 2
  [python] \rightarrow 10 * (25 - 15) + 8/2 + 9 * 9 + 3 + 2
 << x >>
  [raw]-> 5.0 ** {a}
  [python] -> 5.0 ** self.Data["a"]
 << y >>
  [raw] -> \{cnp\} + 0.0
  [python] -> self.Data["cnp"] + 0.0
 << cz1 >>
  [raw] \rightarrow \{czt\}*(1.-(\{beta\}/\{rtd\})**2)-0.19*\{del\}\}
  [python]-> self.Data["czt"]*(1.-(self.Data["beta"]/self.Data["rtd"])**2)-0.19
*self.Data["del"]
 << cnt >>
  [raw]-> -10 if {beta} < 0 else 10
```

```
[python]-> -10 if self.Data["beta"] < 0 else 10
 << ALP >>
  [raw]-> -2.0 if {ALP_UNLIM} < -2 else {ALP_MAX_LIM} if {ALP_UNLIM} >
{ALP_MAX_LIM} else {ALP_UNLIM}
  [python]-> -2.0 if self.Data["ALP_UNLIM"] < -2 else self.Data["ALP_MAX_LIM"]
if self.Data["ALP_UNLIM"] > self.Data["ALP_MAX_LIM"] else self.Data["ALP_UNLIM"]
<< sinTest >>
  [raw] \rightarrow math.sin(0.5235987756)
  [python] \rightarrow math.sin(0.5235987756)
+++++ MODEL INPUTS AND OUTPUTS +++++
++> Input: alpha
++> Input: b
++> Input: ALP_UNLIM
----> depVar: cnp
----> f.dataTable: [0.061, 0.052, 0.052, -0.012, -0.013, -0.024, 0.05, 0.15,
0.13, 0.158, 0.24, 0.15]
----> bpRef: ['ALPHA1']
----> gt.name: Cnp_table
----> f.gtID: Cnp_table
----> f.bp name: ALPHA1
----> f.bpVals: [-10.0, -5.0, 0.0, 5.0, 10.0, 15.0, 20.0, 25.0, 30.0, 35.0,
40.0, 45.0]
----> bpv: [[-10.0, -5.0, 0.0, 5.0, 10.0, 15.0, 20.0, 25.0, 30.0, 35.0, 40.0,
45.011
Number of check cases: 1
--Variables defined in model--
rtd
angleOfAttack
beta
Avariable
Bvariable
cnp
dtr
alpr
dummy
Xvar
У
Cz1
cnt
ALP_UNLIM
ALP_MAX_LIM
Limited_angle_of_attack
sinTest
---- DAVE-ML MODEL PARSE COMPLETE ----
```

```
---- CheckModel -----
     numSignals: [21]
     ---- END CheckModel -----
[11]: ppLoadDml('models/cannonballNoAero/cannonballNoAero.dml')
     CheckModel()
     *************
     Model: Cannon Ball Aerodynamics Model
     creation date: 2010-02-01
     file version: $Revision: 1 $
     ***********
     -variableDef-
     varDefStruct.name: XBodyPositionOfMRC
      varDefStruct.varID: xcgr
      varDefStruct.units: nd
      varDefStruct.axisSystem:
                              None
      varDefStruct.sign: None
      varDefStruct.alias: None
      varDefStruct.symbol: None
      varDefStruct.hasInitialValue: True
     varDefStruct.initialValue: 0.1
      varDefStruct.isStdAIAA: True
      varDefStruct.isOutput: False
      varDefStruct.hasMath: False
      varDefStruct.codeText: None
     -variableDef-
      varDefStruct.name: referenceWingChord
      varDefStruct.varID: cbar
     varDefStruct.units: m
      varDefStruct.axisSystem: None
     varDefStruct.sign: None
      varDefStruct.alias: None
     varDefStruct.symbol: None
     varDefStruct.hasInitialValue: True
      varDefStruct.initialValue: 0.2
      varDefStruct.isStdAIAA: True
      varDefStruct.isOutput: False
      varDefStruct.hasMath: False
     varDefStruct.codeText: None
     -variableDef-
     varDefStruct.name: referenceWingSpan
```

varDefStruct.varID: bspan

varDefStruct.units: m varDefStruct.axisSystem: None varDefStruct.sign: None varDefStruct.alias: None varDefStruct.symbol: None varDefStruct.hasInitialValue: True varDefStruct.initialValue: 0.2 varDefStruct.isStdAIAA: True varDefStruct.isOutput: False varDefStruct.hasMath: False varDefStruct.codeText: None -variableDefvarDefStruct.name: referenceWingArea varDefStruct.varID: swing varDefStruct.units: m2 varDefStruct.axisSystem: None varDefStruct.sign: None varDefStruct.alias: None varDefStruct.symbol: None varDefStruct.hasInitialValue: True varDefStruct.initialValue: 0.0314159 varDefStruct.isStdAIAA: True varDefStruct.isOutput: False varDefStruct.hasMath: False varDefStruct.codeText: None -variableDefvarDefStruct.name: aeroBodyForceCoefficient\_X varDefStruct.varID: cx varDefStruct.units: nd varDefStruct.axisSystem: None varDefStruct.sign: FWD varDefStruct.alias: None varDefStruct.symbol: None varDefStruct.hasInitialValue: True varDefStruct.initialValue: 0.0 varDefStruct.isStdAIAA: True varDefStruct.isOutput: True varDefStruct.hasMath: False varDefStruct.codeText: None -variableDefvarDefStruct.name: aeroBodyForceCoefficient\_Y varDefStruct.varID: су varDefStruct.units: nd varDefStruct.axisSystem: varDefStruct.sign: RIGHT varDefStruct.alias: None

varDefStruct.symbol: None

varDefStruct.hasInitialValue: True

varDefStruct.initialValue: 0.0 varDefStruct.isStdAIAA: True varDefStruct.isOutput: True varDefStruct.hasMath: False varDefStruct.codeText: None -variableDefvarDefStruct.name: aeroBodyForceCoefficient\_Z varDefStruct.varID: cz varDefStruct.units: nd varDefStruct.axisSystem: None varDefStruct.sign: DOWN varDefStruct.alias: None varDefStruct.symbol: None varDefStruct.hasInitialValue: True varDefStruct.initialValue: 0.0 varDefStruct.isStdAIAA: True varDefStruct.isOutput: True varDefStruct.hasMath: False varDefStruct.codeText: None -variableDefvarDefStruct.name: aeroBodyMomentCoefficient\_Roll varDefStruct.varID: cl varDefStruct.units: nd varDefStruct.axisSystem: None varDefStruct.sign: RWD varDefStruct.alias: None varDefStruct.symbol: None varDefStruct.hasInitialValue: True varDefStruct.initialValue: 0.0 varDefStruct.isStdAIAA: True varDefStruct.isOutput: True varDefStruct.hasMath: False varDefStruct.codeText: None -variableDefvarDefStruct.name: aeroBodyMomentCoefficient\_Pitch varDefStruct.varID: cm varDefStruct.units: nd varDefStruct.axisSystem: None varDefStruct.sign: ANU varDefStruct.alias: None varDefStruct.symbol: None varDefStruct.hasInitialValue: True varDefStruct.initialValue: 0.0 varDefStruct.isStdAIAA: True varDefStruct.isOutput: True varDefStruct.hasMath: False varDefStruct.codeText: None -variableDef-

```
varDefStruct.name: aeroBodyMomentCoefficient_Yaw
 varDefStruct.varID: cn
 varDefStruct.units: nd
 varDefStruct.axisSystem:
                         None
 varDefStruct.sign: ANR
 varDefStruct.alias: None
 varDefStruct.symbol: None
 varDefStruct.hasInitialValue: True
varDefStruct.initialValue: 0.0
varDefStruct.isStdAIAA: True
 varDefStruct.isOutput: True
 varDefStruct.hasMath: False
 varDefStruct.codeText: None
-checkData-
staticShot: Internal Constants
 signal type: {http://daveml.org/2010/DAVEML}internalValues
signal name: Wind Chord
 signal varID: cbar
signal value: 0.2
 [localSignal append] -> Internal Constants signal #: 1
 signal type: {http://daveml.org/2010/DAVEML}internalValues
 signal name: Wind Span
 signal varID: bspan
 signal value: 0.2
 [localSignal append] -> Internal Constants signal #: 2
2 signals in Internal Constants
--- PreProcess Equations and Functions ---
+++++ MODEL INPUTS AND OUTPUTS +++++
++> Output: cx
++> Output:
            су
++> Output: cz
++> Output: cl
++> Output:
            cm
++> Output: cn
Number of check cases: 1
--Variables defined in model--
XBodyPositionOfMRC
referenceWingChord
referenceWingSpan
referenceWingArea
aeroBodyForceCoefficient_X
aeroBodyForceCoefficient_Y
aeroBodyForceCoefficient_Z
```

```
aeroBodyMomentCoefficient_Roll
aeroBodyMomentCoefficient_Pitch
aeroBodyMomentCoefficient_Yaw
----- DAVE-ML MODEL PARSE COMPLETE -----
----- CheckModel -----
numSignals: [2]
----- END CheckModel -----
```

#### 1.5.4 F-16 model

```
[12]: printDebugData = False
    ppLoadDml('models/F16/F16_aero.dml', printDebugData)
    CheckModel()
```

ERROR:root:internal: Positive sideslip -> [absCl0] Calculated 0.0, Expected -0.00561599999999995

ERROR:root:internal: Positive sideslip -> [absCn0] Calculated 0.0, Expected
0.00889199999999999

ERROR:root:internal: Positive sideslip -> [clt] Calculated 0.0, Expected -0.00561599999999999

ERROR:root:internal: Positive sideslip -> [cnt] Calculated 0.0, Expected
0.00889199999999999

ERROR:root:internal: Positive sideslip -> [dclda] Calculated -0.052, Expected -0.0512979999999996

ERROR:root:internal: Positive sideslip -> [dcldr] Calculated 0.014, Expected 0.013766

ERROR:root:internal: Positive sideslip  $\rightarrow$  [dcndr] Calculated -0.045, Expected -0.044064

ERROR:root:internal: Positive sideslip -> [cl1] Calculated 0.0, Expected -0.00561599999999995

ERROR:root:internal: Positive sideslip -> [c1] Calculated 0.0, Expected -0.00561599999999999

ERROR:root:internal: Positive sideslip -> [cn] Calculated 0.00176591999999999999999,
Expected 0.010657919999999998

ERROR:root:output: Positive sideslip -> [aeroRollBodyMomentCoefficient]

Calculated 0.0, Expected -0.005616

ERROR:root:output: Positive sideslip -> [aeroYawBodyMomentCoefficient]

Calculated 0.001765919999999999, Expected 0.01065792

ERROR:root:internal: Negative sideslip -> [absCl0] Calculated 0.0, Expected

-0.005615999999999995 ERROR:root:internal: Negative sideslip -> [absCn0] Calculated 0.0, Expected 0.00889199999999999 ERROR:root:internal: Negative sideslip -> [clt] Calculated -0.0, Expected 0.005615999999999999 ERROR:root:internal: Negative sideslip -> [cnt] Calculated -0.0, Expected -0.00889199999999999 ERROR:root:internal: Negative sideslip -> [dclda] Calculated -0.051, Expected ERROR:root:internal: Negative sideslip -> [dcldr] Calculated 0.012, Expected 0.013532 ERROR:root:internal: Negative sideslip -> [dcnda] Calculated -0.006, Expected -0.008298 ERROR:root:internal: Negative sideslip -> [dcndr] Calculated -0.04, Expected ERROR:root:internal: Negative sideslip -> [cl1] Calculated 0.0, Expected 0.005615999999999995 ERROR:root:internal: Negative sideslip -> [cn1] Calculated -0.0, Expected -0.00889199999999999 ERROR:root:internal: Negative sideslip -> [cl] Calculated 0.0, Expected 0.005615999999999999 ERROR:root:internal: Negative sideslip -> [cn] Calculated -0.001765919999999999, Expected -0.01065791999999998 ERROR:root:output: Negative sideslip -> [aeroRollBodyMomentCoefficient] Calculated 0.0, Expected 0.005616 ERROR:root:output: Negative sideslip -> [aeroYawBodyMomentCoefficient] Calculated -0.41530612733186895, Expected -0.01065792 ERROR:root:internal: Positive elevator -> [cxt] Calculated -0.025, Expected -0.028603333333333335 ERROR:root:internal: Positive elevator -> [cmt] Calculated -0.127, Expected -0.13206ERROR:root:internal: Positive elevator -> [cx] Calculated -0.025, Expected -0.02860333333333333 ERROR:root:internal: Positive elevator -> [cm] Calculated -0.1784192, Expected -0.1834792 ERROR:root:output: Positive elevator -> [aeroXBodyForceCoefficient] Calculated -0.025, Expected -0.02860333333333 ERROR:root:output: Positive elevator -> [aeroPitchBodyMomentCoefficient] Calculated -0.1784192, Expected -0.1834792 \*\*\*\*\*\*\*\*\*\*\*\* Model: F-16 Subsonic Aerodynamics Model (a la Garza) creation date: 2003-06-10 file version: \$ Revision: 394 \$ \*\*\*\*\*\*\*\*\*\*\*\*

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+++++ MODEL INPUTS AND OUTPUTS +++++

++> Input: vt

```
++> Input: alpha
++> Input: beta
++> Input: p
++> Input: q
++> Input: r
++> Input: el
++> Input: ail
++> Input: rdr
++> Input: xcg
++> Output: cx
++> Output:
++> Output:
++> Output:
++> Output:
           cm
++> Output: cn
---- DAVE-ML MODEL PARSE COMPLETE ----
---- CheckModel ----
ERROR:root:internal: Negative elevator -> [cxt] Calculated -0.063, Expected
-0.024220000000000005
ERROR:root:internal: Negative elevator -> [cmt] Calculated 0.196, Expected
0.116593333333333334
ERROR:root:internal: Negative elevator -> [cx] Calculated -0.063, Expected
-0.024220000000000005
ERROR:root:internal: Negative elevator -> [cm] Calculated 0.1642192, Expected
0.08481253333333333
ERROR:root:output: Negative elevator -> [aeroXBodyForceCoefficient] Calculated
-0.063, Expected -0.02422
ERROR:root:output: Negative elevator -> [aeroPitchBodyMomentCoefficient]
Calculated 0.1642192, Expected 0.08481253333333
ERROR:root:internal: Skewed inputs -> [cxt] Calculated 0.094, Expected
0.08915927333333333
ERROR:root:internal: Skewed inputs -> [czt] Calculated -1.053, Expected -1.12812
ERROR:root:internal: Skewed inputs -> [cz1] Calculated -1.0843419673257337,
Expected -1.1592217522084587
ERROR:root:internal: Skewed inputs -> [absCl0] Calculated 0.0, Expected
-0.014256
ERROR:root:internal: Skewed inputs -> [absCn0] Calculated 0.0, Expected
ERROR:root:internal: Skewed inputs -> [clt] Calculated -0.0, Expected 0.014256
ERROR:root:internal: Skewed inputs -> [cmt] Calculated 0.01, Expected
-0.03276327333333335
```

```
ERROR:root:internal: Skewed inputs -> [cnt] Calculated -0.0, Expected -0.0108864
ERROR:root:internal: Skewed inputs -> [cxq] Calculated 2.91, Expected 2.874
ERROR:root:internal: Skewed inputs -> [cyr] Calculated 0.974, Expected 0.9368
ERROR:root:internal: Skewed inputs -> [cyp] Calculated 0.226, Expected
0.25432000000000005
ERROR:root:internal: Skewed inputs -> [czq] Calculated -30.7, Expected
-29.97999999999997
ERROR:root:internal: Skewed inputs -> [clr] Calculated 0.23, Expected
0.25136000000000003
ERROR:root:internal: Skewed inputs -> [clp] Calculated -0.375, Expected -0.36396
ERROR:root:internal: Skewed inputs -> [cmq] Calculated -6.64, Expected -6.412
ERROR:root:internal: Skewed inputs -> [cnr] Calculated -0.453, Expected
-0.47628000000000004
ERROR:root:internal: Skewed inputs -> [cnp] Calculated -0.024, Expected
-0.00623999999999982
ERROR:root:internal: Skewed inputs -> [dclda] Calculated -0.049, Expected
-0.04688399999999999
ERROR:root:internal: Skewed inputs -> [dcldr] Calculated 0.009, Expected
0.01230224
ERROR:root:internal: Skewed inputs -> [dcnda] Calculated -0.008, Expected
-0.005441279999999999
ERROR:root:internal: Skewed inputs -> [dcndr] Calculated -0.038, Expected
-0.04297872
ERROR:root:internal: Skewed inputs -> [cl1] Calculated -0.0196496, Expected
-0.004913040127999998
ERROR:root:internal: Skewed inputs -> [cn1] Calculated 0.000726999999999999,
Expected -0.008683799472
ERROR:root:internal: Skewed inputs -> [cx] Calculated 0.05227447999999999,
Expected 0.04794994533333333
ERROR:root:internal: Skewed inputs -> [cy] Calculated 0.0248125, Expected
0.027353860000000008
ERROR:root:internal: Skewed inputs -> [cz] Calculated -0.644144900659067,
Expected -0.7293485255417921
ERROR:root:internal: Skewed inputs -> [cl] Calculated -0.0409596, Expected
-0.026917840128000005
ERROR:root:internal: Skewed inputs -> [cm] Calculated -0.04101214578294153,
Expected -0.10638585796465347
ERROR:root:internal: Skewed inputs -> [cn] Calculated 0.01922069358333333,
Expected 0.011183654767653334
ERROR:root:output: Skewed inputs -> [aeroXBodyForceCoefficient] Calculated
0.05227447999999999, Expected 0.04794994533333
ERROR:root:output: Skewed inputs -> [aeroYBodyForceCoefficient] Calculated
0.0248125, Expected 0.02735386
ERROR:root:output: Skewed inputs -> [aeroZBodyForceCoefficient] Calculated
-0.644144900659067, Expected -0.72934852554344
ERROR:root:output: Skewed inputs -> [aeroRollBodyMomentCoefficient] Calculated
-0.0409596, Expected -0.026917840128
ERROR:root:output: Skewed inputs -> [aeroPitchBodyMomentCoefficient] Calculated
```

-0.04101214578294153, Expected -0.10638585796503 ERROR:root:output: Skewed inputs -> [aeroYawBodyMomentCoefficient] Calculated 0.01922069358333333, Expected 0.01118365476765

---- END CheckModel -----

#### 1.6 Environments

Create a planet class that has an atmosphere and gravity model.

#### 1.6.1 US Standard Atmosphere 1976

The height is geopotential height (Z) in meters above MSL. The reference for the US Standard Atmosphere 1976. The reference for the pressure equation.

Layer	Height (m)	Pressure (Pa)	Temperature (K)	Temperature Lapse Rate (K/m)
0	0	101,325	288.15	-0.0065
1	11,000	22,632.1	216.65	0
2	20,000	5,474.89	216.65	0.001
3	32,000	868.019	228.65	0.0028
4	47,000	110.906	270.65	0
5	51,000	66.9389	270.65	-0.0028
6	71,000	3.95642	214.65	-0.002

### 1.6.2 Gravity

The  $J_2$  gravity model. Reference is Aircraft Control and Simulation, Third Edition on page 25.

$$G^{ecef} = -\frac{GM}{r^2}\mathbf{p}$$

$$\begin{aligned} \mathbf{p_x} &= [1 + 1.5(\frac{a}{r})^2 J_2 (1 - 5 \sin^2 \! \psi)] \; p_x / r \\ \mathbf{p_y} &= [1 + 1.5(\frac{a}{r})^2 J_2 (1 - 5 \sin^2 \! \psi)] \; p_y / r \\ \mathbf{p_z} &= [1 + 1.5(\frac{a}{r})^2 J_2 (3 - 5 \sin^2 \! \psi)] \; p_z / r \end{aligned}$$

$$\mathbf{p_y} = [1 + 1.5(\frac{a}{r})^2 J_2(1 - 5\sin^2 \psi)] p_y/r$$

$$\mathbf{p_z} = [1 + 1.5(\frac{\dot{a}}{r})^2 J_2(3 - 5\sin^2\psi)] p_z/r$$

where  $p_x$ ,  $p_y$  and  $p_z$  are ECEF position components and  $\sin(\psi) = p_z/r$ .

The equations for the WGS84 gravity model.

$$e=\frac{\sqrt{a^2-b^2}}{a},\,a=$$
 semi-major axis,  $b=$  semi-minor axis  $N=\frac{a}{(1-e^2\sin^2\phi)^{1/2}},\,\phi=$  geodetic Latitude

$$N = \frac{a}{(1 - e^2 \sin^2 \phi)^{1/2}}, \ \phi = \text{geodetic Latitude}$$

$$P_r(h,\phi) = (N+h)\cos\phi, h = \text{altitude MSL}$$

$$g_0 = \frac{a(g_e)\cos^2\phi + b(g_p)\sin^2\phi}{\sqrt{a^2\cos^2\phi + b^2\sin^2\phi}}$$

 $P_r(h,\phi) = (N+h)\cos\phi, h = \text{altitude MSL}$   $g_0 = \frac{a(g_e)\cos^2\phi + b(g_p)\sin^2\phi}{\sqrt{a^2\cos^2\phi + b^2\sin^2\phi}}$ where  $g_e$  = gravity at the equator (9.7803253359) and  $g_p$  = gravity at the poles (9.8321849378)  $f = \frac{a-b}{a}, m = \frac{\omega^2 a^2 b}{GM}, \omega = \text{Earth rotation rate}$ 

$$f = \frac{a-b}{a}, m = \frac{\omega^2 a^2 b}{GM}, \omega = \text{Earth rotation rate}$$

$$g_h = g_0 \left[ 1 - \frac{2h}{a} (1 + f + m - 2f \sin^2 \phi) + \frac{3h^2}{a^2} \right]$$

$$g_h = g_0 \left[ 1 - \frac{2h}{a} (1 + f + m - 2f\sin^2\phi) + \frac{3h^2}{a^2} \right]$$
 $\vec{g}_h = -g_h \begin{pmatrix} \cos\phi\cos\lambda\\ \cos\phi\sin\lambda\\ \sin\phi \end{pmatrix}, \lambda = \text{geodetic Longitude}$ 

$$a_{hc} = \omega^2 P_2(h, \phi)$$

$$a_{hc}^{\dagger} = a_{hc} \begin{pmatrix} \cos \lambda \\ \sin \lambda \\ 0 \end{pmatrix}$$

$$g_{hG}^{\dagger} = g_{h}^{\dagger} - a_{hc}^{\dagger}$$

$$g_{hG} = |g_{hG}| = \sqrt{g_{hGx}^2 + g_{hGy}^2 + g_{hGz}^2}$$

#### 1.6.3 ECEF to LLA

J. Zhu. Conversion of earth-centered earth-fixed coordinates to geodetic coordinates. Technical Report IEEE Log NO. T-AES/30/3/1666, IEEE, December 1993.

Reference for PcpfToLlaOsen is: Karl Osen. Accurate Conversion of Earth-Fixed Earth-Centered Coordinates to Geodetic Coordinates. Research Report Norwegian University of Science and Technology. 2017. ffhal-01704943v2f located here.

#### 1.6.4 Planet Base Class

```
[13]: class ppPlanet(ppUnitTest):
          GM = 0
          J2 = 0
          Latitude = 0
          Longitude = 0
          Altitude = 0
          RotationRate = 0
          SemiMajor = 0
          Flattening = 0
          SemiMinor
                     = 0
          Eccentricity = 0
          EccentricitySquared = 0
          def CalcSemiMinor(self):
              self.SemiMinor = self.SemiMajor * ( 1.0 - self.Flattening )
          def CalcEccentricity(self):
              a = self.SemiMajor
              b = self.SemiMinor
              self.Eccentricity = (math.sqrt( a * a - b * b ) / a)
              self.EccentricitySquared = (self.Eccentricity) ** 2
          def LlaToPcpf(self):
             a = self.SemiMajor
              e2 = self.EccentricitySquared
              sinLat = math.sin( self.Latitude )
              N = a / math.sqrt(1.0 - (e2*sinLat*sinLat))
```

```
cosLat = math.cos( self.Latitude )
               # set the planet centered, planet fixed (PCPF) x,y,z vector in meters
               x = (N + self.Altitude) * cosLat * math.cos(self.Longitude)
               y = (N + self.Altitude) * cosLat * math.sin(self.Longitude)
               z = (N*(1.0 - e2) + self.Altitude) * sinLat
               return x, y, z
      def PcpfToLlaZhu(self, x, y, z):
               a = self.SemiMajor
               b = self.SemiMinor
               e = self.Eccentricity
               e2 = self.EccentricitySquared
               assert b != 0, "SemiMinor axis is 0"
               ep = e * a / b
               ep2 = ep * ep
               r = math.sqrt(x*x + y*y)
               F = 54.0 * b*b * z*z
               G = r*r + (1.0 - e2) * z*z - e2*(a*a - b*b)
               c = e2*e2*F*r*r/(G*G*G)
               s = (1.0 + c + math.sqrt(c*c + 2.0*c)) ** (1.0 / 3.0)
               P = F / (3.0*((s + 1.0/s + 1.0)**2.0)*G*G)
               Q = math.sqrt(1.0 + 2.0 * e2*e2 * P)
               r0 = -P*e2*r/(1.0 + Q) + math.sqrt(0.5*a*a*(1.0 + 1.0/Q) - (P*(1.0 + 0.0)) - (P*(1
\rightarrow 0-e2)*z*z)/(Q + Q*Q) - 0.5*P*r*r
               U = math.sqrt((r-e2*r0)**2.0 + z*z)
               V = math.sqrt((r-e2*r0)**2.0 + (1.0 - e2)*z*z)
               z0 = (b*b*z)/(a*V)
               self.Latitude = math.atan((z + ep2*z0)/r)
               self.Longitude = math.atan2(y , x)
               self.Altitude = U * (1.0 - (b*b)/(a*V))
      def PcpfToLlaOsen(self, x, y, z):
               WGS84_INVAA = +2.45817225764733181057e-0014 # 1/(a^2)
               WGS84 EED2 = +3.34718999507065852867e-0003 \# (e^2)/2
               WGS84 EEEE = +4.48147234524044602618e-0005 # e^{2}
               WGS84 EEEED4 = +1.12036808631011150655e-0005 \# (e^4)/4
               WGS84_P1MEE = +9.93305620009858682943e-0001 # 1-(e^2)
               WGS84_P1MEEDAA = +2.44171631847341700642e-0014 # (1-(e^2))/(a^2)
               WGS84_INVCBRT2 = +7.93700525984099737380e-0001 # 1/(2^(1/3))
               WGS84 INV3 = +3.3333333333333333333338e-0001 # 1/3
               WGS84_INV6
                                             ww = x * x + y * y
               m = ww * WGS84 INVAA
```

```
n = z * z * WGS84_P1MEEDAA
mpn = m + n
p = WGS84_INV6 * (mpn - WGS84_EEEE)
G = m * n * WGS84_EEEED4
H = 2 * p * p * p + G
C = ((H + G + 2 * math.sqrt(H * G))**WGS84_INV3) * WGS84_INVCBRT2
assert C != 0, "PcpfToLLaOsen C is 0"
i = -WGS84\_EEEED4 - 0.5 * mpn
P = p * p
beta = WGS84_INV3 * i - C - (P / C)
k = WGS84_EEEED4 * (WGS84_EEEED4 - mpn)
# Compute left part of t
t1 = beta * beta - k
assert t1 >= 0, "PcpfToLLaOsen t1 is negative. t1: {0}".format(t1)
t2 = math.sqrt(t1)
t3 = t2 - 0.5 * (beta + i)
assert t3 >= 0, "PcpfToLLaOsen t3 is negative"
t4 = math.sqrt(t3)
# Compute right part of t
t5 = 0.5 * (beta - i)
# t5 may accidentally drop just below zero due to numeric turbulence
# This only occurs at latitudes close to +- 45.3 degrees
t5 = abs(t5)
t6 = math.sqrt(t5)
t7 = t6 \text{ if } (m < n) \text{ else } -t6
# Add left and right parts
t = t4 + t7
# Use Newton-Raphson's method to compute t correction
j = WGS84\_EED2 * (m - n)
g = 2 * j
tt = t * t
ttt = tt * t
tttt = tt * tt
F = tttt + 2 * i * tt + g * t + k
dFdt = 4 * ttt + 4 * i * t + g;
dt = -F / dFdt
# compute latitude (range -PI/2..PI/2)
u = t + dt + WGS84\_EED2
v = t + dt - WGS84\_EED2
w = math.sqrt(ww)
zu = z * u
wv = w * v
self.Latitude = math.atan2(zu, wv)
```

```
# compute altitude
assert (u*v) != 0, "PcpfToLlaOsen (u*v) is 0"
invuv = 1 / (u * v)
dw = w - wv * invuv
dz = z - zu * WGS84_P1MEE * invuv
da = math.sqrt(dw * dw + dz * dz)
self.Altitude = -da if (u < 1) else da

# compute longitude (range -PI..PI)
self.Longitude = math.atan2(y, x);</pre>
```

# 1.6.5 Earth Class

```
[14]: class ppEarth(ppPlanet):
        def __init__(self):
                                             # GM constant in m3/s2
            self.GM = 3.986004418e14
            self.J2 = 1.082626684e-3
            self.RotationRate = 7.292115e-5 # Earth Rotation Rate (rad/sec, East)
                                             # WGS84 defined
            self.SemiMajor = 6378137.0
            self.Flattening = 1/298.257223563  # WGS84 defined
            self.CalcSemiMinor()
            self.CalcEccentricity()
        def StdAtm1976(self, altitude):
            # Geopotential Alt (m) table ranges for 1976 US standard atmosphere
            # 0 1 2 3 4 5
            Z = [0.0, 11000.0, 20000.0, 32000.0, 47000.0, 51000.0, 71000.0]
            # Temperature (K) at start of air layer
            # 0 11000 20000 32000 47000 51000 71000
            T = [288.15, 216.65, 216.65, 228.65, 270.65, 270.65, 214.65]
            # Pressure (Pa) at start air layer
            # 0 11000 20000 32000 47000 51000 71000
            P = [101325.0, 22632.10, 5474.89, 868.02, 110.91, 66.94, 3.96]
            # Temperature Gradient (K/m) for the altitude ranges
            # 0 11000 20000 32000 47000 51000 71000
            TG = [-6.5e-3, 0, 1.0e-3, 2.8e-3, 0, -2.8e-3, -2.0e-3]
            radiusEarth = 6356766.0 # Earth radius for geopotential alt conversion
            p0 = 101325.0 # pressure at sea level (Pa)
            Rgc = 287.0528
                               # Gas constant (N m / kg K)
# gravity at sea level (m / s^2)
            g0 = 9.806645
            M = 0.0289644 # molar mass of Earth's air (kg/mol)
```

```
Rstar = 8.3144598
                                # universal qas constant [J/(mol \cdot K)]
       airGamma = 1.4
                                 # gamma value for air
       # Convert geometric altitude to geopotential as the standard atmosphere
       # altitude layers are geopotential.
       z0 = radiusEarth * altitude / (radiusEarth + altitude)
       # get the index of the atmosphere layer
       i = -1
       count = 0
       for z in Z:
           if count != 0:
               if z0 < z and i == -1:
                   i = count - 1
           count += 1
       deltaZ = z0 - Z[i]
       temperature = TG[i] * deltaZ + T[i]
       temperature = temperature if (temperature > 0.0) else 0
       pressure = 0
       # The pressure is calculated differently depending
       # on the temperature lapse rate of the air layer.
       if abs(TG[i]) < 1e-12:</pre>
           pressure = P[i] * math.exp( (-g0 * M * deltaZ) / (Rstar * T[i]) )
       else:
           pe = (-g0 * M) / (Rstar * TG[i])
           pressure = P[i] * ((T[i] + TG[i] * deltaZ) / T[i])**pe
       airDensity = (pressure / (Rgc * temperature)) if (temperature > 0.0)
→else 0
       assert temperature >= 0, "temp: {}, alt: {}".format(temperature,_
→altitude)
       speedOfSoundMps = math.sqrt( airGamma * Rgc * temperature )
       return airDensity, temperature, pressure, speedOfSoundMps
   def AirDensity(self, altitude):
       result = self.StdAtm1976(altitude)
       return result[0]
   def GravityConstant(self):
       return 9.80665
   def GravityWgs84(self, latRad, lonRad, h):
```

```
a = self.SemiMajor
       b = self.SemiMinor
       E = self.Eccentricity
       sinPhi = math.sin(latRad)
       sin2Phi = sinPhi**2
       N = a / math.sqrt(1 - E*E*sin2Phi)
       cosPhi = math.cos(latRad)
       cos2Phi = cosPhi**2
       Pr = (N + h) * cosPhi
       ge = 9.7803253359
       gp = 9.8321849378
       g0 = (a*ge + cos2Phi + b*gp*sin2Phi) / math.sqrt(a*a*cos2Phi +
→b*b*sin2Phi)
       f = (a - b) / a
       w = self.RotationRate
       m = w*w*a*a*b / self.GM
       gh = g0*(1 - 2/a * (1 + f + m - 2*f*sin2Phi)*h + (3*h*h)/(a*a))
       cosLambda = math.cos(lonRad)
       sinLambda = math.sin(lonRad)
       #Ghx = -gh * cosPhi
       GhX = -gh*cosPhi*cosLambda
       GhY = -gh*cosPhi*sinLambda
       GhZ = -gh*sinPhi
       ahc = w*w*Pr
       AhcX = ahc*cosLambda
       AhcY = ahc*sinLambda
       AhcZ = 0
       GhGX = GhX - AhcX
       GhGY = GhY - AhcY
       GhGZ = GhZ - AhcZ
       return GhGX, GhGZ, GhGZ
   def GravityJ2(self, x, y, z):
      r2 = x*x + y*y + z*z
       r = math.sqrt(r2)
       assert r != 0, "Gravity J2 r is 0"
       gmOverR3 = -self.GM / (r**3)
       j2Term = (1.5 * self.J2) * (self.SemiMajor)**2 / (r**4)
       z2 = 5.0 * z * z
       gx = x * gmOverR3 * (1 - j2Term*(z2 - r2))
       gy = y * gmOverR3 * (1 - j2Term*(z2 - r2))
       gz = z * gmOverR3 * (1 - j2Term*(z2 - 3*r2))
       return gx, gy, gz
```

```
def GravityJ2SL(self, x, y, z):
       r = math.sqrt(x*x + y*y + z*z)
       assert r != 0, "Gravity J2 r is 0"
       sinPsi2 = (z / r)**2
       aOverR2 = 1.5 * self.J2 * (self.SemiMajor / r)**2
       gmOverR2 = -self.GM/(r**2)
       gx = gmOverR2 * (1 + aOverR2 * (1.0 - 5.0*sinPsi2)) * (x / r)
       gy = gmOverR2 * (1 + aOverR2 * (1.0 - 5.0*sinPsi2)) * (y / r)
       gz = gmOverR2 * (1 + aOverR2 * (3.0 - 5.0*sinPsi2)) * (z / r)
       return gx, gy, gz
   def GravityR2(self, x, y, z):
       r2 = x*x + y*y + z*z
       assert r2 != 0, "GravityR2 r2 is 0"
       return self.GM/r2
   def UnitTest(self):
       self.ClassName = "ppEarth"
       self.TestValue(6356752, self.SemiMinor, "b", 1)
       self.TestValue(8.18191908426e-2, self.Eccentricity, "eccentricity", u
→1e-12)
       # TODO: fix gravity unit tests
       #self.TestValue(9.7879, self.GravityJ2(0,0), "gravity", 1e-4)
       #self.TestValue(9.7848, self.GravityJ2(1000,math.radians(12.34)),
\rightarrow "gravity", 1e-4)
       #self.TestValue(9.7725, self.GravityJ2(5000,math.radians(24.6621)),
→ "gravity", 1e-4)
       #self.TestValue(9.72, self.GravityJ2(25000,math.radians(45.0)),
\rightarrow "gravity", 1e-2)
       #self.TestValue(9.56, self.GravityJ2(75000,math.radians(65.0)),
\rightarrow "gravity", 1e-2)
       self.ClassName = "ppEarth StdAtm1976 Density"
       di = 0 # air density index
       self.TestValue(1.225, self.StdAtm1976(0)[di], "Om", 1e-3)
       self.ClassName = "ppEarth StdAtm1976 Temperature"
       ti = 1 # temperature index
       self.TestValue(288.15, self.StdAtm1976(0)[ti], "Om", 1e-2)
       self.TestValue(275.156, self.StdAtm1976(2000)[ti], "2km", 1e-2)
       self.TestValue(255.676, self.StdAtm1976(5000)[ti], "5km", 1e-2)
       self.TestValue(216.65, self.StdAtm1976(12000)[ti], "12km", 1e-2)
       self.TestValue(222.544, self.StdAtm1976(26000)[ti], "26km", 1e-2)
```

```
self.ClassName = "ppEarth StdAtm1976 Pressure"
pi = 2 # pressure index
self.TestValue(101325, self.StdAtm1976(0)[pi], "Om", 1)
self.TestValue(79505.1, self.StdAtm1976(2000)[pi], "2km", 10)
self.TestValue(54048.8, self.StdAtm1976(5000)[pi], "5km", 10)
self.TestValue(19401, self.StdAtm1976(12000)[pi], "12km", 10)
self.TestValue(2188.41, self.StdAtm1976(26000)[pi], "26km", 1)
self.ClassName = "ppEarth StdAtm1976 Sound"
si = 3 # speed of sound index
self.TestValue(340.294, self.StdAtm1976(0)[si], "Om", 1e-3)
self.ClassName = "ppPlanet Olsen"
self.PcpfToLlaOsen(1191786.0, -5157122.0, 3562840.0)
self.TestValue(34.123456, math.degrees(self.Latitude), "lat", 1e-6)
self.TestValue(-76.987654, math.degrees(self.Longitude), "lon", 1e-6)
self.TestValue(9000.0, self.Altitude, "alt", 1)
self.ClassName = "ppPlanet Zhu"
self.PcpfToLlaZhu(1191786.0, -5157122.0, 3562840.0)
self.TestValue(34.123456, math.degrees(self.Latitude), "lat", 1e-6)
self.TestValue(-76.987654, math.degrees(self.Longitude), "lon", 1e-6)
self.TestValue(9000.0, self.Altitude, "alt", 1)
self.ClassName = "ppPlanet"
self.Latitude = math.radians(34.123456)
self.Longitude = math.radians(-76.987654)
self.Altitude = 9000.0
[x, y, z] = self.LlaToPcpf()
self.TestValue(1191786.0, x, "X", 1)
self.TestValue(-5157122.0, y, "Y", 1)
self.TestValue(3562840.0, z, "Z", 1)
print("Number of ppEarth failed tests: ", self.FailCount)
```

#### 1.6.6 Moon Class

The reference for the moon parameters is NESC Academy Presentation.

```
self.Flattening = 1.0 / 800.98618
self.CalcSemiMinor()
self.CalcEccentricity()

def AirDensity(self, altitude):
    return 0

def Gravity(self, altitude, latRad):
    r = altitude + self.SemiMajor
    gravity = self.GM/r/r
    return gravity

def UnitTest(self):
    self.ClassName = "ppMoon"
    self.TestValue(1.62242, self.Gravity(0,0), "gravity", 1e-6)

    print("Number of ppMoon failed tests: ", self.FailCount)
```

#### 1.6.7 Mars Class

The reference for the Mars atmosphere model.

```
[16]: class ppMars(ppPlanet):
         def init (self):
              self.GM = 42828.371901284e9
             self.RotationRate = 7.0882181e-5 # Mars Rotation Rate (rad/sec, East)
             self.SemiMajor = 3.396196e6
             self.J2 = 0.00195545367944545
              self.CalcSemiMinor()
         def AirDensity(self, altitude):
             temperatureC = 0
             if altitude > 7000:
                  temperatureC = -31 - 0.000998 * altitude
              else:
                  temperatureC = -23.4 - 0.00222 * altitude
             pressureKPa = 0.699 * math.exp(-0.00009 * altitude)
              airDensity = pressurePa / (0.1921 * (temperatureC + 273.1))
             return airDensity
         def Gravity(self, altitude, latRad):
             marsGM = self.GM
             marsRadiusMeter = self.SemiMajor
             J2 = self.J2
```

```
[17]: earth = ppEarth()
      earth.UnitTest()
      moon = ppMoon()
      moon.UnitTest()
      mars = ppMars()
      mars.UnitTest()
      def ToGeopotential(altitude):
          radiusEarth = 6356766.0
          z0 = radiusEarth * altitude / (radiusEarth + altitude)
          return round(z0)
      alt = [0, 2000, 5000, 12000, 26000, 37500, 50000, 60000, 75000]
      print("\nGeopotential table")
      for a in alt:
          geoTable = "{:=6d} -> {:=6d}".format(a, ToGeopotential(a))
          print(geoTable)
      print("=== gravity ===")
      print("-- J2 --")
      myGx, myGy, myGz = earth.GravityJ2( earth.SemiMajor + 9144, 0, 0)
      print(myGx, myGy, myGz)
```

```
print("-- J2 Steven & Lewis --")
      myGx, myGy, myGz = earth.GravityJ2SL( earth.SemiMajor + 9144, 0, 0)
      print(myGx, myGy, myGz)
      print("-- WGS84 --")
      wgx, wgy, wgz = earth. Gravity Wgs84(0.0, 0.0, 9144.0) # 30,000 ft
      print(wgx, wgy, wgz)
     Number of ppEarth failed tests: 0
     Number of ppMoon failed tests: 0
     Number of ppMars failed tests: 0
     Geopotential table
          0 ->
       2000 -> 1999
       5000 -> 4996
      12000 -> 11977
      26000 -> 25894
      37500 -> 37280
      50000 -> 49610
      60000 -> 59439
      75000 -> 74125
     === gravity ===
     -- J2 --
     -9.786072112297859 -0.0 -0.0
     -- J2 Steven & Lewis --
     -9.786072112297857 -0.0 -0.0
     -- WGS84 --
     -9.786116284019755 -0.0 -0.0
[18]: import matplotlib.pyplot as plt
      h = []
      t76 = []
      rho = []
      p = []
      for i in range(90):
         alt = i * 1000.0
          h.append(i)
          [airDensity, temperature, pressure, speedOfSoundMps] = earth.StdAtm1976(alt)
          t76.append(temperature)
          rho.append(airDensity)
          p.append(0.001*pressure)
      altT = (0, 11, 20, 32, 47, 51, 71, 86)
      tmpT = (288.15, 216.65, 216.65, 228.65, 270.65, 270.65, 214.65, 186.9)
```

```
fig, (tp1, dp2, pp3) = plt.subplots(1,3)
fig.suptitle('Standard Atmosphere 1976')

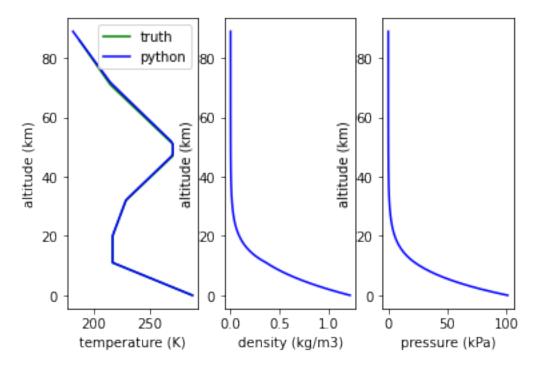
tp1.plot(tmpT, altT, 'g', t76, h, 'b')
tp1.set(xlabel='temperature (K)', ylabel='altitude (km)')
tp1.legend(["truth","python"])
#tp1.grid()

dp2.plot(rho, h, 'b')
dp2.set(xlabel='density (kg/m3)', ylabel='altitude (km)')

pp3.plot(p, h, 'b')
pp3.set(xlabel='pressure (kPa)', ylabel='altitude (km)')
```

[18]: [Text(0.5, 0, 'pressure (kPa)'), Text(0, 0.5, 'altitude (km)')]

# Standard Atmosphere 1976



```
[19]: from IPython.display import Image
Image(url= "images/StdAtm1976.png")
```

[19]: <IPython.core.display.Image object>

# 1.7 Vector Class

Make a vector class for 3 element vectors.

```
[20]: class ppVector3(ppUnitTest):
          def __init__(self, x, y, z):
              self.X = x
              self.Y = y
              self.Z = z
          # defining how to print the class
          def __str__(self):
             return "(%s, %s, %s)" % (self.X, self.Y, self.Z)
          # overloading the + to add vectors
          def __add__(self, o):
             x = self.X + o.X
              y = self.Y + o.Y
              z = self.Z + o.Z
              return ppVector3(x,y,z)
          # overloading the - to subtract vectors
          def __sub__(self, o):
             x = self.X - o.X
              y = self.Y - o.Y
              z = self.Z - o.Z
              return ppVector3(x,y,z)
          # overloading the ^ for cross product
          def __xor__(self, o):
             x = self.Y * o.Z - self.Z * o.Y
              y = self.Z * o.X - self.X * o.Z
              z = self.X * o.Y - self.Y * o.X
              return ppVector3(x,y,z)
          # overloading the * to multiply scalars to a vector
          def __mul__(self, s):
             x = self.X * s
              y = self.Y * s
              z = self.Z * s
              return ppVector3(x,y,z)
          # overloading the / to divide a vector by a scalar
          def __truediv__(self, s):
             x = self.X / s
              y = self.Y / s
              z = self.Z / s
```

```
return ppVector3(x,y,z)
__rmul__ = __mul__
def Set(self, x, y, z):
   self.X = x
   self.Y = y
   self.Z = z
def Magnitude(self):
   magnitude = math.sqrt(self.X*self.X + self.Y*self.Y + self.Z*self.Z)
   return magnitude
def UnitTest(self):
    self.ClassName = "ppVector3"
   v1 = ppVector3(21,33,19)
    self.TestValue(21, v1.X, "init X", 1e-6)
    self.TestValue( 33, v1.Y, "init Y", 1e-6)
   self.TestValue( 19, v1.Z, "init Z", 1e-6)
   v2 = ppVector3(21,33,19)
   v3 = ppVector3(79,67,81)
   v1 = v2 + v3
   self.TestValue( 100, v1.X, "add X", 1e-6)
   self.TestValue( 100, v1.Y, "add Y", 1e-6)
    self.TestValue( 100, v1.Z, "add Z", 1e-6)
   v4 = ppVector3(0,3,-4)
   self.TestValue( 5, v4.Magnitude(), "magnitude", 1e-6)
   v4.Set( 87, 16.9, -3.1)
    self.TestValue( 87, v4.X, "Set X", 1e-6)
    self.TestValue( 16.9, v4.Y, "Set Y", 1e-6)
    self.TestValue( -3.1, v4.Z, "Set Z", 1e-6)
   v5 = v3 - v2
    self.TestValue(58, v5.X, "sub X", 1e-6)
    self.TestValue( 34, v5.Y, "sub Y", 1e-6)
   self.TestValue(62, v5.Z, "sub Z", 1e-6)
   v5 = v2 - v3
   self.TestValue( -58, v5.X, "sub X", 1e-6)
   self.TestValue( -34, v5.Y, "sub Y", 1e-6)
   self.TestValue( -62, v5.Z, "sub Z", 1e-6)
   v6 = ppVector3(4,12,2)
   v7 = ppVector3(13,5,7)
   v8 = v6 ^ v7
   self.TestValue( 74, v8.X, "cross X", 1e-6)
   self.TestValue( -2, v8.Y, "cross Y", 1e-6)
   self.TestValue( -136, v8.Z, "cross Z", 1e-6)
   v9 = 2 * v6
    self.TestValue( 8, v9.X, "mul X", 1e-6)
```

```
self.TestValue( 24, v9.Y, "mul Y", 1e-6)
self.TestValue( 4, v9.Z, "mul Z", 1e-6)
v10 = v7 / 2
self.TestValue( 6.5, v10.X, "div X", 1e-6)
self.TestValue( 2.5, v10.Y, "div Y", 1e-6)
self.TestValue( 3.5, v10.Z, "div Z", 1e-6)
v11 = v10
self.TestValue( 6.5, v11.X, "= X", 1e-6)
self.TestValue( 2.5, v11.Y, "= Y", 1e-6)
self.TestValue( 3.5, v11.Z, "= Z", 1e-6)
print("Number of ppVector3 failed tests: ", self.FailCount)
```

```
[21]: myVector = ppVector3(0,0,0)
myVector.UnitTest()
```

Number of ppVector3 failed tests: (

## 1.8 Quaternion Class

Create a quaternion class. Reference for checking quaternion rotation. Quaternion multiplication checked here.

$$t = q * r = t_0 + \mathbf{i}t_1 + \mathbf{j}t_2 + \mathbf{k}t_3$$

$$t_0 = q_0r_0 - q_1r_1 - q_2r_2 - q_3r_3$$

$$t_1 = q_1r_0 + q_0r_1 - q_3r_2 + q_2r_3$$

$$t_2 = q_2r_0 + q_3r_1 + q_0r_2 - q_1r_3$$

$$t_3 = q_3r_0 - q_2r_1 + q_1r_2 + q_0r_3$$
System b to a ->  $q_{b/a}$ 

$$u^b = q_{b/a}^{-1} * u^a * q_{b/a}, \text{ and } q_{b/a}^{-1} = q_{a/b}$$

$$u^c = q_{c/b}^{-1}q_{b/a}^{-1}u^a q_{b/a}q_{c/b}$$

$$v^{frd} = q_{\phi}^{-1}q_{\theta}^{-1}q_{\psi}^{-1}v^{ned}q_{\psi}q_{\theta}q_{\phi}$$

$$q_{frd/ned} = q_{\psi}q_{\theta}q_{\phi} = \begin{cases} \cos\frac{\phi}{2}\cos\frac{\psi}{2} + \sin\frac{\phi}{2}\sin\frac{\theta}{2}\sin\frac{\psi}{2} \\ \sin\frac{\phi}{2}\cos\frac{\psi}{2}\cos\frac{\psi}{2} + \sin\frac{\phi}{2}\cos\frac{\theta}{2}\sin\frac{\psi}{2} \\ \cos\frac{\phi}{2}\sin\frac{\theta}{2}\cos\frac{\psi}{2} + \sin\frac{\phi}{2}\cos\frac{\theta}{2}\sin\frac{\psi}{2} \\ \cos\frac{\phi}{2}\cos\frac{\psi}{2}\sin\frac{\psi}{2} - \sin\frac{\phi}{2}\sin\frac{\psi}{2}\cos\frac{\psi}{2} \\ \cos\frac{\phi}{2}\cos\frac{\psi}{2}\sin\frac{\psi}{2} - \sin\frac{\phi}{2}\cos\frac{\psi}{2}\sin\frac{\psi}{2} \\ \cos\frac{\phi}{2}\cos\frac{\psi}{2}\sin\frac{\psi}{2} - \sin\frac{\phi}{2}\cos\frac{\psi}{2}\sin\frac{\psi}{2} \\ \cos\frac{\phi}{2}\cos\frac{\psi}{2}\sin\frac{\psi}{2} - \sin\frac{\phi}{2}\cos\frac{\psi}{2}\sin\frac{\psi}{2}\cos\frac{\psi}{2} \end{cases}$$

$$q_{ned/ecf} = \frac{\cos\frac{lon}{2}\cos(\frac{lat}{2} + 45^\circ)}{\sin\frac{lon}{2}\cos(\frac{lat}{2} + 45^\circ)}$$

$$\sin\frac{lon}{2}\cos(\frac{lat}{2} + 45^\circ)$$

$$\sin$$

```
\begin{split} v^{ecf} &= q_{ecf/ned}^{-1} q_{ned/frd}^{-1} \, v^{frd} \, q_{ned/frd} q_{ecf/ned} \\ &= q_{ned/ecf} q_{frd/ned} \, v^{frd} \, q_{frd/ned}^{-1} q_{ned/ecf}^{-1} \\ u^{ned} &= q_{ned/frd}^{-1} \, u^{frd} \, q_{ned/frd} \\ q_{ned/frd}^{-1} &= q_{frd/ned} \\ u^{ned} &= q_{frd/ned} \, \begin{bmatrix} 0 \\ u \end{bmatrix} \, q_{frd/ned}^{-1} \end{split}
```

```
[22]: class ppQuaternion(ppUnitTest):
          def __init__(self, n, x, y, z):
              self.N = n
              self.X = x
              self.Y = y
              self.Z = z
          # defining how to print the class
          def __repr__(self):
              return "(%s, %s, %s, %s)" % (self.N, self.X, self.Y, self.Z)
          # overloading the ~ for quaternion inverse
          def __invert__(self):
              n = self.N
              x = -self.X
              y = -self.Y
              z = -self.Z
              return ppQuaternion(n,x,y,z)
          # overloading the + to add quaternions
          def __add__(self, o):
              n = self.N + o.N
              x = self.X + o.X
              y = self.Y + o.Y
              z = self.Z + o.Z
              return ppQuaternion(n,x,y,z)
          \# overlanding the * to multiply quaternions and multiple scalars and
       \rightarrow quaternions
          def __mul__(self,o):
              n=0
              x=0
              y=0
              z=0
              if isinstance(o, ppQuaternion):
                  n = self.N*o.N - self.X*o.X - self.Y*o.Y - self.Z*o.Z
                  x = self.N*o.X + self.X*o.N + self.Y*o.Z - self.Z*o.Y
                  y = self.N*o.Y + self.Y*o.N + self.Z*o.X - self.X*o.Z
```

```
z = self.N*o.Z + self.Z*o.N + self.X*o.Y - self.Y*o.X
       elif isinstance(o, ppVector3):
           n = -(self.X*o.X + self.Y*o.Y + self.Z*o.Z)
                 self.N*o.X + self.Y*o.Z - self.Z*o.Y
           y = self.N*o.Y + self.Z*o.X - self.X*o.Z
           z = self.N*o.Z + self.X*o.Y - self.Y*o.X
       else:
          n = self.N * o
           x = self.X * o
           y = self.Y * o
           z = self.Z * o
       return ppQuaternion(n,x,y,z)
   \# so that scalar * quaternion is the same as quaternion * scalar
   __rmul__ = __mul__
   def Normalize(self):
       magnitude = math.sqrt(self.N*self.N + self.X*self.X + self.Y*self.Y +
\hookrightarrowself.Z*self.Z)
       if magnitude != 0:
           self.N = self.N / magnitude
           self.X = self.X / magnitude
           self.Y = self.Y / magnitude
           self.Z = self.Z / magnitude
   def SetRollPitchYaw(self, roll, pitch, yaw):
       qroll = ppQuaternion( math.cos(0.5*roll) , math.sin(0.5*roll), 0.0
       qpitch = ppQuaternion( math.cos(0.5*pitch), 0.0
                                                                      , math.
\rightarrowsin(0.5*pitch), 0.0)
       qyaw = ppQuaternion( math.cos(0.5*yaw) , 0.0
                                                                      , 0.0
            , math.sin(0.5*yaw))
       # ZYX rotation
       q = qyaw*qpitch*qroll
       q.Normalize()
       self.N = q.N
       self.X = q.X
       self.Y = q.Y
       self.Z = q.Z
   def SetLatLon(self, lat, lon):
       n = math.cos(0.5*lon)*math.cos(0.5*lat + 0.25*math.pi)
       x = math.sin(0.5*lon)*math.sin(0.5*lat + 0.25*math.pi)
       y = -math.cos(0.5*lon)*math.sin(0.5*lat + 0.25*math.pi)
```

```
z = math.sin(0.5*lon)*math.cos(0.5*lat + 0.25*math.pi)
       q = ppQuaternion( n, x, y, z )
       q.Normalize()
       self.N = q.N
       self.X = q.X
       self.Y = q.Y
       self.Z = q.Z
   def SetQfrdWrtEcf(self, roll, pitch, yaw, lat, lon):
       qroll = ppQuaternion( math.cos(0.5*roll) , math.sin(0.5*roll), 0.0
            , 0.0)
\hookrightarrow
       qpitch = ppQuaternion( math.cos(0.5*pitch), 0.0
                                                                        , math.
\rightarrowsin(0.5*pitch), 0.0)
       qyaw = ppQuaternion( math.cos(0.5*yaw) , 0.0
                                                                       , 0.0 <sub>L</sub>
             , math.sin(0.5*yaw))
       hLon = 0.5*lon
       hLat = 0.5*lat + 0.25*math.pi
       qlon = ppQuaternion(math.cos(hLon), 0, 0, math.sin(hLon))
       qlat = ppQuaternion(math.cos(hLat), 0, -math.sin(hLat), 0)
       # ZYX rotation
       q = qlon*qlat*qyaw*qpitch*qroll
       self.N = q.N
       self.X = q.X
       self.Y = q.Y
       self.Z = q.Z
   def SetPlanetRotation(self, rotationAngle_rad):
       n = math.cos(0.5*rotationAngle_rad)
       z = math.sin(0.5*rotationAngle_rad)
       q = ppQuaternion(n, 0.0, 0.0, z)
       q.Normalize()
       self.N = q.N
       self.X = q.X
       self.Y = q.Y
       self.Z = q.Z
   def EulerAnglesFromQ(self):
       q0 = self.N
       q1 = self.X
       q2 = self.Y
```

```
q3 = self.Z
   c11 = q0*q0 + q1*q1 - q2*q2 - q3*q3
   c12 = 2.0*(q1*q2 + q0*q3)
   c13 = 2.0*(q1*q3 - q0*q2)
   c23 = 2.0*(q2*q3 + q0*q1)
   c33 = q0*q0 - q1*q1 - q2*q2 + q3*q3
   roll = math.atan2(c23,c33)
   pitch = -math.asin(c13)
   yaw = math.atan2(c12,c11)
   return [roll, pitch, yaw]
def EulerAnglesFromQold(self):
   qnn = self.N * self.N
    qxx = self.X * self.X
   qyy = self.Y * self.Y
    qzz = self.Z * self.Z
   img = qxx + qyy + qzz + qnn
   assert img != 0, "EulerAnglesFromQ all elements 0 for quaternion"
   img = 1.0 / img
   m11 = (qnn + qxx - qyy - qzz)*img
   m12 = 2.0*(self.X*self.Y + self.Z*self.N)*img
   m13 = 2.0*(self.X*self.Z - self.Y*self.N)*img
   m23 = 2.0*(self.Y*self.Z + self.X*self.N)*img
   m33 = (qnn - qxx - qyy + qzz)*img
   roll = 0
   pitch = 0
   yaw = 0
    if abs(m13) >= 1.0:
       m21 = 2.0*(self.X*self.Y - self.Z*self.N)*img
       m31 = 2.0*(self.X*self.Z + self.Y*self.N)*img;
       roll = 0.0
       halfPi = 0.5*math.pi
        pitch = -halfPi if (m13 > 0.0) else halfPi
       yaw = math.atan2(-m21, -m31/m13)
    else:
       roll = math.atan2(m23,m33) # Roll
                                 # Pitch
        pitch = math.asin(-m13)
       yaw = math.atan2(m12, m11) # Yaw
   return [roll, pitch, yaw]
```

```
def UnitTest(self):
    self.ClassName = "ppQuaternion"
    q0 = ppQuaternion(4,7,8,9)
    q0i = ~q0
    self.TestValue( 4, q0i.N, "inverse", 1e-6)
    self.TestValue(-7, q0i.X, "inverse", 1e-6)
    self.TestValue(-8, q0i.Y, "inverse", 1e-6)
    self.TestValue(-9, q0i.Z, "inverse", 1e-6)
    q1 = ppQuaternion(2,3,4,5)
    q2 = ppQuaternion(8,9,10,11)
    q3 = q1 + q2
    self.TestValue(10, q3.N, "add", 1e-6)
   self.TestValue(12, q3.X, "add", 1e-6)
   self.TestValue(14, q3.Y, "add", 1e-6)
    self.TestValue(16, q3.Z, "add", 1e-6)
    q4 = q1 * q2
    self.TestValue(-106, q4.N, "multiply", 1e-6)
    self.TestValue(36, q4.X, "multiply", 1e-6)
    self.TestValue(64, q4.Y, "multiply", 1e-6)
    self.TestValue(56, q4.Z, "multiply", 1e-6)
    q5 = 7.0 * q1
    self.TestValue(14, q5.N, "scalar multiply", 1e-6)
    self.TestValue(21, q5.X, "scalar multiply", 1e-6)
    self.TestValue(28, q5.Y, "scalar multiply", 1e-6)
    self.TestValue(35, q5.Z, "scalar multiply", 1e-6)
    q6 = q2 * 10
    self.TestValue(80, q6.N, "scalar multiply", 1e-6)
   self.TestValue(90, q6.X, "scalar multiply", 1e-6)
    self.TestValue(100, q6.Y, "scalar multiply", 1e-6)
    self.TestValue(110, q6.Z, "scalar multiply", 1e-6)
    q6.SetRollPitchYaw(0.3,-0.7,3.11)
    self.TestValue(-0.0365642, q6.N, "Euler", 1e-6)
    self.TestValue(0.3412225, q6.X, "Euler", 1e-6)
    self.TestValue(0.1350051, q6.Y, "Euler", 1e-6)
    self.TestValue(0.9295181, q6.Z, "Euler", 1e-6)
    [roll, pitch, yaw] = q6.EulerAnglesFromQ()
    self.TestValue( 0.3, roll, "EulerFromQ", 1e-6)
    self.TestValue(-0.7, pitch, "EulerFromQ", 1e-6)
    self.TestValue(3.11, yaw, "EulerFromQ", 1e-6)
    q7 = ppQuaternion(0.6680766, 0.2325211, 0.1160514, 0.6972372)
    [roll, pitch, yaw] = q7.EulerAnglesFromQ()
    self.TestValue(0.5, roll, "EulerFromQ", 1e-6)
    self.TestValue(-0.17, pitch, "EulerFromQ", 1e-6)
    self.TestValue(1.57, yaw, "EulerFromQ", 1e-6)
    q8 = ppQuaternion(6,-6,6,6)
    q8.Normalize()
    self.TestValue(0.5, q8.N, "Normalize", 1e-6)
```

```
self.TestValue(-0.5, q8.X, "Normalize", 1e-6)
self.TestValue( 0.5, q8.Y, "Normalize", 1e-6)
self.TestValue( 0.5, q8.Z, "Normalize", 1e-6)
q9 = ppQuaternion(1,3,-2,7)
q9.Normalize()
mag = math.sqrt(1 + 9 + 4 + 49)
self.TestValue( 1.0/mag, q9.N, "Normalize 2", 1e-6)
self.TestValue( 3.0/mag, q9.X, "Normalize 2", 1e-6)
self.TestValue(-2.0/mag, q9.Y, "Normalize 2", 1e-6)
self.TestValue( 7.0/mag, q9.Z, "Normalize 2", 1e-6)
print("Number of ppQuaternion failed tests: ", self.FailCount)
```

```
[23]: q1 = ppQuaternion(9,4,5,6)
q1.UnitTest()
```

Number of ppQuaternion failed tests: 0

#### 1.9 Matrix Class

Create a 3x3 matrix class.

```
[24]: class ppMatrix3x3(ppUnitTest):
          A11 = 1
          A12 = 0
          A13 = 0
          A21 = 0
          A22 = 1
          A23 = 0
          A31 = 0
          A32 = 0
          A33 = 1
          def __mul__(self, v):
              if isinstance(v, ppVector3):
                  x = self.A11 * v.X + self.A12 * v.Y + self.A13 * v.Z
                  y = self.A21 * v.X + self.A22 * v.Y + self.A23 * v.Z
                  z = self.A31 * v.X + self.A32 * v.Y + self.A33 * v.Z
                  return ppVector3(x,y,z)
              elif isinstance(v, ppMatrix3x3):
                  a11 = self.A11*v.A11 + self.A12*v.A21 + self.A13*v.A31
                  a12 = self.A11*v.A12 + self.A12*v.A22 + self.A13*v.A32
                  a13 = self.A11*v.A13 + self.A12*v.A23 + self.A13*v.A33
                  a21 = self.A21*v.A11 + self.A22*v.A21 + self.A23*v.A31
                  a22 = self.A21*v.A12 + self.A22*v.A22 + self.A23*v.A32
```

```
a23 = self.A21*v.A13 + self.A22*v.A23 + self.A23*v.A33
        a31 = self.A31*v.A11 + self.A32*v.A21 + self.A33*v.A31
        a32 = self.A31*v.A12 + self.A32*v.A22 + self.A33*v.A32
        a33 = self.A31*v.A13 + self.A32*v.A23 + self.A33*v.A33
        a = ppMatrix3x3()
        a.SetRow1( a11, a12, a13 )
        a.SetRow2( a21, a22, a23 )
        a.SetRow3( a31, a32, a33 )
        return a
    else:
        a11 = self.A11 * v
        a12 = self.A12 * v
        a13 = self.A13 * v
        a21 = self.A21 * v
        a22 = self.A22 * v
        a23 = self.A23 * v
        a31 = self.A31 * v
        a32 = self.A32 * v
        a33 = self.A33 * v
        a = ppMatrix3x3()
        a.SetRow1( a11, a12, a13 )
        a.SetRow2( a21, a22, a23 )
        a.SetRow3( a31, a32, a33 )
        return a
def SetRow1(self, a11, a12, a13):
    self.A11 = a11
    self.A12 = a12
    self.A13 = a13
def SetRow2(self, a21, a22, a23):
    self.A21 = a21
    self.A22 = a22
    self.A23 = a23
def SetRow3(self, a31, a32, a33):
    self.A31 = a31
    self.A32 = a32
    self.A33 = a33
# defining how to print the class
def __str__(self):
    row1 = "(%s, %s, %s)\n" % (self.A11, self.A12, self.A13)
```

```
row2 = "(\%s, \%s, \%s) \n" \% (self.A21, self.A22, self.A23)
    row3 = "(%s, %s, %s)" % (self.A31, self.A32, self.A33)
    return row1+row2+row3
def Determinant(self):
    d1 = self.A11*(self.A22*self.A33 - self.A23*self.A32)
    d2 = self.A12*(self.A23*self.A31 - self.A21*self.A33)
    d3 = self.A13*(self.A21*self.A32 - self.A22*self.A31)
    return d1+d2+d3
def Inverse(self):
    D = self.Determinant()
    im = ppMatrix3x3()
    # make sure D is not O
    if abs(D) > 1e-12:
        a11 = (self.A22*self.A33 - self.A23*self.A32)/D
        a12 = (self.A13*self.A32 - self.A12*self.A33)/D
        a13 = (self.A12*self.A23 - self.A13*self.A22)/D
        a21 = (self.A23*self.A31 - self.A21*self.A33)/D
        a22 = (self.A11*self.A33 - self.A13*self.A31)/D
        a23 = (self.A13*self.A21 - self.A11*self.A23)/D
        a31 = (self.A21*self.A32 - self.A22*self.A31)/D
        a32 = (self.A12*self.A31 - self.A11*self.A32)/D
        a33 = (self.A11*self.A22 - self.A12*self.A21)/D
        im.SetRow1(a11, a12, a13)
        im.SetRow2(a21, a22, a23)
        im.SetRow3(a31, a32, a33)
    return im
def Transpose(self):
    at = ppMatrix3x3()
    at.SetRow1(self.A11, self.A21, self.A31)
    at.SetRow2(self.A12, self.A22, self.A32)
    at.SetRow3(self.A13, self.A23, self.A33)
    return at
def QuaternionToMatrix(self, q):
    n = q.N;
    x = q.X;
    y = q.Y;
    z = q.Z;
```

```
self.SetRow1( n*n + x*x - y*y - z*z, 2.0*(x*y - n*z),
\rightarrow 0*(x*z + n*y))
                         2.0*(x*y + n*z), n*n - x*x + y*y - z*z,
      self.SetRow2(
                                                                          2.
\hookrightarrow 0*(y*z - n*x))
                       2.0*(x*z - n*y), 2.0*(y*z + n*x), n*n - x*x - 1
      self.SetRow3(
\hookrightarrow y * y + z * z )
  def UnitTest(self):
       self.ClassName = "ppMatrix3"
      m1 = ppMatrix3x3()
      m1.SetRow1(1,2,3)
      m1.SetRow2(4,5,6)
      m1.SetRow3(7,3,9)
      self.TestValue(-30, m1.Determinant(), "Determinant", 1e-6)
      m1.SetRow1(1,2,3)
      m1.SetRow2(0,1,4)
      m1.SetRow3(5,6,0)
      self.TestValue(1, m1.Determinant(), "Determinant", 1e-6)
      m1 = m1.Inverse()
      self.TestValue(-24, m1.A11, "Inverse A11", 1e-6)
      self.TestValue( 18, m1.A12, "Inverse A12", 1e-6)
      self.TestValue( 5, m1.A13, "Inverse A13", 1e-6)
       self.TestValue( 20, m1.A21, "Inverse A21", 1e-6)
      self.TestValue(-15, m1.A22, "Inverse A22", 1e-6)
      self.TestValue( -4, m1.A23, "Inverse A23", 1e-6)
      self.TestValue( -5, m1.A31, "Inverse A31", 1e-6)
      self.TestValue( 4, m1.A32, "Inverse A32", 1e-6)
      self.TestValue( 1, m1.A33, "Inverse A33", 1e-6)
      m2 = ppMatrix3x3()
      m2.SetRow1(1,2,3)
      m2.SetRow2(4,5,6)
      m2.SetRow3(7,2,9)
      m2 = m2.Inverse()
      self.TestValue(-11/12, m2.A11, "Inverse A11", 1e-6)
      self.TestValue( 1/3, m2.A12, "Inverse A12", 1e-6)
      self.TestValue( 1/12, m2.A13, "Inverse A13", 1e-6)
      self.TestValue( -1/6, m2.A21, "Inverse A21", 1e-6)
       self.TestValue( 1/3, m2.A22, "Inverse A22", 1e-6)
      self.TestValue( -1/6, m2.A23, "Inverse A23", 1e-6)
      self.TestValue( 3/4, m2.A31, "Inverse A31", 1e-6)
      self.TestValue( -1/3, m2.A32, "Inverse A32", 1e-6)
      self.TestValue( 1/12, m2.A33, "Inverse A33", 1e-6)
      m3 = ppMatrix3x3()
       m3.SetRow1(1,2,3)
```

```
m3.SetRow2(-4,5,6)
m3.SetRow3(7,8.1,9)
m4 = m3.Transpose()
self.TestValue(
                  1, m3.A11, "Transpose A11", 1e-6)
                  2, m3.A12, "Transpose A12", 1e-6)
self.TestValue(
                3, m3.A13, "Transpose A13", 1e-6)
self.TestValue(
self.TestValue( -4, m3.A21, "Transpose A21", 1e-6)
self.TestValue( 5, m3.A22, "Transpose A22", 1e-6)
                 6, m3.A23, "Transpose A23", 1e-6)
self.TestValue(
self.TestValue( 7, m3.A31, "Transpose A31", 1e-6)
self.TestValue( 8.1, m3.A32, "Transpose A32", 1e-6)
self.TestValue( 9, m3.A33, "Transpose A33", 1e-6)
self.TestValue( 1, m4.A11, "Transpose A11", 1e-6)
self.TestValue( -4, m4.A12, "Transpose A12", 1e-6)
self.TestValue( 7, m4.A13, "Transpose A13", 1e-6)
                2, m4.A21, "Transpose A21", 1e-6)
self.TestValue(
self.TestValue( 5, m4.A22, "Transpose A22", 1e-6)
self.TestValue( 8.1, m4.A23, "Transpose A23", 1e-6)
self.TestValue( 3, m4.A31, "Transpose A31", 1e-6)
self.TestValue( 6, m4.A32, "Transpose A32", 1e-6)
self.TestValue( 9, m4.A33, "Transpose A33", 1e-6)
q = ppQuaternion(0.7, 0.4, 3.2, -0.87)
q.Normalize()
m4.QuaternionToMatrix(q)
self.TestValue( -0.8883823, m4.A11, "Quaternion A11", 1e-7)
self.TestValue( 0.3243782, m4.A12, "Quaternion A12", 1e-7)
self.TestValue( 0.3248933, m4.A13, "Quaternion A13", 1e-7)
self.TestValue( 0.1152238, m4.A21, "Quaternion A21", 1e-7)
self.TestValue( 0.8425504, m4.A22, "Quaternion A22", 1e-7)
self.TestValue( -0.5261486, m4.A23, "Quaternion A23", 1e-7)
self.TestValue( -0.4444101, m4.A31, "Quaternion A31", 1e-7)
self.TestValue( -0.4299857, m4.A32, "Quaternion A32", 1e-7)
self.TestValue( -0.7858829, m4.A33, "Quaternion A33", 1e-7)
m1.SetRow1(2,6,3)
m1.SetRow2(1,1,8)
m1.SetRow3(5,7,-6)
v1 = ppVector3(9,11,-4)
v2 = m1 * v1
self.TestValue( 72, v2.X, "Matrix * Vector X", 1e-7)
self.TestValue( -12, v2.Y, "Matrix * Vector Y", 1e-7)
self.TestValue( 146, v2.Z, "Matrix * Vector Z", 1e-7)
m1.SetRow1(6,3,17)
m1.SetRow2(-4,-0.1,7)
```

```
m1.SetRow3(14,5,-1)
m2.SetRow1(5,0,-6)
m2.SetRow2(3,8,2)
m2.SetRow3(-1,-4,-9)
m3 = m1 * m2
self.TestValue(
                   22, m3.A11, "Matrix * Matrix A11", 1e-7)
self.TestValue( -44, m3.A12, "Matrix * Matrix A12", 1e-7)
self.TestValue( -183, m3.A13, "Matrix * Matrix A13", 1e-7)
self.TestValue( -27.3, m3.A21, "Matrix * Matrix A21", 1e-7)
self.TestValue( -28.8, m3.A22, "Matrix * Matrix A22", 1e-7)
self.TestValue( -39.2, m3.A23, "Matrix * Matrix A23", 1e-7)
self.TestValue( 86, m3.A31, "Matrix * Matrix A31", 1e-7)
self.TestValue(
                  44, m3.A32, "Matrix * Matrix A32", 1e-7)
self.TestValue( -65, m3.A33, "Matrix * Matrix A33", 1e-7)
m5 = m2 * 2
                   10, m5.A11, "Matrix * Scalar A11", 1e-7)
self.TestValue(
                    0, m5.A12, "Matrix * Scalar A12", 1e-7)
self.TestValue(
self.TestValue(
                  -12, m5.A13, "Matrix * Scalar A13", 1e-7)
self.TestValue(
                    6, m5.A21, "Matrix * Scalar A21", 1e-7)
                   16, m5.A22, "Matrix * Scalar A22", 1e-7)
self.TestValue(
self.TestValue(
                   4, m5.A23, "Matrix * Scalar A23", 1e-7)
self.TestValue(
                   -2, m5.A31, "Matrix * Scalar A31", 1e-7)
                   -8, m5.A32, "Matrix * Scalar A32", 1e-7)
self.TestValue(
self.TestValue(
                  -18, m5.A33, "Matrix * Scalar A33", 1e-7)
print("Number of ppMatrix3x3 failed tests: ", self.FailCount)
```

```
[25]: myMatrix = ppMatrix3x3()
print(myMatrix)
myMatrix.UnitTest()
```

```
(1, 0, 0)
(0, 1, 0)
(0, 0, 1)
Number of ppMatrix3x3 failed tests: 0
```

# 1.10 Equations of Motion

A base class for the equations of motion.

```
[26]: class ppSimulation(ppConvert):
    Time = 0.0
    TimeStep = 0.1
    Data = {}
    IC = {}
AeroModelInput = []
```

```
ReferenceWingSpan = 0
  ReferenceWingChord = 0
  ReferenceWingArea = 0
  Position = ppVector3(0, 0, 0)
  TotalMass = 0
  GrossWeight = 0
  TrueAirspeed = 0
  BodyVelocity = ppVector3(0, 0, 0)
  BodyAccel = ppVector3(0, 0, 0)
  BodyForce = ppVector3(0, 0, 0)
  BodyAngle = ppVector3(0, 0, 0)
  BodyAngularRate = ppVector3(0, 0, 0)
  BodyAngularAccel = ppVector3(0, 0, 0)
  gvJx = 0
  gvJy = 0
  gvJz = 0
  gvJxz = 0
  Gamma = 0
  InertiaMatrix = ppMatrix3x3()
  InertiaMatrixInverse = ppMatrix3x3()
  # moment components
  Ml = 0
  Mm = 0
  Mn = 0
  totalCoefficientOfDrag = 0
  # define outputs
  EnglishLabels = ['gePosition_ft_X', 'gePosition_ft_Y', 'gePosition_ft_Z',
                  'feVelocity_ft_s_X', 'feVelocity_ft_s_Y', _
\hookrightarrow 'feVelocity_ft_s_Z',
                   'altitudeMsl_ft', 'longitude_deg', 'latitude_deg', |
'eulerAngle_deg_Yaw', 'eulerAngle_deg_Pitch', __
'bodyAngularRateWrtEi_deg_s_Roll',_
'bodyAngularRateWrtEi_deg_s_Yaw',
                   'altitudeRateWrtMsl_ft_min', 'speedOfSound_ft_s',
'ambientPressure_lbf_ft2', 'ambientTemperature_dgR',
```

```
'aero_bodyForce_lbf_X', 'aero_bodyForce_lbf_Y', __
'aero_bodyMoment_ftlbf_L', 'aero_bodyMoment_ftlbf_M', __
'mach', 'dynamicPressure_lbf_ft2', 'trueAirspeed_nmi_h']
  EnglishData = {}
  time = []
  eiPosition_m_X = []
  eiPosition_m_Y = []
  eiPosition m Z = []
  gePosition_m_X = []
  gePosition_m_Y = []
  gePosition_m_Z = []
  feVelocity_m_s_X = []
  feVelocity_m_s_Y = []
  feVelocity_m_s_Z = []
  altitudeMsl_m = []
  longitude_rad = []
  latitude_rad = []
  localGravity_m_s2 = []
  eulerAngle_Roll = []
  eulerAngle_Pitch = []
  eulerAngle_Yaw = []
  bodyAngularRateWrtEi_rad_s_Roll = []
  bodyAngularRateWrtEi rad s Pitch = []
  bodyAngularRateWrtEi_rad_s_Yaw = []
  trueAirspeed = []
  def AdvanceTime(self):
      self.time.append(self.Time)
      self.Time += self.TimeStep
  def AddAeroModelInput(self, input):
      self.AeroModelInput = input
  def EvaluateAeroModel(self):
      for d in self.AeroModelInput:
          gvAeroModel.Set(d, self.Data[d])
      gvAeroModel.Update()
  def Clear(self):
      self.Time = 0.0
      self.Data.clear()
      self.EnglishData.clear()
      self.AeroModelInput.clear()
```

```
self.time.clear()
       self.eiPosition_m_X.clear()
       self.eiPosition_m_Y.clear()
       self.eiPosition_m_Z.clear()
      self.gePosition_m_X.clear()
      self.gePosition m Y.clear()
      self.gePosition_m_Z.clear()
      self.feVelocity m s X.clear()
       self.feVelocity_m_s_Y.clear()
       self.feVelocity m s Z.clear()
       self.altitudeMsl_m.clear()
      self.longitude_rad.clear()
      self.latitude_rad.clear()
      self.localGravity_m_s2.clear()
      self.eulerAngle_Roll.clear()
       self.eulerAngle_Pitch.clear()
       self.eulerAngle_Yaw.clear()
      self.bodyAngularRateWrtEi_rad_s_Roll.clear()
       self.bodyAngularRateWrtEi_rad_s_Pitch.clear()
       self.bodyAngularRateWrtEi_rad_s_Yaw.clear()
       self.trueAirspeed.clear()
   def GenerateEnglishUnits(self):
      self.EnglishData.clear()
      for key in self.EnglishLabels:
           self.EnglishData[key] = []
       # TODO: extract units from name
       self.EnglishData['gePosition_ft_X'] = self.ToEnglish(self.
→gePosition_m_X,"m")
       self.EnglishData['gePosition_ft_Y'] = self.ToEnglish(self.
→gePosition_m_Y,"m")
       self.EnglishData['gePosition_ft_Z'] = self.ToEnglish(self.
self.EnglishData['feVelocity_m_s_X'] = self.ToEnglish(self.
→feVelocity_m_s_X,"m")
       self.EnglishData['feVelocity_m_s_Y'] = self.ToEnglish(self.
→feVelocity_m_s_Y,"m")
       self.EnglishData['feVelocity_m_s_Z'] = self.ToEnglish(self.
→feVelocity_m_s_Z,"m")
```

```
self.EnglishData['altitudeMsl_ft'] = self.ToEnglish(self.
→altitudeMsl_m,"m")
       self.EnglishData['longitude_deg'] = self.ToEnglish(self.
→longitude rad, "rad")
       self.EnglishData['latitude_deg'] = self.ToEnglish(self.
→latitude_rad, "rad")
       self.EnglishData['localGravity_ft_s2'] = self.ToEnglish(self.
→localGravity m s2,"m")
       self.EnglishData['eulerAngle_deg_Roll'] = self.ToEnglish(self.
→eulerAngle_Roll, "rad")
       self.EnglishData['eulerAngle_deg_Pitch'] = self.ToEnglish(self.
→eulerAngle Pitch, "rad")
       self.EnglishData['eulerAngle_deg_Yaw'] = self.ToEnglish(self.
⇔eulerAngle_Yaw,"rad")
       self.EnglishData['bodyAngularRateWrtEi_deg_s_Roll'] = \
           self.ToEnglish(self.bodyAngularRateWrtEi_rad_s_Roll, "rad")
       self.EnglishData['bodyAngularRateWrtEi deg s Pitch'] = \
           self.ToEnglish(self.bodyAngularRateWrtEi_rad_s_Pitch, "rad")
       self.EnglishData['bodyAngularRateWrtEi deg s Yaw'] = \
           self.ToEnglish(self.bodyAngularRateWrtEi_rad_s_Yaw,"rad")
       self.EnglishData['trueAirspeed_nmi_h'] = self.ToEnglish(self.
→trueAirspeed, "m_s")
   def NormalizeAngle(self, value, lower, upper):
       Returns a value between the range lower and upper.
       Example: NormalizeAngle(181, -180, 180) returns -179
       angleRange = upper - lower
       rangeValue = value - lower
       return (rangeValue - (math.floor(rangeValue / angleRange) *
→angleRange)) + lower
   def SetValue(self, label, defValue = 0):
       value = 0.0
       infoStr = "none"
       if label in self.IC:
           value = self.IC[label]
           infoStr = "[IC case]"
       else:
           value = defValue
           infoStr = "[default]"
```

```
print("++", label, "=", value, infoStr)
      return value
  def ResetSimulation(self, ic):
      self.Clear()
      self.IC.clear()
      self.IC = self.SetIC(ic)
      #self.IC = ic.copy()
      print(self.IC)
      #self.GrossWeight = self.SetValue("grossWeight", 1)
      self.TimeStep = self.SetValue("timeStep", 0.1)
      self.TotalMass = self.SetValue("totalMass", 1)
      assert self.TotalMass != 0, "TotalMass is 0"
      self.ReferenceWingSpan = self.SetValue("referenceWingSpan")
      self.ReferenceWingChord = self.SetValue("referenceWingChord")
      wingArea = self.ReferenceWingSpan * self.ReferenceWingChord
      self.ReferenceWingArea = self.SetValue("referenceWingArea", wingArea)
      gvAeroModel.Set("aeroBodyForceCoefficient_X")
      gvAeroModel.Set("aeroBodyForceCoefficient Y")
      gvAeroModel.Set("aeroBodyForceCoefficient_Z")
      gvAeroModel.Set("aeroBodyMomentCoefficient_Roll")
      gvAeroModel.Set("aeroBodyMomentCoefficient_Pitch")
      gvAeroModel.Set("aeroBodyMomentCoefficient_Yaw")
      self.TrueAirspeed = self.SetValue("trueAirspeed")
      angleOfAttack = self.SetValue("angleOfAttack")
      angleOfSideslip = self.SetValue("angleOfSideslip")
      u = self.TrueAirspeed * math.cos(angleOfAttack) * math.
v = self.TrueAirspeed * math.sin(angleOfSideslip);
      w = self.TrueAirspeed * math.sin(angleOfAttack) * math.
self.BodyVelocity.Set(u, v, w)
      self.BodyAccel.Set(0, 0, 0)
      # Set the rotation quaternion based on the Euler angles
      rollEulerAngle = self.SetValue("eulerAngle_Roll")
```

```
pitchEulerAngle = self.SetValue("eulerAngle_Pitch")
    yawEulerAngle = self.SetValue("eulerAngle_Yaw")
    self.BodyAngle.Set( rollEulerAngle, pitchEulerAngle, yawEulerAngle )
    # Set angular rates
   P = self.SetValue("eulerAngleRate_Roll")
    Q = self.SetValue("eulerAngleRate_Pitch")
   R = self.SetValue("eulerAngleRate_Yaw")
    self.BodyAngularRate.Set( P, Q, R )
    # Set the inertia matrix
    i11 = self.SetValue("bodyMomentOfInertia_X")
    i12 = -self.SetValue("bodyProductOfInertia_XY")
   i13 = -self.SetValue("bodyProductOfInertia_XZ")
    i21 = -self.SetValue("bodyProductOfInertia_YX")
    i22 = self.SetValue("bodyMomentOfInertia_Y")
    i23 = -self.SetValue("bodyProductOfInertia_YZ")
    i31 = -self.SetValue("bodyProductOfInertia_XZ")
    i32 = -self.SetValue("bodyProductOfInertia_YZ")
    i33 = self.SetValue("bodyMomentOfInertia_Z")
    self.InertiaMatrix.SetRow1(i11, i12, i13)
    self.InertiaMatrix.SetRow2(i21, i22, i23)
    self.InertiaMatrix.SetRow3(i31, i32, i33)
   self.InertiaMatrixInverse = self.InertiaMatrix.Inverse()
   self.gvJx = self.InertiaMatrix.A11
   self.gvJy = self.InertiaMatrix.A22
    self.gvJz = self.InertiaMatrix.A33
   self.gvJxz = self.InertiaMatrix.A13
    self.Gamma = (self.gvJx*self.gvJz) - (self.gvJxz*self.gvJxz)
   self.Ml = 0
    self.Mm = 0
    self.Mn = 0
def AdamsBashforth(self, current, past):
   k2 = [1.5, -0.5]
   k3 = [23.0/12.0, -16.0/12.0, 5.0/12.0]
   x = self.TimeStep * (k2[0]*current.X + k2[1]*past.X)
   y = self.TimeStep * (k2[0]*current.Y + k2[1]*past.Y)
   z = self.TimeStep * (k2[0]*current.Z + k2[1]*past.Z)
```

```
return [x, y, z]
def RungeKutta4(self, Fdot, arg):
    h = self.TimeStep
    k1 = []
    arg1 = []
    for (a, f) in zip(arg, Fdot):
        k = h*f(arg)
        k1.append(k)
        arg1.append(a + 0.5*k)
    k2 = []
    arg2 = []
    for (a, f) in zip(arg, Fdot):
        k = h*f(arg1)
        k2.append(k)
        arg2.append(a + 0.5*k)
    k3 = []
    arg3 = []
    for (a, f) in zip(arg, Fdot):
        k = h*f(arg2)
        k3.append(k)
        arg3.append(a + k)
    k4 = \prod
    for f in Fdot:
        k4.append( h*f(arg3))
    result = []
    for (a, kc1, kc2, kc3, kc4) in zip(arg, k1, k2, k3, k4):
        result.append(a + (kc1 + 2.0*kc2 + 2.0*kc3 + kc4) / 6.0)
    return result
def Reset(self, ic):
    pass
def Operate(self):
    pass
def Run(self, numberOfSeconds):
    endTime = int(numberOfSeconds / self.TimeStep) + 1
    for i in range(endTime):
        self.Operate()
    print("=====done=====")
```

```
def UnitTest(self):
    self.ClassName = "ppSimulation"
    # test normalize angle between -180 and 180 (and -pi and pi)
    pi = math.pi
    for i in range(360):
        ang = i
        if ang > 179:
            ang -= 360
        self.TestValue(ang, self.NormalizeAngle(i, -180.0, 180.0),
        "NormalizeAngle", 0.001)

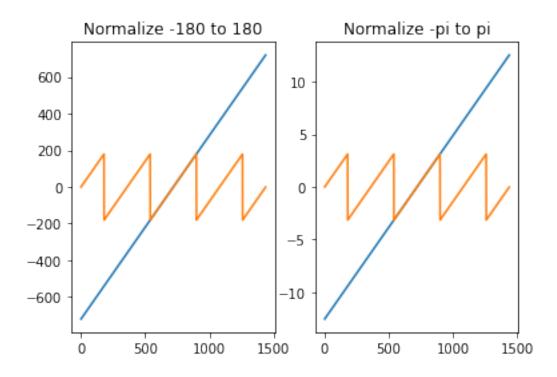
        ri = math.radians(i)
        rang = math.radians(ang)
        self.TestValue(rang, self.NormalizeAngle(ri, -pi, pi),
        "NormalizeAngle", 1e-6)

        print("Number of ppSimulation failed tests: ", self.FailCount)
```

```
[27]: import matplotlib.pyplot as plt
      simUnitTest = ppSimulation()
      simUnitTest.UnitTest()
      # make a plot normalizing the angles between -180 and 180
      t = \prod
      x = []
      nx = \Gamma
      rx = []
      rnx = \Pi
      for a in range(1440):
          t.append(a)
          x.append(a - 720)
          na = simUnitTest.NormalizeAngle( (a-720.0), -180.0, 180.0 )
          nx.append( na )
          rx.append( math.radians(a-720.0) )
          rna = simUnitTest.NormalizeAngle( math.radians(a-720.0), -math.pi, math.pi )
          rnx.append( rna )
      figAng1, (pa1, pa2) = plt.subplots(1,2)
      pa1.plot(t,x,t,nx)
      pa1.set(title='Normalize -180 to 180')
      pa2.plot(t,rx,t,rnx)
      pa2.set(title='Normalize -pi to pi')
```

Number of ppSimulation failed tests: 0

[27]: [Text(0.5, 1.0, 'Normalize -pi to pi')]



## 1.10.1 Flat Earth

Create a simulation for a flat Earth model. Singularities exist at the poles. Vehicle must be symmetric about the x body axis. Vehicle pitch must stay below 90°.

Force Equations

Force Equations 
$$\dot{U} = RV - QW - g_D \sin \theta + \frac{X_A + X_T}{m}$$

$$\dot{V} = PW - RU + g_D \sin \phi \cos \theta + \frac{Y_A + Y_T}{m}$$

$$\dot{W} = QU - PV + g_D \cos \phi \cos \theta + \frac{Z_A + Z_T}{m}$$
In vector form,

$$\dot{\vec{v}} = \frac{F}{m} + R_{n/b} \begin{pmatrix} 0 \\ 0 \\ q_D \end{pmatrix} - \vec{\omega} \times \vec{v}$$

where 
$$R_{n/b}$$
 is the rotation matrix from NED to body.
$$R_{n/b} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos \phi & \sin \phi \\ 0 & -\sin \phi & \cos \phi \end{bmatrix} \begin{bmatrix} \cos \theta & 0 & -\sin \theta \\ 0 & 1 & 0 \\ \sin \theta & 0 & \cos \theta \end{bmatrix} \begin{bmatrix} \cos \psi & \sin \psi & 0 \\ -\sin \psi & \cos \psi & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$R_{b/n} = [R_{n/b}]^T$$

Kinematic equations

$$\begin{split} \dot{\phi} &= P + \tan \hat{\theta} \left( Q \sin \phi + R \cos \phi \right) \\ \dot{\theta} &= Q \cos \phi - R \sin \phi \end{split}$$

$$\dot{\psi} = (Q\sin\phi + R\cos\phi) / \cos\theta$$

```
In vector form,
```

$$\dot{\Phi} = H\omega^b$$
, where  $H = \begin{pmatrix} 1 & \sin\phi\tan\theta & \cos\phi\tan\theta \\ 0 & \cos\phi & -\sin\phi \\ 0 & \sin\phi/\cos\theta & \cos\phi/\cos\theta \end{pmatrix}$ 

## Moment Equations

$$\begin{split} \Gamma \dot{P} &= J_{xz} [J_x - J_y + J_z] PQ - [J_z (J_z - J_y) + J_{xz}^2] QR + lJ_z + nJ_{xz} \\ J_y \dot{Q} &= (J_z - J_x) PR - J_{xz} (P^2 - R^2) + m \\ \Gamma \dot{R} &= [(J_x - J_y) J_x + J_{xz}^2] PQ - J_{xz} [J_x - J_y + J_z] QR + lJ_{xz} nJ_x \\ \Gamma &= J_x J_z - J_{xz}^2 \\ \text{In vector form,} \\ \omega_{b/i}^{\dot{b}} &= J^{-1} (M^b - \omega_{b/i}^b J \omega_{b/i}^b) \end{split}$$

# Navigation Equations

$$\begin{split} \dot{p_N} &= Uc\theta c\psi + V(-c\phi s\psi + s\phi s\theta c\psi) + W(s\phi s\psi + c\phi s\theta c\psi) \\ \dot{p_E} &= Uc\theta s\psi + V(c\phi c\psi + s\phi s\theta s\psi) + W(-s\phi c\psi + c\phi s\theta c\psi) \\ \dot{h} &= Us\theta - Vs\phi c\theta - Wc\phi c\theta \\ \text{In vector form,} \\ \dot{\vec{p}} &= R_{b/n}\vec{v} \end{split}$$

# [28]: class ppFlatEarth(ppSimulation):

```
gD = 0
mass = 0
Planet = ppEarth()
# state values
X = [0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0]
# state DFE
Xdot = []
# the indices of the state list
Ui = 0
Vi = 1
Wi = 2
i = 3
i = 4
i = 5
Pi = 6
Qi = 7
Ri = 8
Ni = 9
Ei = 10
```

```
Zi = 11
# aerodynamic forces in body frame
Ya = 0
Za = 0
# thrust forces in body frame
Xt = 0
Yt = 0
Zt = 0
def Udot(self, state):
   V = state[self.Vi]
   W = state[self.Wi]
   Q = state[self.Qi]
   R = state[self.Ri]
   sin = math.sin(state[self.i])
   assert self.mass != 0.0, "Udot mass is 0"
   value = R*V - Q*W - self.gD*sin + (self.Xa + self.Xt) / self.mass
   return value
def Vdot(self, state):
   U = state[self.Ui]
   W = state[self.Wi]
   P = state[self.Pi]
   R = state[self.Ri]
   sin = math.sin(state[self.i])
   cos = math.cos(state[self.i])
    assert self.mass != 0.0, "Vdot mass is 0"
   value = -R*U + P*W + self.gD*sin*cos + (self.Ya + self.Yt) / self.mass
   return value
def Wdot(self, state):
   U = state[self.Ui]
   V = state[self.Vi]
   P = state[self.Pi]
   Q = state[self.Qi]
   cos = math.cos(state[self.i])
   cos = math.cos(state[self.i])
   assert self.mass != 0.0, "Wdot mass is 0"
   value = Q*U - P*V + self.gD*cos*cos + (self.Za + self.Zt) / self.mass
   return value
```

```
def dot(self, state):
      P = state[self.Pi]
       Q = state[self.Qi]
      R = state[self.Ri]
      assert state[self.i] < abs(math.radians(90.0)), "dot tan is 90"</pre>
      tan = math.tan(state[self.i])
      sin = math.sin(state[self.i])
       cos = math.cos(state[self.i])
      value = P + tan * (Q*sin + R*cos)
      return value
  def dot(self, state):
      Q = state[self.Qi]
      R = state[self.Ri]
      cos = math.cos(state[self.i])
       sin = math.sin(state[self.i])
      value = Q*cos - R*sin
      return value
  def dot(self, state):
      Q = state[self.Qi]
      R = state[self.Ri]
      cos = math.cos(state[self.i])
      sin = math.sin(state[self.i])
      cos = math.cos(state[self.i])
      assert cos != 0.0, "dot cos is 0"
      value = (Q*sin + R*cos) / cos
      return value
  def Pdot(self, state):
      P = state[self.Pi]
      Q = state[self.Qi]
      R = state[self.Ri]
      Jx = self.gvJx
      Jy = self.gvJy
      Jz = self.gvJz
      Jxz = self.gvJxz
      1 = self.Ml
      n = self.Mn
      assert self.Gamma != 0.0, "Pdot Gamma is 0"
       value = (Jxz * (Jx - Jy + Jz)*P*Q - (Jz*(Jz - Jy) + Jxz*Jxz)*Q*R + Jz*l_{\square}
→+ Jxz*n) / self.Gamma
```

```
return value
   def Qdot(self, state):
       P = state[self.Pi]
       Q = state[self.Qi]
       R = state[self.Ri]
       Jx = self.gvJx
       Jy = self.gvJy
       Jz = self.gvJz
       Jxz = self.gvJxz
       m = self.Mm
       assert Jy != 0.0, "Qdot Jy is 0"
       value = ((Jz - Jx)*P*R - Jxz*(P*P - R*R) + m) / Jy
       return value
   def Rdot(self, state):
       P = state[self.Pi]
       Q = state[self.Qi]
       R = state[self.Ri]
       Jx = self.gvJx
       Jy = self.gvJy
       Jz = self.gvJz
       Jxz = self.gvJxz
       l = self.Ml
       n = self.Mn
       assert self.Gamma != 0.0, "Pdot Gamma is 0"
       value = (((Jx - Jy)*Jx + Jxz*Jxz)*P*Q - Jxz*(Jx - Jy + Jz)*Q*R + Jxz*l_{\sqcup}
\rightarrow+ Jx*n) / self.Gamma
       return value
   def Ndot(self, state):
       U = state[self.Ui]
       V = state[self.Vi]
       W = state[self.Wi]
       cos = math.cos(state[self.i])
       sin = math.sin(state[self.i])
       cos = math.cos(state[self.i])
       sin = math.sin(state[self.i])
       cos = math.cos(state[self.i])
       sin = math.sin(state[self.i])
       value = U*cos*cos + V*(-cos*sin + sin*sin*cos)
       + W*(sin *sin + cos *sin *cos)
       return value
```

```
def Edot(self, state):
       U = state[self.Ui]
       V = state[self.Vi]
       W = state[self.Wi]
       cos = math.cos(state[self.i])
       sin = math.sin(state[self.i])
       cos = math.cos(state[self.i])
       sin = math.sin(state[self.i])
       cos = math.cos(state[self.i])
       sin = math.sin(state[self.i])
       value = U*cos *sin + V*(cos *cos + sin *sin *sin)
       + W*(-sin *cos + cos *sin *sin)
       return value
   def Zdot(self, state):
       U = state[self.Ui]
       V = state[self.Vi]
       W = state[self.Wi]
       cos = math.cos(state[self.i])
       sin = math.sin(state[self.i])
       cos = math.cos(state[self.i])
       sin = math.sin(state[self.i])
       value = U*sin - V*sin *cos - W*cos *cos
       return value
   def Reset(self, ic):
       self.ResetSimulation(ic)
       self.gD = self.Planet.GravityConstant()
       self.mass = self.TotalMass
       self.Xdot.clear()
       self.Xdot = [self.Udot, self.Vdot, self.Wdot, self.dot, self.dot, self.
→ dot,
                    self.Pdot, self.Qdot, self.Rdot, self.Ndot, self.Edot, ⊔
→self.Zdot]
       self.X[self.Ui] = self.BodyVelocity.X
       self.X[self.Vi] = self.BodyVelocity.Y
       self.X[self.Wi] = self.BodyVelocity.Z
       self.X[self.i] = self.BodyAngle.X
       self.X[self.i] = self.BodyAngle.Y
       self.X[self.i] = self.BodyAngle.Z
```

```
self.X[self.Pi] = self.BodyAngularRate.X
       self.X[self.Qi] = self.BodyAngularRate.Y
       self.X[self.Ri] = self.BodyAngularRate.Z
       self.X[self.Ni] = self.Position.X
       self.X[self.Ei] = self.Position.Y
       self.X[self.Zi] = self.SetValue("altitudeMsl")
       self.Xa = 0
       self.Ya = 0
       self.Za = 0
       self.Xt = 0
       self.Yt = 0
       self.Zt = 0
   def Operate(self):
       # save output data
       self.localGravity_m_s2.append(self.gD)
       self.altitudeMsl_m.append(self.X[self.Zi])
       self.eulerAngle_Roll.append(self.X[self.i])
       self.eulerAngle_Pitch.append(self.X[self.i])
       self.eulerAngle_Yaw.append( self.NormalizeAngle(self.X[self.i],-math.
→pi,math.pi) )
       self.trueAirspeed.append(self.TrueAirspeed)
       # integrate the equations
       self.X = self.RungeKutta4(self.Xdot, self.X)
       # Now advance time and update state equations
       self.AdvanceTime()
       u = self.X[self.Ui]
       v = self.X[self.Vi]
       w = self.X[self.Wi]
       self.TrueAirspeed = math.sqrt(u*u + v*v + w*w)
       # get dynamic pressure: q = 1/2 rho v^2
       density = self.Planet.AirDensity(self.X[self.Zi])
       dynamicPressure = 0.5 * density * (self.TrueAirspeed)**2
       # Get the qS factor for getting dimensional forces and moments
       qS = dynamicPressure * self.ReferenceWingArea
       # Compute the aerodynamic loads
       assert self.TrueAirspeed != 0, "TrueAirspeed is 0 to model"
```

```
self.Data["trueAirspeed"] = self.TrueAirspeed * self.MeterToFeet
       self.Data["bodyAngularRate_Roll"] = self.X[self.Pi]
       self.Data["bodyAngularRate_Pitch"] = self.X[self.Qi]
       self.Data["bodyAngularRate_Yaw"] = self.X[self.Ri]
       self.EvaluateAeroModel()
       # Aero forces (Newtons) body
       self.Xa = qS * gvAeroModel.DataFromName("aeroBodyForceCoefficient_X")
       self.Ya = qS * gvAeroModel.DataFromName("aeroBodyForceCoefficient_Y")
       self.Za = qS * gvAeroModel.DataFromName("aeroBodyForceCoefficient_Z")
       # Aero moments
       self.Ml = qS * self.ReferenceWingSpan * gvAeroModel.
→DataFromName("aeroBodyMomentCoefficient_Roll")
       self.Mm = qS * self.ReferenceWingChord * gvAeroModel.
→DataFromName("aeroBodyMomentCoefficient_Pitch")
       self.Mn = qS * self.ReferenceWingSpan * gvAeroModel.
→DataFromName("aeroBodyMomentCoefficient_Yaw")
```

## 1.10.2 Oblate, Rotating Earth (Stevens and Lewis)

$$\begin{split} \dot{q_0} &= -0.5* (Pq_1 + Qq_2 + Rq_3) \\ \dot{q_1} &= 0.5* (Pq_0 + Rq_2 - Qq_3) \\ \dot{q_2} &= 0.5* (Qq_0 - Rq_1 + Pq_3) \\ \dot{q_3} &= 0.5* (Rq_0 + Qq_1 - Pq_2) \\ \dot{P_x} &= V_x \\ \dot{P_y} &= V_y \\ \dot{P_z} &= V_z \\ \text{where } P \text{ and } V \text{ are in the ECEF frame.} \\ \dot{v_x} &= \frac{F_x}{m} + 2\omega_e V_y + g_x + P_x \omega_e^2 \\ \dot{v_y} &= \frac{F_y}{m} - 2\omega_e V_x + g_y + P_y \omega_e^2 \\ \dot{v_z} &= \frac{F_z}{m} + g_z \end{split}$$

where  $\omega_e$  is the rotation rate of Earth. The terms  $g_x$ ,  $g_y$ , and  $g_z$  are the  $J_2$  gravity components in ECEF. This acceleration equation is in the ECEF frame.

$$\begin{split} \Gamma \dot{P} &= J_{xz} [J_x - J_y + J_z] PQ - [J_z (J_z - J_y) + J_{xz}^2] QR + lJ_z + nJ_{xz} \\ J_y \dot{Q} &= (J_z - J_x) PR - J_{xz} (P^2 - R^2) + m \\ \Gamma \dot{R} &= [(J_x - J_y) J_x + J_{xz}^2] PQ - J_{xz} [J_x - J_y + J_z] QR + lJ_{xz} nJ_x \\ \text{where } \Gamma &= J_x J_z - J_{xz}^2 \end{split}$$

```
[29]: class slEarthSim(ppSimulation):
    Planet = ppEarth()
    RotationAngle = 0
    EarthRotation = ppQuaternion(0, 0, 0, Planet.RotationRate)

# Earth rotatation in body frame
```

```
Per = 0
Qer = 0
Rer = 0
# quaternion frame rotations
\# i = inertial frame ECI
# e = earth centered, earth fixed ECEF
# n = north \ east \ down \ NED
# b = body forward right down FRD
Qe2n = ppQuaternion(1,0,0,0)
Qn2b = ppQuaternion(1,0,0,0)
Qe2b = ppQuaternion(1,0,0,0)
Qi2e = ppQuaternion(1,0,0,0)
QforceEcf = ppQuaternion(0,0,0,0)
# ECEF gravity components
Gx = 0
Gy = 0
Gz = 0
# state values: quaternion, position, acceleration and angular rates
X = [0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0]
# the state differential equations
Xdot = []
# the indices of the state list
Qni = 0
Qxi = 1
Qyi = 2
Qzi = 3
Xi = 4
Yi = 5
Zi = 6
Vxi = 7
Vyi = 8
Vzi = 9
Pi = 10
Qi = 11
Ri = 12
def Qstate(self,state):
    q0 = state[self.Qni]
```

```
q1 = state[self.Qxi]
    q2 = state[self.Qyi]
    q3 = state[self.Qzi]
   p = state[self.Pi] - self.Per
    q = state[self.Qi] - self.Qer
   r = state[self.Ri] - self.Rer
   return q0, q1, q2, q3, p, q, r
def QnDot(self, state):
    q0, q1, q2, q3, p, q, r = self.Qstate(state)
    qnDot = -0.5*(q1*p + q2*q + q3*r)
   return qnDot
def QxDot(self, state):
    q0, q1, q2, q3, p, q, r = self.Qstate(state)
    qxDot = 0.5*(q0*p + q2*r - q3*q)
   return qxDot
def QyDot(self, state):
    q0, q1, q2, q3, p, q, r = self.Qstate(state)
    qyDot = 0.5*(q0*q - q1*r + q3*p)
   return qyDot
def QzDot(self, state):
    q0, q1, q2, q3, p, q, r = self.Qstate(state)
    qzDot = 0.5*(q0*r + q1*q - q2*p)
   return qzDot
def PxDot(self, state):
   return state[self.Vxi]
def PyDot(self, state):
   return state[self.Vyi]
def PzDot(self, state):
   return state[self.Vzi]
def VxDot(self, state):
   w = self.Planet.RotationRate
    assert self.TotalMass != 0, "VxDot mass is 0"
    ax = self.QforceEcf.X / self.TotalMass
    #if ax != 0:
         print("ax:", ax)
   xDot = ax + 2.0 * w * state[self.Vyi] + self.Gx + state[self.Xi] * w**2
   return xDot
def VyDot(self, state):
   w = self.Planet.RotationRate
   assert self.TotalMass != 0, "VyDot mass is 0"
    ay = self.QforceEcf.Y / self.TotalMass
   yDot = ay - 2.0 * w * state[self.Vxi] + self.Gy + state[self.Yi] * w**2
   return yDot
def VzDot(self, state):
    assert self.TotalMass != 0, "VzDot mass is 0"
```

```
az = self.QforceEcf.Z / self.TotalMass
       return (az + self.Gz)
   def Wstate(self, state):
       P = state[self.Pi]
       Q = state[self.Qi]
       R = state[self.Ri]
       Jx = self.gvJx
       Jy = self.gvJy
       Jz = self.gvJz
       Jxz = self.gvJxz
       Gamma = self.Gamma
       l = self.Ml
       m = self.Mm
       n = self.Mn
       return P, Q, R, Jx, Jy, Jz, Jxz, Gamma, 1, m, n
   def Pdot(self, state):
       P, Q, R, Jx, Jy, Jz, Jxz, Gamma, 1, m, n = self.Wstate(state)
       assert Gamma != 0, "Pdot Gamma is 0"
       pDot = (Jxz * (Jx - Jy + Jz)*P*Q - (Jz*(Jz - Jy) + Jxz*Jxz)*Q*R + Jz*l_{U}
→+ Jxz*n) / Gamma
       return pDot
   def Qdot(self, state):
       P, Q, R, Jx, Jy, Jz, Jxz, Gamma, 1, m, n = self.Wstate(state)
       assert Jy != 0.0, "Qdot Jy is 0"
       qDot = ((Jz - Jx)*P*R - Jxz*(P*P - R*R) + m) / Jy
       return qDot
   def Rdot(self, state):
       P, Q, R, Jx, Jy, Jz, Jxz, Gamma, 1, m, n = self.Wstate(state)
       assert Gamma != 0.0, "Rdot Gamma is 0"
       rDot = (((Jx - Jy)*Jx + Jxz*Jxz)*P*Q - Jxz*(Jx - Jy + Jz)*Q*R + Jxz*1 +_{\sqcup}
→Jx*n) / Gamma
       return rDot
   def Reset(self, ic):
       self.ResetSimulation(ic)
       self.RotationAngle = 0
       self.Planet.Latitude = self.SetValue("latitude")
       self.Planet.Longitude = self.SetValue("longitude")
       self.Planet.Altitude = self.SetValue("altitudeMsl")
       [x, y, z] = self.Planet.LlaToPcpf()
       self.Position.X = x
       self.Position.Y = y
       self.Position.Z = z
```

```
# initialize the frd/ecf quaternion
   roll = self.BodyAngle.X
   pitch = self.BodyAngle.Y
   yaw = self.BodyAngle.Z
   lat = self.Planet.Latitude
   lon = self.Planet.Longitude
   self.Qe2b.SetQfrdWrtEcf(roll , pitch , yaw, lat, lon)
    # transform u,v,w to ECEF velocities
   Vecf = ppVector3(0,0,0)
   Vecf = self.Qe2b * self.BodyVelocity * ~self.Qe2b
    #self.mass = self.GrossWeight / self.gD
    self.Xdot.clear()
    self.Xdot = [self.QnDot, self.QxDot, self.QyDot, self.QzDot,
                 self.PxDot, self.PyDot, self.PzDot,
                 self.VxDot, self.VyDot, self.VzDot,
                 self.Pdot, self.Qdot, self.Rdot]
   self.X[self.Qni] = self.Qe2b.N
    self.X[self.Qxi] = self.Qe2b.X
   self.X[self.Qyi] = self.Qe2b.Y
   self.X[self.Qzi] = self.Qe2b.Z
   self.X[self.Xi] = self.Position.X
   self.X[self.Yi] = self.Position.Y
   self.X[self.Zi] = self.Position.Z
   self.X[self.Vxi] = Vecf.X
   self.X[self.Vyi] = Vecf.Y
    self.X[self.Vzi] = Vecf.Z
   print("Vecf: ", Vecf.X, Vecf.Y, Vecf.Z)
    self.X[self.Pi] = self.BodyAngularRate.X
    self.X[self.Qi] = self.BodyAngularRate.Y
    self.X[self.Ri] = self.BodyAngularRate.Z
def Operate(self):
    # create quaternions
    # TODO: need a check case the Q rotations are correct
    # set q frd/ecf (e2b) ECF to body
   self.Qe2b.N = self.X[self.Qni]
   self.Qe2b.X = self.X[self.Qxi]
    self.Qe2b.Y = self.X[self.Qyi]
    self.Qe2b.Z = self.X[self.Qzi]
```

```
# set q ned/ecf (e2n) ECF to NED
                self.Qe2n.SetLatLon(self.Planet.Latitude, self.Planet.Longitude)
                # set q frd/ned (n2b) NED to body
               self.Qn2b = ~self.Qe2n * self.Qe2b
                # get the euler angles from the quaternion
                [roll, pitch, yaw] = self.Qn2b.EulerAnglesFromQ()
                # rotate the ECF position to ECI to get the inertial position
                self.Qi2e.SetPlanetRotation(self.RotationAngle)
                qgePosition = ppQuaternion( 0, self.X[self.Xi], self.X[self.Yi], self.
→X[self.Zi] )
                qeiPosition = self.Qi2e * qgePosition * ~self.Qi2e
                # save output data
                self.altitudeMsl_m.append(self.Planet.Altitude)
                self.latitude_rad.append(self.Planet.Latitude)
               self.longitude_rad.append(self.Planet.Longitude)
               self.gePosition m X.append(self.X[self.Xi])
                self.gePosition_m_Y.append(self.X[self.Yi])
                self.gePosition_m_Z.append(self.X[self.Zi])
               self.eulerAngle_Roll.append(roll)
               self.eulerAngle_Pitch.append(pitch)
                self.eulerAngle_Yaw.append(yaw)
                self.trueAirspeed.append(self.TrueAirspeed)
                #cosRot = math.cos(self.RotationAngle)
                #sinRot = math.sin(self.RotationAngle)
                \#self.eiPosition\_m\_X.append(cosRot*self.X[self.Xi] - sinRot*self.X[self.Xi] - sinRot*self.X[se
\hookrightarrow Yi7)
               #self.eiPosition m Y.append(sinRot*self.X[self.Xi] + cosRot*self.X[self.
\hookrightarrow Yi)
                #self.eiPosition_m_Z.append(self.X[self.Zi])
               self.eiPosition_m_X.append(qeiPosition.X)
                self.eiPosition_m_Y.append(qeiPosition.Y)
                self.eiPosition_m_Z.append(qeiPosition.Z)
                # get earth rotation in the body frame
               wEarthFrd = ~self.Qe2b * self.EarthRotation * self.Qe2b
                # TODO: need to add body forces and rotate them to ECEF frame
                # set the Earth rotation in the body frame
                self.Per = wEarthFrd.X
```

```
self.Qer = wEarthFrd.Y
       self.Rer = wEarthFrd.Z
       x = self.X[self.Xi]
       y = self.X[self.Yi]
       z = self.X[self.Zi]
       [self.Gx, self.Gy, self.Gz] = self.Planet.GravityJ2(x, y, z)
       g = ppVector3(self.Gx, self.Gy, self.Gz)
       self.localGravity_m_s2.append(g.Magnitude())
       # integrate the equations
       self.X = self.RungeKutta4(self.Xdot, self.X)
       # advance time and set up for next integration
       self.AdvanceTime()
       # get the new true airspeed
       vel = ppVector3(self.X[self.Vxi], self.X[self.Vyi], self.X[self.Vzi])
       self.TrueAirspeed = vel.Magnitude()
       # get dynamic pressure: q = 1/2 rho v^2
       density = self.Planet.AirDensity(self.Planet.Altitude)
       dynamicPressure = 0.5 * density * (self.TrueAirspeed)**2
       # Get the qS factor for getting dimensional forces and moments
       qS = dynamicPressure * self.ReferenceWingArea
       # Compute the aerodynamic loads from the DAVE-ML model
       # set the DAVE-ML model inputs
       assert self.TrueAirspeed != 0, "TrueAirspeed is 0 to model"
       self.Data["trueAirspeed"] = self.TrueAirspeed * self.MeterToFeet
       self.Data["bodyAngularRate_Roll"] = self.X[self.Pi]
       self.Data["bodyAngularRate_Pitch"] = self.X[self.Qi]
       self.Data["bodyAngularRate_Yaw"] = self.X[self.Ri]
       self.EvaluateAeroModel()
       drag = qS * self.totalCoefficientOfDrag
       QforceFrb = ppVector3(-drag, 0, 0)
       self.QforceEcf = self.Qe2b * QforceFrb * ~self.Qe2b
       # calculate the dimentionsal aero moments
       self.Ml = qS * self.ReferenceWingSpan * gvAeroModel.
→DataFromName("aeroBodyMomentCoefficient Roll")
       self.Mm = qS * self.ReferenceWingChord * gvAeroModel.
→DataFromName("aeroBodyMomentCoefficient_Pitch")
       self.Mn = qS * self.ReferenceWingSpan * gvAeroModel.
→DataFromName("aeroBodyMomentCoefficient Yaw")
```

```
# update the latitude, longitude and altitude from ECEF X, Y, Z position
self.Planet.PcpfToLlaZhu(self.X[self.Xi], self.X[self.Yi], self.X[self.
→Zi])

# rotate the earth
self.RotationAngle = self.Planet.RotationRate * self.Time
```

#### 1.11 Check Cases

#### 1.11.1 Check with Kinematics

Using 2D constant acceleration kinematics with no aerodynamic effects, the y displacement (height) equation is:  $v_f = v_0 + gt$ . A true airspeed of 424 m/s at 45° is 300 m/s in x and 300 m/s in y. At the maximum height,  $v_f = 0 \, m/s$ . Using  $g = 9.82 \, m/s^2$ , you get 0 = 300 - (9.82)t. Solving for time to reach the maximum height, t = 300/9.82 = 30.55 seconds.

```
[30]: %%time
      printDebugData = False
      ppLoadDml('models/noAero.dml', printDebugData)
      #
      ic = {
          "timeStep": [0.01, "s"],
          "totalMass": [5.0, "kg"],
          "bodyMomentOfInertia_X": [0.1, "kgm2"],
          "bodyMomentOfInertia_Y": [0.1, "kgm2"],
          "bodyMomentOfInertia_Z": [0.1, "kgm2"],
          "altitudeMsl": [10.0, "m"],
          "referenceWingChord": [0.2, "m"],
          "referenceWingSpan": [0.2, "m"],
          "referenceWingArea": [0.031415, "m2"],
          "trueAirspeed": [424.264, "m_s"],
          "angleOfAttack": [-45, "deg"]
      flatEarthSim = ppFlatEarth()
      flatEarthSim.Reset(ic)
      flatEarthSim.Run(61.0)
```

```
++> Output:
                  CLL
     ++> Output:
                  CLM
     ++> Output:
                  CLN
     ---- DAVE-ML MODEL PARSE COMPLETE ----
     ====== SetIC ========
     {'timeStep': 0.01, 'totalMass': 5.0, 'bodyMomentOfInertia_X': 0.1,
     'bodyMomentOfInertia_Y': 0.1, 'bodyMomentOfInertia_Z': 0.1, 'altitudeMsl': 10.0,
     'referenceWingChord': 0.2, 'referenceWingSpan': 0.2, 'referenceWingArea':
     0.031415, 'trueAirspeed': 424.264, 'angleOfAttack': -0.7853981633974483}
     ++ timeStep = 0.01 [IC case]
     ++ totalMass = 5.0 [IC case]
     ++ referenceWingSpan = 0.2 [IC case]
     ++ referenceWingChord = 0.2 [IC case]
     ++ referenceWingArea = 0.031415 [IC case]
     ++ trueAirspeed = 424.264 [IC case]
     ++ angleOfAttack = -0.7853981633974483 [IC case]
     ++ angleOfSideslip = 0 [default]
     ++ eulerAngle Roll = 0 [default]
     ++ eulerAngle Pitch = 0 [default]
     ++ eulerAngle_Yaw = 0 [default]
     ++ eulerAngleRate_Roll = 0 [default]
     ++ eulerAngleRate_Pitch = 0 [default]
     ++ eulerAngleRate_Yaw = 0 [default]
     ++ bodyMomentOfInertia_X = 0.1 [IC case]
     ++ bodyProductOfInertia_XY = 0 [default]
     ++ bodyProductOfInertia_XZ = 0 [default]
     ++ bodyProductOfInertia_YX = 0 [default]
     ++ bodyMomentOfInertia_Y = 0.1 [IC case]
     ++ bodyProductOfInertia_YZ = 0 [default]
     ++ bodyProductOfInertia_XZ = 0 [default]
     ++ bodyProductOfInertia_YZ = 0 [default]
     ++ bodyMomentOfInertia_Z = 0.1 [IC case]
     ++ altitudeMsl = 10.0 [IC case]
     =====done=====
     CPU times: user 591 ms, sys: 20.5 ms, total: 611 ms
     Wall time: 718 ms
[31]: max(flatEarthSim.altitudeMsl_m)
[31]: 4598.721461301354
[32]: tMax = 300/9.82
     print(tMax)
```

30.54989816700611

The time calculated above (t = 30.55 seconds) matches closely to the plot of the data from the

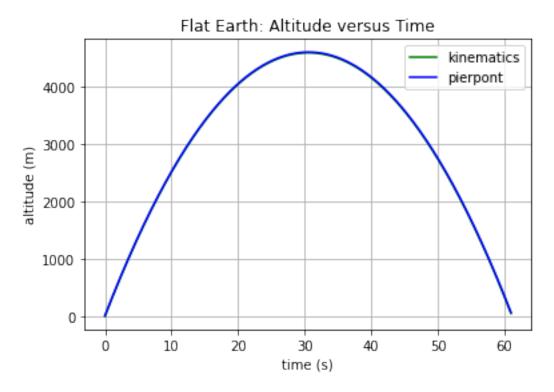
## EOM.

The maximum height in kinematics is:  $y = \frac{1}{2}(v_0y + v_fy)t + y_0$ . Substituting in the equation, you get  $y = \frac{1}{2}(300 + 0)(30.55) + 10$ .

```
[33]: yMax = 0.5*(300 + 0)*(tMax) + 10
print(yMax)
```

#### 4592.4847250509165

The maximum height calculated from the 2D kinematics of 4592 meters is within 10 meters of the EOM calculated value of 4600 meters.



#### 1.11.2 Read NESC Check Cases

Function to read in check cases from NESC. The function gets columns of data from the check case CSV files.

```
[35]: import csv
      def GetCheckCaseData(fileName):
          # open the CSV file as read-only
          csvFile = open(fileName, 'r')
          # strip the newline character from the header line
          headerLine = csvFile.readline().rstrip("\n")
          # make a list of headers
          header = headerLine.split(',')
          print("number of headers: ", len(header))
          print(header)
          # create a data dictionary with header names as keys
          Data = \{\}
          for h in header:
              Data[h] = []
          # read each row in the datafile and add the data to the data dictionary
          for row in csv.reader(csvFile):
              for (i,d) in zip(header, row):
                  Data[i].append( float(d) )
          return Data
```

```
Data checks:
```

```
L-2: D(x, y) = [\Sigma(x_i - y_i)^2]^{1/2}
L-Infinity-Norm: max_i|x_i - y_i|
```

Manhattan distance:  $\Sigma_i |x_i - y_i|$ 

```
[36]: import matplotlib.pyplot as plt
import numpy as np
import math

def MinDeltaAngleDeg(angle1, angle2):
    """
    Returns the minimum delta between two angles.
    Examples:
        20 is returned if angle1=30 and angle2=10
        20 is returned if angle1=-170 and angle2=170
        """
```

```
delta = angle1 - angle2
   twoPi = 360.0
   if abs(delta) > abs( angle1 - (angle2 - twoPi) ):
        delta = angle1 - (angle2 - twoPi)
   if abs(delta) > abs( (angle1 - twoPi) - angle2 ):
        delta = (angle1 - twoPi) - angle2
   return delta
print("MinDeltaAngle check")
print(MinDeltaAngleDeg( 170,160),"=", 10)
print(MinDeltaAngleDeg(-160,170),"=", 30)
print(MinDeltaAngleDeg( 20,-20),"=", 40)
def NescCheckData(data, checkData, isAng):
   12Sum = 0
   manSum = 0
   infNorm = 0
   for (x, y) in zip(data, checkData):
       dxy = x - y
        if isAng:
           dxy = MinDeltaAngleDeg( x, y )
       12Sum += dxy**2
       dist = abs(dxy)
       manSum += dist
        if dist > infNorm:
            infNorm = dist
   return math.sqrt(12Sum), infNorm
def PrintErrorTable(tableTitle, labels, simData, checkData):
   print ("{:<25} {:<25} ".format('Variable', 'L2', 'L-Infinity-Norm'))</pre>
   print ("{:<25} {:<25}".format('----', '--', '-----'))</pre>
   barLinf = {}
   for i in labels:
        tmpDist = NescCheckData(checkData[i], simData.EnglishData[i], i.

→find("_deg_"))
        print ("{:<25} {:<25}".format(i, tmpDist[0], tmpDist[1]))</pre>
       barLinf[i] = tmpDist[1]
   plt.rcdefaults()
   fig, ax = plt.subplots()
   y_pos = np.arange(len(barLinf.keys()))
    #plt.xlim([0, 10])
   ax.barh(y_pos, barLinf.values(), align='center')
   ax.set_yticks(y_pos)
   ax.set_yticklabels(barLinf.keys())
   ax.invert_yaxis() # labels read top-to-bottom
```

```
ax.set_xlabel('L-Infinity Norm')
         ax.set title(tableTitle)
         plt.show()
      data1 = [1, 2, 3, 4, 5]
      data2 = [1.2, 2.2, 3, 3.9, 5]
      dists = NescCheckData(data1, data2, False)
      print("L-2 Norm: ", dists)
      data3 = [160,
      data4 = [-160,
                       30 ]
      dists = NescCheckData(data3, data4, True)
     print("L-2 Norm: ", dists)
     MinDeltaAngle check
     10 = 10
     30.0 = 30
     40 = 40
     L-2 Norm: (0.30000000000001, 0.200000000000018)
     L-2 Norm: (41.23105625617661, 40.0)
[37]: def MakeFlatEarthPlots(simData, checkData, simCheckLabel):
         fig0, ah = plt.subplots()
         ah.plot(checkData['time'], checkData['altitudeMsl_ft'],'g',
                  simData.time, simData.EnglishData['altitudeMsl_ft'], 'b')
         ah.legend([simCheckLabel, "pierpont"])
          ah.set(xlabel='time (s)', ylabel='Altitude (ft)', title='Flat Earth:
      →Altitude versus Time')
         ah.grid()
         fig1, ad = plt.subplots()
         ad.plot(checkData['time'], checkData['eulerAngle deg Roll'],'g',
                  simData.time, simData.EnglishData['eulerAngle_deg_Roll'], 'b')
         ad.legend([simCheckLabel, "pierpont"])
         ad.set(xlabel='time (s)', ylabel='Roll (deg)', title='Flat Earth: Roll
      ⇔versus Time')
         ad.grid()
         fig2, ap = plt.subplots()
         ap.plot(checkData['time'], checkData['eulerAngle_deg_Pitch'],'g',
                  simData.time, simData.EnglishData['eulerAngle_deg_Pitch'], 'b')
         ap.legend([simCheckLabel, "pierpont"])
         ap.set(xlabel='time (s)', ylabel='Pitch (deg)', title='Flat Earth: Pitch⊔
       →versus Time')
```

```
[38]: import matplotlib.pyplot as plt
      import matplotlib.gridspec as gridspec
      def MakePlot(simData, checkData, simCaseLabel):
          fig1 = plt.figure(constrained_layout=True)
          spec1 = gridspec.GridSpec(ncols=2, nrows=1, figure=fig1)
          ax1 = fig1.add_subplot(spec1[0, 0])
          ax1.plot(checkData['time'], checkData['altitudeMsl_ft'],'g',
                   simData.time, simData.EnglishData['altitudeMsl ft'], 'b')
          ax1.set(xlabel='time (s)', ylabel='altitude (ft)', title='Altitude versus_
       →Time')
          ax1.legend([simCaseLabel, "pierpont"])
          ax2 = fig1.add_subplot(spec1[0, 1])
          ax2.yaxis.tick_right()
          ax2.yaxis.set_label_position("right")
          ax2.plot(checkData['time'], checkData['localGravity_ft_s2'],'g',
                   simData.time, simData.EnglishData['localGravity_ft_s2'], 'b')
          ax2.set(xlabel='time (s)', ylabel='localGravity (ft_s2)', title='Gravity_
       \hookrightarrowversus Time')
          ax2.legend([simCaseLabel, "pierpont"])
          fig2, ad = plt.subplots()
          ad.plot(checkData['time'], checkData['gePosition_ft_X'],'g',
                  simData.time, simData.EnglishData['gePosition_ft_X'], 'b')
          ad.legend([simCaseLabel, "pierpont"])
          ad.set(xlabel='time (s)', ylabel='gePosition_ft_X', title='ECEF X versus_
       →Time')
          ad.grid()
```

```
fig5, ad = plt.subplots()
   ad.plot(checkData['time'], checkData['eulerAngle_deg_Roll'],'g',
           simData.time, simData.EnglishData['eulerAngle_deg_Roll'], 'b')
   ad.legend([simCaseLabel, "pierpont"])
   ad.set(xlabel='time (s)', ylabel='Roll (deg)', title='Oblate Earth: Roll
→versus Time')
   ad.grid()
   fig5a, ap = plt.subplots()
   ap.plot(checkData['time'], checkData['eulerAngle_deg_Pitch'],'g',
           simData.time, simData.EnglishData['eulerAngle_deg_Pitch'], 'b')
   ap.legend([simCaseLabel, "pierpont"])
   ap.set(xlabel='time (s)', ylabel='Pitch (deg)', title='Oblate Earth: Pitch⊔
⇔versus Time')
   ap.grid()
   fig5b, ay = plt.subplots()
   ay.plot(checkData['time'], checkData['eulerAngle_deg_Yaw'],'g',
           simData.time, simData.EnglishData['eulerAngle_deg_Yaw'], 'b')
   ay.legend([simCaseLabel, "pierpont"])
   ay.set(xlabel='time (s)', ylabel='Yaw (deg)', title='Oblate Earth: Yaw⊔
⇔versus Time')
   ay.grid()
```

# 1.11.3 Dragless Sphere - 1

Property	English Value	SI Value
$\overline{I_{xx}}$	$3.6 \text{ slug-ft}^2$	$4.881 \text{ kg-m}^2$
$I_{yy}$	$3.6 \text{ slug-ft}^2$	$4.881 \text{ kg-m}^2$
$I_{zz}$	$3.6 \text{ slug-ft}^2$	$4.881 \text{ kg-m}^2$
m	$1.0 \mathrm{slug}$	14.5939  kg
S	$0.1963495 \text{ ft}^{2}$	$0.0182414654525 \text{ m}^2$

```
[39]: ixx = 3.6 * gvConvert.Slugft2ToKgm2
print("ixx=", ixx)
mass = 1.0 * gvConvert.SlugToKg
print("mass=", mass)
S = 0.1963495 * gvConvert.SqFeetToSqMeter
print("S=", S)
```

```
ixx= 4.88094466281336
mass= 14.593902937
S= 0.018241465452480003
```

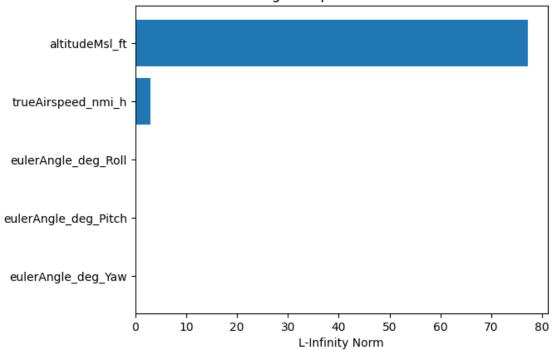
### Flat Earth

```
[40]: %%time
     print("=======")
     checkFile = "NESC-check-cases/Atmospheric_checkcases/Atmos_01_DroppedSphere/
      →Atmos_01_sim_01.csv"
     gvCC1 = GetCheckCaseData(checkFile)
     print("======="")
     printDebugData = False
     ppLoadDml('models/noAero.dml', printDebugData)
     # 1 sluq = 14.5939 kq
     # 3.6 sluq-ft2 = 4.881 kq-m2
      # 30000 ft = 9144 m
      \# 0.1963495 \text{ ft2} = 0.0182414654525 \text{ m2}
     ic = {
         "totalMass": [1.0, "slug"],
         "bodyMomentOfInertia_X": [3.6, "slugft2"],
         "bodyMomentOfInertia_Y": [3.6, "slugft2"],
         "bodyMomentOfInertia_Z": [3.6, "slugft2"],
         "altitudeMsl": [30000, "ft"],
         "referenceWingChord": [0.2, "ft"],
         "referenceWingSpan": [0.2, "ft"],
         "referenceWingArea": [0.1963495, "ft2"]
     }
     gvFlatEarthSim = ppFlatEarth()
     gvFlatEarthSim.Reset(ic)
     gvFlatEarthSim.Run(30.0)
     gvFlatEarthSim.GenerateEnglishUnits()
     number of headers: 31
     ['time', 'gePosition_ft_X', 'gePosition_ft_Y', 'gePosition_ft_Z',
     'feVelocity_ft_s_X', 'feVelocity_ft_s_Y', 'feVelocity_ft_s_Z', 'altitudeMsl_ft',
     'longitude_deg', 'latitude_deg', 'localGravity_ft_s2', 'eulerAngle_deg_Yaw',
     'eulerAngle_deg_Pitch', 'eulerAngle_deg_Roll',
     'bodyAngularRateWrtEi_deg_s_Roll', 'bodyAngularRateWrtEi_deg_s_Pitch',
     'bodyAngularRateWrtEi_deg_s_Yaw', 'altitudeRateWrtMsl_ft_min',
     'speedOfSound_ft_s', 'airDensity_slug_ft3', 'ambientPressure_lbf_ft2',
     'ambientTemperature_dgR', 'aero_bodyForce_lbf_X', 'aero_bodyForce_lbf_Y',
     'aero_bodyForce_lbf_Z', 'aero_bodyMoment_ftlbf_L', 'aero_bodyMoment_ftlbf_M',
     'aero_bodyMoment_ftlbf_N', 'mach', 'dynamicPressure_lbf_ft2',
     'trueAirspeed nmi h']
       ===========
     *************
     Model: Zero Aero Output
     creation date: 2021-07-21
```

```
file version: Initial version
************
+++++ MODEL INPUTS AND OUTPUTS +++++
++> Output: CX
++> Output: CY
++> Output: CZ
++> Output:
            CLL
++> Output:
            CLM
++> Output:
            CLN
---- DAVE-ML MODEL PARSE COMPLETE ----
====== SetIC ========
{'totalMass': 14.593902937, 'bodyMomentOfInertia_X': 4.88094466281336,
'bodyMomentOfInertia_Y': 4.88094466281336, 'bodyMomentOfInertia_Z':
4.88094466281336, 'altitudeMsl': 9144.0, 'referenceWingChord':
0.0609600000000001, 'referenceWingSpan': 0.0609600000000001,
'referenceWingArea': 0.018241465452480003}
++ timeStep = 0.1 [default]
++ totalMass = 14.593902937 [IC case]
++ referenceWingSpan = 0.0609600000000001 [IC case]
++ referenceWingChord = 0.0609600000000001 [IC case]
++ referenceWingArea = 0.018241465452480003 [IC case]
++ trueAirspeed = 0 [default]
++ angleOfAttack = 0 [default]
++ angleOfSideslip = 0 [default]
++ eulerAngle_Roll = 0 [default]
++ eulerAngle_Pitch = 0 [default]
++ eulerAngle_Yaw = 0 [default]
++ eulerAngleRate_Roll = 0 [default]
++ eulerAngleRate_Pitch = 0 [default]
++ eulerAngleRate_Yaw = 0 [default]
++ bodyMomentOfInertia_X = 4.88094466281336 [IC case]
++ bodyProductOfInertia XY = 0 [default]
++ bodyProductOfInertia_XZ = 0 [default]
++ bodyProductOfInertia YX = 0 [default]
++ bodyMomentOfInertia_Y = 4.88094466281336 [IC case]
++ bodyProductOfInertia_YZ = 0 [default]
++ bodyProductOfInertia_XZ = 0 [default]
++ bodyProductOfInertia_YZ = 0 [default]
++ bodyMomentOfInertia_Z = 4.88094466281336 [IC case]
++ altitudeMsl = 9144.0 [IC case]
====done=====
CPU times: user 39.2 ms, sys: 6.46 ms, total: 45.7 ms
Wall time: 44 ms
```

```
Variable
                          L2
                                                     L-Infinity-Norm
                                                     77.22782399366406
altitudeMsl ft
                          607.9947105071013
trueAirspeed_nmi_h
                          30.300144397638636
                                                     2.917378529996199
eulerAngle_deg_Roll
                                                     0.1253996817079627
                          1.2569020649616232
eulerAngle_deg_Pitch
                          0.0
eulerAngle_deg_Yaw
                          0.0
                                                     0
```



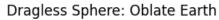


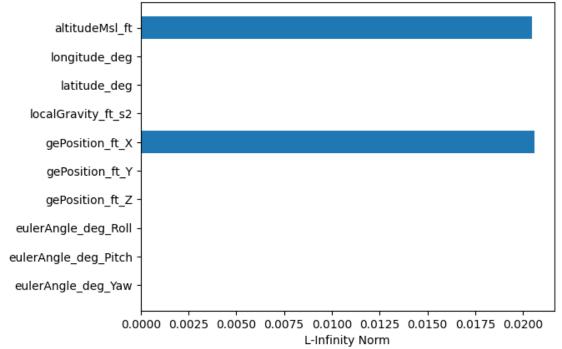
# Oblate, Rotating Earth

```
[42]: %%time
ic = {
    "totalMass": [1.0, "slug"],
    "bodyMomentOfInertia_X": [3.6, "slugft2"],
    "bodyMomentOfInertia_Y": [3.6, "slugft2"],
```

```
"bodyMomentOfInertia_Z": [3.6, "slugft2"],
    "altitudeMsl": [30000, "ft"],
    "referenceWingChord": [0.2, "ft"],
    "referenceWingSpan": [0.2, "ft"],
    "referenceWingArea": [0.1963495, "ft2"]
}
gvOblateRotatingEarth = slEarthSim()
gvOblateRotatingEarth.Reset(ic)
gvOblateRotatingEarth.Run(30)
gvOblateRotatingEarth.GenerateEnglishUnits()
====== SetIC ========
{'totalMass': 14.593902937, 'bodyMomentOfInertia_X': 4.88094466281336,
'bodyMomentOfInertia_Y': 4.88094466281336, 'bodyMomentOfInertia_Z':
4.88094466281336, 'altitudeMsl': 9144.0, 'referenceWingChord':
0.0609600000000001, 'referenceWingSpan': 0.0609600000000001,
'referenceWingArea': 0.018241465452480003}
++ timeStep = 0.1 [default]
++ totalMass = 14.593902937 [IC case]
++ referenceWingSpan = 0.060960000000001 [IC case]
++ referenceWingChord = 0.0609600000000001 [IC case]
++ referenceWingArea = 0.018241465452480003 [IC case]
++ trueAirspeed = 0 [default]
++ angleOfAttack = 0 [default]
++ angleOfSideslip = 0 [default]
++ eulerAngle_Roll = 0 [default]
++ eulerAngle_Pitch = 0 [default]
++ eulerAngle_Yaw = 0 [default]
++ eulerAngleRate_Roll = 0 [default]
++ eulerAngleRate_Pitch = 0 [default]
++ eulerAngleRate_Yaw = 0 [default]
++ bodyMomentOfInertia_X = 4.88094466281336 [IC case]
++ bodyProductOfInertia_XY = 0 [default]
++ bodyProductOfInertia_XZ = 0 [default]
++ bodyProductOfInertia_YX = 0 [default]
++ bodyMomentOfInertia_Y = 4.88094466281336 [IC case]
++ bodyProductOfInertia_YZ = 0 [default]
++ bodyProductOfInertia_XZ = 0 [default]
++ bodyProductOfInertia_YZ = 0 [default]
++ bodyMomentOfInertia_Z = 4.88094466281336 [IC case]
++ latitude = 0 [default]
++ longitude = 0 [default]
++ altitudeMsl = 9144.0 [IC case]
Vecf: 0.0 0.0 0.0
=====done=====
CPU times: user 43.8 ms, sys: 3.9 ms, total: 47.7 ms
Wall time: 49.2 ms
```

Variable	L2	t L-Infinity-Norm
altitudeMsl_ft	0.132658503212526	0.02048735810240032
longitude_deg	1.840404761574716e-10	3.1393235335245e-11
latitude_deg	0.0	0
localGravity_ft_s2	8.923802486168079e-05	9.171880087421869e-06
<pre>gePosition_ft_X</pre>	0.1343059782985569	0.020609743893146515
<pre>gePosition_ft_Y</pre>	6.717601039765743e-05	1.1453364173519276e-05
gePosition_ft_Z	0.0	0
eulerAngle_deg_Roll	2.5437306921432218e-08	2.5527049640761135e-09
${\tt eulerAngle\_deg\_Pitch}$	3.632333118253663e-19	4.079942171239155e-20
eulerAngle_deg_Yaw	4.446769635169959e-16	3.7272118343786e-17





# 1.11.4 Dragless Tumbling Brick - 2

Property	English Value	SI Value
$\overline{I_{xx}}$	$0.001894220 \text{ slug-ft}^2$	$0.002568217477249 \text{ kg-m}^2$
$I_{yy}$	$0.006211019 \text{ slug-ft}^2$	$0.00842101104799105 \text{ kg-m}^2$
$I_{zz}$	$0.007194665 \text{ slug-ft}^2$	$0.00975465595123675~\rm kg\text{-}m^2$
m	0.155404754  slug	2.2679619056149  kg
S	$0.22222 \; \mathrm{ft}^2$	$0.020644913548800003 \text{ m}^2$
b	$0.33333 \; \mathrm{ft}$	0.1016  m
$\bar{c}$	0.66667  ft	0.2032  m

```
[44]: mass = 0.155404754 * gvConvert.SlugToKg
    print("m=", mass)

    ixx = 0.001894220 * gvConvert.Slugft2ToKgm2
    print("Ixx=", ixx)
    iyy = 0.006211019 * gvConvert.Slugft2ToKgm2
    print("Iyy=", iyy)
    izz = 0.007194665 * gvConvert.Slugft2ToKgm2
    print("Izz=", izz)

    cbar = 0.66667 * gvConvert.FeetToMeter
    print("cbar=", cbar)
    b = 0.33333 * gvConvert.FeetToMeter
    print("b=", b)
    s = 0.22222 * gvConvert.SqFeetToSqMeter
    print("s=", s)
```

```
m= 2.2679618958243624

Ixx= 0.002568217499776201

Iyy= 0.008421011121856215

Izz= 0.009754656036800024

cbar= 0.203201016

b= 0.101598984
```

s= 0.020644913548800003

## Flat Earth

```
ic = {
    "totalMass": [0.155404754, "slug"],
    "bodyMomentOfInertia_X": [0.001894220, "slugft2"],
    "bodyMomentOfInertia_Y": [0.006211019, "slugft2"],
    "bodyMomentOfInertia_Z": [0.007194665, "slugft2"],
    "altitudeMsl": [30000, "ft"],
    "referenceWingChord": [0.66667, "ft"],
    "referenceWingSpan": [0.33333, "ft"],
    "referenceWingArea": [0.22222, "ft"],
    "eulerAngleRate_Roll": [10, "deg_s"],
    "eulerAngleRate_Pitch": [20, "deg_s"],
    "eulerAngleRate_Yaw": [30, "deg_s"]
}
gvFlatEarthSim.Reset(ic)
gvFlatEarthSim.Run(30.0)
gvFlatEarthSim.GenerateEnglishUnits()
number of headers: 31
['time', 'gePosition_ft_X', 'gePosition_ft_Y', 'gePosition_ft_Z',
'feVelocity_ft_s_X', 'feVelocity_ft_s_Y', 'feVelocity_ft_s_Z', 'altitudeMsl_ft',
'longitude_deg', 'latitude_deg', 'localGravity_ft_s2', 'eulerAngle_deg_Yaw',
'eulerAngle_deg_Pitch', 'eulerAngle_deg_Roll',
'bodyAngularRateWrtEi_deg_s_Roll', 'bodyAngularRateWrtEi_deg_s_Pitch',
'bodyAngularRateWrtEi_deg_s_Yaw', 'altitudeRateWrtMsl_ft_min',
'speedOfSound_ft_s', 'airDensity_slug_ft3', 'ambientPressure_lbf_ft2',
'ambientTemperature_dgR', 'aero_bodyForce_lbf_X', 'aero_bodyForce_lbf_Y',
'aero_bodyForce_lbf_Z', 'aero_bodyMoment_ftlbf_L', 'aero_bodyMoment_ftlbf_M',
'aero_bodyMoment_ftlbf_N', 'mach', 'dynamicPressure_lbf_ft2',
'trueAirspeed_nmi_h']
===============
*************
Model: Zero Aero Output
creation date: 2021-07-21
file version: Initial version
*************
+++++ MODEL INPUTS AND OUTPUTS +++++
++> Output: CX
++> Output:
            CY
++> Output: CZ
++> Output:
            CLL
++> Output:
            CLM
++> Output: CLN
---- DAVE-ML MODEL PARSE COMPLETE ----
```

```
---- CheckModel ----
     numSignals:
                  ---- END CheckModel ----
     ====== SetIC ========
     {'totalMass': 2.2679618958243624, 'bodyMomentOfInertia_X': 0.002568217499776201,
     'bodyMomentOfInertia_Y': 0.008421011121856215, 'bodyMomentOfInertia_Z':
     0.009754656036800024, 'altitudeMsl': 9144.0, 'referenceWingChord': 0.203201016,
     'referenceWingSpan': 0.101598984, 'referenceWingArea': 0.067732656,
     'eulerAngleRate_Roll': 0.17453292519943295, 'eulerAngleRate_Pitch':
     0.3490658503988659, 'eulerAngleRate_Yaw': 0.5235987755982988}
     ++ timeStep = 0.1 [default]
     ++ totalMass = 2.2679618958243624 [IC case]
     ++ referenceWingSpan = 0.101598984 [IC case]
     ++ referenceWingChord = 0.203201016 [IC case]
     ++ referenceWingArea = 0.067732656 [IC case]
     ++ trueAirspeed = 0 [default]
     ++ angleOfAttack = 0 [default]
     ++ angleOfSideslip = 0 [default]
     ++ eulerAngle_Roll = 0 [default]
     ++ eulerAngle_Pitch = 0 [default]
     ++ eulerAngle_Yaw = 0 [default]
     ++ eulerAngleRate_Roll = 0.17453292519943295 [IC case]
     ++ eulerAngleRate_Pitch = 0.3490658503988659 [IC case]
     ++ eulerAngleRate_Yaw = 0.5235987755982988 [IC case]
     ++ bodyMomentOfInertia_X = 0.002568217499776201 [IC case]
     ++ bodyProductOfInertia_XY = 0 [default]
     ++ bodyProductOfInertia_XZ = 0 [default]
     ++ bodyProductOfInertia_YX = 0 [default]
     ++ bodyMomentOfInertia_Y = 0.008421011121856215 [IC case]
     ++ bodyProductOfInertia_YZ = 0 [default]
     ++ bodyProductOfInertia_XZ = 0 [default]
     ++ bodyProductOfInertia YZ = 0 [default]
     ++ bodyMomentOfInertia_Z = 0.009754656036800024 [IC case]
     ++ altitudeMsl = 9144.0 [IC case]
     =====done=====
     CPU times: user 60.6 ms, sys: 10.1 ms, total: 70.7 ms
     Wall time: 105 ms
[46]: | PrintErrorTable("Tumbling Brick: Flat Earth", gvFlatEarthLabel, gvFlatEarthSim, u
      ⇒gvCC2)
     Variable
                               L2
                                                         L-Infinity-Norm
     _____
                                                         _____
                              607.972054527782
                                                         77.22500410951034
     altitudeMsl_ft
```

2.9152206853502776

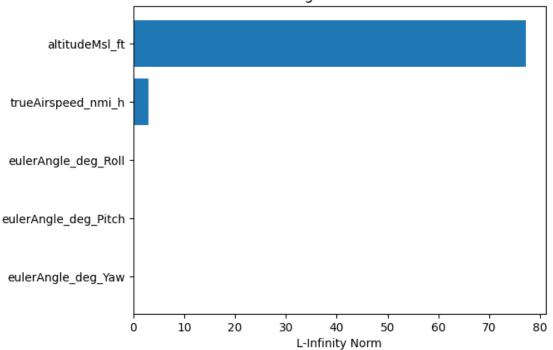
30.278799187883976

trueAirspeed\_nmi\_h

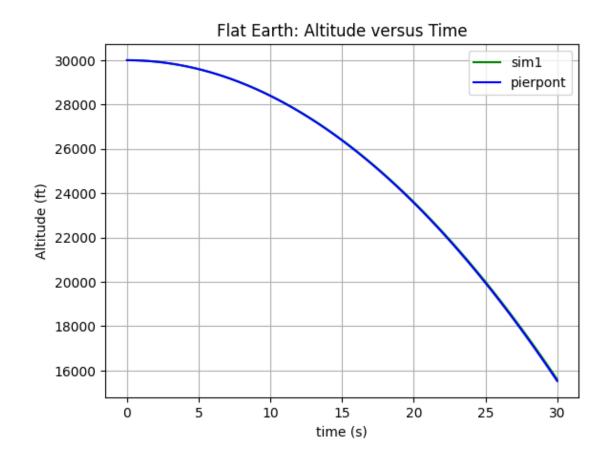
eulerAngle\_deg\_Roll
eulerAngle\_deg\_Pitch
eulerAngle\_deg\_Yaw

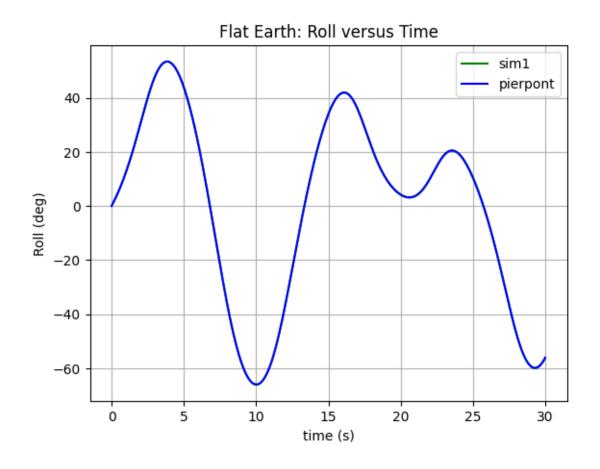
- 0.9696393367797839
- 0.8533820770548749
- 0.2980578394513776
- 0.1253209330691334
- 0.11575857923671151
- 0.03904847535990541

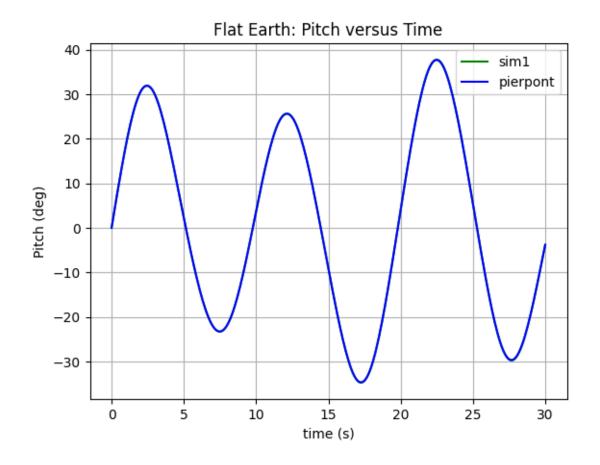


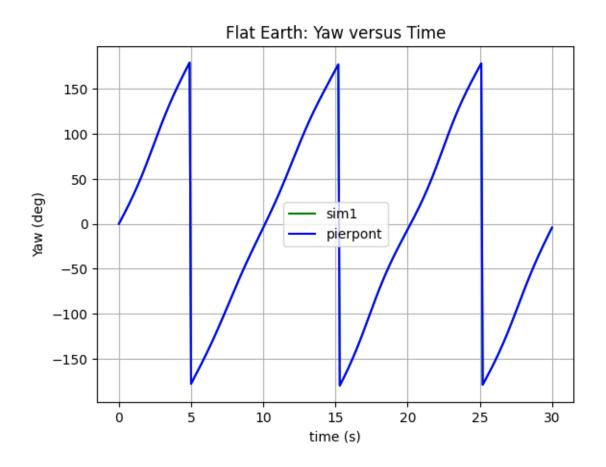


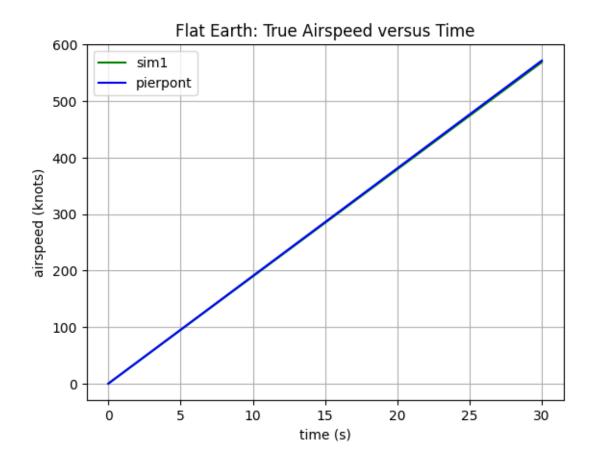
[47]: MakeFlatEarthPlots(gvFlatEarthSim, gvCC2, "sim1")









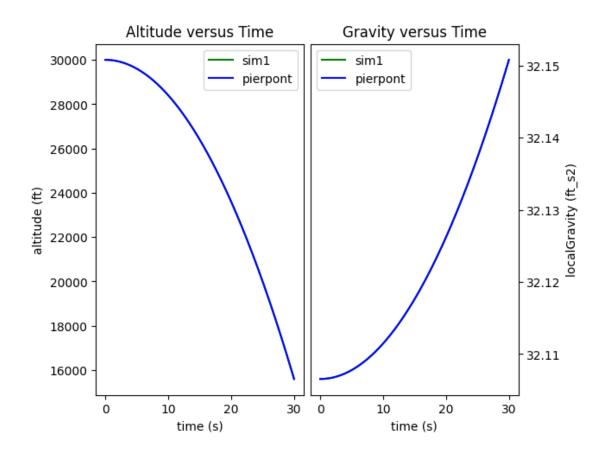


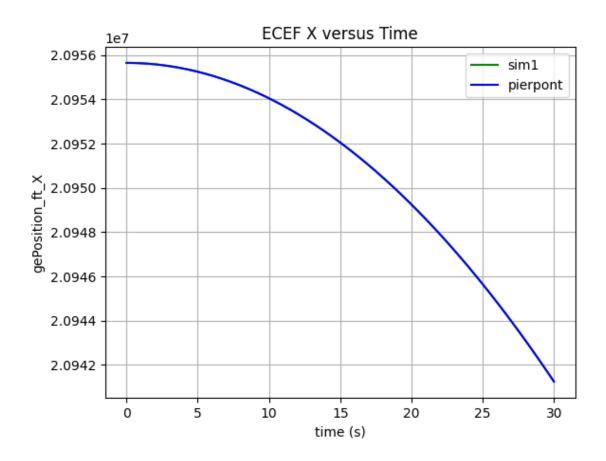
## Oblate, Rotating Earth

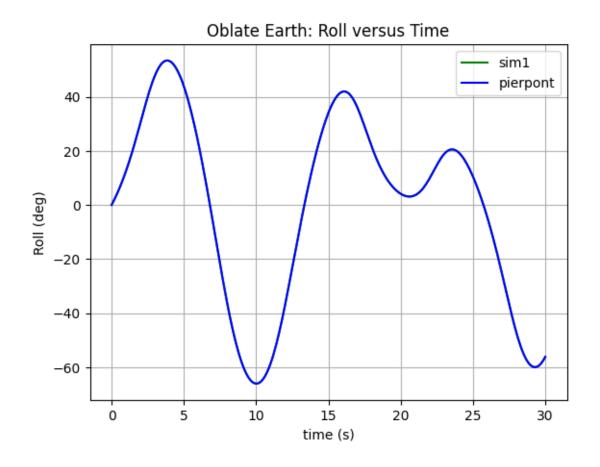
```
[48]: %%time
      ic = {
          "totalMass": [0.155404754, "slug"],
          "bodyMomentOfInertia_X": [0.001894220, "slugft2"],
          "bodyMomentOfInertia_Y": [0.006211019, "slugft2"],
          "bodyMomentOfInertia_Z": [0.007194665, "slugft2"],
          "altitudeMsl": [30000, "ft"],
          "referenceWingChord": [0.66667, "ft"],
          "referenceWingSpan": [0.33333, "ft"],
          "referenceWingArea": [0.22222, "ft"],
          "eulerAngleRate_Roll": [10, "deg_s"],
          "eulerAngleRate_Pitch": [20, "deg_s"],
          "eulerAngleRate_Yaw": [30, "deg_s"]
      gvOblateRotatingEarth.Reset(ic)
      gvOblateRotatingEarth.Run(30)
      gvOblateRotatingEarth.GenerateEnglishUnits()
```

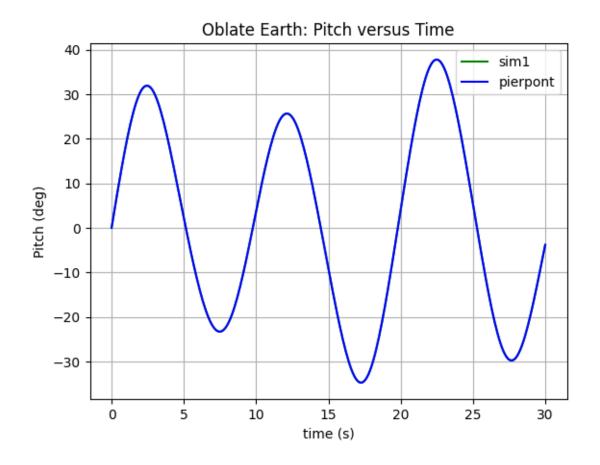
====== SetIC ========

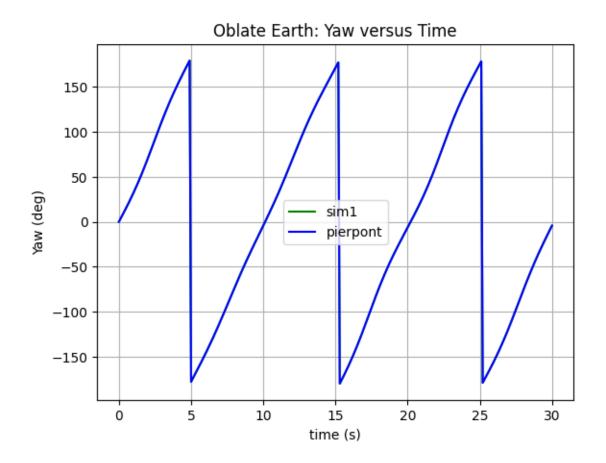
```
{'totalMass': 2.2679618958243624, 'bodyMomentOfInertia_X': 0.002568217499776201,
     'bodyMomentOfInertia_Y': 0.008421011121856215, 'bodyMomentOfInertia_Z':
     0.009754656036800024, 'altitudeMsl': 9144.0, 'referenceWingChord': 0.203201016,
     'referenceWingSpan': 0.101598984, 'referenceWingArea': 0.067732656,
     'eulerAngleRate Roll': 0.17453292519943295, 'eulerAngleRate Pitch':
     0.3490658503988659, 'eulerAngleRate_Yaw': 0.5235987755982988}
     ++ timeStep = 0.1 [default]
     ++ totalMass = 2.2679618958243624 [IC case]
     ++ referenceWingSpan = 0.101598984 [IC case]
     ++ referenceWingChord = 0.203201016 [IC case]
     ++ referenceWingArea = 0.067732656 [IC case]
     ++ trueAirspeed = 0 [default]
     ++ angleOfAttack = 0 [default]
     ++ angleOfSideslip = 0 [default]
     ++ eulerAngle_Roll = 0 [default]
     ++ eulerAngle_Pitch = 0 [default]
     ++ eulerAngle_Yaw = 0 [default]
     ++ eulerAngleRate_Roll = 0.17453292519943295 [IC case]
     ++ eulerAngleRate_Pitch = 0.3490658503988659 [IC case]
     ++ eulerAngleRate Yaw = 0.5235987755982988 [IC case]
     ++ bodyMomentOfInertia X = 0.002568217499776201 [IC case]
     ++ bodyProductOfInertia XY = 0 [default]
     ++ bodyProductOfInertia_XZ = 0 [default]
     ++ bodyProductOfInertia_YX = 0 [default]
     ++ bodyMomentOfInertia_Y = 0.008421011121856215 [IC case]
     ++ bodyProductOfInertia_YZ = 0 [default]
     ++ bodyProductOfInertia_XZ = 0 [default]
     ++ bodyProductOfInertia_YZ = 0 [default]
     ++ bodyMomentOfInertia_Z = 0.009754656036800024 [IC case]
     ++ latitude = 0 [default]
     ++ longitude = 0 [default]
     ++ altitudeMsl = 9144.0 [IC case]
     Vecf: 0.0 0.0 0.0
     =====done=====
     CPU times: user 59.5 ms, sys: 10.3 ms, total: 69.7 ms
     Wall time: 80.8 ms
[49]: MakePlot(gvOblateRotatingEarth, gvCC2, "sim1")
```











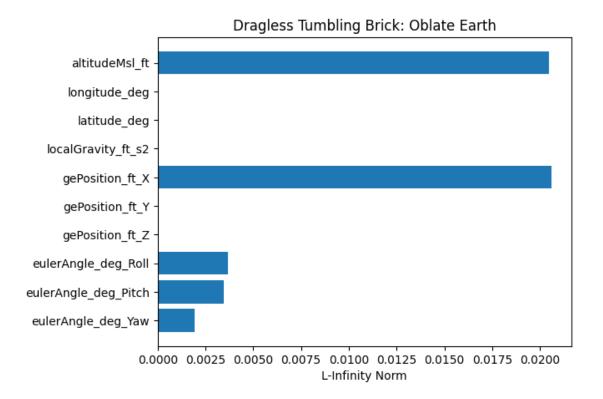
[50]: PrintErrorTable("Dragless Tumbling Brick: Oblate Earth", gvOblateEarthLabel, u →gvOblateRotatingEarth, gvCC2)

L-Infinity-Norm

L2

Variable

altitudeMsl_ft	0.132658503212526	0.02048735810240032
longitude_deg	1.840404761574716e-10	3.1393235335245e-11
latitude_deg	0.0	0
localGravity_ft_s2	8.923802486168079e-05	9.171880087421869e-06
<pre>gePosition_ft_X</pre>	0.1343059782985569	0.020609743893146515
<pre>gePosition_ft_Y</pre>	6.717601039765743e-05	1.1453364173519276e-05
<pre>gePosition_ft_Z</pre>	0.0	0
eulerAngle_deg_Roll	0.02661660809333747	0.003678374584481503
eulerAngle_deg_Pitch	0.025190519978331032	0.0034552761165556056
eulerAngle_deg_Yaw	0.01326283857690703	0.0019424252806858888



## 1.11.5 Tumbling Brick Damping - 3

Tumbling brick with damping check case

**Flat Earth** Run a simulation for 30 seconds at a time step of 0.1 seconds.

```
"referenceWingArea": [0.22222, "ft2"],
    "eulerAngleRate_Roll": [10, "deg_s"],
    "eulerAngleRate_Pitch": [20, "deg_s"],
    "eulerAngleRate_Yaw": [30, "deg_s"]
}
inputs = ["trueAirspeed", "bodyAngularRate_Roll", "bodyAngularRate_Pitch", __
 →"bodyAngularRate Yaw"]
gvFlatEarthSim.Reset(ic)
gvFlatEarthSim.AddAeroModelInput(inputs)
gvFlatEarthSim.Run(30.0)
gvFlatEarthSim.GenerateEnglishUnits()
number of headers: 31
['time', 'gePosition ft X', 'gePosition ft Y', 'gePosition ft Z',
'feVelocity_ft_s_X', 'feVelocity_ft_s_Y', 'feVelocity_ft_s_Z', 'altitudeMsl_ft',
'longitude_deg', 'latitude_deg', 'localGravity_ft_s2', 'eulerAngle_deg_Yaw',
'eulerAngle_deg_Pitch', 'eulerAngle_deg_Roll',
'bodyAngularRateWrtEi_deg_s_Roll', 'bodyAngularRateWrtEi_deg_s_Pitch',
'bodyAngularRateWrtEi_deg_s_Yaw', 'altitudeRateWrtMsl_ft_min',
'speedOfSound ft_s', 'airDensity slug ft3', 'ambientPressure_lbf_ft2',
'ambientTemperature_dgR', 'aero_bodyForce_lbf_X', 'aero_bodyForce_lbf_Y',
'aero bodyForce lbf Z', 'aero bodyMoment ftlbf L', 'aero bodyMoment ftlbf M',
'aero_bodyMoment_ftlbf_N', 'mach', 'dynamicPressure_lbf_ft2',
'trueAirspeed_nmi_h']
***********
Model: Example brick aerodynamic model
creation date: 2012-10-05
file version: Mod D, 2021-05-01
************
-variableDef-
 varDefStruct.name: referenceWingArea
 varDefStruct.varID: SWING
 varDefStruct.units: ft2
 varDefStruct.axisSystem: None
varDefStruct.sign: None
 varDefStruct.alias: None
varDefStruct.symbol: None
 varDefStruct.hasInitialValue: True
 varDefStruct.initialValue: 0.22222
varDefStruct.isStdAIAA: True
 varDefStruct.isOutput: False
varDefStruct.hasMath: False
varDefStruct.codeText: None
-variableDef-
 varDefStruct.name: referenceWingSpan
 varDefStruct.varID: BSPAN
```

varDefStruct.axisSystem: None varDefStruct.sign: None varDefStruct.alias: None varDefStruct.symbol: None varDefStruct.hasInitialValue: True varDefStruct.initialValue: 0.33333 varDefStruct.isStdAIAA: True varDefStruct.isOutput: False varDefStruct.hasMath: False varDefStruct.codeText: None -variableDefvarDefStruct.name: referenceWingChord varDefStruct.varID: CBAR varDefStruct.units: ft varDefStruct.axisSystem: None varDefStruct.sign: None varDefStruct.alias: None varDefStruct.symbol: None varDefStruct.hasInitialValue: True varDefStruct.initialValue: 0.66667 varDefStruct.isStdAIAA: True varDefStruct.isOutput: False varDefStruct.hasMath: False varDefStruct.codeText: None -variableDefvarDefStruct.name: roll damping from roll rate varDefStruct.varID: CLP\_DAMPING varDefStruct.units: \_rad varDefStruct.axisSystem: varDefStruct.sign: None varDefStruct.alias: None varDefStruct.symbol: None varDefStruct.hasInitialValue: True

varDefStruct.units: ft

-variableDef-

varDefStruct.name: roll damping from yaw rate

varDefStruct.varID: CLR\_DAMPING

varDefStruct.initialValue: -1.0
varDefStruct.isStdAIAA: False
varDefStruct.isOutput: False
varDefStruct.hasMath: False
varDefStruct.codeText: None

varDefStruct.units: \_rad
varDefStruct.axisSystem: None

varDefStruct.sign: None
varDefStruct.alias: None
varDefStruct.symbol: None

varDefStruct.hasInitialValue: True

varDefStruct.initialValue: 0.0 varDefStruct.isStdAIAA: False varDefStruct.isOutput: False varDefStruct.hasMath: False varDefStruct.codeText: None -variableDefvarDefStruct.name: pitch damping from pitch rate varDefStruct.varID: CMQ\_DAMPING varDefStruct.units: \_rad varDefStruct.axisSystem: None varDefStruct.sign: None varDefStruct.alias: None varDefStruct.symbol: None varDefStruct.hasInitialValue: True varDefStruct.initialValue: -1.0 varDefStruct.isStdAIAA: False varDefStruct.isOutput: False varDefStruct.hasMath: False varDefStruct.codeText: None -variableDefvarDefStruct.name: yaw damping from roll rate varDefStruct.varID: CNP\_DAMPING varDefStruct.units: \_rad varDefStruct.axisSystem: None varDefStruct.sign: None varDefStruct.alias: None varDefStruct.symbol: None varDefStruct.hasInitialValue: True varDefStruct.initialValue: 0.0 varDefStruct.isStdAIAA: False varDefStruct.isOutput: False varDefStruct.hasMath: False varDefStruct.codeText: None -variableDefvarDefStruct.name: yaw damping from yaw rate varDefStruct.varID: CNR\_DAMPING varDefStruct.units: rad varDefStruct.axisSystem: None varDefStruct.sign: None varDefStruct.alias: None varDefStruct.symbol: None varDefStruct.hasInitialValue: True varDefStruct.initialValue: -1.0 varDefStruct.isStdAIAA: False varDefStruct.isOutput: False varDefStruct.hasMath: False varDefStruct.codeText: None

-variableDef-

varDefStruct.name: trueAirspeed

varDefStruct.varID: VRW
varDefStruct.units: ft\_s

varDefStruct.axisSystem: None

varDefStruct.sign: None
varDefStruct.alias: None
varDefStruct.symbol: None

varDefStruct.hasInitialValue: False
varDefStruct.initialValue: None
varDefStruct.isStdAIAA: True
varDefStruct.isOutput: False
varDefStruct.hasMath: False
varDefStruct.codeText: None

-variableDef-

varDefStruct.name: bodyAngularRate\_Roll

varDefStruct.varID: PB
varDefStruct.units: rad\_s
varDefStruct.axisSystem: None

varDefStruct.sign: RWD
varDefStruct.alias: None
varDefStruct.symbol: None

varDefStruct.hasInitialValue: False
varDefStruct.initialValue: None
varDefStruct.isStdAIAA: True
varDefStruct.isOutput: False
varDefStruct.hasMath: False
varDefStruct.codeText: None

-variableDef-

varDefStruct.name: bodyAngularRate\_Pitch

varDefStruct.varID: QB
varDefStruct.units: rad\_s
varDefStruct.axisSystem: None

varDefStruct.sign: ANU
varDefStruct.alias: None
varDefStruct.symbol: None

varDefStruct.hasInitialValue: False
varDefStruct.initialValue: None
varDefStruct.isStdAIAA: True
varDefStruct.isOutput: False
varDefStruct.hasMath: False
varDefStruct.codeText: None

-variableDef-

varDefStruct.name: bodyAngularRate\_Yaw

varDefStruct.varID: RB
varDefStruct.units: rad\_s
varDefStruct.axisSystem: None

varDefStruct.sign: ANR
varDefStruct.alias: None

varDefStruct.symbol: None varDefStruct.hasInitialValue: False varDefStruct.initialValue: None varDefStruct.isStdAIAA: True varDefStruct.isOutput: False varDefStruct.hasMath: False varDefStruct.codeText: None -variableDefvarDefStruct.name: PBO2V varDefStruct.varID: PBO2V varDefStruct.units: nd varDefStruct.axisSystem: None varDefStruct.sign: None varDefStruct.alias: None varDefStruct.symbol: None varDefStruct.hasInitialValue: False varDefStruct.initialValue: None varDefStruct.isStdAIAA: False varDefStruct.isOutput: False varDefStruct.hasMath: True varDefStruct.codeText: ({BSPAN} \* {PB}) / (2.0 \* {VRW}) -variableDefvarDefStruct.name: QCO2V varDefStruct.varID: QCO2V varDefStruct.units: nd varDefStruct.axisSystem: None varDefStruct.sign: None varDefStruct.alias: None varDefStruct.symbol: None varDefStruct.hasInitialValue: False varDefStruct.initialValue: None varDefStruct.isStdAIAA: False varDefStruct.isOutput: False varDefStruct.hasMath: True varDefStruct.codeText: ({CBAR} \* {QB}) / (2.0 \* {VRW}) -variableDefvarDefStruct.name: RBO2V varDefStruct.varID: RBO2V varDefStruct.units: nd varDefStruct.axisSystem: None varDefStruct.sign: None varDefStruct.alias: None varDefStruct.symbol: None varDefStruct.hasInitialValue: False varDefStruct.initialValue: None varDefStruct.isStdAIAA: False varDefStruct.isOutput: False varDefStruct.hasMath: True

```
varDefStruct.codeText: ({BSPAN} * {RB}) / (2.0 * {VRW})
-variableDef-
varDefStruct.name: totalCoefficientOfLift
varDefStruct.varID: CL
varDefStruct.units: nd
varDefStruct.axisSystem:
varDefStruct.sign: None
varDefStruct.alias: None
varDefStruct.symbol: None
varDefStruct.hasInitialValue: True
varDefStruct.initialValue: 0.0
varDefStruct.isStdAIAA: True
varDefStruct.isOutput: True
varDefStruct.hasMath: False
varDefStruct.codeText: None
-variableDef-
varDefStruct.name: totalCoefficientOfDrag
varDefStruct.varID: CD
varDefStruct.units: nd
varDefStruct.axisSystem:
                          None
varDefStruct.sign: None
varDefStruct.alias: None
varDefStruct.symbol: None
varDefStruct.hasInitialValue: True
varDefStruct.initialValue: 0.01
varDefStruct.isStdAIAA: True
varDefStruct.isOutput: True
varDefStruct.hasMath: False
varDefStruct.codeText: None
-variableDef-
varDefStruct.name: aeroBodyForceCoefficient_Y
varDefStruct.varID: CY
varDefStruct.units: nd
varDefStruct.axisSystem:
                          None
varDefStruct.sign: None
varDefStruct.alias: None
varDefStruct.symbol: None
varDefStruct.hasInitialValue: True
varDefStruct.initialValue: 0.0
varDefStruct.isStdAIAA: True
varDefStruct.isOutput: True
varDefStruct.hasMath: False
varDefStruct.codeText: None
-variableDef-
varDefStruct.name: aeroBodyMomentCoefficient_Roll
varDefStruct.varID: Cl
varDefStruct.units: nd
varDefStruct.axisSystem: None
```

```
varDefStruct.sign: None
varDefStruct.alias: None
varDefStruct.symbol: None
varDefStruct.hasInitialValue: False
varDefStruct.initialValue: None
varDefStruct.isStdAIAA: True
varDefStruct.isOutput: True
varDefStruct.hasMath: True
varDefStruct.codeText: ({CLP_DAMPING} * {PBO2V}) + ({CLR_DAMPING} * {RBO2V})
-variableDef-
varDefStruct.name: aeroBodyMomentCoefficient_Pitch
varDefStruct.varID: Cm
varDefStruct.units: nd
varDefStruct.axisSystem:
varDefStruct.sign: None
varDefStruct.alias: None
varDefStruct.symbol: None
varDefStruct.hasInitialValue: False
varDefStruct.initialValue: None
varDefStruct.isStdAIAA: True
varDefStruct.isOutput: True
varDefStruct.hasMath: True
varDefStruct.codeText: ({CMQ_DAMPING} * {QCO2V})
-variableDef-
varDefStruct.name: aeroBodyMomentCoefficient_Yaw
varDefStruct.varID: Cn
varDefStruct.units: nd
varDefStruct.axisSystem:
                          None
varDefStruct.sign: None
varDefStruct.alias: None
varDefStruct.symbol: None
varDefStruct.hasInitialValue: False
varDefStruct.initialValue: None
varDefStruct.isStdAIAA: True
varDefStruct.isOutput: True
varDefStruct.hasMath: True
varDefStruct.codeText: ({CNP_DAMPING} * {PBO2V}) + ({CNR_DAMPING} * {RBO2V})
-checkData-
staticShot: Nominal
signal type: {http://daveml.org/2010/DAVEML}checkInputs
signal name: trueAirspeed
signal varID: VRW
signal value:
               10.0
signal units: ft_s
[localSignal append] -> Nominal signal #: 1
signal type: {http://daveml.org/2010/DAVEML}checkInputs
signal name: bodyAngularRate_Roll
signal varID: PB
```

```
signal value: 0.3
signal units: rad_s
[localSignal append] -> Nominal signal #: 2
signal type: {http://daveml.org/2010/DAVEML}checkInputs
signal name: bodyAngularRate Pitch
signal varID: QB
signal value: 1.5
signal units: rad_s
[localSignal append] -> Nominal signal #: 3
signal type: {http://daveml.org/2010/DAVEML}checkInputs
signal name: bodyAngularRate_Yaw
signal varID: RB
signal value: 0.6
signal units: rad_s
[localSignal append] -> Nominal signal #: 4
signal type: {http://daveml.org/2010/DAVEML}internalValues
signal name: referenceWingArea
signal varID: SWING
signal value: 0.22222
signal units: ft2
signal tol: 1e-5
[localSignal append] -> Nominal signal #: 5
signal type: {http://daveml.org/2010/DAVEML}internalValues
signal name: referenceWingSpan
signal varID: BSPAN
signal value: 0.33333
signal units: ft
signal tol: 1e-5
[localSignal append] -> Nominal signal #: 6
signal type: {http://daveml.org/2010/DAVEML}internalValues
signal name: referenceWingChord
signal varID: CBAR
signal value: 0.66667
signal units: ft
signal tol: 1e-5
[localSignal append] -> Nominal signal #: 7
signal type: {http://daveml.org/2010/DAVEML}internalValues
signal name: roll damping from roll rate
signal varID: CLP_DAMPING
signal value: -1.0
[localSignal append] -> Nominal signal #: 8
signal type: {http://daveml.org/2010/DAVEML}internalValues
signal name: roll damping from yaw rate
signal varID: CLR_DAMPING
signal value: 0.0
[localSignal append] -> Nominal signal #: 9
signal type: {http://daveml.org/2010/DAVEML}internalValues
signal name: pitch damping from pitch rate
```

```
signal varID: CMQ_DAMPING
signal value:
              -1.0
[localSignal append] -> Nominal signal #: 10
signal type: {http://daveml.org/2010/DAVEML}internalValues
signal name: yaw damping from roll rate
signal varID: CNP_DAMPING
signal value: 0.0
[localSignal append] -> Nominal signal #: 11
signal type: {http://daveml.org/2010/DAVEML}internalValues
signal name: yaw damping from yaw rate
signal varID: CNR_DAMPING
signal value: -1.0
[localSignal append] -> Nominal signal #: 12
             {http://daveml.org/2010/DAVEML}internalValues
signal type:
signal name: PBO2V
signal varID: PBO2V
signal value: 0.005
[localSignal append] -> Nominal signal #: 13
signal type: {http://daveml.org/2010/DAVEML}internalValues
signal name: QCO2V
signal varID: QCO2V
signal value: 0.05
[localSignal append] -> Nominal signal #: 14
signal type: {http://daveml.org/2010/DAVEML}internalValues
signal name: RBO2V
signal varID: RBO2V
signal value: 0.01
[localSignal append] -> Nominal signal #: 15
signal type: {http://daveml.org/2010/DAVEML}checkOutputs
signal name: totalCoefficientOfLift
signal varID: CL
signal value: 0.0
[localSignal append] -> Nominal signal #: 16
signal type: {http://daveml.org/2010/DAVEML}checkOutputs
signal name: totalCoefficientOfDrag
signal varID: CD
signal value: 0.01
[localSignal append] -> Nominal signal #: 17
signal type: {http://daveml.org/2010/DAVEML}checkOutputs
signal name: aeroBodyForceCoefficient_Y
signal varID: CY
signal value: 0.0
[localSignal append] -> Nominal signal #: 18
signal type: {http://daveml.org/2010/DAVEML}checkOutputs
signal name: aeroBodyMomentCoefficient_Roll
signal varID: Cl
signal value: -0.005
[localSignal append] -> Nominal signal #: 19
```

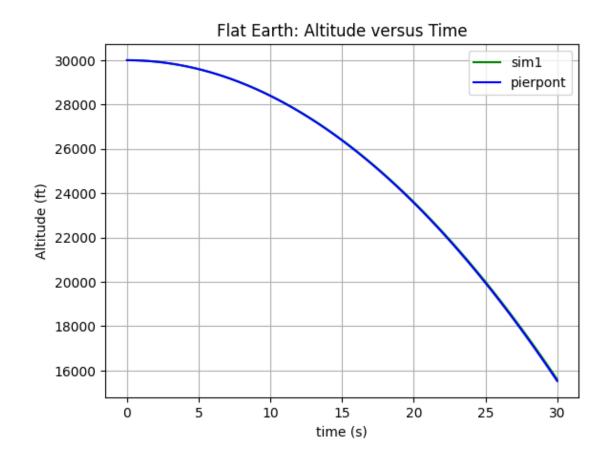
```
signal type: {http://daveml.org/2010/DAVEML}checkOutputs
 signal name: aeroBodyMomentCoefficient_Pitch
 signal varID: Cm
 signal value: -0.05
 [localSignal append] -> Nominal signal #: 20
 signal type: {http://daveml.org/2010/DAVEML}checkOutputs
 signal name: aeroBodyMomentCoefficient Yaw
 signal varID: Cn
 signal value: -0.01
 [localSignal append] -> Nominal signal #: 21
21 signals in Nominal
--- PreProcess Equations and Functions ---
 << PBO2V >>
  [raw] \rightarrow (\{BSPAN\} * \{PB\}) / (2.0 * \{VRW\})
  [python] -> (self.Data["BSPAN"] * self.Data["PB"]) / (2.0 * self.Data["VRW"])
 << QCO2V >>
  [raw] \rightarrow (\{CBAR\} * \{QB\}) / (2.0 * \{VRW\})
  [python] -> (self.Data["CBAR"] * self.Data["QB"]) / (2.0 * self.Data["VRW"])
 << RBO2V >>
  [raw] \rightarrow (\{BSPAN\} * \{RB\}) / (2.0 * \{VRW\})
  [python] -> (self.Data["BSPAN"] * self.Data["RB"]) / (2.0 * self.Data["VRW"])
 << Cl >>
  [raw]-> ({CLP_DAMPING} * {PBO2V}) + ({CLR_DAMPING} * {RBO2V})
  [python]-> (self.Data["CLP_DAMPING"] * self.Data["PB02V"]) +
(self.Data["CLR_DAMPING"] * self.Data["RB02V"])
 << Cm >>
  [raw]-> ({CMQ_DAMPING} * {QCO2V})
  [python] -> (self.Data["CMQ_DAMPING"] * self.Data["QCO2V"])
 << Cn >>
  [raw] \rightarrow (\{CNP\_DAMPING\} * \{PBO2V\}) + (\{CNR\_DAMPING\} * \{RBO2V\})
  [python]-> (self.Data["CNP_DAMPING"] * self.Data["PB02V"]) +
(self.Data["CNR_DAMPING"] * self.Data["RBO2V"])
+++++ MODEL INPUTS AND OUTPUTS +++++
++> Input: VRW
++> Input: PB
++> Input: QB
++> Input: RB
++> Output: CL
++> Output: CD
++> Output: CY
++> Output:
            Cl
++> Output: Cm
++> Output:
            Cn
Number of check cases: 1
```

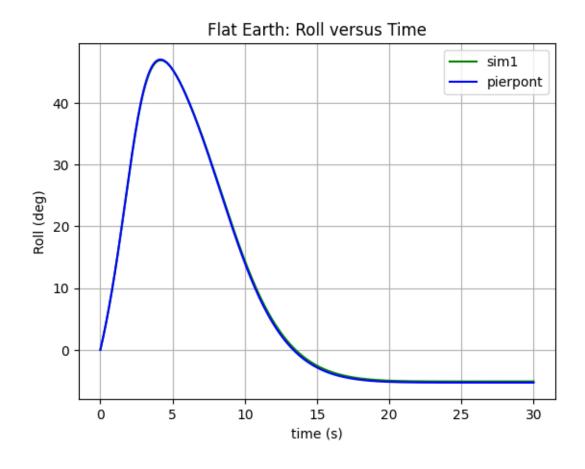
```
--Variables defined in model--
referenceWingArea
referenceWingSpan
referenceWingChord
roll damping from roll rate
roll damping from yaw rate
pitch damping from pitch rate
yaw damping from roll rate
yaw damping from yaw rate
trueAirspeed
bodyAngularRate_Roll
bodyAngularRate_Pitch
bodyAngularRate_Yaw
PB02V
QCO2V
RB02V
totalCoefficientOfLift
totalCoefficientOfDrag
aeroBodyForceCoefficient Y
aeroBodyMomentCoefficient_Roll
aeroBodyMomentCoefficient Pitch
aeroBodyMomentCoefficient_Yaw
---- DAVE-ML MODEL PARSE COMPLETE ----
---- CheckModel -----
numSignals: [21]
---- END CheckModel ----
====== SetIC ========
{'totalMass': 2.2679618958243624, 'bodyMomentOfInertia_X': 0.002568217499776201,
'bodyMomentOfInertia Y': 0.008421011121856215, 'bodyMomentOfInertia Z':
0.009754656036800024, 'altitudeMsl': 9144.0, 'referenceWingChord': 0.203201016,
'referenceWingSpan': 0.101598984, 'referenceWingArea': 0.020644913548800003,
'eulerAngleRate_Roll': 0.17453292519943295, 'eulerAngleRate_Pitch':
0.3490658503988659, 'eulerAngleRate_Yaw': 0.5235987755982988}
++ timeStep = 0.1 [default]
++ totalMass = 2.2679618958243624 [IC case]
++ referenceWingSpan = 0.101598984 [IC case]
++ referenceWingChord = 0.203201016 [IC case]
++ referenceWingArea = 0.020644913548800003 [IC case]
++ trueAirspeed = 0 [default]
++ angleOfAttack = 0 [default]
++ angleOfSideslip = 0 [default]
++ eulerAngle_Roll = 0 [default]
```

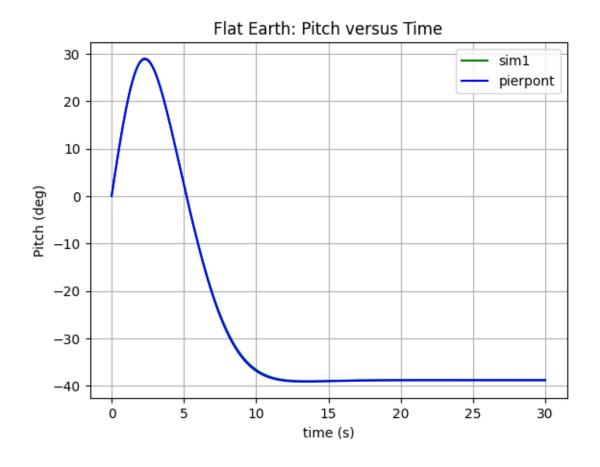
```
++ eulerAngle_Pitch = 0 [default]
++ eulerAngle_Yaw = 0 [default]
++ eulerAngleRate_Roll = 0.17453292519943295 [IC case]
++ eulerAngleRate_Pitch = 0.3490658503988659 [IC case]
++ eulerAngleRate Yaw = 0.5235987755982988 [IC case]
++ bodyMomentOfInertia_X = 0.002568217499776201 [IC case]
++ bodyProductOfInertia_XY = 0 [default]
++ bodyProductOfInertia_XZ = 0 [default]
++ bodyProductOfInertia_YX = 0 [default]
++ bodyMomentOfInertia_Y = 0.008421011121856215 [IC case]
++ bodyProductOfInertia_YZ = 0 [default]
++ bodyProductOfInertia_XZ = 0 [default]
++ bodyProductOfInertia_YZ = 0 [default]
++ bodyMomentOfInertia_Z = 0.009754656036800024 [IC case]
++ altitudeMsl = 9144.0 [IC case]
=====done=====
CPU times: user 113 ms, sys: 40.6 ms, total: 154 ms
Wall time: 177 ms
```

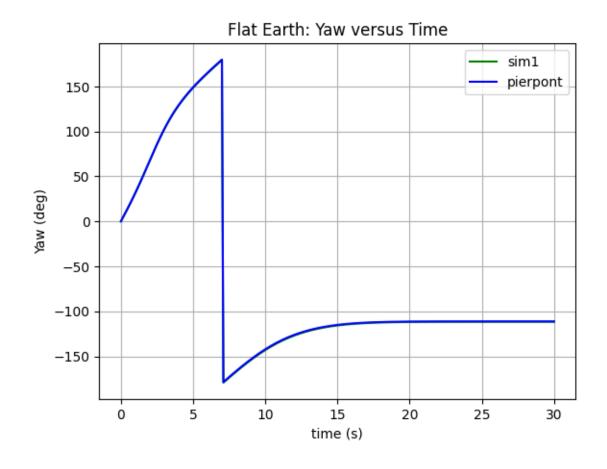
## [52]: MakeFlatEarthPlots(gvFlatEarthSim, gvCC3, "sim1") PrintErrorTable("Tumbling Brick Damping: Flat Earth", gvFlatEarthLabel, →gvFlatEarthSim, gvCC3)

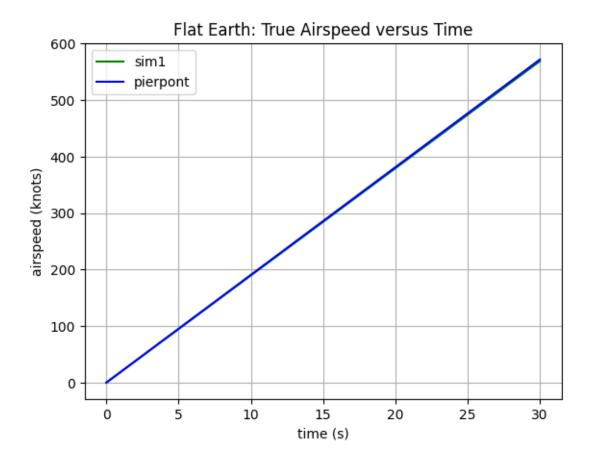
Variable	L2	L-Infinity-Norm
altitudeMsl_ft	607.9829747297271	77.22618244503428
${\tt trueAirspeed\_nmi\_h}$	30.29929205599481	2.9172948944294603
eulerAngle_deg_Roll	4.115319287865008	0.4073996530980857
eulerAngle_deg_Pitch	2.4299375300946506	0.3420764770645235
eulerAngle_deg_Yaw	9.941099709629293	0.8768454992063255

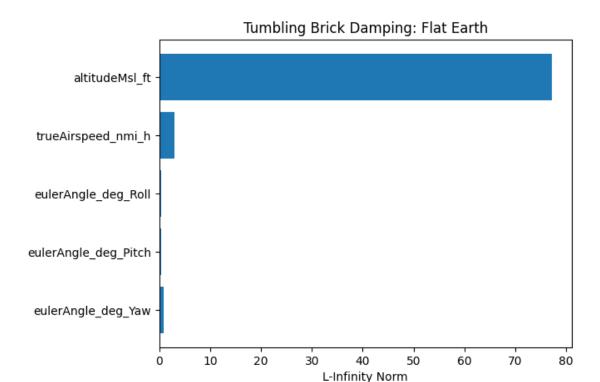












## Oblate, Rotating Earth

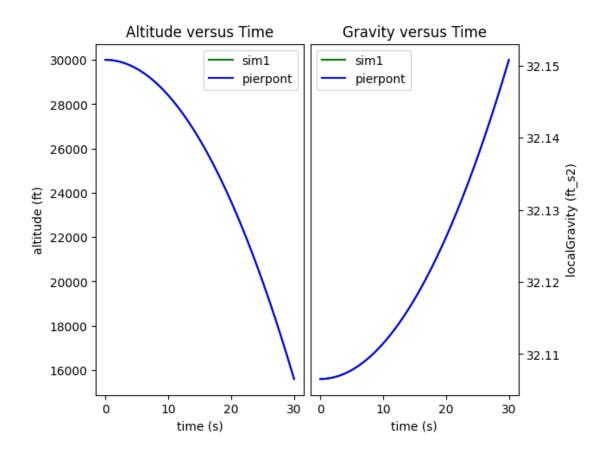
```
[53]: %%time
     ic = {
        "totalMass": [0.155404754, "slug"],
        "bodyMomentOfInertia_X": [0.001894220, "slugft2"],
        "bodyMomentOfInertia_Y": [0.006211019, "slugft2"],
        "bodyMomentOfInertia_Z": [0.007194665, "slugft2"],
        "altitudeMsl": [30000, "ft"],
        "referenceWingChord": [0.66667, "ft"],
        "referenceWingSpan": [0.33333, "ft"],
        "referenceWingArea": [0.22222, "ft2"],
        "eulerAngleRate Roll": [10.0, "deg s"],
        "eulerAngleRate_Pitch": [20.0, "deg_s"],
        "eulerAngleRate_Yaw": [30.0, "deg_s"]
     }
     gvOblateRotatingEarth.Reset(ic)
     gvOblateRotatingEarth.AddAeroModelInput(inputs)
     gvOblateRotatingEarth.Run(30)
     gvOblateRotatingEarth.GenerateEnglishUnits()
```

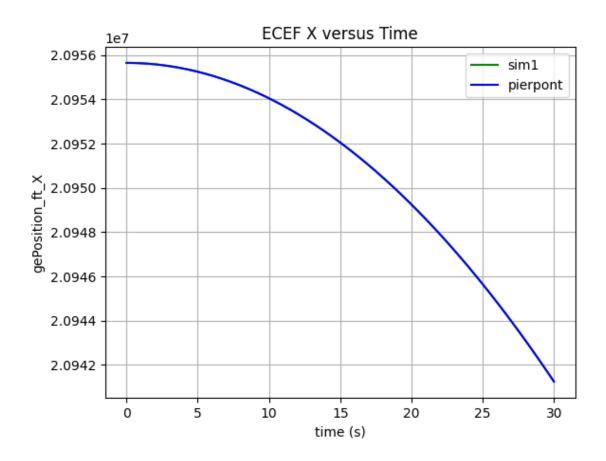
====== SetIC ========

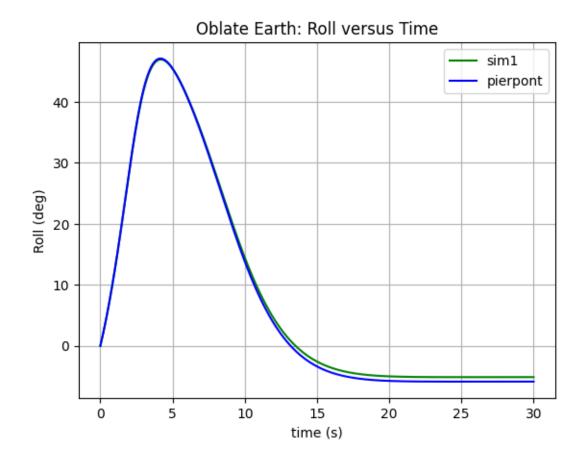
```
{'totalMass': 2.2679618958243624, 'bodyMomentOfInertia_X': 0.002568217499776201,
'bodyMomentOfInertia_Y': 0.008421011121856215, 'bodyMomentOfInertia_Z':
0.009754656036800024, 'altitudeMsl': 9144.0, 'referenceWingChord': 0.203201016,
'referenceWingSpan': 0.101598984, 'referenceWingArea': 0.020644913548800003,
'eulerAngleRate Roll': 0.17453292519943295, 'eulerAngleRate Pitch':
0.3490658503988659, 'eulerAngleRate_Yaw': 0.5235987755982988}
++ timeStep = 0.1 [default]
++ totalMass = 2.2679618958243624 [IC case]
++ referenceWingSpan = 0.101598984 [IC case]
++ referenceWingChord = 0.203201016 [IC case]
++ referenceWingArea = 0.020644913548800003 [IC case]
++ trueAirspeed = 0 [default]
++ angleOfAttack = 0 [default]
++ angleOfSideslip = 0 [default]
++ eulerAngle_Roll = 0 [default]
++ eulerAngle_Pitch = 0 [default]
++ eulerAngle_Yaw = 0 [default]
++ eulerAngleRate_Roll = 0.17453292519943295 [IC case]
++ eulerAngleRate_Pitch = 0.3490658503988659 [IC case]
++ eulerAngleRate Yaw = 0.5235987755982988 [IC case]
++ bodyMomentOfInertia X = 0.002568217499776201 [IC case]
++ bodyProductOfInertia XY = 0 [default]
++ bodyProductOfInertia_XZ = 0 [default]
++ bodyProductOfInertia_YX = 0 [default]
++ bodyMomentOfInertia_Y = 0.008421011121856215 [IC case]
++ bodyProductOfInertia_YZ = 0 [default]
++ bodyProductOfInertia_XZ = 0 [default]
++ bodyProductOfInertia_YZ = 0 [default]
++ bodyMomentOfInertia_Z = 0.009754656036800024 [IC case]
++ latitude = 0 [default]
++ longitude = 0 [default]
++ altitudeMsl = 9144.0 [IC case]
Vecf: 0.0 0.0 0.0
=====done=====
CPU times: user 58.4 ms, sys: 4.2 ms, total: 62.6 ms
Wall time: 72.7 ms
```

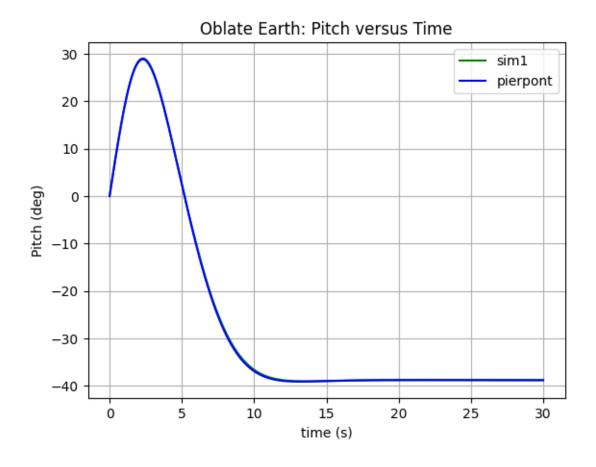
100

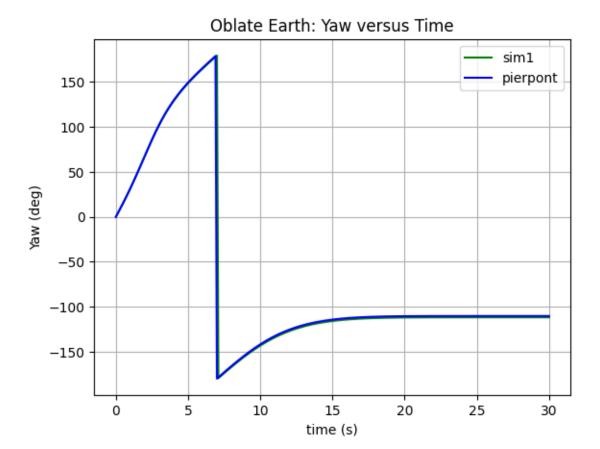
[54]: MakePlot(gvOblateRotatingEarth, gvCC3, "sim1")





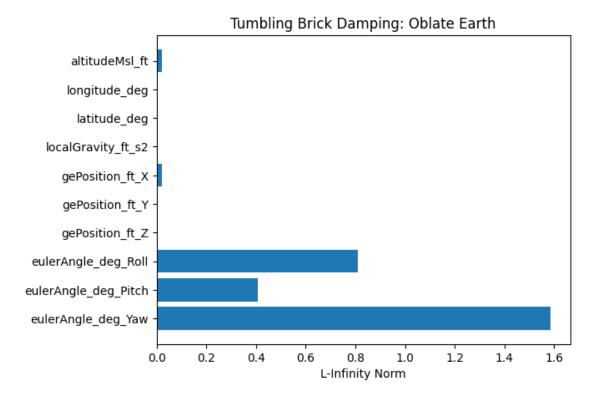






[55]: PrintErrorTable("Tumbling Brick Damping: Oblate Earth", gvOblateEarthLabel, ⊔ ⇔gvOblateRotatingEarth, gvCC3)

Variable	L2	L-Infinity-Norm
altitudeMsl_ft	0.1411402118223276	0.02177725810179254
longitude_deg	2.0649476793351563e-10	3.5255035334494107e-11
latitude_deg	0.0	0
localGravity_ft_s2	1.0779742411875166e-05	6.735199136187475e-07
<pre>gePosition_ft_X</pre>	0.14276753800880893	0.021909743547439575
<pre>gePosition_ft_Y</pre>	7.537771387173215e-05	1.2863464171175565e-05
<pre>gePosition_ft_Z</pre>	0.0	0
eulerAngle_deg_Roll	11.016248991497301	0.8105152209742009
eulerAngle_deg_Pitch	2.787238066784144	0.40670110493388734
eulerAngle_deg_Yaw	22.54233249753164	1.5857787776205328



## 1.11.6 Sphere dropping over rotating, ellipsoidal Earth - $6\,$

```
[56]: %%time
      ppLoadDml('models/NESC/cannonball_inertia.dml')
      CheckModel()
      ic = {
          "totalMass": [1.0, "slug"],
          "bodyMomentOfInertia_X": [3.6, "slugft2"],
          "bodyMomentOfInertia_Y": [3.6, "slugft2"],
          "bodyMomentOfInertia_Z": [3.6, "slugft2"],
          "altitudeMsl": [30000, "ft"],
          "referenceWingChord": [0.2, "ft"],
          "referenceWingSpan": [0.2, "ft"],
          "referenceWingArea": [0.1963495, "ft2"]
      }
      gvOblateRotatingEarth.Reset(ic)
      gvOblateRotatingEarth.totalCoefficientOfDrag = 0.1
      gvOblateRotatingEarth.Run(30)
      gvOblateRotatingEarth.GenerateEnglishUnits()
```

\*\*\*\*\*\*\*\*\*\*\*

Model: Example cannonball inertia model

creation date: 2012-10-04 file version: Initial version

\*\*\*\*\*\*\*\*\*\*\*\*

```
-variableDef-
```

varDefStruct.name: bodyMomentOfInertia\_Roll

varDefStruct.varID: XIXX
varDefStruct.units: slugft2
varDefStruct.axisSystem: None

varDefStruct.sign: None
varDefStruct.alias: None
varDefStruct.symbol: None

varDefStruct.hasInitialValue: True
varDefStruct.initialValue: 3.6
varDefStruct.isStdAIAA: True
varDefStruct.isOutput: True
varDefStruct.hasMath: False
varDefStruct.codeText: None

-variableDef-

varDefStruct.name: bodyMomentOfInertia\_Pitch

varDefStruct.varID: XIYY
varDefStruct.units: slugft2
varDefStruct.axisSystem: None

varDefStruct.sign: None
varDefStruct.alias: None
varDefStruct.symbol: None

varDefStruct.hasInitialValue: True
varDefStruct.initialValue: 3.6
varDefStruct.isStdAIAA: True
varDefStruct.isOutput: True

varDefStruct.hasMath: False
varDefStruct.codeText: None

-variableDef-

varDefStruct.name: bodyMomentOfInertia\_Yaw

varDefStruct.varID: XIZZ
varDefStruct.units: slugft2
varDefStruct.axisSystem: None

varDefStruct.sign: None
varDefStruct.alias: None
varDefStruct.symbol: None

varDefStruct.hasInitialValue: True
varDefStruct.initialValue: 3.6
varDefStruct.isStdAIAA: True
varDefStruct.isOutput: True
varDefStruct.hasMath: False
varDefStruct.codeText: None

-variableDef-

varDefStruct.name: bodyProductOfInertia\_ZX

varDefStruct.varID: XIZX
varDefStruct.units: slugft2
varDefStruct.axisSystem: None

varDefStruct.sign: None
varDefStruct.alias: None
varDefStruct.symbol: None

varDefStruct.hasInitialValue: True
varDefStruct.initialValue: 0.0
varDefStruct.isStdAIAA: True
varDefStruct.isOutput: True
varDefStruct.hasMath: False

-variableDef-

varDefStruct.name: bodyProductOfInertia\_XY

varDefStruct.varID: XIXY
varDefStruct.units: slugft2
varDefStruct.axisSystem: None

varDefStruct.codeText: None

varDefStruct.sign: None
varDefStruct.alias: None
varDefStruct.symbol: None

varDefStruct.hasInitialValue: True
varDefStruct.initialValue: 0.0
varDefStruct.isStdAIAA: True
varDefStruct.isOutput: True
varDefStruct.hasMath: False
varDefStruct.codeText: None

-variableDef-

varDefStruct.name: bodyProductOfInertia\_YZ

varDefStruct.varID: XIYZ
varDefStruct.units: slugft2
varDefStruct.axisSystem: None

varDefStruct.sign: None
varDefStruct.alias: None
varDefStruct.symbol: None

varDefStruct.hasInitialValue: True
varDefStruct.initialValue: 0.0
varDefStruct.isStdAIAA: True
varDefStruct.isOutput: True
varDefStruct.hasMath: False
varDefStruct.codeText: None

-variableDef-

varDefStruct.name: totalMass
varDefStruct.varID: XMASS
varDefStruct.units: slug
varDefStruct.axisSystem: None

varDefStruct.sign: None
varDefStruct.alias: None
varDefStruct.symbol: None

varDefStruct.hasInitialValue: True varDefStruct.initialValue: 1.0 varDefStruct.isStdAIAA: True varDefStruct.isOutput: True varDefStruct.hasMath: False varDefStruct.codeText: None -variableDefvarDefStruct.name: bodyPositionOfCmWrtMrc\_X varDefStruct.varID: DXCG varDefStruct.units: ft varDefStruct.axisSystem: None varDefStruct.sign: FWD varDefStruct.alias: None varDefStruct.symbol: None varDefStruct.hasInitialValue: True varDefStruct.initialValue: 0.0 varDefStruct.isStdAIAA: True varDefStruct.isOutput: True varDefStruct.hasMath: False varDefStruct.codeText: None -variableDefvarDefStruct.name: bodyPositionOfCmWrtMrc\_Y varDefStruct.varID: DYCG varDefStruct.units: ft varDefStruct.axisSystem: None varDefStruct.sign: RT varDefStruct.alias: None varDefStruct.symbol: None varDefStruct.hasInitialValue: True varDefStruct.initialValue: 0. varDefStruct.isStdAIAA: True varDefStruct.isOutput: True varDefStruct.hasMath: False varDefStruct.codeText: None -variableDefvarDefStruct.name: bodyPositionOfCmWrtMrc\_Z varDefStruct.varID: DZCG varDefStruct.units: ft varDefStruct.axisSystem: None varDefStruct.sign: DOWN varDefStruct.alias: None varDefStruct.symbol: None varDefStruct.hasInitialValue: True varDefStruct.initialValue: 0. varDefStruct.isStdAIAA: True varDefStruct.isOutput: True varDefStruct.hasMath: False varDefStruct.codeText: None

```
--- PreProcess Equations and Functions ---
+++++ MODEL INPUTS AND OUTPUTS +++++
++> Output: XIXX
++> Output: XIYY
++> Output: XIZZ
++> Output: XIZX
++> Output: XIXY
++> Output: XIYZ
++> Output: XMASS
++> Output: DXCG
++> Output: DYCG
++> Output: DZCG
Number of check cases: 0
--Variables defined in model--
bodyMomentOfInertia Roll
bodyMomentOfInertia_Pitch
bodyMomentOfInertia Yaw
bodyProductOfInertia_ZX
bodyProductOfInertia_XY
bodyProductOfInertia_YZ
totalMass
bodyPositionOfCmWrtMrc_X
bodyPositionOfCmWrtMrc_Y
{\tt bodyPositionOfCmWrtMrc\_Z}
---- DAVE-ML MODEL PARSE COMPLETE ----
---- CheckModel -----
numSignals: []
---- END CheckModel -----
====== SetIC ========
{'totalMass': 14.593902937, 'bodyMomentOfInertia_X': 4.88094466281336,
'bodyMomentOfInertia_Y': 4.88094466281336, 'bodyMomentOfInertia_Z':
4.88094466281336, 'altitudeMsl': 9144.0, 'referenceWingChord':
0.0609600000000001, 'referenceWingSpan': 0.060960000000001,
'referenceWingArea': 0.018241465452480003}
++ timeStep = 0.1 [default]
++ totalMass = 14.593902937 [IC case]
++ referenceWingSpan = 0.0609600000000001 [IC case]
++ referenceWingChord = 0.0609600000000001 [IC case]
```

```
++ trueAirspeed = 0 [default]
     ++ angleOfAttack = 0 [default]
     ++ angleOfSideslip = 0 [default]
     ++ eulerAngle Roll = 0 [default]
     ++ eulerAngle Pitch = 0 [default]
     ++ eulerAngle Yaw = 0 [default]
     ++ eulerAngleRate_Roll = 0 [default]
     ++ eulerAngleRate_Pitch = 0 [default]
     ++ eulerAngleRate_Yaw = 0 [default]
     ++ bodyMomentOfInertia_X = 4.88094466281336 [IC case]
     ++ bodyProductOfInertia_XY = 0 [default]
     ++ bodyProductOfInertia_XZ = 0 [default]
     ++ bodyProductOfInertia_YX = 0 [default]
     ++ bodyMomentOfInertia_Y = 4.88094466281336 [IC case]
     ++ bodyProductOfInertia_YZ = 0 [default]
     ++ bodyProductOfInertia_XZ = 0 [default]
     ++ bodyProductOfInertia_YZ = 0 [default]
     ++ bodyMomentOfInertia_Z = 4.88094466281336 [IC case]
     ++ latitude = 0 [default]
     ++ longitude = 0 [default]
     ++ altitudeMsl = 9144.0 [IC case]
     Vecf: 0.0 0.0 0.0
     =====done=====
     CPU times: user 53.1 ms, sys: 10.7 ms, total: 63.8 ms
     Wall time: 62.1 ms
[57]: checkFile = "NESC-check-cases/Atmospheric_checkcases/
      →Atmos_06_DroppedSphereEllipsoidalNoWind/Atmos_06_sim_01.csv"
      gvCC6 = GetCheckCaseData(checkFile)
      MakePlot(gvOblateRotatingEarth, gvCC6, "sim1")
      PrintErrorTable("Dropped Sphere (Cd=0.1): Oblate Earth", gvOblateEarthLabel,

→gvOblateRotatingEarth, gvCC6)

     number of headers: 31
     ['time', 'gePosition_ft_X', 'gePosition_ft_Y', 'gePosition_ft_Z',
     'feVelocity_ft_s_X', 'feVelocity_ft_s_Y', 'feVelocity_ft_s_Z', 'altitudeMsl_ft',
     'longitude_deg', 'latitude_deg', 'localGravity_ft_s2', 'eulerAngle_deg_Yaw',
     'eulerAngle_deg_Pitch', 'eulerAngle_deg_Roll',
     'bodyAngularRateWrtEi_deg_s_Roll', 'bodyAngularRateWrtEi_deg_s_Pitch',
     'bodyAngularRateWrtEi_deg_s_Yaw', 'altitudeRateWrtMsl_ft_min',
     'speedOfSound_ft_s', 'airDensity_slug_ft3', 'ambientPressure_lbf_ft2',
     'ambientTemperature_dgR', 'aero_bodyForce_lbf_X', 'aero_bodyForce_lbf_Y',
     'aero_bodyForce_lbf_Z', 'aero_bodyMoment_ftlbf_L', 'aero_bodyMoment_ftlbf_M',
     'aero_bodyMoment_ftlbf_N', 'mach', 'dynamicPressure_lbf_ft2',
     'trueAirspeed nmi h']
     Variable
                               L2
                                                          L-Infinity-Norm
```

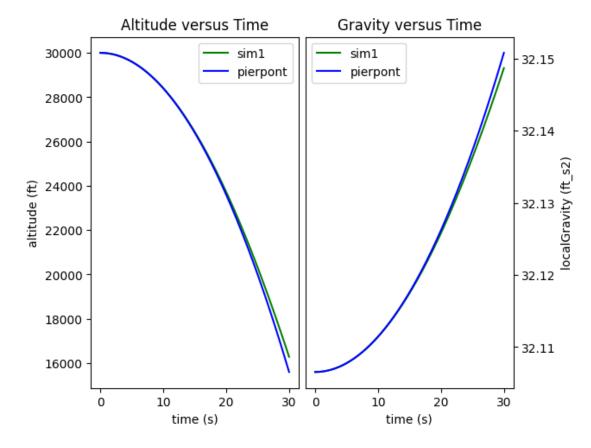
++ referenceWingArea = 0.018241465452480003 [IC case]

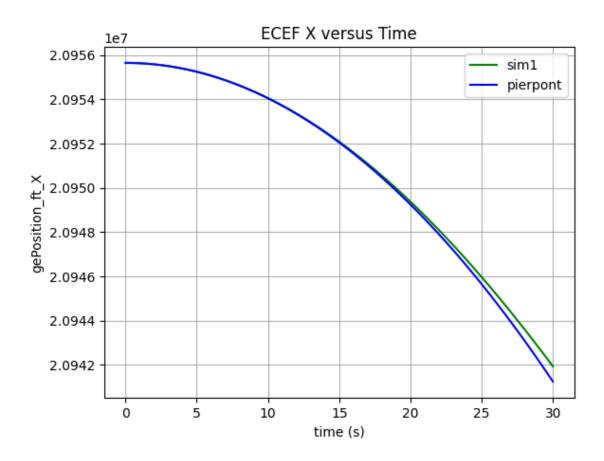
altitudeMsl\_ft
longitude\_deg
latitude\_deg
localGravity\_ft\_s2
gePosition\_ft\_X
gePosition\_ft\_Y
gePosition\_ft\_Z
eulerAngle\_deg\_Roll
eulerAngle\_deg\_Pitch
eulerAngle\_deg\_Yaw

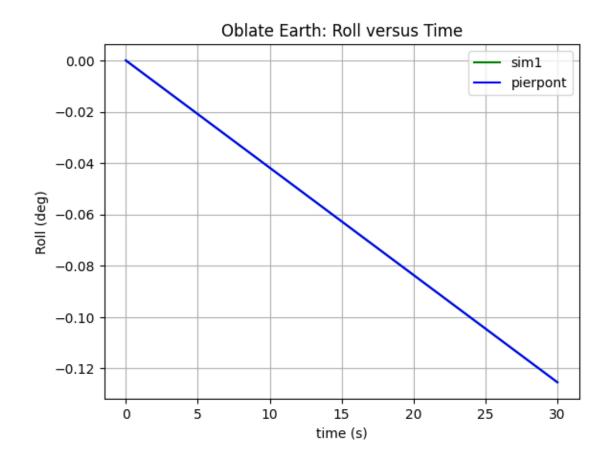
1623.5211209467052 2.1152199421624644e-05 0.011438977033606221 0.012094173192575396 1623.5553863119626 7.728925068145949 1789.84846464604 2.113161955755363e-05 0.011438987196216907

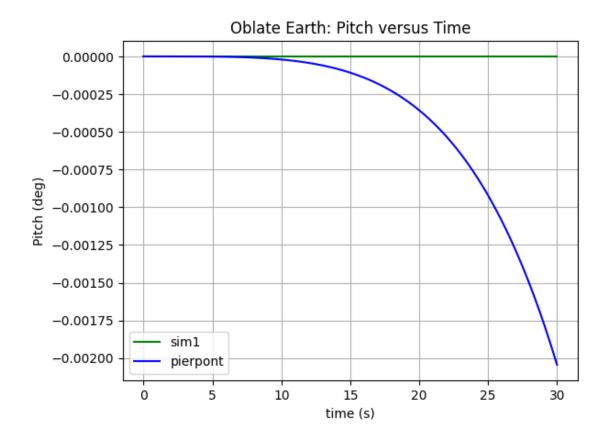
2.1936332786621488e-11

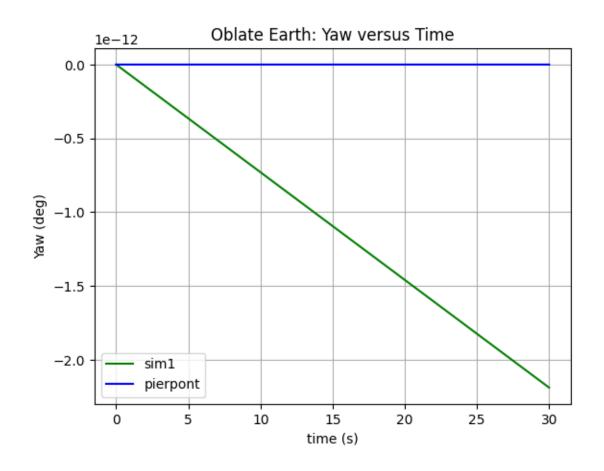
325.78303920045437 4.07701616992336e-06 0.002043666399558951 0.0021173060586789916 325.79608972370625 1.4894847350385234 381.95052822030516 4.074494643513393e-06 0.00204366739954423 2.188531823772246e-12

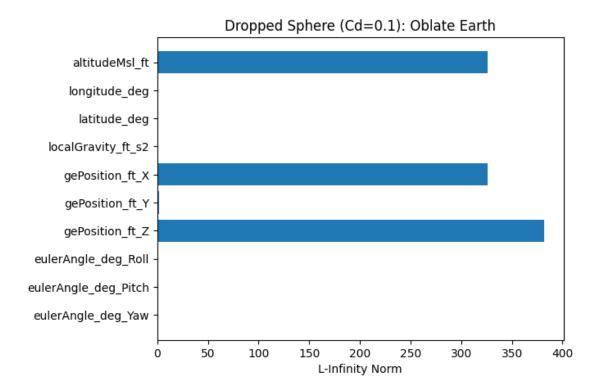












## 1.11.7 Stevens and Lewis Orbit

```
[58]: h = 422000  # height in meters (about ISS alt); sim on pg 43 uses 100km
gd = 9.80665
a = 6378137.0
wz = 7.292115e-5

vy = math.sqrt(gd*(a + h)) - wz*(a + h)
print("vy: ", vy, " m/s")
```

vy: 7670.310336088278 m/s

The vehicle in the simulation is a solid brick. The moment of inertia of a brick:

$$\begin{split} I_x &= \frac{1}{12} m(y^2 + z^2) \\ I_y &= \frac{1}{12} m(x^2 + z^2) \\ I_z &= \frac{1}{12} m(x^2 + y^2) \end{split}$$

The dimensions of the brick as stated by Stevens and Lewis is 2 x 5 x 8 units.

Coordinate origin is at the center of mass of the brick.

8 unit side is parallel to the x-axis

5 unit side is parallel to the y-axis

For simplicity, the brick units are meters and the mass is 1kg.

$$I_x = \frac{1}{12}(1)(2^2 + 5^2) = 2.41667$$

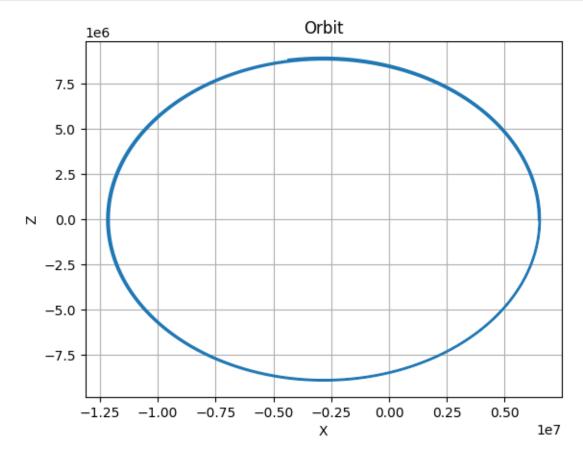
```
I_y = \frac{1}{12}(1)(8^2 + 2^2) = 5.667

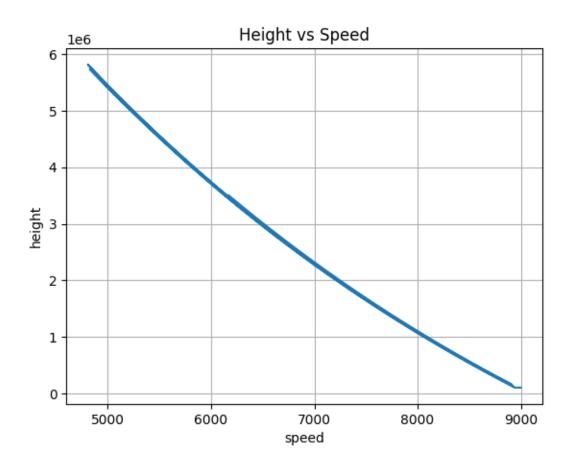
I_z = \frac{1}{12}(1)(8^2 + 5^2) = 7.41667
[59]: %%time
     printDebugData = False
     ppLoadDml('models/noAero.dml', printDebugData)
     ic = {
         "timeStep": [1.0, "s"],
         "totalMass": [1.0, "kg"],
         "bodyMomentOfInertia X": [2.4167, "kgm2"],
         "bodyMomentOfInertia_Y": [5.667, "kgm2"],
          "bodyMomentOfInertia_Z": [7.417, "kgm2"],
         "altitudeMsl": [100000.0, "m"],
         "trueAirspeed": [9000.0, "m_s"],
         "referenceWingChord": [8.0, "m"],
         "referenceWingSpan": [5.0, "m"]
     }
     gvOblateRotatingEarth.Reset(ic)
     gvOblateRotatingEarth.totalCoefficientOfDrag = 0.0
     gvOblateRotatingEarth.Run(20000)
     gvOblateRotatingEarth.GenerateEnglishUnits()
     *************
     Model: Zero Aero Output
     creation date: 2021-07-21
     file version: Initial version
     ************
     +++++ MODEL INPUTS AND OUTPUTS +++++
     ++> Output: CX
     ++> Output: CY
     ++> Output: CZ
     ++> Output:
                 CLL
     ++> Output: CLM
     ++> Output:
                 CLN
     ---- DAVE-ML MODEL PARSE COMPLETE ----
     ======= SetIC ========
     {'timeStep': 1.0, 'totalMass': 1.0, 'bodyMomentOfInertia X': 2.4167,
     'bodyMomentOfInertia Y': 5.667, 'bodyMomentOfInertia Z': 7.417, 'altitudeMsl':
     100000.0, 'trueAirspeed': 9000.0, 'referenceWingChord': 8.0,
     'referenceWingSpan': 5.0}
     ++ timeStep = 1.0 [IC case]
     ++ totalMass = 1.0 [IC case]
     ++ referenceWingSpan = 5.0 [IC case]
```

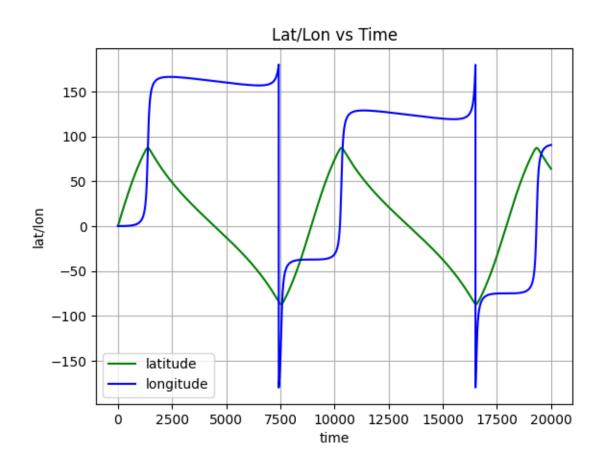
```
++ referenceWingChord = 8.0 [IC case]
     ++ referenceWingArea = 40.0 [default]
     ++ trueAirspeed = 9000.0 [IC case]
     ++ angleOfAttack = 0 [default]
     ++ angleOfSideslip = 0 [default]
     ++ eulerAngle_Roll = 0 [default]
     ++ eulerAngle Pitch = 0 [default]
     ++ eulerAngle_Yaw = 0 [default]
     ++ eulerAngleRate Roll = 0 [default]
     ++ eulerAngleRate_Pitch = 0 [default]
     ++ eulerAngleRate_Yaw = 0 [default]
     ++ bodyMomentOfInertia_X = 2.4167 [IC case]
     ++ bodyProductOfInertia_XY = 0 [default]
     ++ bodyProductOfInertia_XZ = 0 [default]
     ++ bodyProductOfInertia_YX = 0 [default]
     ++ bodyMomentOfInertia_Y = 5.667 [IC case]
     ++ bodyProductOfInertia_YZ = 0 [default]
     ++ bodyProductOfInertia_XZ = 0 [default]
     ++ bodyProductOfInertia_YZ = 0 [default]
     ++ bodyMomentOfInertia Z = 7.417 [IC case]
     ++ latitude = 0 [default]
     ++ longitude = 0 [default]
     ++ altitudeMsl = 100000.0 [IC case]
     Vecf: 1.8189894035458565e-12 0.0 9000.0
     =====done=====
     CPU times: user 2.49 s, sys: 41.4 ms, total: 2.53 s
     Wall time: 2.62 s
[60]: from IPython.display import Image
      Image(url= "images/SLOrbit.JPG", width=600, height=600)
[60]: <IPython.core.display.Image object>
[61]: fig1, a = plt.subplots()
      a.plot(gvOblateRotatingEarth.eiPosition_m_X, gvOblateRotatingEarth.
      →eiPosition_m_Z)
      a.set(xlabel='X', ylabel='Z', title='Orbit')
      a.grid()
      fig2, b = plt.subplots()
      b.plot(gv0blateRotatingEarth.trueAirspeed, gv0blateRotatingEarth.altitudeMsl_m)
      b.set(xlabel='speed', ylabel='height', title='Height vs Speed')
      b.grid()
      fig3, c = plt.subplots()
      c.plot(gvOblateRotatingEarth.time, gvOblateRotatingEarth.
       →EnglishData['latitude_deg'],'g',
```

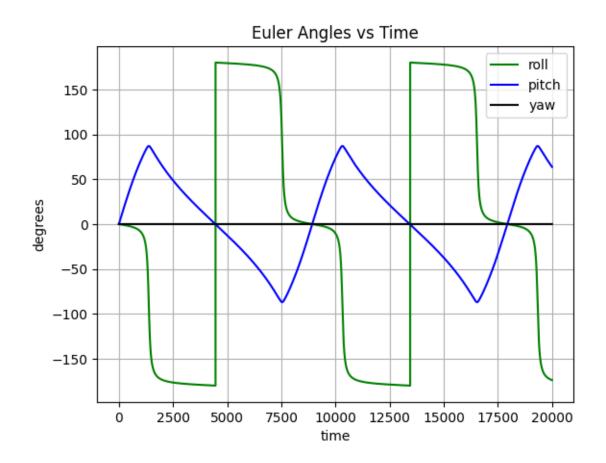
```
gvOblateRotatingEarth.time, gvOblateRotatingEarth.

Description of the property of the pr
```









[]: