EOM

December 6, 2021

1 6DOF Equations of Motion

Acronym	Meaning
CG	Center of Gravity
DAVE-ML	Dynamic Aerospace Vehicle Exchange Markup Language
EOM	Equations of Motion
FRD	Forward Right Down
IC	Initial Condition
LLA	Latitude, Longitude and Altitude
MathML	Mathematical Markup Language
NED	North, East, Down
PCPF	Planet Centered, Planet Fixed (aka, ECEF on Earth)
SI	International System of Units

The goal of the software is to implement the EOM for DAVE-ML models in Python.

The software implements flat earth and the oblate rotating planet EOM as derived in Stevens and Lewis. Stevens and Lewis derives the equations of motion; therefore they are not described here.

All internal calculations are in SI.

To upgrade jupyter: **pip3 install –upgrade jupyterlab**

1.1 Greek Alphabet

alpha $\alpha\,A$	beta $\beta\,B$	gamma $\gamma\,\Gamma$
delta $\delta\,\Delta$	epsilon $\epsilon\,E$	zeta $\zeta\,Z$
eta $\eta\,H$	theta $\theta\,\Theta$	iota $\iota\,I$
kappa $\kappa\,K$	lambda $\lambda\,\Lambda$	mu $\mu\,M$
nu $\nu\,N$	xi $\xi\,\Xi$	omicron $o\,O$
pi $\pi\,\Pi$	rho $\rho\,P$	sigma $\sigma\,\Sigma$
tau $\tau\,T$	upsilon $\upsilon\,\Upsilon$	phi $\phi\,\Phi$
chi $\chi\,X$	psi $\psi\,\Psi$	omega $\omega\,\Omega$

1.2 Unit Test Class

A base class is created for doing unit testing. Other classes derive from this class.

[1]:
```python
import logging

class ppUnitTest:
    """A unit test class.

    A base class that other classes derive for unit testing.

    Attributes:
        FailCount: The number of failed tests.
        ClassName: A string to identify the class performing the unit test.
    """

    FailCount = 0
    ClassName = " "

    def TestValue(self, actualValue, testValue, label, tol):
        """Tests whether two values match within a tolerance.

        A string label is passed to identify the test. An error via logging is
        printed if the test fails.

        Example:
            TestValue( 2, 1+1, "SimpleAdd", 1e-2)

        Args:
            actualValue: The expected value from doing the test.
            testValue: The computed value from doing the test.
            label: A label to identify the specific test.
            tol: The tolerance for the actual and test values to match.
        """
        if abs(actualValue - testValue) > tol:
            self.FailCount += 1
            labelStr = "[" + self.ClassName + "]" + label
            actStr = labelStr + ": Test failed. Expected: {};".
 ↪format(actualValue)
            calStr = " Calculated: {}".format(testValue)
            logging.error(actStr+calStr)

    def TestArray(self, arrayData):
        """Performs multiple tests on an array of data.

        Example:
            Create a data array containing the desired value, the test value,
            the identity string, and the tolerance.

            checkData = (
              (  4.0, 2.0*2.0, "MultipleTest", 1e-6),
```

```
            ( 16.0, 8.0+8.0, "Add-Example",  1e-5)
          )

          TestArray(checkData)
      """
      for d in arrayData:
          self.TestValue( d[0], d[1], d[2], d[3] )
```

Test the ppUnitTest class.

[2]:
```
class ppUnitTestCheck(ppUnitTest):
    def UnitTest(self):
        self.ClassName = "UnitCheck"
        # Same test, different tolerances
        self.TestValue(9.9, 10.1, "Should Pass", 1.0)
        self.TestValue(9.9, 10.1, "Should Fail", 1e-1)

        print("Number of ppUnitTestCheck failed tests: ", self.FailCount)

unitTestCheck = ppUnitTestCheck()
unitTestCheck.UnitTest()
help(ppUnitTest)
```

```
ERROR:root:[UnitCheck]Should Fail: Test failed. Expected: 9.9; Calculated: 10.1
```

```
Number of ppUnitTestCheck failed tests:  1
Help on class ppUnitTest in module __main__:

class ppUnitTest(builtins.object)
 |  A unit test class.
 |
 |  A base class that other classes derive for unit testing.
 |
 |  Attributes:
 |      FailCount: The number of failed tests.
 |      ClassName: A string to identify the class performing the unit test.
 |
 |  Methods defined here:
 |
 |  TestArray(self, arrayData)
 |      Performs multiple tests on an array of data.
 |
 |      Example:
 |          Create a data array containing the desired value, the test value,
 |          the identity string, and the tolerance.
 |
 |          checkData = (
 |            (  4.0, 2.0*2.0, "MultipleTest", 1e-6),
 |            ( 16.0, 8.0+8.0, "Add-Example",  1e-5)
```

```
|         )
|
|         TestArray(checkData)
|
|  TestValue(self, actualValue, testValue, label, tol)
|      Tests whether two values match within a tolerance.
|
|      A string label is passed to identify the test. An error via logging is
|      printed if the test fails.
|
|      Example:
|          TestValue( 2, 1+1, "SimpleAdd", 1e-2)
|
|      Args:
|          actualValue: The expected value from doing the test.
|          testValue: The computed value from doing the test.
|          label: A label to identify the specific test.
|          tol: The tolerance for the actual and test values to match.
|
|  ----------------------------------------------------------------------
|  Data descriptors defined here:
|
|  __dict__
|      dictionary for instance variables (if defined)
|
|  __weakref__
|      list of weak references to the object (if defined)
|
|  ----------------------------------------------------------------------
|  Data and other attributes defined here:
|
|  ClassName = ' '
|
|  FailCount = 0
```

1.3 Conversion Class

Create a class for doing common conversions.

unit	abbreviation
second	s
minute	min
inch	inch
foot	ft
meter	m
nautical mile	nmi
statute mile	smi

unit	abbreviation
kilometer	km
centimeter	cm
millimeter	mm
pound force	lbf
Newton	N
kilogram force	kgf
kilogram	kg
pound mass	lbm
slug	slug
degree	deg
radian	rad
knot (nmi/hr)	kt
nondimensional	nd

[3]:
```
import math

class ppConvert(ppUnitTest):
    """A class for performing unit conversions.

    Attributes:
        KnotToFps: scale factor to convert knots to feet per second.
        FpsToKnot: scale factor to convert feet per second to knots.
        MinToSec: scale factor to convert minutes to seconds.
        FeetToMeter: scale factor to convert feet to meters.
        MeterToFeet: scale factor to convert meters to feet.
        NmToFeet: scale factor to convert nautical miles to feet.
        FeetToNm: scale factor to convert feet to nautical miles.
        SqMeterToSqFeet: scale factor to convert square meters to square feet.
        SqFeetToSqMeter: scale factor to convert square feet to square meters.
        PoundToNewton: scale factor to convert pounds to Newtons.
        NewtonToPound: scale factor to convert Newtons to pounds.
        SlugToKg: scale factor to convert slugs to kilograms.
        KgToSlug: scale factor to convert kilograms to slugs.
        Slugft2ToKgm2: scale factor to convert slug-ft2 to kg-m2.
        Kgm2ToSlugft2: scale factor to convert kg-m2 to slug-ft2.
        DegToRad: scale factor to convert degrees to radians.
        RadToDeg: scale factor to convert radians to degrees.
        MpsToKt: scale factor to convert meters per second to knots.
    """
    KnotToFps = 1.6878097112860893
    FpsToKnot = (1.0 / KnotToFps)
    MinToSec = 60.0
    FeetToMeter = 0.3048
    MeterToFeet = (1.0 / FeetToMeter)
    NmToFeet = 6076.115485564304
```

```
FeetToNm = (1.0 / NmToFeet)
SqMeterToSqFeet = (MeterToFeet*MeterToFeet)
SqFeetToSqMeter = 1.0 / SqMeterToSqFeet
PoundToNewton = 4.4482216152605
NewtonToPound = 1.0 / PoundToNewton
SlugToKg = 14.593902937
KgToSlug = 1.0 / SlugToKg
Slugft2ToKgm2 = 1.3558179618926
Kgm2ToSlugft2 = 1.0 / Slugft2ToKgm2
DegToRad = math.radians(1.0)
RadToDeg = math.degrees(1.0)
MpsToKt = 1.94384

EnglishToSI = {
    "lbf": PoundToNewton,
    "slug": SlugToKg,
    "slugft2": Slugft2ToKgm2,
    "ft": FeetToMeter,
    "ft_s": FeetToMeter,
    "ft2": SqFeetToSqMeter,
    "deg": DegToRad,
    "deg_s": DegToRad,
    "km": 1000.0,
    "km_s": 1000.0,
    "nd": 1
}

SiToEnglish = {
    "m": MeterToFeet,
    "m2": SqMeterToSqFeet,
    "rad": RadToDeg,
    "rad_s": RadToDeg,
    "m_s": MpsToKt,
    "n": NewtonToPound,
    "kg": KgToSlug,
    "kgm2": Kgm2ToSlugft2,
    "s": 1
}

def LogWarn(self, inUnit):
    """Log a warning for unrecognized unit.

    Args:
        inUnit: unit string value not recognized.
    """
    warnStr = units + " not recognized in ppConvert.  No conversion done."
    logging.warning(warnStr)
```

```python
def SetIC(self, inIC):
    """Convert initial conditions to SI.

    Args:
        inIC: initial conditions.
    """
    print("========= SetIC ==============")
    icData = {}
    for key,value in inIC.items():
        units = value[1].lower()
        factor = 1
        if units in self.EnglishToSI:
            factor = self.EnglishToSI[units]
        elif units not in self.SiToEnglish:
            self.LogWarn(units)
        icData[key] = factor * value[0]
    return icData

def ToSI(self, value, inUnits):
    """Convert a value from Imperial to SI units.

    Args:
        value: Imperial value to convert.
        inUnits: string value of the English unit value.

    Returns:
        SI value.
    """
    units = inUnits.lower()
    factor = 1
    if units in self.EnglishToSI:
        factor = self.EnglishToSI[units]
    elif units not in self.SiToEnglish:
        self.LogWarn(inUnits)
    return value*factor

def ToEnglish(self, value, inUnits):
    """Convert a value from SI to Imperial units.

    Args:
        value: SI value to convert.
        inUnits: string value of the SI unit value.

    Returns:
        Imperial value.
    """
```

```
        units = inUnits.lower()
        factor = 1
        if units in self.SiToEnglish:
            factor = self.SiToEnglish[units]
        elif units in self.EnglishToSI:
            self.LogWarn(inUnits)

        convertedValue = []
        for v in value:
            convertedValue.append(v*factor)
        return convertedValue

    def UnitTest(self):
        """Perform tests to check unit conversions."""
        self.ClassName = "ppConvert"
        checkData = (
            (     123.0,          72.876*self.KnotToFps, "KnotToFps", 1e-3),
            (      78.8,           133.0*self.FpsToKnot, "FpsToKnot", 1e-3),
            (     300.0,              5.0*self.MinToSec, "MinToSec", 1e-12),
            (  395.9352,       1299.0*self.FeetToMeter, "FeetToMeter", 1e-4),
            (    1299.0,     395.9352*self.MeterToFeet, "MeterToFeet", 1e-4),
            ( 3967703.4,            653.0*self.NmToFeet, "NmToFeet", 0.1),
            (     653.0,       3967703.4*self.FeetToNm, "FeetToNm", 0.1),
            ( 10.763910,           self.SqMeterToSqFeet, "SqMeterToSqFeet",␣
↪1e-6),
            (      13.0,    2.92252*self.PoundToNewton, "PoundToNewton", 1e-4),
            (       1.0, 0.73756215*self.Slugft2ToKgm2, "Slugft2ToKgm2", 1e-7),
            (    54.864,         self.FeetToMeter*180.0, "FeetToMeters", 1e-4),
            (1742.12598,         self.MeterToFeet*531.0, "MetersToFeet", 1e-5),
            (  178.5596, self.SqFeetToSqMeter*1922.0, "SqFeetToSqMeters", 1e-4),
            (   4412.64, self.PoundToNewton*992.0, "PoundsToNewtons", 0.01),
            (21.6930872, self.Slugft2ToKgm2*16.0, "SlugFt2ToKgM2", 1e-6),
            (161.0256, self.ToSI(36.2,"lbf"), "ToSI lbf->N", 1e-3),
            (161.0256, self.ToSI(36.2,"LBf"), "ToSI lbf->N", 1e-3),
            (105.7538001, self.ToSI(78.0,"slugft2"), "ToSI slugf2->kgm2", 1e-6),
            (531.0, self.ToSI(1742.12598,"ft"), "ToSI f->m", 1e-5),
            (9.7536, self.ToSI(32.0,"ft_s"), "ToSI fps-mps", 1e-4),
            (140.6552, self.ToSI(1514,"ft2"), "ToSI f2-m2", 1e-4),
            (math.pi, self.ToSI(180.0,"deg"), "ToSI deg->rad", 1e-6),
            (0.25*math.pi, self.ToSI(45.0,"deg_s"), "ToSI dps->rps", 1e-6),
            (93200, self.ToSI(93.2,"km"), "ToSI km->m", 1e-6),
            (4221, self.ToSI(4.221,"km_s"), "ToSI km_s->m_s", 1e-6)
        )

        self.TestArray(checkData)

        self.ClassName = "ppConvert IC"
```

```
        icTest = {
            "newtonTest": [36.2, "lbf"],
            "inertiaTest": [78.0, "slugft2"],
            "feetTest": [1742.12598, "ft"],
            "fpsTest": [32.0, "ft_s"],
            "ft2Test": [1514, "ft2"],
            "degTest": [180, "deg"],
            "dpsTest": [45.0, "deg_s"],
            "kmTest": [93.2, "km"],
            "kpsTest": [4.221, "km_s"]
        }
        icData = self.SetIC(icTest)
        self.TestValue(161.0256, icData["newtonTest"], "lbf->N", 1e-3)
        self.TestValue(105.7538001, icData["inertiaTest"], "slugf2->kgm2", 1e-6)
        self.TestValue(531.0, icData["feetTest"], "f->m", 1e-5)
        self.TestValue(9.7536, icData["fpsTest"], "fps-mps", 1e-4)
        self.TestValue(140.6552, icData["ft2Test"], "f2-m2", 1e-4)
        self.TestValue(math.pi, icData["degTest"], "deg->rad", 1e-6)
        self.TestValue(0.25*math.pi, icData["dpsTest"], "dps->rps", 1e-6)
        self.TestValue(93200, icData["kmTest"], "km->m", 1e-6)
        self.TestValue(4221, icData["kpsTest"], "km_s->m_s", 1e-6)

        self.ClassName = "ppConvert ToEnglish"

        fa = self.ToEnglish([100, 1000],"m")
        aa = self.ToEnglish([100, 1000],"m2")
        da = self.ToEnglish([0.5*math.pi, math.pi],"rad")
        dsa = self.ToEnglish([1.5*math.pi],"rad_s")
        ka = self.ToEnglish([100, 1000],"m_s")
        na = self.ToEnglish([25, 250],"N")
        sa = self.ToEnglish([16, 160],"kg")
        ga = self.ToEnglish([59, 590],"kgm2")

        checkEnglish = (
            (328.084, fa[0], "m->ft", 1e-3),
            (3280.84, fa[1], "m->ft", 1e-2),
            (1076.39, aa[0], "m2->ft2", 1e-2),
            (10763.9, aa[1], "m2->ft2", 0.1),
            (   90.0, da[0], "rad->deg", 1e-6),
            (  180.0, da[1], "rad->deg", 1e-6),
            (  270.0, dsa[0], "rad_s->deg_s", 1e-6),
            (194.384, ka[0], "m_s->knot", 1e-3),
            (1943.84, ka[1], "m_s->knot", 1e-2),
            (5.62022, na[0], "N->lbf", 1e-5),
            (56.2022, na[1], "N->lbf", 1e-4),
            (1.09635, sa[0], "kg->slug", 1e-5),
            (10.9635, sa[1], "kg->slug", 1e-4),
```

```
            (43.5161664, ga[0], "kgm2->slugft2", 1e-7),
            (435.161664, ga[1], "kgm2->slugft2", 1e-6)
        )

        self.TestArray(checkEnglish)

        print("Number of ppConvert failed tests: ", self.FailCount)
```

Create an instance of the conversion class to be used globally.

[4]:
```
gvConvert = ppConvert()
gvConvert.UnitTest()
```

```
========= SetIC ==============
Number of ppConvert failed tests:  0
```

1.4 DAVE Model Parser

Parse a DAVE-ML model and store as a Python class.

In XML, you must escape:

```
" with &quot;
< with &lt;
& with &amp;
```

[5]:
```
import xml.etree.ElementTree as ET
import math
import logging

class ppDaveModel:
    """A class hold the DAVE-ML aerodynamic data

    Attributes:
        Data : a key-value pair containing aero data.
        NameToId : a key-value pair to find varID given a name.
        IdToName : a key-value pair to find a variable name from a varID.
        VarDef : contains all the variables defined in the DAVE model.
        BpDef : contains all of the breakpoints of the DAVE model.
        GtDef : all gridded tables in the DAVE model.
    """
    Data = {}

    NameToId = {}
    IdToName = {}

    VarDef = []
    BpDef = []
    GtDef = []
```

```
FunctionDef = []

class ppVariableDef:
    name = None
    varID = None
    units = None
    axisSystem = None
    sign = None
    alias = None
    symbol = None
    hasInitialValue = False
    initialValue = 0

    hasMath = False
    code = compile("1", "<string>", "eval")
    codeText = None

    isInput = True
    isOutput = False
    isStdAIAA = False
    isState = False
    isStateDeriv = False

class ppBreakpointDef:
    name = None
    bpID = None
    units = None
    bpVals = []

    def Clear(self):
        self.bpVals.clear()

class ppGriddedTableDef:
    name = None
    gtID = None
    units = None
    bpRef = []
    dataTableStr = None
    dataTable = []

    def Clear(self):
        self.bpRef.clear()
        self.dataTable.clear()

class ppFunction:
    name = None
    fdName = None
```

```
    gtID = None
    numBreakPts = 0
    dependentVarID = None
    independentVarRef = []
    bpVals = []
    dataTable = []

    def Clear(self):
        self.independentVarRef.clear()
        self.bpVals.clear()
        self.dataTable.clear()

    def Evaluate(self, data):
        index = 0
        if self.numBreakPts == 1:
            inValue = data[self.independentVarRef[0].varID]
            i = 0
            for v in self.bpVals[0]:
                if v <= inValue:
                    #print("i: ", i, "v: ", v, "inValue: ", inValue)
                    i += 1
            index = i - 1
        elif self.numBreakPts == 2:
            x = data[self.independentVarRef[1].varID]
            y = data[self.independentVarRef[0].varID]

            #print("::: iv x: ", self.independentVarRef[1].varID)
            #print("::: iv y: ", self.independentVarRef[0].varID)
            jmax = len(self.bpVals[1])

            i = 0
            for a in self.bpVals[1]:
                if a <= x:
                    #print("i: ", i, "a: ", a, "x: ", x)
                    i += 1
            i -= 1
            j = 0
            for b in self.bpVals[0]:
                if b <= y:
                    #print("j: ", j, "b: ", b, "y: ", y)
                    j += 1
            j -= 1

            index = j*jmax + i

            #print("i: ", i, " j: ", j, " jmax: ", jmax, " index: ", index)
```

```python
        data[self.dependentVarID] = self.dataTable[index]
        return

class ppFunctionVar:
    varID = None
    fmin = 0
    fmax = 0
    extrapolate = "neither"
    interpolate = "linear"

class ppSignal:
    signalType = None
    signalName = None
    signalUnits = None
    varID = None
    signalID = None
    signalValue = 0
    tol = 1e-6

class ppCheckData:
    name = []
    signal = []
    numSignals = []

    def Clear(self):
        self.name.clear()
        self.signal.clear()
        self.numSignals.clear()

CheckData = ppCheckData()

def Clear(self):
    """Clear all of the data in the class."""
    self.Data.clear()
    self.NameToId.clear()
    self.IdToName.clear()
    self.VarDef.clear()
    for b in self.BpDef:
        b.Clear()
    self.BpDef.clear()
    for g in self.GtDef:
        g.Clear()
    self.GtDef.clear()
    for f in self.FunctionDef:
        f.Clear()
    self.FunctionDef.clear()
    self.CheckData.Clear()
```

```
    def HasName(self, inName):
        """Check if name is in the model."""
        return inName in self.NameToId

    def DataFromName(self, inName):
        """Get the data value given a variable name."""
        outVarID = self.NameToId[inName]
        return self.Data[outVarID]

    def Set(self, inName, inValue = 0):
        """Set the value of a model value given a name."""
        if not (inName in self.NameToId):
            infoStr = inName + " not in DAVE model. Value set to " +␣
↪str(inValue)
            logging.info(infoStr)
            self.NameToId[inName] = inName
            self.Data[inName] = inValue
        else:
            self.Data[self.NameToId[inName]] = inValue

    def PreProcess(self, printDebugData):
        """Preprocess the model."""
        # Change variable names in equations to self.Data[] dictionary
        for v in self.VarDef:
            # function variables are not inputs
            for f in self.FunctionDef:
                if v.varID == f.dependentVarID:
                    v.isInput = False
            if v.hasMath:
                newText = v.codeText.replace("{", "self.Data[\"")
                newText = newText.replace("}", "\"]")
                v.code = compile(newText, "<string>", "eval")

                if printDebugData:
                    print(" <<", v.varID, ">>")
                    print("  [raw]-> ", v.codeText)
                    print("  [python]-> ", newText)

        print("+++++ MODEL INPUTS AND OUTPUTS +++++")
        for v in self.VarDef:
            if v.isInput:
                print("++> Input: ", v.varID)
            if v.isOutput:
                print("++> Output: ", v.varID)
        print("++++++++++++++++++++++++++++++++++++")
```

```
        # connect the gridded tables with break points to functions
        for f in self.FunctionDef:
            for gt in self.GtDef:
                if f.gtID == gt.name:
                    f.dataTable = gt.dataTable
                    if printDebugData:
                        print("------> depVar: ", f.dependentVarID)
                        print("----> f.dataTable: ", f.dataTable)
                        print("----> bpRef: ", gt.bpRef)
                        print("----> gt.name: ", gt.name)
                        print("----> f.gtID: ", f.gtID)

                    bpv = []
                    for bpr in gt.bpRef:
                        for bp in self.BpDef:
                            if bp.bpID == bpr:
                                bpv.append(bp.bpVals)
                                if printDebugData:
                                    print("----> f.bp name: ", bpr)
                                    print("----> f.bpVals: ", bp.bpVals)
                                    print("----> bpv: ", bpv)
                    f.bpVals = bpv

    def Update(self):
        """Update all the values in the model."""
        for f in self.FunctionDef:
            f.Evaluate(self.Data)

        # Evaluate the MATH-ML equations
        for v in self.VarDef:
            if v.hasMath:
                self.Data[v.varID] = eval(v.code)

    def Tag(self, name):
        """Put the namespace prefix on DAVE-ML tags

        Args:
            name: string tag to add DAVE-ML namespace

        Returns:
            full tag name
        """
        daveNs = "{http://daveml.org/2010/DAVEML}"
        return (daveNs + name)

    def FileHeader(self, e):
        print("********************************************")
```

```
        print("Model: ", e.get('name'))
        for fhTag in e:
            if fhTag.tag == self.Tag("creationDate"):
                print("creation date: ", fhTag.get('date'))
            if fhTag.tag == self.Tag("fileVersion"):
                print("file version: ", fhTag.text)
        print("*******************************************\n")

    def VariableDef(self, e, printDebugData):
        varDefStruct = self.ppVariableDef();
        varDefStruct.name = e.get('name')
        varDefStruct.varID = e.get('varID')
        varDefStruct.units = e.get('units')
        varDefStruct.axisSystem = e.get('axisSystem')
        varDefStruct.sign = e.get('sign')
        varDefStruct.alias = e.get('alias')
        varDefStruct.symbol = e.get('symbol')
        varDefStruct.initialValue = e.get('initialValue')

        value = 0
        if varDefStruct.initialValue != None:
            value = varDefStruct.initialValue
            varDefStruct.hasInitialValue = True
            varDefStruct.isInput = False

        self.Data[varDefStruct.varID] = float(value)
        self.NameToId[varDefStruct.name]  = varDefStruct.varID
        self.IdToName[varDefStruct.varID] = varDefStruct.name

        for label in e:
            if label.tag == self.Tag("isStdAIAA"):
                varDefStruct.isStdAIAA = True
            if label.tag == self.Tag("isOutput"):
                varDefStruct.isOutput = True
                varDefStruct.isInput = False
            if label.tag == self.Tag("isState"):
                varDefStruct.isState = True
            if label.tag == self.Tag("isStateDeriv"):
                varDefStruct.isStateDeriv = True
            if label.tag == self.Tag("calculation"):
                varDefStruct.hasMath = True
                varDefStruct.isInput = False
                for pl in label:
                    if pl.tag == self.Tag("python"):
                        varDefStruct.codeText = pl.text

        # TODO: add MathML
```

```
        self.VarDef.append(varDefStruct)

        if printDebugData:
            print("-variableDef-")
            print(" varDefStruct.name: ", varDefStruct.name)
            print(" varDefStruct.varID: ", varDefStruct.varID)
            print(" varDefStruct.units: ", varDefStruct.units)
            print(" varDefStruct.axisSystem: ", varDefStruct.axisSystem)
            print(" varDefStruct.sign: ", varDefStruct.sign)
            print(" varDefStruct.alias: ", varDefStruct.alias)
            print(" varDefStruct.symbol: ", varDefStruct.symbol)
            print(" varDefStruct.hasInitialValue: ", varDefStruct.
↪hasInitialValue)
            print(" varDefStruct.initialValue: ", varDefStruct.initialValue)
            print(" varDefStruct.isStdAIAA: ", varDefStruct.isStdAIAA)
            print(" varDefStruct.isOutput: ", varDefStruct.isOutput)
            print(" varDefStruct.hasMath: ", varDefStruct.hasMath)
            print(" varDefStruct.codeText: ", varDefStruct.codeText)

    def BreakpointDef(self, e, printDebugData):
        bpDefStruct = self.ppBreakpointDef()
        bpDefStruct.name = e.get('name')
        bpDefStruct.bpID = e.get('bpID')
        bpDefStruct.units = e.get('units')

        for label in e:
            if label.tag == self.Tag("bpVals"):
                bpList = []
                for i in label.text.split(','):
                    bpList.append( float(i) )
                bpDefStruct.bpVals = bpList

        self.BpDef.append(bpDefStruct)

        if printDebugData:
            print("-bpDefStruct-")
            print(" bpDefStruct.name: ", bpDefStruct.name)
            print(" bpDefStruct.bpID: ", bpDefStruct.bpID)
            print(" bpDefStruct.units: ", bpDefStruct.units)
            print(" bpDefStruct.bpVals: ", bpDefStruct.bpVals)

    def GriddedTableDef(self, e, printDebugData):
        gtDefStruct = self.ppGriddedTableDef()
        gtDefStruct.name = e.get('name')
        gtDefStruct.gtID = e.get('gtID')
        gtDefStruct.units = e.get('units')
        gtDefStruct.bpRef.clear()
```

```
        bpr = []
        for label in e:
            if label.tag == self.Tag("breakpointRefs"):
                for refs in label:
                    if refs.tag == self.Tag("bpRef"):
                        bpr.append( refs.get('bpID') )
            if label.tag == self.Tag("dataTable"):
                gtDefStruct.dataTableStr = label.text
        gtDefStruct.bpRef = bpr

        gtDefStruct.dataTable.clear()
        dt = []
        for i in gtDefStruct.dataTableStr.split(','):
            dt.append( float(i) )
        gtDefStruct.dataTable = dt

        self.GtDef.append(gtDefStruct)

        if printDebugData:
            print("-gtDefStruct-")
            print(" gtDefStruct.name: ", gtDefStruct.name)
            print(" gtDefStruct.gtID: ", gtDefStruct.gtID)
            print(" gtDefStruct.units: ", gtDefStruct.units)
            print(" gtDefStruct.bpRef: ", gtDefStruct.bpRef)
            print(" gtDefStruct.dataTableStr: ", gtDefStruct.dataTableStr)
            print(" gtDefStruct.dataTable: ", gtDefStruct.dataTable)

    # TODO: add ungridded table parsing
    def UngriddedTableDef(self, e):
        logging.info("-UngriddedTableDef-:  COMING SOON")

    def Function(self, e, printDebugData):
        funDefStruct = self.ppFunction()
        funDefStruct.name = e.get('name')
        funDefStruct.independentVarRef.clear()
        iVar = []
        for label in e:
            if label.tag == self.Tag("independentVarRef"):
                indVar = self.ppFunctionVar()
                indVar.varID = label.get('varID')
                indVar.min = float( label.get('min') )
                indVar.max = float( label.get('max') )
                indVar.extrapolate = label.get('extrapolate')
                indVar.interpolate = label.get('interpolate')
                iVar.append(indVar)
            if label.tag == self.Tag("dependentVarRef"):
                funDefStruct.dependentVarID = label.get('varID')
```

```
            if label.tag == self.Tag("functionDefn"):
                funDefStruct.fdName = label.get('name')
                for tVar in label:
                    if tVar.tag == self.Tag("griddedTableRef"):
                        funDefStruct.gtID = tVar.get('gtID')
                    if tVar.tag == self.Tag("griddedTable"):
                        funDefStruct.gtID = tVar.get('name')
                        self.GriddedTableDef(tVar, printDebugData)

        funDefStruct.independentVarRef = iVar
        funDefStruct.numBreakPts = len(funDefStruct.independentVarRef)
        self.FunctionDef.append(funDefStruct)

        if printDebugData:
            print("-functionStruct-")
            print(" funDefStruct.name: ", funDefStruct.name)
            print(" funDefStruct.fdName: ", funDefStruct.name)
            print(" funDefStruct.gtID: ", funDefStruct.gtID)
            print(" funDefStruct.numBreakPts: ", funDefStruct.numBreakPts)
            print(" funDefStruct.dependentVarID: ", funDefStruct.dependentVarID)
            for iv in funDefStruct.independentVarRef:
                print(" independentVarRef.varID: ", iv.varID)
                print(" independentVarRef.min: ", iv.min)
                print(" independentVarRef.max: ", iv.max)
                print(" independentVarRef.extrapolate: ", iv.extrapolate)
                print(" independentVarRef.interpolate: ", iv.interpolate)

    def ppPrint(self, str1, str2, printDebugData):
        if printDebugData:
            print(str1, str2)

    def CheckDataFx(self, e, printDebugData):
        pd = printDebugData
        self.ppPrint("-checkData-", "", pd)
        for ssTag in e:
            if ssTag.tag == self.Tag("staticShot"):
                self.ppPrint("staticShot: ", ssTag.get('name'), pd)
                self.CheckData.name.append(ssTag.get('name'))
                numSignals = 0
                for signalType in ssTag:
                    for signal in signalType:
                        if signal.tag == self.Tag("signal"):
                            localSignal = self.ppSignal()
                            self.ppPrint(" signal type: ", signalType.tag, pd)
                            localSignal.signalType = signalType.tag
                            numSignals += 1
                            for oneSignal in signal:
```

```
                                if oneSignal.tag == self.Tag("signalName"):
                                    localSignal.signalName = oneSignal.text
                                    self.ppPrint(" signal name: ", localSignal.
↪signalName, pd)
                                if oneSignal.tag == self.Tag("signalID"):
                                    localSignal.signalID = oneSignal.text
                                    self.ppPrint(" signal ID: ", localSignal.
↪signalID, pd)
                                if oneSignal.tag == self.Tag("varID"):
                                    localSignal.varID = oneSignal.text
                                    self.ppPrint(" signal varID: ", localSignal.
↪varID, pd)
                                if oneSignal.tag == self.Tag("signalUnits"):
                                    localSignal.signalUnits = oneSignal.text
                                    self.ppPrint(" signal units: ", localSignal.
↪signalUnits, pd)
                                if oneSignal.tag == self.Tag("signalValue"):
                                    localSignal.signalValue = float(oneSignal.
↪text)
                                    self.ppPrint(" signal value: ", localSignal.
↪signalValue, pd)
                                if oneSignal.tag == self.Tag("tol"):
                                    localSignal.tol = oneSignal.text
                                    self.ppPrint(" signal tol: ", localSignal.
↪tol, pd)

                            sigStr = "{} signal #: {}".format(ssTag.
↪get('name'), numSignals)
                            self.ppPrint(" [ localSignal append ] -> ", sigStr,␣
↪pd)
                            self.CheckData.signal.append(localSignal)
                self.CheckData.numSignals.append(numSignals)
                numStr = "{} signals in ".format(numSignals)
                self.ppPrint(numStr, ssTag.get('name'), pd)

    def CheckModel(self):
        print("\n----- CheckModel -----\n")
        print("numSignals: ", self.CheckData.numSignals)
        i = 0
        shotCount = 0
        for ss in self.CheckData.name:
            prevSignalType = self.Tag("checkInputs")
            for si in range(self.CheckData.numSignals[shotCount]):
                signal = self.CheckData.signal[i]
                name = signal.varID if signal.signalName == None else signal.
↪signalName
```

```
                i += 1
                if signal.signalType == self.Tag("checkInputs"):
                    self.Data[signal.varID] = float(signal.signalValue)

                if signal.signalType != prevSignalType:
                    self.Update()

                if signal.signalType == self.Tag("internalValues"):
                    modelValue = self.Data[signal.varID]
                    checkValue = signal.signalValue
                    if abs(modelValue - checkValue) > float(signal.tol):
                        errStr = "internal: {} -> [{}] Calculated {}, Expected␣
↪{}".format(ss, name, modelValue, checkValue)
                        logging.error(errStr)

                if signal.signalType == self.Tag("checkOutputs"):
                    modelValue = self.Data[signal.varID]
                    checkValue = signal.signalValue
                    if abs(modelValue - checkValue) > float(signal.tol):
                        errStr = "output: {} -> [{}] Calculated {}, Expected␣
↪{}".format(ss, name, modelValue, checkValue)
                        logging.error(errStr)

                prevSignalType = signal.signalType
            shotCount += 1
        print("\n----- END CheckModel -----\n")

    def LoadDml(self, dmlFile, printDebugData=True):
        """Pass in DAVE-ML model format file"""

        self.Clear()

        root = ET.parse(dmlFile).getroot()

        if root.tag == self.Tag("DAVEfunc"):
            for daveFcn in root:
                if daveFcn.tag == self.Tag("fileHeader"):
                    self.FileHeader(daveFcn)
                if daveFcn.tag == self.Tag("variableDef"):
                    self.VariableDef(daveFcn, printDebugData)
                if daveFcn.tag == self.Tag("breakpointDef"):
                    self.BreakpointDef(daveFcn, printDebugData)
                if daveFcn.tag == self.Tag("griddedTableDef"):
                    self.GriddedTableDef(daveFcn, printDebugData)
                if daveFcn.tag == self.Tag("ungriddedTableDef"):
                    self.UngriddedTableDef(daveFcn)
```

```
                if daveFcn.tag == self.Tag("function"):
                    self.Function(daveFcn, printDebugData)
                if daveFcn.tag == self.Tag("checkData"):
                    self.CheckDataFx(daveFcn, printDebugData)

        if printDebugData:
            print("\n--- PreProcess Equations and Functions ---\n")

        self.PreProcess(printDebugData)

        if printDebugData:
            print("Number of check cases: ", len(self.CheckData.name))

            print("\n--Variables defined in model--\n")
            for i in self.VarDef:
                print(i.name)

        print("\n----- DAVE-ML MODEL PARSE COMPLETE -----")
```

1.4.1 1-D Gridded Table

Load and Check the DAVE-ML model. Start with some test models.

[6]:
```
gvDaveModel = ppDaveModel()
gvDaveModel.LoadDml('models/tests/oneD_table.dml')
gvDaveModel.CheckModel()
```

```
*******************************************
Model:  One Dimensional Table Test
creation date:  2021-04-26
file version:  $Revision: 100 $
*******************************************

-variableDef-
 varDefStruct.name:  angleOfAttack
 varDefStruct.varID:  alpha
 varDefStruct.units:  deg
 varDefStruct.axisSystem:  None
 varDefStruct.sign:  None
 varDefStruct.alias:  None
 varDefStruct.symbol:  None
 varDefStruct.hasInitialValue:  False
 varDefStruct.initialValue:  None
 varDefStruct.isStdAIAA:  False
 varDefStruct.isOutput:  False
 varDefStruct.hasMath:  False
 varDefStruct.codeText:  None
-variableDef-
```

```
 varDefStruct.name:  Cnp
 varDefStruct.varID:  cnp
 varDefStruct.units:  nd
 varDefStruct.axisSystem:  None
 varDefStruct.sign:  None
 varDefStruct.alias:  None
 varDefStruct.symbol:  None
 varDefStruct.hasInitialValue:  False
 varDefStruct.initialValue:  None
 varDefStruct.isStdAIAA:  False
 varDefStruct.isOutput:  False
 varDefStruct.hasMath:  False
 varDefStruct.codeText:  None
-bpDefStruct-
 bpDefStruct.name:  alpha
 bpDefStruct.bpID:  ALPHA1
 bpDefStruct.units:  deg
 bpDefStruct.bpVals:  [-10.0, -5.0, 0.0, 5.0, 10.0, 15.0, 20.0, 25.0, 30.0,
35.0, 40.0, 45.0]
-gtDefStruct-
 gtDefStruct.name:  Cnp_table
 gtDefStruct.gtID:  None
 gtDefStruct.units:  None
 gtDefStruct.bpRef:  ['ALPHA1']
 gtDefStruct.dataTableStr:

             .061, .052, .052, -.012, -.013, -.024, .050, .150, .130, .158,
.240, .150

 gtDefStruct.dataTable:  [0.061, 0.052, 0.052, -0.012, -0.013, -0.024, 0.05,
0.15, 0.13, 0.158, 0.24, 0.15]
-functionStruct-
 funDefStruct.name:  Cnp
 funDefStruct.fdName:  Cnp
 funDefStruct.gtID:  Cnp_table
 funDefStruct.numBreakPts:  1
 funDefStruct.dependentVarID:  cnp
 independentVarRef.varID:  alpha
 independentVarRef.min:  -10.0
 independentVarRef.max:  45.0
 independentVarRef.extrapolate:  neither
 independentVarRef.interpolate:  None
-checkData-
staticShot:  AOA 5 deg
 signal type:  {http://daveml.org/2010/DAVEML}checkInputs
 signal name:  angleOfAttack 5 deg
 signal varID:  alpha
 signal value:  5.0
```

```
 [ localSignal append ] ->  AOA 5 deg signal #: 1
 signal type:  {http://daveml.org/2010/DAVEML}checkOutputs
 signal name:  Cnp at 5 deg
 signal varID:  cnp
 signal value:  -0.012
 signal tol:  0.000001
 [ localSignal append ] ->  AOA 5 deg signal #: 2
2 signals in  AOA 5 deg
staticShot:  AOA 10 deg
 signal type:  {http://daveml.org/2010/DAVEML}checkInputs
 signal name:  angleOfAttack 10 deg
 signal varID:  alpha
 signal value:  10.0
 [ localSignal append ] ->  AOA 10 deg signal #: 1
 signal type:  {http://daveml.org/2010/DAVEML}checkOutputs
 signal name:  Cnp at 10 deg
 signal varID:  cnp
 signal value:  -0.013
 signal tol:  0.000001
 [ localSignal append ] ->  AOA 10 deg signal #: 2
2 signals in  AOA 10 deg
staticShot:  AOA 29 deg
 signal type:  {http://daveml.org/2010/DAVEML}checkInputs
 signal name:  AOA at 29 deg
 signal varID:  alpha
 signal value:  29.0
 [ localSignal append ] ->  AOA 29 deg signal #: 1
 signal type:  {http://daveml.org/2010/DAVEML}checkOutputs
 signal name:  Cnp at 29 deg AOA
 signal varID:  cnp
 signal value:  0.15
 signal tol:  0.000001
 [ localSignal append ] ->  AOA 29 deg signal #: 2
2 signals in  AOA 29 deg

--- PreProcess Equations and Functions ---

+++++ MODEL INPUTS AND OUTPUTS +++++
++> Input:  alpha
++++++++++++++++++++++++++++++++++++
------> depVar:  cnp
----> f.dataTable:  [0.061, 0.052, 0.052, -0.012, -0.013, -0.024, 0.05, 0.15,
0.13, 0.158, 0.24, 0.15]
----> bpRef:  ['ALPHA1']
----> gt.name:  Cnp_table
----> f.gtID:  Cnp_table
----> f.bp name:  ALPHA1
----> f.bpVals:  [-10.0, -5.0, 0.0, 5.0, 10.0, 15.0, 20.0, 25.0, 30.0, 35.0,
```

```
40.0, 45.0]
----> bpv:  [[-10.0, -5.0, 0.0, 5.0, 10.0, 15.0, 20.0, 25.0, 30.0, 35.0, 40.0,
45.0]]
Number of check cases:  3

--Variables defined in model--

angleOfAttack
Cnp

----- DAVE-ML MODEL PARSE COMPLETE -----

----- CheckModel -----

numSignals:  [2, 2, 2]

----- END CheckModel -----
```

Do a short test to check of the function cnp which is a function of alpha.

[7]:
```
#gvAeroModel.Data["alpha"] = 29.0
#gvAeroModel.Update()
#print("f cnp: ", gvAeroModel.Data["cnp"])
gvDaveModel.Data["alpha"] = 29.0
gvDaveModel.Update()
print("f cnp: ", gvDaveModel.Data["cnp"])
```

```
f cnp:  0.15
```

1.4.2 2-D Gridded Table

[8]:
```
gvDaveModel.LoadDml('models/tests/twoD_table.dml')
gvDaveModel.CheckModel()
```

```
******************************************
Model:  2D gridded table
file version:  $Revision: 108 $
******************************************

-variableDef-
 varDefStruct.name:  angleOfAttack
 varDefStruct.varID:  ALPHA
 varDefStruct.units:  deg
 varDefStruct.axisSystem:  None
 varDefStruct.sign:  None
 varDefStruct.alias:  None
 varDefStruct.symbol:  #x3B1
 varDefStruct.hasInitialValue:  False
```

```
 varDefStruct.initialValue:  None
 varDefStruct.isStdAIAA:  False
 varDefStruct.isOutput:  False
 varDefStruct.hasMath:  False
 varDefStruct.codeText:  None
-variableDef-
 varDefStruct.name:  Mach
 varDefStruct.varID:  MACH
 varDefStruct.units:  ND
 varDefStruct.axisSystem:  None
 varDefStruct.sign:  None
 varDefStruct.alias:  None
 varDefStruct.symbol:  M
 varDefStruct.hasInitialValue:  False
 varDefStruct.initialValue:  None
 varDefStruct.isStdAIAA:  False
 varDefStruct.isOutput:  False
 varDefStruct.hasMath:  False
 varDefStruct.codeText:  None
-variableDef-
 varDefStruct.name:  coefficientOfLift
 varDefStruct.varID:  CL
 varDefStruct.units:  ND
 varDefStruct.axisSystem:  None
 varDefStruct.sign:  None
 varDefStruct.alias:  None
 varDefStruct.symbol:  CL
 varDefStruct.hasInitialValue:  False
 varDefStruct.initialValue:  None
 varDefStruct.isStdAIAA:  False
 varDefStruct.isOutput:  False
 varDefStruct.hasMath:  False
 varDefStruct.codeText:  None
-bpDefStruct-
 bpDefStruct.name:  alpha
 bpDefStruct.bpID:  ALPHA1
 bpDefStruct.units:  deg
 bpDefStruct.bpVals:  [-4.0, 0.0, 4.0, 8.0, 12.0, 16.0]
-bpDefStruct-
 bpDefStruct.name:  mach
 bpDefStruct.bpID:  MACH1
 bpDefStruct.units:  ND
 bpDefStruct.bpVals:  [0.0, 0.4, 0.8, 0.9, 0.95, 0.99, 1.0, 1.01, 1.05, 1.2]
-gtDefStruct-
 gtDefStruct.name:  CL_TABLE
 gtDefStruct.gtID:  None
 gtDefStruct.units:  None
 gtDefStruct.bpRef:  ['MACH1', 'ALPHA1']
```

```
 gtDefStruct.dataTableStr:


      9.5013e-01, 6.1543e-01, 5.7891e-02, 1.5274e-02, 8.3812e-01, 1.9343e-01,
      2.3114e-01, 7.9194e-01, 3.5287e-01, 7.4679e-01, 1.9640e-02, 6.8222e-01,
      6.0684e-01, 9.2181e-01, 8.1317e-01, 4.4510e-01, 6.8128e-01, 3.0276e-01,
      4.8598e-01, 7.3821e-01, 9.8613e-03, 9.3181e-01, 3.7948e-01, 5.4167e-01,
      8.9130e-01, 1.7627e-01, 1.3889e-01, 4.6599e-01, 8.3180e-01, 1.5087e-01,
      7.6210e-01, 4.0571e-01, 2.0277e-01, 4.1865e-01, 5.0281e-01, 6.9790e-01,
      4.5647e-01, 9.3547e-01, 1.9872e-01, 8.4622e-01, 7.0947e-01, 3.7837e-01,
      1.8504e-02, 9.1690e-01, 6.0379e-01, 5.2515e-01, 4.2889e-01, 8.6001e-01,
      8.2141e-01, 4.1027e-01, 2.7219e-01, 2.0265e-01, 3.0462e-01, 8.5366e-01,
      4.4470e-01, 8.9365e-01, 1.9881e-01, 6.7214e-01, 1.8965e-01, 5.9356e-01

 gtDefStruct.dataTable:  [0.95013, 0.61543, 0.057891, 0.015274, 0.83812,
0.19343, 0.23114, 0.79194, 0.35287, 0.74679, 0.01964, 0.68222, 0.60684, 0.92181,
0.81317, 0.4451, 0.68128, 0.30276, 0.48598, 0.73821, 0.0098613, 0.93181,
0.37948, 0.54167, 0.8913, 0.17627, 0.13889, 0.46599, 0.8318, 0.15087, 0.7621,
0.40571, 0.20277, 0.41865, 0.50281, 0.6979, 0.45647, 0.93547, 0.19872, 0.84622,
0.70947, 0.37837, 0.018504, 0.9169, 0.60379, 0.52515, 0.42889, 0.86001, 0.82141,
0.41027, 0.27219, 0.20265, 0.30462, 0.85366, 0.4447, 0.89365, 0.19881, 0.67214,
0.18965, 0.59356]
-functionStruct-
 funDefStruct.name:  Basic CL
 funDefStruct.fdName:  Basic CL
 funDefStruct.gtID:  CL_TABLE
 funDefStruct.numBreakPts:  2
 funDefStruct.dependentVarID:  CL
 independentVarRef.varID:  MACH
 independentVarRef.min:  0.3
 independentVarRef.max:  0.95
 independentVarRef.extrapolate:  max
 independentVarRef.interpolate:  None
 independentVarRef.varID:  ALPHA
 independentVarRef.min:  0.0
 independentVarRef.max:  15.0
 independentVarRef.extrapolate:  both
 independentVarRef.interpolate:  None
-checkData-
staticShot:  AOA 4 deg; Mach 0.9
 signal type:  {http://daveml.org/2010/DAVEML}checkInputs
 signal name:  AOA
 signal varID:  ALPHA
 signal value:  4.0
 [ localSignal append ] ->  AOA 4 deg; Mach 0.9 signal #: 1
 signal type:  {http://daveml.org/2010/DAVEML}checkInputs
 signal name:  MACH number
 signal varID:  MACH
```

```
 signal value:  0.9
 [ localSignal append ] ->  AOA 4 deg; Mach 0.9 signal #: 2
 signal type:  {http://daveml.org/2010/DAVEML}checkOutputs
 signal name:  CL
 signal varID:  CL
 signal value:  0.0098613
 signal tol:  0.000001
 [ localSignal append ] ->  AOA 4 deg; Mach 0.9 signal #: 3
3 signals in  AOA 4 deg; Mach 0.9

--- PreProcess Equations and Functions ---

+++++ MODEL INPUTS AND OUTPUTS +++++
++> Input:  ALPHA
++> Input:  MACH
++++++++++++++++++++++++++++++++++++
------> depVar:  CL
----> f.dataTable:  [0.95013, 0.61543, 0.057891, 0.015274, 0.83812, 0.19343,
0.23114, 0.79194, 0.35287, 0.74679, 0.01964, 0.68222, 0.60684, 0.92181, 0.81317,
0.4451, 0.68128, 0.30276, 0.48598, 0.73821, 0.0098613, 0.93181, 0.37948,
0.54167, 0.8913, 0.17627, 0.13889, 0.46599, 0.8318, 0.15087, 0.7621, 0.40571,
0.20277, 0.41865, 0.50281, 0.6979, 0.45647, 0.93547, 0.19872, 0.84622, 0.70947,
0.37837, 0.018504, 0.9169, 0.60379, 0.52515, 0.42889, 0.86001, 0.82141, 0.41027,
0.27219, 0.20265, 0.30462, 0.85366, 0.4447, 0.89365, 0.19881, 0.67214, 0.18965,
0.59356]
----> bpRef:  ['MACH1', 'ALPHA1']
----> gt.name:  CL_TABLE
----> f.gtID:  CL_TABLE
----> f.bp name:  MACH1
----> f.bpVals:  [0.0, 0.4, 0.8, 0.9, 0.95, 0.99, 1.0, 1.01, 1.05, 1.2]
----> bpv:  [[0.0, 0.4, 0.8, 0.9, 0.95, 0.99, 1.0, 1.01, 1.05, 1.2]]
----> f.bp name:  ALPHA1
----> f.bpVals:  [-4.0, 0.0, 4.0, 8.0, 12.0, 16.0]
----> bpv:  [[0.0, 0.4, 0.8, 0.9, 0.95, 0.99, 1.0, 1.01, 1.05, 1.2], [-4.0, 0.0,
4.0, 8.0, 12.0, 16.0]]
Number of check cases:  1

--Variables defined in model--

angleOfAttack
Mach
coefficientOfLift

----- DAVE-ML MODEL PARSE COMPLETE -----

----- CheckModel -----

numSignals:  [3]
```

```
----- END CheckModel -----
```

1.4.3 Various DAVE-ML Tests

[9]:
```
gvDaveModel.LoadDml('models/tests/test.dml')
gvDaveModel.CheckModel()
```

```
*******************************************
Model:  Test File
creation date:  2009-06-01
file version:  $Revision: 1 $
*******************************************

-variableDef-
 varDefStruct.name:  rtd
 varDefStruct.varID:  rtd
 varDefStruct.units:  deg_rad
 varDefStruct.axisSystem:  None
 varDefStruct.sign:  None
 varDefStruct.alias:  None
 varDefStruct.symbol:  None
 varDefStruct.hasInitialValue:  True
 varDefStruct.initialValue:  57.29577951
 varDefStruct.isStdAIAA:  False
 varDefStruct.isOutput:  False
 varDefStruct.hasMath:  False
 varDefStruct.codeText:  None
-variableDef-
 varDefStruct.name:  angleOfAttack
 varDefStruct.varID:  alpha
 varDefStruct.units:  nd
 varDefStruct.axisSystem:  None
 varDefStruct.sign:  None
 varDefStruct.alias:  None
 varDefStruct.symbol:  None
 varDefStruct.hasInitialValue:  False
 varDefStruct.initialValue:  None
 varDefStruct.isStdAIAA:  False
 varDefStruct.isOutput:  False
 varDefStruct.hasMath:  False
 varDefStruct.codeText:  None
-variableDef-
 varDefStruct.name:  beta
 varDefStruct.varID:  beta
 varDefStruct.units:  deg
 varDefStruct.axisSystem:  None
```

```
 varDefStruct.sign:  None
 varDefStruct.alias:  None
 varDefStruct.symbol:  None
 varDefStruct.hasInitialValue:  True
 varDefStruct.initialValue:  5.0
 varDefStruct.isStdAIAA:  False
 varDefStruct.isOutput:  False
 varDefStruct.hasMath:  False
 varDefStruct.codeText:  None
-variableDef-
 varDefStruct.name:  Avariable
 varDefStruct.varID:  a
 varDefStruct.units:  nd
 varDefStruct.axisSystem:  None
 varDefStruct.sign:  None
 varDefStruct.alias:  None
 varDefStruct.symbol:  None
 varDefStruct.hasInitialValue:  True
 varDefStruct.initialValue:  2.0
 varDefStruct.isStdAIAA:  False
 varDefStruct.isOutput:  False
 varDefStruct.hasMath:  False
 varDefStruct.codeText:  None
-variableDef-
 varDefStruct.name:  Bvariable
 varDefStruct.varID:  b
 varDefStruct.units:  nd
 varDefStruct.axisSystem:  None
 varDefStruct.sign:  None
 varDefStruct.alias:  None
 varDefStruct.symbol:  None
 varDefStruct.hasInitialValue:  False
 varDefStruct.initialValue:  None
 varDefStruct.isStdAIAA:  False
 varDefStruct.isOutput:  False
 varDefStruct.hasMath:  False
 varDefStruct.codeText:  None
-variableDef-
 varDefStruct.name:  cnp
 varDefStruct.varID:  cnp
 varDefStruct.units:  nd
 varDefStruct.axisSystem:  None
 varDefStruct.sign:  None
 varDefStruct.alias:  None
 varDefStruct.symbol:  None
 varDefStruct.hasInitialValue:  False
 varDefStruct.initialValue:  None
 varDefStruct.isStdAIAA:  False
```

```
 varDefStruct.isOutput:  False
 varDefStruct.hasMath:  False
 varDefStruct.codeText:  None
-variableDef-
 varDefStruct.name:  dtr
 varDefStruct.varID:  dtr
 varDefStruct.units:  r_d
 varDefStruct.axisSystem:  None
 varDefStruct.sign:  None
 varDefStruct.alias:  None
 varDefStruct.symbol:  None
 varDefStruct.hasInitialValue:  False
 varDefStruct.initialValue:  None
 varDefStruct.isStdAIAA:  False
 varDefStruct.isOutput:  False
 varDefStruct.hasMath:  True
 varDefStruct.codeText:  3.14159265/180.0
-variableDef-
 varDefStruct.name:  alpr
 varDefStruct.varID:  ALPR
 varDefStruct.units:  rad
 varDefStruct.axisSystem:  None
 varDefStruct.sign:  None
 varDefStruct.alias:  None
 varDefStruct.symbol:  None
 varDefStruct.hasInitialValue:  False
 varDefStruct.initialValue:  None
 varDefStruct.isStdAIAA:  False
 varDefStruct.isOutput:  False
 varDefStruct.hasMath:  True
 varDefStruct.codeText:  {alpha} * {dtr}
-variableDef-
 varDefStruct.name:  dummy
 varDefStruct.varID:  dummy
 varDefStruct.units:  nd
 varDefStruct.axisSystem:  None
 varDefStruct.sign:  None
 varDefStruct.alias:  None
 varDefStruct.symbol:  None
 varDefStruct.hasInitialValue:  False
 varDefStruct.initialValue:  None
 varDefStruct.isStdAIAA:  False
 varDefStruct.isOutput:  False
 varDefStruct.hasMath:  True
 varDefStruct.codeText:  10 * (25 - 15) + 8/2 + 9 * 9 + 3 + 2
-variableDef-
 varDefStruct.name:  Xvar
 varDefStruct.varID:  x
```

```
 varDefStruct.units:  nd
 varDefStruct.axisSystem:  None
 varDefStruct.sign:  None
 varDefStruct.alias:  None
 varDefStruct.symbol:  None
 varDefStruct.hasInitialValue:  False
 varDefStruct.initialValue:  None
 varDefStruct.isStdAIAA:  False
 varDefStruct.isOutput:  False
 varDefStruct.hasMath:  True
 varDefStruct.codeText:  5.0 ** {a}
-variableDef-
 varDefStruct.name:  y
 varDefStruct.varID:  y
 varDefStruct.units:  nd
 varDefStruct.axisSystem:  None
 varDefStruct.sign:  None
 varDefStruct.alias:  None
 varDefStruct.symbol:  None
 varDefStruct.hasInitialValue:  False
 varDefStruct.initialValue:  None
 varDefStruct.isStdAIAA:  False
 varDefStruct.isOutput:  False
 varDefStruct.hasMath:  True
 varDefStruct.codeText:  {cnp} + 0.0
-variableDef-
 varDefStruct.name:  Cz1
 varDefStruct.varID:  cz1
 varDefStruct.units:  nd
 varDefStruct.axisSystem:  None
 varDefStruct.sign:  down
 varDefStruct.alias:  None
 varDefStruct.symbol:  None
 varDefStruct.hasInitialValue:  False
 varDefStruct.initialValue:  None
 varDefStruct.isStdAIAA:  False
 varDefStruct.isOutput:  False
 varDefStruct.hasMath:  True
 varDefStruct.codeText:  {czt}*(1.-({beta}/{rtd})**2)-0.19*{del}
-variableDef-
 varDefStruct.name:  cnt
 varDefStruct.varID:  cnt
 varDefStruct.units:  nd
 varDefStruct.axisSystem:  None
 varDefStruct.sign:  nose right
 varDefStruct.alias:  None
 varDefStruct.symbol:  None
 varDefStruct.hasInitialValue:  False
```

```
 varDefStruct.initialValue:  None
 varDefStruct.isStdAIAA:  False
 varDefStruct.isOutput:  False
 varDefStruct.hasMath:  True
 varDefStruct.codeText:  -10 if {beta} < 0 else 10
-variableDef-
 varDefStruct.name:  ALP_UNLIM
 varDefStruct.varID:  ALP_UNLIM
 varDefStruct.units:  deg
 varDefStruct.axisSystem:  None
 varDefStruct.sign:  None
 varDefStruct.alias:  None
 varDefStruct.symbol:  None
 varDefStruct.hasInitialValue:  False
 varDefStruct.initialValue:  None
 varDefStruct.isStdAIAA:  False
 varDefStruct.isOutput:  False
 varDefStruct.hasMath:  False
 varDefStruct.codeText:  None
-variableDef-
 varDefStruct.name:  ALP_MAX_LIM
 varDefStruct.varID:  ALP_MAX_LIM
 varDefStruct.units:  deg
 varDefStruct.axisSystem:  None
 varDefStruct.sign:  None
 varDefStruct.alias:  None
 varDefStruct.symbol:  None
 varDefStruct.hasInitialValue:  True
 varDefStruct.initialValue:  5.0
 varDefStruct.isStdAIAA:  False
 varDefStruct.isOutput:  False
 varDefStruct.hasMath:  False
 varDefStruct.codeText:  None
-variableDef-
 varDefStruct.name:  Limited_angle_of_attack
 varDefStruct.varID:  ALP
 varDefStruct.units:  deg
 varDefStruct.axisSystem:  None
 varDefStruct.sign:  anu
 varDefStruct.alias:  None
 varDefStruct.symbol:  None
 varDefStruct.hasInitialValue:  False
 varDefStruct.initialValue:  None
 varDefStruct.isStdAIAA:  False
 varDefStruct.isOutput:  False
 varDefStruct.hasMath:  True
 varDefStruct.codeText:  -2.0 if {ALP_UNLIM} < -2 else {ALP_MAX_LIM} if
{ALP_UNLIM} > {ALP_MAX_LIM} else {ALP_UNLIM}
```

```
-variableDef-
 varDefStruct.name:  sinTest
 varDefStruct.varID:  sinTest
 varDefStruct.units:  nd
 varDefStruct.axisSystem:  None
 varDefStruct.sign:  none
 varDefStruct.alias:  None
 varDefStruct.symbol:  None
 varDefStruct.hasInitialValue:  False
 varDefStruct.initialValue:  None
 varDefStruct.isStdAIAA:  False
 varDefStruct.isOutput:  False
 varDefStruct.hasMath:  True
 varDefStruct.codeText:  math.sin(0.5235987756)
-bpDefStruct-
 bpDefStruct.name:  alpha
 bpDefStruct.bpID:  ALPHA1
 bpDefStruct.units:  deg
 bpDefStruct.bpVals:  [-10.0, -5.0, 0.0, 5.0, 10.0, 15.0, 20.0, 25.0, 30.0,
35.0, 40.0, 45.0]
-gtDefStruct-
 gtDefStruct.name:  Cnp_table
 gtDefStruct.gtID:  None
 gtDefStruct.units:  None
 gtDefStruct.bpRef:  ['ALPHA1']
 gtDefStruct.dataTableStr:   .061, .052, .052, -.012, -.013, -.024, .050, .150,
.130, .158, .240,
        .150
 gtDefStruct.dataTable:  [0.061, 0.052, 0.052, -0.012, -0.013, -0.024, 0.05,
0.15, 0.13, 0.158, 0.24, 0.15]
-functionStruct-
 funDefStruct.name:  Cnp
 funDefStruct.fdName:  Cnp
 funDefStruct.gtID:  Cnp_table
 funDefStruct.numBreakPts:  1
 funDefStruct.dependentVarID:  cnp
 independentVarRef.varID:  alpha
 independentVarRef.min:  -10.0
 independentVarRef.max:  45.0
 independentVarRef.extrapolate:  neither
 independentVarRef.interpolate:  None
-checkData-
staticShot:  Nominal
 signal type:  {http://daveml.org/2010/DAVEML}checkInputs
 signal name:  Bvariable
 signal varID:  b
 signal value:  -2.5
 [ localSignal append ] ->  Nominal signal #: 1
```

```
signal type:  {http://daveml.org/2010/DAVEML}checkInputs
signal name:  angleOfAttack
signal varID:  alpha
signal value:  5.0
[ localSignal append ] ->  Nominal signal #: 2
signal type:  {http://daveml.org/2010/DAVEML}checkInputs
signal name:  ALP_UNLIM
signal varID:  ALP_UNLIM
signal value:  -1.0
[ localSignal append ] ->  Nominal signal #: 3
signal type:  {http://daveml.org/2010/DAVEML}checkInputs
signal name:  angleOfSideslip
signal varID:  beta
signal value:  5.0
[ localSignal append ] ->  Nominal signal #: 4
signal type:  {http://daveml.org/2010/DAVEML}checkInputs
signal name:  elevatorDeflection
signal varID:  del
signal units:  d
signal value:  0.0
[ localSignal append ] ->  Nominal signal #: 5
signal type:  {http://daveml.org/2010/DAVEML}checkInputs
signal name:  czt
signal varID:  czt
signal units:  d
signal value:  -49.5
[ localSignal append ] ->  Nominal signal #: 6
signal type:  {http://daveml.org/2010/DAVEML}internalValues
signal name:  Radian to degree
signal varID:  rtd
signal value:  57.29577951
[ localSignal append ] ->  Nominal signal #: 7
signal type:  {http://daveml.org/2010/DAVEML}internalValues
signal name:  Degree to radian
signal varID:  dtr
signal value:  0.01745329252
[ localSignal append ] ->  Nominal signal #: 8
signal type:  {http://daveml.org/2010/DAVEML}internalValues
signal name:  a
signal varID:  a
signal value:  2.0
[ localSignal append ] ->  Nominal signal #: 9
signal type:  {http://daveml.org/2010/DAVEML}internalValues
signal name:  b
signal varID:  b
signal value:  -2.5
[ localSignal append ] ->  Nominal signal #: 10
signal type:  {http://daveml.org/2010/DAVEML}internalValues
```

```
signal name:  beta
signal varID:  beta
signal value:  5.0
[ localSignal append ] ->  Nominal signal #: 11
signal type:  {http://daveml.org/2010/DAVEML}internalValues
signal name:  ALP_UNLIM
signal varID:  ALP_UNLIM
signal value:  -1.0
[ localSignal append ] ->  Nominal signal #: 12
signal type:  {http://daveml.org/2010/DAVEML}internalValues
signal name:  sinTest
signal varID:  sinTest
signal value:  0.5
[ localSignal append ] ->  Nominal signal #: 13
signal type:  {http://daveml.org/2010/DAVEML}internalValues
signal name:  ALPR
signal varID:  ALPR
signal value:  0.0872665
[ localSignal append ] ->  Nominal signal #: 14
signal type:  {http://daveml.org/2010/DAVEML}checkOutputs
signal name:  dummy
signal varID:  dummy
signal value:  190.0
signal tol:  0.000001
[ localSignal append ] ->  Nominal signal #: 15
signal type:  {http://daveml.org/2010/DAVEML}checkOutputs
signal name:  Xvar
signal varID:  x
signal value:  25.0
signal tol:  0.000001
[ localSignal append ] ->  Nominal signal #: 16
signal type:  {http://daveml.org/2010/DAVEML}checkOutputs
signal name:  cz1
signal varID:  cz1
signal value:  -49.123036
signal tol:  0.000001
[ localSignal append ] ->  Nominal signal #: 17
signal type:  {http://daveml.org/2010/DAVEML}checkOutputs
signal name:  y
signal varID:  y
signal value:  -0.012
signal tol:  0.000001
[ localSignal append ] ->  Nominal signal #: 18
signal type:  {http://daveml.org/2010/DAVEML}checkOutputs
signal name:  cnt
signal varID:  cnt
signal value:  10.0
signal tol:  0.000001
```

```
 [ localSignal append ] ->  Nominal signal #: 19
 signal type:  {http://daveml.org/2010/DAVEML}checkOutputs
 signal name:  Limited_angle_of_attack
 signal varID:  ALP
 signal value:  -1.0
 signal tol:  0.000001
 [ localSignal append ] ->  Nominal signal #: 20
 signal type:  {http://daveml.org/2010/DAVEML}checkOutputs
 signal name:  Cnp
 signal varID:  cnp
 signal value:  -0.012
 signal tol:  0.000001
 [ localSignal append ] ->  Nominal signal #: 21
21 signals in  Nominal

--- PreProcess Equations and Functions ---

 << dtr >>
  [raw]->  3.14159265/180.0
  [python]->  3.14159265/180.0
 << ALPR >>
  [raw]->  {alpha} * {dtr}
  [python]->  self.Data["alpha"] * self.Data["dtr"]
 << dummy >>
  [raw]->  10 * (25 - 15) + 8/2 + 9 * 9 + 3 + 2
  [python]->  10 * (25 - 15) + 8/2 + 9 * 9 + 3 + 2
 << x >>
  [raw]->  5.0 ** {a}
  [python]->  5.0 ** self.Data["a"]
 << y >>
  [raw]->  {cnp} + 0.0
  [python]->  self.Data["cnp"] + 0.0
 << cz1 >>
  [raw]->  {czt}*(1.-({beta}/{rtd})**2)-0.19*{del}
  [python]->  self.Data["czt"]*(1.-(self.Data["beta"]/self.Data["rtd"])**2)-0.19
*self.Data["del"]
 << cnt >>
  [raw]->  -10 if {beta} < 0 else 10
  [python]->  -10 if self.Data["beta"] < 0 else 10
 << ALP >>
  [raw]->  -2.0 if {ALP_UNLIM} < -2 else {ALP_MAX_LIM} if {ALP_UNLIM} >
{ALP_MAX_LIM} else {ALP_UNLIM}
  [python]->  -2.0 if self.Data["ALP_UNLIM"] < -2 else self.Data["ALP_MAX_LIM"]
if self.Data["ALP_UNLIM"] > self.Data["ALP_MAX_LIM"] else self.Data["ALP_UNLIM"]
 << sinTest >>
  [raw]->  math.sin(0.5235987756)
  [python]->  math.sin(0.5235987756)
+++++ MODEL INPUTS AND OUTPUTS +++++
```

```
++> Input:  alpha
++> Input:  b
++> Input:  ALP_UNLIM
++++++++++++++++++++++++++++++++++++
------> depVar:  cnp
----> f.dataTable:  [0.061, 0.052, 0.052, -0.012, -0.013, -0.024, 0.05, 0.15,
0.13, 0.158, 0.24, 0.15]
----> bpRef:  ['ALPHA1']
----> gt.name:  Cnp_table
----> f.gtID:  Cnp_table
----> f.bp name:  ALPHA1
----> f.bpVals:  [-10.0, -5.0, 0.0, 5.0, 10.0, 15.0, 20.0, 25.0, 30.0, 35.0,
40.0, 45.0]
----> bpv:  [[-10.0, -5.0, 0.0, 5.0, 10.0, 15.0, 20.0, 25.0, 30.0, 35.0, 40.0,
45.0]]
Number of check cases:  1

--Variables defined in model--

rtd
angleOfAttack
beta
Avariable
Bvariable
cnp
dtr
alpr
dummy
Xvar
y
Cz1
cnt
ALP_UNLIM
ALP_MAX_LIM
Limited_angle_of_attack
sinTest

----- DAVE-ML MODEL PARSE COMPLETE -----

----- CheckModel -----

numSignals:  [21]

----- END CheckModel -----
```

[10]:
```python
gvDaveModel.LoadDml('models/cannonballNoAero/cannonballNoAero.dml')
gvDaveModel.CheckModel()
```

```
*******************************************
Model:  Cannon Ball Aerodynamics Model
creation date:  2010-02-01
file version:  $Revision: 1 $
*******************************************

-variableDef-
 varDefStruct.name:  XBodyPositionOfMRC
 varDefStruct.varID:  xcgr
 varDefStruct.units:  nd
 varDefStruct.axisSystem:  None
 varDefStruct.sign:  None
 varDefStruct.alias:  None
 varDefStruct.symbol:  None
 varDefStruct.hasInitialValue:  True
 varDefStruct.initialValue:  0.1
 varDefStruct.isStdAIAA:  True
 varDefStruct.isOutput:  False
 varDefStruct.hasMath:  False
 varDefStruct.codeText:  None
-variableDef-
 varDefStruct.name:  referenceWingChord
 varDefStruct.varID:  cbar
 varDefStruct.units:  m
 varDefStruct.axisSystem:  None
 varDefStruct.sign:  None
 varDefStruct.alias:  None
 varDefStruct.symbol:  None
 varDefStruct.hasInitialValue:  True
 varDefStruct.initialValue:  0.2
 varDefStruct.isStdAIAA:  True
 varDefStruct.isOutput:  False
 varDefStruct.hasMath:  False
 varDefStruct.codeText:  None
-variableDef-
 varDefStruct.name:  referenceWingSpan
 varDefStruct.varID:  bspan
 varDefStruct.units:  m
 varDefStruct.axisSystem:  None
 varDefStruct.sign:  None
 varDefStruct.alias:  None
 varDefStruct.symbol:  None
 varDefStruct.hasInitialValue:  True
 varDefStruct.initialValue:  0.2
 varDefStruct.isStdAIAA:  True
```

```
 varDefStruct.isOutput:  False
 varDefStruct.hasMath:  False
 varDefStruct.codeText:  None
-variableDef-
 varDefStruct.name:  referenceWingArea
 varDefStruct.varID:  swing
 varDefStruct.units:  m2
 varDefStruct.axisSystem:  None
 varDefStruct.sign:  None
 varDefStruct.alias:  None
 varDefStruct.symbol:  None
 varDefStruct.hasInitialValue:  True
 varDefStruct.initialValue:  0.0314159
 varDefStruct.isStdAIAA:  True
 varDefStruct.isOutput:  False
 varDefStruct.hasMath:  False
 varDefStruct.codeText:  None
-variableDef-
 varDefStruct.name:  aeroBodyForceCoefficient_X
 varDefStruct.varID:  cx
 varDefStruct.units:  nd
 varDefStruct.axisSystem:  None
 varDefStruct.sign:  FWD
 varDefStruct.alias:  None
 varDefStruct.symbol:  None
 varDefStruct.hasInitialValue:  True
 varDefStruct.initialValue:  0.0
 varDefStruct.isStdAIAA:  True
 varDefStruct.isOutput:  True
 varDefStruct.hasMath:  False
 varDefStruct.codeText:  None
-variableDef-
 varDefStruct.name:  aeroBodyForceCoefficient_Y
 varDefStruct.varID:  cy
 varDefStruct.units:  nd
 varDefStruct.axisSystem:  None
 varDefStruct.sign:  RIGHT
 varDefStruct.alias:  None
 varDefStruct.symbol:  None
 varDefStruct.hasInitialValue:  True
 varDefStruct.initialValue:  0.0
 varDefStruct.isStdAIAA:  True
 varDefStruct.isOutput:  True
 varDefStruct.hasMath:  False
 varDefStruct.codeText:  None
-variableDef-
 varDefStruct.name:  aeroBodyForceCoefficient_Z
 varDefStruct.varID:  cz
```

```
 varDefStruct.units:  nd
 varDefStruct.axisSystem:  None
 varDefStruct.sign:  DOWN
 varDefStruct.alias:  None
 varDefStruct.symbol:  None
 varDefStruct.hasInitialValue:  True
 varDefStruct.initialValue:  0.0
 varDefStruct.isStdAIAA:  True
 varDefStruct.isOutput:  True
 varDefStruct.hasMath:  False
 varDefStruct.codeText:  None
-variableDef-
 varDefStruct.name:  aeroBodyMomentCoefficient_Roll
 varDefStruct.varID:  cl
 varDefStruct.units:  nd
 varDefStruct.axisSystem:  None
 varDefStruct.sign:  RWD
 varDefStruct.alias:  None
 varDefStruct.symbol:  None
 varDefStruct.hasInitialValue:  True
 varDefStruct.initialValue:  0.0
 varDefStruct.isStdAIAA:  True
 varDefStruct.isOutput:  True
 varDefStruct.hasMath:  False
 varDefStruct.codeText:  None
-variableDef-
 varDefStruct.name:  aeroBodyMomentCoefficient_Pitch
 varDefStruct.varID:  cm
 varDefStruct.units:  nd
 varDefStruct.axisSystem:  None
 varDefStruct.sign:  ANU
 varDefStruct.alias:  None
 varDefStruct.symbol:  None
 varDefStruct.hasInitialValue:  True
 varDefStruct.initialValue:  0.0
 varDefStruct.isStdAIAA:  True
 varDefStruct.isOutput:  True
 varDefStruct.hasMath:  False
 varDefStruct.codeText:  None
-variableDef-
 varDefStruct.name:  aeroBodyMomentCoefficient_Yaw
 varDefStruct.varID:  cn
 varDefStruct.units:  nd
 varDefStruct.axisSystem:  None
 varDefStruct.sign:  ANR
 varDefStruct.alias:  None
 varDefStruct.symbol:  None
 varDefStruct.hasInitialValue:  True
```

```
 varDefStruct.initialValue:  0.0
 varDefStruct.isStdAIAA:  True
 varDefStruct.isOutput:  True
 varDefStruct.hasMath:  False
 varDefStruct.codeText:  None
-checkData-
staticShot:  Internal Constants
 signal type:  {http://daveml.org/2010/DAVEML}internalValues
 signal name:  Wind Chord
 signal varID:  cbar
 signal value:  0.2
 [ localSignal append ] ->  Internal Constants signal #: 1
 signal type:  {http://daveml.org/2010/DAVEML}internalValues
 signal name:  Wind Span
 signal varID:  bspan
 signal value:  0.2
 [ localSignal append ] ->  Internal Constants signal #: 2
2 signals in  Internal Constants

--- PreProcess Equations and Functions ---

+++++ MODEL INPUTS AND OUTPUTS +++++
++> Output:  cx
++> Output:  cy
++> Output:  cz
++> Output:  cl
++> Output:  cm
++> Output:  cn
++++++++++++++++++++++++++++++++++++
Number of check cases:  1

--Variables defined in model--

XBodyPositionOfMRC
referenceWingChord
referenceWingSpan
referenceWingArea
aeroBodyForceCoefficient_X
aeroBodyForceCoefficient_Y
aeroBodyForceCoefficient_Z
aeroBodyMomentCoefficient_Roll
aeroBodyMomentCoefficient_Pitch
aeroBodyMomentCoefficient_Yaw

----- DAVE-ML MODEL PARSE COMPLETE -----

----- CheckModel -----
```

```
numSignals:  [2]

----- END CheckModel -----
```

1.4.4 F-16 model

[11]:
```python
gvAeroModel = ppDaveModel()
printDebugData = False
gvAeroModel.LoadDml('models/F16/F16_aero.dml', printDebugData)
gvAeroModel.CheckModel()
```

```
ERROR:root:internal: Positive sideslip -> [absCl0] Calculated 0.0, Expected
-0.005615999999999995
ERROR:root:internal: Positive sideslip -> [absCn0] Calculated 0.0, Expected
0.008891999999999999
ERROR:root:internal: Positive sideslip -> [clt] Calculated 0.0, Expected
-0.005615999999999995
ERROR:root:internal: Positive sideslip -> [cnt] Calculated 0.0, Expected
0.008891999999999999
ERROR:root:internal: Positive sideslip -> [dclda] Calculated -0.052, Expected
-0.051297999999999996
ERROR:root:internal: Positive sideslip -> [dcldr] Calculated 0.014, Expected
0.013766
ERROR:root:internal: Positive sideslip -> [dcnda] Calculated -0.009, Expected
-0.009701999999999999
ERROR:root:internal: Positive sideslip -> [dcndr] Calculated -0.045, Expected
-0.044064
ERROR:root:internal: Positive sideslip -> [cl1] Calculated 0.0, Expected
-0.005615999999999995
ERROR:root:internal: Positive sideslip -> [cn1] Calculated 0.0, Expected
0.008891999999999999
ERROR:root:internal: Positive sideslip -> [cl] Calculated 0.0, Expected
-0.005615999999999995
ERROR:root:internal: Positive sideslip -> [cn] Calculated 0.0017659199999999998,
Expected 0.010657919999999998
ERROR:root:output: Positive sideslip -> [aeroRollBodyMomentCoefficient]
Calculated 0.0, Expected -0.005616
ERROR:root:output: Positive sideslip -> [aeroYawBodyMomentCoefficient]
Calculated 0.0017659199999999998, Expected 0.01065792
ERROR:root:internal: Negative sideslip -> [absCl0] Calculated 0.0, Expected
-0.005615999999999995
ERROR:root:internal: Negative sideslip -> [absCn0] Calculated 0.0, Expected
0.008891999999999999
ERROR:root:internal: Negative sideslip -> [clt] Calculated -0.0, Expected
0.005615999999999995
ERROR:root:internal: Negative sideslip -> [cnt] Calculated -0.0, Expected
-0.008891999999999999
```

```
ERROR:root:internal: Negative sideslip -> [dclda] Calculated -0.051, Expected
-0.051766
ERROR:root:internal: Negative sideslip -> [dcldr] Calculated 0.012, Expected
0.013532
ERROR:root:internal: Negative sideslip -> [dcnda] Calculated -0.006, Expected
-0.008298
ERROR:root:internal: Negative sideslip -> [dcndr] Calculated -0.04, Expected
-0.04383
ERROR:root:internal: Negative sideslip -> [cl1] Calculated 0.0, Expected
0.005615999999999995
ERROR:root:internal: Negative sideslip -> [cn1] Calculated -0.0, Expected
-0.008891999999999999
ERROR:root:internal: Negative sideslip -> [cl] Calculated 0.0, Expected
0.005615999999999995
ERROR:root:internal: Negative sideslip -> [cn] Calculated
-0.0017659199999999998, Expected -0.010657919999999998
ERROR:root:output: Negative sideslip -> [aeroRollBodyMomentCoefficient]
Calculated 0.0, Expected 0.005616
ERROR:root:output: Negative sideslip -> [aeroYawBodyMomentCoefficient]
Calculated -0.41530612733186895, Expected -0.01065792
ERROR:root:internal: Positive elevator -> [cxt] Calculated -0.025, Expected
-0.028603333333333335
ERROR:root:internal: Positive elevator -> [cmt] Calculated -0.127, Expected
-0.13206
ERROR:root:internal: Positive elevator -> [cx] Calculated -0.025, Expected
-0.028603333333333335
ERROR:root:internal: Positive elevator -> [cm] Calculated -0.1784192, Expected
-0.1834792

*******************************************
Model:  F-16 Subsonic Aerodynamics Model (a la Garza)
creation date:  2003-06-10
file version:  $ Revision: 394 $
*******************************************

+++++ MODEL INPUTS AND OUTPUTS +++++
++> Input:  vt
++> Input:  alpha
++> Input:  beta
++> Input:  p
++> Input:  q
++> Input:  r
++> Input:  el
++> Input:  ail
++> Input:  rdr
++> Input:  xcg
++> Output:  cx
++> Output:  cy
```

```
++> Output:  cz
++> Output:  cl
++> Output:  cm
++> Output:  cn
++++++++++++++++++++++++++++++++++++

----- DAVE-ML MODEL PARSE COMPLETE -----

----- CheckModel -----

numSignals:  [67, 67, 67, 16, 67, 67, 67, 67, 67, 67, 67, 67, 67, 67, 67, 67,
67]
ERROR:root:output: Positive elevator -> [aeroXBodyForceCoefficient] Calculated
-0.025, Expected -0.0286033333333
ERROR:root:output: Positive elevator -> [aeroPitchBodyMomentCoefficient]
Calculated -0.1784192, Expected -0.1834792
ERROR:root:internal: Negative elevator -> [cxt] Calculated -0.063, Expected
-0.024220000000000005
ERROR:root:internal: Negative elevator -> [cmt] Calculated 0.196, Expected
0.11659333333333334
ERROR:root:internal: Negative elevator -> [cx] Calculated -0.063, Expected
-0.024220000000000005
ERROR:root:internal: Negative elevator -> [cm] Calculated 0.1642192, Expected
0.08481253333333336
ERROR:root:output: Negative elevator -> [aeroXBodyForceCoefficient] Calculated
-0.063, Expected -0.02422
ERROR:root:output: Negative elevator -> [aeroPitchBodyMomentCoefficient]
Calculated 0.1642192, Expected 0.08481253333333
ERROR:root:internal: Skewed inputs -> [cxt] Calculated 0.094, Expected
0.08915927333333333
ERROR:root:internal: Skewed inputs -> [czt] Calculated -1.053, Expected -1.12812
ERROR:root:internal: Skewed inputs -> [cz1] Calculated -1.0843419673257337,
Expected -1.1592217522084587
ERROR:root:internal: Skewed inputs -> [absCl0] Calculated 0.0, Expected
-0.014256
ERROR:root:internal: Skewed inputs -> [absCn0] Calculated 0.0, Expected
0.0108864
ERROR:root:internal: Skewed inputs -> [clt] Calculated -0.0, Expected 0.014256
ERROR:root:internal: Skewed inputs -> [cmt] Calculated 0.01, Expected
-0.03276327333333335
ERROR:root:internal: Skewed inputs -> [cnt] Calculated -0.0, Expected -0.0108864
ERROR:root:internal: Skewed inputs -> [cxq] Calculated 2.91, Expected 2.874
ERROR:root:internal: Skewed inputs -> [cyr] Calculated 0.974, Expected 0.9368
ERROR:root:internal: Skewed inputs -> [cyp] Calculated 0.226, Expected
0.25432000000000005
ERROR:root:internal: Skewed inputs -> [czq] Calculated -30.7, Expected
-29.979999999999997
```

```
ERROR:root:internal: Skewed inputs -> [clr] Calculated 0.23, Expected
0.25136000000000003
ERROR:root:internal: Skewed inputs -> [clp] Calculated -0.375, Expected -0.36396
ERROR:root:internal: Skewed inputs -> [cmq] Calculated -6.64, Expected -6.412
ERROR:root:internal: Skewed inputs -> [cnr] Calculated -0.453, Expected
-0.47628000000000004
ERROR:root:internal: Skewed inputs -> [cnp] Calculated -0.024, Expected
-0.006239999999999982
ERROR:root:internal: Skewed inputs -> [dclda] Calculated -0.049, Expected
-0.046883999999999995
ERROR:root:internal: Skewed inputs -> [dcldr] Calculated 0.009, Expected
0.01230224
ERROR:root:internal: Skewed inputs -> [dcnda] Calculated -0.008, Expected
-0.005441279999999999
ERROR:root:internal: Skewed inputs -> [dcndr] Calculated -0.038, Expected
-0.04297872
ERROR:root:internal: Skewed inputs -> [cl1] Calculated -0.0196496, Expected
-0.004913040127999998
ERROR:root:internal: Skewed inputs -> [cn1] Calculated 0.0007269999999999998,
Expected -0.008683799472
ERROR:root:internal: Skewed inputs -> [cx] Calculated 0.05227447999999999,
Expected 0.04794994533333333
ERROR:root:internal: Skewed inputs -> [cy] Calculated 0.0248125, Expected
0.027353860000000008
ERROR:root:internal: Skewed inputs -> [cz] Calculated -0.644144900659067,
Expected -0.7293485255417921
ERROR:root:internal: Skewed inputs -> [cl] Calculated -0.0409596, Expected
-0.026917840128000005
ERROR:root:internal: Skewed inputs -> [cm] Calculated -0.04101214578294153,
Expected -0.10638585796465347
ERROR:root:internal: Skewed inputs -> [cn] Calculated 0.01922069358333333,
Expected 0.011183654767653334
ERROR:root:output: Skewed inputs -> [aeroXBodyForceCoefficient] Calculated
0.05227447999999999, Expected 0.04794994533333
ERROR:root:output: Skewed inputs -> [aeroYBodyForceCoefficient] Calculated
0.0248125, Expected 0.02735386
ERROR:root:output: Skewed inputs -> [aeroZBodyForceCoefficient] Calculated
-0.644144900659067, Expected -0.72934852554344
ERROR:root:output: Skewed inputs -> [aeroRollBodyMomentCoefficient] Calculated
-0.0409596, Expected -0.026917840128
ERROR:root:output: Skewed inputs -> [aeroPitchBodyMomentCoefficient] Calculated
-0.04101214578294153, Expected -0.10638585796503
ERROR:root:output: Skewed inputs -> [aeroYawBodyMomentCoefficient] Calculated
0.01922069358333333, Expected 0.01118365476765

----- END CheckModel -----
```

1.5 Environments

Create a planet class that has an atmosphere and gravity model.

1.5.1 US Standard Atmosphere 1976

The height is geopotential height (Z) in meters above MSL. The reference for the US Standard Atmosphere 1976. The refernence for the pressure equation.

Layer	Height (m)	Pressure (Pa)	Temperature (K)	Temperature Lapse Rate (K/m)
0	0	101,325	288.15	-0.0065
1	11,000	22,632.1	216.65	0
2	20,000	5,474.89	216.65	0.001
3	32,000	868.019	228.65	0.0028
4	47,000	110.906	270.65	0
5	51,000	66.9389	270.65	-0.0028
6	71,000	3.95642	214.65	-0.002

1.5.2 Gravity

The J_2 gravity model. Reference is Aircraft Control and Simulation, Third Edition on page 25.

$G^{ecef} = -\frac{GM}{r^2}\mathbf{p}$

$\mathbf{p_x} = [1 + 1.5(\frac{a}{r})^2 J_2(1 - 5\sin^2\psi)]\ p_x/r$
$\mathbf{p_y} = [1 + 1.5(\frac{a}{r})^2 J_2(1 - 5\sin^2\psi)]\ p_y/r$
$\mathbf{p_z} = [1 + 1.5(\frac{a}{r})^2 J_2(3 - 5\sin^2\psi)]\ p_z/r$

where p_x, p_y and p_z are ECEF position components and $\sin(\psi) = p_z/r$.

The equations for the WGS84 gravity model.

$e = \frac{\sqrt{a^2-b^2}}{a}$, a = semi-major axis, b = semi-minor axis
$N = \frac{a}{(1-e^2\sin^2\phi)^{1/2}}$, ϕ = geodetic Latitude
$P_r(h,\phi) = (N+h)\cos\phi$, h = altitude MSL
$g_0 = \frac{a(g_e)\cos^2\phi + b(g_p)\sin^2\phi}{\sqrt{a^2\cos^2\phi + b^2\sin^2\phi}}$
where g_e = gravity at the equator (9.7803253359) and g_p = gravity at the poles (9.8321849378)
$f = \frac{a-b}{a}, m = \frac{\omega^2 a^2 b}{GM}$, ω = Earth rotation rate
$g_h = g_0[1 - \frac{2h}{a}(1 + f + m - 2f\sin^2\phi) + \frac{3h^2}{a^2}]$
$\vec{g_h} = -g_h \begin{pmatrix} \cos\phi\ \cos\lambda \\ \cos\phi\ \sin\lambda \\ \sin\phi \end{pmatrix}$, λ = geodetic Longitude
$a_{hc} = \omega^2 P_2(h,\phi)$
$\vec{a_{hc}} = a_{hc} \begin{pmatrix} \cos\lambda \\ \sin\lambda \\ 0 \end{pmatrix}$
$\vec{g_{hG}} = \vec{g_h} - \vec{a_{hc}}$
$g_{hG} = |\vec{g_{hG}}| = \sqrt{g_{hGx}^2 + g_{hGy}^2 + g_{hGz}^2}$

1.5.3 ECEF to LLA

J. Zhu. Conversion of earth-centered earth-fixed coordinates to geodetic coordinates. Technical Report IEEE Log NO. T-AES/30/3/1666, IEEE, December 1993.

Reference for PcpfToLlaOsen is: Karl Osen. Accurate Conversion of Earth-Fixed Earth-Centered Coordinates to Geodetic Coordinates. Research Report Norwegian University of Science and Technology. 2017. ffhal-01704943v2f located here.

1.5.4 Planet Base Class

[12]:
```
class ppPlanet(ppUnitTest):
    GM = 0
    J2 = 0

    Latitude = 0
    Longitude = 0
    Altitude = 0

    RotationRate = 0
    SemiMajor    = 0
    Flattening   = 0
    SemiMinor    = 0
    Eccentricity = 0
    EccentricitySquared = 0

    def CalcSemiMinor(self):
        self.SemiMinor = self.SemiMajor * ( 1.0 - self.Flattening )

    def CalcEccentricity(self):
        a = self.SemiMajor
        b = self.SemiMinor
        self.Eccentricity = (math.sqrt( a * a - b * b ) /  a)
        self.EccentricitySquared = (self.Eccentricity) ** 2

    def LlaToPcpf(self):
        a  = self.SemiMajor
        e2 = self.EccentricitySquared
        sinLat = math.sin( self.Latitude )
        N = a / math.sqrt( 1.0 - (e2*sinLat*sinLat) )

        cosLat = math.cos( self.Latitude )
        # set the planet centered, planet fixed (PCPF) x,y,z vector in meters
        x = (N + self.Altitude) * cosLat * math.cos(self.Longitude)
        y = (N + self.Altitude) * cosLat * math.sin(self.Longitude)
        z = (N*(1.0 - e2) + self.Altitude) * sinLat
        return x, y, z
```

```
    def PcpfToLlaZhu(self, x, y, z):
        a  = self.SemiMajor
        b  = self.SemiMinor
        e  = self.Eccentricity
        e2 = self.EccentricitySquared

        assert b != 0, "SemiMinor axis is 0"
        ep  = e * a / b
        ep2 = ep * ep

        r = math.sqrt( x*x + y*y )
        F = 54.0 * b*b * z*z
        G = r*r + (1.0 - e2) * z*z - e2*(a*a - b*b)
        c = e2*e2*F*r*r/(G*G*G)
        s = ( 1.0 + c + math.sqrt(c*c + 2.0*c) ) ** (1.0 / 3.0)
        P = F / ( 3.0*( (s + 1.0/s + 1.0)**2.0 )*G*G )
        Q = math.sqrt(1.0 + 2.0 * e2*e2 * P)
        r0 = -P*e2*r/(1.0 + Q) + math.sqrt( 0.5*a*a*(1.0 + 1.0/Q) - (P*(1.0-e2)*z*z)/(Q + Q*Q) - 0.5*P*r*r )
        U = math.sqrt( (r-e2*r0)**2.0 + z*z )
        V = math.sqrt( (r-e2*r0)**2.0 + (1.0 - e2)*z*z )
        z0 = (b*b*z)/(a*V)

        self.Latitude  = math.atan((z + ep2*z0)/r)
        self.Longitude = math.atan2(y , x)
        self.Altitude  = U * ( 1.0 - (b*b)/(a*V) )

    def PcpfToLlaOsen(self, x, y, z):
        WGS84_INVAA = +2.45817225764733181057e-0014 # 1/(a^2)
        WGS84_EED2  = +3.34718999507065852867e-0003 # (e^2)/2
        WGS84_EEEE  = +4.48147234524044602618e-0005 # e^4
        WGS84_EEEED4 = +1.12036808631011150655e-0005 # (e^4)/4
        WGS84_P1MEE  = +9.93305620009858682943e-0001 # 1-(e^2)
        WGS84_P1MEEDAA = +2.44171631847341700642e-0014 # (1-(e^2))/(a^2)
        WGS84_INVCBRT2 = +7.93700525984099737380e-0001 # 1/(2^(1/3))
        WGS84_INV3     = +3.33333333333333333333e-0001 # 1/3
        WGS84_INV6     = +1.66666666666666666667e-0001 # 1/6

        ww = x * x + y * y
        m = ww * WGS84_INVAA
        n = z * z * WGS84_P1MEEDAA
        mpn = m + n
        p = WGS84_INV6 * (mpn - WGS84_EEEE)
        G = m * n * WGS84_EEEED4
        H = 2 * p * p * p + G

        C = ((H + G + 2 * math.sqrt(H * G))**WGS84_INV3) * WGS84_INVCBRT2
```

```
        assert C != 0, "PcpfToLLaOsen C is 0"
        i = -WGS84_EEEED4 - 0.5 * mpn
        P = p * p
        beta = WGS84_INV3 * i - C - (P / C)
        k = WGS84_EEEED4 * (WGS84_EEEED4 - mpn)

        # Compute left part of t
        t1 = beta * beta - k
        assert t1 >= 0, "PcpfToLLaOsen t1 is negative. t1: {0}".format(t1)
        t2 = math.sqrt(t1)
        t3 = t2 - 0.5 * (beta + i)
        assert t3 >= 0, "PcpfToLLaOsen t3 is negative"
        t4 = math.sqrt(t3)
        # Compute right part of t
        t5 = 0.5 * (beta - i)
        # t5 may accidentally drop just below zero due to numeric turbulence
        # This only occurs at latitudes close to +- 45.3 degrees
        t5 = abs(t5)
        t6 = math.sqrt(t5)
        t7 = t6 if (m < n) else -t6
        # Add left and right parts
        t = t4 + t7
        # Use Newton-Raphson's method to compute t correction
        j = WGS84_EED2 * (m - n)
        g = 2 * j
        tt = t * t
        ttt = tt * t
        tttt = tt * tt
        F = tttt + 2 * i * tt + g * t + k
        dFdt = 4 * ttt + 4 * i * t + g;
        dt = -F / dFdt

        # compute latitude (range -PI/2..PI/2)
        u = t + dt + WGS84_EED2
        v = t + dt - WGS84_EED2
        w = math.sqrt(ww)
        zu = z * u
        wv = w * v
        self.Latitude = math.atan2(zu, wv)

        # compute altitude
        assert (u*v) != 0, "PcpfToLlaOsen (u*v) is 0"
        invuv = 1 / (u * v)
        dw = w - wv * invuv
        dz = z - zu * WGS84_P1MEE * invuv
        da = math.sqrt(dw * dw + dz * dz)
        self.Altitude = -da if (u < 1) else da
```

```
        # compute longitude (range -PI..PI)
        self.Longitude = math.atan2(y, x);
```

1.5.5 Earth Class

[13]:
```
class ppEarth(ppPlanet):

    def __init__(self):
        self.GM = 3.986004418e14              # GM constant in m3/s2
        self.J2 = 1.082626684e-3
        self.RotationRate = 7.292115e-5  # Earth Rotation Rate (rad/sec, East)

        self.SemiMajor   = 6378137.0          # WGS84 defined
        self.Flattening  = 1/298.257223563    # WGS84 defined
        self.CalcSemiMinor()
        self.CalcEccentricity()

    def StdAtm1976(self, altitude):
        # Geopotential Alt (m) table ranges for 1976 US standard atmosphere
        #      0        1        2        3        4        5        6
        Z = [0.0, 11000.0, 20000.0, 32000.0, 47000.0, 51000.0, 71000.0]

        # Temperature (K) at start of air layer
        #          0    11000   20000   32000   47000   51000   71000
        T  = [288.15, 216.65, 216.65, 228.65, 270.65, 270.65, 214.65]

        # Pressure (Pa) at start air layer
        #           0      11000     20000    32000    47000   51000 71000
        P = [101325.0, 22632.10, 5474.89, 868.02, 110.91, 66.94, 3.96]

        # Temperature Gradient (K/m) for the altitude ranges
        #           0  11000   20000   32000   47000     51000     71000
        TG = [-6.5e-3,     0, 1.0e-3, 2.8e-3,      0, -2.8e-3, -2.0e-3]

        radiusEarth = 6356766.0  # Earth radius for geopotential alt conversion
        p0 = 101325.0            # pressure at sea level (Pa)
        Rgc  = 287.0528          # Gas constant (N m / kg K)
        g0 = 9.806645            # gravity at sea level (m / s^2)
        M = 0.0289644            # molar mass of Earth's air (kg/mol)
        Rstar = 8.3144598        # universal gas constant [J/(mol·K)]
        airGamma = 1.4           # gamma value for air

        # Convert geometric altitude to geopotential as the standard atmosphere
        # altitude layers are geopotential.
        z0 = radiusEarth * altitude / (radiusEarth + altitude)
```

```
        # get the index of the atmosphere layer
        i = -1
        count = 0
        for z in Z:
            if count != 0:
                if z0 < z and i == -1:
                    i = count - 1
            count += 1

        deltaZ = z0 - Z[i]

        temperature = TG[i] * deltaZ + T[i]
        temperature = temperature if (temperature > 0.0) else 0

        pressure = 0
        # The pressure is calculated differently depending
        # on the temperature lapse rate of the air layer.
        if abs(TG[i]) < 1e-12:
            pressure = P[i] * math.exp( (-g0 * M * deltaZ) / (Rstar * T[i]) )
        else:
            pe = (-g0 * M) / (Rstar * TG[i])
            pressure = P[i] * ((T[i] + TG[i] * deltaZ) / T[i])**pe

        airDensity = (pressure / (Rgc * temperature)) if (temperature > 0.0)␣
↪else 0

        assert temperature >= 0, "temp: {}, alt: {}".format(temperature,␣
↪altitude)
        speedOfSoundMps = math.sqrt( airGamma * Rgc * temperature )

        return airDensity, temperature, pressure, speedOfSoundMps

    def AirDensity(self, altitude):
        result = self.StdAtm1976(altitude)
        return result[0]

    def GravityConstant(self):
        return 9.80665

    def GravityWgs84(self, latRad, lonRad, h):
        a = self.SemiMajor
        b = self.SemiMinor
        E = self.Eccentricity
        sinPhi = math.sin(latRad)
        sin2Phi = sinPhi**2
        N = a / math.sqrt(1 - E*E*sin2Phi)
        cosPhi = math.cos(latRad)
```

```python
        cos2Phi = cosPhi**2
        Pr = (N + h) * cosPhi
        ge = 9.7803253359
        gp = 9.8321849378
        g0 = (a*ge + cos2Phi + b*gp*sin2Phi) / math.sqrt(a*a*cos2Phi + b*b*sin2Phi)
        f = (a - b) / a
        w = self.RotationRate
        m = w*w*a*a*b / self.GM
        gh = g0*(1 - 2/a * (1 + f + m - 2*f*sin2Phi)*h + (3*h*h)/(a*a))
        cosLambda = math.cos(lonRad)
        sinLambda = math.sin(lonRad)
        #Ghx = -gh * cosPhi
        GhX = -gh*cosPhi*cosLambda
        GhY = -gh*cosPhi*sinLambda
        GhZ = -gh*sinPhi
        ahc = w*w*Pr
        AhcX = ahc*cosLambda
        AhcY = ahc*sinLambda
        AhcZ = 0

        GhGX = GhX - AhcX
        GhGY = GhY - AhcY
        GhGZ = GhZ - AhcZ

        return GhGX, GhGZ, GhGZ

    def GravityJ2(self, x, y, z):
        r2 = x*x + y*y + z*z
        r = math.sqrt(r2)
        assert r != 0, "Gravity J2 r is 0"
        gmOverR3 = -self.GM / (r**3)
        j2Term = (1.5 * self.J2) * (self.SemiMajor)**2 / (r**4)
        z2 = 5.0 * z * z

        gx = x * gmOverR3 * (1 - j2Term*(z2 - r2))
        gy = y * gmOverR3 * (1 - j2Term*(z2 - r2))
        gz = z * gmOverR3 * (1 - j2Term*(z2 - 3*r2))

        return gx, gy, gz

    def GravityJ2SL(self, x, y, z):
        r = math.sqrt(x*x + y*y + z*z)
        assert r != 0, "Gravity J2 r is 0"
        sinPsi2 = (z / r)**2
        aOverR2 = 1.5 * self.J2 * (self.SemiMajor / r)**2
        gmOverR2 = -self.GM/(r**2)
```

```
        gx = gmOverR2 * (1 + aOverR2 * (1.0 - 5.0*sinPsi2)) * (x / r)
        gy = gmOverR2 * (1 + aOverR2 * (1.0 - 5.0*sinPsi2)) * (y / r)
        gz = gmOverR2 * (1 + aOverR2 * (3.0 - 5.0*sinPsi2)) * (z / r)

        return gx, gy, gz

    def GravityR2(self, x, y, z):
        r2 = x*x + y*y + z*z
        assert r2 != 0, "GravityR2 r2 is 0"
        return self.GM/r2

    def UnitTest(self):
        self.ClassName = "ppEarth"
        self.TestValue(6356752, self.SemiMinor, "b", 1)
        self.TestValue(8.18191908426e-2, self.Eccentricity, "eccentricity",␣
↪1e-12)

        # TODO: fix gravity unit tests
        #self.TestValue(9.7879, self.GravityJ2(0,0), "gravity", 1e-4)
        #self.TestValue(9.7848, self.GravityJ2(1000,math.radians(12.34)),␣
↪"gravity", 1e-4)
        #self.TestValue(9.7725, self.GravityJ2(5000,math.radians(24.6621)),␣
↪"gravity", 1e-4)
        #self.TestValue(9.72, self.GravityJ2(25000,math.radians(45.0)),␣
↪"gravity", 1e-2)
        #self.TestValue(9.56, self.GravityJ2(75000,math.radians(65.0)),␣
↪"gravity", 1e-2)

        self.ClassName = "ppEarth StdAtm1976 Density"
        di = 0  # air density index
        self.TestValue(1.225, self.StdAtm1976(0)[di], "0m", 1e-3)

        self.ClassName = "ppEarth StdAtm1976 Temperature"
        ti = 1  # temperature index
        self.TestValue(288.15, self.StdAtm1976(0)[ti], "0m", 1e-2)
        self.TestValue(275.156, self.StdAtm1976(2000)[ti], "2km", 1e-2)
        self.TestValue(255.676, self.StdAtm1976(5000)[ti], "5km", 1e-2)
        self.TestValue(216.65, self.StdAtm1976(12000)[ti], "12km", 1e-2)
        self.TestValue(222.544, self.StdAtm1976(26000)[ti], "26km", 1e-2)

        self.ClassName = "ppEarth StdAtm1976 Pressure"
        pi = 2  # pressure index
        self.TestValue(101325, self.StdAtm1976(0)[pi], "0m", 1)
        self.TestValue(79505.1, self.StdAtm1976(2000)[pi], "2km", 10)
        self.TestValue(54048.8, self.StdAtm1976(5000)[pi], "5km", 10)
        self.TestValue(19401, self.StdAtm1976(12000)[pi], "12km", 10)
```

```
        self.TestValue(2188.41, self.StdAtm1976(26000)[pi], "26km", 1)

        self.ClassName = "ppEarth StdAtm1976 Sound"
        si = 3  # speed of sound index
        self.TestValue(340.294, self.StdAtm1976(0)[si], "0m", 1e-3)

        self.ClassName = "ppPlanet Olsen"
        self.PcpfToLlaOsen(1191786.0, -5157122.0, 3562840.0)
        self.TestValue(34.123456, math.degrees(self.Latitude), "lat", 1e-6)
        self.TestValue(-76.987654, math.degrees(self.Longitude), "lon", 1e-6)
        self.TestValue(9000.0, self.Altitude, "alt", 1)

        self.ClassName = "ppPlanet Zhu"
        self.PcpfToLlaZhu(1191786.0, -5157122.0, 3562840.0)
        self.TestValue(34.123456, math.degrees(self.Latitude), "lat", 1e-6)
        self.TestValue(-76.987654, math.degrees(self.Longitude), "lon", 1e-6)
        self.TestValue(9000.0, self.Altitude, "alt", 1)

        self.ClassName = "ppPlanet"
        self.Latitude = math.radians(34.123456)
        self.Longitude = math.radians(-76.987654)
        self.Altitude = 9000.0
        [x, y, z] = self.LlaToPcpf()
        self.TestValue(1191786.0, x, "X", 1)
        self.TestValue(-5157122.0, y, "Y", 1)
        self.TestValue(3562840.0, z, "Z", 1)

        print("Number of ppEarth failed tests: ", self.FailCount)
```

1.5.6 Moon Class

The reference for the moon parameters is NESC Academy Presentation.

[14]:
```
class ppMoon(ppPlanet):

    def __init__(self):
        self.RotationRate = 2.6617072235e-6  # Moon Rotation Rate (rad/sec,␣
 ↪East)
        self.GM = 4.90154449e12
        self.SemiMajor = 1738140.0
        self.Flattening  = 1.0 / 800.98618
        self.CalcSemiMinor()
        self.CalcEccentricity()

    def AirDensity(self, altitude):
        return 0
```

```
    def Gravity(self, altitude, latRad):
        r = altitude + self.SemiMajor
        gravity = self.GM/r/r
        return gravity

    def UnitTest(self):
        self.ClassName = "ppMoon"
        self.TestValue(1.62242, self.Gravity(0,0), "gravity", 1e-6)

        print("Number of ppMoon failed tests: ", self.FailCount)
```

1.5.7 Mars Class

The reference for the Mars atmosphere model.

[15]:
```
class ppMars(ppPlanet):

    def __init__(self):
        self.GM = 42828.371901284e9
        self.RotationRate = 7.0882181e-5  # Mars Rotation Rate (rad/sec, East)
        self.SemiMajor = 3.396196e6
        self.J2 = 0.00195545367944545
        self.CalcSemiMinor()

    def AirDensity(self, altitude):
        temperatureC = 0
        if altitude > 7000:
            temperatureC = -31 - 0.000998 * altitude
        else:
            temperatureC = -23.4 - 0.00222 * altitude

        pressureKPa = 0.699 * math.exp( -0.00009 * altitude )
        airDensity  = pressurePa / (0.1921 * (temperatureC + 273.1))
        return airDensity

    def Gravity(self, altitude, latRad):
        marsGM = self.GM
        marsRadiusMeter = self.SemiMajor
        J2 = self.J2
        J3 = 3.14498094262035e-5
        J4 = -1.53773961526397e-5
        cosPhi = math.cos( 0.5*math.pi - latRad )

        r  = altitude + marsRadiusMeter
        rr = marsRadiusMeter / r
```

```
        gravity = marsGM*(1.0 - 1.5 * J2 * ( 3.0 * cosPhi*cosPhi - 1.0 ) *␣
↪rr*rr - 2.0 * J3 * cosPhi
            * ( 5.0 * cosPhi*cosPhi - 3.0 ) * rr*rr*rr - (5.0/8.0) * J4 * ( 35.
↪0 * cosPhi**4
            - 30.0 * cosPhi*cosPhi + 3.0 ) * rr**4.0 ) / (r*r);

        return gravity

    def UnitTest(self):
        self.ClassName = "ppMars"
        self.TestValue(3.724179, self.Gravity(0,0), "gravity", 1e-6)

        print("Number of ppMars failed tests: ", self.FailCount)
```

[16]:

```
earth = ppEarth()
earth.UnitTest()

moon = ppMoon()
moon.UnitTest()

mars = ppMars()
mars.UnitTest()


def ToGeopotential(altitude):
    radiusEarth = 6356766.0
    z0 = radiusEarth * altitude / (radiusEarth + altitude)
    return round(z0)

alt = [0, 2000, 5000, 12000, 26000, 37500, 50000, 60000, 75000]
print("\nGeopotential table")
for a in alt:
    geoTable = "{:=6d} -> {:=6d}".format(a, ToGeopotential(a))
    print(geoTable)

print("=== gravity ===")
print("-- J2 --")
myGx, myGy, myGz = earth.GravityJ2( earth.SemiMajor + 9144, 0, 0)
print(myGx, myGy, myGz)

print("-- J2 Steven & Lewis --")
myGx, myGy, myGz = earth.GravityJ2SL( earth.SemiMajor + 9144, 0, 0)
print(myGx, myGy, myGz)

print("-- WGS84 --")
wgx, wgy, wgz = earth.GravityWgs84(0.0, 0.0, 9144.0) # 30,000 ft
print(wgx, wgy, wgz)
```

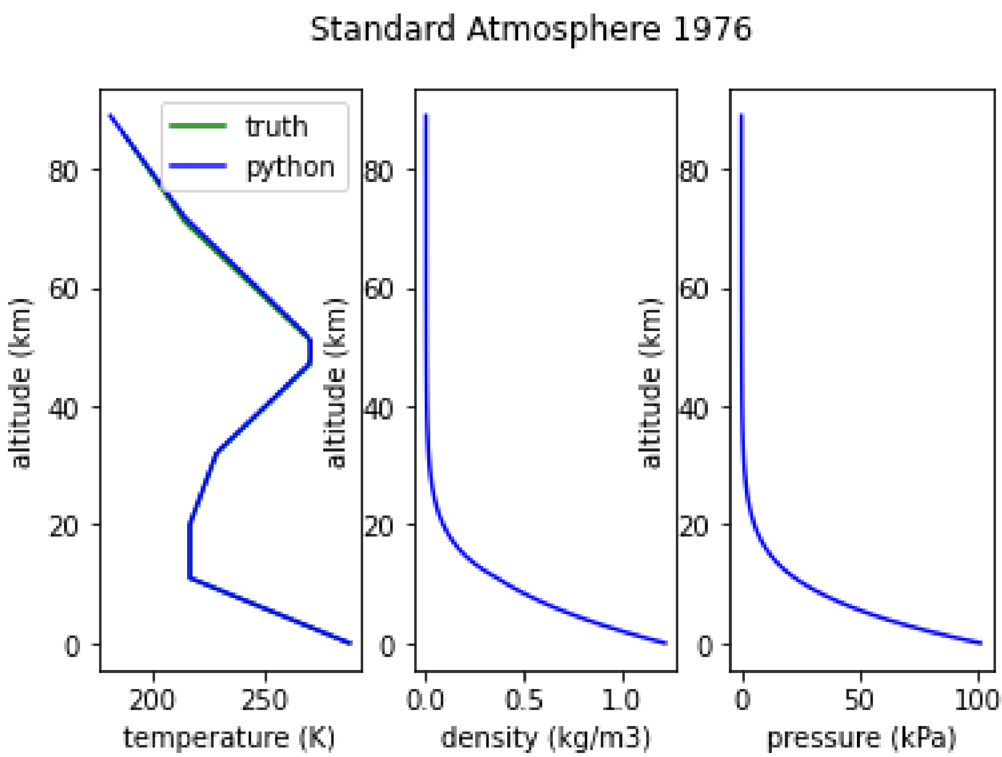
```
Number of ppEarth failed tests:  0
Number of ppMoon failed tests:  0
Number of ppMars failed tests:  0

Geopotential table
     0 ->      0
  2000 ->   1999
  5000 ->   4996
 12000 ->  11977
 26000 ->  25894
 37500 ->  37280
 50000 ->  49610
 60000 ->  59439
 75000 ->  74125
=== gravity ===
-- J2 --
-9.786072112297859 -0.0 -0.0
-- J2 Steven & Lewis --
-9.786072112297857 -0.0 -0.0
-- WGS84 --
-9.786116284019755 -0.0 -0.0
```

[17]:

```python
import matplotlib.pyplot as plt

h = []
t76 = []
rho = []
p = []
for i in range(90):
    alt = i * 1000.0
    h.append(i)
    [airDensity, temperature, pressure, speedOfSoundMps] = earth.StdAtm1976(alt)
    t76.append(temperature)
    rho.append(airDensity)
    p.append(0.001*pressure)

altT = (0, 11, 20, 32, 47, 51, 71, 86)
tmpT = (288.15, 216.65, 216.65, 228.65, 270.65, 270.65, 214.65, 186.9)

fig, (tp1, dp2, pp3) = plt.subplots(1,3)
fig.suptitle('Standard Atmosphere 1976')

tp1.plot(tmpT, altT, 'g', t76, h, 'b')
tp1.set(xlabel='temperature (K)', ylabel='altitude (km)')
tp1.legend(["truth","python"])
#tp1.grid()
```

```
dp2.plot(rho, h, 'b')
dp2.set(xlabel='density (kg/m3)', ylabel='altitude (km)')

pp3.plot(p, h, 'b')
pp3.set(xlabel='pressure (kPa)', ylabel='altitude (km)')
```

[17]: [Text(0.5, 0, 'pressure (kPa)'), Text(0, 0.5, 'altitude (km)')]



[18]:
```
from IPython.display import Image
Image(url= "images/StdAtm1976.png")
```

[18]: <IPython.core.display.Image object>

1.6 Vector Class

Make a vector class for 3 element vectors.

[19]:
```
class ppVector3(ppUnitTest):

    def __init__(self, x, y, z):
        self.X = x
        self.Y = y
        self.Z = z
```

```
    # defining how to print the class
    def __str__(self):
        return "(%s, %s, %s)" % (self.X, self.Y, self.Z)

    # overloading the + to add vectors
    def __add__(self, o):
        x = self.X + o.X
        y = self.Y + o.Y
        z = self.Z + o.Z
        return ppVector3(x,y,z)

    # overloading the - to subtract vectors
    def __sub__(self, o):
        x = self.X - o.X
        y = self.Y - o.Y
        z = self.Z - o.Z
        return ppVector3(x,y,z)

    # overloading the ^ for cross product
    def __xor__(self, o):
        x = self.Y * o.Z - self.Z * o.Y
        y = self.Z * o.X - self.X * o.Z
        z = self.X * o.Y - self.Y * o.X
        return ppVector3(x,y,z)

    # overloading the * to multiply scalars to a vector
    def __mul__(self, s):
        x = self.X * s
        y = self.Y * s
        z = self.Z * s
        return ppVector3(x,y,z)

    # overloading the / to divide a vector by a scalar
    def __truediv__(self, s):
        x = self.X / s
        y = self.Y / s
        z = self.Z / s
        return ppVector3(x,y,z)

    __rmul__ = __mul__

    def Set(self, x, y, z):
        self.X = x
        self.Y = y
        self.Z = z
```

```python
    def Magnitude(self):
        magnitude = math.sqrt(self.X*self.X + self.Y*self.Y + self.Z*self.Z)
        return magnitude

    def UnitTest(self):
        self.ClassName = "ppVector3"
        v1 = ppVector3(21,33,19)
        self.TestValue( 21, v1.X, "init X", 1e-6)
        self.TestValue( 33, v1.Y, "init Y", 1e-6)
        self.TestValue( 19, v1.Z, "init Z", 1e-6)
        v2 = ppVector3(21,33,19)
        v3 = ppVector3(79,67,81)
        v1 = v2 + v3
        self.TestValue( 100, v1.X, "add X", 1e-6)
        self.TestValue( 100, v1.Y, "add Y", 1e-6)
        self.TestValue( 100, v1.Z, "add Z", 1e-6)
        v4 = ppVector3(0,3,-4)
        self.TestValue( 5, v4.Magnitude(), "magnitude", 1e-6)
        v4.Set( 87, 16.9, -3.1 )
        self.TestValue(   87, v4.X, "Set X", 1e-6)
        self.TestValue( 16.9, v4.Y, "Set Y", 1e-6)
        self.TestValue( -3.1, v4.Z, "Set Z", 1e-6)
        v5 = v3 - v2
        self.TestValue( 58, v5.X, "sub X", 1e-6)
        self.TestValue( 34, v5.Y, "sub Y", 1e-6)
        self.TestValue( 62, v5.Z, "sub Z", 1e-6)
        v5 = v2 - v3
        self.TestValue( -58, v5.X, "sub X", 1e-6)
        self.TestValue( -34, v5.Y, "sub Y", 1e-6)
        self.TestValue( -62, v5.Z, "sub Z", 1e-6)
        v6 = ppVector3(4,12,2)
        v7 = ppVector3(13,5,7)
        v8 = v6 ^ v7
        self.TestValue(   74, v8.X, "cross X", 1e-6)
        self.TestValue(   -2, v8.Y, "cross Y", 1e-6)
        self.TestValue( -136, v8.Z, "cross Z", 1e-6)
        v9 = 2 * v6
        self.TestValue(   8, v9.X, "mul X", 1e-6)
        self.TestValue(  24, v9.Y, "mul Y", 1e-6)
        self.TestValue(   4, v9.Z, "mul Z", 1e-6)
        v10 = v7 / 2
        self.TestValue(  6.5, v10.X, "div X", 1e-6)
        self.TestValue(  2.5, v10.Y, "div Y", 1e-6)
        self.TestValue(  3.5, v10.Z, "div Z", 1e-6)
        v11 = v10
        self.TestValue(  6.5, v11.X, "= X", 1e-6)
        self.TestValue(  2.5, v11.Y, "= Y", 1e-6)
```

```
        self.TestValue(  3.5, v11.Z, "= Z", 1e-6)

        print("Number of ppVector3 failed tests: ", self.FailCount)
```

[20]:
```
myVector = ppVector3(0,0,0)
myVector.UnitTest()
```

```
Number of ppVector3 failed tests:  0
```

1.7 Quaternion Class

Create a quaternion class. Reference for checking quaternion rotation. Quaternion multiplication checked here.

$$
\begin{aligned}
t &= q * r = t_0 + \mathbf{i}t_1 + \mathbf{j}t_2 + \mathbf{k}t_3 \\
t_0 &= q_0 r_0 - q_1 r_1 - q_2 r_2 - q_3 r_3 \\
t_1 &= q_1 r_0 + q_0 r_1 - q_3 r_2 + q_2 r_3 \\
t_2 &= q_2 r_0 + q_3 r_1 + q_0 r_2 - q_1 r_3 \\
t_3 &= q_3 r_0 - q_2 r_1 + q_1 r_2 + q_0 r_3
\end{aligned}
$$

System b to a -> $q_{b/a}$

$$
\begin{aligned}
u^b &= q_{b/a}^{-1} * u^a * q_{b/a}, \text{ and } q_{b/a}^{-1} = q_{a/b} \\
u^c &= q_{c/b}^{-1} q_{b/a}^{-1}\, u^a\, q_{b/a} q_{c/b}
\end{aligned}
$$

$$v^{frd} = q_\phi^{-1} q_\theta^{-1} q_\psi^{-1}\, v^{ned}\, q_\psi q_\theta q_\phi$$

$$
q_{frd/ned} = q_\psi q_\theta q_\phi = \begin{matrix}
\cos\frac{\phi}{2}\cos\frac{\theta}{2}\cos\frac{\psi}{2} + \sin\frac{\phi}{2}\sin\frac{\theta}{2}\sin\frac{\psi}{2} \\
\sin\frac{\phi}{2}\cos\frac{\theta}{2}\cos\frac{\psi}{2} - \cos\frac{\phi}{2}\sin\frac{\theta}{2}\sin\frac{\psi}{2} \\
\cos\frac{\phi}{2}\sin\frac{\theta}{2}\cos\frac{\psi}{2} + \sin\frac{\phi}{2}\cos\frac{\theta}{2}\sin\frac{\psi}{2} \\
\cos\frac{\phi}{2}\cos\frac{\theta}{2}\sin\frac{\psi}{2} - \sin\frac{\phi}{2}\sin\frac{\theta}{2}\cos\frac{\psi}{2}
\end{matrix}
$$

$$
q_{ned/ecf} = \begin{matrix}
\cos\frac{lon}{2}\cos(\frac{lat}{2} + 45°) \\
\sin\frac{lon}{2}\sin(\frac{lat}{2} + 45°) \\
-\cos\frac{lon}{2}\sin(\frac{lat}{2} + 45°) \\
\sin\frac{lon}{2}\cos(\frac{lat}{2} + 45°)
\end{matrix}
$$

$$\omega^{frd} = q_{frd/ned}^{-1} q_{ned/ecf}^{-1}\, \omega^{ecf}\, q_{ned/ecf} q_{frd/ned}$$

$$
\begin{aligned}
F^{ecf} &= q_{ecf/ned}^{-1} q_{ned/frd}^{-1}\, F^{frd}\, q_{ned/frd} q_{ecf/ned} \\
&= q_{ned/ecf} q_{frd/ned}\, F^{frd}\, q_{frd/ned}^{-1} q_{ned/ecf}^{-1}
\end{aligned}
$$

$$
\begin{aligned}
v^{ecf} &= q_{ecf/ned}^{-1} q_{ned/frd}^{-1}\, v^{frd}\, q_{ned/frd} q_{ecf/ned} \\
&= q_{ned/ecf} q_{frd/ned}\, v^{frd}\, q_{frd/ned}^{-1} q_{ned/ecf}^{-1}
\end{aligned}
$$

$$
\begin{aligned}
u^{ned} &= q_{ned/frd}^{-1}\, u^{frd}\, q_{ned/frd} \\
q_{ned/frd}^{-1} &= q_{frd/ned} \\
u^{ned} &= q_{frd/ned} \begin{bmatrix} 0 \\ u \end{bmatrix} q_{frd/ned}^{-1}
\end{aligned}
$$

[21]:
```python
class ppQuaternion(ppUnitTest):

    def __init__(self, n, x, y, z):
        self.N = n
        self.X = x
        self.Y = y
        self.Z = z

    # defining how to print the class
    def __repr__(self):
        return "(%s, %s, %s, %s)" % (self.N, self.X, self.Y, self.Z)

    # overloading the ~ for quaternion inverse
    def __invert__(self):
        n = self.N
        x = -self.X
        y = -self.Y
        z = -self.Z
        return ppQuaternion(n,x,y,z)

    # overloading the + to add quaternions
    def __add__(self, o):
        n = self.N + o.N
        x = self.X + o.X
        y = self.Y + o.Y
        z = self.Z + o.Z
        return ppQuaternion(n,x,y,z)

    # overlaoding the * to multiply quaternions and multiple scalars and quaternions
    def __mul__(self,o):
        n=0
        x=0
        y=0
        z=0
        if isinstance(o, ppQuaternion):
            n = self.N*o.N - self.X*o.X - self.Y*o.Y - self.Z*o.Z
            x = self.N*o.X + self.X*o.N + self.Y*o.Z - self.Z*o.Y
            y = self.N*o.Y + self.Y*o.N + self.Z*o.X - self.X*o.Z
            z = self.N*o.Z + self.Z*o.N + self.X*o.Y - self.Y*o.X
        elif isinstance(o, ppVector3):
            n = -(self.X*o.X + self.Y*o.Y + self.Z*o.Z)
            x =   self.N*o.X + self.Y*o.Z - self.Z*o.Y
            y =   self.N*o.Y + self.Z*o.X - self.X*o.Z
            z =   self.N*o.Z + self.X*o.Y - self.Y*o.X
        else:
            n = self.N * o
```

```
            x = self.X * o
            y = self.Y * o
            z = self.Z * o
        return ppQuaternion(n,x,y,z)

    # so that scalar * quaternion is the same as quaternion * scalar
    __rmul__ = __mul__

    def Normalize(self):
        magnitude = math.sqrt(self.N*self.N + self.X*self.X + self.Y*self.Y + self.Z*self.Z)

        if magnitude != 0:
            self.N = self.N / magnitude
            self.X = self.X / magnitude
            self.Y = self.Y / magnitude
            self.Z = self.Z / magnitude

    def SetRollPitchYaw(self, roll, pitch, yaw):
        qroll  = ppQuaternion( math.cos(0.5*roll) , math.sin(0.5*roll), 0.0             , 0.0)
        qpitch = ppQuaternion( math.cos(0.5*pitch), 0.0               , math.sin(0.5*pitch), 0.0)
        qyaw   = ppQuaternion( math.cos(0.5*yaw)  , 0.0               , 0.0             , math.sin(0.5*yaw))

        # ZYX rotation
        q = qyaw*qpitch*qroll
        q.Normalize()

        self.N = q.N
        self.X = q.X
        self.Y = q.Y
        self.Z = q.Z

    def SetLatLon(self, lat, lon):
        n =  math.cos(0.5*lon)*math.cos(0.5*lat + 0.25*math.pi)
        x =  math.sin(0.5*lon)*math.sin(0.5*lat + 0.25*math.pi)
        y = -math.cos(0.5*lon)*math.sin(0.5*lat + 0.25*math.pi)
        z =  math.sin(0.5*lon)*math.cos(0.5*lat + 0.25*math.pi)

        q = ppQuaternion( n, x, y, z )
        q.Normalize()

        self.N = q.N
        self.X = q.X
        self.Y = q.Y
```

```
        self.Z = q.Z

    def SetQfrdWrtEcf(self, roll, pitch, yaw, lat, lon):
        qroll  = ppQuaternion( math.cos(0.5*roll) , math.sin(0.5*roll), 0.0              , 0.0)
        qpitch = ppQuaternion( math.cos(0.5*pitch), 0.0               , math.sin(0.5*pitch), 0.0)
        qyaw   = ppQuaternion( math.cos(0.5*yaw)  , 0.0               , 0.0              , math.sin(0.5*yaw))

        hLon = 0.5*lon
        hLat = 0.5*lat + 0.25*math.pi
        qlon = ppQuaternion(math.cos(hLon), 0, 0, math.sin(hLon))
        qlat = ppQuaternion(math.cos(hLat), 0, -math.sin(hLat), 0)

        # ZYX rotation
        q = qlon*qlat*qyaw*qpitch*qroll

        self.N = q.N
        self.X = q.X
        self.Y = q.Y
        self.Z = q.Z

    def SetPlanetRotation(self, rotationAngle_rad):
        n = math.cos(0.5*rotationAngle_rad)
        z = math.sin(0.5*rotationAngle_rad)

        q = ppQuaternion(n, 0.0, 0.0, z)
        q.Normalize()

        self.N = q.N
        self.X = q.X
        self.Y = q.Y
        self.Z = q.Z

    def EulerAnglesFromQ(self):
        q0 = self.N
        q1 = self.X
        q2 = self.Y
        q3 = self.Z

        c11 = q0*q0 + q1*q1 - q2*q2 - q3*q3
        c12 = 2.0*(q1*q2 + q0*q3)
        c13 = 2.0*(q1*q3 - q0*q2)
        c23 = 2.0*(q2*q3 + q0*q1)
        c33 = q0*q0 - q1*q1 - q2*q2 + q3*q3
```

```
        roll  =  math.atan2(c23,c33)
        pitch = -math.asin(c13)
        yaw   =  math.atan2(c12,c11)

        return [roll, pitch, yaw]

    def EulerAnglesFromQold(self):
        qnn = self.N * self.N
        qxx = self.X * self.X
        qyy = self.Y * self.Y
        qzz = self.Z * self.Z

        img = qxx + qyy + qzz + qnn
        assert img != 0, "EulerAnglesFromQ all elements 0 for quaternion"
        img = 1.0 / img

        m11 = (qnn + qxx - qyy - qzz)*img
        m12 = 2.0*(self.X*self.Y + self.Z*self.N)*img
        m13 = 2.0*(self.X*self.Z - self.Y*self.N)*img
        m23 = 2.0*(self.Y*self.Z + self.X*self.N)*img
        m33 = (qnn - qxx - qyy + qzz)*img

        roll = 0
        pitch = 0
        yaw = 0
        if abs(m13) >= 1.0:
            m21 = 2.0*(self.X*self.Y - self.Z*self.N)*img
            m31 = 2.0*(self.X*self.Z + self.Y*self.N)*img;
            roll  = 0.0
            halfPi = 0.5*math.pi
            pitch = -halfPi if (m13 > 0.0) else halfPi
            yaw   = math.atan2(-m21, -m31/m13)
        else:
            roll  = math.atan2(m23,m33)  # Roll
            pitch = math.asin(-m13)      # Pitch
            yaw   = math.atan2(m12,m11)  # Yaw

        return [roll, pitch, yaw]

    def UnitTest(self):
        self.ClassName = "ppQuaternion"
        q0 = ppQuaternion(4,7,8,9)
        q0i = ~q0
        self.TestValue( 4, q0i.N, "inverse", 1e-6)
        self.TestValue(-7, q0i.X, "inverse", 1e-6)
        self.TestValue(-8, q0i.Y, "inverse", 1e-6)
        self.TestValue(-9, q0i.Z, "inverse", 1e-6)
```

```
        q1 = ppQuaternion(2,3,4,5)
        q2 = ppQuaternion(8,9,10,11)
        q3 = q1 + q2
        self.TestValue(10, q3.N, "add", 1e-6)
        self.TestValue(12, q3.X, "add", 1e-6)
        self.TestValue(14, q3.Y, "add", 1e-6)
        self.TestValue(16, q3.Z, "add", 1e-6)
        q4 = q1 * q2
        self.TestValue(-106, q4.N, "multiply", 1e-6)
        self.TestValue(36, q4.X, "multiply", 1e-6)
        self.TestValue(64, q4.Y, "multiply", 1e-6)
        self.TestValue(56, q4.Z, "multiply", 1e-6)
        q5 = 7.0 * q1
        self.TestValue(14, q5.N, "scalar multiply", 1e-6)
        self.TestValue(21, q5.X, "scalar multiply", 1e-6)
        self.TestValue(28, q5.Y, "scalar multiply", 1e-6)
        self.TestValue(35, q5.Z, "scalar multiply", 1e-6)
        q6 = q2 * 10
        self.TestValue(80, q6.N, "scalar multiply", 1e-6)
        self.TestValue(90, q6.X, "scalar multiply", 1e-6)
        self.TestValue(100, q6.Y, "scalar multiply", 1e-6)
        self.TestValue(110, q6.Z, "scalar multiply", 1e-6)
        q6.SetRollPitchYaw(0.3,-0.7,3.11)
        self.TestValue(-0.0365642, q6.N, "Euler", 1e-6)
        self.TestValue(0.3412225, q6.X, "Euler", 1e-6)
        self.TestValue(0.1350051, q6.Y, "Euler", 1e-6)
        self.TestValue(0.9295181, q6.Z, "Euler", 1e-6)
        [roll, pitch, yaw] = q6.EulerAnglesFromQ()
        self.TestValue( 0.3, roll, "EulerFromQ", 1e-6)
        self.TestValue(-0.7, pitch, "EulerFromQ", 1e-6)
        self.TestValue(3.11, yaw, "EulerFromQ", 1e-6)
        q7 = ppQuaternion(0.6680766, 0.2325211, 0.1160514, 0.6972372)
        [roll, pitch, yaw] = q7.EulerAnglesFromQ()
        self.TestValue( 0.5, roll, "EulerFromQ", 1e-6)
        self.TestValue(-0.17, pitch, "EulerFromQ", 1e-6)
        self.TestValue(1.57, yaw, "EulerFromQ", 1e-6)
        q8 = ppQuaternion(6,-6,6,6)
        q8.Normalize()
        self.TestValue( 0.5, q8.N, "Normalize", 1e-6)
        self.TestValue(-0.5, q8.X, "Normalize", 1e-6)
        self.TestValue( 0.5, q8.Y, "Normalize", 1e-6)
        self.TestValue( 0.5, q8.Z, "Normalize", 1e-6)
        q9 = ppQuaternion(1,3,-2,7)
        q9.Normalize()
        mag = math.sqrt(1 + 9 + 4 + 49)
        self.TestValue( 1.0/mag, q9.N, "Normalize 2", 1e-6)
        self.TestValue( 3.0/mag, q9.X, "Normalize 2", 1e-6)
```

```
        self.TestValue(-2.0/mag, q9.Y, "Normalize 2", 1e-6)
        self.TestValue( 7.0/mag, q9.Z, "Normalize 2", 1e-6)

        print("Number of ppQuaternion failed tests: ", self.FailCount)
```

[22]:
```
q1 = ppQuaternion(9,4,5,6)
q1.UnitTest()
```

```
Number of ppQuaternion failed tests:  0
```

1.8 Matrix Class

Create a 3x3 matrix class.

[23]:
```
class ppMatrix3x3(ppUnitTest):
    A11 = 1
    A12 = 0
    A13 = 0

    A21 = 0
    A22 = 1
    A23 = 0

    A31 = 0
    A32 = 0
    A33 = 1

    def __mul__(self, v):
        if isinstance(v, ppVector3):
            x = self.A11 * v.X + self.A12 * v.Y + self.A13 * v.Z
            y = self.A21 * v.X + self.A22 * v.Y + self.A23 * v.Z
            z = self.A31 * v.X + self.A32 * v.Y + self.A33 * v.Z
            return ppVector3(x,y,z)
        elif isinstance(v, ppMatrix3x3):
            a11 = self.A11*v.A11 + self.A12*v.A21 + self.A13*v.A31
            a12 = self.A11*v.A12 + self.A12*v.A22 + self.A13*v.A32
            a13 = self.A11*v.A13 + self.A12*v.A23 + self.A13*v.A33

            a21 = self.A21*v.A11 + self.A22*v.A21 + self.A23*v.A31
            a22 = self.A21*v.A12 + self.A22*v.A22 + self.A23*v.A32
            a23 = self.A21*v.A13 + self.A22*v.A23 + self.A23*v.A33

            a31 = self.A31*v.A11 + self.A32*v.A21 + self.A33*v.A31
            a32 = self.A31*v.A12 + self.A32*v.A22 + self.A33*v.A32
            a33 = self.A31*v.A13 + self.A32*v.A23 + self.A33*v.A33

            a = ppMatrix3x3()
            a.SetRow1( a11, a12, a13 )
```

```
            a.SetRow2( a21, a22, a23 )
            a.SetRow3( a31, a32, a33 )
            return a

        else:
            a11 = self.A11 * v
            a12 = self.A12 * v
            a13 = self.A13 * v
            a21 = self.A21 * v
            a22 = self.A22 * v
            a23 = self.A23 * v
            a31 = self.A31 * v
            a32 = self.A32 * v
            a33 = self.A33 * v
            a = ppMatrix3x3()
            a.SetRow1( a11, a12, a13 )
            a.SetRow2( a21, a22, a23 )
            a.SetRow3( a31, a32, a33 )
            return a


    def SetRow1(self, a11, a12, a13):
        self.A11 = a11
        self.A12 = a12
        self.A13 = a13

    def SetRow2(self, a21, a22, a23):
        self.A21 = a21
        self.A22 = a22
        self.A23 = a23

    def SetRow3(self, a31, a32, a33):
        self.A31 = a31
        self.A32 = a32
        self.A33 = a33

    # defining how to print the class
    def __str__(self):
        row1 = "(%s, %s, %s)\n" % (self.A11, self.A12, self.A13)
        row2 = "(%s, %s, %s)\n" % (self.A21, self.A22, self.A23)
        row3 = "(%s, %s, %s)" % (self.A31, self.A32, self.A33)
        return row1+row2+row3

    def Determinant(self):
        d1 = self.A11*(self.A22*self.A33 - self.A23*self.A32)
        d2 = self.A12*(self.A23*self.A31 - self.A21*self.A33)
        d3 = self.A13*(self.A21*self.A32 - self.A22*self.A31)
```

```
        return d1+d2+d3

    def Inverse(self):
        D = self.Determinant()

        im = ppMatrix3x3()

        # make sure D is not 0
        if abs(D) > 1e-12:
            a11 = (self.A22*self.A33 - self.A23*self.A32)/D
            a12 = (self.A13*self.A32 - self.A12*self.A33)/D
            a13 = (self.A12*self.A23 - self.A13*self.A22)/D

            a21 = (self.A23*self.A31 - self.A21*self.A33)/D
            a22 = (self.A11*self.A33 - self.A13*self.A31)/D
            a23 = (self.A13*self.A21 - self.A11*self.A23)/D

            a31 = (self.A21*self.A32 - self.A22*self.A31)/D
            a32 = (self.A12*self.A31 - self.A11*self.A32)/D
            a33 = (self.A11*self.A22 - self.A12*self.A21)/D

            im.SetRow1(a11, a12, a13)
            im.SetRow2(a21, a22, a23)
            im.SetRow3(a31, a32, a33)

        return im

    def Transpose(self):
        at = ppMatrix3x3()
        at.SetRow1(self.A11, self.A21, self.A31)
        at.SetRow2(self.A12, self.A22, self.A32)
        at.SetRow3(self.A13, self.A23, self.A33)
        return at

    def QuaternionToMatrix(self, q):
        n = q.N;
        x = q.X;
        y = q.Y;
        z = q.Z;

        self.SetRow1( n*n + x*x - y*y - z*z,       2.0*(x*y - n*z),       2.0*(x*z + n*y) )
        self.SetRow2(       2.0*(x*y + n*z), n*n - x*x + y*y - z*z,       2.0*(y*z - n*x) )
        self.SetRow3(       2.0*(x*z - n*y),       2.0*(y*z + n*x), n*n - x*x - y*y + z*z )
```

```python
    def UnitTest(self):
        self.ClassName = "ppMatrix3"
        m1 = ppMatrix3x3()
        m1.SetRow1(1,2,3)
        m1.SetRow2(4,5,6)
        m1.SetRow3(7,3,9)
        self.TestValue(-30, m1.Determinant(), "Determinant", 1e-6)
        m1.SetRow1(1,2,3)
        m1.SetRow2(0,1,4)
        m1.SetRow3(5,6,0)
        self.TestValue(1, m1.Determinant(), "Determinant", 1e-6)
        m1 = m1.Inverse()
        self.TestValue(-24, m1.A11, "Inverse A11", 1e-6)
        self.TestValue( 18, m1.A12, "Inverse A12", 1e-6)
        self.TestValue(  5, m1.A13, "Inverse A13", 1e-6)
        self.TestValue( 20, m1.A21, "Inverse A21", 1e-6)
        self.TestValue(-15, m1.A22, "Inverse A22", 1e-6)
        self.TestValue( -4, m1.A23, "Inverse A23", 1e-6)
        self.TestValue( -5, m1.A31, "Inverse A31", 1e-6)
        self.TestValue(  4, m1.A32, "Inverse A32", 1e-6)
        self.TestValue(  1, m1.A33, "Inverse A33", 1e-6)

        m2 = ppMatrix3x3()
        m2.SetRow1(1,2,3)
        m2.SetRow2(4,5,6)
        m2.SetRow3(7,2,9)
        m2 = m2.Inverse()
        self.TestValue(-11/12, m2.A11, "Inverse A11", 1e-6)
        self.TestValue(   1/3, m2.A12, "Inverse A12", 1e-6)
        self.TestValue(  1/12, m2.A13, "Inverse A13", 1e-6)
        self.TestValue(  -1/6, m2.A21, "Inverse A21", 1e-6)
        self.TestValue(   1/3, m2.A22, "Inverse A22", 1e-6)
        self.TestValue(  -1/6, m2.A23, "Inverse A23", 1e-6)
        self.TestValue(   3/4, m2.A31, "Inverse A31", 1e-6)
        self.TestValue(  -1/3, m2.A32, "Inverse A32", 1e-6)
        self.TestValue(  1/12, m2.A33, "Inverse A33", 1e-6)

        m3 = ppMatrix3x3()
        m3.SetRow1( 1,2,3)
        m3.SetRow2(-4,5,6)
        m3.SetRow3( 7,8.1,9)
        m4 = m3.Transpose()
        self.TestValue(   1, m3.A11, "Transpose A11", 1e-6)
        self.TestValue(   2, m3.A12, "Transpose A12", 1e-6)
        self.TestValue(   3, m3.A13, "Transpose A13", 1e-6)
        self.TestValue(  -4, m3.A21, "Transpose A21", 1e-6)
        self.TestValue(   5, m3.A22, "Transpose A22", 1e-6)
```

```
        self.TestValue(   6, m3.A23, "Transpose A23", 1e-6)
        self.TestValue(   7, m3.A31, "Transpose A31", 1e-6)
        self.TestValue( 8.1, m3.A32, "Transpose A32", 1e-6)
        self.TestValue(   9, m3.A33, "Transpose A33", 1e-6)
        self.TestValue(   1, m4.A11, "Transpose A11", 1e-6)
        self.TestValue(  -4, m4.A12, "Transpose A12", 1e-6)
        self.TestValue(   7, m4.A13, "Transpose A13", 1e-6)
        self.TestValue(   2, m4.A21, "Transpose A21", 1e-6)
        self.TestValue(   5, m4.A22, "Transpose A22", 1e-6)
        self.TestValue( 8.1, m4.A23, "Transpose A23", 1e-6)
        self.TestValue(   3, m4.A31, "Transpose A31", 1e-6)
        self.TestValue(   6, m4.A32, "Transpose A32", 1e-6)
        self.TestValue(   9, m4.A33, "Transpose A33", 1e-6)

        q = ppQuaternion(0.7, 0.4, 3.2, -0.87)
        q.Normalize()
        m4.QuaternionToMatrix(q)
        self.TestValue( -0.8883823, m4.A11, "Quaternion A11", 1e-7)
        self.TestValue(  0.3243782, m4.A12, "Quaternion A12", 1e-7)
        self.TestValue(  0.3248933, m4.A13, "Quaternion A13", 1e-7)
        self.TestValue(  0.1152238, m4.A21, "Quaternion A21", 1e-7)
        self.TestValue(  0.8425504, m4.A22, "Quaternion A22", 1e-7)
        self.TestValue( -0.5261486, m4.A23, "Quaternion A23", 1e-7)
        self.TestValue( -0.4444101, m4.A31, "Quaternion A31", 1e-7)
        self.TestValue( -0.4299857, m4.A32, "Quaternion A32", 1e-7)
        self.TestValue( -0.7858829, m4.A33, "Quaternion A33", 1e-7)

        m1.SetRow1(2,6,3)
        m1.SetRow2(1,1,8)
        m1.SetRow3(5,7,-6)

        v1 = ppVector3(9,11,-4)
        v2 = m1 * v1
        self.TestValue(  72, v2.X, "Matrix * Vector X", 1e-7)
        self.TestValue( -12, v2.Y, "Matrix * Vector Y", 1e-7)
        self.TestValue( 146, v2.Z, "Matrix * Vector Z", 1e-7)

        m1.SetRow1(6,3,17)
        m1.SetRow2(-4,-0.1,7)
        m1.SetRow3(14,5,-1)
        m2.SetRow1(5,0,-6)
        m2.SetRow2(3,8,2)
        m2.SetRow3(-1,-4,-9)
        m3 = m1 * m2
        self.TestValue(    22, m3.A11, "Matrix * Matrix A11", 1e-7)
        self.TestValue(   -44, m3.A12, "Matrix * Matrix A12", 1e-7)
        self.TestValue(  -183, m3.A13, "Matrix * Matrix A13", 1e-7)
```

```
        self.TestValue( -27.3, m3.A21, "Matrix * Matrix A21", 1e-7)
        self.TestValue( -28.8, m3.A22, "Matrix * Matrix A22", 1e-7)
        self.TestValue( -39.2, m3.A23, "Matrix * Matrix A23", 1e-7)
        self.TestValue(    86, m3.A31, "Matrix * Matrix A31", 1e-7)
        self.TestValue(    44, m3.A32, "Matrix * Matrix A32", 1e-7)
        self.TestValue(   -65, m3.A33, "Matrix * Matrix A33", 1e-7)

        m5 = m2 * 2
        self.TestValue(    10, m5.A11, "Matrix * Scalar A11", 1e-7)
        self.TestValue(     0, m5.A12, "Matrix * Scalar A12", 1e-7)
        self.TestValue(   -12, m5.A13, "Matrix * Scalar A13", 1e-7)
        self.TestValue(     6, m5.A21, "Matrix * Scalar A21", 1e-7)
        self.TestValue(    16, m5.A22, "Matrix * Scalar A22", 1e-7)
        self.TestValue(     4, m5.A23, "Matrix * Scalar A23", 1e-7)
        self.TestValue(    -2, m5.A31, "Matrix * Scalar A31", 1e-7)
        self.TestValue(    -8, m5.A32, "Matrix * Scalar A32", 1e-7)
        self.TestValue(   -18, m5.A33, "Matrix * Scalar A33", 1e-7)

        print("Number of ppMatrix3x3 failed tests: ", self.FailCount)
```

[24]:
```
myMatrix = ppMatrix3x3()
print(myMatrix)
myMatrix.UnitTest()
```

```
(1, 0, 0)
(0, 1, 0)
(0, 0, 1)
Number of ppMatrix3x3 failed tests:  0
```

1.9 Equations of Motion

A base class for the equations of motion.

[25]:
```
class ppSimulation(ppConvert):
    Time = 0.0
    TimeStep = 0.1
    Data = {}
    IC = {}

    AeroModelInput = []

    ReferenceWingSpan = 0
    ReferenceWingChord = 0
    ReferenceWingArea = 0

    Position = ppVector3(0, 0, 0)

    TotalMass = 0
```

```
    GrossWeight = 0
    TrueAirspeed = 0
    BodyVelocity = ppVector3(0, 0, 0)
    BodyAccel = ppVector3(0, 0, 0)
    BodyForce = ppVector3(0, 0, 0)
    BodyAngle = ppVector3(0, 0, 0)
    BodyAngularRate = ppVector3(0, 0, 0)
    BodyAngularAccel = ppVector3(0, 0, 0)

    gvJx = 0
    gvJy = 0
    gvJz = 0
    gvJxz = 0
    Gamma = 0
    InertiaMatrix = ppMatrix3x3()
    InertiaMatrixInverse = ppMatrix3x3()

    # moment components
    Ml = 0
    Mm = 0
    Mn = 0

    totalCoefficientOfDrag = 0

    # define outputs
    EnglishLabels = ['gePosition_ft_X', 'gePosition_ft_Y', 'gePosition_ft_Z',
                     'feVelocity_ft_s_X', 'feVelocity_ft_s_Y',␣
↪'feVelocity_ft_s_Z',
                     'altitudeMsl_ft', 'longitude_deg', 'latitude_deg',␣
↪'localGravity_ft_s2',
                     'eulerAngle_deg_Yaw', 'eulerAngle_deg_Pitch',␣
↪'eulerAngle_deg_Roll',
                     'bodyAngularRateWrtEi_deg_s_Roll',␣
↪'bodyAngularRateWrtEi_deg_s_Pitch',
                     'bodyAngularRateWrtEi_deg_s_Yaw',
                     'altitudeRateWrtMsl_ft_min', 'speedOfSound_ft_s',␣
↪'airDensity_slug_ft3',
                     'ambientPressure_lbf_ft2', 'ambientTemperature_dgR',
                     'aero_bodyForce_lbf_X', 'aero_bodyForce_lbf_Y',␣
↪'aero_bodyForce_lbf_Z',
                     'aero_bodyMoment_ftlbf_L', 'aero_bodyMoment_ftlbf_M',␣
↪'aero_bodyMoment_ftlbf_N',
                     'mach', 'dynamicPressure_lbf_ft2', 'trueAirspeed_nmi_h']

    EnglishData = {}
```

```
    time = []
    eiPosition_m_X = []
    eiPosition_m_Y = []
    eiPosition_m_Z = []
    gePosition_m_X = []
    gePosition_m_Y = []
    gePosition_m_Z = []
    feVelocity_m_s_X = []
    feVelocity_m_s_Y = []
    feVelocity_m_s_Z = []
    altitudeMsl_m = []
    longitude_rad = []
    latitude_rad = []
    localGravity_m_s2 = []
    eulerAngle_Roll = []
    eulerAngle_Pitch = []
    eulerAngle_Yaw = []
    bodyAngularRateWrtEi_rad_s_Roll = []
    bodyAngularRateWrtEi_rad_s_Pitch = []
    bodyAngularRateWrtEi_rad_s_Yaw = []
    trueAirspeed = []

    def AdvanceTime(self):
        self.time.append(self.Time)
        self.Time += self.TimeStep

    def AddAeroModelInput(self, input):
        self.AeroModelInput = input

    def EvaluateAeroModel(self):
        for d in self.AeroModelInput:
            gvAeroModel.Set(d, self.Data[d])
        gvAeroModel.Update()

    def Clear(self):
        self.Time = 0.0
        self.Data.clear()
        self.EnglishData.clear()
        self.AeroModelInput.clear()

        self.time.clear()
        self.eiPosition_m_X.clear()
        self.eiPosition_m_Y.clear()
        self.eiPosition_m_Z.clear()
        self.gePosition_m_X.clear()
        self.gePosition_m_Y.clear()
        self.gePosition_m_Z.clear()
```

```
        self.feVelocity_m_s_X.clear()
        self.feVelocity_m_s_Y.clear()
        self.feVelocity_m_s_Z.clear()
        self.altitudeMsl_m.clear()
        self.longitude_rad.clear()
        self.latitude_rad.clear()
        self.localGravity_m_s2.clear()

        self.eulerAngle_Roll.clear()
        self.eulerAngle_Pitch.clear()
        self.eulerAngle_Yaw.clear()

        self.bodyAngularRateWrtEi_rad_s_Roll.clear()
        self.bodyAngularRateWrtEi_rad_s_Pitch.clear()
        self.bodyAngularRateWrtEi_rad_s_Yaw.clear()

        self.trueAirspeed.clear()

    def GenerateEnglishUnits(self):
        self.EnglishData.clear()
        for key in self.EnglishLabels:
            self.EnglishData[key] = []

        # TODO: extract units from name
        self.EnglishData['gePosition_ft_X'] = self.ToEnglish(self.
↪gePosition_m_X,"m")
        self.EnglishData['gePosition_ft_Y'] = self.ToEnglish(self.
↪gePosition_m_Y,"m")
        self.EnglishData['gePosition_ft_Z'] = self.ToEnglish(self.
↪gePosition_m_Z,"m")
        self.EnglishData['feVelocity_m_s_X'] = self.ToEnglish(self.
↪feVelocity_m_s_X,"m")
        self.EnglishData['feVelocity_m_s_Y'] = self.ToEnglish(self.
↪feVelocity_m_s_Y,"m")
        self.EnglishData['feVelocity_m_s_Z'] = self.ToEnglish(self.
↪feVelocity_m_s_Z,"m")

        self.EnglishData['altitudeMsl_ft'] = self.ToEnglish(self.
↪altitudeMsl_m,"m")
        self.EnglishData['longitude_deg'] = self.ToEnglish(self.
↪longitude_rad,"rad")
        self.EnglishData['latitude_deg'] = self.ToEnglish(self.
↪latitude_rad,"rad")
        self.EnglishData['localGravity_ft_s2'] = self.ToEnglish(self.
↪localGravity_m_s2,"m")
```

```
        self.EnglishData['eulerAngle_deg_Roll'] = self.ToEnglish(self.
↪eulerAngle_Roll,"rad")
        self.EnglishData['eulerAngle_deg_Pitch'] = self.ToEnglish(self.
↪eulerAngle_Pitch,"rad")
        self.EnglishData['eulerAngle_deg_Yaw'] = self.ToEnglish(self.
↪eulerAngle_Yaw,"rad")

        self.EnglishData['bodyAngularRateWrtEi_deg_s_Roll'] = \
            self.ToEnglish(self.bodyAngularRateWrtEi_rad_s_Roll,"rad")
        self.EnglishData['bodyAngularRateWrtEi_deg_s_Pitch'] = \
            self.ToEnglish(self.bodyAngularRateWrtEi_rad_s_Pitch,"rad")
        self.EnglishData['bodyAngularRateWrtEi_deg_s_Yaw'] = \
            self.ToEnglish(self.bodyAngularRateWrtEi_rad_s_Yaw,"rad")

        self.EnglishData['trueAirspeed_nmi_h'] = self.ToEnglish(self.
↪trueAirspeed,"m_s")

    def NormalizeAngle(self, value, lower, upper):
        """
        Returns a value between the range lower and upper.
        Example: NormalizeAngle(181, -180, 180) returns -179
        """
        angleRange = upper - lower
        rangeValue = value - lower
        return (rangeValue - (math.floor(rangeValue / angleRange) *␣
↪angleRange)) + lower

    def SetValue(self, label, defValue = 0):
        value = 0.0
        infoStr = "none"

        if label in self.IC:
            value = self.IC[label]
            infoStr = "[IC case]"
        else:
            value = defValue
            infoStr = "[default]"
        print("++", label, "=", value, infoStr)
        return value

    def ResetSimulation(self, ic):
        self.Clear()

        self.IC.clear()
        self.IC = self.SetIC(ic)
        #self.IC = ic.copy()
        print(self.IC)
```

```
        #self.GrossWeight = self.SetValue("grossWeight", 1)

        self.TimeStep = self.SetValue("timeStep", 0.1)

        self.TotalMass = self.SetValue("totalMass", 1)
        assert self.TotalMass != 0, "TotalMass is 0"

        self.ReferenceWingSpan = self.SetValue("referenceWingSpan")
        self.ReferenceWingChord = self.SetValue("referenceWingChord")

        wingArea = self.ReferenceWingSpan * self.ReferenceWingChord
        self.ReferenceWingArea = self.SetValue("referenceWingArea", wingArea)

        gvAeroModel.Set("aeroBodyForceCoefficient_X")
        gvAeroModel.Set("aeroBodyForceCoefficient_Y")
        gvAeroModel.Set("aeroBodyForceCoefficient_Z")

        gvAeroModel.Set("aeroBodyMomentCoefficient_Roll")
        gvAeroModel.Set("aeroBodyMomentCoefficient_Pitch")
        gvAeroModel.Set("aeroBodyMomentCoefficient_Yaw")

        self.TrueAirspeed = self.SetValue("trueAirspeed")

        angleOfAttack = self.SetValue("angleOfAttack")
        angleOfSideslip = self.SetValue("angleOfSideslip")
        u = self.TrueAirspeed * math.cos(angleOfAttack) * math.
↪cos(angleOfSideslip);
        v = self.TrueAirspeed * math.sin(angleOfSideslip);
        w = self.TrueAirspeed * math.sin(angleOfAttack) * math.
↪cos(angleOfSideslip);
        self.BodyVelocity.Set(u, v, w)

        self.BodyAccel.Set(0, 0, 0)

        # Set the rotation quaternion based on the Euler angles
        rollEulerAngle  = self.SetValue("eulerAngle_Roll")
        pitchEulerAngle = self.SetValue("eulerAngle_Pitch")
        yawEulerAngle   = self.SetValue("eulerAngle_Yaw")
        self.BodyAngle.Set( rollEulerAngle, pitchEulerAngle, yawEulerAngle )

        # Set angular rates
        P = self.SetValue("eulerAngleRate_Roll")
        Q = self.SetValue("eulerAngleRate_Pitch")
        R = self.SetValue("eulerAngleRate_Yaw")
        self.BodyAngularRate.Set( P, Q, R )
```

```
        # Set the inertia matrix
        i11 =  self.SetValue("bodyMomentOfInertia_X")
        i12 = -self.SetValue("bodyProductOfInertia_XY")
        i13 = -self.SetValue("bodyProductOfInertia_XZ")

        i21 = -self.SetValue("bodyProductOfInertia_YX")
        i22 =  self.SetValue("bodyMomentOfInertia_Y")
        i23 = -self.SetValue("bodyProductOfInertia_YZ")

        i31 = -self.SetValue("bodyProductOfInertia_XZ")
        i32 = -self.SetValue("bodyProductOfInertia_YZ")
        i33 =  self.SetValue("bodyMomentOfInertia_Z")

        self.InertiaMatrix.SetRow1(i11, i12, i13)
        self.InertiaMatrix.SetRow2(i21, i22, i23)
        self.InertiaMatrix.SetRow3(i31, i32, i33)

        self.InertiaMatrixInverse = self.InertiaMatrix.Inverse()

        self.gvJx = self.InertiaMatrix.A11
        self.gvJy = self.InertiaMatrix.A22
        self.gvJz = self.InertiaMatrix.A33
        self.gvJxz = self.InertiaMatrix.A13
        self.Gamma = (self.gvJx*self.gvJz) - (self.gvJxz*self.gvJxz)

        self.Ml = 0
        self.Mm = 0
        self.Mn = 0

    def AdamsBashforth(self, current, past):
        k2 = [1.5, -0.5]
        k3 = [23.0/12.0, -16.0/12.0, 5.0/12.0]

        x = self.TimeStep * (k2[0]*current.X + k2[1]*past.X)
        y = self.TimeStep * (k2[0]*current.Y + k2[1]*past.Y)
        z = self.TimeStep * (k2[0]*current.Z + k2[1]*past.Z)

        return [x, y, z]

    def RungeKutta4(self, Fdot, arg):
        h = self.TimeStep

        k1 = []
        arg1 = []
        for (a, f) in zip(arg, Fdot):
            k = h*f(arg)
            k1.append(k)
```

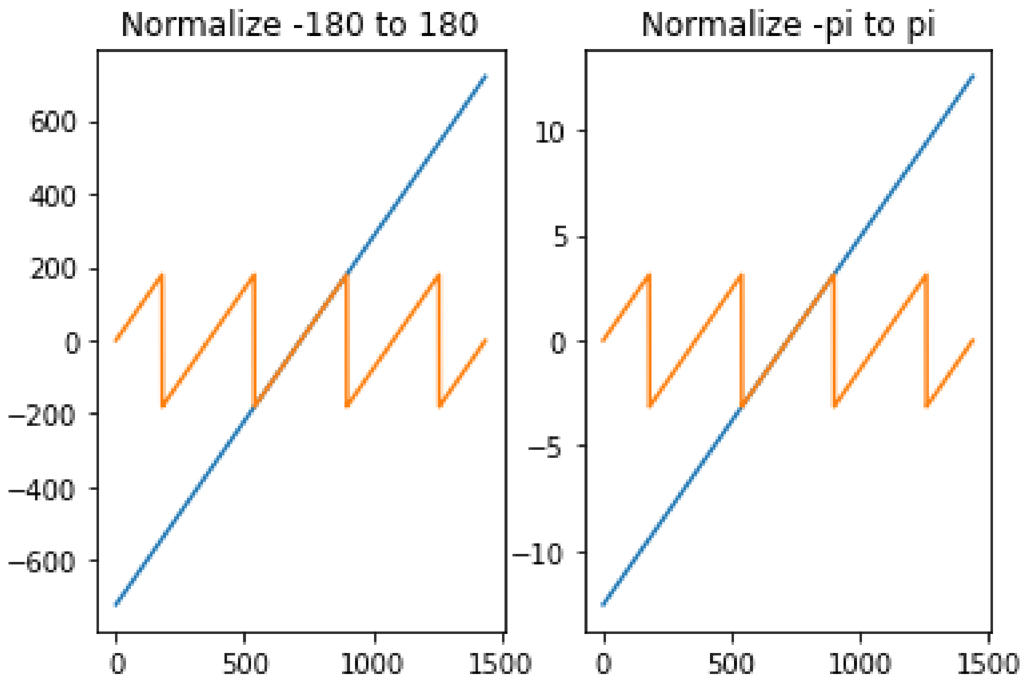
```
            arg1.append(a + 0.5*k)

        k2 = []
        arg2 = []
        for (a, f) in zip(arg, Fdot):
            k = h*f(arg1)
            k2.append(k)
            arg2.append(a + 0.5*k)

        k3 = []
        arg3 = []
        for (a, f) in zip(arg, Fdot):
            k = h*f(arg2)
            k3.append(k)
            arg3.append(a + k)

        k4 = []
        for f in Fdot:
            k4.append( h*f(arg3))

        result = []
        for (a, kc1, kc2, kc3, kc4) in zip(arg, k1, k2, k3, k4):
            result.append(a + (kc1 + 2.0*kc2 + 2.0*kc3 + kc4) / 6.0)

        return result

    def Reset(self, ic):
        pass

    def Operate(self):
        pass

    def Run(self, numberOfSeconds):
        endTime = int(numberOfSeconds / self.TimeStep) + 1
        for i in range(endTime):
            self.Operate()
        print("======done=======")

    def UnitTest(self):
        self.ClassName = "ppSimulation"
        # test normalize angle between -180 and 180 (and -pi and pi)
        pi = math.pi
        for i in range(360):
            ang = i
            if ang > 179:
                ang -= 360
```

```
            self.TestValue(ang, self.NormalizeAngle(i, -180.0, 180.0),␣
↪"NormalizeAngle", 0.001)

            ri = math.radians(i)
            rang = math.radians(ang)
            self.TestValue(rang, self.NormalizeAngle(ri, -pi, pi),␣
↪"NormalizeAngle", 1e-6)

        print("Number of ppSimulation failed tests: ", self.FailCount)
```

[26]:
```
import matplotlib.pyplot as plt

simUnitTest = ppSimulation()
simUnitTest.UnitTest()

# make a plot normalizing the angles between -180 and 180
t = []
x = []
nx = []
rx = []
rnx = []
for a in range(1440):
    t.append(a)
    x.append(a - 720)
    na = simUnitTest.NormalizeAngle( (a-720.0), -180.0, 180.0 )
    nx.append( na )

    rx.append( math.radians(a-720.0) )
    rna = simUnitTest.NormalizeAngle( math.radians(a-720.0), -math.pi, math.pi )
    rnx.append( rna )

figAng1, (pa1, pa2) = plt.subplots(1,2)
pa1.plot(t,x,t,nx)
pa1.set(title='Normalize -180 to 180')
pa2.plot(t,rx,t,rnx)
pa2.set(title='Normalize -pi to pi')
```

```
Number of ppSimulation failed tests:  0
```

[26]:
```
[Text(0.5, 1.0, 'Normalize -pi to pi')]
```



1.9.1 Flat Earth

Create a simulation for a flat Earth model. Singularities exist at the poles. Vehicle must be symmetric about the x body axis. Vehicle pitch must stay below 90°.

Force Equations
$\dot{U} = RV - QW - g_D \sin\theta + \frac{X_A + X_T}{m}$
$\dot{V} = PW - RU + g_D \sin\phi \cos\theta + \frac{Y_A + Y_T}{m}$
$\dot{W} = QU - PV + g_D \cos\phi \cos\theta + \frac{Z_A + Z_T}{m}$
In vector form,

$$\dot{\vec{v}} = \frac{F}{m} + R_{n/b} \begin{pmatrix} 0 \\ 0 \\ g_D \end{pmatrix} - \vec{\omega} \times \vec{v}$$

where $R_{n/b}$ is the rotation matrix from NED to body.

$$R_{n/b} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\phi & \sin\phi \\ 0 & -\sin\phi & \cos\phi \end{bmatrix} \begin{bmatrix} \cos\theta & 0 & -\sin\theta \\ 0 & 1 & 0 \\ \sin\theta & 0 & \cos\theta \end{bmatrix} \begin{bmatrix} \cos\psi & \sin\psi & 0 \\ -\sin\psi & \cos\psi & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$R_{b/n} = [R_{n/b}]^T$

Kinematic equations
$\dot{\phi} = P + \tan\theta\,(Q\sin\phi + R\cos\phi)$
$\dot{\theta} = Q\cos\phi - R\sin\phi$
$\dot{\psi} = (Q\sin\phi + R\cos\phi)\,/\,\cos\theta$
In vector form,

$\dot{\Phi} = H\omega^b$, where $H = \begin{pmatrix} 1 & \sin\phi\tan\theta & \cos\phi\tan\theta \\ 0 & \cos\phi & -\sin\phi \\ 0 & \sin\phi/\cos\theta & \cos\phi/\cos\theta \end{pmatrix}$

Moment Equations
$\Gamma\dot{P} = J_{xz}[J_x - J_y + J_z]PQ - [J_z(J_z - J_y) + J_{xz}^2]QR + lJ_z + nJ_{xz}$
$J_y\dot{Q} = (J_z - J_x)PR - J_{xz}(P^2 - R^2) + m$
$\Gamma\dot{R} = [(J_x - J_y)J_x + J_{xz}^2]PQ - J_{xz}[J_x - J_y + J_z]QR + lJ_{xz}nJ_x$
$\Gamma = J_xJ_z - J_{xz}^2$
In vector form,
$\dot{\omega^b_{b/i}} = J^{-1}(M^b - \omega^b_{b/i}J\omega^b_{b/i})$

Navigation Equations
$\dot{p_N} = Uc\theta c\psi + V(-c\phi s\psi + s\phi s\theta c\psi) + W(s\phi s\psi + c\phi s\theta c\psi)$
$\dot{p_E} = Uc\theta s\psi + V(c\phi c\psi + s\phi s\theta s\psi) + W(-s\phi c\psi + c\phi s\theta c\psi)$
$\dot{h} = Us\theta - Vs\phi c\theta - Wc\phi c\theta$
In vector form,
$\dot{\vec{p}} = R_{b/n}\vec{v}$

[27]:
```
class ppFlatEarth(ppSimulation):

    gD = 0
    mass = 0

    Planet = ppEarth()

    # state values
    X = [0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0]

    # state DFE
    Xdot = []

    # the indices of the state list
    Ui = 0
    Vi = 1
    Wi = 2

     i = 3
     i = 4
     i = 5

    Pi = 6
    Qi = 7
    Ri = 8

    Ni = 9
    Ei = 10
    Zi = 11
```

```
# aerodynamic forces in body frame
Xa = 0
Ya = 0
Za = 0

# thrust forces in body frame
Xt = 0
Yt = 0
Zt = 0

def Udot(self, state):
    V = state[self.Vi]
    W = state[self.Wi]
    Q = state[self.Qi]
    R = state[self.Ri]
    sin  = math.sin(state[self. i])

    assert self.mass != 0.0, "Udot mass is 0"
    value =  R*V - Q*W - self.gD*sin  + (self.Xa + self.Xt) / self.mass
    return value

def Vdot(self, state):
    U = state[self.Ui]
    W = state[self.Wi]
    P = state[self.Pi]
    R = state[self.Ri]
    sin  = math.sin(state[self. i])
    cos  = math.cos(state[self. i])

    assert self.mass != 0.0, "Vdot mass is 0"
    value = -R*U + P*W + self.gD*sin *cos  + (self.Ya + self.Yt) / self.mass
    return value

def Wdot(self, state):
    U = state[self.Ui]
    V = state[self.Vi]
    P = state[self.Pi]
    Q = state[self.Qi]
    cos  = math.cos(state[self. i])
    cos  = math.cos(state[self. i])

    assert self.mass != 0.0, "Wdot mass is 0"
    value =  Q*U - P*V + self.gD*cos *cos  + (self.Za + self.Zt) / self.mass
    return value

def  dot(self, state):
```

```
        P = state[self.Pi]
        Q = state[self.Qi]
        R = state[self.Ri]

        assert state[self. i] < abs(math.radians(90.0)), " dot tan  is 90"
        tan  = math.tan(state[self. i])
        sin  = math.sin(state[self. i])
        cos  = math.cos(state[self. i])

        value = P + tan  * (Q*sin  + R*cos )
        return value

    def  dot(self, state):
        Q = state[self.Qi]
        R = state[self.Ri]
        cos  = math.cos(state[self. i])
        sin  = math.sin(state[self. i])

        value = Q*cos  - R*sin
        return value

    def  dot(self, state):
        Q = state[self.Qi]
        R = state[self.Ri]
        cos  = math.cos(state[self. i])
        sin  = math.sin(state[self. i])
        cos  = math.cos(state[self. i])

        assert cos  != 0.0, " dot cos  is 0"
        value = (Q*sin  + R*cos ) / cos
        return value

    def Pdot(self, state):
        P = state[self.Pi]
        Q = state[self.Qi]
        R = state[self.Ri]
        Jx = self.gvJx
        Jy = self.gvJy
        Jz = self.gvJz
        Jxz = self.gvJxz
        l = self.Ml
        n = self.Mn

        assert self.Gamma != 0.0, "Pdot Gamma is 0"
        value = (Jxz * (Jx - Jy + Jz)*P*Q - (Jz*(Jz - Jy) + Jxz*Jxz)*Q*R + Jz*l + Jxz*n) / self.Gamma
        return value
```

```
    def Qdot(self, state):
        P = state[self.Pi]
        Q = state[self.Qi]
        R = state[self.Ri]
        Jx = self.gvJx
        Jy = self.gvJy
        Jz = self.gvJz
        Jxz = self.gvJxz
        m = self.Mm

        assert Jy != 0.0, "Qdot Jy is 0"
        value = ((Jz - Jx)*P*R - Jxz*(P*P - R*R) + m) / Jy
        return value

    def Rdot(self, state):
        P = state[self.Pi]
        Q = state[self.Qi]
        R = state[self.Ri]
        Jx = self.gvJx
        Jy = self.gvJy
        Jz = self.gvJz
        Jxz = self.gvJxz
        l = self.Ml
        n = self.Mn

        assert self.Gamma != 0.0, "Pdot Gamma is 0"
        value = (((Jx - Jy)*Jx + Jxz*Jxz)*P*Q - Jxz*(Jx - Jy + Jz)*Q*R + Jxz*l + Jx*n) / self.Gamma
        return value

    def Ndot(self, state):
        U = state[self.Ui]
        V = state[self.Vi]
        W = state[self.Wi]
        cos  = math.cos(state[self. i])
        sin  = math.sin(state[self. i])
        cos  = math.cos(state[self. i])
        sin  = math.sin(state[self. i])
        cos  = math.cos(state[self. i])
        sin  = math.sin(state[self. i])

        value = U*cos *cos  + V*(-cos *sin  + sin *sin *cos )
        + W*(sin *sin  + cos *sin *cos )
        return value

    def Edot(self, state):
```

```
        U = state[self.Ui]
        V = state[self.Vi]
        W = state[self.Wi]
        cos  = math.cos(state[self. i])
        sin  = math.sin(state[self. i])
        cos  = math.cos(state[self. i])
        sin  = math.sin(state[self. i])
        cos  = math.cos(state[self. i])
        sin  = math.sin(state[self. i])

        value = U*cos *sin  + V*(cos *cos  + sin *sin *sin )
        + W*(-sin *cos  + cos *sin *sin )
        return value

    def Zdot(self, state):
        U = state[self.Ui]
        V = state[self.Vi]
        W = state[self.Wi]
        cos  = math.cos(state[self. i])
        sin  = math.sin(state[self. i])
        cos  = math.cos(state[self. i])
        sin  = math.sin(state[self. i])

        value = U*sin  - V*sin *cos  - W*cos *cos
        return value

    def Reset(self, ic):
        self.ResetSimulation(ic)

        self.gD = self.Planet.GravityConstant()
        self.mass = self.TotalMass

        self.Xdot.clear()
        self.Xdot = [self.Udot, self.Vdot, self.Wdot, self. dot, self. dot, self.
↪ dot,
                     self.Pdot, self.Qdot, self.Rdot, self.Ndot, self.Edot,␣
↪self.Zdot]

        self.X[self.Ui] = self.BodyVelocity.X
        self.X[self.Vi] = self.BodyVelocity.Y
        self.X[self.Wi] = self.BodyVelocity.Z

        self.X[self. i] = self.BodyAngle.X
        self.X[self. i] = self.BodyAngle.Y
        self.X[self. i] = self.BodyAngle.Z

        self.X[self.Pi] = self.BodyAngularRate.X
```

```python
        self.X[self.Qi] = self.BodyAngularRate.Y
        self.X[self.Ri] = self.BodyAngularRate.Z

        self.X[self.Ni] = self.Position.X
        self.X[self.Ei] = self.Position.Y
        self.X[self.Zi] = self.SetValue("altitudeMsl")

        self.Xa = 0
        self.Ya = 0
        self.Za = 0

        self.Xt = 0
        self.Yt = 0
        self.Zt = 0

    def Operate(self):

        # save output data
        self.localGravity_m_s2.append(self.gD)
        self.altitudeMsl_m.append(self.X[self.Zi])
        self.eulerAngle_Roll.append(self.X[self. i])
        self.eulerAngle_Pitch.append(self.X[self. i])
        self.eulerAngle_Yaw.append( self.NormalizeAngle(self.X[self. i],-math.
↪pi,math.pi) )
        self.trueAirspeed.append(self.TrueAirspeed)

        # integrate the equations
        self.X = self.RungeKutta4(self.Xdot, self.X)

        # Now advance time and update state equations
        self.AdvanceTime()

        u = self.X[self.Ui]
        v = self.X[self.Vi]
        w = self.X[self.Wi]
        self.TrueAirspeed = math.sqrt(u*u + v*v + w*w)

        # get dynamic pressure:  q = 1/2 rho v^2
        density = self.Planet.AirDensity(self.X[self.Zi])
        dynamicPressure = 0.5 * density * (self.TrueAirspeed)**2

        # Get the qS factor for getting dimensional forces and moments
        qS = dynamicPressure * self.ReferenceWingArea

        # Compute the aerodynamic loads
        assert self.TrueAirspeed != 0, "TrueAirspeed is 0 to model"
        self.Data["trueAirspeed"] = self.TrueAirspeed * self.MeterToFeet
```

```
        self.Data["bodyAngularRate_Roll"] = self.X[self.Pi]
        self.Data["bodyAngularRate_Pitch"] = self.X[self.Qi]
        self.Data["bodyAngularRate_Yaw"] = self.X[self.Ri]
        self.EvaluateAeroModel()

        # Aero forces (Newtons) body
        self.Xa = qS * gvAeroModel.DataFromName("aeroBodyForceCoefficient_X")
        self.Ya = qS * gvAeroModel.DataFromName("aeroBodyForceCoefficient_Y")
        self.Za = qS * gvAeroModel.DataFromName("aeroBodyForceCoefficient_Z")

        # Aero moments
        self.Ml = qS * self.ReferenceWingSpan  * gvAeroModel.
↪DataFromName("aeroBodyMomentCoefficient_Roll")
        self.Mm = qS * self.ReferenceWingChord * gvAeroModel.
↪DataFromName("aeroBodyMomentCoefficient_Pitch")
        self.Mn = qS * self.ReferenceWingSpan  * gvAeroModel.
↪DataFromName("aeroBodyMomentCoefficient_Yaw")
```

1.9.2 Oblate, Rotating Earth (Stevens and Lewis)

$\dot{q_0} = -0.5 * (Pq_1 + Qq_2 + Rq_3)$
$\dot{q_1} = 0.5 * (Pq_0 + Rq_2 - Qq_3)$
$\dot{q_2} = 0.5 * (Qq_0 - Rq_1 + Pq_3)$
$\dot{q_3} = 0.5 * (Rq_0 + Qq_1 - Pq_2)$

$\dot{P_x} = V_x$
$\dot{P_y} = V_y$
$\dot{P_z} = V_z$
where P and V are in the ECEF frame.

$\dot{v_x} = \frac{F_x}{m} + 2\omega_e V_y + g_x + P_x \omega_e^2$
$\dot{v_y} = \frac{F_y}{m} - 2\omega_e V_x + g_y + P_y \omega_e^2$
$\dot{v_z} = \frac{F_z}{m} + g_z$
where ω_e is the rotation rate of Earth. The terms g_x, g_y, and g_z are the J_2 gravity components in ECEF. This acceleration equation is in the ECEF frame.

$\Gamma\dot{P} = J_{xz}[J_x - J_y + J_z]PQ - [J_z(J_z - J_y) + J_{xz}^2]QR + lJ_z + nJ_{xz}$
$J_y\dot{Q} = (J_z - J_x)PR - J_{xz}(P^2 - R^2) + m$
$\Gamma\dot{R} = [(J_x - J_y)J_x + J_{xz}^2]PQ - J_{xz}[J_x - J_y + J_z]QR + lJ_{xz}nJ_x$
where $\Gamma = J_xJ_z - J_{xz}^2$

[28]:
```
class slEarthSim(ppSimulation):
    Planet = ppEarth()
    RotationAngle = 0
    EarthRotation = ppQuaternion(0, 0, 0, Planet.RotationRate)

    # Earth rotatation in body frame
    Per = 0
```

```
    Qer = 0
    Rer = 0

    # quaternion frame rotations
    #  i = inertial frame ECI
    #  e = earth centered, earth fixed ECEF
    #  n = north east down NED
    #  b = body forward right down FRD
    Qe2n = ppQuaternion(1,0,0,0)
    Qn2b = ppQuaternion(1,0,0,0)
    Qe2b = ppQuaternion(1,0,0,0)
    Qi2e = ppQuaternion(1,0,0,0)

    QforceEcf = ppQuaternion(0,0,0,0)

    # ECEF gravity components
    Gx = 0
    Gy = 0
    Gz = 0

    # state values: quaternion, position, acceleration and angular rates
    X = [0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0]

    # the state differential equations
    Xdot = []

    # the indices of the state list
    Qni = 0
    Qxi = 1
    Qyi = 2
    Qzi = 3

    Xi = 4
    Yi = 5
    Zi = 6

    Vxi = 7
    Vyi = 8
    Vzi = 9

    Pi = 10
    Qi = 11
    Ri = 12

    def Qstate(self,state):
        q0 = state[self.Qni]
        q1 = state[self.Qxi]
```

```
        q2 = state[self.Qyi]
        q3 = state[self.Qzi]
        p = state[self.Pi] - self.Per
        q = state[self.Qi] - self.Qer
        r = state[self.Ri] - self.Rer
        return q0, q1, q2, q3, p, q, r
    def QnDot(self, state):
        q0, q1, q2, q3, p, q, r = self.Qstate(state)
        qnDot = -0.5*(q1*p + q2*q + q3*r)
        return qnDot
    def QxDot(self, state):
        q0, q1, q2, q3, p, q, r = self.Qstate(state)
        qxDot = 0.5*(q0*p + q2*r - q3*q)
        return qxDot
    def QyDot(self, state):
        q0, q1, q2, q3, p, q, r = self.Qstate(state)
        qyDot = 0.5*(q0*q - q1*r + q3*p )
        return qyDot
    def QzDot(self, state):
        q0, q1, q2, q3, p, q, r = self.Qstate(state)
        qzDot = 0.5*(q0*r + q1*q - q2*p)
        return qzDot

    def PxDot(self, state):
        return state[self.Vxi]
    def PyDot(self, state):
        return state[self.Vyi]
    def PzDot(self, state):
        return state[self.Vzi]

    def VxDot(self, state):
        w = self.Planet.RotationRate
        assert self.TotalMass != 0, "VxDot mass is 0"
        ax = self.QforceEcf.X / self.TotalMass
        #if ax != 0:
        #    print("ax:",ax)
        xDot = ax + 2.0 * w * state[self.Vyi] + self.Gx + state[self.Xi] * w**2
        return xDot
    def VyDot(self, state):
        w = self.Planet.RotationRate
        assert self.TotalMass != 0, "VyDot mass is 0"
        ay = self.QforceEcf.Y / self.TotalMass
        yDot = ay - 2.0 * w * state[self.Vxi] + self.Gy + state[self.Yi] * w**2
        return yDot
    def VzDot(self, state):
        assert self.TotalMass != 0, "VzDot mass is 0"
        az = self.QforceEcf.Z / self.TotalMass
```

```
        return (az + self.Gz)

    def Wstate(self, state):
        P = state[self.Pi]
        Q = state[self.Qi]
        R = state[self.Ri]
        Jx = self.gvJx
        Jy = self.gvJy
        Jz = self.gvJz
        Jxz = self.gvJxz
        Gamma = self.Gamma
        l = self.Ml
        m = self.Mm
        n = self.Mn
        return P, Q, R, Jx, Jy, Jz, Jxz, Gamma, l, m, n
    def Pdot(self, state):
        P, Q, R, Jx, Jy, Jz, Jxz, Gamma, l, m, n = self.Wstate(state)
        assert Gamma != 0, "Pdot Gamma is 0"
        pDot = (Jxz * (Jx - Jy + Jz)*P*Q - (Jz*(Jz - Jy) + Jxz*Jxz)*Q*R + Jz*l + Jxz*n) / Gamma
        return pDot
    def Qdot(self, state):
        P, Q, R, Jx, Jy, Jz, Jxz, Gamma, l, m, n = self.Wstate(state)
        assert Jy != 0.0, "Qdot Jy is 0"
        qDot = ((Jz - Jx)*P*R - Jxz*(P*P - R*R) + m) / Jy
        return qDot
    def Rdot(self, state):
        P, Q, R, Jx, Jy, Jz, Jxz, Gamma, l, m, n = self.Wstate(state)
        assert Gamma != 0.0, "Rdot Gamma is 0"
        rDot = (((Jx - Jy)*Jx + Jxz*Jxz)*P*Q - Jxz*(Jx - Jy + Jz)*Q*R + Jxz*l +Jx*n) / Gamma
        return rDot

    def Reset(self, ic):
        self.ResetSimulation(ic)

        self.RotationAngle = 0

        self.Planet.Latitude = self.SetValue("latitude")
        self.Planet.Longitude = self.SetValue("longitude")
        self.Planet.Altitude = self.SetValue("altitudeMsl")
        [x, y, z] = self.Planet.LlaToPcpf()
        self.Position.X = x
        self.Position.Y = y
        self.Position.Z = z

        # initialize the frd/ecf quaternion
```

```
        roll  = self.BodyAngle.X
        pitch = self.BodyAngle.Y
        yaw   = self.BodyAngle.Z
        lat = self.Planet.Latitude
        lon = self.Planet.Longitude
        self.Qe2b.SetQfrdWrtEcf(roll , pitch , yaw, lat, lon)

        # transform u,v,w to ECEF velocities
        Vecf = ppVector3(0,0,0)
        Vecf = self.Qe2b * self.BodyVelocity * ~self.Qe2b

        #self.mass = self.GrossWeight / self.gD

        self.Xdot.clear()
        self.Xdot = [self.QnDot, self.QxDot, self.QyDot, self.QzDot,
                     self.PxDot, self.PyDot, self.PzDot,
                     self.VxDot, self.VyDot, self.VzDot,
                     self.Pdot, self.Qdot, self.Rdot]

        self.X[self.Qni] = self.Qe2b.N
        self.X[self.Qxi] = self.Qe2b.X
        self.X[self.Qyi] = self.Qe2b.Y
        self.X[self.Qzi] = self.Qe2b.Z

        self.X[self.Xi] = self.Position.X
        self.X[self.Yi] = self.Position.Y
        self.X[self.Zi] = self.Position.Z

        self.X[self.Vxi] = Vecf.X
        self.X[self.Vyi] = Vecf.Y
        self.X[self.Vzi] = Vecf.Z
        print("Vecf: ", Vecf.X, Vecf.Y, Vecf.Z)

        self.X[self.Pi] = self.BodyAngularRate.X
        self.X[self.Qi] = self.BodyAngularRate.Y
        self.X[self.Ri] = self.BodyAngularRate.Z

    def Operate(self):
        # create quaternions

        # TODO: need a check case the Q rotations are correct
        # set q frd/ecf (e2b) ECF to body
        self.Qe2b.N = self.X[self.Qni]
        self.Qe2b.X = self.X[self.Qxi]
        self.Qe2b.Y = self.X[self.Qyi]
        self.Qe2b.Z = self.X[self.Qzi]
```

```
        # set q ned/ecf (e2n) ECF to NED
        self.Qe2n.SetLatLon(self.Planet.Latitude, self.Planet.Longitude)

        # set q frd/ned (n2b) NED to body
        self.Qn2b = ~self.Qe2n * self.Qe2b

        # get the euler angles from the quaternion
        [roll, pitch, yaw] = self.Qn2b.EulerAnglesFromQ()

        # rotate the ECF position to ECI to get the inertial position
        self.Qi2e.SetPlanetRotation(self.RotationAngle)
        qgePosition = ppQuaternion( 0, self.X[self.Xi], self.X[self.Yi], self.
↪X[self.Zi] )
        qeiPosition = self.Qi2e * qgePosition * ~self.Qi2e

        # save output data
        self.altitudeMsl_m.append(self.Planet.Altitude)
        self.latitude_rad.append(self.Planet.Latitude)
        self.longitude_rad.append(self.Planet.Longitude)
        self.gePosition_m_X.append(self.X[self.Xi])
        self.gePosition_m_Y.append(self.X[self.Yi])
        self.gePosition_m_Z.append(self.X[self.Zi])

        self.eulerAngle_Roll.append(roll)
        self.eulerAngle_Pitch.append(pitch)
        self.eulerAngle_Yaw.append(yaw)

        self.trueAirspeed.append(self.TrueAirspeed)
        #cosRot = math.cos(self.RotationAngle)
        #sinRot = math.sin(self.RotationAngle)
        #self.eiPosition_m_X.append(cosRot*self.X[self.Xi] - sinRot*self.X[self.
↪Yi])
        #self.eiPosition_m_Y.append(sinRot*self.X[self.Xi] + cosRot*self.X[self.
↪Yi])
        #self.eiPosition_m_Z.append(self.X[self.Zi])
        self.eiPosition_m_X.append(qeiPosition.X)
        self.eiPosition_m_Y.append(qeiPosition.Y)
        self.eiPosition_m_Z.append(qeiPosition.Z)

        # get earth rotation in the body frame
        wEarthFrd = ~self.Qe2b * self.EarthRotation * self.Qe2b

        # TODO: need to add body forces and rotate them to ECEF frame

        # set the Earth rotation in the body frame
        self.Per = wEarthFrd.X
        self.Qer = wEarthFrd.Y
```

```
        self.Rer = wEarthFrd.Z

        x = self.X[self.Xi]
        y = self.X[self.Yi]
        z = self.X[self.Zi]
        [self.Gx, self.Gy, self.Gz] = self.Planet.GravityJ2( x, y, z )
        g = ppVector3(self.Gx, self.Gy, self.Gz)
        self.localGravity_m_s2.append(g.Magnitude())

        # integrate the equations
        self.X = self.RungeKutta4(self.Xdot, self.X)

        # advance time and set up for next integration
        self.AdvanceTime()

        # get the new true airspeed
        vel = ppVector3(self.X[self.Vxi], self.X[self.Vyi], self.X[self.Vzi])
        self.TrueAirspeed = vel.Magnitude()

        # get dynamic pressure:  q = 1/2 rho v^2
        density = self.Planet.AirDensity(self.Planet.Altitude)
        dynamicPressure = 0.5 * density * (self.TrueAirspeed)**2

        # Get the qS factor for getting dimensional forces and moments
        qS = dynamicPressure * self.ReferenceWingArea

        # Compute the aerodynamic loads from the DAVE-ML model
        #  set the DAVE-ML model inputs
        assert self.TrueAirspeed != 0, "TrueAirspeed is 0 to model"
        self.Data["trueAirspeed"] = self.TrueAirspeed * self.MeterToFeet
        self.Data["bodyAngularRate_Roll"] = self.X[self.Pi]
        self.Data["bodyAngularRate_Pitch"] = self.X[self.Qi]
        self.Data["bodyAngularRate_Yaw"] = self.X[self.Ri]
        self.EvaluateAeroModel()

        drag = qS * self.totalCoefficientOfDrag
        QforceFrb = ppVector3(-drag, 0, 0)
        self.QforceEcf = self.Qe2b * QforceFrb * ~self.Qe2b

        # calculate the dimentionsal aero moments
        self.Ml = qS * self.ReferenceWingSpan  * gvAeroModel.
↪DataFromName("aeroBodyMomentCoefficient_Roll")
        self.Mm = qS * self.ReferenceWingChord * gvAeroModel.
↪DataFromName("aeroBodyMomentCoefficient_Pitch")
        self.Mn = qS * self.ReferenceWingSpan  * gvAeroModel.
↪DataFromName("aeroBodyMomentCoefficient_Yaw")
```

```
        # update the latitude, longitude and altitude from ECEF X, Y, Z position
        self.Planet.PcpfToLlaZhu(self.X[self.Xi], self.X[self.Yi], self.X[self.
↪Zi])

        # rotate the earth
        self.RotationAngle = self.Planet.RotationRate * self.Time
```

1.10 Check Cases

1.10.1 Check with Kinematics

Using 2D constant acceleration kinematics with no aerodynamic effects, the y displacement (height) equation is: $v_f = v_0 + gt$. A true airspeed of 424 m/s at 45° is 300 m/s in x and 300 m/s in y. At the maximum height, $v_f = 0\,m/s$. Using $g = 9.82\,m/s^2$, you get $0 = 300 - (9.82)t$. Solving for time to reach the maximum height, $t = 300/9.82 = 30.55$ seconds.

[29]:
```
%%time
printDebugData = False
gvAeroModel.LoadDml('models/noAero.dml', printDebugData)
#
ic = {
    "timeStep": [0.01, "s"],
    "totalMass": [5.0, "kg"],
    "bodyMomentOfInertia_X": [0.1, "kgm2"],
    "bodyMomentOfInertia_Y": [0.1, "kgm2"],
    "bodyMomentOfInertia_Z": [0.1, "kgm2"],
    "altitudeMsl": [10.0, "m"],
    "referenceWingChord": [0.2, "m"],
    "referenceWingSpan": [0.2, "m"],
    "referenceWingArea": [0.031415, "m2"],
    "trueAirspeed": [424.264, "m_s"],
    "angleOfAttack": [-45, "deg"]
}
flatEarthSim = ppFlatEarth()
flatEarthSim.Reset(ic)
flatEarthSim.Run(61.0)
```

```
*******************************************
Model:  Zero Aero Output
creation date:  2021-07-21
file version:  Initial version
*******************************************

+++++ MODEL INPUTS AND OUTPUTS +++++
++> Output:  CX
++> Output:  CY
++> Output:  CZ
++> Output:  CLL
```

```
++> Output:  CLM
++> Output:  CLN
++++++++++++++++++++++++++++++++++++

----- DAVE-ML MODEL PARSE COMPLETE -----
========= SetIC ==============
{'timeStep': 0.01, 'totalMass': 5.0, 'bodyMomentOfInertia_X': 0.1,
'bodyMomentOfInertia_Y': 0.1, 'bodyMomentOfInertia_Z': 0.1, 'altitudeMsl': 10.0,
'referenceWingChord': 0.2, 'referenceWingSpan': 0.2, 'referenceWingArea':
0.031415, 'trueAirspeed': 424.264, 'angleOfAttack': -0.7853981633974483}
++ timeStep = 0.01 [IC case]
++ totalMass = 5.0 [IC case]
++ referenceWingSpan = 0.2 [IC case]
++ referenceWingChord = 0.2 [IC case]
++ referenceWingArea = 0.031415 [IC case]
++ trueAirspeed = 424.264 [IC case]
++ angleOfAttack = -0.7853981633974483 [IC case]
++ angleOfSideslip = 0 [default]
++ eulerAngle_Roll = 0 [default]
++ eulerAngle_Pitch = 0 [default]
++ eulerAngle_Yaw = 0 [default]
++ eulerAngleRate_Roll = 0 [default]
++ eulerAngleRate_Pitch = 0 [default]
++ eulerAngleRate_Yaw = 0 [default]
++ bodyMomentOfInertia_X = 0.1 [IC case]
++ bodyProductOfInertia_XY = 0 [default]
++ bodyProductOfInertia_XZ = 0 [default]
++ bodyProductOfInertia_YX = 0 [default]
++ bodyMomentOfInertia_Y = 0.1 [IC case]
++ bodyProductOfInertia_YZ = 0 [default]
++ bodyProductOfInertia_XZ = 0 [default]
++ bodyProductOfInertia_YZ = 0 [default]
++ bodyMomentOfInertia_Z = 0.1 [IC case]
++ altitudeMsl = 10.0 [IC case]
======done=======
CPU times: user 590 ms, sys: 22.7 ms, total: 613 ms
Wall time: 881 ms
```

[30]:
```
max(flatEarthSim.altitudeMsl_m)
```

[30]: 4598.721461301354

[31]:
```
tMax = 300/9.82
print(tMax)
```

```
30.54989816700611
```

The time calculated above ($t = 30.55$ seconds) matches closely to the plot of the data from the EOM.

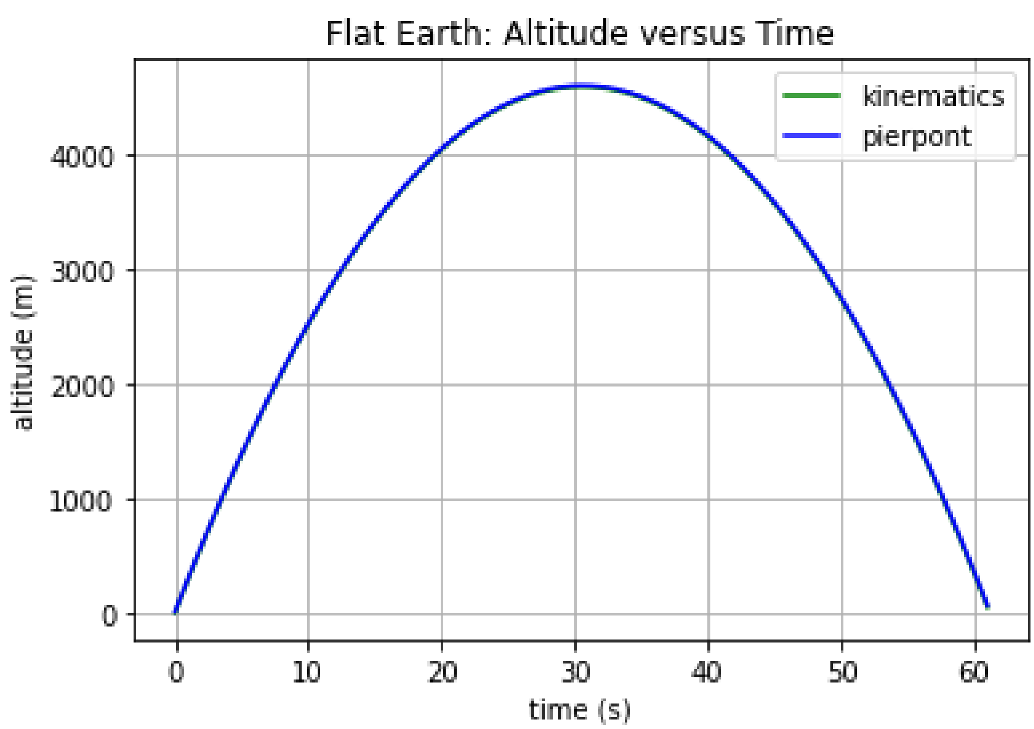
The maximum height in kinematics is: $y = \frac{1}{2}(v_0 y + v_f y)t + y_0$. Substituting in the equation, you get $y = \frac{1}{2}(300 + 0)(30.55) + 10$.

[32]:
```
yMax = 0.5*(300 + 0)*(tMax) + 10
print(yMax)
```

```
4592.4847250509165
```

The maximum height calculated from the 2D kinematics of 4592 meters is within 10 meters of the EOM calculated value of 4600 meters.

[33]:
```
import matplotlib.pyplot as plt

y = []
for t in  flatEarthSim.time:
    y.append(300*t - 0.5*9.81*t*t)

fig, ck = plt.subplots()
ck.plot( flatEarthSim.time, y, 'g', flatEarthSim.time, flatEarthSim.
 ↪altitudeMsl_m, 'b')
ck.legend(["kinematics","pierpont"])

ck.set(xlabel='time (s)', ylabel='altitude (m)',
       title='Flat Earth: Altitude versus Time')
ck.grid()
```



1.10.2 Read NESC Check Cases

Function to read in check cases from NESC. The function gets columns of data from the check case CSV files.

[34]:
```
import csv

def GetCheckCaseData(fileName):
    # open the CSV file as read-only
    csvFile = open(fileName,'r')
    # strip the newline character from the header line
    headerLine = csvFile.readline().rstrip("\n")
    # make a list of headers
    header = headerLine.split(',')
    print("number of headers: ", len(header))
    print(header)

    # create a data dictionary with header names as keys
    Data = {}
    for h in header:
        Data[h] = []

    # read each row in the datafile and add the data to the data dictionary
    for row in csv.reader(csvFile):
        for (i,d) in zip(header, row):
            Data[i].append( float(d) )

    return Data
```

Data checks:
L-2: $D(x,y) = [\Sigma(x_i - y_i)^2]^{1/2}$
L-Infinity-Norm: $max_i|x_i - y_i|$

Manhattan distance: $\Sigma_i|x_i - y_i|$

[35]:
```
import matplotlib.pyplot as plt
import numpy as np
import math

def MinDeltaAngleDeg(angle1, angle2):
    """
    Returns the minimum delta between two angles.
    Examples:
      20 is returned if angle1=30 and angle2=10
      20 is returned if angle1=-170 and angle2=170
    """
    delta = angle1 - angle2
    twoPi = 360.0
    if abs(delta) > abs( angle1 - (angle2 - twoPi) ):
```

```
        delta = angle1 - (angle2 - twoPi)
    if abs(delta) > abs( (angle1 - twoPi) - angle2 ):
        delta = (angle1 - twoPi) - angle2
    return delta

print("MinDeltaAngle check")
print(MinDeltaAngleDeg( 170,160),"=", 10)
print(MinDeltaAngleDeg(-160,170),"=", 30)
print(MinDeltaAngleDeg(  20,-20),"=", 40)

def NescCheckData(data, checkData, isAng):
    l2Sum = 0
    manSum = 0
    infNorm = 0
    for (x, y) in zip(data, checkData):
        dxy = x - y
        if isAng:
            dxy = MinDeltaAngleDeg( x, y )
        l2Sum += dxy**2
        dist = abs(dxy)

        manSum += dist
        if dist > infNorm:
            infNorm = dist
    return math.sqrt(l2Sum), infNorm

def PrintErrorTable(tableTitle, labels, simData, checkData):
    print ("{:<25} {:<25} {:<25}".format('Variable', 'L2', 'L-Infinity-Norm'))
    print ("{:<25} {:<25} {:<25}".format('--------', '--', '---------------'))
    barLinf = {}
    for i in labels:
        tmpDist = NescCheckData(checkData[i], simData.EnglishData[i], i.
↪find("_deg_"))
        print ("{:<25} {:<25} {:<25}".format(i, tmpDist[0], tmpDist[1]))
        barLinf[i] = tmpDist[1]

    plt.rcdefaults()
    fig, ax = plt.subplots()
    y_pos = np.arange(len(barLinf.keys()))
    #plt.xlim([0, 10])
    ax.barh(y_pos, barLinf.values(), align='center')
    ax.set_yticks(y_pos)
    ax.set_yticklabels(barLinf.keys())
    ax.invert_yaxis()  # labels read top-to-bottom
    ax.set_xlabel('L-Infinity Norm')
    ax.set_title(tableTitle)
```

```
    plt.show()

data1 = [  1,    2, 3,    4, 5]
data2 = [1.2, 2.2, 3, 3.9, 5]

dists = NescCheckData(data1, data2, False)
print("L-2 Norm: ", dists)

data3 = [  160,    20 ]
data4 = [ -160,    30 ]

dists = NescCheckData(data3, data4, True)
print("L-2 Norm: ", dists)
```

```
MinDeltaAngle check
10 = 10
30.0 = 30
40 = 40
L-2 Norm:  (0.30000000000000001, 0.20000000000000018)
L-2 Norm:  (41.23105625617661, 40.0)
```

[36]:
```
def MakeFlatEarthPlots(simData, checkData, simCheckLabel):
    fig0, ah = plt.subplots()
    ah.plot(checkData['time'], checkData['altitudeMsl_ft'],'g',
            simData.time, simData.EnglishData['altitudeMsl_ft'], 'b')
    ah.legend([simCheckLabel,"pierpont"])
    ah.set(xlabel='time (s)', ylabel='Altitude (ft)', title='Flat Earth:␣
↪Altitude versus Time')
    ah.grid()

    fig1, ad = plt.subplots()
    ad.plot(checkData['time'], checkData['eulerAngle_deg_Roll'],'g',
            simData.time, simData.EnglishData['eulerAngle_deg_Roll'], 'b')
    ad.legend([simCheckLabel,"pierpont"])
    ad.set(xlabel='time (s)', ylabel='Roll (deg)', title='Flat Earth: Roll␣
↪versus Time')
    ad.grid()

    fig2, ap = plt.subplots()
    ap.plot(checkData['time'], checkData['eulerAngle_deg_Pitch'],'g',
            simData.time, simData.EnglishData['eulerAngle_deg_Pitch'], 'b')
    ap.legend([simCheckLabel,"pierpont"])
    ap.set(xlabel='time (s)', ylabel='Pitch (deg)', title='Flat Earth: Pitch␣
↪versus Time')
    ap.grid()

    fig3, ay = plt.subplots()
```

```
    ay.plot(checkData['time'], checkData['eulerAngle_deg_Yaw'],'g',
            simData.time, simData.EnglishData['eulerAngle_deg_Yaw'], 'b')
    ay.legend([simCheckLabel,"pierpont"])
    ay.set(xlabel='time (s)', ylabel='Yaw (deg)', title='Flat Earth: Yaw versus␣
↪Time')
    ay.grid()

    fig4, asp = plt.subplots()
    asp.plot(checkData['time'], checkData['trueAirspeed_nmi_h'],'g',
             simData.time, simData.EnglishData['trueAirspeed_nmi_h'], 'b')
    asp.legend([simCheckLabel,"pierpont"])
    asp.set(xlabel='time (s)', ylabel='airspeed (knots)', title='Flat Earth:␣
↪True Airspeed versus Time')
    asp.grid()
```

[37]:
```
import matplotlib.pyplot as plt
import matplotlib.gridspec as gridspec

def MakePlot(simData, checkData, simCaseLabel):
    fig1 = plt.figure(constrained_layout=True)
    spec1 = gridspec.GridSpec(ncols=2, nrows=1, figure=fig1)

    ax1 = fig1.add_subplot(spec1[0, 0])
    ax1.plot(checkData['time'], checkData['altitudeMsl_ft'],'g',
             simData.time, simData.EnglishData['altitudeMsl_ft'], 'b')
    ax1.set(xlabel='time (s)', ylabel='altitude (ft)', title='Altitude versus␣
↪Time')
    ax1.legend([simCaseLabel,"pierpont"])
    ax2 = fig1.add_subplot(spec1[0, 1])
    ax2.yaxis.tick_right()
    ax2.yaxis.set_label_position("right")
    ax2.plot(checkData['time'], checkData['localGravity_ft_s2'],'g',
             simData.time, simData.EnglishData['localGravity_ft_s2'], 'b')
    ax2.set(xlabel='time (s)', ylabel='localGravity (ft_s2)', title='Gravity␣
↪versus Time')
    ax2.legend([simCaseLabel,"pierpont"])

    fig2, ad = plt.subplots()
    ad.plot(checkData['time'], checkData['gePosition_ft_X'],'g',
            simData.time, simData.EnglishData['gePosition_ft_X'], 'b')
    ad.legend([simCaseLabel,"pierpont"])
    ad.set(xlabel='time (s)', ylabel='gePosition_ft_X', title='ECEF X versus␣
↪Time')
    ad.grid()

    fig5, ad = plt.subplots()
    ad.plot(checkData['time'], checkData['eulerAngle_deg_Roll'],'g',
```

```
            simData.time, simData.EnglishData['eulerAngle_deg_Roll'], 'b')
    ad.legend([simCaseLabel,"pierpont"])
    ad.set(xlabel='time (s)', ylabel='Roll (deg)', title='Oblate Earth: Roll␣
↪versus Time')
    ad.grid()

    fig5a, ap = plt.subplots()
    ap.plot(checkData['time'], checkData['eulerAngle_deg_Pitch'],'g',
            simData.time, simData.EnglishData['eulerAngle_deg_Pitch'], 'b')
    ap.legend([simCaseLabel,"pierpont"])
    ap.set(xlabel='time (s)', ylabel='Pitch (deg)', title='Oblate Earth: Pitch␣
↪versus Time')
    ap.grid()

    fig5b, ay = plt.subplots()
    ay.plot(checkData['time'], checkData['eulerAngle_deg_Yaw'],'g',
            simData.time, simData.EnglishData['eulerAngle_deg_Yaw'], 'b')
    ay.legend([simCaseLabel,"pierpont"])
    ay.set(xlabel='time (s)', ylabel='Yaw (deg)', title='Oblate Earth: Yaw␣
↪versus Time')
    ay.grid()
```

1.10.3 Dragless Sphere - 1

Property	English Value	SI Value
I_{xx}	3.6 slug-ft^2	4.881 kg-m^2
I_{yy}	3.6 slug-ft^2	4.881 kg-m^2
I_{zz}	3.6 slug-ft^2	4.881 kg-m^2
m	1.0 slug	14.5939 kg
S	0.1963495 ft^2	0.0182414654525 m^2

[38]:
```
ixx = 3.6 * gvConvert.Slugft2ToKgm2
print("ixx=", ixx)
mass = 1.0 * gvConvert.SlugToKg
print("mass=", mass)
S = 0.1963495 * gvConvert.SqFeetToSqMeter
print("S=", S)
```

```
ixx= 4.88094466281336
mass= 14.593902937
S= 0.018241465452480003
```

Flat Earth

[39]:
```
%%time
#
print("====================")
```

```python
checkFile = "NESC-check-cases/Atmospheric_checkcases/Atmos_01_DroppedSphere/
↪Atmos_01_sim_01.csv"
gvCC1 = GetCheckCaseData(checkFile)
#
print("====================")
printDebugData = False
gvAeroModel.LoadDml('models/noAero.dml', printDebugData)
#
# 1 slug = 14.5939 kg
# 3.6 slug-ft2 = 4.881 kg-m2
# 30000 ft = 9144 m
# 0.1963495 ft2 = 0.0182414654525 m2
#
ic = {
    "totalMass": [1.0, "slug"],
    "bodyMomentOfInertia_X": [3.6, "slugft2"],
    "bodyMomentOfInertia_Y": [3.6, "slugft2"],
    "bodyMomentOfInertia_Z": [3.6, "slugft2"],
    "altitudeMsl": [30000, "ft"],
    "referenceWingChord": [0.2, "ft"],
    "referenceWingSpan": [0.2, "ft"],
    "referenceWingArea": [0.1963495, "ft2"]
}
gvFlatEarthSim = ppFlatEarth()
gvFlatEarthSim.Reset(ic)
gvFlatEarthSim.Run(30.0)
gvFlatEarthSim.GenerateEnglishUnits()
```

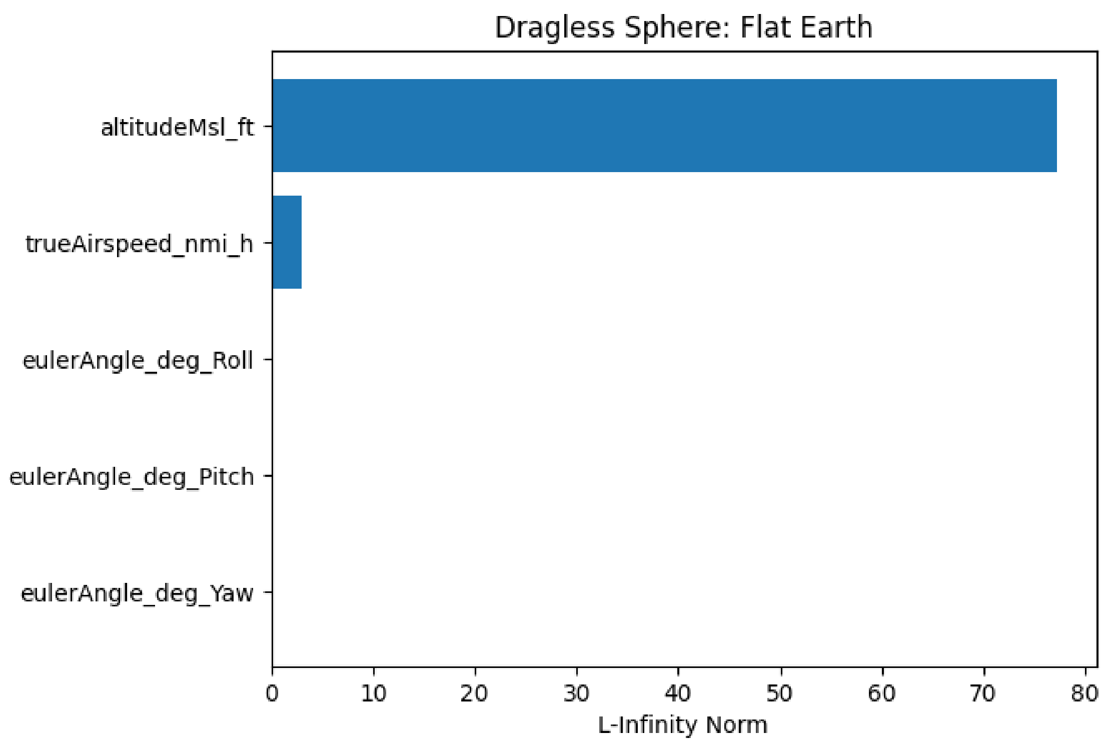
```
====================
number of headers:  31
['time', 'gePosition_ft_X', 'gePosition_ft_Y', 'gePosition_ft_Z',
'feVelocity_ft_s_X', 'feVelocity_ft_s_Y', 'feVelocity_ft_s_Z', 'altitudeMsl_ft',
'longitude_deg', 'latitude_deg', 'localGravity_ft_s2', 'eulerAngle_deg_Yaw',
'eulerAngle_deg_Pitch', 'eulerAngle_deg_Roll',
'bodyAngularRateWrtEi_deg_s_Roll', 'bodyAngularRateWrtEi_deg_s_Pitch',
'bodyAngularRateWrtEi_deg_s_Yaw', 'altitudeRateWrtMsl_ft_min',
'speedOfSound_ft_s', 'airDensity_slug_ft3', 'ambientPressure_lbf_ft2',
'ambientTemperature_dgR', 'aero_bodyForce_lbf_X', 'aero_bodyForce_lbf_Y',
'aero_bodyForce_lbf_Z', 'aero_bodyMoment_ftlbf_L', 'aero_bodyMoment_ftlbf_M',
'aero_bodyMoment_ftlbf_N', 'mach', 'dynamicPressure_lbf_ft2',
'trueAirspeed_nmi_h']
====================
*******************************************
Model:  Zero Aero Output
creation date:  2021-07-21
file version:  Initial version
*******************************************
```

```
+++++ MODEL INPUTS AND OUTPUTS +++++
++> Output:  CX
++> Output:  CY
++> Output:  CZ
++> Output:  CLL
++> Output:  CLM
++> Output:  CLN
++++++++++++++++++++++++++++++++++++

----- DAVE-ML MODEL PARSE COMPLETE -----
========= SetIC ==============
{'totalMass': 14.593902937, 'bodyMomentOfInertia_X': 4.88094466281336,
'bodyMomentOfInertia_Y': 4.88094466281336, 'bodyMomentOfInertia_Z':
4.88094466281336, 'altitudeMsl': 9144.0, 'referenceWingChord':
0.06096000000000001, 'referenceWingSpan': 0.06096000000000001,
'referenceWingArea': 0.018241465452480003}
++ timeStep = 0.1 [default]
++ totalMass = 14.593902937 [IC case]
++ referenceWingSpan = 0.06096000000000001 [IC case]
++ referenceWingChord = 0.06096000000000001 [IC case]
++ referenceWingArea = 0.018241465452480003 [IC case]
++ trueAirspeed = 0 [default]
++ angleOfAttack = 0 [default]
++ angleOfSideslip = 0 [default]
++ eulerAngle_Roll = 0 [default]
++ eulerAngle_Pitch = 0 [default]
++ eulerAngle_Yaw = 0 [default]
++ eulerAngleRate_Roll = 0 [default]
++ eulerAngleRate_Pitch = 0 [default]
++ eulerAngleRate_Yaw = 0 [default]
++ bodyMomentOfInertia_X = 4.88094466281336 [IC case]
++ bodyProductOfInertia_XY = 0 [default]
++ bodyProductOfInertia_XZ = 0 [default]
++ bodyProductOfInertia_YX = 0 [default]
++ bodyMomentOfInertia_Y = 4.88094466281336 [IC case]
++ bodyProductOfInertia_YZ = 0 [default]
++ bodyProductOfInertia_XZ = 0 [default]
++ bodyProductOfInertia_YZ = 0 [default]
++ bodyMomentOfInertia_Z = 4.88094466281336 [IC case]
++ altitudeMsl = 9144.0 [IC case]
======done=======
CPU times: user 34.2 ms, sys: 2.99 ms, total: 37.1 ms
Wall time: 40.3 ms
```

[40]:
```
gvFlatEarthLabel = \
[
    'altitudeMsl_ft', 'trueAirspeed_nmi_h',
```

```
    'eulerAngle_deg_Roll', 'eulerAngle_deg_Pitch', 'eulerAngle_deg_Yaw'
]

PrintErrorTable("Dragless Sphere: Flat Earth", gvFlatEarthLabel,␣
↪gvFlatEarthSim, gvCC1)
```

```
Variable                  L2                        L-Infinity-Norm
--------                  --                        ---------------
altitudeMsl_ft            607.9947105071013         77.22782399366406
trueAirspeed_nmi_h        30.300144397638636        2.917378529996199
eulerAngle_deg_Roll       1.2569020649616232        0.1253996817079627
eulerAngle_deg_Pitch      0.0                       0
eulerAngle_deg_Yaw        0.0                       0
```



Oblate, Rotating Earth

[41]:
```
%%time
ic = {
    "totalMass": [1.0, "slug"],
    "bodyMomentOfInertia_X": [3.6, "slugft2"],
    "bodyMomentOfInertia_Y": [3.6, "slugft2"],
    "bodyMomentOfInertia_Z": [3.6, "slugft2"],
    "altitudeMsl": [30000, "ft"],
    "referenceWingChord": [0.2, "ft"],
```

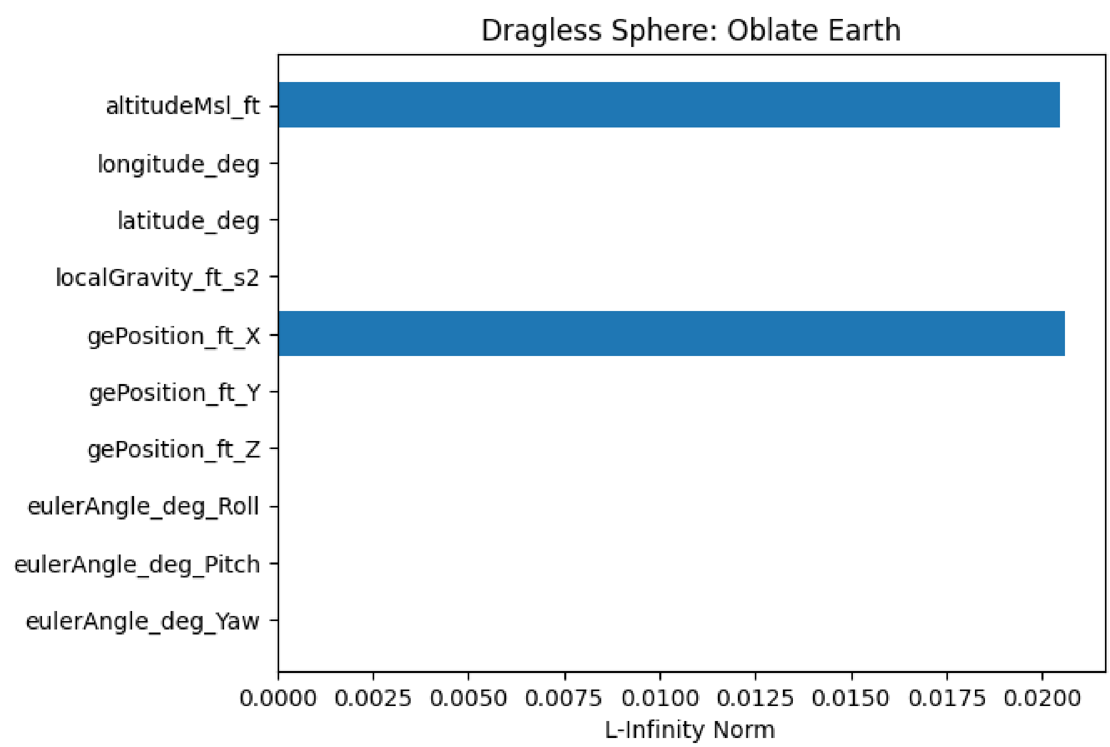
```python
    "referenceWingSpan": [0.2, "ft"],
    "referenceWingArea": [0.1963495, "ft2"]
}
gvOblateRotatingEarth = slEarthSim()
gvOblateRotatingEarth.Reset(ic)
gvOblateRotatingEarth.Run(30)
gvOblateRotatingEarth.GenerateEnglishUnits()
```

```
========= SetIC ==============
{'totalMass': 14.593902937, 'bodyMomentOfInertia_X': 4.88094466281336,
'bodyMomentOfInertia_Y': 4.88094466281336, 'bodyMomentOfInertia_Z':
4.88094466281336, 'altitudeMsl': 9144.0, 'referenceWingChord':
0.06096000000000001, 'referenceWingSpan': 0.06096000000000001,
'referenceWingArea': 0.018241465452480003}
++ timeStep = 0.1 [default]
++ totalMass = 14.593902937 [IC case]
++ referenceWingSpan = 0.06096000000000001 [IC case]
++ referenceWingChord = 0.06096000000000001 [IC case]
++ referenceWingArea = 0.018241465452480003 [IC case]
++ trueAirspeed = 0 [default]
++ angleOfAttack = 0 [default]
++ angleOfSideslip = 0 [default]
++ eulerAngle_Roll = 0 [default]
++ eulerAngle_Pitch = 0 [default]
++ eulerAngle_Yaw = 0 [default]
++ eulerAngleRate_Roll = 0 [default]
++ eulerAngleRate_Pitch = 0 [default]
++ eulerAngleRate_Yaw = 0 [default]
++ bodyMomentOfInertia_X = 4.88094466281336 [IC case]
++ bodyProductOfInertia_XY = 0 [default]
++ bodyProductOfInertia_XZ = 0 [default]
++ bodyProductOfInertia_YX = 0 [default]
++ bodyMomentOfInertia_Y = 4.88094466281336 [IC case]
++ bodyProductOfInertia_YZ = 0 [default]
++ bodyProductOfInertia_XZ = 0 [default]
++ bodyProductOfInertia_YZ = 0 [default]
++ bodyMomentOfInertia_Z = 4.88094466281336 [IC case]
++ latitude = 0 [default]
++ longitude = 0 [default]
++ altitudeMsl = 9144.0 [IC case]
Vecf:  0.0 0.0 0.0
======done=======
CPU times: user 41.5 ms, sys: 3.5 ms, total: 45 ms
Wall time: 45.1 ms
```

[42]:
```python
gvOblateEarthLabel = \
[
```

```
    'altitudeMsl_ft', 'longitude_deg', 'latitude_deg', 'localGravity_ft_s2',
    'gePosition_ft_X', 'gePosition_ft_Y', 'gePosition_ft_Z',
    'eulerAngle_deg_Roll', 'eulerAngle_deg_Pitch', 'eulerAngle_deg_Yaw'
]

PrintErrorTable("Dragless Sphere: Oblate Earth", gvOblateEarthLabel,␣
 ↪gvOblateRotatingEarth, gvCC1)
```

```
Variable                  L2                        L-Infinity-Norm
--------                  --                        ---------------
altitudeMsl_ft            0.132658503212526         0.02048735810240032
longitude_deg             1.840404761574716e-10     3.1393235335245e-11
latitude_deg              0.0                       0
localGravity_ft_s2        8.923802486168079e-05     9.171880087421869e-06
gePosition_ft_X           0.1343059782985569        0.020609743893146515
gePosition_ft_Y           6.717601039765743e-05     1.1453364173519276e-05
gePosition_ft_Z           0.0                       0
eulerAngle_deg_Roll       2.5437306921432218e-08    2.5527049640761135e-09
eulerAngle_deg_Pitch      3.632333118253663e-19     4.079942171239155e-20
eulerAngle_deg_Yaw        4.446769635169959e-16     3.7272118343786e-17
```



1.10.4 Dragless Tumbling Brick - 2

Property	English Value	SI Value
I_{xx}	0.001894220 slug-ft^2	0.002568217477249 kg-m^2
I_{yy}	0.006211019 slug-ft^2	0.00842101104799105 kg-m^2
I_{zz}	0.007194665 slug-ft^2	0.00975465595123675 kg-m^2
m	0.155404754 slug	2.2679619056149 kg
S	0.22222 ft^2	0.020644913548800003 m^2
b	0.33333 ft	0.1016 m
$\bar{c}$	0.66667 ft	0.2032 m

[43]:

```
mass = 0.155404754 * gvConvert.SlugToKg
print("m=", mass)

ixx = 0.001894220 * gvConvert.Slugft2ToKgm2
print("Ixx=", ixx)
iyy = 0.006211019 * gvConvert.Slugft2ToKgm2
print("Iyy=", iyy)
izz = 0.007194665 * gvConvert.Slugft2ToKgm2
print("Izz=", izz)

cbar = 0.66667 * gvConvert.FeetToMeter
print("cbar=", cbar)
b = 0.33333 * gvConvert.FeetToMeter
print("b=", b)
s = 0.22222 * gvConvert.SqFeetToSqMeter
print("s=", s)
```

```
m= 2.2679618958243624
Ixx= 0.002568217499776201
Iyy= 0.008421011121856215
Izz= 0.009754656036800024
cbar= 0.203201016
b= 0.101598984
s= 0.020644913548800003
```

Flat Earth

[44]:

```
%%time
checkFile = "NESC-check-cases/Atmospheric_checkcases/
↪Atmos_02_TumblingBrickNoDamping/Atmos_02_sim_01.csv"
gvCC2 = GetCheckCaseData(checkFile)
#
print("===================")
printDebugData = False
gvAeroModel.LoadDml('models/noAero.dml', printDebugData)
gvAeroModel.CheckModel()
#
ic = {
```

```
    "totalMass": [0.155404754, "slug"],
    "bodyMomentOfInertia_X": [0.001894220, "slugft2"],
    "bodyMomentOfInertia_Y": [0.006211019, "slugft2"],
    "bodyMomentOfInertia_Z": [0.007194665, "slugft2"],
    "altitudeMsl": [30000, "ft"],
    "referenceWingChord": [0.66667, "ft"],
    "referenceWingSpan": [0.33333, "ft"],
    "referenceWingArea": [0.22222, "ft"],
    "eulerAngleRate_Roll": [10, "deg_s"],
    "eulerAngleRate_Pitch": [20, "deg_s"],
    "eulerAngleRate_Yaw": [30, "deg_s"]
}
gvFlatEarthSim.Reset(ic)
gvFlatEarthSim.Run(30.0)
gvFlatEarthSim.GenerateEnglishUnits()
```

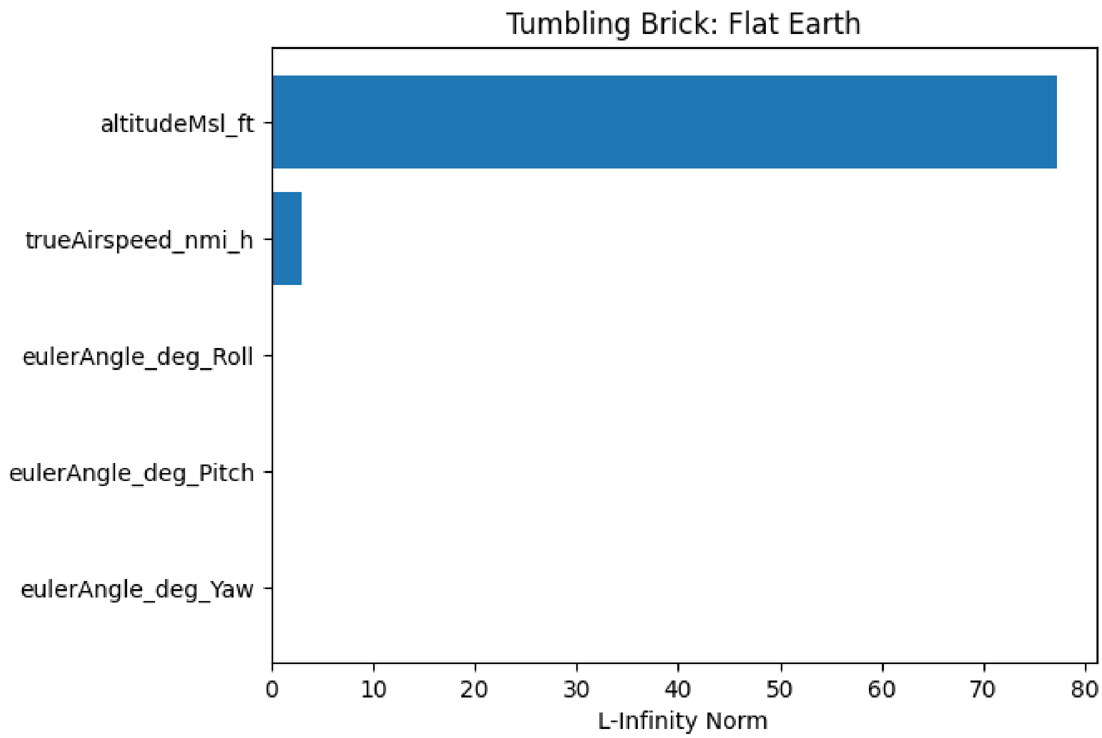
```
number of headers:  31
['time', 'gePosition_ft_X', 'gePosition_ft_Y', 'gePosition_ft_Z',
'feVelocity_ft_s_X', 'feVelocity_ft_s_Y', 'feVelocity_ft_s_Z', 'altitudeMsl_ft',
'longitude_deg', 'latitude_deg', 'localGravity_ft_s2', 'eulerAngle_deg_Yaw',
'eulerAngle_deg_Pitch', 'eulerAngle_deg_Roll',
'bodyAngularRateWrtEi_deg_s_Roll', 'bodyAngularRateWrtEi_deg_s_Pitch',
'bodyAngularRateWrtEi_deg_s_Yaw', 'altitudeRateWrtMsl_ft_min',
'speedOfSound_ft_s', 'airDensity_slug_ft3', 'ambientPressure_lbf_ft2',
'ambientTemperature_dgR', 'aero_bodyForce_lbf_X', 'aero_bodyForce_lbf_Y',
'aero_bodyForce_lbf_Z', 'aero_bodyMoment_ftlbf_L', 'aero_bodyMoment_ftlbf_M',
'aero_bodyMoment_ftlbf_N', 'mach', 'dynamicPressure_lbf_ft2',
'trueAirspeed_nmi_h']
====================
*******************************************
Model:  Zero Aero Output
creation date:  2021-07-21
file version:  Initial version
*******************************************

+++++ MODEL INPUTS AND OUTPUTS +++++
++> Output:  CX
++> Output:  CY
++> Output:  CZ
++> Output:  CLL
++> Output:  CLM
++> Output:  CLN
++++++++++++++++++++++++++++++++++++

----- DAVE-ML MODEL PARSE COMPLETE -----

----- CheckModel -----
```

```
numSignals:  []

----- END CheckModel -----

========= SetIC ==============
{'totalMass': 2.2679618958243624, 'bodyMomentOfInertia_X': 0.002568217499776201,
'bodyMomentOfInertia_Y': 0.008421011121856215, 'bodyMomentOfInertia_Z':
0.009754656036800024, 'altitudeMsl': 9144.0, 'referenceWingChord': 0.203201016,
'referenceWingSpan': 0.101598984, 'referenceWingArea': 0.067732656,
'eulerAngleRate_Roll': 0.17453292519943295, 'eulerAngleRate_Pitch':
0.3490658503988659, 'eulerAngleRate_Yaw': 0.5235987755982988}
++ timeStep = 0.1 [default]
++ totalMass = 2.2679618958243624 [IC case]
++ referenceWingSpan = 0.101598984 [IC case]
++ referenceWingChord = 0.203201016 [IC case]
++ referenceWingArea = 0.067732656 [IC case]
++ trueAirspeed = 0 [default]
++ angleOfAttack = 0 [default]
++ angleOfSideslip = 0 [default]
++ eulerAngle_Roll = 0 [default]
++ eulerAngle_Pitch = 0 [default]
++ eulerAngle_Yaw = 0 [default]
++ eulerAngleRate_Roll = 0.17453292519943295 [IC case]
++ eulerAngleRate_Pitch = 0.3490658503988659 [IC case]
++ eulerAngleRate_Yaw = 0.5235987755982988 [IC case]
++ bodyMomentOfInertia_X = 0.002568217499776201 [IC case]
++ bodyProductOfInertia_XY = 0 [default]
++ bodyProductOfInertia_XZ = 0 [default]
++ bodyProductOfInertia_YX = 0 [default]
++ bodyMomentOfInertia_Y = 0.008421011121856215 [IC case]
++ bodyProductOfInertia_YZ = 0 [default]
++ bodyProductOfInertia_XZ = 0 [default]
++ bodyProductOfInertia_YZ = 0 [default]
++ bodyMomentOfInertia_Z = 0.009754656036800024 [IC case]
++ altitudeMsl = 9144.0 [IC case]
======done=======
CPU times: user 51.9 ms, sys: 5.86 ms, total: 57.7 ms
Wall time: 69.5 ms
```

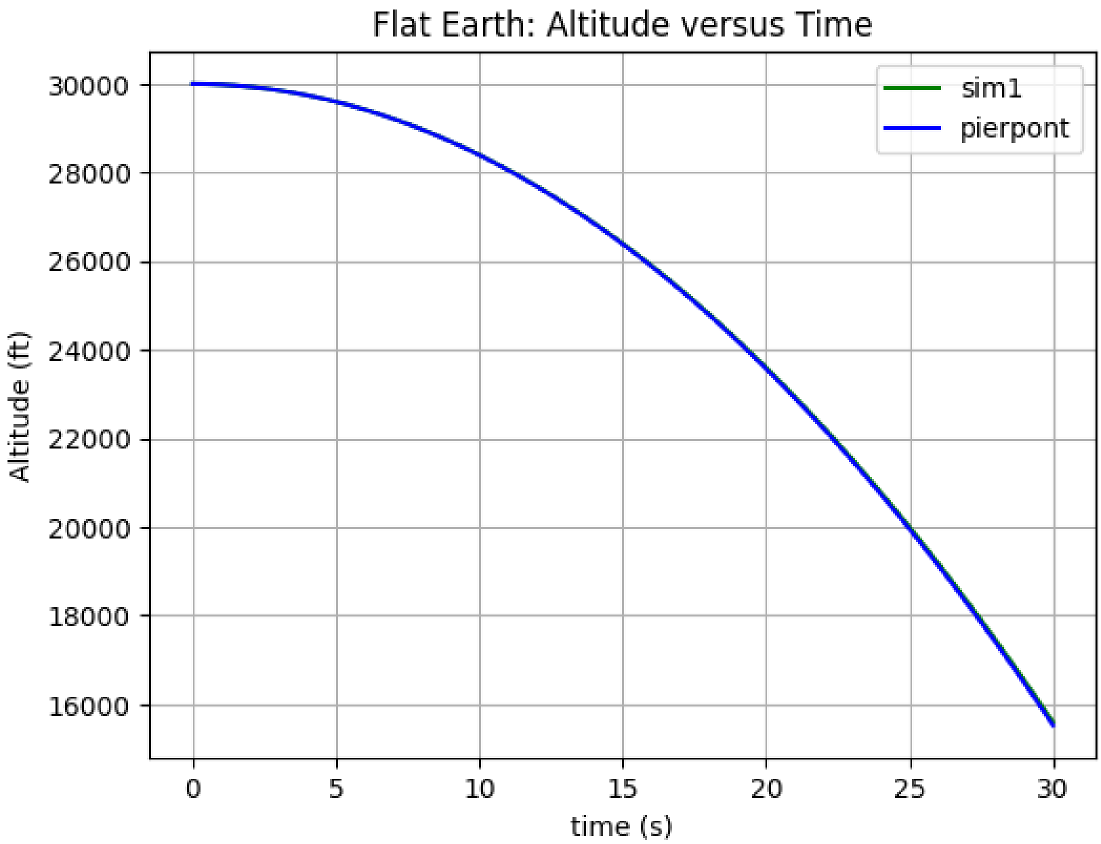
[45]:
```
PrintErrorTable("Tumbling Brick: Flat Earth", gvFlatEarthLabel, gvFlatEarthSim,␣
↪gvCC2)
```

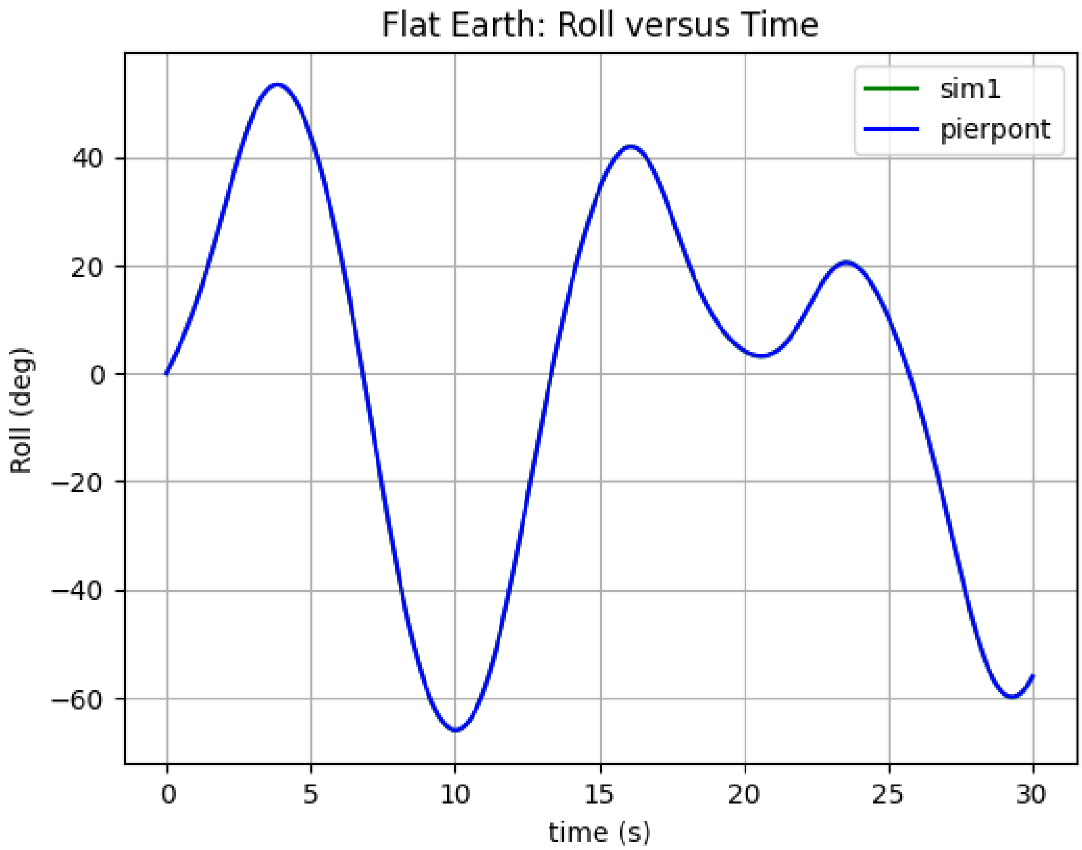
```
Variable                  L2                        L-Infinity-Norm
--------                  --                        ---------------
altitudeMsl_ft            607.972054527782          77.22500410951034
trueAirspeed_nmi_h        30.278799187883976        2.9152206853502776
eulerAngle_deg_Roll       0.9696393367797839        0.1253209330691334
eulerAngle_deg_Pitch      0.8533820770548749        0.11575857923671151
```

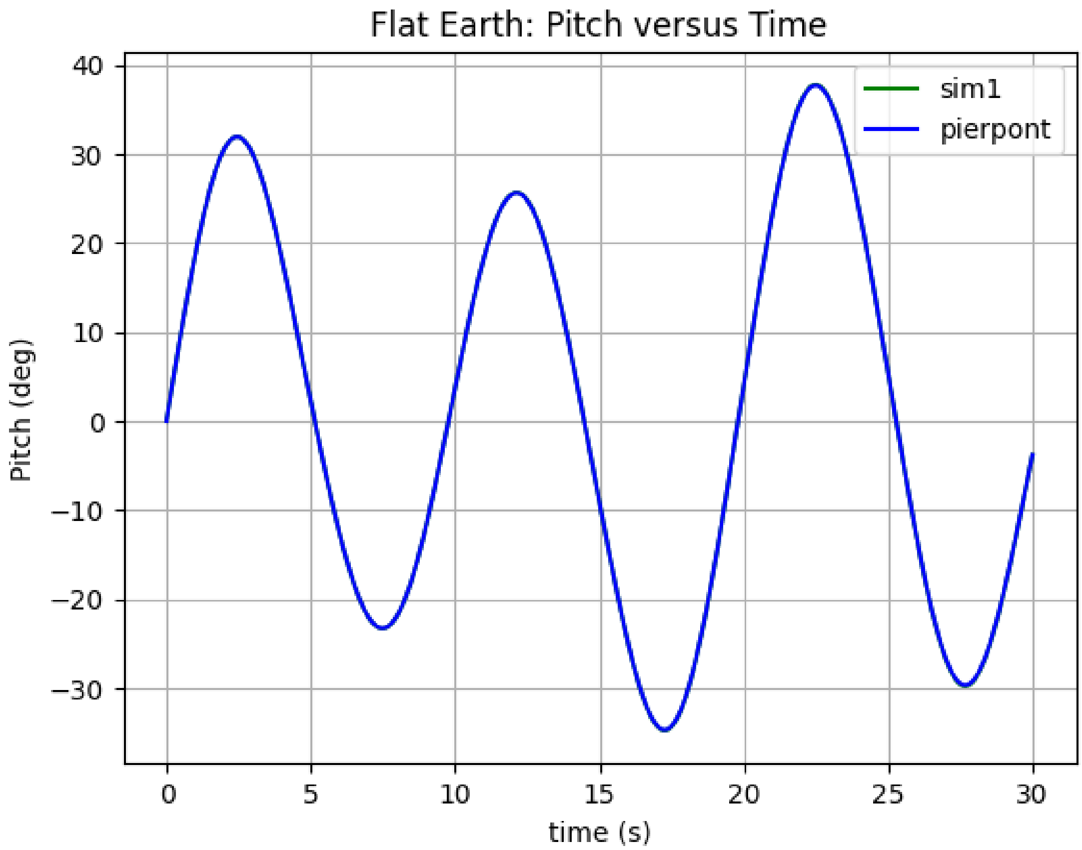
```
eulerAngle_deg_Yaw          0.2980578394513776          0.03904847535990541
```



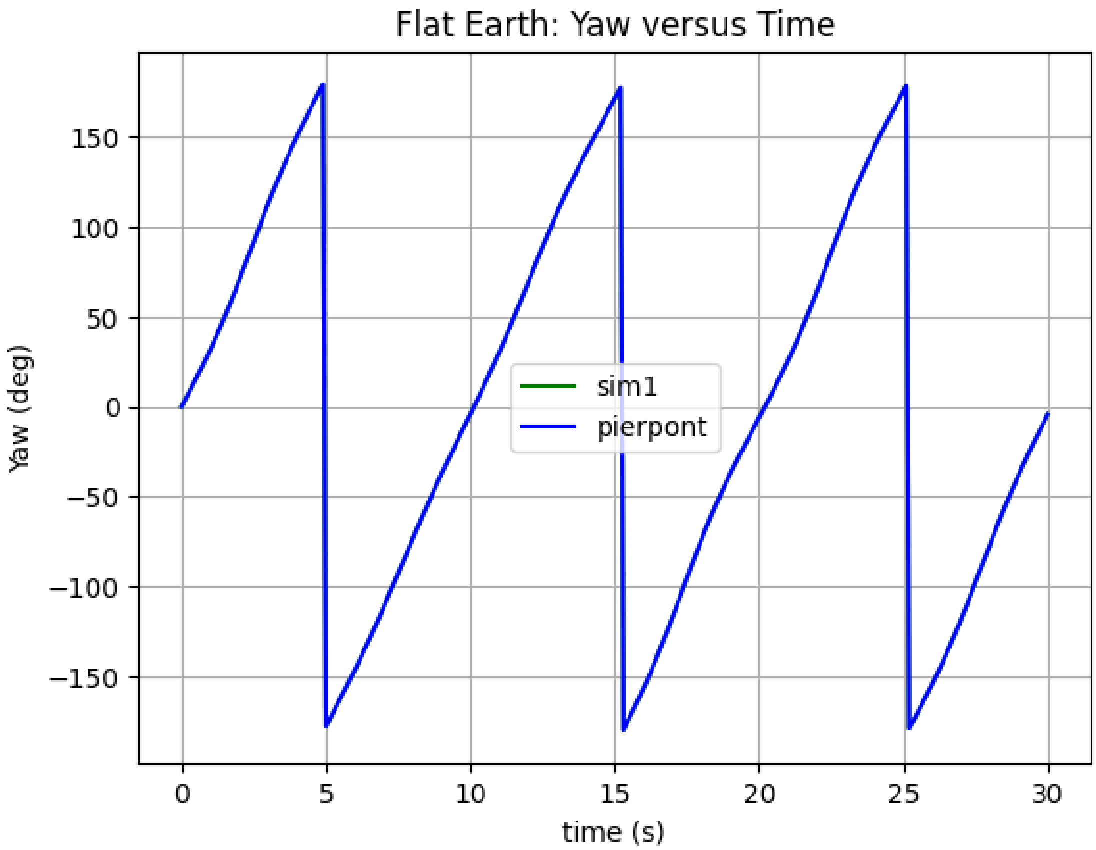
[46]:
```
MakeFlatEarthPlots(gvFlatEarthSim, gvCC2, "sim1")
```


Flat Earth: Altitude versus Time
sim1
pierpont
Altitude (ft)
30000
28000
26000
24000
22000
20000
18000
16000
0
5
10
15
20
25
30
time (s)

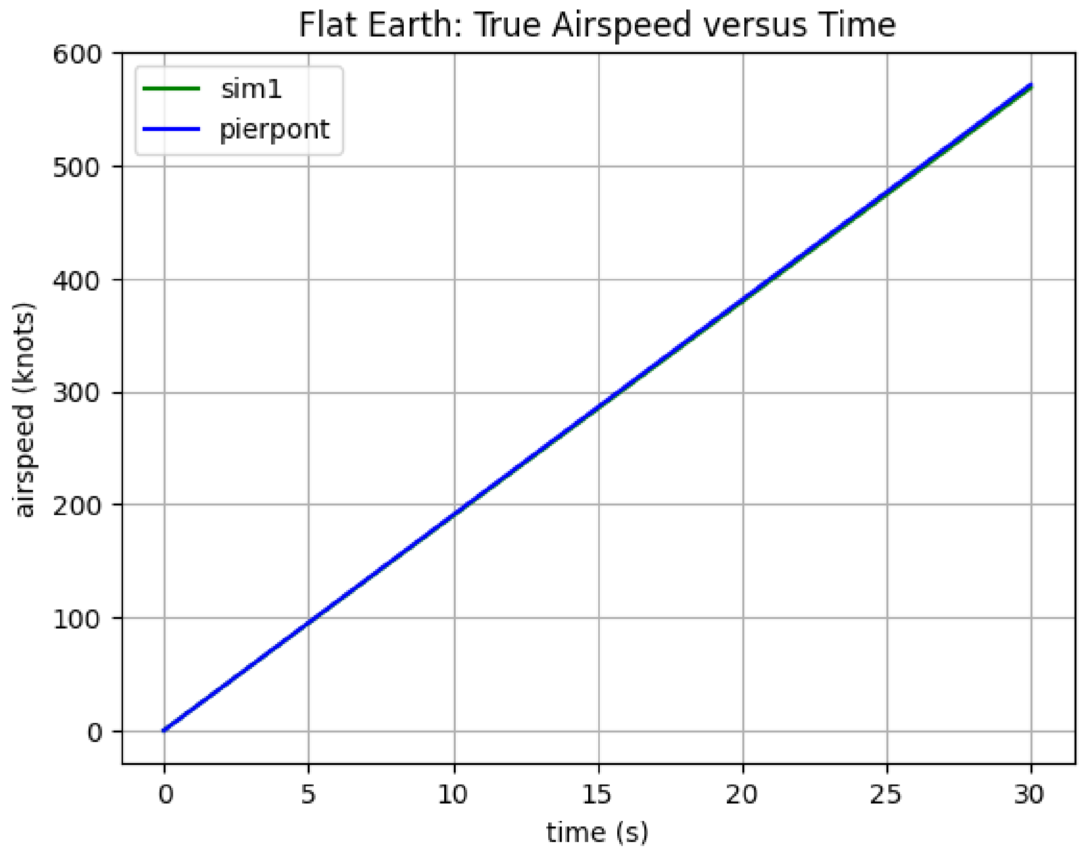

Flat Earth: Roll versus Time
sim1
pierpont
Roll (deg)
time (s)
40
20
0
−20
−40
−60
0
5
10
15
20
25
30



Flat Earth: Pitch versus Time
sim1
pierpont
Pitch (deg)
40
30
20
10
0
−10
−20
−30
time (s)
0
5
10
15
20
25
30



Flat Earth: Yaw versus Time
Yaw (deg)
150
100
50
0
−50
−100
−150
0
5
10
15
20
25
30
time (s)
sim1
pierpont



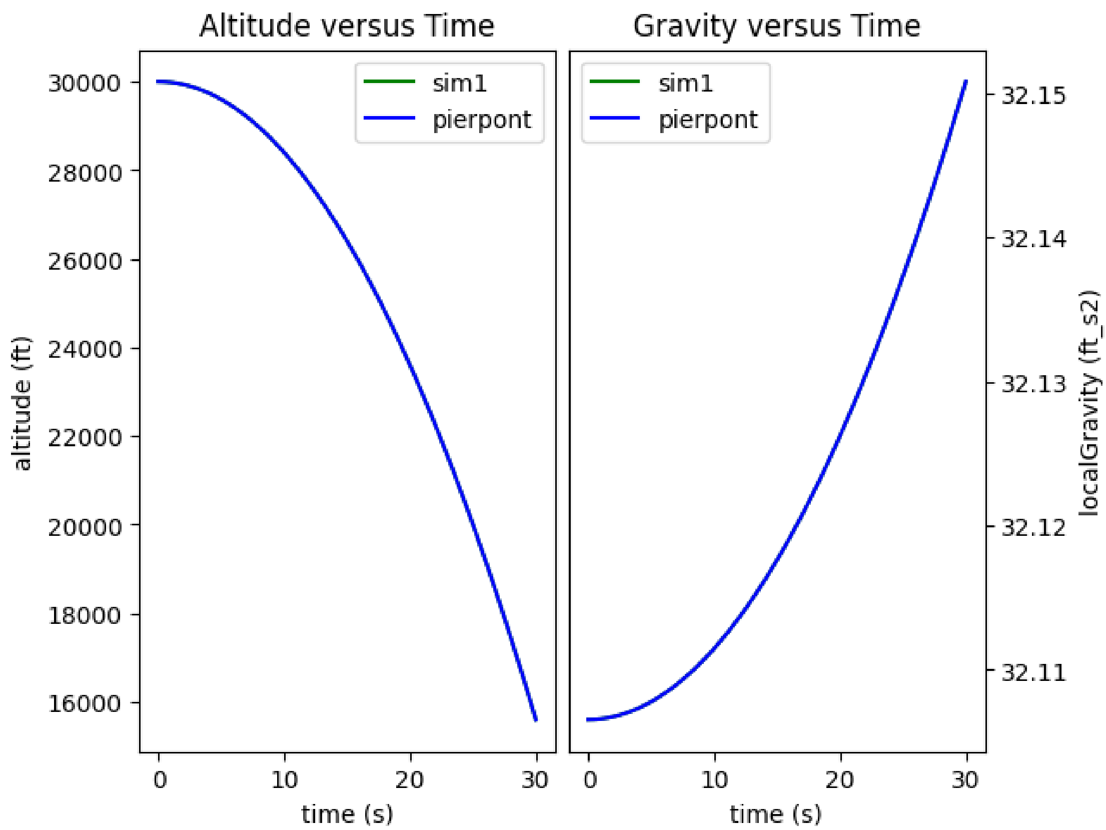
Oblate, Rotating Earth

[47]:
```
%%time
ic = {
    "totalMass": [0.155404754, "slug"],
    "bodyMomentOfInertia_X": [0.001894220, "slugft2"],
    "bodyMomentOfInertia_Y": [0.006211019, "slugft2"],
    "bodyMomentOfInertia_Z": [0.007194665, "slugft2"],
    "altitudeMsl": [30000, "ft"],
    "referenceWingChord": [0.66667, "ft"],
    "referenceWingSpan": [0.33333, "ft"],
    "referenceWingArea": [0.22222, "ft"],
    "eulerAngleRate_Roll": [10, "deg_s"],
    "eulerAngleRate_Pitch": [20, "deg_s"],
    "eulerAngleRate_Yaw": [30, "deg_s"]
}
gvOblateRotatingEarth.Reset(ic)
gvOblateRotatingEarth.Run(30)
gvOblateRotatingEarth.GenerateEnglishUnits()
```

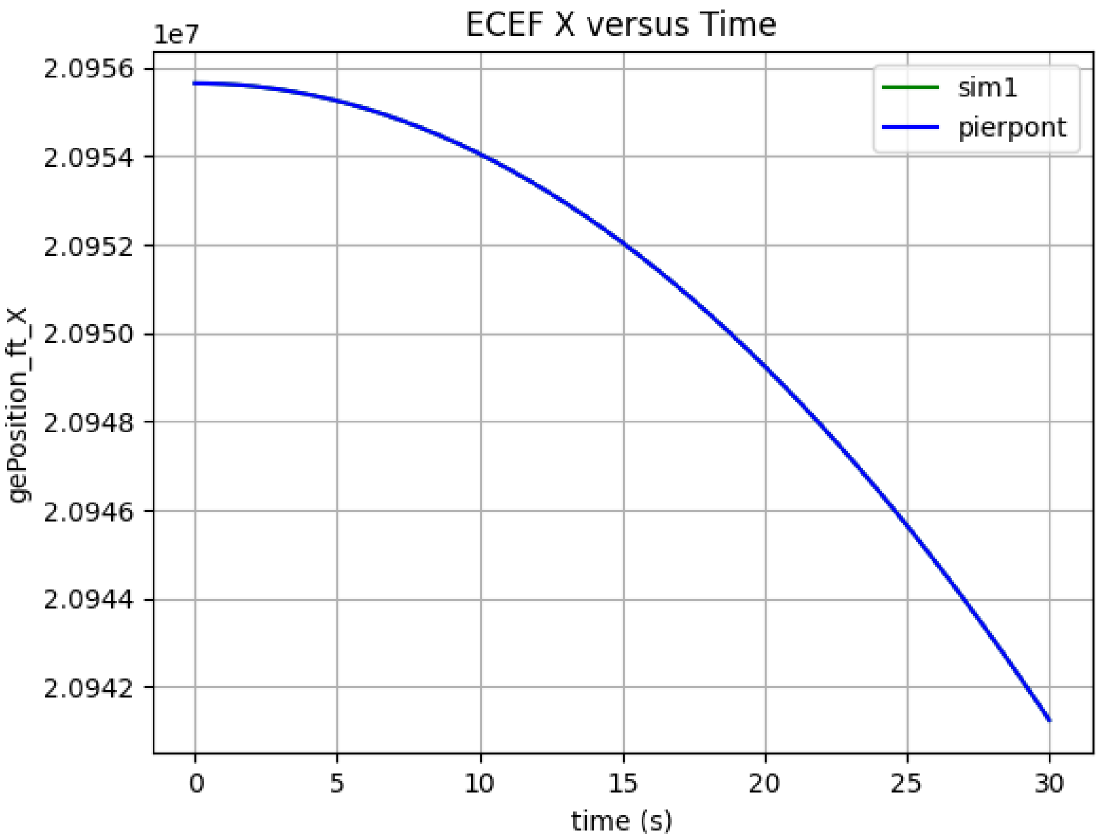
```
========= SetIC ==============
```

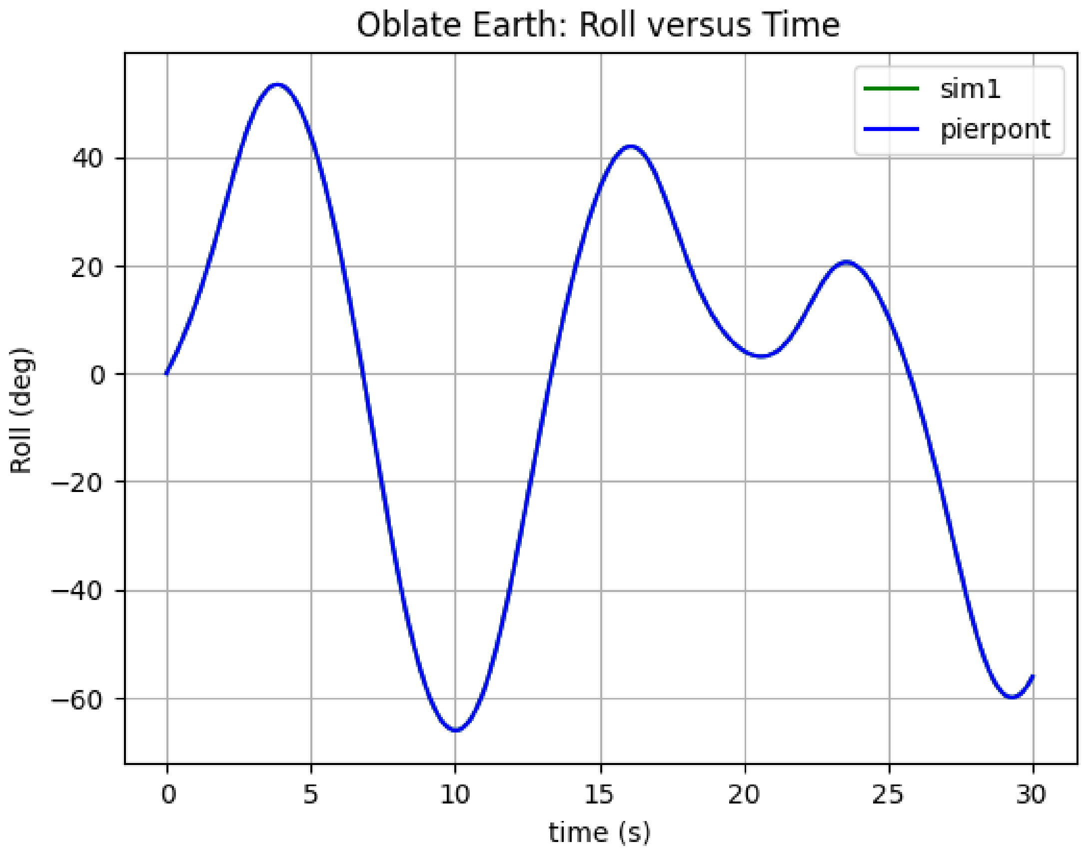
```
{'totalMass': 2.2679618958243624, 'bodyMomentOfInertia_X': 0.002568217499776201,
'bodyMomentOfInertia_Y': 0.008421011121856215, 'bodyMomentOfInertia_Z':
0.009754656036800024, 'altitudeMsl': 9144.0, 'referenceWingChord': 0.203201016,
'referenceWingSpan': 0.101598984, 'referenceWingArea': 0.067732656,
'eulerAngleRate_Roll': 0.17453292519943295, 'eulerAngleRate_Pitch':
0.3490658503988659, 'eulerAngleRate_Yaw': 0.5235987755982988}
++ timeStep = 0.1 [default]
++ totalMass = 2.2679618958243624 [IC case]
++ referenceWingSpan = 0.101598984 [IC case]
++ referenceWingChord = 0.203201016 [IC case]
++ referenceWingArea = 0.067732656 [IC case]
++ trueAirspeed = 0 [default]
++ angleOfAttack = 0 [default]
++ angleOfSideslip = 0 [default]
++ eulerAngle_Roll = 0 [default]
++ eulerAngle_Pitch = 0 [default]
++ eulerAngle_Yaw = 0 [default]
++ eulerAngleRate_Roll = 0.17453292519943295 [IC case]
++ eulerAngleRate_Pitch = 0.3490658503988659 [IC case]
++ eulerAngleRate_Yaw = 0.5235987755982988 [IC case]
++ bodyMomentOfInertia_X = 0.002568217499776201 [IC case]
++ bodyProductOfInertia_XY = 0 [default]
++ bodyProductOfInertia_XZ = 0 [default]
++ bodyProductOfInertia_YX = 0 [default]
++ bodyMomentOfInertia_Y = 0.008421011121856215 [IC case]
++ bodyProductOfInertia_YZ = 0 [default]
++ bodyProductOfInertia_XZ = 0 [default]
++ bodyProductOfInertia_YZ = 0 [default]
++ bodyMomentOfInertia_Z = 0.009754656036800024 [IC case]
++ latitude = 0 [default]
++ longitude = 0 [default]
++ altitudeMsl = 9144.0 [IC case]
Vecf:  0.0 0.0 0.0
======done=======
CPU times: user 62.6 ms, sys: 7.18 ms, total: 69.8 ms
Wall time: 141 ms
```

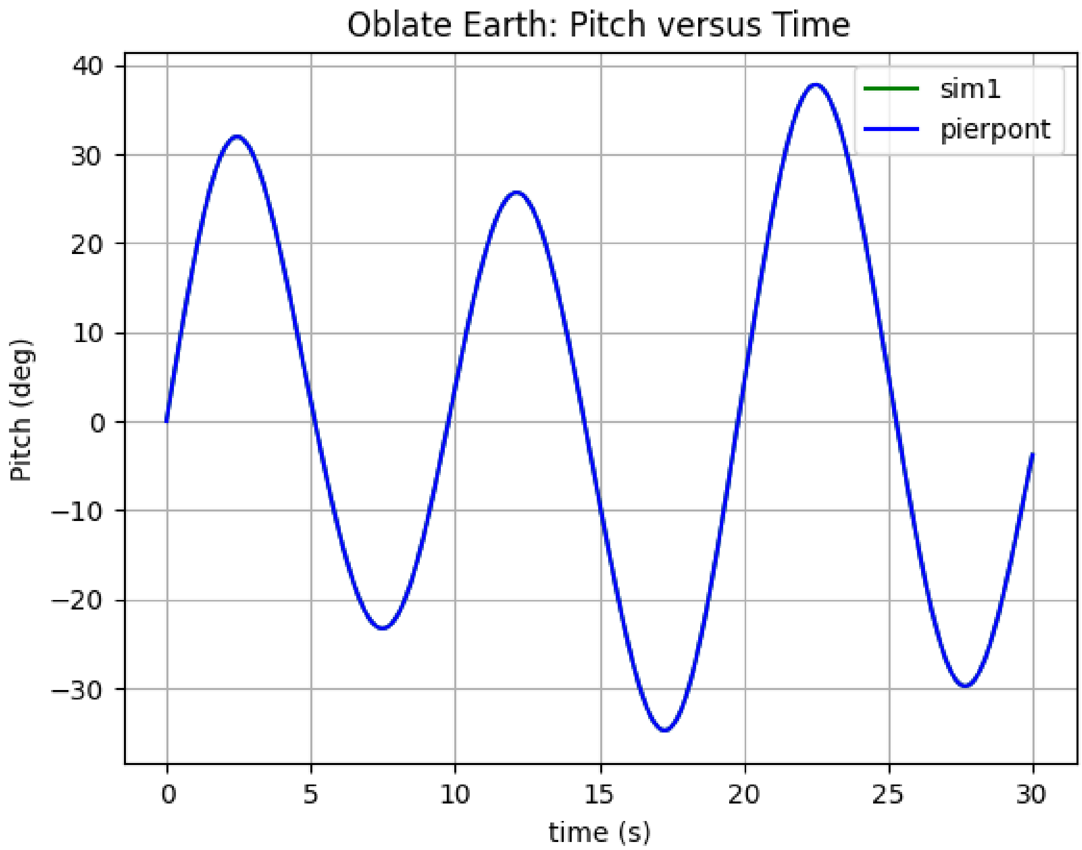
[48]:
```
MakePlot(gvOblateRotatingEarth, gvCC2, "sim1")
```

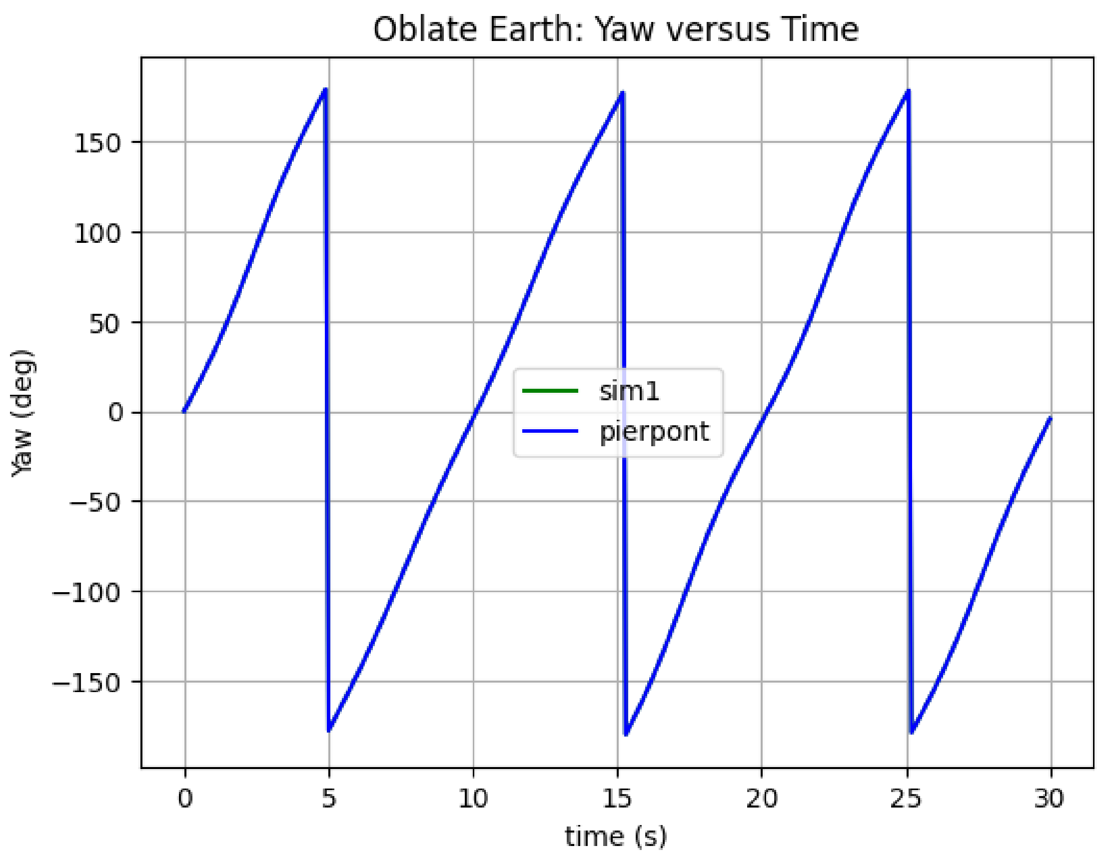


Altitude versus Time
Gravity versus Time
sim1
pierpont
sim1
pierpont
30000
28000
26000
24000
22000
20000
18000
16000
altitude (ft)
0
10
20
30
time (s)
0
10
20
30
time (s)
32.15
32.14
32.13
32.12
32.11
localGravity (ft_s2)


ECEF X versus Time
1e7
2.0956
2.0954
2.0952
2.0950
2.0948
2.0946
2.0944
2.0942
gePosition_ft_X
sim1
pierpont
0
5
10
15
20
25
30
time (s)

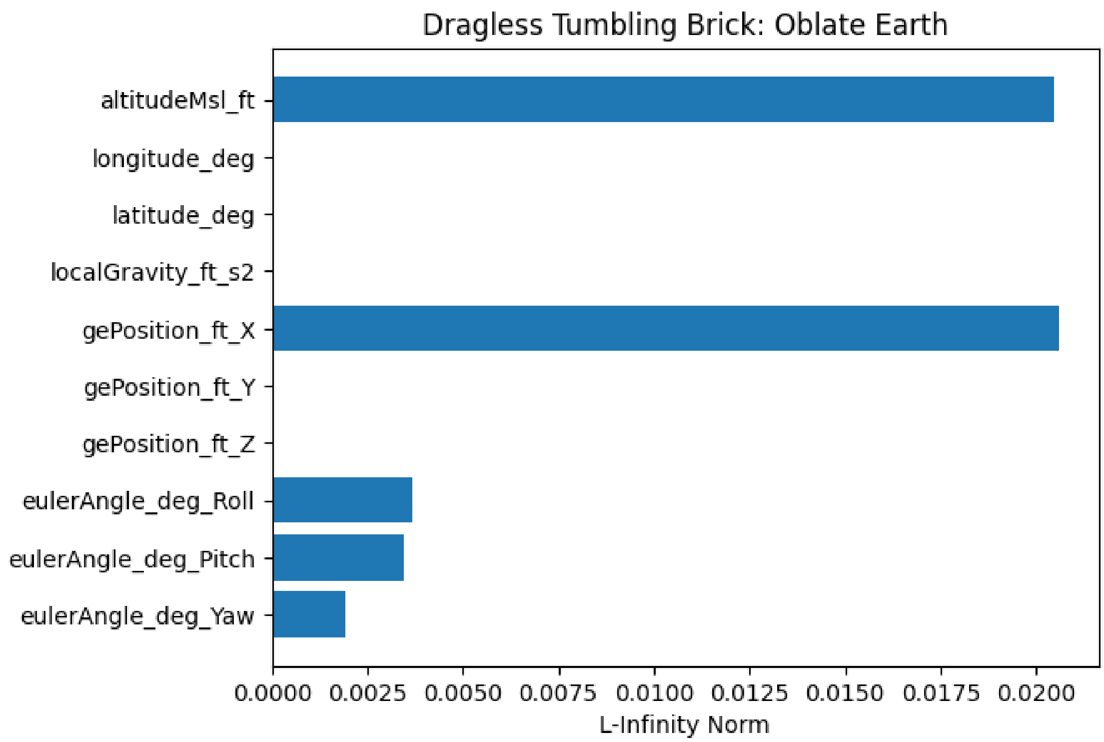

Oblate Earth: Roll versus Time
sim1
pierpont
Roll (deg)
time (s)
40
20
0
−20
−40
−60
0
5
10
15
20
25
30


Oblate Earth: Pitch versus Time
sim1
pierpont
Pitch (deg)
time (s)
40
30
20
10
0
−10
−20
−30
0
5
10
15
20
25
30



[49]:
```
PrintErrorTable("Dragless Tumbling Brick: Oblate Earth", gvOblateEarthLabel,␣
 ↪gvOblateRotatingEarth, gvCC2)
```

```
Variable                  L2                        L-Infinity-Norm
--------                  --                        ---------------
altitudeMsl_ft            0.132658503212526         0.02048735810240032
longitude_deg             1.840404761574716e-10     3.1393235335245e-11
latitude_deg              0.0                       0
localGravity_ft_s2        8.923802486168079e-05     9.171880087421869e-06
gePosition_ft_X           0.1343059782985569        0.020609743893146515
gePosition_ft_Y           6.717601039765743e-05     1.1453364173519276e-05
gePosition_ft_Z           0.0                       0
eulerAngle_deg_Roll       0.02661660809333747       0.003678374584481503
eulerAngle_deg_Pitch      0.025190519978331032      0.0034552761165556056
eulerAngle_deg_Yaw        0.01326283857690703       0.0019424252806858888
```



1.10.5 Tumbling Brick Damping - 3

Tumbling brick with damping check case

Flat Earth Run a simulation for 30 seconds at a time step of 0.1 seconds.

[50]:
```
%%time
checkFile = "NESC-check-cases/Atmospheric_checkcases/
↪Atmos_03_TumblingBrickDamping/Atmos_03_sim_01.csv"
gvCC3 = GetCheckCaseData(checkFile)
#
print("====================")
gvAeroModel.LoadDml('models/NESC/brick_aero_mod.dml')
gvAeroModel.CheckModel()
#
ic = {
    "totalMass": [0.155404754, "slug"],
    "bodyMomentOfInertia_X": [0.001894220, "slugft2"],
    "bodyMomentOfInertia_Y": [0.006211019, "slugft2"],
    "bodyMomentOfInertia_Z": [0.007194665, "slugft2"],
    "altitudeMsl": [30000, "ft"],
    "referenceWingChord": [0.66667, "ft"],
    "referenceWingSpan": [0.33333, "ft"],
```

```
    "referenceWingArea": [0.22222, "ft2"],
    "eulerAngleRate_Roll": [10, "deg_s"],
    "eulerAngleRate_Pitch": [20, "deg_s"],
    "eulerAngleRate_Yaw": [30, "deg_s"]
}
inputs = ["trueAirspeed", "bodyAngularRate_Roll", "bodyAngularRate_Pitch",␣
 ↪"bodyAngularRate_Yaw"]
gvFlatEarthSim.Reset(ic)
gvFlatEarthSim.AddAeroModelInput(inputs)
gvFlatEarthSim.Run(30.0)
gvFlatEarthSim.GenerateEnglishUnits()
```

```
number of headers:  31
['time', 'gePosition_ft_X', 'gePosition_ft_Y', 'gePosition_ft_Z',
'feVelocity_ft_s_X', 'feVelocity_ft_s_Y', 'feVelocity_ft_s_Z', 'altitudeMsl_ft',
'longitude_deg', 'latitude_deg', 'localGravity_ft_s2', 'eulerAngle_deg_Yaw',
'eulerAngle_deg_Pitch', 'eulerAngle_deg_Roll',
'bodyAngularRateWrtEi_deg_s_Roll', 'bodyAngularRateWrtEi_deg_s_Pitch',
'bodyAngularRateWrtEi_deg_s_Yaw', 'altitudeRateWrtMsl_ft_min',
'speedOfSound_ft_s', 'airDensity_slug_ft3', 'ambientPressure_lbf_ft2',
'ambientTemperature_dgR', 'aero_bodyForce_lbf_X', 'aero_bodyForce_lbf_Y',
'aero_bodyForce_lbf_Z', 'aero_bodyMoment_ftlbf_L', 'aero_bodyMoment_ftlbf_M',
'aero_bodyMoment_ftlbf_N', 'mach', 'dynamicPressure_lbf_ft2',
'trueAirspeed_nmi_h']
====================
*******************************************
Model:  Example brick aerodynamic model
creation date:  2012-10-05
file version:  Mod D, 2021-05-01
*******************************************

-variableDef-
 varDefStruct.name:  referenceWingArea
 varDefStruct.varID:  SWING
 varDefStruct.units:  ft2
 varDefStruct.axisSystem:  None
 varDefStruct.sign:  None
 varDefStruct.alias:  None
 varDefStruct.symbol:  None
 varDefStruct.hasInitialValue:  True
 varDefStruct.initialValue:  0.22222
 varDefStruct.isStdAIAA:  True
 varDefStruct.isOutput:  False
 varDefStruct.hasMath:  False
 varDefStruct.codeText:  None
-variableDef-
 varDefStruct.name:  referenceWingSpan
 varDefStruct.varID:  BSPAN
```

```
 varDefStruct.units:  ft
 varDefStruct.axisSystem:  None
 varDefStruct.sign:  None
 varDefStruct.alias:  None
 varDefStruct.symbol:  None
 varDefStruct.hasInitialValue:  True
 varDefStruct.initialValue:  0.33333
 varDefStruct.isStdAIAA:  True
 varDefStruct.isOutput:  False
 varDefStruct.hasMath:  False
 varDefStruct.codeText:  None
-variableDef-
 varDefStruct.name:  referenceWingChord
 varDefStruct.varID:  CBAR
 varDefStruct.units:  ft
 varDefStruct.axisSystem:  None
 varDefStruct.sign:  None
 varDefStruct.alias:  None
 varDefStruct.symbol:  None
 varDefStruct.hasInitialValue:  True
 varDefStruct.initialValue:  0.66667
 varDefStruct.isStdAIAA:  True
 varDefStruct.isOutput:  False
 varDefStruct.hasMath:  False
 varDefStruct.codeText:  None
-variableDef-
 varDefStruct.name:  roll damping from roll rate
 varDefStruct.varID:  CLP_DAMPING
 varDefStruct.units:  _rad
 varDefStruct.axisSystem:  None
 varDefStruct.sign:  None
 varDefStruct.alias:  None
 varDefStruct.symbol:  None
 varDefStruct.hasInitialValue:  True
 varDefStruct.initialValue:  -1.0
 varDefStruct.isStdAIAA:  False
 varDefStruct.isOutput:  False
 varDefStruct.hasMath:  False
 varDefStruct.codeText:  None
-variableDef-
 varDefStruct.name:  roll damping from yaw rate
 varDefStruct.varID:  CLR_DAMPING
 varDefStruct.units:  _rad
 varDefStruct.axisSystem:  None
 varDefStruct.sign:  None
 varDefStruct.alias:  None
 varDefStruct.symbol:  None
 varDefStruct.hasInitialValue:  True
```

```
 varDefStruct.initialValue:  0.0
 varDefStruct.isStdAIAA:  False
 varDefStruct.isOutput:  False
 varDefStruct.hasMath:  False
 varDefStruct.codeText:  None
-variableDef-
 varDefStruct.name:  pitch damping from pitch rate
 varDefStruct.varID:  CMQ_DAMPING
 varDefStruct.units:  _rad
 varDefStruct.axisSystem:  None
 varDefStruct.sign:  None
 varDefStruct.alias:  None
 varDefStruct.symbol:  None
 varDefStruct.hasInitialValue:  True
 varDefStruct.initialValue:  -1.0
 varDefStruct.isStdAIAA:  False
 varDefStruct.isOutput:  False
 varDefStruct.hasMath:  False
 varDefStruct.codeText:  None
-variableDef-
 varDefStruct.name:  yaw damping from roll rate
 varDefStruct.varID:  CNP_DAMPING
 varDefStruct.units:  _rad
 varDefStruct.axisSystem:  None
 varDefStruct.sign:  None
 varDefStruct.alias:  None
 varDefStruct.symbol:  None
 varDefStruct.hasInitialValue:  True
 varDefStruct.initialValue:  0.0
 varDefStruct.isStdAIAA:  False
 varDefStruct.isOutput:  False
 varDefStruct.hasMath:  False
 varDefStruct.codeText:  None
-variableDef-
 varDefStruct.name:  yaw damping from yaw rate
 varDefStruct.varID:  CNR_DAMPING
 varDefStruct.units:  _rad
 varDefStruct.axisSystem:  None
 varDefStruct.sign:  None
 varDefStruct.alias:  None
 varDefStruct.symbol:  None
 varDefStruct.hasInitialValue:  True
 varDefStruct.initialValue:  -1.0
 varDefStruct.isStdAIAA:  False
 varDefStruct.isOutput:  False
 varDefStruct.hasMath:  False
 varDefStruct.codeText:  None
-variableDef-
```

```
 varDefStruct.name:  trueAirspeed
 varDefStruct.varID:  VRW
 varDefStruct.units:  ft_s
 varDefStruct.axisSystem:  None
 varDefStruct.sign:  None
 varDefStruct.alias:  None
 varDefStruct.symbol:  None
 varDefStruct.hasInitialValue:  False
 varDefStruct.initialValue:  None
 varDefStruct.isStdAIAA:  True
 varDefStruct.isOutput:  False
 varDefStruct.hasMath:  False
 varDefStruct.codeText:  None
-variableDef-
 varDefStruct.name:  bodyAngularRate_Roll
 varDefStruct.varID:  PB
 varDefStruct.units:  rad_s
 varDefStruct.axisSystem:  None
 varDefStruct.sign:  RWD
 varDefStruct.alias:  None
 varDefStruct.symbol:  None
 varDefStruct.hasInitialValue:  False
 varDefStruct.initialValue:  None
 varDefStruct.isStdAIAA:  True
 varDefStruct.isOutput:  False
 varDefStruct.hasMath:  False
 varDefStruct.codeText:  None
-variableDef-
 varDefStruct.name:  bodyAngularRate_Pitch
 varDefStruct.varID:  QB
 varDefStruct.units:  rad_s
 varDefStruct.axisSystem:  None
 varDefStruct.sign:  ANU
 varDefStruct.alias:  None
 varDefStruct.symbol:  None
 varDefStruct.hasInitialValue:  False
 varDefStruct.initialValue:  None
 varDefStruct.isStdAIAA:  True
 varDefStruct.isOutput:  False
 varDefStruct.hasMath:  False
 varDefStruct.codeText:  None
-variableDef-
 varDefStruct.name:  bodyAngularRate_Yaw
 varDefStruct.varID:  RB
 varDefStruct.units:  rad_s
 varDefStruct.axisSystem:  None
 varDefStruct.sign:  ANR
 varDefStruct.alias:  None
```

```
 varDefStruct.symbol:  None
 varDefStruct.hasInitialValue:  False
 varDefStruct.initialValue:  None
 varDefStruct.isStdAIAA:  True
 varDefStruct.isOutput:  False
 varDefStruct.hasMath:  False
 varDefStruct.codeText:  None
-variableDef-
 varDefStruct.name:  PBO2V
 varDefStruct.varID:  PBO2V
 varDefStruct.units:  nd
 varDefStruct.axisSystem:  None
 varDefStruct.sign:  None
 varDefStruct.alias:  None
 varDefStruct.symbol:  None
 varDefStruct.hasInitialValue:  False
 varDefStruct.initialValue:  None
 varDefStruct.isStdAIAA:  False
 varDefStruct.isOutput:  False
 varDefStruct.hasMath:  True
 varDefStruct.codeText:  ({BSPAN} * {PB}) / (2.0 * {VRW})
-variableDef-
 varDefStruct.name:  QCO2V
 varDefStruct.varID:  QCO2V
 varDefStruct.units:  nd
 varDefStruct.axisSystem:  None
 varDefStruct.sign:  None
 varDefStruct.alias:  None
 varDefStruct.symbol:  None
 varDefStruct.hasInitialValue:  False
 varDefStruct.initialValue:  None
 varDefStruct.isStdAIAA:  False
 varDefStruct.isOutput:  False
 varDefStruct.hasMath:  True
 varDefStruct.codeText:  ({CBAR} * {QB}) / (2.0 * {VRW})
-variableDef-
 varDefStruct.name:  RBO2V
 varDefStruct.varID:  RBO2V
 varDefStruct.units:  nd
 varDefStruct.axisSystem:  None
 varDefStruct.sign:  None
 varDefStruct.alias:  None
 varDefStruct.symbol:  None
 varDefStruct.hasInitialValue:  False
 varDefStruct.initialValue:  None
 varDefStruct.isStdAIAA:  False
 varDefStruct.isOutput:  False
 varDefStruct.hasMath:  True
```

```
 varDefStruct.codeText:  ({BSPAN} * {RB}) / (2.0 * {VRW})
-variableDef-
 varDefStruct.name:  totalCoefficientOfLift
 varDefStruct.varID:  CL
 varDefStruct.units:  nd
 varDefStruct.axisSystem:  None
 varDefStruct.sign:  None
 varDefStruct.alias:  None
 varDefStruct.symbol:  None
 varDefStruct.hasInitialValue:  True
 varDefStruct.initialValue:  0.0
 varDefStruct.isStdAIAA:  True
 varDefStruct.isOutput:  True
 varDefStruct.hasMath:  False
 varDefStruct.codeText:  None
-variableDef-
 varDefStruct.name:  totalCoefficientOfDrag
 varDefStruct.varID:  CD
 varDefStruct.units:  nd
 varDefStruct.axisSystem:  None
 varDefStruct.sign:  None
 varDefStruct.alias:  None
 varDefStruct.symbol:  None
 varDefStruct.hasInitialValue:  True
 varDefStruct.initialValue:  0.01
 varDefStruct.isStdAIAA:  True
 varDefStruct.isOutput:  True
 varDefStruct.hasMath:  False
 varDefStruct.codeText:  None
-variableDef-
 varDefStruct.name:  aeroBodyForceCoefficient_Y
 varDefStruct.varID:  CY
 varDefStruct.units:  nd
 varDefStruct.axisSystem:  None
 varDefStruct.sign:  None
 varDefStruct.alias:  None
 varDefStruct.symbol:  None
 varDefStruct.hasInitialValue:  True
 varDefStruct.initialValue:  0.0
 varDefStruct.isStdAIAA:  True
 varDefStruct.isOutput:  True
 varDefStruct.hasMath:  False
 varDefStruct.codeText:  None
-variableDef-
 varDefStruct.name:  aeroBodyMomentCoefficient_Roll
 varDefStruct.varID:  Cl
 varDefStruct.units:  nd
 varDefStruct.axisSystem:  None
```

```
 varDefStruct.sign:  None
 varDefStruct.alias:  None
 varDefStruct.symbol:  None
 varDefStruct.hasInitialValue:  False
 varDefStruct.initialValue:  None
 varDefStruct.isStdAIAA:  True
 varDefStruct.isOutput:  True
 varDefStruct.hasMath:  True
 varDefStruct.codeText:  ({CLP_DAMPING} * {PBO2V}) + ({CLR_DAMPING} * {RBO2V})
-variableDef-
 varDefStruct.name:  aeroBodyMomentCoefficient_Pitch
 varDefStruct.varID:  Cm
 varDefStruct.units:  nd
 varDefStruct.axisSystem:  None
 varDefStruct.sign:  None
 varDefStruct.alias:  None
 varDefStruct.symbol:  None
 varDefStruct.hasInitialValue:  False
 varDefStruct.initialValue:  None
 varDefStruct.isStdAIAA:  True
 varDefStruct.isOutput:  True
 varDefStruct.hasMath:  True
 varDefStruct.codeText:  ({CMQ_DAMPING} * {QCO2V})
-variableDef-
 varDefStruct.name:  aeroBodyMomentCoefficient_Yaw
 varDefStruct.varID:  Cn
 varDefStruct.units:  nd
 varDefStruct.axisSystem:  None
 varDefStruct.sign:  None
 varDefStruct.alias:  None
 varDefStruct.symbol:  None
 varDefStruct.hasInitialValue:  False
 varDefStruct.initialValue:  None
 varDefStruct.isStdAIAA:  True
 varDefStruct.isOutput:  True
 varDefStruct.hasMath:  True
 varDefStruct.codeText:  ({CNP_DAMPING} * {PBO2V}) + ({CNR_DAMPING} * {RBO2V})
-checkData-
staticShot:  Nominal
 signal type:  {http://daveml.org/2010/DAVEML}checkInputs
 signal name:  trueAirspeed
 signal varID:  VRW
 signal value:  10.0
 signal units:  ft_s
 [ localSignal append ] ->  Nominal signal #: 1
 signal type:  {http://daveml.org/2010/DAVEML}checkInputs
 signal name:  bodyAngularRate_Roll
 signal varID:  PB
```

```
signal value:  0.3
signal units:  rad_s
[ localSignal append ] ->  Nominal signal #: 2
signal type:  {http://daveml.org/2010/DAVEML}checkInputs
signal name:  bodyAngularRate_Pitch
signal varID:  QB
signal value:  1.5
signal units:  rad_s
[ localSignal append ] ->  Nominal signal #: 3
signal type:  {http://daveml.org/2010/DAVEML}checkInputs
signal name:  bodyAngularRate_Yaw
signal varID:  RB
signal value:  0.6
signal units:  rad_s
[ localSignal append ] ->  Nominal signal #: 4
signal type:  {http://daveml.org/2010/DAVEML}internalValues
signal name:  referenceWingArea
signal varID:  SWING
signal value:  0.22222
signal units:  ft2
signal tol:  1e-5
[ localSignal append ] ->  Nominal signal #: 5
signal type:  {http://daveml.org/2010/DAVEML}internalValues
signal name:  referenceWingSpan
signal varID:  BSPAN
signal value:  0.33333
signal units:  ft
signal tol:  1e-5
[ localSignal append ] ->  Nominal signal #: 6
signal type:  {http://daveml.org/2010/DAVEML}internalValues
signal name:  referenceWingChord
signal varID:  CBAR
signal value:  0.66667
signal units:  ft
signal tol:  1e-5
[ localSignal append ] ->  Nominal signal #: 7
signal type:  {http://daveml.org/2010/DAVEML}internalValues
signal name:  roll damping from roll rate
signal varID:  CLP_DAMPING
signal value:  -1.0
[ localSignal append ] ->  Nominal signal #: 8
signal type:  {http://daveml.org/2010/DAVEML}internalValues
signal name:  roll damping from yaw rate
signal varID:  CLR_DAMPING
signal value:  0.0
[ localSignal append ] ->  Nominal signal #: 9
signal type:  {http://daveml.org/2010/DAVEML}internalValues
signal name:  pitch damping from pitch rate
```

```
signal varID:  CMQ_DAMPING
signal value:  -1.0
[ localSignal append ] ->  Nominal signal #: 10
signal type:  {http://daveml.org/2010/DAVEML}internalValues
signal name:  yaw damping from roll rate
signal varID:  CNP_DAMPING
signal value:  0.0
[ localSignal append ] ->  Nominal signal #: 11
signal type:  {http://daveml.org/2010/DAVEML}internalValues
signal name:  yaw damping from yaw rate
signal varID:  CNR_DAMPING
signal value:  -1.0
[ localSignal append ] ->  Nominal signal #: 12
signal type:  {http://daveml.org/2010/DAVEML}internalValues
signal name:  PBO2V
signal varID:  PBO2V
signal value:  0.005
[ localSignal append ] ->  Nominal signal #: 13
signal type:  {http://daveml.org/2010/DAVEML}internalValues
signal name:  QCO2V
signal varID:  QCO2V
signal value:  0.05
[ localSignal append ] ->  Nominal signal #: 14
signal type:  {http://daveml.org/2010/DAVEML}internalValues
signal name:  RBO2V
signal varID:  RBO2V
signal value:  0.01
[ localSignal append ] ->  Nominal signal #: 15
signal type:  {http://daveml.org/2010/DAVEML}checkOutputs
signal name:  totalCoefficientOfLift
signal varID:  CL
signal value:  0.0
[ localSignal append ] ->  Nominal signal #: 16
signal type:  {http://daveml.org/2010/DAVEML}checkOutputs
signal name:  totalCoefficientOfDrag
signal varID:  CD
signal value:  0.01
[ localSignal append ] ->  Nominal signal #: 17
signal type:  {http://daveml.org/2010/DAVEML}checkOutputs
signal name:  aeroBodyForceCoefficient_Y
signal varID:  CY
signal value:  0.0
[ localSignal append ] ->  Nominal signal #: 18
signal type:  {http://daveml.org/2010/DAVEML}checkOutputs
signal name:  aeroBodyMomentCoefficient_Roll
signal varID:  Cl
signal value:  -0.005
[ localSignal append ] ->  Nominal signal #: 19
```

```
 signal type:  {http://daveml.org/2010/DAVEML}checkOutputs
 signal name:  aeroBodyMomentCoefficient_Pitch
 signal varID:  Cm
 signal value:  -0.05
 [ localSignal append ] ->  Nominal signal #: 20
 signal type:  {http://daveml.org/2010/DAVEML}checkOutputs
 signal name:  aeroBodyMomentCoefficient_Yaw
 signal varID:  Cn
 signal value:  -0.01
 [ localSignal append ] ->  Nominal signal #: 21
21 signals in  Nominal

--- PreProcess Equations and Functions ---

 << PBO2V >>
  [raw]->  ({BSPAN} * {PB}) / (2.0 * {VRW})
  [python]->  (self.Data["BSPAN"] * self.Data["PB"]) / (2.0 * self.Data["VRW"])
 << QCO2V >>
  [raw]->  ({CBAR} * {QB}) / (2.0 * {VRW})
  [python]->  (self.Data["CBAR"] * self.Data["QB"]) / (2.0 * self.Data["VRW"])
 << RBO2V >>
  [raw]->  ({BSPAN} * {RB}) / (2.0 * {VRW})
  [python]->  (self.Data["BSPAN"] * self.Data["RB"]) / (2.0 * self.Data["VRW"])
 << Cl >>
  [raw]->  ({CLP_DAMPING} * {PBO2V}) + ({CLR_DAMPING} * {RBO2V})
  [python]->  (self.Data["CLP_DAMPING"] * self.Data["PBO2V"]) +
(self.Data["CLR_DAMPING"] * self.Data["RBO2V"])
 << Cm >>
  [raw]->  ({CMQ_DAMPING} * {QCO2V})
  [python]->  (self.Data["CMQ_DAMPING"] * self.Data["QCO2V"])
 << Cn >>
  [raw]->  ({CNP_DAMPING} * {PBO2V}) + ({CNR_DAMPING} * {RBO2V})
  [python]->  (self.Data["CNP_DAMPING"] * self.Data["PBO2V"]) +
(self.Data["CNR_DAMPING"] * self.Data["RBO2V"])
+++++ MODEL INPUTS AND OUTPUTS +++++
++> Input:  VRW
++> Input:  PB
++> Input:  QB
++> Input:  RB
++> Output:  CL
++> Output:  CD
++> Output:  CY
++> Output:  Cl
++> Output:  Cm
++> Output:  Cn
++++++++++++++++++++++++++++++++++++
Number of check cases:  1
```

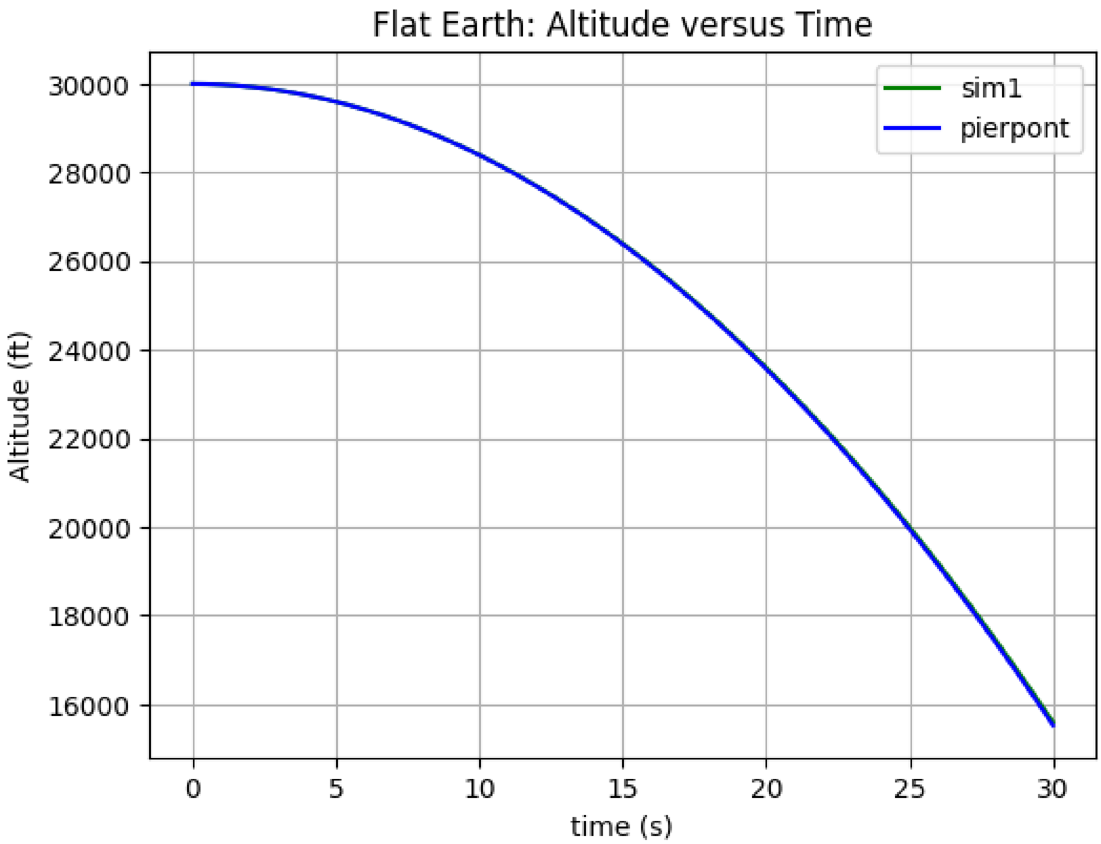
```
--Variables defined in model--

referenceWingArea
referenceWingSpan
referenceWingChord
roll damping from roll rate
roll damping from yaw rate
pitch damping from pitch rate
yaw damping from roll rate
yaw damping from yaw rate
trueAirspeed
bodyAngularRate_Roll
bodyAngularRate_Pitch
bodyAngularRate_Yaw
PBO2V
QCO2V
RBO2V
totalCoefficientOfLift
totalCoefficientOfDrag
aeroBodyForceCoefficient_Y
aeroBodyMomentCoefficient_Roll
aeroBodyMomentCoefficient_Pitch
aeroBodyMomentCoefficient_Yaw

----- DAVE-ML MODEL PARSE COMPLETE -----

----- CheckModel -----

numSignals:  [21]

----- END CheckModel -----

========= SetIC ==============
{'totalMass': 2.2679618958243624, 'bodyMomentOfInertia_X': 0.002568217499776201,
'bodyMomentOfInertia_Y': 0.008421011121856215, 'bodyMomentOfInertia_Z':
0.009754656036800024, 'altitudeMsl': 9144.0, 'referenceWingChord': 0.203201016,
'referenceWingSpan': 0.101598984, 'referenceWingArea': 0.020644913548800003,
'eulerAngleRate_Roll': 0.17453292519943295, 'eulerAngleRate_Pitch':
0.3490658503988659, 'eulerAngleRate_Yaw': 0.5235987755982988}
++ timeStep = 0.1 [default]
++ totalMass = 2.2679618958243624 [IC case]
++ referenceWingSpan = 0.101598984 [IC case]
++ referenceWingChord = 0.203201016 [IC case]
++ referenceWingArea = 0.020644913548800003 [IC case]
++ trueAirspeed = 0 [default]
++ angleOfAttack = 0 [default]
++ angleOfSideslip = 0 [default]
++ eulerAngle_Roll = 0 [default]
```

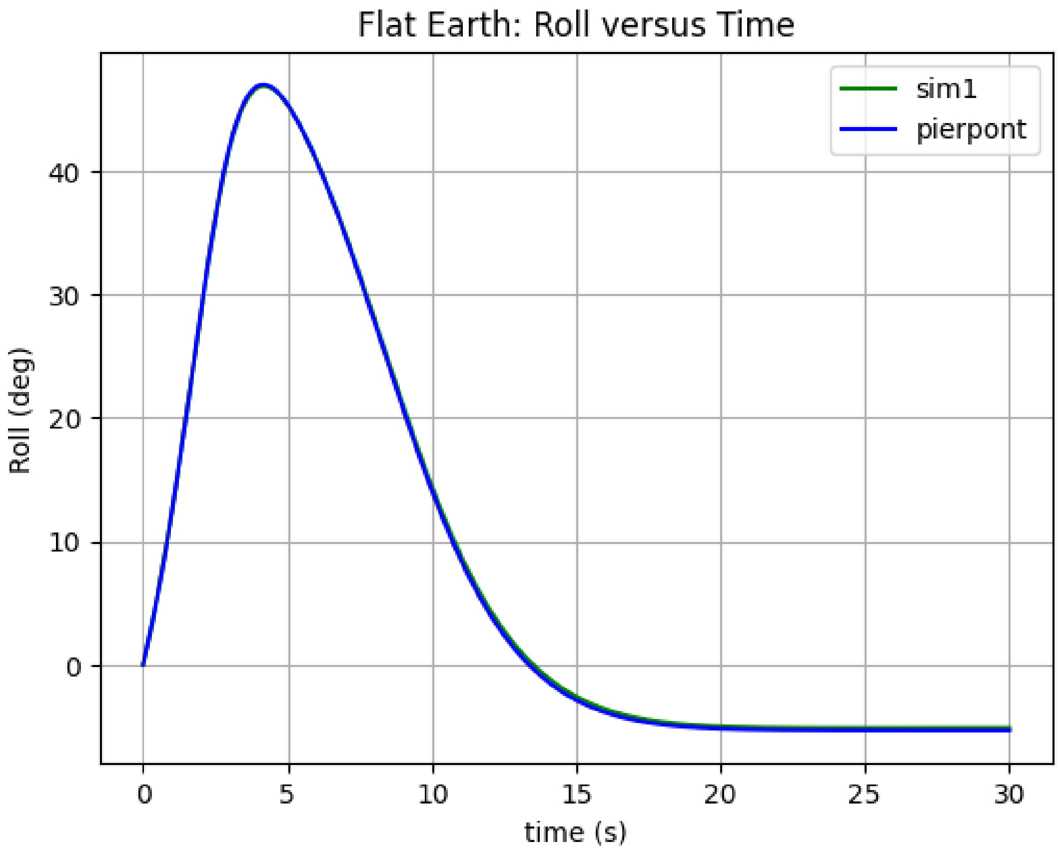
```
++ eulerAngle_Pitch = 0 [default]
++ eulerAngle_Yaw = 0 [default]
++ eulerAngleRate_Roll = 0.17453292519943295 [IC case]
++ eulerAngleRate_Pitch = 0.3490658503988659 [IC case]
++ eulerAngleRate_Yaw = 0.5235987755982988 [IC case]
++ bodyMomentOfInertia_X = 0.002568217499776201 [IC case]
++ bodyProductOfInertia_XY = 0 [default]
++ bodyProductOfInertia_XZ = 0 [default]
++ bodyProductOfInertia_YX = 0 [default]
++ bodyMomentOfInertia_Y = 0.008421011121856215 [IC case]
++ bodyProductOfInertia_YZ = 0 [default]
++ bodyProductOfInertia_XZ = 0 [default]
++ bodyProductOfInertia_YZ = 0 [default]
++ bodyMomentOfInertia_Z = 0.009754656036800024 [IC case]
++ altitudeMsl = 9144.0 [IC case]
======done=======
CPU times: user 104 ms, sys: 27.1 ms, total: 131 ms
Wall time: 196 ms
```

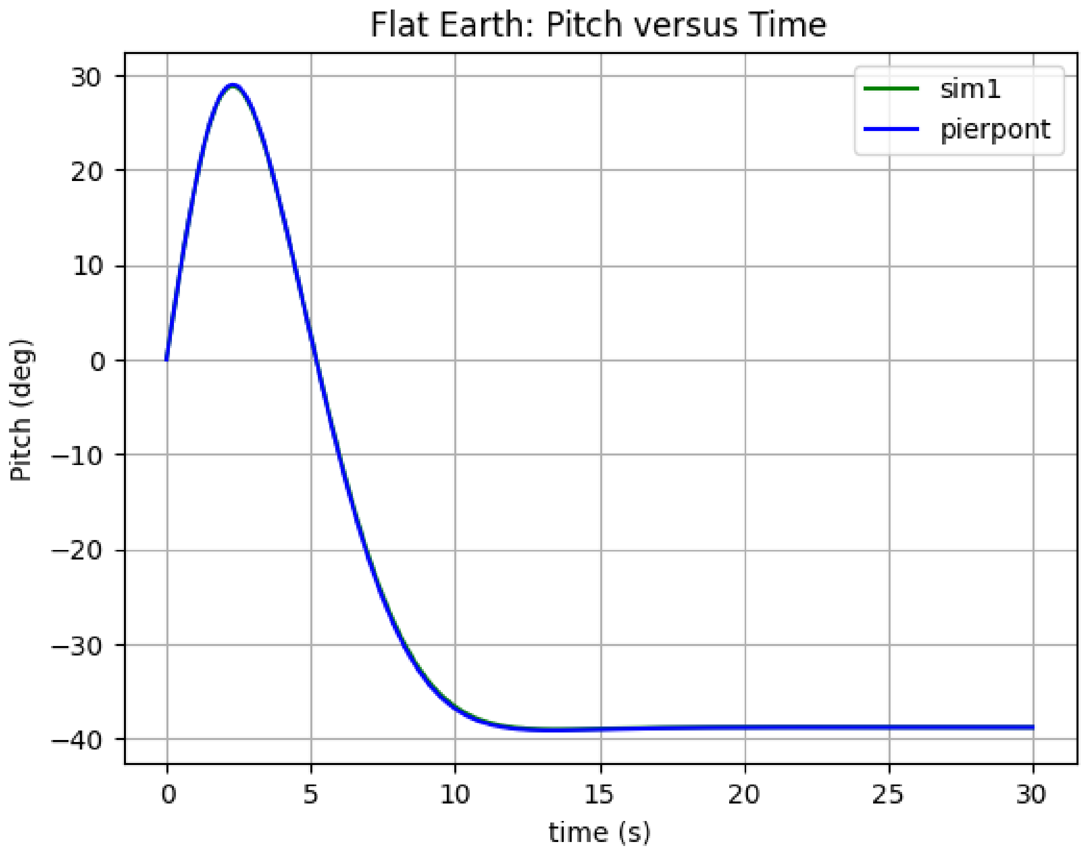
[51]:

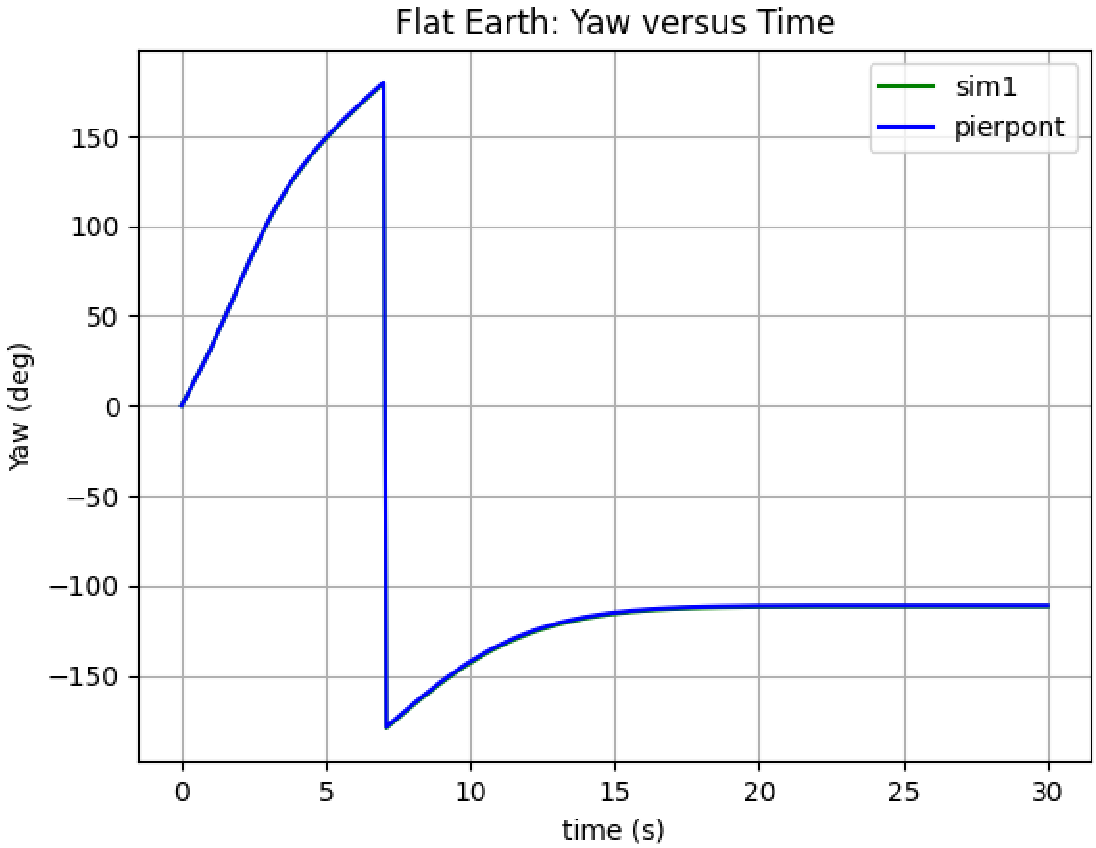
```
MakeFlatEarthPlots(gvFlatEarthSim, gvCC3, "sim1")
PrintErrorTable("Tumbling Brick Damping: Flat Earth", gvFlatEarthLabel,␣
↪gvFlatEarthSim, gvCC3)
```

```
Variable                 L2                       L-Infinity-Norm
--------                 --                       ---------------
altitudeMsl_ft           607.9829747297271        77.22618244503428
trueAirspeed_nmi_h       30.29929205599481        2.9172948944294603
eulerAngle_deg_Roll      4.115319287865008        0.4073996530980857
eulerAngle_deg_Pitch     2.4299375300946506       0.3420764770645235
eulerAngle_deg_Yaw       9.941099709629293        0.8768454992063255
```

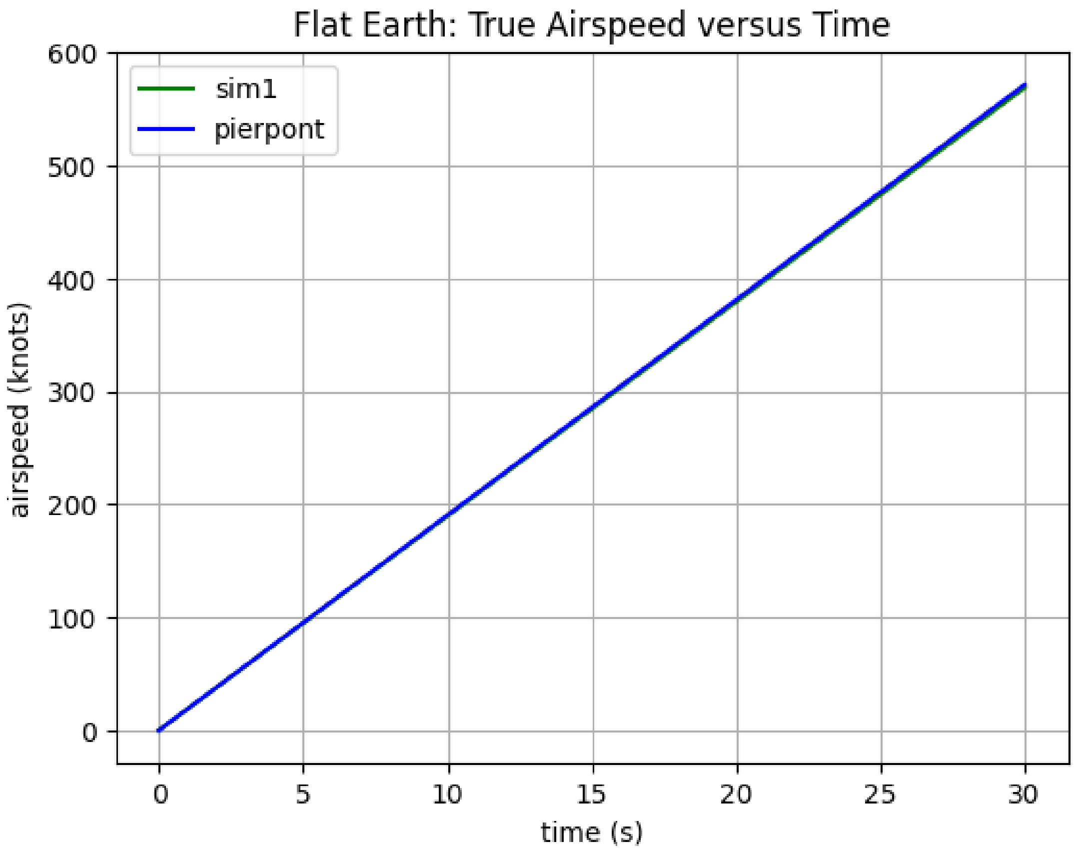

Flat Earth: Altitude versus Time
sim1
pierpont
Altitude (ft)
30000
28000
26000
24000
22000
20000
18000
16000
time (s)
0
5
10
15
20
25
30

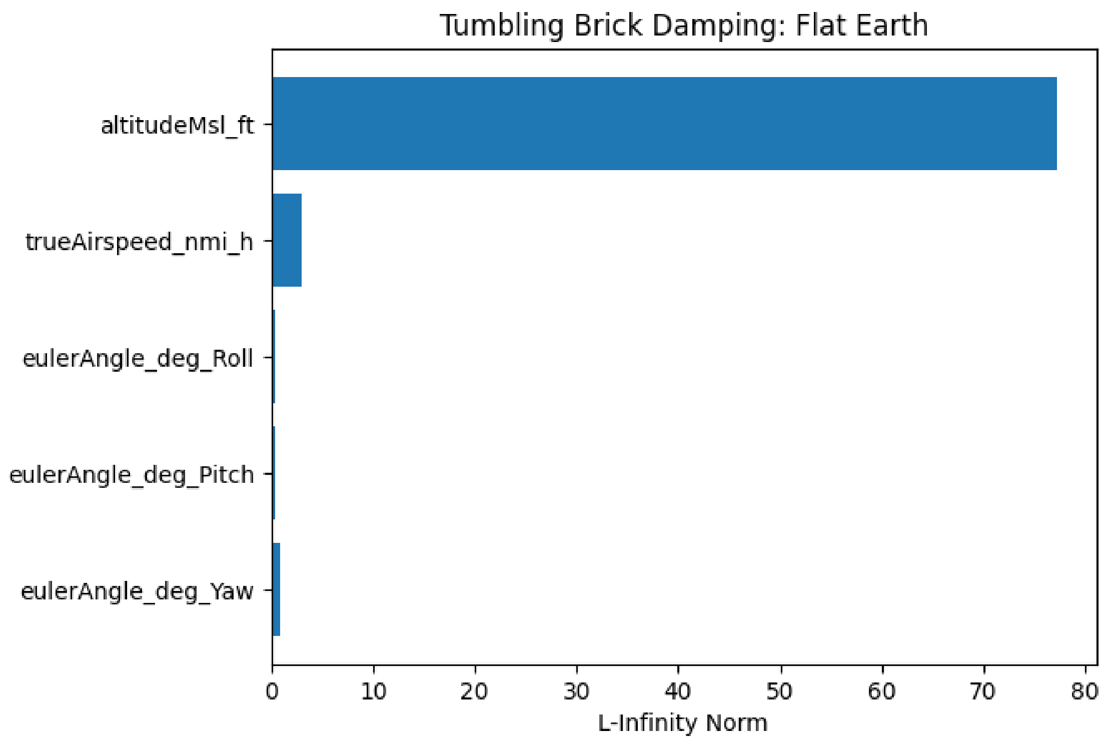

Flat Earth: Roll versus Time
sim1
pierpont
Roll (deg)
40
30
20
10
0
0
5
10
15
20
25
30
time (s)


Flat Earth: Pitch versus Time
sim1
pierpont
Pitch (deg)
time (s)
30
20
10
0
−10
−20
−30
−40
0
5
10
15
20
25
30



Flat Earth: Yaw versus Time
sim1
pierpont
Yaw (deg)
150
100
50
0
−50
−100
−150
0
5
10
15
20
25
30
time (s)


Flat Earth: True Airspeed versus Time
sim1
pierpont
airspeed (knots)
600
500
400
300
200
100
0
0
5
10
15
20
25
30
time (s)



Oblate, Rotating Earth

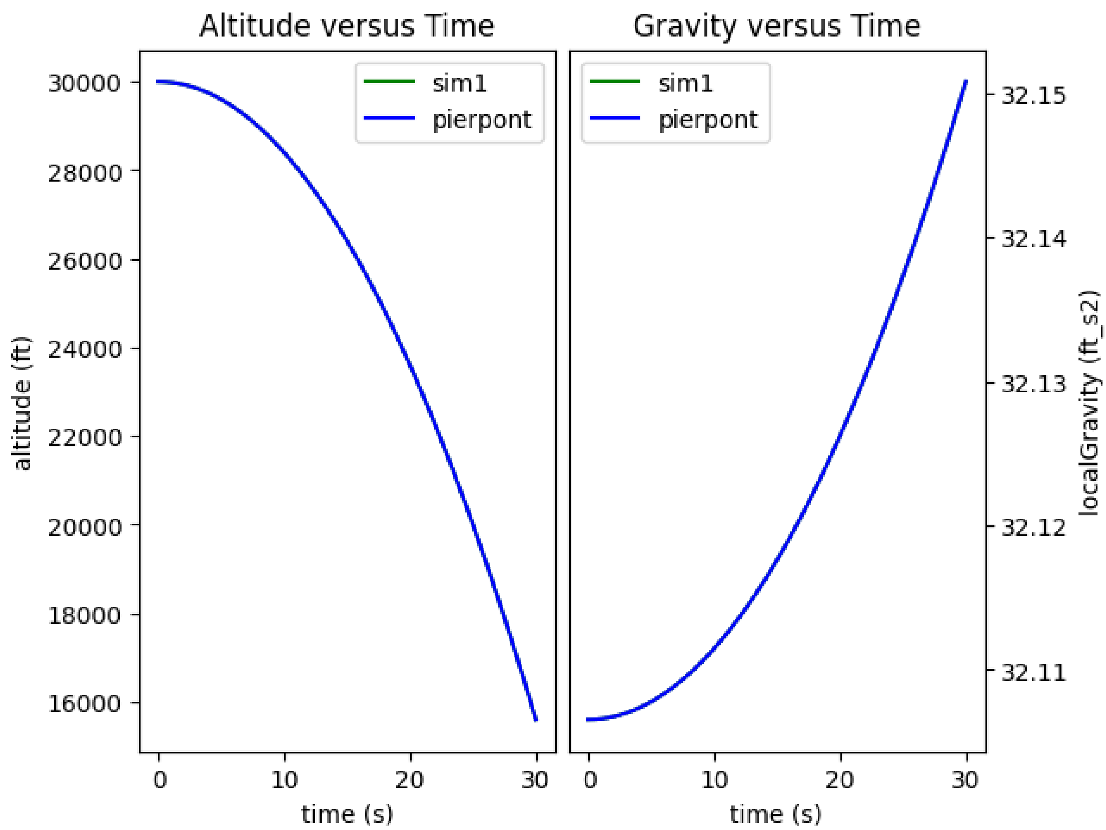
[52]:

```
%%time
ic = {
    "totalMass": [0.155404754, "slug"],
    "bodyMomentOfInertia_X": [0.001894220, "slugft2"],
    "bodyMomentOfInertia_Y": [0.006211019, "slugft2"],
    "bodyMomentOfInertia_Z": [0.007194665, "slugft2"],
    "altitudeMsl": [30000, "ft"],
    "referenceWingChord": [0.66667, "ft"],
    "referenceWingSpan": [0.33333, "ft"],
    "referenceWingArea": [0.22222, "ft2"],
    "eulerAngleRate_Roll": [10.0, "deg_s"],
    "eulerAngleRate_Pitch": [20.0, "deg_s"],
    "eulerAngleRate_Yaw": [30.0, "deg_s"]
}
inputs = ["trueAirspeed", "bodyAngularRate_Roll", "bodyAngularRate_Pitch",␣
 ↪"bodyAngularRate_Yaw"]
gvOblateRotatingEarth.Reset(ic)
gvOblateRotatingEarth.AddAeroModelInput(inputs)
gvOblateRotatingEarth.Run(30)
gvOblateRotatingEarth.GenerateEnglishUnits()
```

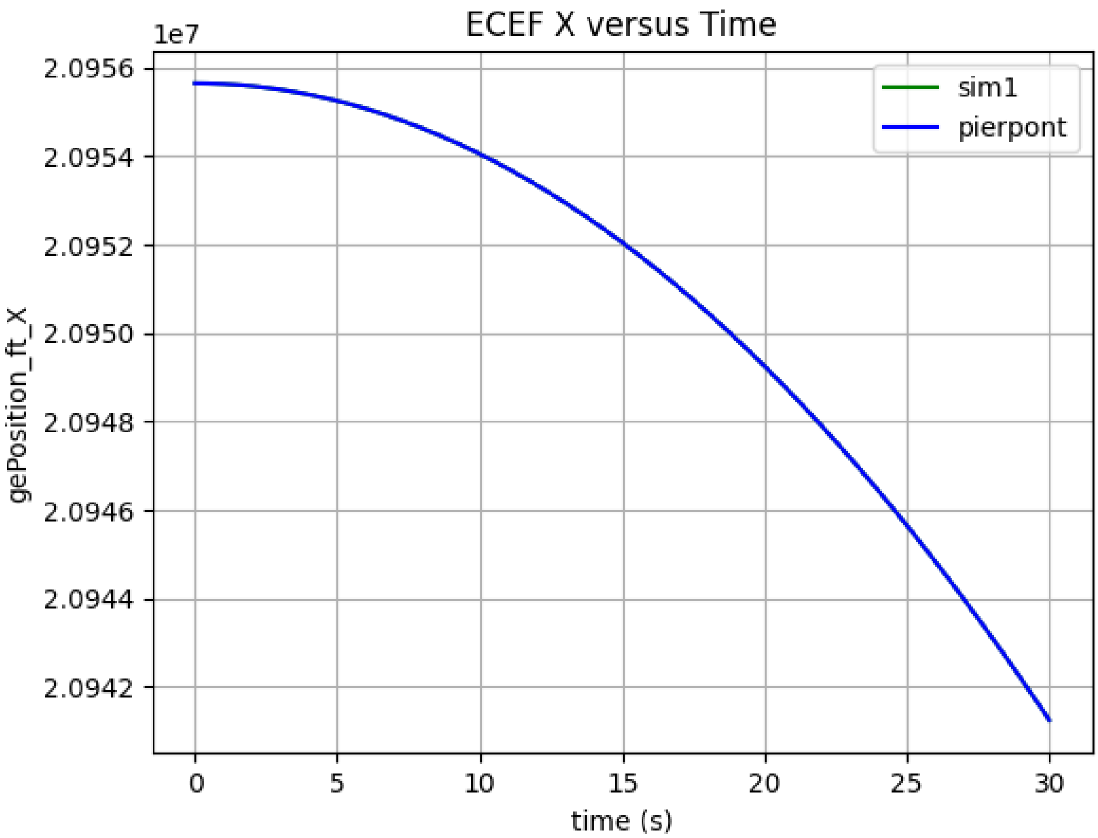
```
========= SetIC ==============
```

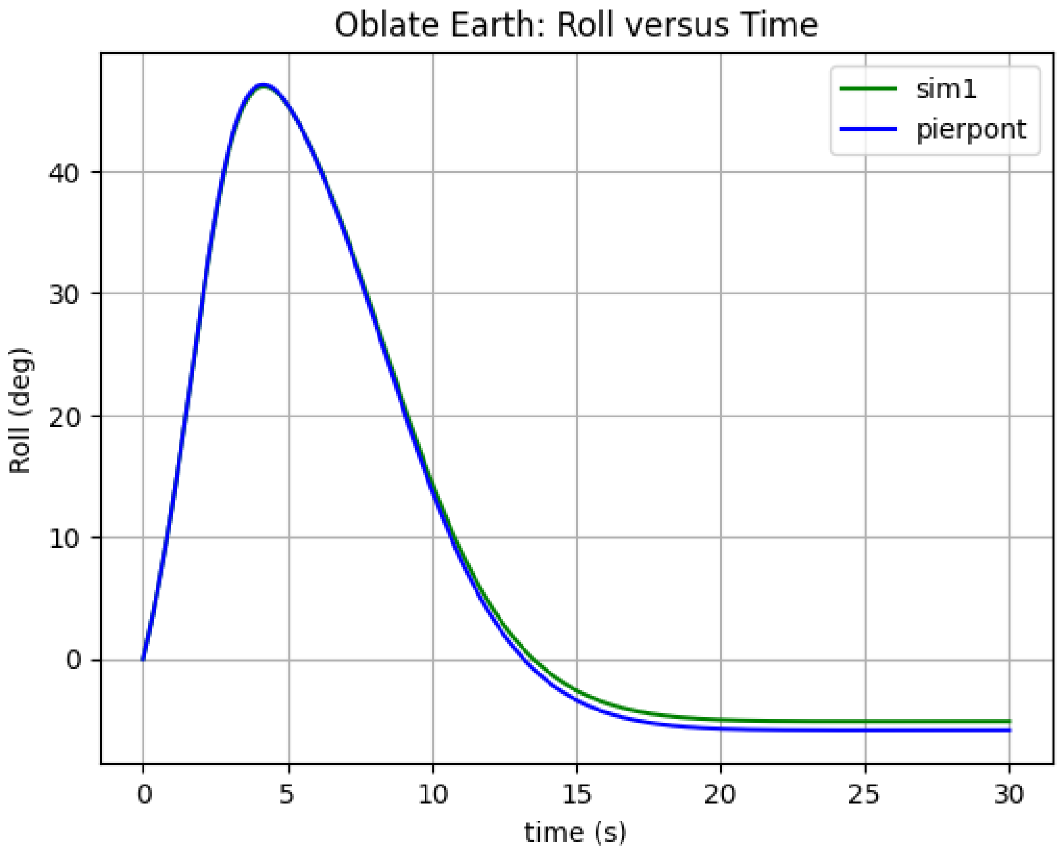
```
{'totalMass': 2.2679618958243624, 'bodyMomentOfInertia_X': 0.002568217499776201,
'bodyMomentOfInertia_Y': 0.008421011121856215, 'bodyMomentOfInertia_Z':
0.009754656036800024, 'altitudeMsl': 9144.0, 'referenceWingChord': 0.203201016,
'referenceWingSpan': 0.101598984, 'referenceWingArea': 0.020644913548800003,
'eulerAngleRate_Roll': 0.17453292519943295, 'eulerAngleRate_Pitch':
0.3490658503988659, 'eulerAngleRate_Yaw': 0.5235987755982988}
++ timeStep = 0.1 [default]
++ totalMass = 2.2679618958243624 [IC case]
++ referenceWingSpan = 0.101598984 [IC case]
++ referenceWingChord = 0.203201016 [IC case]
++ referenceWingArea = 0.020644913548800003 [IC case]
++ trueAirspeed = 0 [default]
++ angleOfAttack = 0 [default]
++ angleOfSideslip = 0 [default]
++ eulerAngle_Roll = 0 [default]
++ eulerAngle_Pitch = 0 [default]
++ eulerAngle_Yaw = 0 [default]
++ eulerAngleRate_Roll = 0.17453292519943295 [IC case]
++ eulerAngleRate_Pitch = 0.3490658503988659 [IC case]
++ eulerAngleRate_Yaw = 0.5235987755982988 [IC case]
++ bodyMomentOfInertia_X = 0.002568217499776201 [IC case]
++ bodyProductOfInertia_XY = 0 [default]
++ bodyProductOfInertia_XZ = 0 [default]
++ bodyProductOfInertia_YX = 0 [default]
++ bodyMomentOfInertia_Y = 0.008421011121856215 [IC case]
++ bodyProductOfInertia_YZ = 0 [default]
++ bodyProductOfInertia_XZ = 0 [default]
++ bodyProductOfInertia_YZ = 0 [default]
++ bodyMomentOfInertia_Z = 0.009754656036800024 [IC case]
++ latitude = 0 [default]
++ longitude = 0 [default]
++ altitudeMsl = 9144.0 [IC case]
Vecf:  0.0 0.0 0.0
======done=======
CPU times: user 56.3 ms, sys: 5.17 ms, total: 61.5 ms
Wall time: 67.1 ms
```

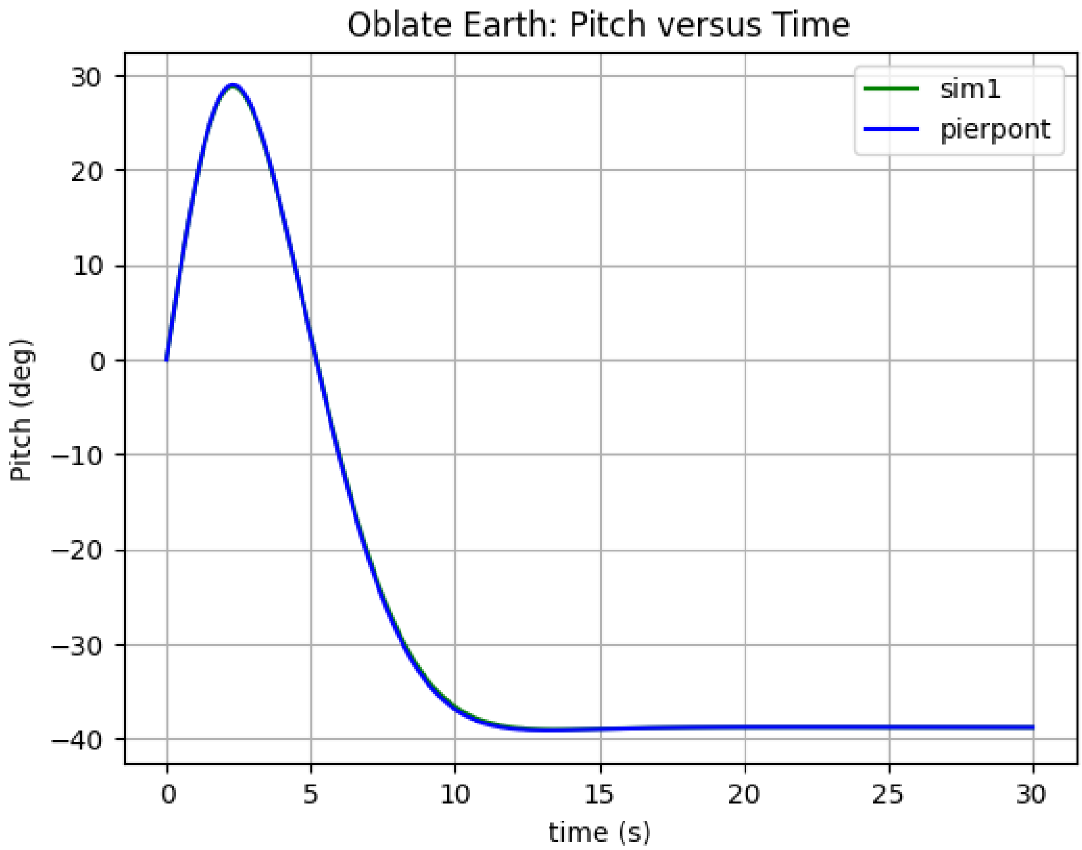
[53]:
```
MakePlot(gvOblateRotatingEarth, gvCC3, "sim1")
```

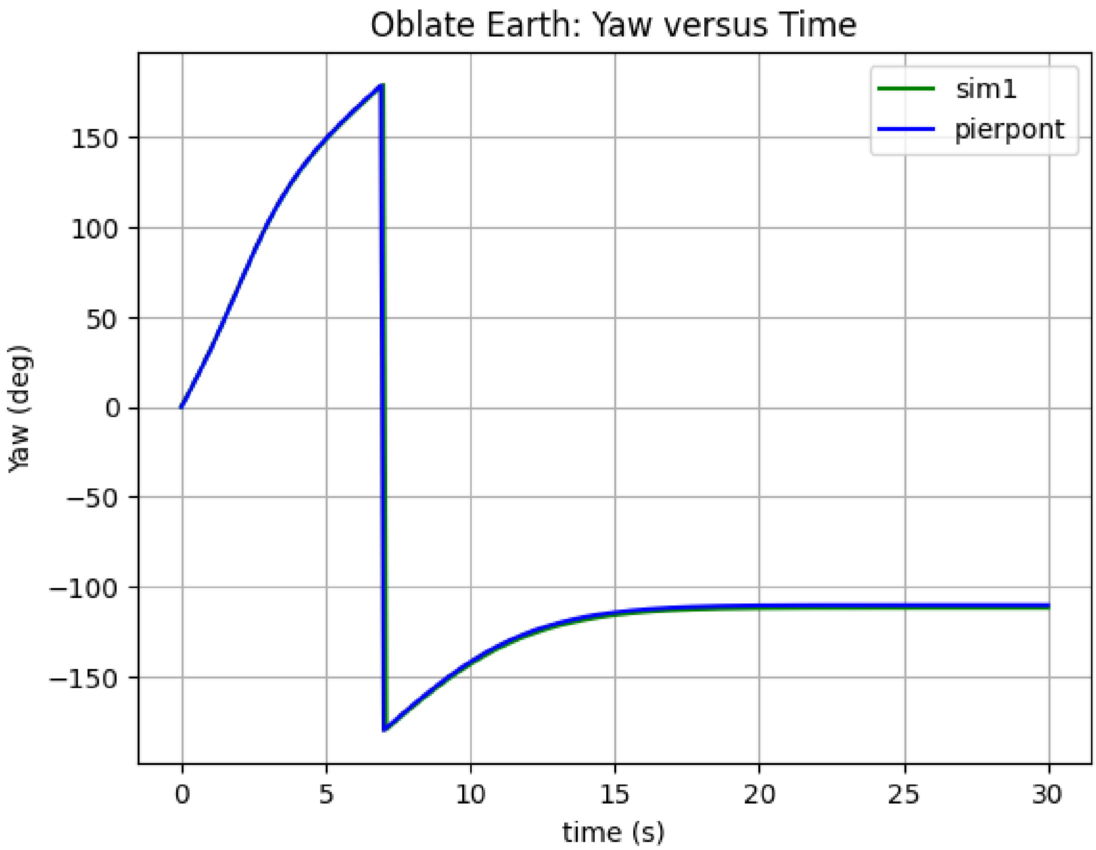


Altitude versus Time
Gravity versus Time
sim1
pierpont
sim1
pierpont
30000
28000
26000
24000
22000
20000
18000
16000
altitude (ft)
0
10
20
30
time (s)
0
10
20
30
time (s)
32.15
32.14
32.13
32.12
32.11
localGravity (ft_s2)


ECEF X versus Time
1e7
2.0956
2.0954
2.0952
2.0950
2.0948
2.0946
2.0944
2.0942
gePosition_ft_X
0
5
10
15
20
25
30
time (s)
sim1
pierpont

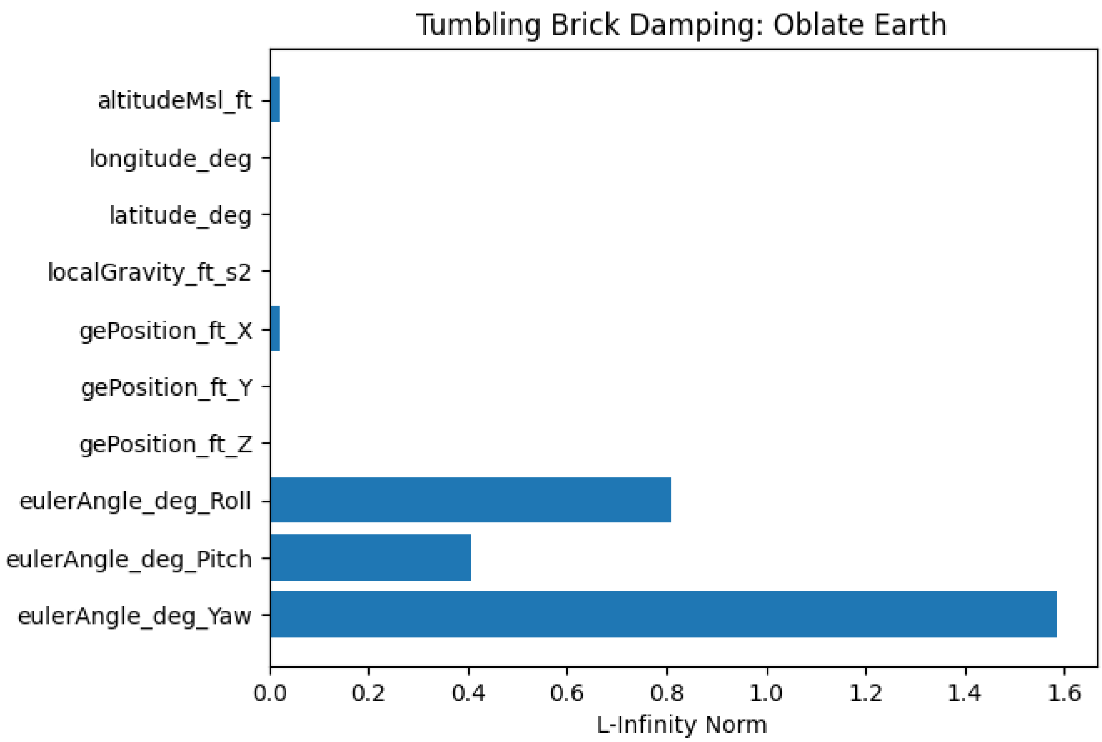

Oblate Earth: Roll versus Time
sim1
pierpont
Roll (deg)
40
30
20
10
0
0
5
10
15
20
25
30
time (s)


Oblate Earth: Pitch versus Time
sim1
pierpont
Pitch (deg)
time (s)
30
20
10
0
−10
−20
−30
−40
0
5
10
15
20
25
30



[54]:
```
PrintErrorTable("Tumbling Brick Damping: Oblate Earth", gvOblateEarthLabel,␣
↪gvOblateRotatingEarth, gvCC3)
```

```
Variable                  L2                        L-Infinity-Norm
--------                  --                        ---------------
altitudeMsl_ft            0.1411402118223276        0.02177725810179254
longitude_deg             2.0649476793351563e-10    3.5255035334494107e-11
latitude_deg              0.0                       0
localGravity_ft_s2        1.0779742411875166e-05    6.735199136187475e-07
gePosition_ft_X           0.14276753800880893       0.021909743547439575
gePosition_ft_Y           7.537771387173215e-05     1.2863464171175565e-05
gePosition_ft_Z           0.0                       0
eulerAngle_deg_Roll       11.016248991497301        0.8105152209742009
eulerAngle_deg_Pitch      2.787238066784144         0.40670110493388734
eulerAngle_deg_Yaw        22.54233249753164         1.5857787776205328
```



1.10.6 Sphere dropping over rotating, ellipsoidal Earth - 6

[55]:
```
%%time
gvAeroModel.LoadDml('models/NESC/cannonball_inertia.dml')
gvAeroModel.CheckModel()
#
ic = {
    "totalMass": [1.0, "slug"],
    "bodyMomentOfInertia_X": [3.6, "slugft2"],
    "bodyMomentOfInertia_Y": [3.6, "slugft2"],
    "bodyMomentOfInertia_Z": [3.6, "slugft2"],
    "altitudeMsl": [30000, "ft"],
    "referenceWingChord": [0.2, "ft"],
    "referenceWingSpan": [0.2, "ft"],
    "referenceWingArea": [0.1963495, "ft2"]
}
gvOblateRotatingEarth.Reset(ic)
gvOblateRotatingEarth.totalCoefficientOfDrag = 0.1
gvOblateRotatingEarth.Run(30)
gvOblateRotatingEarth.GenerateEnglishUnits()
```

```
*********************************************
Model:  Example cannonball inertia model
```

```
creation date:  2012-10-04
file version:  Initial version
*******************************************

-variableDef-
 varDefStruct.name:  bodyMomentOfInertia_Roll
 varDefStruct.varID:  XIXX
 varDefStruct.units:  slugft2
 varDefStruct.axisSystem:  None
 varDefStruct.sign:  None
 varDefStruct.alias:  None
 varDefStruct.symbol:  None
 varDefStruct.hasInitialValue:  True
 varDefStruct.initialValue:  3.6
 varDefStruct.isStdAIAA:  True
 varDefStruct.isOutput:  True
 varDefStruct.hasMath:  False
 varDefStruct.codeText:  None
-variableDef-
 varDefStruct.name:  bodyMomentOfInertia_Pitch
 varDefStruct.varID:  XIYY
 varDefStruct.units:  slugft2
 varDefStruct.axisSystem:  None
 varDefStruct.sign:  None
 varDefStruct.alias:  None
 varDefStruct.symbol:  None
 varDefStruct.hasInitialValue:  True
 varDefStruct.initialValue:  3.6
 varDefStruct.isStdAIAA:  True
 varDefStruct.isOutput:  True
 varDefStruct.hasMath:  False
 varDefStruct.codeText:  None
-variableDef-
 varDefStruct.name:  bodyMomentOfInertia_Yaw
 varDefStruct.varID:  XIZZ
 varDefStruct.units:  slugft2
 varDefStruct.axisSystem:  None
 varDefStruct.sign:  None
 varDefStruct.alias:  None
 varDefStruct.symbol:  None
 varDefStruct.hasInitialValue:  True
 varDefStruct.initialValue:  3.6
 varDefStruct.isStdAIAA:  True
 varDefStruct.isOutput:  True
 varDefStruct.hasMath:  False
 varDefStruct.codeText:  None
-variableDef-
 varDefStruct.name:  bodyProductOfInertia_ZX
```

```
 varDefStruct.varID:  XIZX
 varDefStruct.units:  slugft2
 varDefStruct.axisSystem:  None
 varDefStruct.sign:  None
 varDefStruct.alias:  None
 varDefStruct.symbol:  None
 varDefStruct.hasInitialValue:  True
 varDefStruct.initialValue:  0.0
 varDefStruct.isStdAIAA:  True
 varDefStruct.isOutput:  True
 varDefStruct.hasMath:  False
 varDefStruct.codeText:  None
-variableDef-
 varDefStruct.name:  bodyProductOfInertia_XY
 varDefStruct.varID:  XIXY
 varDefStruct.units:  slugft2
 varDefStruct.axisSystem:  None
 varDefStruct.sign:  None
 varDefStruct.alias:  None
 varDefStruct.symbol:  None
 varDefStruct.hasInitialValue:  True
 varDefStruct.initialValue:  0.0
 varDefStruct.isStdAIAA:  True
 varDefStruct.isOutput:  True
 varDefStruct.hasMath:  False
 varDefStruct.codeText:  None
-variableDef-
 varDefStruct.name:  bodyProductOfInertia_YZ
 varDefStruct.varID:  XIYZ
 varDefStruct.units:  slugft2
 varDefStruct.axisSystem:  None
 varDefStruct.sign:  None
 varDefStruct.alias:  None
 varDefStruct.symbol:  None
 varDefStruct.hasInitialValue:  True
 varDefStruct.initialValue:  0.0
 varDefStruct.isStdAIAA:  True
 varDefStruct.isOutput:  True
 varDefStruct.hasMath:  False
 varDefStruct.codeText:  None
-variableDef-
 varDefStruct.name:  totalMass
 varDefStruct.varID:  XMASS
 varDefStruct.units:  slug
 varDefStruct.axisSystem:  None
 varDefStruct.sign:  None
 varDefStruct.alias:  None
 varDefStruct.symbol:  None
```

```
 varDefStruct.hasInitialValue:  True
 varDefStruct.initialValue:  1.0
 varDefStruct.isStdAIAA:  True
 varDefStruct.isOutput:  True
 varDefStruct.hasMath:  False
 varDefStruct.codeText:  None
-variableDef-
 varDefStruct.name:  bodyPositionOfCmWrtMrc_X
 varDefStruct.varID:  DXCG
 varDefStruct.units:  ft
 varDefStruct.axisSystem:  None
 varDefStruct.sign:  FWD
 varDefStruct.alias:  None
 varDefStruct.symbol:  None
 varDefStruct.hasInitialValue:  True
 varDefStruct.initialValue:  0.0
 varDefStruct.isStdAIAA:  True
 varDefStruct.isOutput:  True
 varDefStruct.hasMath:  False
 varDefStruct.codeText:  None
-variableDef-
 varDefStruct.name:  bodyPositionOfCmWrtMrc_Y
 varDefStruct.varID:  DYCG
 varDefStruct.units:  ft
 varDefStruct.axisSystem:  None
 varDefStruct.sign:  RT
 varDefStruct.alias:  None
 varDefStruct.symbol:  None
 varDefStruct.hasInitialValue:  True
 varDefStruct.initialValue:  0.
 varDefStruct.isStdAIAA:  True
 varDefStruct.isOutput:  True
 varDefStruct.hasMath:  False
 varDefStruct.codeText:  None
-variableDef-
 varDefStruct.name:  bodyPositionOfCmWrtMrc_Z
 varDefStruct.varID:  DZCG
 varDefStruct.units:  ft
 varDefStruct.axisSystem:  None
 varDefStruct.sign:  DOWN
 varDefStruct.alias:  None
 varDefStruct.symbol:  None
 varDefStruct.hasInitialValue:  True
 varDefStruct.initialValue:  0.
 varDefStruct.isStdAIAA:  True
 varDefStruct.isOutput:  True
 varDefStruct.hasMath:  False
 varDefStruct.codeText:  None
```

```
--- PreProcess Equations and Functions ---

+++++ MODEL INPUTS AND OUTPUTS +++++
++> Output:  XIXX
++> Output:  XIYY
++> Output:  XIZZ
++> Output:  XIZX
++> Output:  XIXY
++> Output:  XIYZ
++> Output:  XMASS
++> Output:  DXCG
++> Output:  DYCG
++> Output:  DZCG
++++++++++++++++++++++++++++++++++++
Number of check cases:  0

--Variables defined in model--

bodyMomentOfInertia_Roll
bodyMomentOfInertia_Pitch
bodyMomentOfInertia_Yaw
bodyProductOfInertia_ZX
bodyProductOfInertia_XY
bodyProductOfInertia_YZ
totalMass
bodyPositionOfCmWrtMrc_X
bodyPositionOfCmWrtMrc_Y
bodyPositionOfCmWrtMrc_Z

----- DAVE-ML MODEL PARSE COMPLETE -----

----- CheckModel -----

numSignals:  []

----- END CheckModel -----

========= SetIC ==============
{'totalMass': 14.593902937, 'bodyMomentOfInertia_X': 4.88094466281336,
'bodyMomentOfInertia_Y': 4.88094466281336, 'bodyMomentOfInertia_Z':
4.88094466281336, 'altitudeMsl': 9144.0, 'referenceWingChord':
0.06096000000000001, 'referenceWingSpan': 0.06096000000000001,
'referenceWingArea': 0.018241465452480003}
++ timeStep = 0.1 [default]
++ totalMass = 14.593902937 [IC case]
++ referenceWingSpan = 0.06096000000000001 [IC case]
++ referenceWingChord = 0.06096000000000001 [IC case]
```

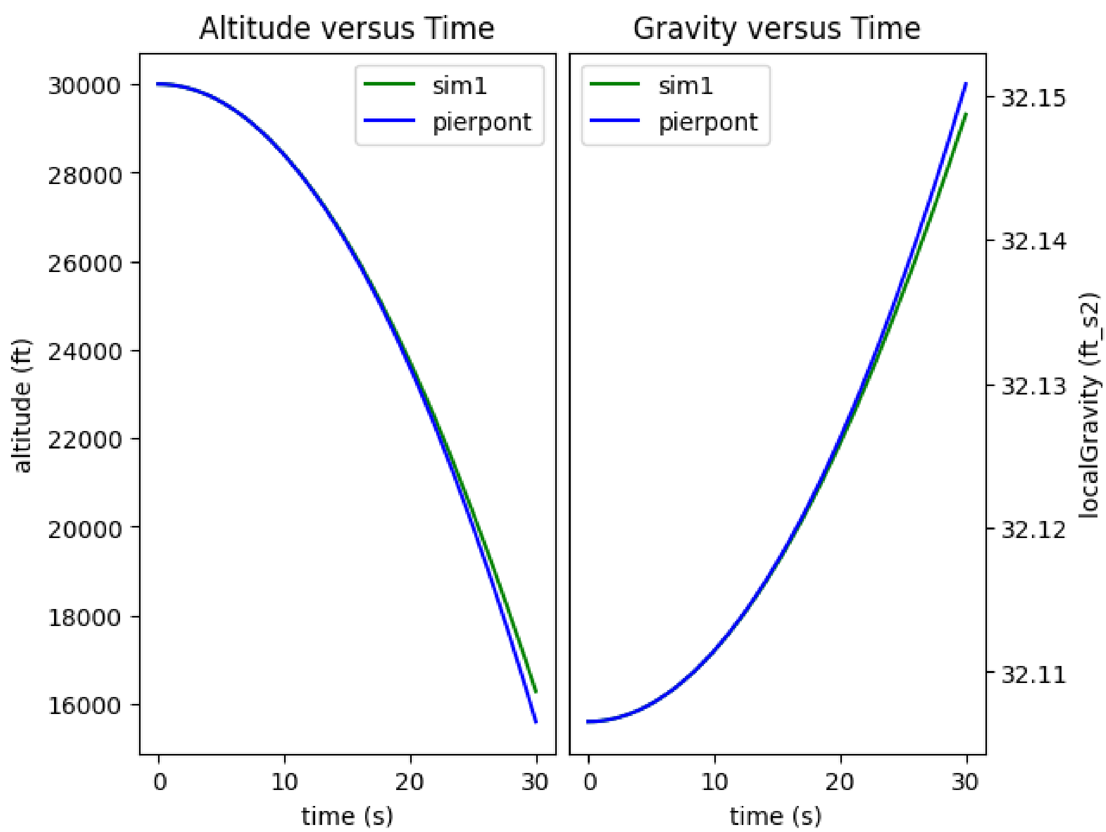
```
++ referenceWingArea = 0.018241465452480003 [IC case]
++ trueAirspeed = 0 [default]
++ angleOfAttack = 0 [default]
++ angleOfSideslip = 0 [default]
++ eulerAngle_Roll = 0 [default]
++ eulerAngle_Pitch = 0 [default]
++ eulerAngle_Yaw = 0 [default]
++ eulerAngleRate_Roll = 0 [default]
++ eulerAngleRate_Pitch = 0 [default]
++ eulerAngleRate_Yaw = 0 [default]
++ bodyMomentOfInertia_X = 4.88094466281336 [IC case]
++ bodyProductOfInertia_XY = 0 [default]
++ bodyProductOfInertia_XZ = 0 [default]
++ bodyProductOfInertia_YX = 0 [default]
++ bodyMomentOfInertia_Y = 4.88094466281336 [IC case]
++ bodyProductOfInertia_YZ = 0 [default]
++ bodyProductOfInertia_XZ = 0 [default]
++ bodyProductOfInertia_YZ = 0 [default]
++ bodyMomentOfInertia_Z = 4.88094466281336 [IC case]
++ latitude = 0 [default]
++ longitude = 0 [default]
++ altitudeMsl = 9144.0 [IC case]
Vecf:  0.0 0.0 0.0
======done=======
CPU times: user 48.4 ms, sys: 6.85 ms, total: 55.3 ms
Wall time: 51.3 ms
```

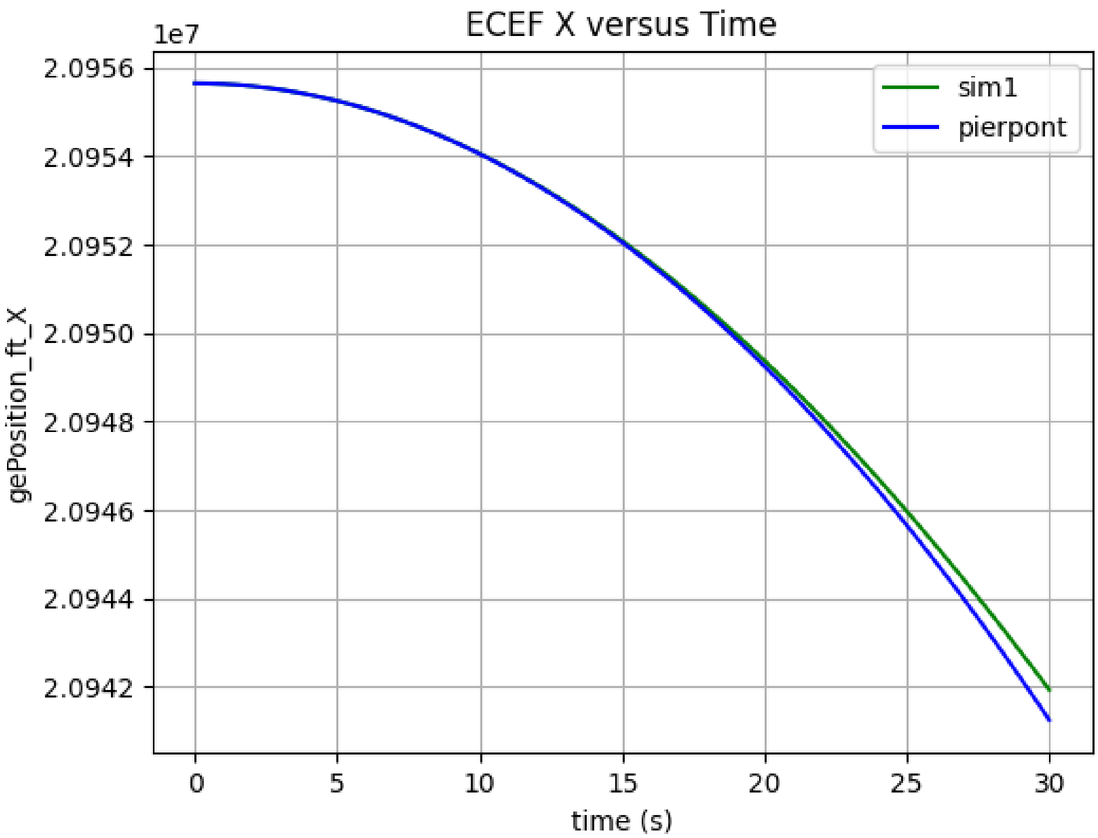
[56]:

```
checkFile = "NESC-check-cases/Atmospheric_checkcases/
 ↪Atmos_06_DroppedSphereEllipsoidalNoWind/Atmos_06_sim_01.csv"
gvCC6 = GetCheckCaseData(checkFile)
MakePlot(gvOblateRotatingEarth, gvCC6, "sim1")
PrintErrorTable("Dropped Sphere (Cd=0.1): Oblate Earth", gvOblateEarthLabel,␣
 ↪gvOblateRotatingEarth, gvCC6)
```

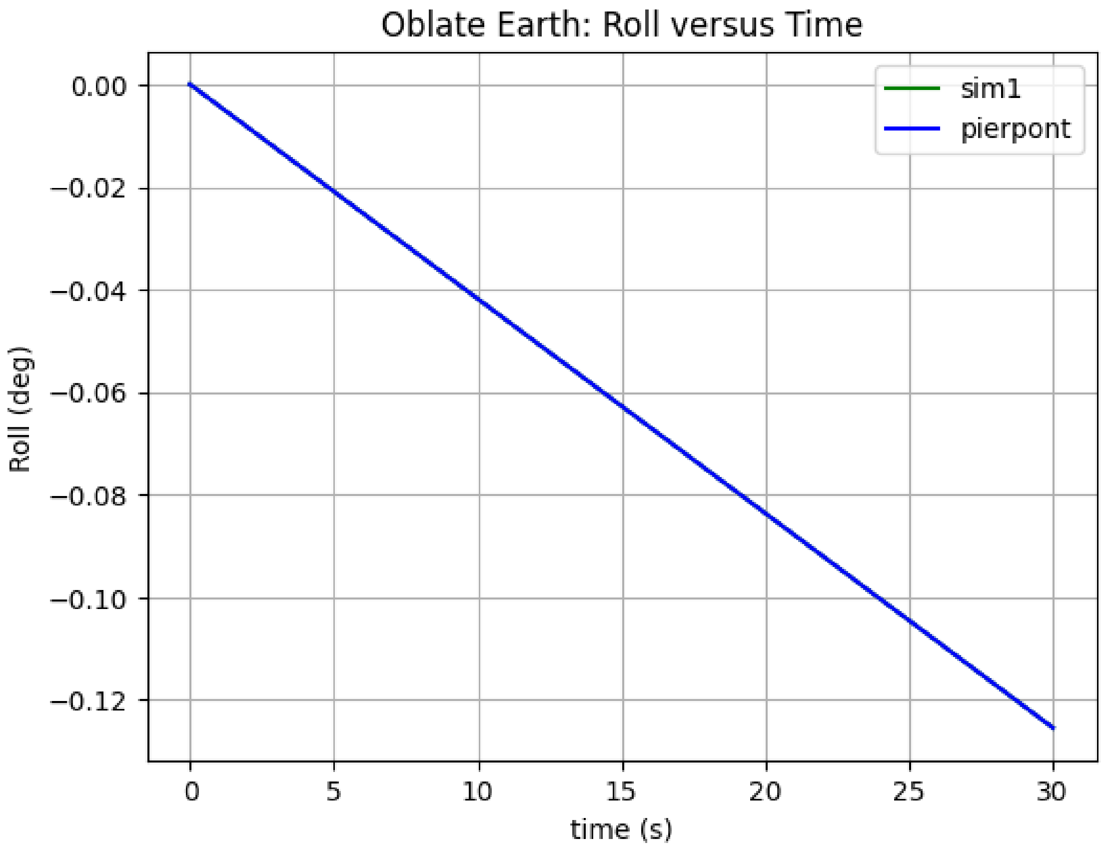
```
number of headers:  31
['time', 'gePosition_ft_X', 'gePosition_ft_Y', 'gePosition_ft_Z',
'feVelocity_ft_s_X', 'feVelocity_ft_s_Y', 'feVelocity_ft_s_Z', 'altitudeMsl_ft',
'longitude_deg', 'latitude_deg', 'localGravity_ft_s2', 'eulerAngle_deg_Yaw',
'eulerAngle_deg_Pitch', 'eulerAngle_deg_Roll',
'bodyAngularRateWrtEi_deg_s_Roll', 'bodyAngularRateWrtEi_deg_s_Pitch',
'bodyAngularRateWrtEi_deg_s_Yaw', 'altitudeRateWrtMsl_ft_min',
'speedOfSound_ft_s', 'airDensity_slug_ft3', 'ambientPressure_lbf_ft2',
'ambientTemperature_dgR', 'aero_bodyForce_lbf_X', 'aero_bodyForce_lbf_Y',
'aero_bodyForce_lbf_Z', 'aero_bodyMoment_ftlbf_L', 'aero_bodyMoment_ftlbf_M',
'aero_bodyMoment_ftlbf_N', 'mach', 'dynamicPressure_lbf_ft2',
'trueAirspeed_nmi_h']
Variable                  L2                         L-Infinity-Norm
--------                  --                         ---------------
```

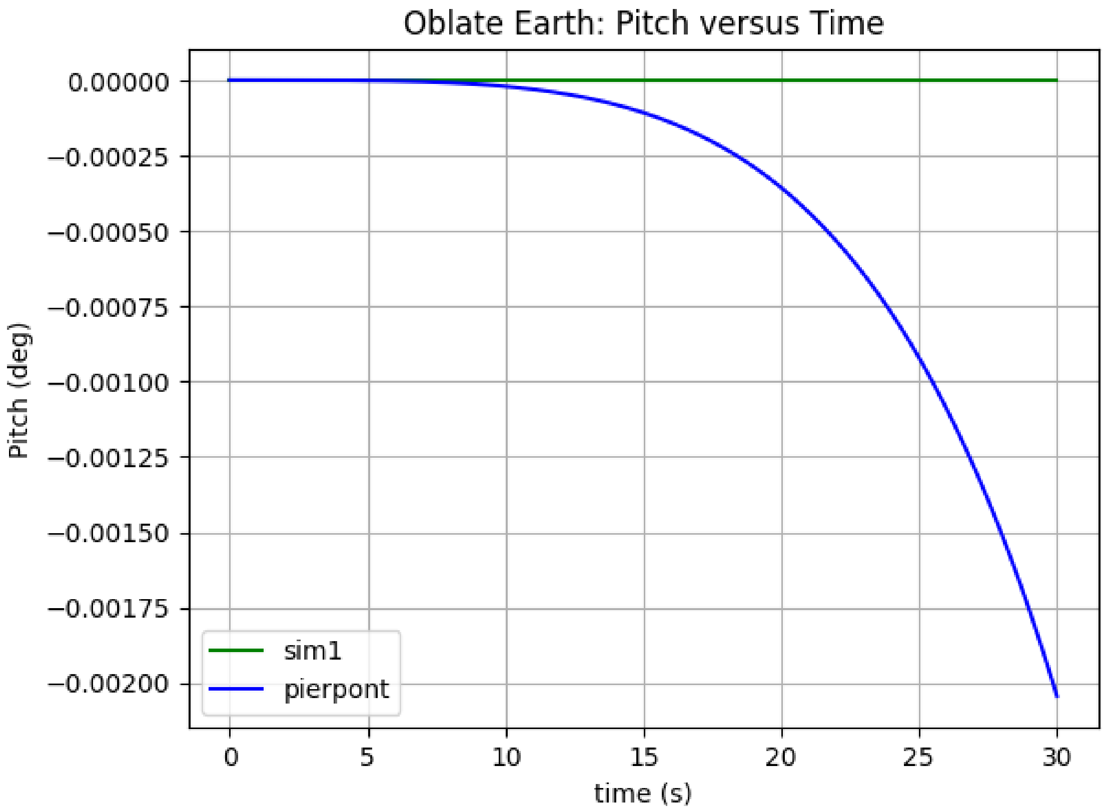
altitudeMsl_ft	1623.5211209467052	325.78303920045437
longitude_deg	2.1152199421624644e-05	4.07701616992336e-06
latitude_deg	0.011438977033606221	0.002043666399558951
localGravity_ft_s2	0.012094173192575396	0.0021173060586789916
gePosition_ft_X	1623.5553863119626	325.79608972370625
gePosition_ft_Y	7.728925068145949	1.4894847350385234
gePosition_ft_Z	1789.84846464604	381.95052822030516
eulerAngle_deg_Roll	2.113161955755363e-05	4.074494643513393e-06
eulerAngle_deg_Pitch	0.011438987196216907	0.00204366739954423
eulerAngle_deg_Yaw	2.1936332786621488e-11	2.188531823772246e-12

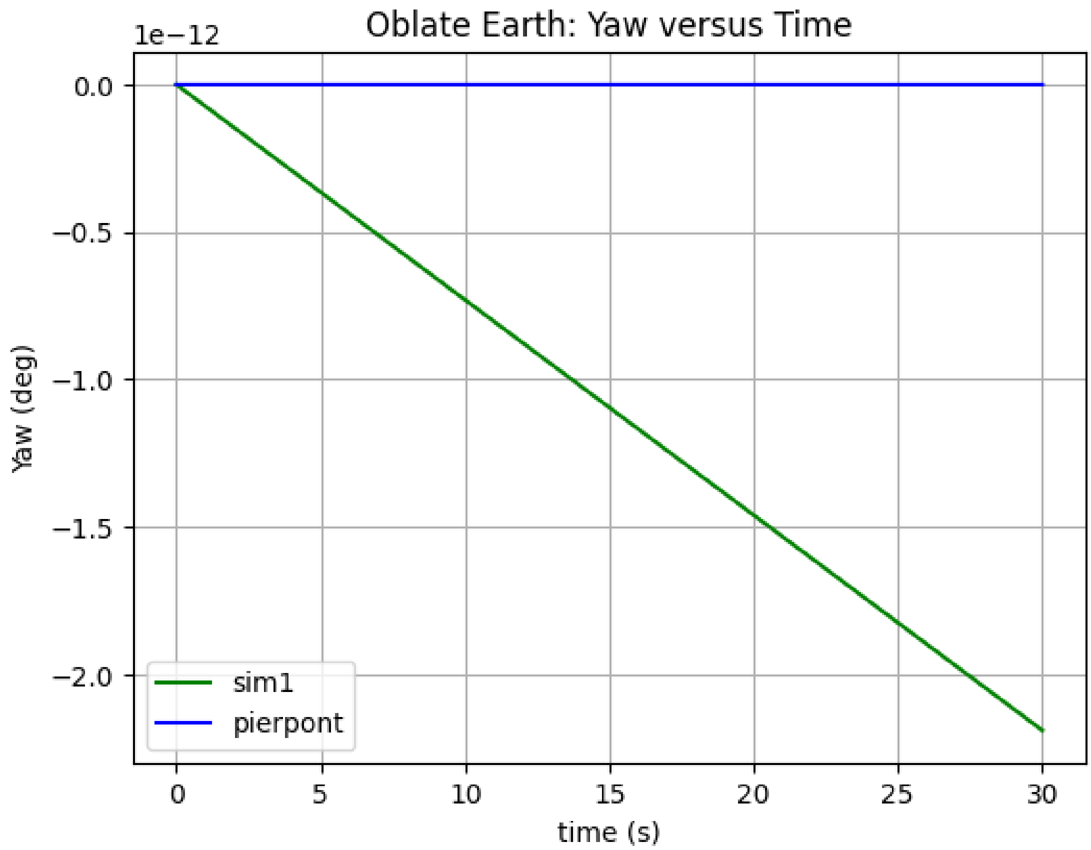


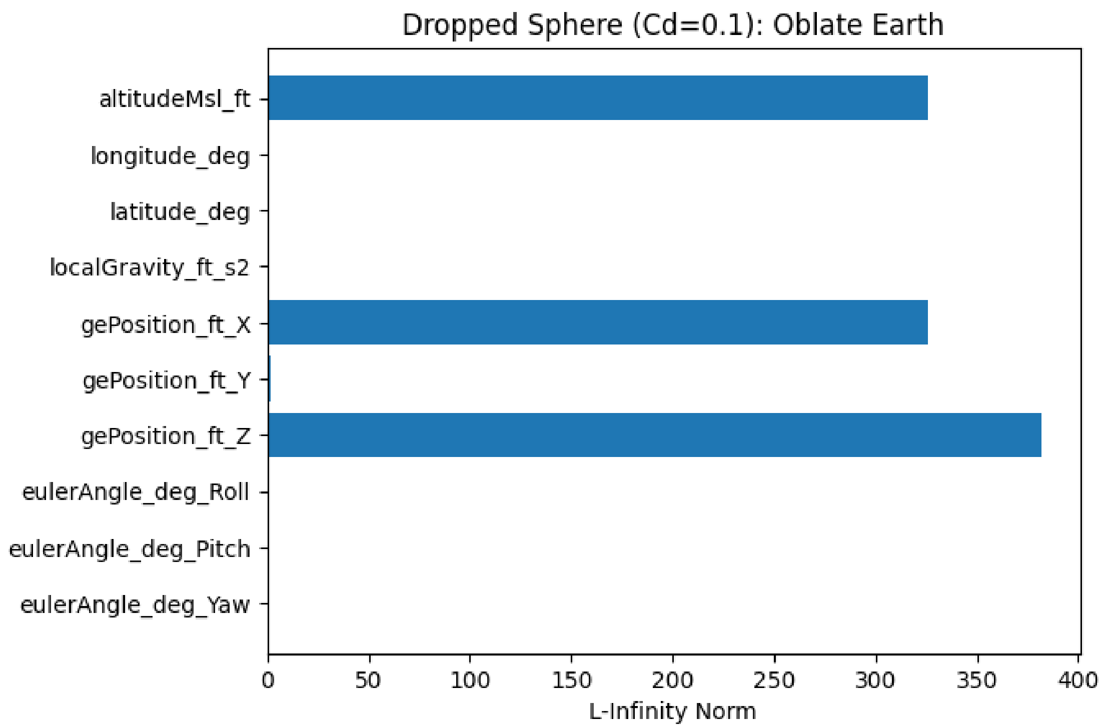


ECEF X versus Time
1e7
2.0956
2.0954
2.0952
2.0950
2.0948
2.0946
2.0944
2.0942
gePosition_ft_X
0
5
10
15
20
25
30
time (s)
sim1
pierpont


Oblate Earth: Roll versus Time
sim1
pierpont
Roll (deg)
0.00
−0.02
−0.04
−0.06
−0.08
−0.10
−0.12
0
5
10
15
20
25
30
time (s)


Oblate Earth: Pitch versus Time
0.00000
−0.00025
−0.00050
−0.00075
−0.00100
−0.00125
−0.00150
−0.00175
−0.00200
Pitch (deg)
0
5
10
15
20
25
30
time (s)
sim1
pierpont


Oblate Earth: Yaw versus Time
1e−12
0.0
−0.5
−1.0
−1.5
−2.0
Yaw (deg)
0
5
10
15
20
25
30
time (s)
sim1
pierpont



1.10.7 Stevens and Lewis Orbit

[57]:
```
h = 422000  # height in meters (about ISS alt); sim on pg 43 uses 100km
gd = 9.80665
a = 6378137.0
wz = 7.292115e-5

vy = math.sqrt(gd*(a + h)) - wz*(a + h)
print("vy: ", vy, " m/s")
```

```
vy:  7670.310336088278  m/s
```

The vehicle in the simulation is a solid brick. The moment of inertia of a brick:

$I_x = \frac{1}{12}m(y^2 + z^2)$
$I_y = \frac{1}{12}m(x^2 + z^2)$
$I_z = \frac{1}{12}m(x^2 + y^2)$

The dimensions of the brick as stated by Stevens and Lewis is 2 x 5 x 8 units.
Coordinate origin is at the center of mass of the brick.
8 unit side is parallel to the x-axis
5 unit side is parallel to the y-axis

For simplicity, the brick units are meters and the mass is 1kg.

$I_x = \frac{1}{12}(1)(2^2 + 5^2) = 2.41667$

$I_y = \frac{1}{12}(1)(8^2 + 2^2) = 5.667$
$I_z = \frac{1}{12}(1)(8^2 + 5^2) = 7.41667$

[58]:
```
%%time
printDebugData = False
gvAeroModel.LoadDml('models/noAero.dml', printDebugData)
#
ic = {
    "timeStep": [1.0, "s"],
    "totalMass": [1.0, "kg"],
    "bodyMomentOfInertia_X": [2.4167, "kgm2"],
    "bodyMomentOfInertia_Y": [5.667, "kgm2"],
    "bodyMomentOfInertia_Z": [7.417, "kgm2"],
    "altitudeMsl": [100000.0, "m"],
    "trueAirspeed": [9000.0, "m_s"],
    "referenceWingChord": [8.0, "m"],
    "referenceWingSpan": [5.0, "m"]
}
#
gvOblateRotatingEarth.Reset(ic)
gvOblateRotatingEarth.totalCoefficientOfDrag = 0.0
gvOblateRotatingEarth.Run(20000)
gvOblateRotatingEarth.GenerateEnglishUnits()
```

```
*******************************************
Model:  Zero Aero Output
creation date:  2021-07-21
file version:  Initial version
*******************************************

+++++ MODEL INPUTS AND OUTPUTS +++++
++> Output:  CX
++> Output:  CY
++> Output:  CZ
++> Output:  CLL
++> Output:  CLM
++> Output:  CLN
++++++++++++++++++++++++++++++++++++

----- DAVE-ML MODEL PARSE COMPLETE -----
========= SetIC ==============
{'timeStep': 1.0, 'totalMass': 1.0, 'bodyMomentOfInertia_X': 2.4167,
'bodyMomentOfInertia_Y': 5.667, 'bodyMomentOfInertia_Z': 7.417, 'altitudeMsl':
100000.0, 'trueAirspeed': 9000.0, 'referenceWingChord': 8.0,
'referenceWingSpan': 5.0}
++ timeStep = 1.0 [IC case]
++ totalMass = 1.0 [IC case]
++ referenceWingSpan = 5.0 [IC case]
```

```
++ referenceWingChord = 8.0 [IC case]
++ referenceWingArea = 40.0 [default]
++ trueAirspeed = 9000.0 [IC case]
++ angleOfAttack = 0 [default]
++ angleOfSideslip = 0 [default]
++ eulerAngle_Roll = 0 [default]
++ eulerAngle_Pitch = 0 [default]
++ eulerAngle_Yaw = 0 [default]
++ eulerAngleRate_Roll = 0 [default]
++ eulerAngleRate_Pitch = 0 [default]
++ eulerAngleRate_Yaw = 0 [default]
++ bodyMomentOfInertia_X = 2.4167 [IC case]
++ bodyProductOfInertia_XY = 0 [default]
++ bodyProductOfInertia_XZ = 0 [default]
++ bodyProductOfInertia_YX = 0 [default]
++ bodyMomentOfInertia_Y = 5.667 [IC case]
++ bodyProductOfInertia_YZ = 0 [default]
++ bodyProductOfInertia_XZ = 0 [default]
++ bodyProductOfInertia_YZ = 0 [default]
++ bodyMomentOfInertia_Z = 7.417 [IC case]
++ latitude = 0 [default]
++ longitude = 0 [default]
++ altitudeMsl = 100000.0 [IC case]
Vecf:  1.8189894035458565e-12 0.0 9000.0
======done=======
CPU times: user 2.49 s, sys: 45.2 ms, total: 2.53 s
Wall time: 2.64 s
```

[ ]:
```
from IPython.display import Image
Image(url= "images/SLOrbit.JPG", width=600, height=600)
```

[ ]:
```
fig1, a = plt.subplots()
a.plot(gvOblateRotatingEarth.eiPosition_m_X, gvOblateRotatingEarth.
 ↪eiPosition_m_Z)
a.set(xlabel='X', ylabel='Z', title='Orbit')
a.grid()

fig2, b = plt.subplots()
b.plot(gvOblateRotatingEarth.trueAirspeed, gvOblateRotatingEarth.altitudeMsl_m)
b.set(xlabel='speed', ylabel='height', title='Height vs Speed')
b.grid()

fig3, c = plt.subplots()
c.plot(gvOblateRotatingEarth.time, gvOblateRotatingEarth.
 ↪EnglishData['latitude_deg'],'g',
       gvOblateRotatingEarth.time, gvOblateRotatingEarth.
 ↪EnglishData['longitude_deg'], 'b')
```

```
c.legend(["latitude","longitude"])
c.set(xlabel='time', ylabel='lat/lon', title='Lat/Lon vs Time')
c.grid()

fig4, d = plt.subplots()
d.plot(gvOblateRotatingEarth.time, gvOblateRotatingEarth.
 ↪EnglishData['eulerAngle_deg_Roll'],'g',
       gvOblateRotatingEarth.time, gvOblateRotatingEarth.
 ↪EnglishData['eulerAngle_deg_Pitch'], 'b',
       gvOblateRotatingEarth.time, gvOblateRotatingEarth.
 ↪EnglishData['eulerAngle_deg_Yaw'], 'k')
d.legend(["roll","pitch","yaw"])
d.set(xlabel='time', ylabel='degrees', title='Euler Angles vs Time')
d.grid()
```

[ ]:

```

```