Author: Trey Chase Date: sys.date()

Driveline Open Biomechanics Project

```
## Rows: 411 Columns: 81
## -- Column specification --------------------------------------------------------
## Delimiter: ","
## chr  (3): session_pitch, p_throws, pitch_type
## dbl (78): session, pitch_speed_mph, max_shoulder_internal_rotational_velo, m...
##
## i Use `spec()` to retrieve the full column specification for this data.
## i Specify the column types or set `show_col_types = FALSE` to quiet this message.
```

```
dim(df)
```

```
## [1] 411  81
```

```
head(df)
```

```
## # A tibble: 6 x 81
##   session_pitch session p_throws pitch_type pitch_speed_mph
##   <chr>           <dbl> <chr>    <chr>                <dbl>
## 1 1031_2           1031 R        FF                    90.4
## 2 1031_3           1031 R        FF                    90.4
## 3 1097_1           1097 R        FF                    77.6
## 4 1097_2           1097 R        FF                    77
## 5 1097_3           1097 R        FF                    76.1
## 6 1170_1           1170 R        FF                    80.9
## # i 76 more variables: max_shoulder_internal_rotational_velo <dbl>,
## #   max_elbow_extension_velo <dbl>, max_torso_rotational_velo <dbl>,
## #   max_rotation_hip_shoulder_separation <dbl>, max_elbow_flexion <dbl>,
## #   max_shoulder_external_rotation <dbl>, elbow_flexion_fp <dbl>,
## #   elbow_pronation_fp <dbl>, rotation_hip_shoulder_separation_fp <dbl>,
## #   shoulder_horizontal_abduction_fp <dbl>, shoulder_abduction_fp <dbl>,
## #   shoulder_external_rotation_fp <dbl>, ...
```

```
summary(df)
```

```
##  session_pitch         session       p_throws          pitch_type
##  Length:411         Min.   :1031   Length:411         Length:411
##  Class :character   1st Qu.:1830   Class :character   Class :character
##  Mode  :character   Median :2861   Mode  :character   Mode  :character
##                     Mean   :2601
##                     3rd Qu.:2999
##                     Max.   :3252
##
##  pitch_speed_mph max_shoulder_internal_rotational_velo max_elbow_extension_velo
##  Min.   :69.5    Min.   :2718                          Min.   :1772
##  1st Qu.:81.4    1st Qu.:4326                          1st Qu.:2308
##  Median :85.3    Median :4533                          Median :2446
##  Mean   :84.7    Mean   :4531                          Mean   :2465
```

```
##  3rd Qu.:87.9    3rd Qu.:4746                          3rd Qu.:2611
##  Max.   :94.4    Max.   :5413                          Max.   :3100
##
##  max_torso_rotational_velo max_rotation_hip_shoulder_separation
##  Min.   : 848.4            Min.   :13.16
##  1st Qu.: 997.3            1st Qu.:27.57
##  Median :1049.7            Median :32.32
##  Mean   :1054.8            Mean   :32.25
##  3rd Qu.:1110.1            3rd Qu.:36.74
##  Max.   :1383.2            Max.   :50.51
##
##  max_elbow_flexion max_shoulder_external_rotation elbow_flexion_fp
##  Min.   : 82.43    Min.   :143.3                  Min.   : 50.33
##  1st Qu.:110.75    1st Qu.:163.5                  1st Qu.: 92.09
##  Median :119.27    Median :169.3                  Median :102.41
##  Mean   :117.99    Mean   :169.1                  Mean   :103.24
##  3rd Qu.:124.14    3rd Qu.:176.4                  3rd Qu.:115.73
##  Max.   :142.38    Max.   :191.5                  Max.   :137.98
##
##  elbow_pronation_fp rotation_hip_shoulder_separation_fp
##  Min.   :-38.3447   Min.   : 8.404
##  1st Qu.:  0.4375   1st Qu.:25.196
##  Median : 12.2514   Median :29.991
##  Mean   : 11.3472   Mean   :29.795
##  3rd Qu.: 23.2280   3rd Qu.:34.697
##  Max.   : 64.7835   Max.   :48.764
##
##  shoulder_horizontal_abduction_fp shoulder_abduction_fp
##  Min.   :-6.529                   Min.   : 63.37
##  1st Qu.:32.691                   1st Qu.: 80.25
##  Median :43.955                   Median : 85.92
##  Mean   :41.964                   Mean   : 86.65
##  3rd Qu.:50.900                   3rd Qu.: 93.26
##  Max.   :71.291                   Max.   :118.30
##
##  shoulder_external_rotation_fp lead_knee_extension_angular_velo_fp
##  Min.   :-17.45                Min.   :-480.273
##  1st Qu.: 33.68                1st Qu.:   5.865
##  Median : 47.24                Median : 121.201
##  Mean   : 47.40                Mean   : 127.209
##  3rd Qu.: 61.91                3rd Qu.: 245.734
##  Max.   :114.83                Max.   : 632.028
##
##  lead_knee_extension_angular_velo_br lead_knee_extension_angular_velo_max
##  Min.   :-14.93                      Min.   : 77.97
##  1st Qu.:178.42                      1st Qu.:261.04
##  Median :267.92                      Median :347.84
##  Mean   :288.24                      Mean   :373.07
##  3rd Qu.:375.46                      3rd Qu.:471.87
##  Max.   :954.92                      Max.   :955.15
##
##  torso_anterior_tilt_fp torso_lateral_tilt_fp torso_rotation_fp
##  Min.   :-35.8486       Min.   :-23.2480      Min.   :-34.422
##  1st Qu.:-14.6965       1st Qu.: -9.2243      1st Qu.: -2.703
```

```
##  Median : -6.4603       Median : -4.3166      Median :  5.535
##  Mean   : -7.3104       Mean   : -4.2617      Mean   :  4.813
##  3rd Qu.: -0.2376       3rd Qu.: -0.0921      3rd Qu.: 12.295
##  Max.   : 16.7567       Max.   : 16.0880      Max.   : 63.155
##
##  pelvis_anterior_tilt_fp pelvis_lateral_tilt_fp pelvis_rotation_fp
##  Min.   :-12.289         Min.   :-12.72950      Min.   : 1.877
##  1st Qu.:  2.548         1st Qu.: -3.34955      1st Qu.:27.095
##  Median :  7.563         Median : -0.11740      Median :34.314
##  Mean   :  7.358         Mean   : -0.09267      Mean   :34.726
##  3rd Qu.: 12.577         3rd Qu.:  2.47725      3rd Qu.:42.092
##  Max.   : 24.884         Max.   : 16.87230      Max.   :73.480
##
##  max_cog_velo_x  torso_rotation_min max_pelvis_rotational_velo
##  Min.   :2.274   Min.   :-75.70     Min.   : 505.2
##  1st Qu.:2.911   1st Qu.:-43.63     1st Qu.: 687.6
##  Median :3.075   Median :-37.54     Median : 742.1
##  Mean   :3.056   Mean   :-38.53     Mean   : 751.5
##  3rd Qu.:3.239   3rd Qu.:-31.14     3rd Qu.: 806.5
##  Max.   :3.583   Max.   :-19.09     Max.   :1125.4
##
##  glove_shoulder_horizontal_abduction_fp glove_shoulder_abduction_fp
##  Min.   :-3.224                         Min.   : 42.54
##  1st Qu.:31.587                         1st Qu.: 68.15
##  Median :37.559                         Median : 75.00
##  Mean   :38.002                         Mean   : 74.46
##  3rd Qu.:44.933                         3rd Qu.: 81.61
##  Max.   :75.990                         Max.   :109.83
##
##  glove_shoulder_external_rotation_fp glove_shoulder_abduction_mer
##  Min.   :-81.81                      Min.   :12.28
##  1st Qu.:-52.50                      1st Qu.:29.43
##  Median :-40.31                      Median :35.83
##  Mean   :-39.41                      Mean   :36.09
##  3rd Qu.:-29.93                      3rd Qu.:42.61
##  Max.   : 30.67                      Max.   :72.33
##
##  elbow_flexion_mer torso_anterior_tilt_mer torso_lateral_tilt_mer
##  Min.   : 66.76    Min.   :-9.104          Min.   :-2.248
##  1st Qu.: 87.71    1st Qu.: 9.265          1st Qu.:17.041
##  Median : 92.64    Median :17.538          Median :22.909
##  Mean   : 92.17    Mean   :17.262          Mean   :23.329
##  3rd Qu.: 97.02    3rd Qu.:25.139          3rd Qu.:30.258
##  Max.   :116.95    Max.   :42.742          Max.   :45.517
##
##  torso_rotation_mer elbow_varus_moment shoulder_internal_rotation_moment
##  Min.   : 76.98     Min.   : 67.85     Min.   : 63.24
##  1st Qu.: 98.55     1st Qu.: 96.90     1st Qu.: 91.47
##  Median :105.56     Median :110.95     Median :107.44
##  Mean   :104.62     Mean   :111.27     Mean   :106.12
##  3rd Qu.:112.65     3rd Qu.:123.17     3rd Qu.:116.85
##  Max.   :132.07     Max.   :199.93     Max.   :181.46
##
##  torso_anterior_tilt_br torso_lateral_tilt_br torso_rotation_br
```

```
##  Min.   : 6.018         Min.   :-5.933        Min.   : 93.69
##  1st Qu.:27.204         1st Qu.:11.489        1st Qu.:113.23
##  Median :34.698         Median :18.068        Median :118.53
##  Mean   :35.092         Mean   :17.115        Mean   :119.55
##  3rd Qu.:44.105         3rd Qu.:23.494        3rd Qu.:126.31
##  Max.   :60.478         Max.   :37.928        Max.   :144.43
##
##  lead_knee_extension_from_fp_to_br  cog_velo_pkh    stride_length
##  Min.   :-17.71                    Min.   :0.0539   Min.   :0.6980
##  1st Qu.:  1.88                    1st Qu.:0.2500   1st Qu.:0.7986
##  Median : 10.02                    Median :0.3214   Median :0.8374
##  Mean   : 10.85                    Mean   :0.3173   Mean   :0.8354
##  3rd Qu.: 17.52                    3rd Qu.:0.3750   3rd Qu.:0.8766
##  Max.   : 55.19                    Max.   :0.8571   Max.   :0.9956
##
##   stride_angle        arm_slot     timing_peak_torso_to_peak_pelvis_rot_velo
##  Min.   :-12.286   Min.   :14.21   Min.   :-0.02500
##  1st Qu.: -2.706   1st Qu.:37.49   1st Qu.: 0.00000
##  Median :  2.595   Median :42.10   Median : 0.00830
##  Mean   :  1.919   Mean   :41.93   Mean   : 0.01214
##  3rd Qu.:  5.928   3rd Qu.:46.57   3rd Qu.: 0.01670
##  Max.   : 16.343   Max.   :60.38   Max.   : 0.11670
##
##  max_shoulder_horizontal_abduction shoulder_transfer_fp_br
##  Min.   :15.75                     Min.   :170.6
##  1st Qu.:40.01                     1st Qu.:295.1
##  Median :50.68                     Median :329.8
##  Mean   :48.24                     Mean   :338.0
##  3rd Qu.:56.05                     3rd Qu.:385.9
##  Max.   :77.30                     Max.   :503.3
##
##  shoulder_generation_fp_br shoulder_absorption_fp_br elbow_transfer_fp_br
##  Min.   :  1.139           Min.   : 0.00             Min.   :195.9
##  1st Qu.: 21.739           1st Qu.:11.96             1st Qu.:297.7
##  Median : 32.111           Median :20.26             Median :336.6
##  Mean   : 33.209           Mean   :22.28             Mean   :341.4
##  3rd Qu.: 42.273           3rd Qu.:30.30             3rd Qu.:388.7
##  Max.   :110.680           Max.   :78.32             Max.   :482.1
##
##  elbow_generation_fp_br elbow_absorption_fp_br lead_hip_transfer_fp_br
##  Min.   : 0.000         Min.   : 13.31         Min.   :  5.881
##  1st Qu.: 2.938         1st Qu.: 40.52         1st Qu.: 27.837
##  Median : 4.108         Median : 52.31         Median : 42.778
##  Mean   : 4.683         Mean   : 51.90         Mean   : 48.542
##  3rd Qu.: 5.764         3rd Qu.: 62.91         3rd Qu.: 67.810
##  Max.   :29.892         Max.   :116.60         Max.   :152.273
##                                                NA's   :8
##  lead_hip_generation_fp_br lead_hip_absorption_fp_br lead_knee_transfer_fp_br
##  Min.   :  0.000           Min.   :  0.8008          Min.   :  1.876
##  1st Qu.:  9.179           1st Qu.: 70.5898          1st Qu.: 28.617
##  Median : 17.434           Median : 96.8691          Median : 42.865
##  Mean   : 22.713           Mean   : 99.1174          Mean   : 44.547
##  3rd Qu.: 29.978           3rd Qu.:125.5535          3rd Qu.: 57.724
##  Max.   :138.125           Max.   :233.1741          Max.   :144.053
```

```
##  NA's   :8                NA's   :8                NA's   :8
##  lead_knee_generation_fp_br lead_knee_absorption_fp_br rear_hip_transfer_pkh_fp
##  Min.   : 0.0981           Min.   :  0.00            Min.   :  1.677
##  1st Qu.:18.6322           1st Qu.: 11.14            1st Qu.: 74.847
##  Median :32.4933           Median : 18.70            Median : 99.057
##  Mean   :33.4674           Mean   : 20.73            Mean   : 99.357
##  3rd Qu.:44.3273           3rd Qu.: 26.89            3rd Qu.:123.640
##  Max.   :92.5344           Max.   :110.92            Max.   :270.675
##  NA's   :8                 NA's   :8                 NA's   :8
##  rear_hip_generation_pkh_fp rear_hip_absorption_pkh_fp
##  Min.   :  3.929           Min.   :  0.2457
##  1st Qu.:117.425           1st Qu.: 32.3544
##  Median :152.299           Median : 46.7506
##  Mean   :149.814           Mean   : 51.5963
##  3rd Qu.:185.740           3rd Qu.: 67.3661
##  Max.   :300.149           Max.   :175.7362
##  NA's   :8                 NA's   :8
##  rear_knee_transfer_pkh_fp rear_knee_generation_pkh_fp
##  Min.   :  0.00            Min.   :  0.0933
##  1st Qu.: 36.48            1st Qu.: 30.1995
##  Median : 48.86            Median : 56.3184
##  Mean   : 52.66            Mean   : 58.1526
##  3rd Qu.: 63.72            3rd Qu.: 78.5082
##  Max.   :161.49            Max.   :210.7238
##  NA's   :8                 NA's   :8
##  rear_knee_absorption_pkh_fp pelvis_lumbar_transfer_fp_br
##  Min.   :  0.189             Min.   : 14.27
##  1st Qu.: 26.458             1st Qu.: 91.00
##  Median : 42.712             Median :120.30
##  Mean   : 44.620             Mean   :139.15
##  3rd Qu.: 60.303             3rd Qu.:170.34
##  Max.   :104.741             Max.   :639.76
##  NA's   :8                   NA's   :8
##  thorax_distal_transfer_fp_br rear_grf_x_max   rear_grf_y_max
##  Min.   :207.9                Min.   : 263.2   Min.   :  3.239
##  1st Qu.:308.3                1st Qu.: 703.4   1st Qu.: 77.683
##  Median :354.4                Median : 795.1   Median :111.171
##  Mean   :359.7                Mean   : 801.8   Mean   :121.749
##  3rd Qu.:405.2                3rd Qu.: 877.7   3rd Qu.:161.773
##  Max.   :537.0                Max.   :1238.3   Max.   :367.228
##                               NA's   :8        NA's   :8
##  rear_grf_z_max   rear_grf_mag_max rear_grf_angle_at_max lead_grf_x_max
##  Min.   : 712.8   Min.   : 831.9   Min.   :46.75         Min.   : 553.5
##  1st Qu.:1123.1   1st Qu.:1338.2   1st Qu.:56.63         1st Qu.: 919.2
##  Median :1282.8   Median :1468.5   Median :58.87         Median :1038.2
##  Mean   :1283.0   Mean   :1479.9   Mean   :59.51         Mean   :1055.7
##  3rd Qu.:1395.7   3rd Qu.:1587.7   3rd Qu.:62.63         3rd Qu.:1171.7
##  Max.   :1892.6   Max.   :2183.7   Max.   :87.05         Max.   :1821.3
##  NA's   :8        NA's   :8        NA's   :8             NA's   :8
##  lead_grf_y_max   lead_grf_z_max lead_grf_mag_max lead_grf_angle_at_max
##  Min.   : 51.17   Min.   :1115   Min.   :1162     Min.   :48.72
##  1st Qu.:147.24   1st Qu.:1629   1st Qu.:1861     1st Qu.:57.62
##  Median :205.27   Median :1772   Median :2067     Median :60.17
##  Mean   :223.65   Mean   :1818   Mean   :2095     Mean   :60.45
```

```
##  3rd Qu.:271.23   3rd Qu.:1946   3rd Qu.:2273     3rd Qu.:63.03
##  Max.   :820.48   Max.   :3273   Max.   :3633     Max.   :78.32
##  NA's   :8        NA's   :8      NA's   :8        NA's   :8
##  peak_rfd_rear       peak_rfd_lead
##  Min.   :    2.501   Min.   :    14.32
##  1st Qu.:    6.977   1st Qu.:    51.91
##  Median :    9.716   Median :    73.93
##  Mean   :  720.711   Mean   :  6742.60
##  3rd Qu.:   14.272   3rd Qu.:   116.01
##  Max.   :16303.170   Max.   :144366.68
##  NA's   :8           NA's   :8
```

```
#obtaining all quantitative variables, dropping all rows with na
q_df = df |>
  select_if(
    is.numeric,
  ) |>
  drop_na() |>
  select(
    -(session) #removing session column
  )
```

```
## [1] 0.6822101
```

```
## [1] "elbow_transfer_fp_br"
```

```
cor_plot = ggcorrplot(corr_matrix)
ggsave("dl_corr_plot.png", plot = cor_plot, width = 30, height = 30)
```

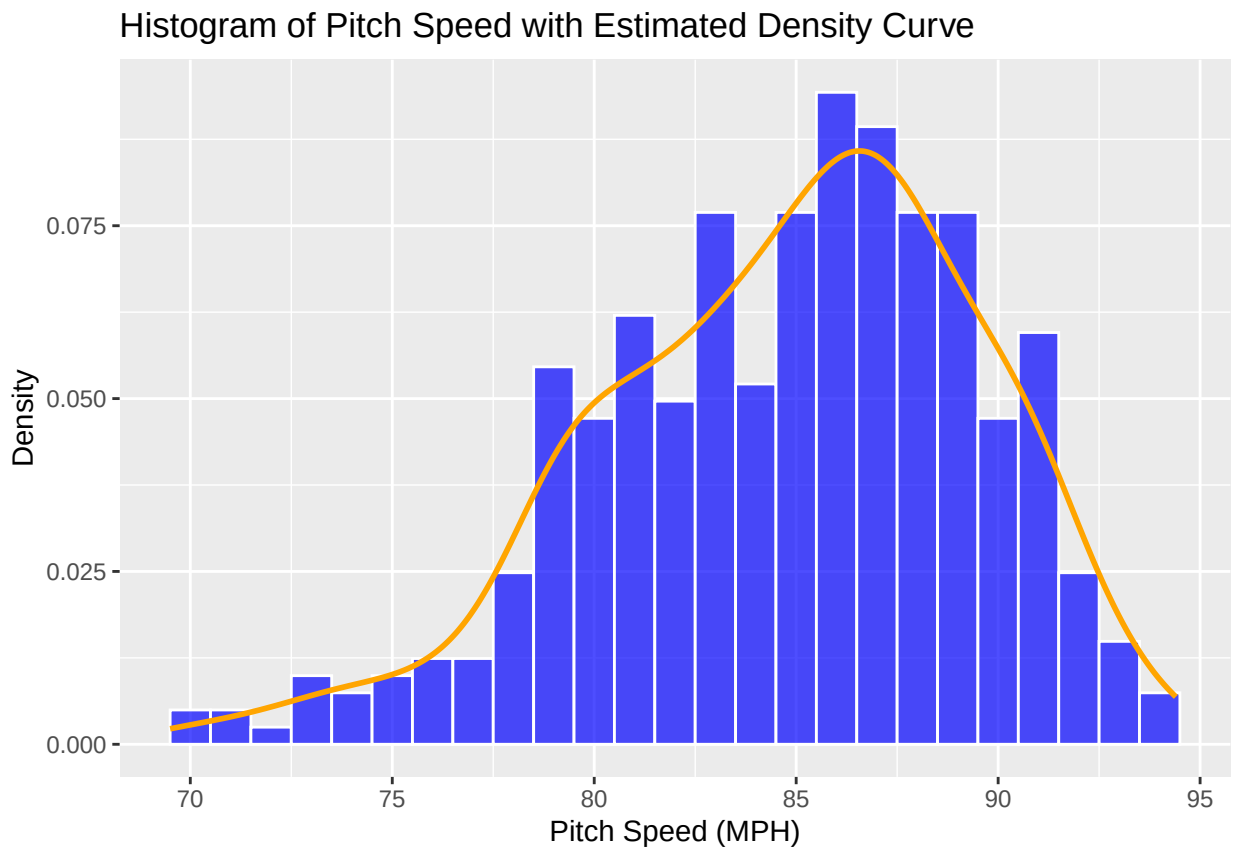
Saving the correlation plot to examine which variables are correlated with pitch speed.

```
pitch_speed_plot = ggplot(q_df, aes(x = pitch_speed_mph)) +
  geom_histogram(aes(y = ..density..), binwidth = 1, color = "white", fill = "blue", alpha = 0.7) +
  geom_density(color = "orange", size = 1) +
  labs(x = "Pitch Speed (MPH)",
       y = "Density",
       title = "Histogram of Pitch Speed with Estimated Density Curve")
```

```
## Warning: Using ‘size‘ aesthetic for lines was deprecated in ggplot2 3.4.0.
## i Please use ‘linewidth‘ instead.
## This warning is displayed once every 8 hours.
## Call ‘lifecycle::last_lifecycle_warnings()‘ to see where this warning was
## generated.
```

```
pitch_speed_plot
```

```
## Warning: The dot-dot notation (‘..density..‘) was deprecated in ggplot2 3.4.0.
## i Please use ‘after_stat(density)‘ instead.
## This warning is displayed once every 8 hours.
## Call ‘lifecycle::last_lifecycle_warnings()‘ to see where this warning was
## generated.
```




```
ggsave("pitch_speed_plot.png", pitch_speed_plot)
```

```
## Saving 6.5 x 4.5 in image
```

We can see that the distribution of pitch speeds are slightly skewed left but approximately normal.

```
set.seed(123)  # For reproducibility
split <- initial_split(q_df, prop = 0.8) #splitting data into test and training data
train_df <- training(split)
test_df <- testing(split)
```

This step-wise regression will start by constructing regression models with all variables present, and will iterate and remove one variable based on the lowest AIC of the models.

```
library(car)
```

```
## Loading required package: carData
```

```
##
## Attaching package: 'car'
```

```
## The following object is masked from 'package:purrr':
##
##     some
```

```
## The following object is masked from 'package:dplyr':
##
##     recode
```

```
vifs = car::vif(selected_mod)
print(vifs)
```

```
##  max_shoulder_internal_rotational_velo             max_torso_rotational_velo
##                                2.285484                                4.913453
##                      max_elbow_flexion        max_shoulder_external_rotation
##                                8.020336                                4.750964
##                       elbow_flexion_fp                    elbow_pronation_fp
##                                7.500229                                2.112016
##                  shoulder_abduction_fp         shoulder_external_rotation_fp
##                                3.745980                                5.228102
##    lead_knee_extension_angular_velo_br   lead_knee_extension_angular_velo_max
##                                5.542250                                5.259915
##                 torso_anterior_tilt_fp                torso_lateral_tilt_fp
##                                7.294458                                4.063381
##                     torso_rotation_fp              pelvis_anterior_tilt_fp
##                               19.919216                                2.320841
##                    pelvis_rotation_fp                       max_cog_velo_x
##                                9.159052                                2.842627
## glove_shoulder_horizontal_abduction_fp          glove_shoulder_abduction_fp
##                                1.913885                                2.270621
##    glove_shoulder_external_rotation_fp                    elbow_flexion_mer
##                                2.635181                                4.420009
##                 torso_lateral_tilt_mer                   torso_rotation_mer
##                               63.322493                               12.151858
##                    elbow_varus_moment     shoulder_internal_rotation_moment
##                               84.124006                               88.902903
##                 torso_lateral_tilt_br                    torso_rotation_br
##                               42.101716                               19.335710
##     lead_knee_extension_from_fp_to_br                        cog_velo_pkh
##                                5.791639                                2.422215
##              shoulder_absorption_fp_br                 elbow_transfer_fp_br
##                                4.264534                                7.313654
##                elbow_generation_fp_br              lead_hip_transfer_fp_br
##                                2.127481                                5.031992
##              lead_hip_generation_fp_br            lead_hip_absorption_fp_br
##                                3.662829                                2.964820
##             lead_knee_generation_fp_br             rear_hip_transfer_pkh_fp
##                                4.297034                                5.275631
##              rear_knee_transfer_pkh_fp          rear_knee_absorption_pkh_fp
##                                4.218693                                2.661234
##           pelvis_lumbar_transfer_fp_br                       rear_grf_z_max
##                                2.603382                                3.442771
##                          lead_grf_y_max                       lead_grf_z_max
##                                2.434598                             159.316201
##                       lead_grf_mag_max                lead_grf_angle_at_max
##                              148.874592                                9.805557
```

Recursively remove VIFs > 4 to ensure the multi-collinearity check is confirmed.

```r
signif_all <- names(vifs)

while(any(vifs >= 10)) {  # while any of the vifs have a value greater than 10
  var_with_max_vif <- names(which.max(vifs))  # get the var with max vif
  signif_all <- signif_all[!signif_all %in% var_with_max_vif]  # remove
  myForm <- as.formula(paste("pitch_speed_mph ~ ", paste(signif_all, collapse=" + "), sep=""))  # new f
  selected_mod <- lm(myForm, data=train_df)  # re-build model with new formula
  vifs <- car::vif(selected_mod)  # update VIFs
}

summary(selected_mod)
```

```
##
## Call:
## lm(formula = myForm, data = train_df)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -7.1096 -1.0396  0.1078  1.0884  7.2857
##
## Coefficients:
##                                          Estimate Std. Error t value Pr(>|t|)
## (Intercept)                             12.0132319  7.3157384   1.642 0.101682
## max_shoulder_internal_rotational_velo    0.0008071  0.0004879   1.654 0.099187
## max_torso_rotational_velo                0.0205134  0.0022403   9.156  < 2e-16
## max_elbow_flexion                        0.0616693  0.0291114   2.118 0.035017
## max_shoulder_external_rotation           0.1011347  0.0237768   4.254 2.87e-05
## elbow_flexion_fp                        -0.0248327  0.0174797  -1.421 0.156522
## elbow_pronation_fp                       0.0242014  0.0085995   2.814 0.005233
## shoulder_abduction_fp                    0.0265147  0.0203532   1.303 0.193729
## shoulder_external_rotation_fp           -0.0372884  0.0085800  -4.346 1.94e-05
## lead_knee_extension_angular_velo_br      0.0061943  0.0016651   3.720 0.000240
## lead_knee_extension_angular_velo_max    -0.0032685  0.0017547  -1.863 0.063543
## torso_anterior_tilt_fp                   0.0318379  0.0218459   1.457 0.146123
## torso_lateral_tilt_fp                   -0.0670205  0.0283167  -2.367 0.018617
## pelvis_anterior_tilt_fp                  0.0567593  0.0227372   2.496 0.013120
## pelvis_rotation_fp                       0.1672631  0.0184424   9.069  < 2e-16
## max_cog_velo_x                           4.4812459  0.7156129   6.262 1.41e-09
## glove_shoulder_horizontal_abduction_fp -0.0216585  0.0108643  -1.994 0.047164
## glove_shoulder_abduction_fp              0.0163906  0.0155897   1.051 0.293987
## glove_shoulder_external_rotation_fp     -0.0278044  0.0093638  -2.969 0.003241
## elbow_flexion_mer                       -0.0785751  0.0215555  -3.645 0.000318
## torso_rotation_mer                      -0.1218707  0.0198981  -6.125 3.04e-09
## elbow_varus_moment                       0.0384731  0.0126068   3.052 0.002492
## torso_lateral_tilt_br                    0.1053713  0.0241265   4.367 1.77e-05
## lead_knee_extension_from_fp_to_br        0.0534942  0.0223974   2.388 0.017579
## cog_velo_pkh                            -2.2527959  1.7012471  -1.324 0.186508
## shoulder_absorption_fp_br               -0.0442418  0.0138573  -3.193 0.001569
## elbow_transfer_fp_br                     0.0469873  0.0038665  12.152  < 2e-16
## elbow_generation_fp_br                   0.0531901  0.0480555   1.107 0.269303
## lead_hip_transfer_fp_br                 -0.0180542  0.0091067  -1.983 0.048392
## lead_hip_generation_fp_br               -0.0402842  0.0099407  -4.052 6.56e-05
## lead_hip_absorption_fp_br               -0.0132945  0.0044685  -2.975 0.003182
```

```
## lead_knee_generation_fp_br             -0.0051314  0.0115253  -0.445 0.656493
## rear_hip_transfer_pkh_fp               -0.0198730  0.0058429  -3.401 0.000768
## rear_knee_transfer_pkh_fp               0.0068513  0.0084418   0.812 0.417711
## rear_knee_absorption_pkh_fp            -0.0206865  0.0074977  -2.759 0.006176
## pelvis_lumbar_transfer_fp_br            0.0086582  0.0024337   3.558 0.000439
## rear_grf_z_max                         -0.0004112  0.0009492  -0.433 0.665192
## lead_grf_y_max                         -0.0041456  0.0015010  -2.762 0.006124
## lead_grf_mag_max                        0.0002301  0.0005435   0.423 0.672370
## lead_grf_angle_at_max                   0.1172674  0.0361342   3.245 0.001315
##
## (Intercept)
## max_shoulder_internal_rotational_velo  .
## max_torso_rotational_velo              ***
## max_elbow_flexion                      *
## max_shoulder_external_rotation         ***
## elbow_flexion_fp
## elbow_pronation_fp                     **
## shoulder_abduction_fp
## shoulder_external_rotation_fp          ***
## lead_knee_extension_angular_velo_br    ***
## lead_knee_extension_angular_velo_max   .
## torso_anterior_tilt_fp
## torso_lateral_tilt_fp                  *
## pelvis_anterior_tilt_fp                *
## pelvis_rotation_fp                     ***
## max_cog_velo_x                         ***
## glove_shoulder_horizontal_abduction_fp *
## glove_shoulder_abduction_fp
## glove_shoulder_external_rotation_fp    **
## elbow_flexion_mer                      ***
## torso_rotation_mer                     ***
## elbow_varus_moment                     **
## torso_lateral_tilt_br                  ***
## lead_knee_extension_from_fp_to_br      *
## cog_velo_pkh
## shoulder_absorption_fp_br              **
## elbow_transfer_fp_br                   ***
## elbow_generation_fp_br
## lead_hip_transfer_fp_br                *
## lead_hip_generation_fp_br              ***
## lead_hip_absorption_fp_br              **
## lead_knee_generation_fp_br
## rear_hip_transfer_pkh_fp               ***
## rear_knee_transfer_pkh_fp
## rear_knee_absorption_pkh_fp            **
## pelvis_lumbar_transfer_fp_br           ***
## rear_grf_z_max
## lead_grf_y_max                         **
## lead_grf_mag_max
## lead_grf_angle_at_max                  **
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 2.003 on 282 degrees of freedom
```

```
## Multiple R-squared:  0.8441, Adjusted R-squared:  0.8225
## F-statistic: 39.15 on 39 and 282 DF,  p-value: < 2.2e-16
```

Multicollinearity has been removed. Now we need to remove the variables in the model that are not statistically significant from the t-test.

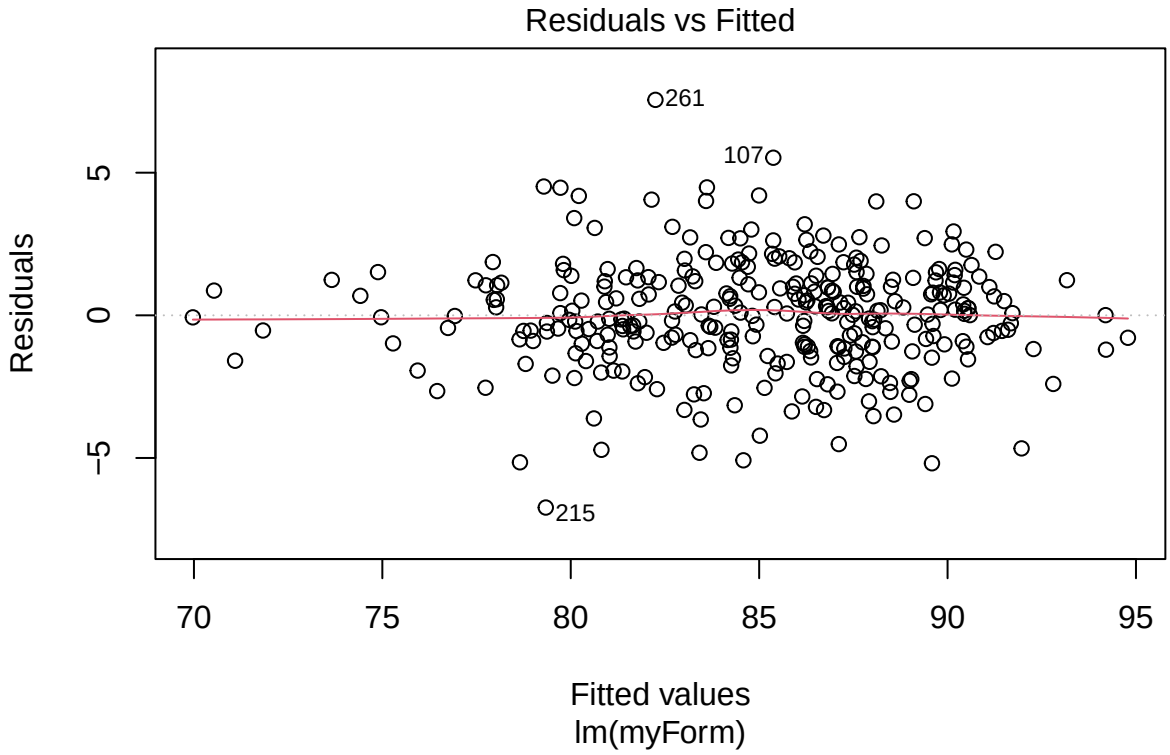
```
all_vars <- names(selected_mod[[1]])[-1]  # names of all X variables
# Get the non-significant vars
summ <- summary(selected_mod)  # model summary
pvals <- summ[[4]][, 4]  # get all p values
not_significant <- character()  # init variables that aren't statistically significant
not_significant <- names(which(pvals > 0.05))
not_significant <- not_significant[!not_significant %in% "(Intercept)"]  # remove 'intercept'. Optional

# If there are any non-significant variables,
while(length(not_significant) > 0){
  all_vars <- all_vars[!all_vars %in% not_significant[1]]
  myForm <- as.formula(paste("pitch_speed_mph ~ ", paste (all_vars, collapse=" + "), sep=""))  # new fo
  selected_mod <- lm(myForm, data=train_df)  # re-build model with new formula

  # Get the non-significant vars.
  summ <- summary(selected_mod)
  pvals <- summ[[4]][, 4]
  not_significant <- character()
  not_significant <- names(which(pvals > 0.1))
  not_significant <- not_significant[!not_significant %in% "(Intercept)"]
}
summary(selected_mod)
```

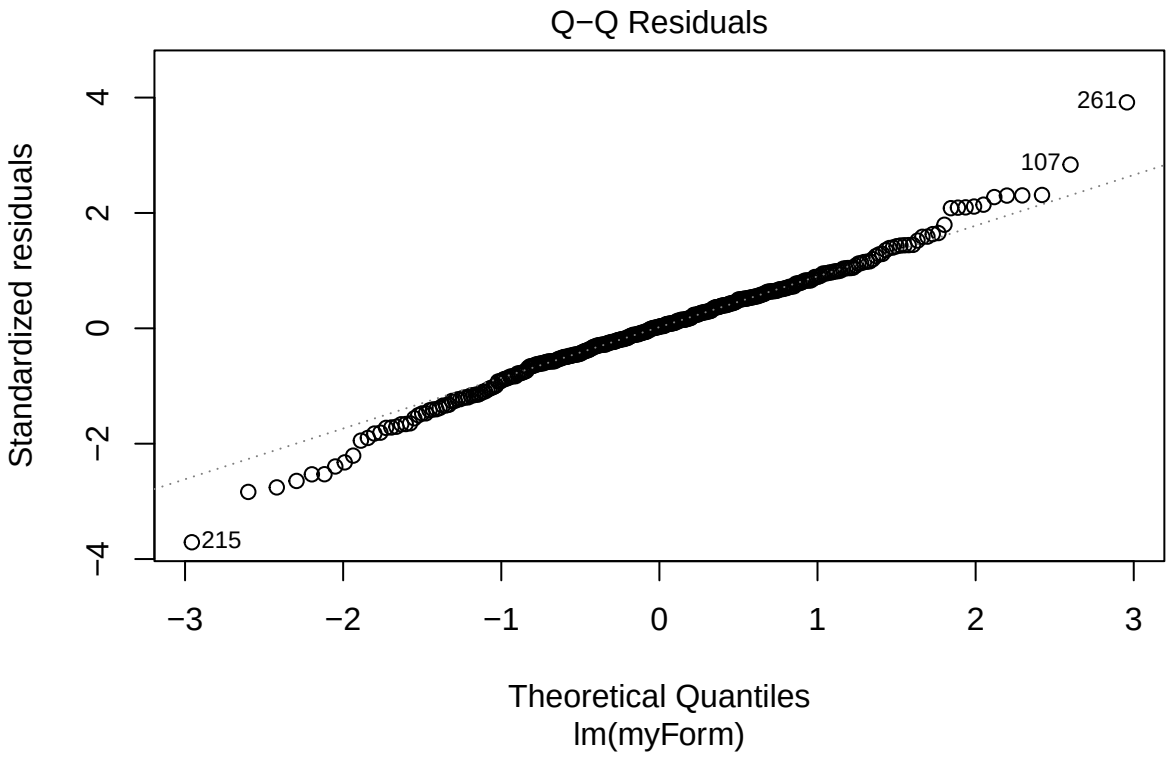
```
##
## Call:
## lm(formula = myForm, data = train_df)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -6.7371 -1.0973  0.0581  1.1605  7.5529
##
## Coefficients:
##                                      Estimate Std. Error t value Pr(>|t|)
## (Intercept)                         16.280539   6.134699   2.654 0.008394
## max_torso_rotational_velo            0.021577   0.001832  11.779  < 2e-16
## max_elbow_flexion                    0.080054   0.027607   2.900 0.004018
## max_shoulder_external_rotation       0.113679   0.019702   5.770 2.02e-08
## elbow_flexion_fp                    -0.041197   0.015181  -2.714 0.007048
## elbow_pronation_fp                   0.027486   0.008048   3.415 0.000728
## shoulder_external_rotation_fp       -0.038954   0.007089  -5.495 8.54e-08
## lead_knee_extension_angular_velo_br  0.005459   0.001513   3.607 0.000364
## lead_knee_extension_angular_velo_max -0.003993   0.001648  -2.423 0.015980
## torso_lateral_tilt_fp               -0.091676   0.027036  -3.391 0.000793
## pelvis_anterior_tilt_fp              0.062023   0.020879   2.971 0.003218
## pelvis_rotation_fp                   0.164132   0.015582  10.534  < 2e-16
## max_cog_velo_x                       4.202985   0.633500   6.635 1.58e-10
## glove_shoulder_horizontal_abduction_fp -0.020555   0.010334  -1.989 0.047622
```

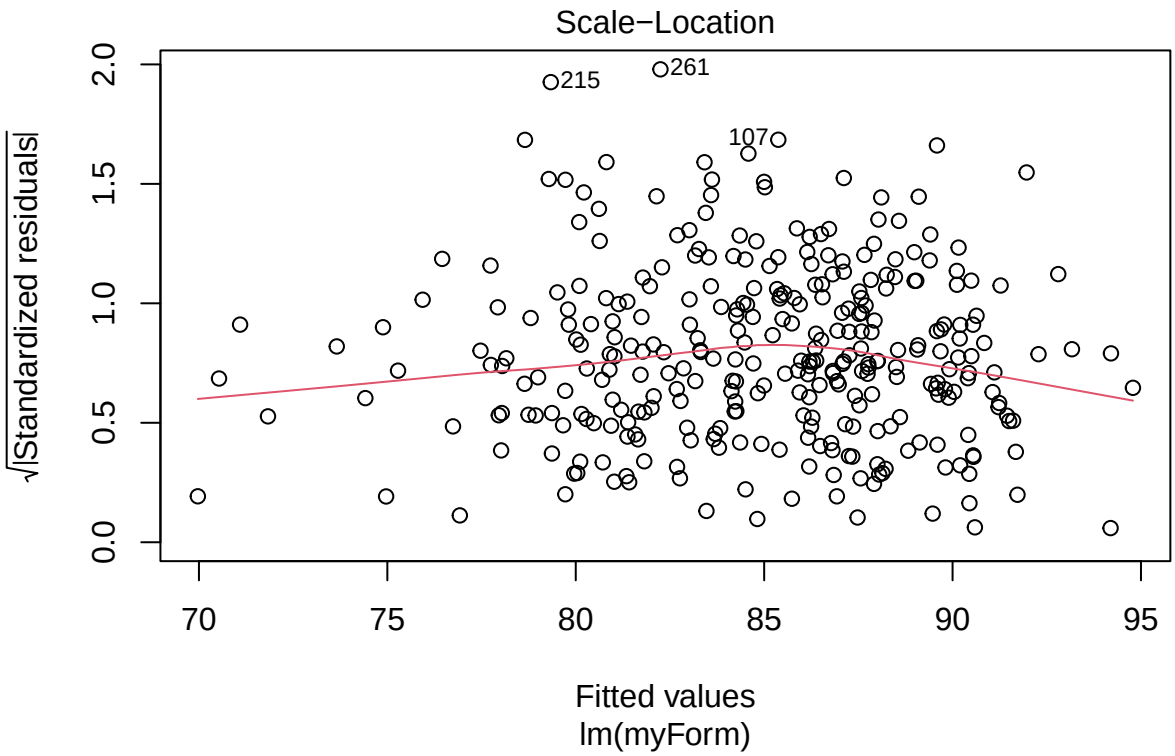
```
## glove_shoulder_external_rotation_fp    -0.025859   0.008446  -3.062 0.002405
## elbow_flexion_mer                      -0.076919   0.019464  -3.952 9.74e-05
## torso_rotation_mer                     -0.120593   0.018019  -6.693 1.12e-10
## elbow_varus_moment                      0.052714   0.011049   4.771 2.90e-06
## torso_lateral_tilt_br                   0.084919   0.016643   5.102 6.06e-07
## lead_knee_extension_from_fp_to_br       0.062547   0.021158   2.956 0.003369
## shoulder_absorption_fp_br              -0.038536   0.010559  -3.649 0.000311
## elbow_transfer_fp_br                    0.044617   0.003291  13.559  < 2e-16
## lead_hip_transfer_fp_br                -0.015724   0.007560  -2.080 0.038409
## lead_hip_generation_fp_br              -0.041649   0.008247  -5.050 7.78e-07
## lead_hip_absorption_fp_br              -0.015213   0.004065  -3.743 0.000219
## rear_hip_transfer_pkh_fp               -0.018309   0.003977  -4.604 6.20e-06
## rear_knee_absorption_pkh_fp            -0.017192   0.006802  -2.528 0.012009
## pelvis_lumbar_transfer_fp_br            0.009474   0.002199   4.308 2.25e-05
## lead_grf_y_max                         -0.004798   0.001325  -3.621 0.000345
## lead_grf_angle_at_max                   0.099055   0.034547   2.867 0.004442
##
## (Intercept)                            **
## max_torso_rotational_velo              ***
## max_elbow_flexion                      **
## max_shoulder_external_rotation         ***
## elbow_flexion_fp                       **
## elbow_pronation_fp                     ***
## shoulder_external_rotation_fp          ***
## lead_knee_extension_angular_velo_br    ***
## lead_knee_extension_angular_velo_max   *
## torso_lateral_tilt_fp                  ***
## pelvis_anterior_tilt_fp                **
## pelvis_rotation_fp                     ***
## max_cog_velo_x                         ***
## glove_shoulder_horizontal_abduction_fp *
## glove_shoulder_external_rotation_fp    **
## elbow_flexion_mer                      ***
## torso_rotation_mer                     ***
## elbow_varus_moment                     ***
## torso_lateral_tilt_br                  ***
## lead_knee_extension_from_fp_to_br      **
## shoulder_absorption_fp_br              ***
## elbow_transfer_fp_br                   ***
## lead_hip_transfer_fp_br                *
## lead_hip_generation_fp_br              ***
## lead_hip_absorption_fp_br              ***
## rear_hip_transfer_pkh_fp               ***
## rear_knee_absorption_pkh_fp            *
## pelvis_lumbar_transfer_fp_br           ***
## lead_grf_y_max                         ***
## lead_grf_angle_at_max                  **
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 2.008 on 292 degrees of freedom
## Multiple R-squared:  0.8377, Adjusted R-squared:  0.8216
## F-statistic: 51.97 on 29 and 292 DF,  p-value: < 2.2e-16
```

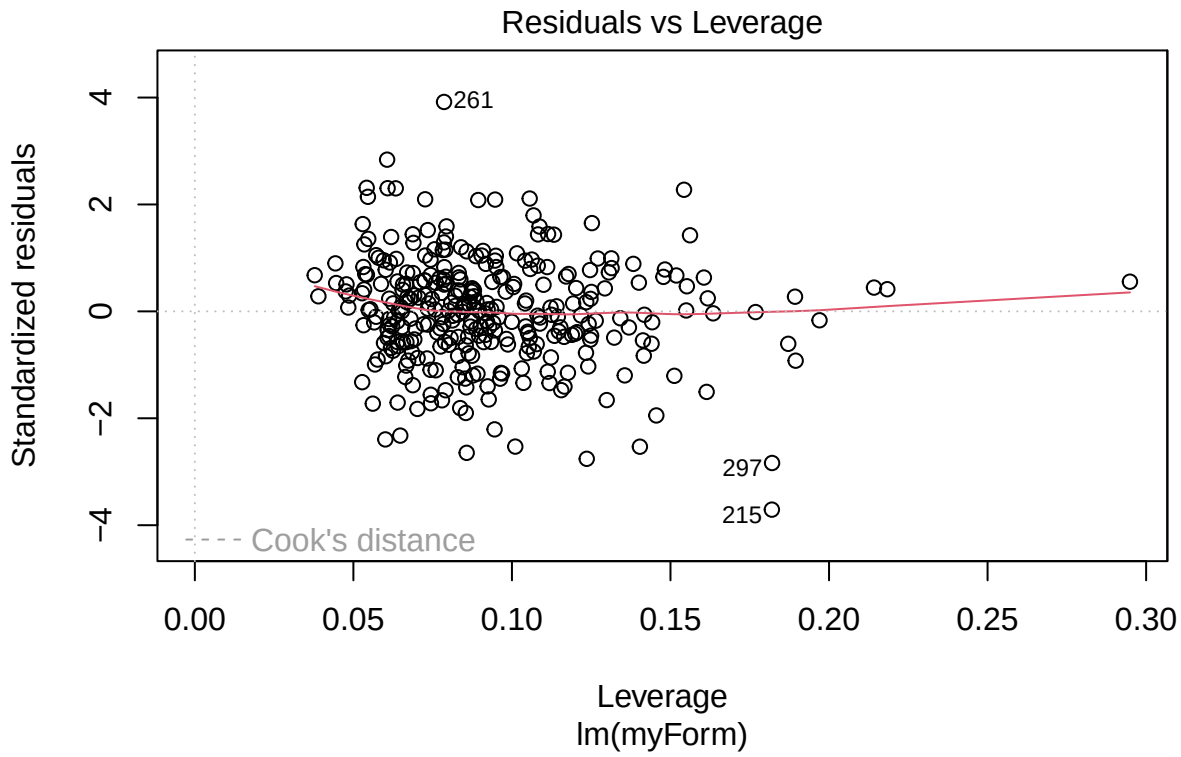
Multi-collinearity and statistical significance ($p < 0.05$) have been confirmed.

```
model_plot = plot(selected_mod)
```




Q−Q Residuals
Standardized residuals
Theoretical Quantiles
lm(myForm)
261
107
215
−4
−2
0
2
4
−3
−2
−1
0
1
2
3


Scale−Location
√|Standardized residuals|
0.0
0.5
1.0
1.5
2.0
215
261
107
70
75
80
85
90
95
Fitted values
lm(myForm)




```
model_plot
```

```
## NULL
```

```
ggsave("model_plot.png", model_plot)
```

```
## Saving 6.5 x 4.5 in image
```

```
predict_df = as.data.frame(predict(selected_mod, test_df, level = 0.95, interval = "predict"))
```

```
head(test_df |>
       select(pitch_speed_mph))
```

```
## # A tibble: 6 x 1
##   pitch_speed_mph
##             <dbl>
## 1            90.4
## 2            77.6
## 3            80.9
## 4            84.8
## 5            86.9
## 6            87.4
```

```r
head(predict_df)
```

```
##        fit      lwr      upr
## 1 92.84310 88.72892 96.95729
## 2 81.46993 77.26061 85.67925
## 3 84.80432 80.63517 88.97347
## 4 82.43143 78.20383 86.65903
## 5 87.81619 83.66195 91.97043
## 6 88.92070 84.81302 93.02838
```

```r
combined_df = bind_cols(predict_df, test_df) #combining the two data sets together by observation numbe

combined_df = combined_df |>
  select(c(pitch_speed_mph, fit, upr, lwr)) |>
  rename(
    predicted_mph = fit,
    upper_bound = upr,
    lower_bound = lwr
  )

write.csv(combined_df, "prediction_data.csv", row.names = FALSE)
```