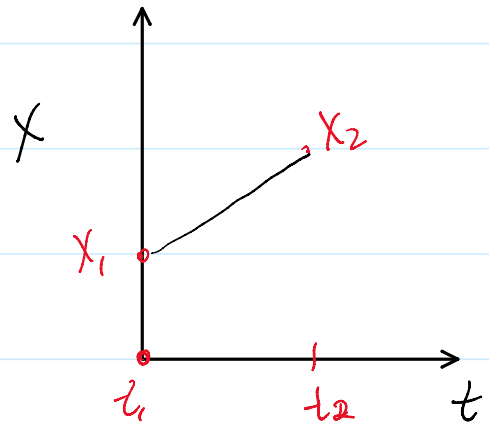
Path Generation Lecture (Canvas)



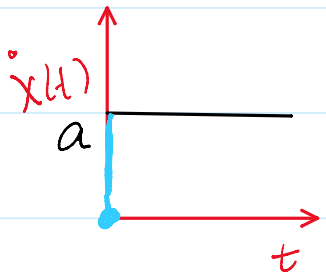
$$X(t) = \overset{?}{a}t + \overset{?}{b}$$

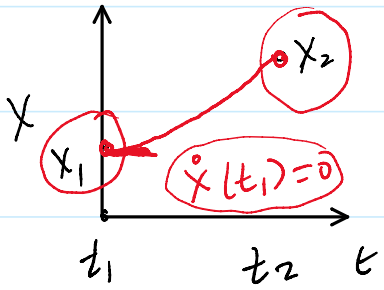
$$X(t_1 = 0) = X_1 \rightarrow b = X_1$$

$$X(t_2) = X_2 \rightarrow at_2 + b = X_2$$

$b \downarrow X_1$

$$\dot{X}(t) = a \qquad \ddot{X}(t) = 0$$





$$X(t) = \overset{?}{a}t^2 + \overset{?}{b}t + \overset{?}{C}$$

$$\dot{X}(t) = 2at + b$$

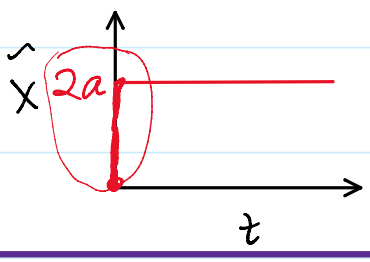
$$X(t_1 = 0) = X_1 \rightarrow C = X_1$$

$$\dot{X}(t_1 = 0) = 0 \rightarrow 2a(0) + b = 0 \rightarrow b = 0.$$

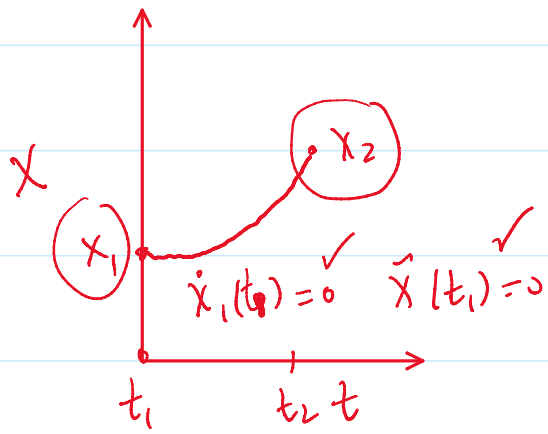
$$X(t_2) = X_2 \rightarrow at_2^2 + \cancel{b}t_2 + C = 0$$

$\downarrow 0.$

$$\ddot{X} = 2a$$

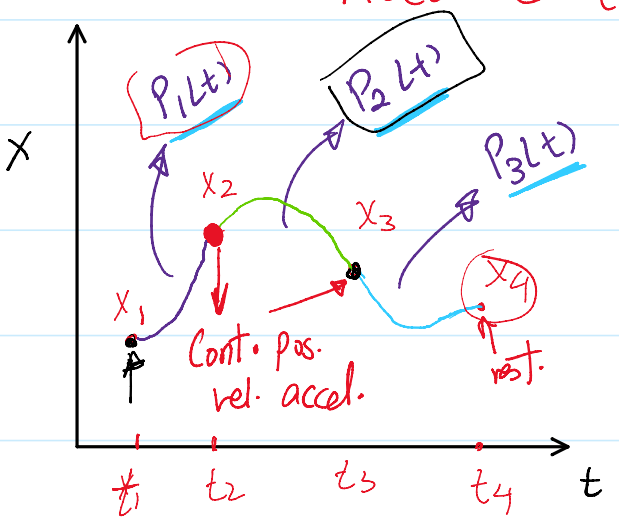


$$\text{Jerk} = \frac{d(\text{Accel})}{dt}$$



$$x(t) = at^3 + bt^2 + ct + d$$

Piecewise curve fitting



$P_1(t_1) = X_1$ (1)

$\dot{P}_1(t_1) = 0$ (2)

$\ddot{P}_1(t_1) = 0$ (3)

$P_1(t_2) = X_2$ (4)

(5) $P_1(t_2) = P_2(t_2) \rightarrow P_1(t_2) - P_2(t_2) = 0$

(6) $\dot{P}_1(t_2) = \dot{P}_2(t_2)$

(7) $\ddot{P}_1(t_2) = \ddot{P}_2(t_2)$

$P_2(t_3) = X_3$ ✓ (8)

$P_2(t_3) = P_3(t_3)$ (9)

$\dot{P}_2(t_3) = \dot{P}_3(t_3)$ (10)

$\ddot{P}_2(t_3) = \ddot{P}_3(t_3)$ (11)

14 Conditions

we need 14 unknowns:

$P_3(t_4) = X_4$ (12)

$\dot{P}_3(t_4) = 0$ (13)

$\ddot{P}_3(t_4) = 0$ (14)

linear in coeff $a_1 \rightarrow a_5$ $c_1 \rightarrow c_5$ $b_1 \rightarrow b_4$

nonlinear in t

$$P_1(t) = a_1 + a_2 t + a_3 t^2 + a_4 t^3 + a_5 t^4 \quad (5)$$

$$P_2(t) = b_1 + b_2 t + b_3 t^2 + b_4 t^3 \quad (4)$$

$$P_3(t) = c_1 + c_2 t + c_3 t^2 + c_4 t^3 + c_5 t^4 \quad (5)$$

14

$$P_1(t_1) = x_1 \rightarrow a_1 + a_2 t_1 + a_3 t_1^2 + a_4 t_1^3 + a_5 t_1^4 = x_1$$

$$\vdots$$

$$P_1(t_2) = P_2(t_2) \quad (5)$$

$$a_1 + a_2 t_2 + a_3 t_2^2 + a_4 t_2^3 + a_5 t_2^4 = b_1 + b_2 t_2 + b_3 t_2^2 + b_4 t_2^3$$

$$\vdots$$

$$P_1(t) = \begin{bmatrix} 1 & t & t^2 & t^3 & t^4 \end{bmatrix} \begin{Bmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \\ a_5 \end{Bmatrix} \rightarrow a$$

1×5 $\quad$ 5×1

$$a_1 + a_2 t + a_3 t^2 + a_4 t^3 + a_5 t^4$$

$$P_1(t) = \begin{bmatrix} 1 & t & t^2 & t^3 & t^4 \end{bmatrix} a = F_4(t)\, a$$

$F_4(t)$ → 4th order poly.

$$\dot{P}_1(t) = \begin{bmatrix} 0 & 1 & 2t & 3t^2 & 4t^3 \end{bmatrix} a = \dot{F}_4(t)\, a$$

4th order

$\dot{F}_4(t)$

$$\ddot{P}_1(t) = \begin{bmatrix} 0 & 0 & 2 & 6t & 12t^2 \end{bmatrix} a = \ddot{F}_4(t)\, a$$

$$P_2(t) = \begin{bmatrix} 1 & t & t^2 & t^3 \end{bmatrix} b \qquad b = \begin{Bmatrix} b_1 \\ b_2 \\ b_3 \\ b_4 \end{Bmatrix} \quad b_1 + b_2 t + b_3 t^2 + b_4 t^3$$

$$P_2(t) = F_3(t)\, b$$

$F_3(t)$ → 3rd order

$$\dot{P}_2(t) = \begin{bmatrix} 0 & 1 & 2t & 3t^2 \end{bmatrix} b \qquad \ddot{P}_2(t) = \begin{bmatrix} 0 & 0 & 2 & 6t \end{bmatrix} b$$

$\dot{F}_3(t)$ $\qquad$ $\dot{F}_3(t)\, b$ $\qquad$ $\ddot{F}_3(t)\, b$

$$P_3(t) = \begin{bmatrix} 1 & \underline{t} & \underline{t^2} & \underline{t^3} & \underline{t^4} \end{bmatrix} \underline{c} = F_4(t)\, C \qquad c = \begin{Bmatrix} c_1 \\ c_2 \\ c_3 \\ c_4 \\ c_5 \end{Bmatrix}$$

$F_4(t)$ $\qquad$ $\dot{F}_4(t)$

$$\dot{P}_3(t) = \begin{bmatrix} 0 & 1 & 2t & 3t^2 & 4t^3 \end{bmatrix} c = \dot{F}_4(t)\, C$$

$$\ddot{P}_3(t) = \begin{bmatrix} 0 & 0 & 2 & 6t & 12t^2 \end{bmatrix} c = \ddot{F}_4(t)\, C$$

$F_4(t)\ a$

① $P_1(t_1) = x_1 \rightarrow F_4(t_1)\, a = x_1$

② $\dot{P}_1(t_1) = 0 \rightarrow \dot{F}_4(t_1)\, a = 0$

③ $\ddot{P}_1(t_1) = 0 \rightarrow \ddot{F}_4(t_1)\, a = 0$

④ $P_1(t_2) = x_2 \rightarrow F_4(t_2)\, a = x_2$

$P_1(t) = F_4(t)\, a$

$\dot{P}_1(t) = \dot{F}_4(t)\, a$

$\ddot{P}_1(t) = \ddot{F}_4(t)\, a$

$P_2(t) = F_3(t)\, b$

$\dot{P}_2(t) = \dot{F}_3(t)\, b$

⑤ $P_1(t_2) - P_2(t_2) = 0 \rightarrow F_4(t_2)\, a - F_3(t_2)\, b = 0.$

⑥ $\dot{P}_1(t_2) - \dot{P}_2(t_2) = 0 \rightarrow \dot{F}_4(t_2)\, a - \dot{F}_3(t_2)\, b = 0.$ ✓

⑦ $\ddot{P}_1(t_2) - \ddot{P}_2(t_2) = 0 \rightarrow \ddot{F}_4(t_2)\, a - \ddot{F}_3(t_2)\, b = 0.$ ✓

⑧ $P_2(t_3) = x_3 \rightarrow F_3(t_3)\, b = x_2$

⑨ $P_2(t_3) - P_3(t_3) = 0 \rightarrow F_3(t_3)\, b - F_4(t_3)\, c = 0$

⑩ $\dot{P}_2(t_3) - \dot{P}_3(t_3) = 0 \rightarrow \dot{F}_3(t_3)\, b - \dot{F}_4(t_3)\, c = 0$

⑪ $\ddot{P}_2(t_3) - \ddot{P}_3(t_3) = 0 \rightarrow \ddot{F}_3(t_3)\, b - \ddot{F}_4(t_3)\, c = 0$

⑫ $P_3(t_4) = x_4 \rightarrow F_4(t_4)\, c = x_4$

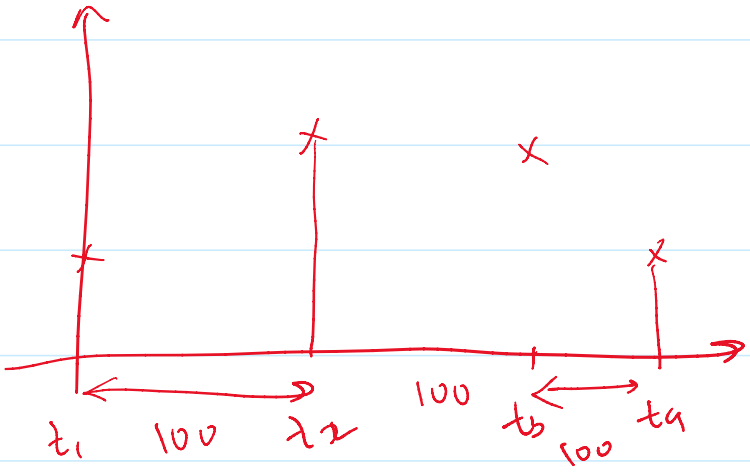
⑬ $\dot{P}_3(t_4) = 0 \rightarrow \dot{F}_4(t_4)\, c = 0$

⑭ $\ddot{P}(t_4) = 0 \rightarrow \ddot{F}_4(t_4)\, c = 0.$

1 : 5	6 : 9	10 : 14		
$F_4(t_1)$			a_1	x_1
$\dot{F}_4(t_1)$			a_2	0
$\ddot{F}_4(t_1)$			a_3	0
$F_4(t_2)$			a_4	x_2
$F_4(t_2)$	$-F_3(t_2)$		a_5	0
$\dot{F}_4(t_2)$	$-\dot{F}_3(t_2)$		b_1	0
$\ddot{F}_4(t_2)$	$-\ddot{F}_3(t_2)$		b_2	0
	$F_3(t_3)$		b_3	x_3
	$F_3(t_3)$	$-F_4(t_3)$	b_4	0
	$\dot{F}_3(t_3)$	$-\dot{F}_4(t_3)$	c_1	0
	$\ddot{F}_3(t_3)$	$-\ddot{F}_4(t_3)$	c_2	0
		$F_4(t_4)$	c_3	x_4
		$\dot{F}_4(t_4)$	c_4	0
		$\ddot{F}_4(t_4)$	c_5	0

AA

BB


100
100
100