

Image Compression

- * Stored in form of pixels
- * Pixels are arranged in form of 2d-array
- Amplitude (value stored in each picture)
- * No. of pixels \propto Resolution

Eg. 512×512 — Higher resolution compared to
 8×8 image

- * Better resolution \rightarrow Occupies more space
- Storage & Transmission is diff
- \therefore We need to compress.

Types of Images

① Bilevel / Monochrome Image

All Pixel values : 1 bit only 2^1 different values
either 0 or 1

② Grayscale Image

(Based on no. of bits for each pixel. no. of different values is decided, improves quality of image)

③ Continuous Tone (natural image)

No abrupt frequency

④ Discrete Tone (sharp edges)

⑤ Cartoon like (purely man-made)

Grayscale:
Bit plane: No. of bits for each pixel

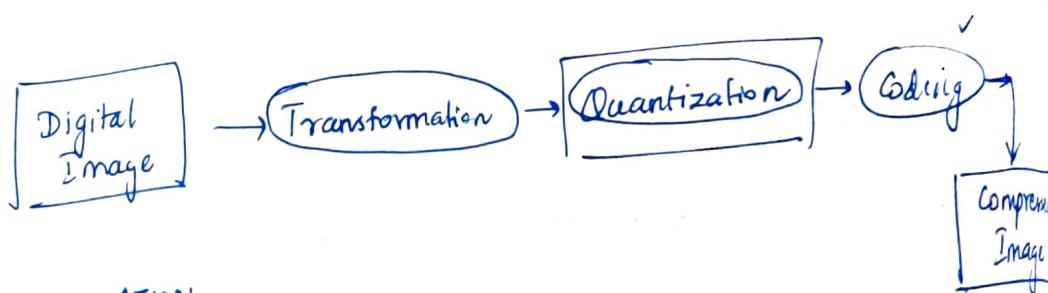
e.g. 8 bits, each bit plane is of size 8

If we remove few LSB, we can perform compression.

Some compression technique cannot be used for all types of images.

Sampling: Technique to convert continuous to discrete data.

Loss of data is there when continuous is converted to digital image.



QUANTIZATION

Scalar Quantization

simply remove certain bitplanes.

Not optimized

8 bit image

203 - 214

203 :

2	203	
2	101	1
2	50	1
2	25	0
2	12	1
2	6	0
2	3	0
	1	1

214

2

11001011

(Remove L8B 4 bits)

<i>One now 2 pixels</i>		
203	11001011	11000000
204	11001100	11000000
205	11001101	11000000
206	11001110	11000000
207	11001111	11000000
208	11010000	11010000
209	11010001	11010000
210	11010010	11010000
211	11010011	11010000
212	11010100	11010000
213	11010101	11010000
214	11010110	11010000

We obtain sharp boundary. ~~∴ Decompressed image will not like a natural image~~
 \therefore This is not an optimized method

Grayscale Quantization

* SNo	* Value	* rsm (random no. based on neighbouring pixels)
1	11010101	0000 0000

+ Compressed Stream

SNO	Value	rsm Previous RSM	Compressed Stream
1	11010101	0000 0000	1101 (1st step alone 4 bits)
2	10010111	1001 0111	1001 (DPCM wise: First 4 bits of RSM)
3	10101010	10110001	1011

In case value is of the form

1111 xxxx :- (RSM updated with value of pixel itself (no addition with prev RSM))

Compressed Stream:

1111

No Banding Effect

Because neighbourhood pixel values incorporated for next pixel.

Better than scalar quantization

Principles of Image Compression

- * Neighborhood pixels carry similar values (almost equal)

Difference Matrix

: Instead of coding each pixel value as it is we can code diff b/w consecutive pixels

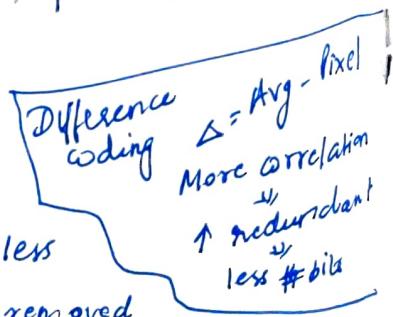
Eg: 212 215 216 216 217 219 220 218

Code: 212 3 1 0 1 2 -1 -2

→ Lossless compression

Difference Matrix

- * Range has become less
- * Correlation has been removed
- * If there are more # zeroes or ones, we can



Code it efficiently

Bilevel Image

Run length Encoding

Scan through entire image (Σ or ∇)

Count # consecutive zeroes & ones

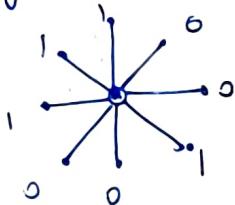
Store it with freq.

Eg: 0 0 0 0 0 . . . 0 1 1 1 1 1 1

Store it as

0 10 1 9 (0 - 10 times
 1 - 9 times)

Bilevel Image Context



(Based on neighbourhood context is created for a pixel)

Represented as a n-bit number

This bit-sequence might appear k_o of times.

Calc freq. prob. Do arithmetic coding

Context-based coding can be applied to any type of data, each bit plane to be compressed

Transform based Compression

Spatial domain

data of image
(2-D space)



Frequency domain

data

(Can identify uniform (low freq.) and non-uniform (high freq.) areas of an image)

* This removes inter-pixel redundancy

Transform is not compression

It just represents image in a different format

Only when we apply quantization, we achieve compression.

Continuous Tone Image

Natural Images: No sharp edges

Color Image - 3 channels $\begin{matrix} R \\ G \\ B \end{matrix}$

Divide each pixel into three components

Apply the compression technique for each component separately

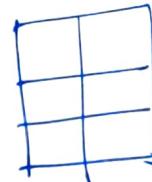
Eg:	R	G	B
P ₁ :	1000	0011	0000
P ₂ :	1001	0011	0000

Both pixels almost identical
Overall value significantly diff

But R G B separately are very similar

Fractal Image Compression

Divide Image into blocks (regions)



Code a block.

If other blocks are similar, no need to code them separately & pointer to the coded block is stored.

JPEG

- Compression Technique for images & videos.
- Cannot be used for bilevel image
- Can be used for grayscale images & colour images.

Merits

- * It has parameters to decide quality of decompressed image (block size, Quantization bit)
- * It can be used as either a lossy or lossless technique.
- * Better Quality
- * JPEG works efficiently in all image features like size, colourspace, aspect ratio . evaluated from
- * Good to Excellent
- * Progressive Transmission

chrominance Hue -
Prominence : Dominant wavelength of spectrum

Saturation - Amt of white light

luminous Brightness

Downsampling :

Subtract 128 from all pixels



Good Dates

Char	Frequency	Prob	Interval
G	1	1/6	[0, 1)
O	1	1/6	[1, 2)
D	1	1/6	[2, 3)
A	1	1/6	[3, 4)
T	1	1/6	[4, 5)
E	1	1/6	[5, 6)
S	1	1/6	[6, 7)

RGB to YCrCb

color info

Y Cr Cb

Luminance: Gives the picture (grayscale image).

For each pixel:

$$Y = \left(\frac{77}{256}\right)R + \left(\frac{150}{256}\right)G + \left(\frac{29}{256}\right)B$$

$$C_b = \left(\frac{44}{256}\right)R - \left(\frac{87}{256}\right)G + \left(\frac{131}{256}\right)B + 128$$

$$C_r = \left(\frac{131}{256}\right)R - \left(\frac{110}{256}\right)G - \left(\frac{21}{256}\right)B + 128$$

To convert YCrCb to RGB:

$$R : Y + 1.371(C_r - 128)$$

$$G : Y - 0.698(C_r - 128) - 0.366(C_b - 128)$$

$$B : Y + 1.732(C_b - 128)$$

Chromaticity Diagram (3D Diagram)

DCT

\equiv (Discrete Cosine Transform)

2-dimensional DCT

Blockwise only DCT is applied because

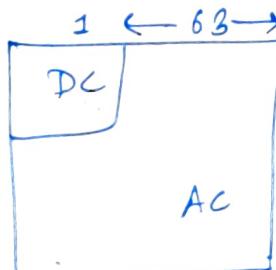
- * Only neighbouring pixels are correlated
- * High no. of operations in DCT & IDCT if applied for entire image.

$$\text{DCT}(x, y) = \frac{1}{4} (c_i c_j) \sum_{x=0}^7 \sum_{y=0}^7 P(x, y) \left(\cos\left(\frac{2x+1}{16}j\pi\right) \right) \left(\cos\left(\frac{2y+1}{16}j\pi\right) \right)$$

$$c_i, c_j \Rightarrow [0, 7]$$

$$C_f = \begin{cases} 1/2, & f=0 \\ 1, & f>0 \end{cases}$$

Most values accumulated in upper-left corner. Less value in bottom right corner



Quantization

Divide image after DCT into 8×8 blocks and apply quantization using basis table (8×8)

Upper left - more info \Rightarrow less quantization

Lower right - less info \Rightarrow More quantization

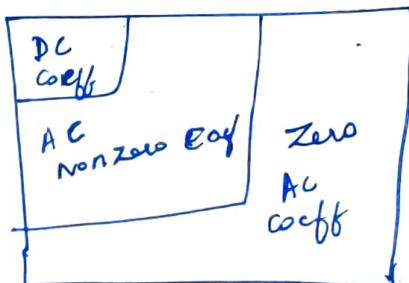
JPEG Parameters : 8×8 quantization table

To create own quantization table :

$$(i) 1 + (i+j) * R$$

(ii) Simply divide by $\frac{4}{6}$

Entropy Coding



Different entropy coding for 3 components

Two AC Coefficients

- Highly correlated
- Mostly 0s
- We can use RLE

000 1 2 2 2 2 2 3 3 3 5
3, 0, 1, 1, 5, 2, 3, 3, 1, 5
(skip, value) pairs

Skip $\#x$ values and
1st occurrence of a non-zero
value

Non-zero AC coefficients

Huffman Coding Technique

DC Coefficients

DPCM : Differential Pulse
code Modulation (Lossless Technique)

$$DP(i) = DC(i) - DC(i-1)$$

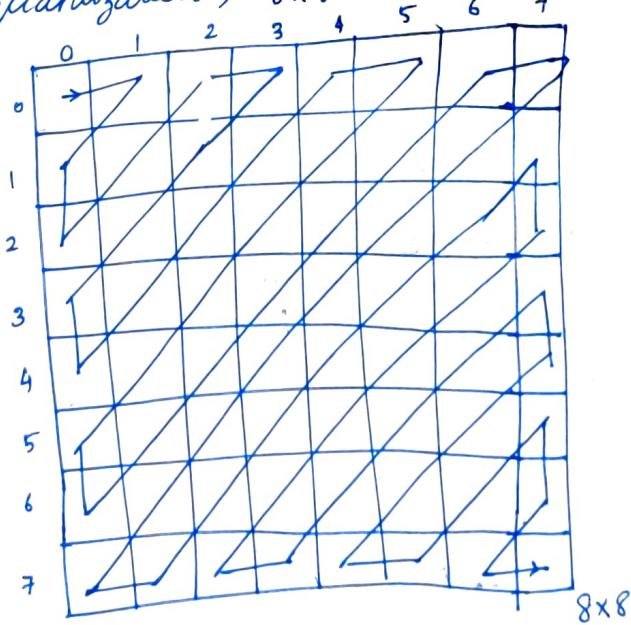
1118, 1114, 1112, ...

6 1118, -4, -2, ...

After DPCM, perform Huffman Coding

specialised table only of DC coefficients is created.

After quantization, 8x8



If we read  mixture of zero & non-zero values

∴ We use zig-zag scan 

Hence, we convert 8×8 block into 1-D data with 64 comp.

Co-ordinates

(0, 0) ~~(0, 0)~~

(0, 1) (1, 0)

(2, 0), (1, 1), (0, 2)

\leftrightarrow (0, 3), (1, 2), (2, 1), (3, 0)

.

.

.

After Huffman, we obtain output stream

To decode, we need to send few tables

① Huffman table and

② DC&BM Huffman Table :

Size	Value	Code
0	0	0
1	-1, 1	$0 \rightarrow 1^0$
2	-3, -2, -1, 0, 1, 2, 3	$00, 01, 10, 11$
3	-7, -6, -5, -4, -3, -2, -1, 0, 1, 2, 3, 4, 5, 6, 7	110 1110

$\{ (2^i - 1) \text{ to } + (2^i - 1) \} = \{ (2^{i+1} - 1) \text{ to } (2^{i+1} - 1) \}$

16 \rightarrow 05536 \dots 65536 $\underbrace{\dots 111111 \dots 1}_\text{15 15} 0$

Row	value	code
i	$-(2^i - 1) \text{ to } (2^i - 1)$	$(i-1's) 0$

Represented as (R, c)

g₁ 1118

$R = 11$ Code
 $C = \boxed{1}$ ~~11111110~~

g₂ -2

$R = +2$
 $C = 2$

Code : $(110, 10)$

From table

g₃ -1

$R : 1$ $(10, 1)$
 $C : 1$

Code value
from table

(R, c)

Code c in
binary R-bit

g₄ -7

$R = 3$ $(1110, 001)$
 $C = 1$

AC Components : Low value, close to zero

JPEG has separate table for luminance & chrominance components.

(R, z) No. of zero coefficients appearing before

it

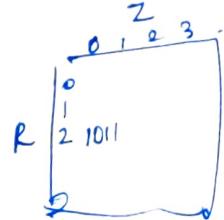
1118, -2, 0, 2, 0, 0, 0, 1

$R : 2$ $C : 2$

$z = 0$

Table

$(1011, 10)$



Code :

(Table R, Z , C in binary R bits)

Q

$$\downarrow \quad R = 2 \quad C = 3 \quad Z = 1$$

From table

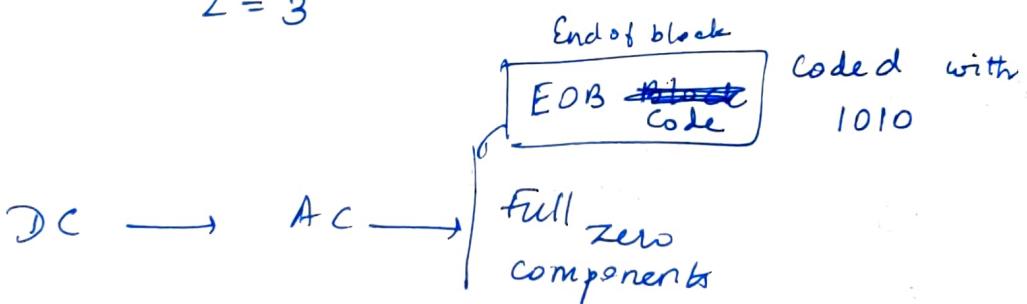
($\begin{array}{c} R=2 \\ Z=1 \\ \downarrow \\ \boxed{} \end{array}$, C in 2-bits
11)

Q →

$$R = 2$$

$$C = 3$$

$$Z = 3$$



Before compression

Each 8x8 block

$$64 \times 24 = 1536$$

8 8
4 8
4 or 9_B

$$\text{Ratio} = \frac{1536}{128}$$

$$= 11$$

With JPEG

DC Component : 23 bits

$$46 \times 3 = 138$$

AC Components 23

Video Compression

→ Spatial Redundancy

- Coding
- Interpixel
- Psychovisual

(16 redund. no difference to eye)

→ Temporal Redundancy

remove Coding Redundancy : Any encoding technique (Huffman, arithmetic, RLE)

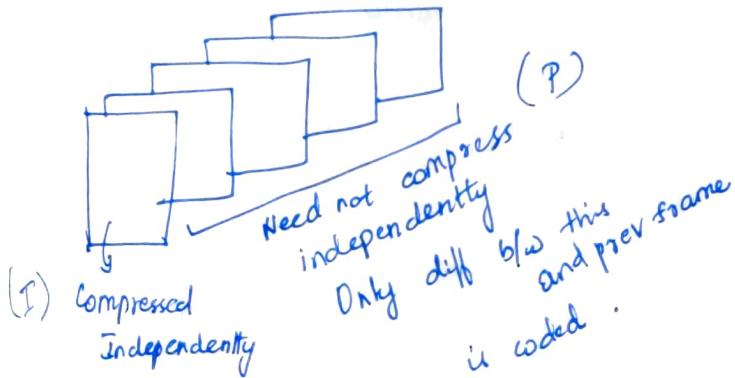
Interpixel : DCT, wavelet Transformation, etc.

Psychovisual : Quantization

Video : Comprises of Frames

Neighbourhood frames are very similar (almost same)

* First frame will be compressed independently



Interframe : Which depends on prev. frame for encoding & decoding. Also called as Predictive frame

Intraframe: Coded / Decoded independently

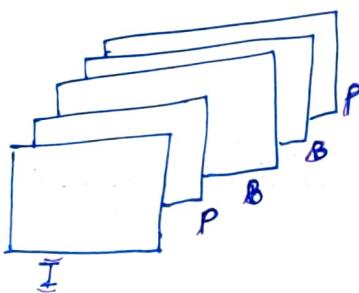
In case of error in k^{th} frame it will be carried in $k+1, k+2, \dots$ frames

We need B-frame (Bidirectional frame)
depends on prev & next frame.

I frame \rightarrow Coded independently

P frame \rightarrow Coded by previous I / P frame

B frame \rightarrow coded with previous and successive P frame over I frame.



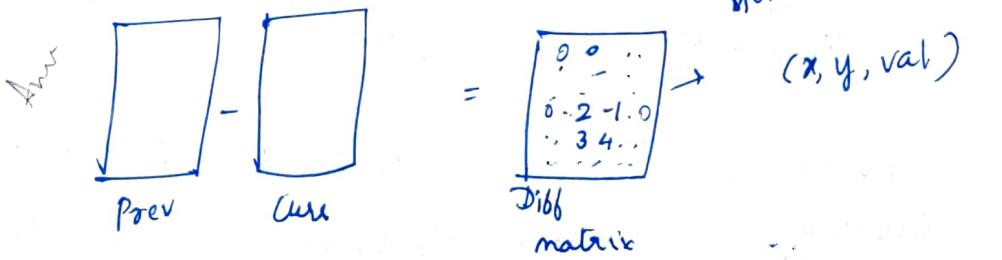
Techniques :

① Subsampling

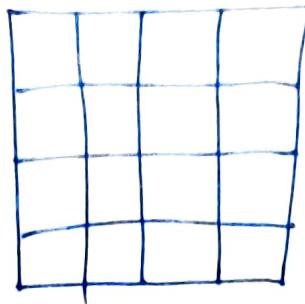
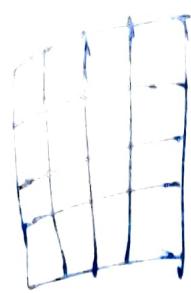
* Encode alternate frames only

* Decoding: do duplication: 1, 1, 3, 3, 5, 5, ...

② Differencing



Block differencing



divide frame into blocks

Find diff b/w blocks

Motion Compression

$$(C_x - b_x, C_y - b_y) = \Delta x, \Delta y$$

↓
 x-coord
 in prev
 frame

↓
 x-coord
 in current
 frame

↓
 y-coord
 in prev
 frame

↓
 y-coord
 in cur
 frame

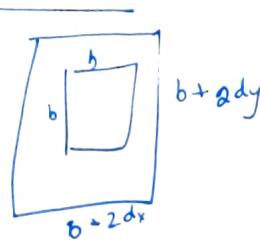
Motion vector

- * Segregate image into blocks
- * Check if block is present in prev image
- * Block size should be selected optimally
 - If block size is very large, entire block should be searched. To find absolute match for large block is hard

Threshold Value

Limit to which diff can be accepted

Block Search Procedure



$$(2dx+1) \cdot (2dy+1) \quad \# \text{blocks}$$

Each block : $b \times b$ block

Distortion measure

$$\text{Mean pixel difference} = \frac{1}{b^2} \sum_{i=1}^b \sum_{j=1}^b [c_{ij} - b_{ij}]$$

↓
 pixel in
 block in
 prev
 frame

↓
 pixel in
 block in
 current
 frame

First frame + Motion vector \Rightarrow Second frame

↳ Diff b/w x & y word

Motion compensation of upper left corner of matching frame in current frame or previous frame.

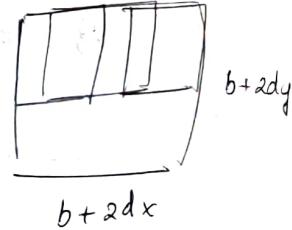
H261 Standard

- * Video compression Technique
- ↳ * Motion compensation to create predicted image
- ↳ * Diff b/w predicted & original frame
- ↳ * DCT
- ↳ * quantization
- ↳ * Entropy Coding

search area

Search for

b \square block in prev frame



Each block in search area, overlapping search criterion is been create motion vector.

$$\# \text{ Comparisons} : (2dx+1) (2dy+1)$$

If $dx = dy = 20$,

$$\text{No. of comparisons} = 1681$$

for one 8×8 block

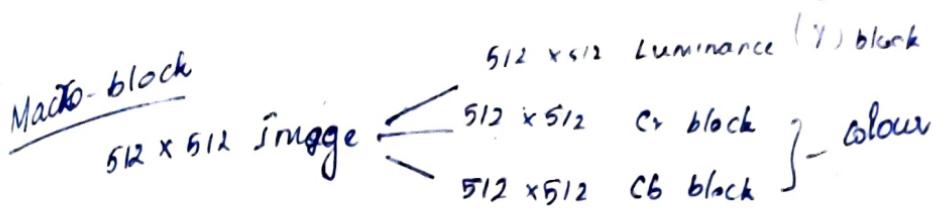
Solutions

* Increase block size

But block may comprise of > 1 object

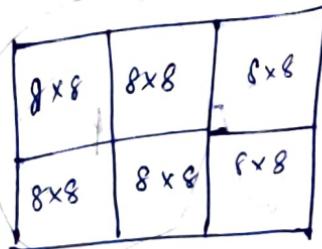
41
41
41
164
1681

Outline



More important: Luminance component

Macro Block (4 - Luminance
2 - Cr,Cb blocks)



- + Image is divided into Macro-blocks
- + Motion-compensation done w.r.t luminance component only

Block To be encoded

(x_c, y_c)

Block Used for Prediction

(x_p, y_p)

Diff should be < 15

- + Filtering operation (Preprocessing) - Before motion compensation

To remove noise (sharp edges)

DO filtering

for each pixel

$$\boxed{1 \ 1 \ 2 \ 1} \quad \text{New val}(2) = \frac{1}{4}(1) + \frac{1}{2}(2) + \frac{1}{4}(3)$$

- * Row-wise filtering separate
- * Column-wise filtering separate

If no. of I frames is more, time will be saved
Delay reduced.

Store and retrieve - H261 - caused delay.

Moving Picture Expert Group (MPEG-I)

I - Independent frame / Intraframe

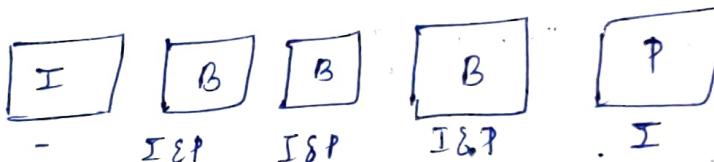
Less

P - Predictive coded frame

More

B - Bidirectional Predictive Coded frame

High



B frames tolerate error. B frames not used for prediction of other frames. ∵ error doesn't propagate.

I frame cannot have error

P frame can have error. This gets propagated

Display Order

I	B	B	P	B	B	P	B	B	P	B	B	I
1	2	3	4	5	6	7	8	9	10	11	12	13

Processing order / Bitstream Order

1, 4, 2, 3, 7, 5, 6, 10, 8, 9, 13, 11, 12

Frames are sent to decoder in the bitstream order

Optimal size of frame: 768 x 576

Macro blocks: 396 25 frames or less / sec
 330 30 frames or less / sec

MPEG :
good for slow / Medium moving video
Fast moving images - doesn't work well

STATISTICAL MODELLING

* Zero - Order Model:

① The accurate probability table

② Deviation of the data from uniform distribution

These 2 factors are imp. for performing compression.

* Finite Context Modelling

Instead of considering individual char & their prob, find the prob of each char based on its context.

↳ Order 0: That symbol alone (1 Table)

↳ Order 1: Symbol & its prev char

suppose, Total = 2^{8b} (For each char, 256 possibilities for prev char)

∴ 256 Tables

↳ Order 2: ----- Each char based on prev 2 char

Order 0:

Order 1 - 8 bit \rightarrow 2 bit

-- 8+8 \rightarrow 2 bit

Order 2 --- 8+8+8 \rightarrow 2 bits

↓
compression increases

Problem:

All tables should be sent to decoder

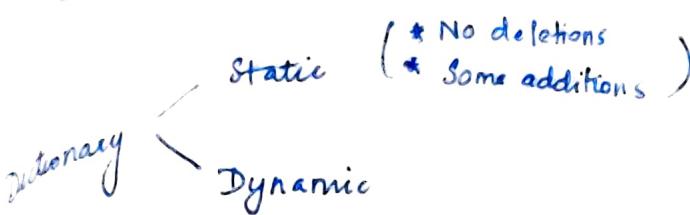
Solution:

Adaptive Modelling

No need of to send static prob table to decoder

Disadv: Not need of scanning data before encoding / decoding
Less info abt data

Dictionary Based Compression Techniques



Static Dictionary

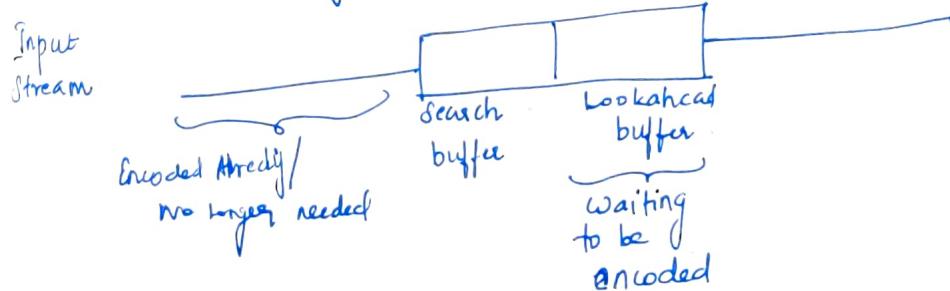
- * 2^{19} records \Rightarrow 19 bits required to represent index
- * Restricted to certain words
- * Not customizable for demand
- General purpose static dictionary : Not used much
Special purpose static dictionaries created to fulfill demands of specific applications.
Eg. Medical field, programming, etc. \rightarrow consumes more space
- We use Dynamic Dictionary

Adaptive Dictionary

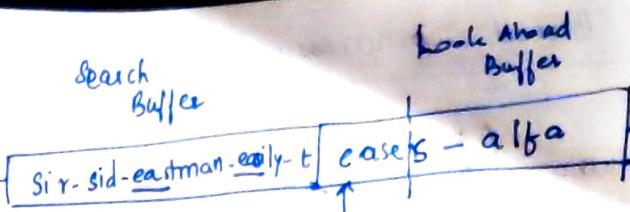
offset	length of string	First non-match char
		match

LZ77 - Algorithm / Sliding Window Algorithm

- * Uses Sliding Window :



Eg:



Check if e is present in search buffer.

offset	Length of match	First non match
--------	-----------------	-----------------

16	3	e
----	---	---

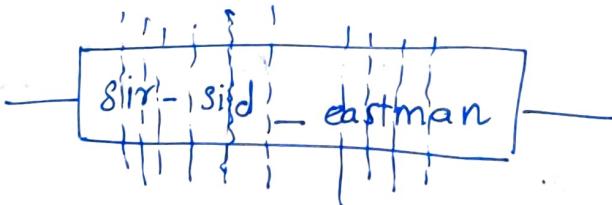
Move sliding window

28	1	—
----	---	---

Start of I/P Stream

Token will be

0	0	st letter (s)
---	---	------------------



0	0	s
---	---	---

0	0	i
---	---	---

0	0	r
---	---	---

0	0	i
---	---	---

4	2	d
---	---	---

4	1	e
---	---	---

0	0	a
---	---	---

10	1	t
----	---	---

0	0	m
---	---	---

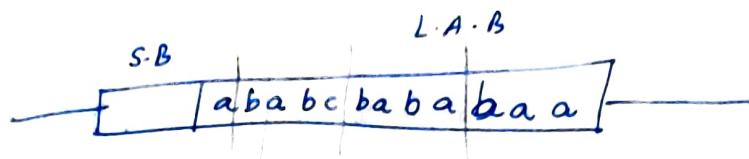
4	1	n
---	---	---

offset
 \downarrow
 $\log_2 S$

String Match length
 $\log_2 (L-1)$ Next char to encode
 ↓
 8 bit

Total each token ≈ 24 bits

Ex: I/p string: ababc bababaa



Compressed data:

[0, 0, 'a']

[0, 0, 'b']

[2, 2, 'c']

[4, 3, 'a']

[8, 2, 'a']

→ Decoder

Decoder

ab ab. c bababaa

Eg:



[3, 4, '-']

Eg:



[0, 0, 'M']

[0, 0, 'I']

[0, 1, 'I']

[3, 3, 'P']

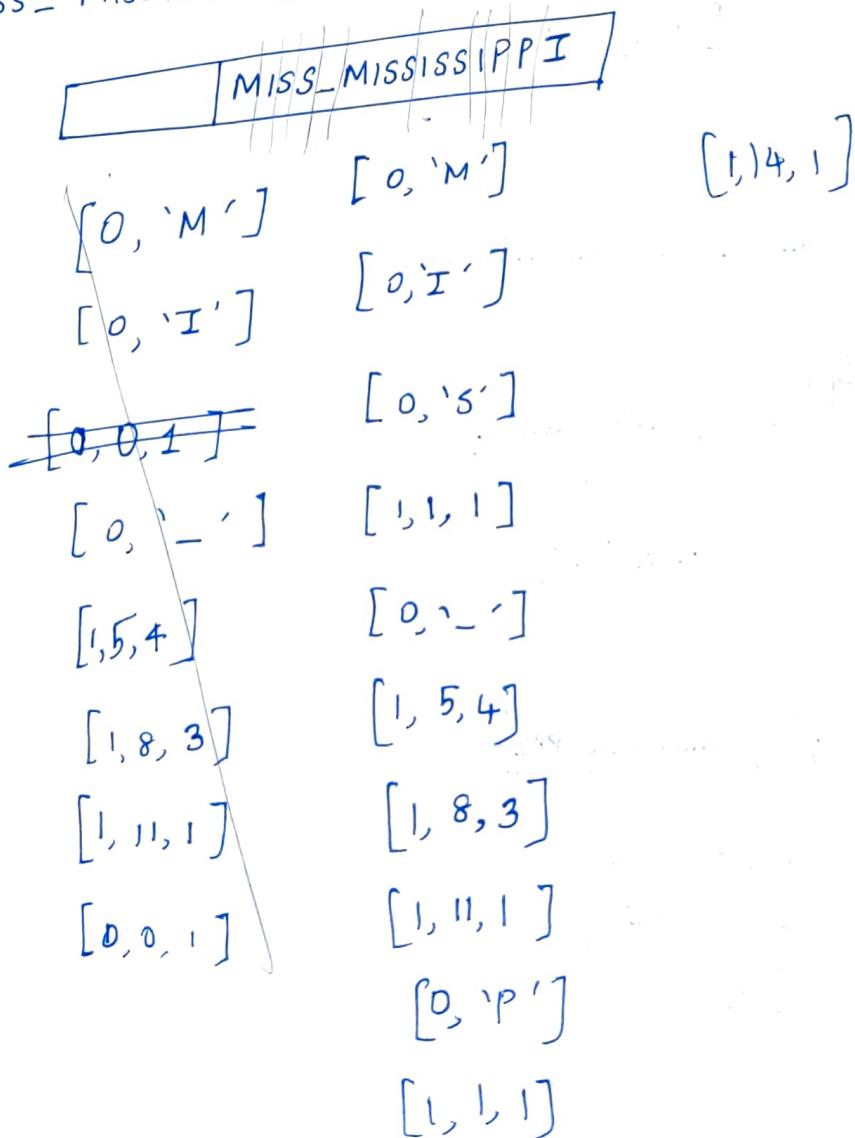
Decoding

MISSISSIPPI

LZSS Algorithm

- * Variant of LZ77 algorithm
- * LZ77 :- DS used was Circular Queue
- * To optimize search procedure, LZSS uses BST
- * Search Buffer: 16 bytes
- * Token only for matching sequence : Token (offset, length)
- * Flag Bit also stored in token
- Uncompressed : 2 bytes + 2 bits
Compressed 2 bytes + 1 bit (Flag, offset, Length)
$$\begin{matrix} 1 & 1 & 1 \\ \downarrow & \downarrow & \downarrow \\ \text{Flag} & \text{Offset} & \text{Length} \end{matrix}$$

Eg.: MISS-MISSISSIPPI



L17.8 Algorithm

Adaptive ~~Algorithm~~, Dictionary based alg.

3 types of tokens

Unmatched char : (0, unmatched char)

"Good Morning."

h) dictionary . O/P string

index string

NULL

0

G

(0, 'G')

1

(0, 'O')

(matches → read next
char also)

2

O

(2, 'ad')

3

od

0's
index

(0, '_')

4

_

(0, 'M')

5

M

(0, 'r')

6

or

(0, 'n')

7

n

(0, 'i')

8

i

(7, 'g')

9

ng

bits for index depends on max value required for index

$$\text{# bits} = \log_2 M \quad , \quad M - \text{max index value}$$

Stream of bits sent to decoder

Decoder should construct dictionary

Decoding

Index	Char	of string
0	NULL	
1	G	G
2	O	GO
3	D	GOOD
4	-	GOOD-
5	M	GOOD-M
6	R	GOOD-MOR
7	N	GOOD-MORN
8	I	GOOD-MORNI
9	NG	GOOD-MORNING

No comp.,

each char: 8 bits

After

$$\left[(\log_2 9) \times \# \text{entries} \right] + \left[8 \times \# \text{char} \right]$$

abababacbacacabac

Compression

Index	Char	o/p stream
0	NULL	
1	a	(0, 'a')
2	ab	(1, 'b')
3	aba	(2, 'a')
4	c	(0, 'c')
5	b	(0, 'b')
6	ac	(1, 'c')
7	aca	(6, 'a')
<u>8</u>	<u>ca</u>	<u>(4, a)</u>
8	ba	(5, 'a')
		(4)

Decoding

Index	Char	o/p string
0	NULL	
1	a	a
2	ab	aab
3	aba	aababa
4	c	aababac
5	b	aababacb
6	ac	aababacbac
7	aca	aababacbacaca
8	ba	aababacbacaca b aababacbacacabac

Before compression

$$16 \times 8 = 128 \text{ bits}$$

After compression

$$(8 \times 9) + (9 \times 8) = 72 + 72 = 96 \text{ bits}$$

9×12

$$46 \text{ bits} + 8$$

Each token

$$(8 + \log_2 9) = 12$$

$$(12 \times 8) + (4) = 96 + 4 = 100 \text{ bits}$$

LZW - Algorithm

(ε)

Initially, dictionary is not empty

It has $2^8 = 256$ entries (for each char of alphabet)

ASCII Table

<u>a</u> <u>b</u> <u>c</u>	x	256 ab
	*	257 ba
<u>a</u> <u>b</u> <u>a</u> <u>c</u>		258 aba
0 1 256 0 2		259 ac

Decoding

0 1 256 0 2
a b ab a c

b
a

Dict

0 a

1 b

ba b a a b a a a.

256 ba

" "

257 ab.

258 baa

259 aba

260 ~~a~~ aa

! reading
ba ba ab a aa

ababb abbabb abbbbbb

1 0 256 257 0 260

b a / b a a b a a a

b a ba ab a a a