Problem 1:

p(D|theta) = $\prod_{i=1}^{n} p(xi|theta)$

l(theta) = ln p(D|theta)

gradient_theta(l(theta)) = $\sum_{i=1}^{n} gradient\_theta(\ln p(xi|theta))$

= $\sum_{i=1}^{n} gradient\_theta(\ln(theta)) - gradient\_theta(theta * xi)$

0 = n / theta_hat - $\sum_{i=1}^{n} xi$

sum(xi) = n / theta_hat

theta_hat = n / sum(xi)

theta_hat = 1 / (sum(xi) * 1/n)

Averages for 10 runs each (code attached):

Max Likelihood Estimation
- Average: 75.78%
- Std. Dev: 2.302

Naive-Bayes
- Average: 73.65%
- Std. Dev: 2.409

k-Nearest Neighbor k = 1
- Average: 64.92%
- Std. Dev: 1.680

k-Nearest Neighbor k = 5
- Average: 65.49%
- Std. Dev: 1.100

k-Nearest Neighbor k = 11
- Average: 64.97%
- Std. Dev: 2.832

Parzen Window edge length = 20
- Average: 65.03%
- Std. Dev: 1.610

Problem 2 (Maximum Likelihood Estimation):

```
import csv
import math
import numpy as np

runs = 0
accuracy = []
while (runs < 10):
      raw_data = np.genfromtxt("pima-indians-diabetes.csv", delimiter =
",", usecols = (1, 2, 3, 8))
      np.random.shuffle(raw_data) #randomize data

      train_data = raw_data[0:len(raw_data)/2] #assign 1st half of data
to training data
      test_data = raw_data[len(raw_data)/2:] #assign 2nd half of data
to test data
      correct = 0
      wrong = 0

      train_data_filter1 = train_data[:, 3] = train_data[:, 3] == 1
#setting filter for training data when column 9 = 1
      mean1 = train_data[train_data_filter1].mean(0) #calculate mean of
each attribute
      mean1 = np.delete(mean1, 3, 0) #remove the 4th column from mean
vector
      var1 = np.cov(train_data[train_data_filter1].transpose())
#calculate covariance matric
      var1 = np.delete(var1, 3, 1) #removes the 4th column from the
covariance matrix
      var1 = np.delete(var1, 3, 0) #removes the 4th row from the
covariance matrix

      train_data_filter0 = train_data[:, 3] = train_data[:, 3] == 0
      mean0 = train_data[train_data_filter0].mean(0)
      mean0 = np.delete(mean0, 3, 0)
      var0 = np.cov(train_data[train_data_filter0].transpose())
      var0 = np.delete(var0, 3, 1)
      var0 = np.delete(var0, 3, 0)

      priortmp1 = train_data[train_data_filter1].shape[0] #gives the
count where training data column 4 equals 1
      priortmp0 = train_data[train_data_filter0].shape[0]

      prior0 = float(priortmp0) / (priortmp0 + priortmp1) #calculates
prior probability of trainging data where column 4 equals 0
      prior1 = float(priortmp1) / (priortmp0 + priortmp1)

      constant1 = 1 / math.sqrt(np.linalg.det(var1)) #calculates
1/sqrt(covariance)
      constant0 = 1 / math.sqrt(np.linalg.det(var0))
      inv_var0 = np.linalg.inv(var0) #inverse of covariance matrix
```

```
      inv_var1 = np.linalg.inv(var1)

      for i in test_data:
            j = np.delete(i, 3, 0)
            diff0 = j - mean0
            lklhood0 = constant0 * np.exp(-np.dot(np.dot(diff0,
inv_var0), diff0) / 2)

            diff1 = j - mean1
            lklhood1 = constant1 * np.exp(-np.dot(np.dot(diff1,
inv_var1), diff1) / 2)

            post0 = prior0 * lklhood0
            post1 = prior1 * lklhood1

            if(post0 > post1 and i[3] == 0):
                  correct += 1
            elif(post1 < post0 and i[3] == 1):
                  correct += 1
            else:
                  wrong += 1

      accuracy.append(100 * float(correct) / (correct + wrong))
      runs += 1

print "Average Accuracy: " + str(np.average(accuracy))
print "Standard Deviation of Accuracy: " + str(np.std(accuracy))
```

Problem 2 (Naive Bayes):

```
import csv
import math
import numpy as np

runs = 0
accuracy = []
while(runs < 10):
      raw_data = np.genfromtxt("pima-indians-diabetes.csv", delimiter =
",", usecols = (1, 2, 3, 8))
      np.random.shuffle(raw_data) #randomize raw data

      train_data = raw_data[0:len(raw_data)/2] #separate 1st half to
training data
      test_data = raw_data[len(raw_data)/2:] #separate 2nd half to test
data
      correct = 0
      wrong = 0

      train_data_filter1 = train_data[:, 3] = train_data[:, 3] == 1
#setup filter where training data column 4 = 1
      mean1 = train_data[train_data_filter1].mean(0) #calculate mean
for each attribute in training data
      mean1 = np.delete(mean1, 3, 0) #delete the 4th column from mean
vector
      var1 = np.cov(train_data[train_data_filter1].transpose())
#calculate covariance matrix on training data
      var1 = np.delete(var1, 3, 1) #delete column 4 from covariance
matrix
      var1 = np.delete(var1, 3, 0) #delete row 4 from covariance matrix

      train_data_filter0 = train_data[:, 3] = train_data[:, 3] == 0
#setup filter where training data column 4 = 0
      mean0 = train_data[train_data_filter0].mean(0)
      mean0 = np.delete(mean0, 3, 0)
      var0 = np.cov(train_data[train_data_filter0].transpose())
      var0 = np.delete(var0, 3, 1)
      var0 = np.delete(var0, 3, 0)

      priortmp1 = train_data[train_data_filter1].shape[0] #number of
training data records where column 4 = 1
      priortmp0 = train_data[train_data_filter0].shape[0] #number of
training data records where column 4 = 0

      #calculate priors
      prior0 = float(priortmp0) / (priortmp0 + priortmp1)
      prior1 = float(priortmp1) / (priortmp0 + priortmp1)

      for i in test_data:
            #calculate the independent likelihoods
```

```
            lklhood00 = np.exp(-(i[0] - mean0[0]) * (i[0] - mean0[0]) /
(2 * var0[0,0])) / math.sqrt(var0[0,0])
            lklhood01 = np.exp(-(i[1] - mean0[1]) * (i[1] - mean0[1]) /
(2 * var0[1,1])) / math.sqrt(var0[1,1])
            lklhood02 = np.exp(-(i[2] - mean0[2]) * (i[2] - mean0[2]) /
(2 * var0[2,2])) / math.sqrt(var0[2,2])

            lklhood10 = np.exp(-(i[0] - mean1[0]) * (i[0] - mean1[0]) /
(2 * var1[0,0])) / math.sqrt(var1[0,0])
            lklhood11 = np.exp(-(i[1] - mean1[1]) * (i[1] - mean1[1]) /
(2 * var1[1,1])) / math.sqrt(var1[1,1])
            lklhood12 = np.exp(-(i[2] - mean1[2]) * (i[2] - mean1[2]) /
(2 * var1[2,2])) / math.sqrt(var1[2,2])

            #calculate the posteriors
            post0 = prior0 * lklhood00 * lklhood01 * lklhood02
            post1 = prior1 * lklhood10 * lklhood11 * lklhood12

            #make decisions based on the posterior and count correct vs
wrong on test data
            if(post0 > post1 and i[3] == 0):
                  correct += 1
            elif(post1 < post0 and i[3] == 1):
                  correct += 1
            else:
                  wrong += 1

      runs += 1
      accuracy.append(100 * float(correct) / (correct + wrong))

print "Accuracy Average: " + str(np.average(accuracy))
print "Accuracy Standard Deviation: " + str(np.std(accuracy))
```

Problem 3 (k-Nearest Neighbor):

```
import csv
import math
import numpy as np
from scipy import spatial

runs = 0
accuracy = []
while(runs < 10):
      raw_data = np.genfromtxt("pima-indians-diabetes.csv", delimiter =
",", usecols = (1, 2, 3, 8))
      np.random.shuffle(raw_data) #shuffle data
      num_samples = 1 #value of k

      train_data = raw_data[0:len(raw_data)/2] #separate 1st half of
raw data into training data
      test_data = raw_data[len(raw_data)/2:] #separate 2nd half of raw
data into test data
      correct = 0
      wrong = 0
      post0 = 0
      post1 = 0

      train = np.delete(train_data, 3, 1) #assign train to train_data
without column 4
      tree = spatial.KDTree(train) #create kd tree with training data

      for i in test_data:
            distance, closest = tree.query(np.delete(i, 3, 0), k =
num_samples) #find k nearest neighbor(s) and assign it/them to closest
            j = 0
            #closest will be a list if k is greater than 1 and integer
if equal to 1, so this conditional separates them
            if(num_samples > 1):
                  while(j < num_samples): #this checks each of the
closest neighbor
                        if(train_data[closest[j], 3] == 1):
                              post1 += 1
                        else:
                              post0 += 1
                        j += 1
            else:
                  if(train_data[closest, 3] == 1):
                        post1 += 1
                  else:
                        post0 += 1

            if(post1 > post0 and i[3] == 1):
                  correct += 1
            elif(post0 > post1 and i[3] == 0):
                  correct += 1
```

```
            else:
                wrong += 1

        runs += 1
        accuracy.append(100 * float(correct) / (correct + wrong))

print "Accuracy Average: " + str(np.average(accuracy))
print "Accuracy Standard Deviation: " + str(np.std(accuracy))
```

Problem 4 (Parzen Window):

```
import csv
import math
import numpy as np
from scipy import spatial

runs = 0
accuracy = []
while(runs < 10):
      raw_data = np.genfromtxt("pima-indians-diabetes.csv", delimiter =
",", usecols = (1, 2, 3, 8))
      np.random.shuffle(raw_data) #randomizes data
      window_size = 20 #sets radius of hypercube

      train_data = raw_data[0:len(raw_data)/2] #separates the 1st half
of data to training data
      test_data = raw_data[len(raw_data)/2:] #separates the 2nd half of
data to testing data
      correct = 0
      wrong = 0
      post0 = 0
      post1 = 0

      train = np.delete(train_data, 3, 1) #assign training data without
column 4 to train
      tree = spatial.KDTree(train) #assign training data to kd tree

      for i in test_data:
            closest = tree.query_ball_point(np.delete(i, 3, 0),
window_size) #returns the indexes for neighbors inside of window
            j = 0
            while(j < len(closest)):
                  if(train_data[closest[j], 3] == 1):
                        post1 += 1
                  else:
                        post0 += 1
                  j += 1

            if(post1 > post0 and i[3] == 1):
                  correct += 1
            elif(post0 > post1 and i[3] == 0):
                  correct += 1
            else:
                  wrong += 1

      runs += 1
      accuracy.append(100 * float(correct) / (correct + wrong))

print "Accuracy Average: " + str(np.average(accuracy))
print "Accuracy Standard Deviation: " + str(np.std(accuracy))
```