Matt Hall mhall4@stevens.edu

1.) $P(\text{Dime} \mid \text{Green Wallet}) = \frac{4}{10} = \frac{2}{5} = 0.4$

$P(\text{Penny} \mid \text{Dime already pulled and Green Wallet}) = \frac{6}{9} = \frac{2}{3}$

$P(\text{Penny} \mid \text{1 penny and 1 dime already and Green}) = \frac{5}{8}$

$$= 0.167 = \frac{\cancel{2}\cdot\cancel{2}\cdot\cancel{5}}{\cancel{5}\cdot 3\cdot 2\cdot \cancel{2}\cdot\cancel{2}} = \frac{1}{6}$$

$P(\text{Dime} \mid \text{Black Wallet}) = \frac{2}{10} = \frac{1}{5}$

$P(\text{Penny} \mid \text{Dime already \& black wallet}) = \frac{8}{9}$

$P(\text{Penny} \mid \text{1 Dime \& 1 penny already \& Black wallet}) = \frac{7}{8}$

$P(1D \,\&\, 2P \mid B) = \frac{7}{45} = 0.156$

$$P(B \mid 1D \,\&\, 2P) = \frac{P(B)\,P(1D \,\&\, 2P \mid B)}{P(B)\,P(1D \,\&\, 2P \mid B) + P(G)\,P(1D \,\&\, 2P \mid G)}$$

$P(B) = 0.25$

$P(G) = 0.75$

$$= \frac{\left(\frac{1}{4}\right)\left(\frac{7}{45}\right)}{\frac{1}{4}\left(\frac{7}{45}\right) + \frac{3}{4}\left(\frac{1}{6}\right)} = \frac{\frac{7}{180}}{\frac{7}{180} + \frac{3}{24}} = \frac{0.0389}{\underset{0.125}{0.0389+}}$$

$$= 0.237$$

$P(G \mid 1D \,\&\, 2P) = 1 - P(B \mid 1D \,\&\, 2P)$

$= 0.763$

$P(\text{error} \mid G) = P(B \mid 1D \,\&\, 2P)$

$= 0.237$

$_3/x)$

2.) when $i=j$ $\lambda=0$

if $\lambda_r=0$, it will always reject. if $\lambda_r>\lambda_s$ it will never reject.

$$R(\alpha_i|x) = \sum_{j=1}^{c} \lambda(\alpha_i|\omega_j)P(\omega_j|x)$$

$$= \lambda_r + \lambda_s - P(\omega_i|x)$$

$\therefore$ Maximize $P(\omega_i/x)$

Problem 3

(a) $p(x|\omega_1) \sim N\left(\begin{bmatrix}0\\0\end{bmatrix}, \begin{bmatrix}1&0\\0&1\end{bmatrix}\right)$

$p(x|\omega_2) \sim N\left(\begin{bmatrix}2\\1\end{bmatrix}, \begin{bmatrix}1&0\\0&1\end{bmatrix}\right)$

$p(\omega_1) = p(\omega_2) = \frac{1}{2}$

$\mu_1 = \begin{bmatrix}0\\0\end{bmatrix} \quad \sigma_1^2 = 1$

$w_1 = \frac{[0\ 0]}{1}$

$w_{10} = -\frac{1}{2(1)} [0\ 0] \begin{bmatrix}0\\0\end{bmatrix} + \ln\left(\frac{1}{2}\right)$

$= -0.693$

$g_1(x) = [0\ 0]x - 0.693$

$\mu_2 = \begin{bmatrix}2\\1\end{bmatrix} \quad \sigma_2^2 = 1$

$w_2 = \begin{bmatrix}2\\1\end{bmatrix}$

$w_{20} = -\frac{1}{2} [2\ 1] \begin{bmatrix}2\\1\end{bmatrix} + \ln \frac{1}{2}$

$= -\frac{1}{2} \cdot 5 - 0.693$

$= -3.193$

$g_2(x) = [2\ 1]x - 3.193$

$g_1(x) = g_2(x)$

$-0.693 = 2x_1 + x_2 - 3.193$

$\boxed{2x_1 + x_2 = 2.5}$

(b) $p(x|\omega_1) \sim N\left(\begin{bmatrix}0\\0\end{bmatrix}, \begin{bmatrix}2&0\\0&2\end{bmatrix}\right)$

$p(x|\omega_2) \sim N\left(\begin{bmatrix}2\\1\end{bmatrix}, \begin{bmatrix}4&3\\2&4\end{bmatrix}\right)$

$p(\omega_1) = p(\omega_2) = \frac{1}{2}$

$$W_1 = \frac{-1}{2}\frac{1}{4}\begin{bmatrix}2&0\\0&2\end{bmatrix} = \begin{bmatrix}-1/4&0\\0&-1/4\end{bmatrix}$$

$$w_1 = \begin{bmatrix}-1/4&0\\0&-1/4\end{bmatrix}\begin{bmatrix}0\\0\end{bmatrix}$$

$$w_{10} = -\frac{1}{2}\cdot\ln\begin{vmatrix}2&0\\0&2\end{vmatrix} + \ln\frac{1}{2}$$

$$= -\frac{1}{2}\ln 4 + \ln\frac{1}{2}$$

$$= -1.386$$

$$g_1(x) = \begin{bmatrix}x_1 & x_2\end{bmatrix}\begin{bmatrix}-1/4&0\\0&-1/4\end{bmatrix}\begin{bmatrix}x_1\\x_2\end{bmatrix} + \overset{0}{\cancel{w_1^t x}} - 1.386$$

$$= \begin{bmatrix}-1/4\,x_1 & -1/4\,x_2\end{bmatrix}\begin{bmatrix}x_1\\x_2\end{bmatrix}$$

$$= -\frac{x_1^2}{4} - \frac{x_2^2}{4} - 1.386$$

$$W_1 = -\frac{1}{2}\begin{bmatrix} 2/5 & -3/10 \\ -1/5 & 2/5 \end{bmatrix} = \begin{bmatrix} -1/5 & 3/20 \\ 1/10 & -1/5 \end{bmatrix}$$

$$w_2 = \begin{bmatrix} 2/5 & -3/10 \\ -1/5 & 2/5 \end{bmatrix}\begin{bmatrix} 2 \\ 1 \end{bmatrix} = \begin{bmatrix} 4/5 - 3/10 \\ -2/5 + 2/5 \end{bmatrix} = \begin{bmatrix} 1/2 \\ 0 \end{bmatrix}$$

$$w_{20} = -\frac{1}{2}\begin{bmatrix} 2 & 1 \end{bmatrix}\begin{bmatrix} 2/5 & -3/10 \\ -1/5 & 2/5 \end{bmatrix}\begin{bmatrix} 2 \\ 1 \end{bmatrix} - \frac{1}{2}\ln\begin{vmatrix} 4 & 3 \\ 2 & 4 \end{vmatrix} + \ln\frac{1}{2}$$

$$= -\frac{1}{2}\begin{bmatrix} 3/5 & -1/5 \end{bmatrix}\begin{bmatrix} 2 \\ 1 \end{bmatrix} - \frac{1}{2}\ln 10 + \ln\frac{1}{2}$$

$$= -\frac{1}{2}(1) - \frac{1}{2}\ln 10 + \ln\frac{1}{2}$$

$$= -\frac{1}{2} - 1.151 - 0.693$$

.386

$$= -2.344$$

$$g_2(x) = \begin{bmatrix} x_1 & x_2 \end{bmatrix}\begin{bmatrix} -1/5 & 3/20 \\ 1/10 & -1/5 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 1/2 & 0 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} - 2.344$$

$$= \begin{bmatrix} -1/5\, x_1 + 1/10\, x_2 & 3/20\, x_1 - 1/5\, x_2 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + 1/2\, x_1 - 2.344$$

$$= \frac{-x_1^2}{5} + \frac{x_1 x_2}{10} + \frac{3x_1 x_2}{20} - \frac{x_2^2}{5} + 1/2\, x_1 - 2.344$$

$$g_1(x) = g_2(x) \Rightarrow -\frac{x_1^2}{4} - \frac{x_2^2}{4} - 1.386 = \frac{-x_1^2}{5} + \frac{x_1 x_2}{10} + \frac{3x_1 x_2}{20} - \frac{x_2^2}{5} + \frac{1}{2}x_1 - 2.344$$

$$0.958 = \frac{x_1^2}{20} + \frac{x_2^2}{20} + \frac{x_1 x_2}{4} + \frac{1}{2}x_1$$

$$\boxed{19.16 = x_1^2 + x_2^2 + 5x_1 x_2 + 10x_1}$$

Problem 4 (2nd 3) part A


```
#include <random>
#include <vector>

std::vector<double> custom_normal_distribution(double mean, double var, int N) {
    std::random_device rd;
    std::mt19937 gen(rd());

    std::vector<double> dist;
    std::normal_distribution<> base(0, 1);

    for (int i(0); i < N; ++i)
        dist.push_back(mean + sqrt(var) * base(gen));

    return dist;
}
```

2nd Part

$$mean_{total} = \frac{N_1 mean_1 + N_2 mean_2}{N_1 + N_2}$$

$$= \frac{\cancel{N_1} \frac{\sum x_{i1}}{\cancel{N_1}} + \cancel{N_2} \frac{\sum x_{j2}}{\cancel{N_2}}}{N_1 + N_2}$$

$$= \frac{\sum x_{i1} + \sum x_{j2}}{N_1 + N_2}$$

$\therefore$

$$mean_{total} = \frac{N_1 mean_1 + N_2 mean_2}{N_1 + N_2}$$

$$\frac{1}{\sqrt{2\pi}\,\sigma_1} e^{\frac{-(x_i-\mu_1)^2}{2\sigma_1{}_2}} \cdot \frac{1}{\sqrt{2\pi}\sigma_2} \cdot e^{\frac{-(x_i-\mu_2)^2}{2\sigma_2{}^2}}$$

$$\frac{1}{2\pi\sqrt{\sigma_1\sigma_2}} \cdot e^{\frac{-(x-\mu_1{}^2)\sigma_2^2 - \sigma_1{}^2(x-\mu_2)^2}{2\sigma_1^2\sigma_2{}^2}}$$

$$-\left(x - \left(\frac{\Sigma x}{N_1}\right)^2\right)\left(\Sigma x - \left(\frac{\Sigma x_2}{N_2}\right)^2\right.$$

$$\text{mean}_t = \frac{N_1\,\text{mean}_1 + N_2\,\text{mean}_2}{N_1 + N_2}$$

$$\frac{\Sigma x_{1i}}{N_1} + \frac{\Sigma x_{2i}}{N_2}$$