Übung Serie 5

Aufgabe No-Cloning Theorem

Wir betrachten

$$K(|0\rangle \otimes |0\rangle) = |0\rangle \otimes |0\rangle, \quad K(|1\rangle \otimes |0\rangle) = |1\rangle \otimes |1\rangle \quad (*)$$

$$K\left(\frac{|1\rangle + |0\rangle}{\sqrt{2}} \otimes |0\rangle\right) = \frac{|1\rangle + |0\rangle}{\sqrt{2}} \otimes \frac{|1\rangle + |0\rangle}{\sqrt{2}} \quad (**)$$

Da K linear ist, gilt in (**) auch

$$K\left(\frac{|1\rangle + |0\rangle}{\sqrt{2}} \otimes |0\rangle\right) = K\left(\frac{|1\rangle \otimes |0\rangle}{\sqrt{2}} + \frac{|0\rangle \otimes |0\rangle}{\sqrt{2}}\right)$$

$$\overset{K \text{ linear}}{=} \frac{1}{\sqrt{2}} K\left(|1\rangle \otimes |0\rangle + |0\rangle \otimes |0\rangle\right)$$

$$\overset{K \text{ linear}}{=} \frac{1}{\sqrt{2}} \left(K(|1\rangle \otimes |0\rangle) + K(|0\rangle \otimes |0\rangle)\right)$$

$$\overset{(*)}{=} \frac{1}{\sqrt{2}} \left(|1\rangle \otimes |1\rangle + |0\rangle \otimes |0\rangle\right)$$

$$= \frac{|1\rangle \otimes |1\rangle}{\sqrt{2}} + \frac{|0\rangle \otimes |0\rangle}{\sqrt{2}} \quad \lightning \text{ zu } (**)$$

Aufgabe SWAP-Gatter

zu (i) Für SWAP gilt

$$\text{SWAP}: \begin{cases} |0\rangle|0\rangle \longmapsto |0\rangle|0\rangle \\ |0\rangle|1\rangle \longmapsto |1\rangle|0\rangle \\ |1\rangle|0\rangle \longmapsto |0\rangle|1\rangle \\ |1\rangle|1\rangle \longmapsto |1\rangle|1\rangle \end{cases}$$

Die dazugehörige Matrix

$$\begin{array}{c} \\ |00\rangle \\ |01\rangle \\ |10\rangle \\ |11\rangle \end{array} \begin{array}{c} \begin{array}{cccc} |00\rangle & |01\rangle & |10\rangle & |11\rangle \end{array} \\ \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \end{array} = A_{SWAP}$$

ist eine Permutationsmatrix und als solche unitär

<u>zu (ii)</u> Es gilt

$$(\alpha|0\rangle + \beta|1\rangle)(\gamma|0\rangle + \delta|1\rangle) =$$

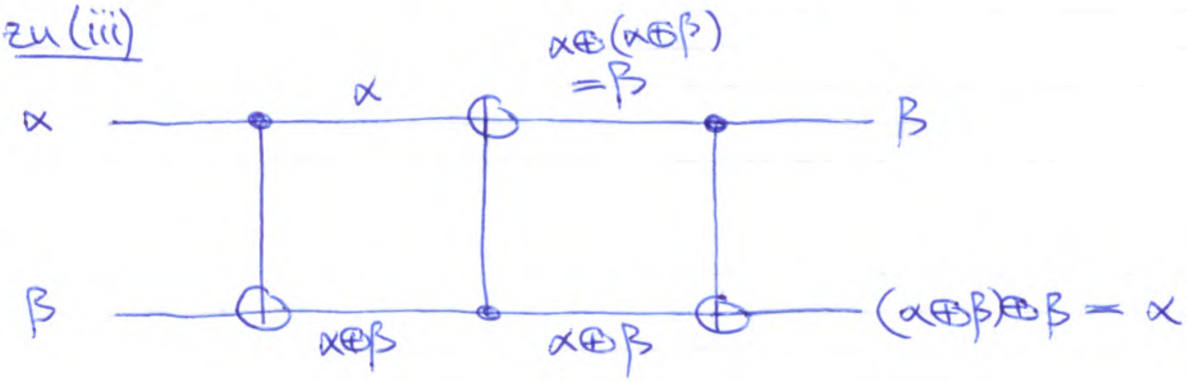
$$= \alpha\gamma|00\rangle + \alpha\delta|01\rangle + \beta\gamma|10\rangle + \beta\delta|11\rangle$$

$$\xmapsto{\text{SWAP}} \alpha\gamma|00\rangle + \alpha\delta|10\rangle + \gamma\beta|01\rangle + \beta\delta|11\rangle$$

$$= \gamma\alpha|00\rangle + \gamma\beta|01\rangle + \delta\alpha|10\rangle + \delta\beta|11\rangle$$

$$= (\gamma|0\rangle + \delta|1\rangle)(\alpha|0\rangle + \beta|1\rangle)$$

<u>zu (iii)</u>



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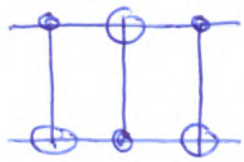

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