

Appendix

Definition 1 (Buyers/Sellers). *Each buyer, B_i , has a true cost for the item they are buying, c_i , and a bid b_i such that $b_i \geq c_i$. Each seller, S_j , has a true price for the item they are selling, p_j , and an ask a_j such that $a_j \leq p_j$.*

Definition 2 (Clearing Price). *Consider some bid b and ask a . The clearing price, $cp(b, a)$, a function of the bid and ask that maps to the price at which the transaction clears. We require that $cp(b, a) \in [a, b]$.*

Definition 3 (Surplus). *Let B_i denote a buyer with cost c_i and bid b_i . Let S_j denote a seller with price p_j and ask a_j . Consider some clearing price $cp(b_i, a_j)$. The surplus for the buyer is defined as $s_{B_i} = c_i - cp(b_i, a_j)$ and the surplus for the seller is defined as $s_{S_j} = p_j - cp(b_i, a_j)$. The total surplus of the transaction, defined $s_{B_i, S_j} = s_{B_i} + s_{S_j}$, is the sum of the surplus for the buyer and the seller. The surplus of any buyer or seller not involved in a transaction is 0.*

Theorem 1 (Surplus Calculation). *The calculation of surplus in the market does not depend on the clearing price.*

Proof. We define the surplus of the market as the summation of the surplus for each transaction. Note that for each transaction with buyer and seller B_i, S_j , the surplus is defined as $s_{B_i, S_j} = c_i - p_j$ and thus does not depend on the clearing price. \square