Date: * *
Page:

a) Both person meet at same place after n steps.

For both meeting at same places, they both should move left and right equal no. of times (but in arbitary order)

Let i be the number of times they move left

$\because$ moving left and right are equally probable

$$P(x) = \begin{cases} 1/2 & , x = -1 \\ 1/2 & , x = 1 \\ 0 & , \text{otherwise}. \end{cases}$$

Let Probability both meeting at same place be p after i left steps.

$$p_i = \underbrace{\left[{}^{n}C_i \left(\frac{1}{2}\right)^{n-i} \left(\frac{1}{2}\right)^{i} \right]}_{\text{first person}} \underbrace{\left[{}^{n}C_i \left(\frac{1}{2}\right)^{n-i} \left(\frac{1}{2}\right)^{i} \right]}_{\text{second person}}$$

$$p_i = \left({}^{n}C_i\right)^2 \frac{1}{2^{2n}}$$

$$\text{Total probability} = \sum_{i=0}^{n} p_i = \frac{1}{2^{2n}} \sum_{i=0}^{i=n} \left({}^{n}C_i\right)^2$$

Using Vandermonde's Identity i.e.

$$\sum_{i=0}^{n} {}^{n}C_i^{\,2} = {}^{2n}C_n$$

$$\boxed{p = \frac{{}^{2n}C_n}{2^{2n}}} \quad \forall\ n \in \mathbb{N} \cup \{0\}.$$

else zero

b) For a person to reach at origin after ~~n~~ steps, he/she must cover equal no. of steps in left and right direction.

Case-1 $\quad n$ is odd.

then person can never reach zero, therefore $\quad p = 0$

Case-2 $\quad n$ is even.

$\frac{n}{2}$ left steps + $\frac{n}{2}$ right steps

So $\quad p = {}^{n}C_{n/2} \left(\frac{1}{2}\right)^{n/2} \left(\frac{1}{2}\right)^{n/2}$

$$\boxed{p = \frac{{}^{n}C_{n/2}}{2^{n}}}$$

$$\therefore \quad \boxed{P(x=0 \mid n) = \begin{cases} \dfrac{{}^{n}C_{n/2}}{2^{n}} & n \in \mathbb{N} \cup \{0\} \text{ and } n \text{ is even} \\ 0 & , \text{ otherwise} \end{cases}}$$

c) Mean displacement of a person

Let x_i be random variable denoting the ~~positive~~ i^{th} step to be taken.

~~also~~ $x_i = \begin{cases} +1 & , \text{ right} \\ -1 & , \text{ left.} \end{cases}$

$\because$ $x_i = 1$ & $x_i = -1$ are equally probable.

$$E[x_i] = \frac{1}{2}(1) + \frac{1}{2}(-1) = 0$$

~~&~~ $E[x] = E[\Sigma x_i] = \Sigma E[x_i]$

$\boxed{E[x] = 0}$ → mean displacement equals zero. independent of n

d) Mean square displacement

Now we need to calculate

$E[Y]$ where $Y = (\Sigma x_i)^2$

$$E[x_i^2] = \frac{1}{2}(1)^2 + \frac{1}{2}(-1)^2 = 1$$

and we know that, i^{th} step and j^{th} independent. ~~therefo~~ as long as $i \neq j$

So $E[x_i x_j] = E[x_i]\, E[x_j]$

(Due to independence)

$E[x_i x_j] = 0 \qquad \because \quad E[x_i] = 0 \ \forall i$

Now

$$Y = \left(\Sigma x_i\right)^2 = \sum_{i=1}^{n} x_i^2 + \sum_{\substack{i,j \\ i \neq j}} x_i x_j$$

Using above results.

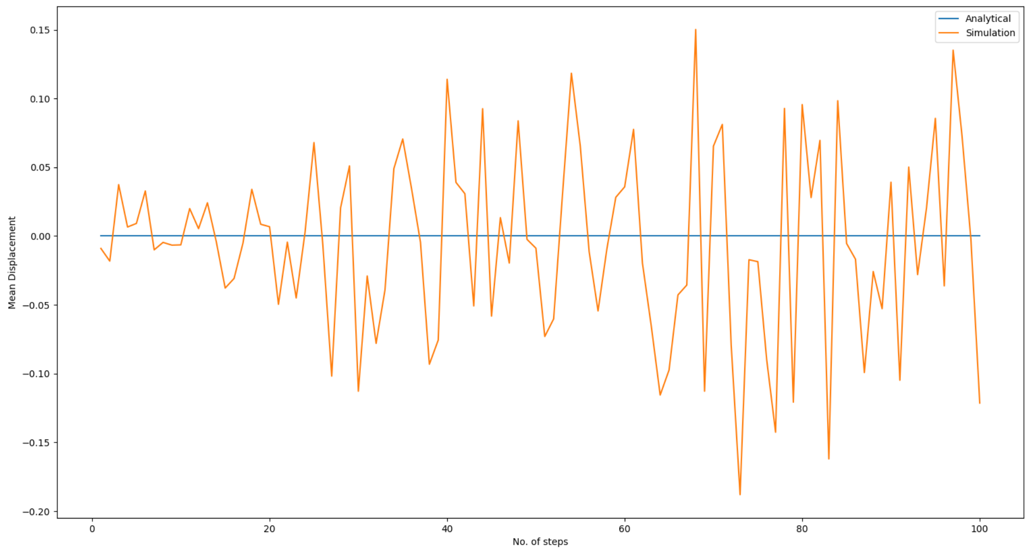
$$E[Y] = E[\Sigma x_i^2] + E[\Sigma x_i x_j]$$

$$E[Y] = \sum_{i=1}^{n} E[x_i^2] = n$$

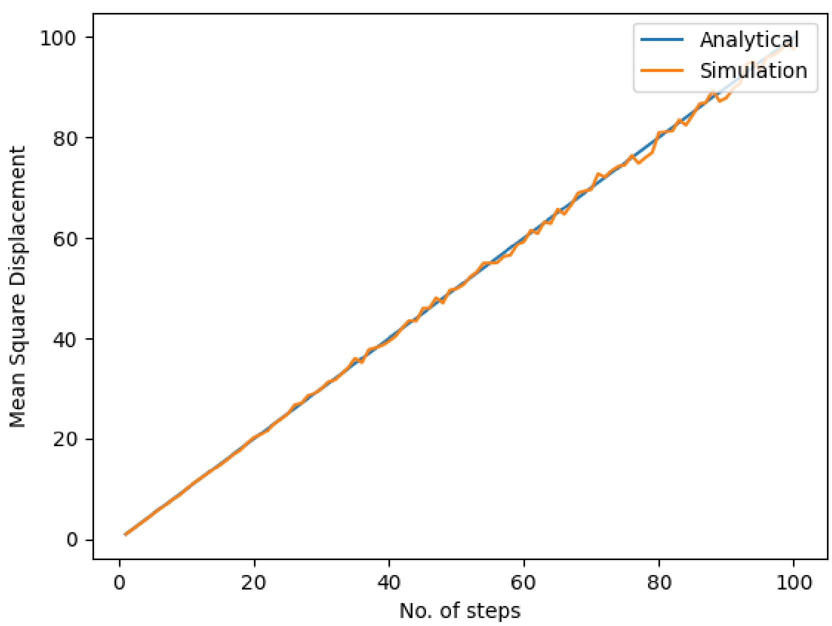
$$\boxed{E[Y] = N}$$

mean square displacement is linear in n

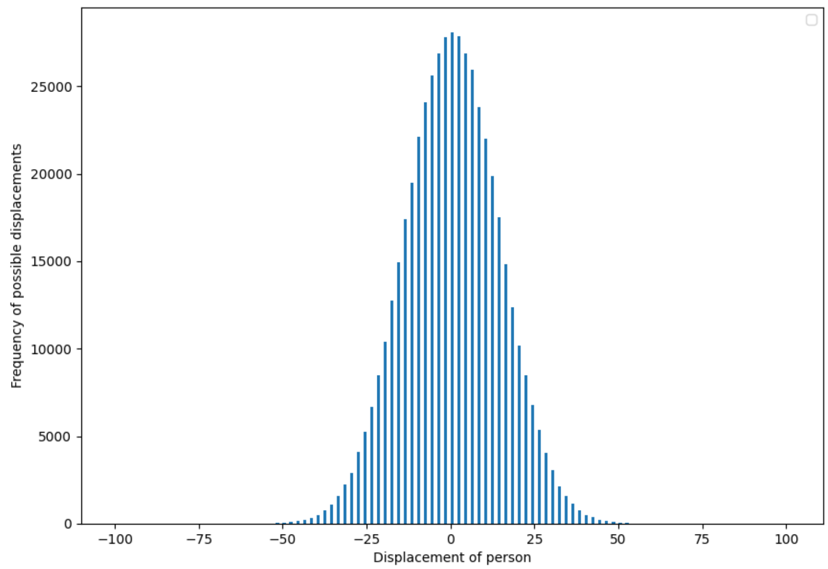
Plot of mean displacement



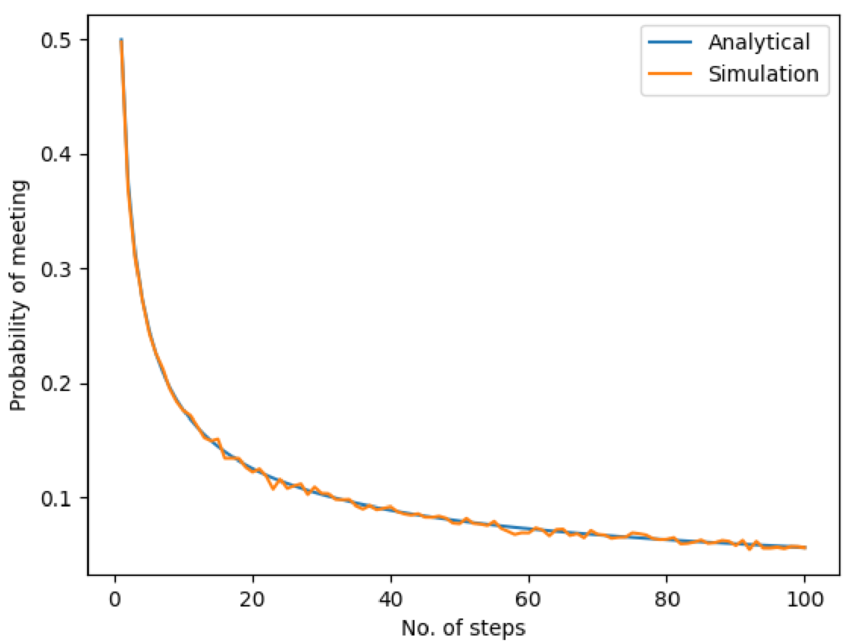
Plot of mean square displacement



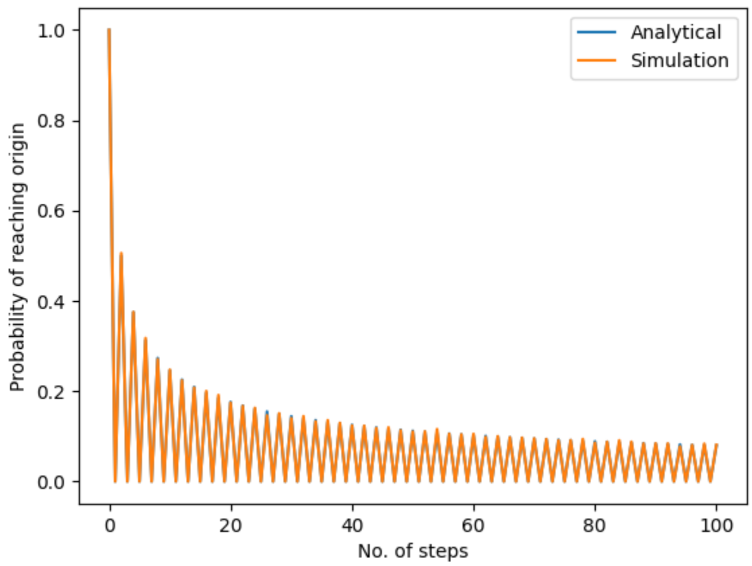
Bell curve for mean displacement



Plot for probability of meeting



Plot for probability of meeting at origin



Plot for probability of meeting at origin at even points