Theoretical Computerscience - Summary

WS 24/25

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1 Words

A word w (also called String) has length l and consists of symbols $\sigma \in \Sigma$. The empty word ε has length 0.

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2 Regular Languages

3 Regular Expressions

A regular expression always describes a regular language. If we can build a regular expression E, then $L(E) \in \mathsf{REG}$.

4 Common Proof Techniques

4.1 Pumping Lemma

The pumping lemma can only be used to show that a language is not REG. We take a word $w \in L$ and split it into multiple sub words u,v,w where $|v| \ge n$ with $n \ge 0$. Now there are words x,y,z with v = xyz and $|y| \ge 0$ such that $uxy^izw \in L$ for all $i \in \mathbb{N}$. Afterwards we try to find an i such that the pumped word is no longer i and thus we proved that the language is not regular.

4.1.1 Example

Exercise:

Show that the following language is not regular.

Let
$$A = \{1^{(3n)} \mid n \in \mathbb{N}\}$$

Solution:

Let $n \ge 0$ be given.

We choose $u = 1^n$, $v = 1^n$, $w = 1^n$ such that $uvw = 1^{3n}$ and |v| = n.

Let x,y,z be given as $x=1^r$, $y=1^s$, $z=1^t$ with xyz=v and $s\geq 0$ since $y\neq \varepsilon$ and r+s+t=n.

$$uxy^{i}zw = 1^{n}1^{r}1^{s \cdot i}1^{t}1^{n} = 1^{n}1^{r+s+t}1^{s \cdot (i-1)}1^{n}$$

We choose i=0, therefore $1^n1^{r+s+t}1^{s\cdot(i-1)}1^n=1^n1^{n-s}1^n\notin A$ since it is not of the form 1^{3m} anymore for any $m\in\mathbb{N}$.

Therefore we cannot pump language A and thus it is not regular.

4.2 Myhill Nerode

4.2.1 Example

5 Useful Proofs

5.1 Regular Languages

5.1.1 Finite Set

Exercise:

Show that the following language is regular over the alphabet $\{0,1\}$.

 $L = \{x \mid x \text{ is prime and } x < 1'000'000'000\}$

Solution:

Since there are only finitely many prime numbers between 0 and 1'000'000'000, the set of the words that are accepted by L is finite and thus the language is regular.

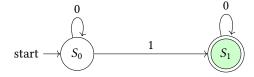
5.1.2 Finite Automaton

Exercise:

Show that the following language is regular over the alphabet $\{0,1\}$.

$$L = \{0^n 10^m \mid n, m \in \mathbb{N}\}$$

Solution:



Since we can describe the language L by the finite automaton given above, the language is regular.

5.1.3 Regular Expression

Exercise:

Show that the following language is regular over the alphabet $\{0,1\}$.

$$L = \{0^n 1^m \mid n, m \in \mathbb{N}\}$$

Solution:

Let $E = 0^*1^*$ be the regular expression describing the language L.

Since we can describe the language L by the regular expression given above, the language is regular.

5.1.4 Closure Properties

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5.2 Non-Regular Languages	

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