

# **Theoretical Computerscience – Summary**

WS 24/25

---

## Contents

## 1 Words

A word  $w$  (also called String) has length  $l$  and consists of symbols  $\sigma \in \Sigma$ .

The empty word  $\varepsilon$  has length 0.

## 2 Regular Languages

### 3 Regular Expressions

A regular expression always describes a regular language. If we can build a regular expression  $E$ , then  $L(E) \in \text{REG}$ .

## 4 Common Proof Techniques

### 4.1 Pumping Lemma

The pumping lemma can only be used to show that a language is not REG. We take a word  $w \in L$  and split it into multiple sub words  $u, v, w$  where  $|v| \geq n$  with  $n \geq 0$ . Now there are words  $x, y, z$  with  $v = xyz$  and  $|y| \geq 0$  such that  $uxy^i zw \in L$  for all  $i \in \mathbb{N}$ . Afterwards we try to find an  $i$  such that the pumped word is no longer  $\in L$  and thus we proved that the language is not regular.

#### 4.1.1 Example

##### Exercise:

Show that the following language is not regular.

Let  $A = \{1^{(3n)} \mid n \in \mathbb{N}\}$

##### Solution:

Let  $n \geq 0$  be given.

We choose  $u = 1^n$ ,  $v = 1^n$ ,  $w = 1^n$  such that  $uvw = 1^{3n}$  and  $|v| = n$ .

Let  $x, y, z$  be given as  $x = 1^r$ ,  $y = 1^s$ ,  $z = 1^t$  with  $xyz = v$  and  $s \geq 0$  since  $y \neq \epsilon$  and  $r + s + t = n$ .

$$uxy^i zw = 1^n 1^r 1^{s \cdot i} 1^t 1^n = 1^n 1^{r+s+t} 1^{s \cdot (i-1)} 1^n$$

We choose  $i = 0$ , therefore  $1^n 1^{r+s+t} 1^{s \cdot (i-1)} 1^n = 1^n 1^{n-s} 1^n \notin A$  since it is not of the form  $1^{3m}$  anymore for any  $m \in \mathbb{N}$ .

Therefore we cannot pump language  $A$  and thus it is not regular.

### 4.2 Myhill Nerode

#### 4.2.1 Example

TODO: Add example for Myhill Nerode

## 5 Useful Proofs

### 5.1 Regular Languages

#### 5.1.1 Finite Set

**Exercise:**

Show that the following language is regular over the alphabet  $\{0,1\}$ .

$$L = \{x \mid x \text{ is prime and } x < 1'000'000'000\}$$

**Solution:**

Since there are only finitely many prime numbers between 0 and  $1'000'000'000$ , the set of the words that are accepted by  $L$  is finite and thus the language is regular.

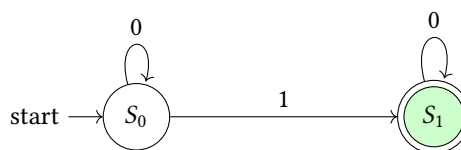
#### 5.1.2 Finite Automaton

**Exercise:**

Show that the following language is regular over the alphabet  $\{0,1\}$ .

$$L = \{0^n 10^m \mid n, m \in \mathbb{N}\}$$

**Solution:**



Since we can describe the language  $L$  by the finite automaton given above, the language is regular.

#### 5.1.3 Regular Expression

**Exercise:**

Show that the following language is regular over the alphabet  $\{0,1\}$ .

$$L = \{0^n 1^m \mid n, m \in \mathbb{N}\}$$

**Solution:**

Let  $E = 0^*1^*$  be the regular expression describing the language  $L$ .

Since we can describe the language  $L$  by the regular expression given above, the language is regular.

#### 5.1.4 Closure Properties

## 5.2 Non-Regular Languages

### 5.2.1 Pumping Lemma

**Exercise:**

Show that the following language is not regular over the alphabet  $\{0,1\}$ .

$$L = \{0^n 1^n \mid n \in \mathbb{N}\}$$

**Solution:**

Let  $n \geq 0$  be given.

We choose  $u = 1^n$ ,  $v = 1^n$ ,  $w = 1^n$  such that  $uvw = 1^{3n}$  and  $|v| = n$ .

Let  $x,y,z$  be given as  $x = 1^r$ ,  $y = 1^s$ ,  $z = 1^t$  with  $xyz = v$  and  $s \geq 0$  since  $y \neq \varepsilon$  and  $r + s + t = n$ .

$$uxy^i z w = 1^n 1^r 1^{s \cdot i} 1^t 1^n = 1^n 1^{r+s+t} 1^{s \cdot (i-1)} 1^n$$

We choose  $i = 0$ , therefore  $1^n 1^{r+s+t} 1^{s \cdot (i-1)} 1^n = 1^n 1^{n-s} 1^n \notin A$  since it is not of the form  $1^{3m}$  anymore for any  $m \in \mathbb{N}$ .

Therefore we cannot pump language  $A$  and thus it is not regular.

### 5.2.2 Myhill Nerode

**Exercise:**

Show that the following language is not regular over the alphabet  $\{0,1\}$ .

$$L = \{0^n 1^n \mid n \in \mathbb{N}\}$$

**Solution:**

**TODO: Add solution for Myhill Nerode**