# Theoretical Computerscience - Summary

WS 24/25

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# 1 Words

A word w (also called String) has length l and consists of symbols  $\sigma \in \Sigma$ . The empty word  $\varepsilon$  has length 0.

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# 2 Regular Languages

# 3 Regular Expressions

A regular expression always describes a regular language. If we can build a regular expression E, then  $L(E) \in \mathsf{REG}$ .

## 4 Common Proof Techniques

## 4.1 Pumping Lemma

The pumping lemma can only be used to show that a language is not REG. We take a word  $w \in L$  and split it into multiple sub words u,v,w where  $|v| \ge n$  with  $n \ge 0$ . Now there are words x,y,z with v = xyz and  $|y| \ge 0$  such that  $uxy^izw \in L$  for all  $i \in \mathbb{N}$ . Afterwards we try to find an i such that the pumped word is no longer i and thus we proved that the language is not regular.

## 4.1.1 Example

### Exercise:

Show that the following language is not regular.

Let 
$$A = \{1^{(3n)} \mid n \in \mathbb{N}\}$$

#### Solution:

Let  $n \ge 0$  be given.

We choose  $u = 1^n$ ,  $v = 1^n$ ,  $w = 1^n$  such that  $uvw = 1^{3n}$  and |v| = n.

Let x,y,z be given as  $x=1^r$ ,  $y=1^s$ ,  $z=1^t$  with xyz=v and  $s\geq 0$  since  $y\neq \varepsilon$  and r+s+t=n.

$$uxy^{i}zw = 1^{n}1^{r}1^{s \cdot i}1^{t}1^{n} = 1^{n}1^{r+s+t}1^{s \cdot (i-1)}1^{n}$$

We choose i=0, therefore  $1^n1^{r+s+t}1^{s\cdot(i-1)}1^n=1^n1^{n-s}1^n\notin A$  since it is not of the form  $1^{3m}$  anymore for any  $m\in\mathbb{N}$ .

Therefore we cannot pump language A and thus it is not regular.

## 4.2 Myhill Nerode

## 4.2.1 Example

## 5 Useful Proofs

## 5.1 Regular Languages

### 5.1.1 Finite Set

### Exercise:

Show that the following language is regular over the alphabet  $\{0,1\}$ .

 $L = \{x \mid x \text{ is prime and } x < 1'000'000'000\}$ 

### **Solution**:

Since there are only finitely many prime numbers between 0 and 1'000'000'000, the set of the words that are accepted by L is finite and thus the language is regular.

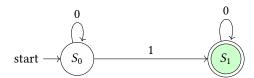
### 5.1.2 Finite Automaton

### Exercise:

Show that the following language is regular over the alphabet  $\{0,1\}$ .

$$L = \{0^n 10^m \mid n, m \in \mathbb{N}\}$$

### **Solution**:



Since we can describe the language L by the finite automaton given above, the language is regular.

## 5.1.3 Regular Expression

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5.2 Non-Regular Languages	

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