Theoretical Computerscience – Summary

WS 24/25

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1 Words

A word w (also called String) has length l and consists of symbols $\sigma \in \Sigma$.
The empty word ε has length 0.

2 Regular Languages

3 Regular Expressions

A regular expression always describes a regular language. If we can build a regular expression E, then L(E) ∈ REG.

4 Common Proof Techniques

4.1 Pumping Lemma

The pumping lemma can only be used to show that a language is not REG. We take a word $w \in L$ and split it into multiple sub words u,v,w where $|v| \geq n$ with $n \geq 0$. Now there are words x,y,z with $v = xyz$ and $|y| \geq 0$ such that $uxy^izw \in L$ for all $i \in \mathbb{N}$. Afterwards we try to find an i such that the pumped word is no longer $\in L$ and thus we proved that the language is not regular.

4.1.1 Example

Exercise:
Show that the following language is not regular.
Let $A = \{1^{(3n)} \mid n \in \mathbb{N}\}$

Solution:
Let $n \geq 0$ be given.
We choose $u = 1^n$, $v = 1^n$, $w = 1^n$ such that $uvw = 1^{3n}$ and $|v| = n$.
Let x,y,z be given as $x = 1^r$, $y = 1^s$, $z = 1^t$ with $xyz = v$ and $s \geq 0$ since $y \neq \varepsilon$ and $r + s + t = n$.
$uxy^izw = 1^n 1^r 1^{s \cdot i} 1^t 1^n = 1^n 1^{r+s+t} 1^{s \cdot (i-1)} 1^n$
We choose $i = 0$, therefore $1^n 1^{r+s+t} 1^{s \cdot (i-1)} 1^n = 1^n 1^{n-s} 1^n \notin A$ since it is not of the form 1^{3m} anymore for any $m \in \mathbb{N}$.
Therefore we cannot pump language A and thus it is not regular.

4.2 Myhill Nerode

4.2.1 Example

5 Useful Proofs

5.1 Regular Languages

5.1.1 Finite Set

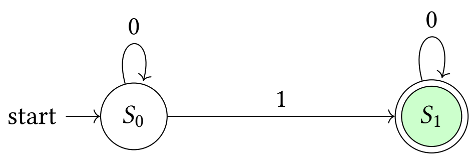
Exercise:
Show that the following language is regular over the alphabet $\{0,1\}$.
$L = \{x \mid x \text{ is prime and } x < 1'000'000'000\}$

Solution:
Since there are only finitely many prime numbers between 0 and $1'000'000'000$, the set of the words that are accepted by L is finite and thus the language is regular.

5.1.2 Finite Automaton

Exercise:
Show that the following language is regular over the alphabet $\{0,1\}$.
$L = \{0^n 1 0^m \mid n,m \in \mathbb{N}\}$

Solution:



Since we can describe the language L by the finite automaton given above, the language is regular.

5.1.3 Regular Expression

Exercise:
Show that the following language is regular over the alphabet $\{0,1\}$.
$L = \{0^n 1^m \mid n,m \in \mathbb{N}\}$

Solution:
Let $E = 0^* 1^*$ be the regular expression describing the language L.
Since we can describe the language L by the regular expression given above, the language is regular.

5.1.4 Closure Properties

5.2 Non-Regular Languages

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