PROJECT

Fundamentals of Optimization

TABLE OF CONTENTS

Bin packing lower, upper bound

Tran Le My Linh 20210535
Nguyen Ba Thiem 20214931
Phan Dinh Truong 20214937
Nguyen Minh Tuan 20214940

01

Problem description

IDEA FOR THE PROBLEM

TRUCK DATA

Quantity K trucks

Load limit max, min

CUSTOMER DATA

Quantity N customers

Order quantities of goods

Value Total value

GOALS

- each customer takes goods from only 1 truck
- total amount of goods/truck in range [min,max load]
- total values of delivered goods must be maximized

02

MODELLING

FIRST ANNOUNCEMENT

D[1],
D[2]
...
D[N]

goods_quantity
= [d[1], d[2], d[3],..., d[N]]

the vector containing the quantity of goods that customers ordered

C[1],
C[2]
...
C[N]

values
= [c[1], c[2], c[3],..., c[N]]

the vector containing goods' values

SECOND ANNOUNCEMENT

Load of each truck

weighted sum of goods' quantities
indicating either that truck contains
goods for that particular customer.

Variables

weight[1:K, 1:N]: weight[i, j]

packages of customer j
are contained on truck j

weight[i, j] == 1

truck i contains packages
of customer j

weight[i, j] != 1

truck i doesn't contains packages
of customer j

THIRD ANNOUNCEMENT

Constraints

For each truck i

load must be between lower bound
and upper bound

$c1[i] <= weight[i,:]^{T}\ x\ goods\_quantity <= c2[i]$

For each customer j

his/her packages can only be
delivered on one single truck

`sum(weight[i, j]) <= 1 for i in [1, K]`

LAST ANNOUNCEMENT

Objective function to maximize

Totals values of
deliveried packages

sum(weight x values)

03

PROPOSED METHODS

PROPOSED ALGORITHMS

Greedy

Local search

Ortools ILP

Ortools CP-SAT

PROPOSED ALGORITHMS

sort goods

put them into trucks

until the lower load bound is reached

next truck

Next, we packages (goods) into trucks until the trucks are full or there is no package left.

PROPOSED ALGORITHMS

sort goods

put them into trucks

until the lower load bound
is reached

next truck

PROBLEM

in which way
should we sort packages and trucks?

PROPOSED ALGORITHMS

sort goods

put them into trucks

until the lower load bound
is reached

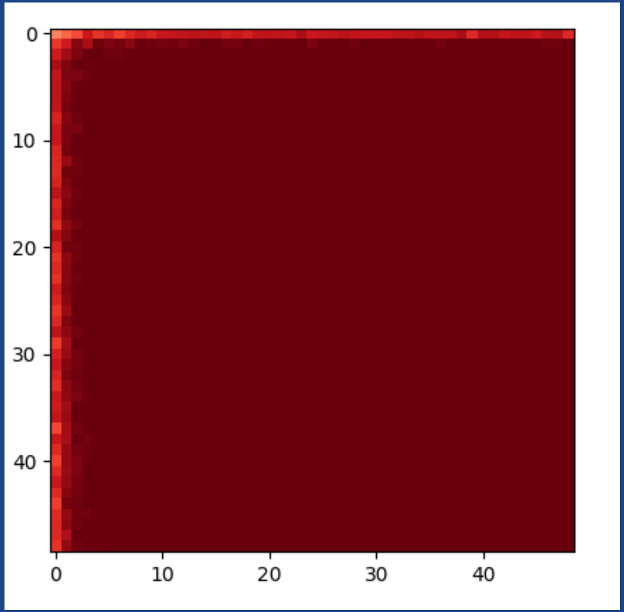
next truck

▲ PROBLEM

with packages sorting,
3 ways to sort: their quantities, values
and by their efficiency ▼

SORTING

efficiency = value / quantity

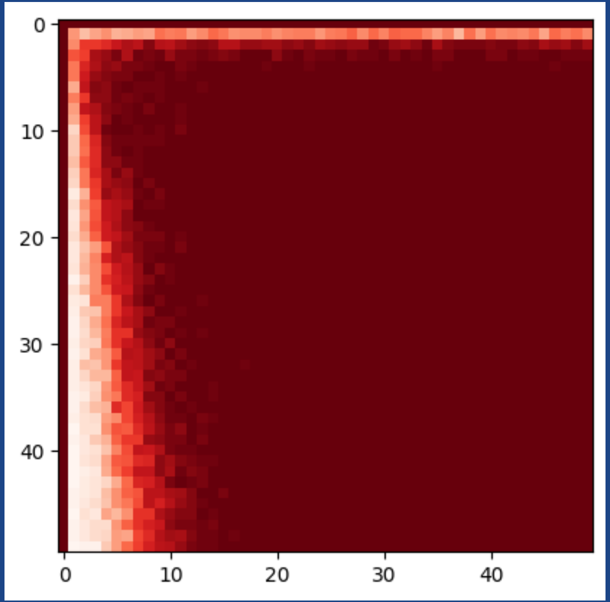


Most of the time, we don't have a feasible solution.
Otherwise, the algorithm fails.

Sort by efficiency

SORTING

prioritizing the values of goods
instead of efficiency



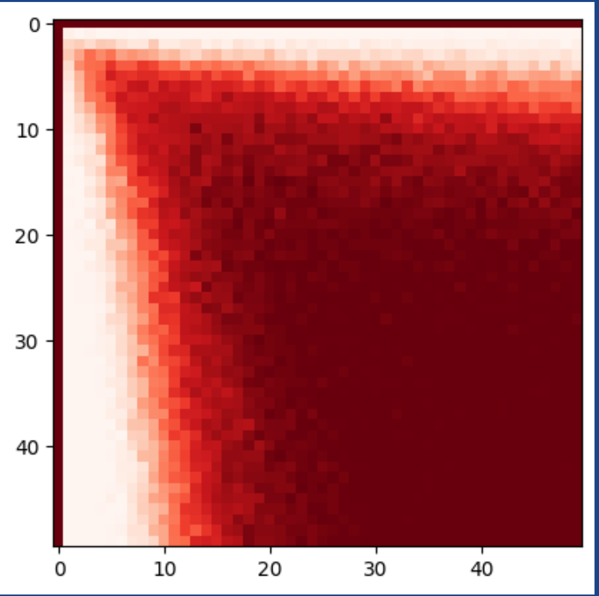
Test A: 99.62% **goods delivered**
92.89% **total values**

Test B: 90.54% **goods delivered**
99.75% **total values**

Sort by value

SORTING

we think that sorting by the quantity of goods
might be a better bet



our algorithm gives us
a feasible solution every time

Test A: 93.97% goods delivered
93.97% total values

Test B: 97.59% goods delivered
97.59% total values

Sort by quantity

SORTING

lower bound

First, we scale both values and quantities to a range of [min_importance, 1], here min_importance (> 0)

define importance of a package

$$\text{importance} = \text{value}^{\text{order}} \times \text{quantity}$$

order: scale down the importance of value comparing to quantity
we choose order = 2

Test A: 92.38% packages delivered
94.17% value rate

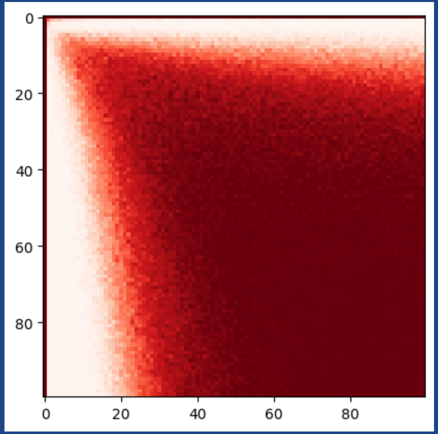
Test B: 97.98% packages delivered
98.66% value rate

Sort by a combination of value and quantity

SORTING

we sort those approaches by their rates of values delivered,
try to use the one with the highest rate
and check if it fails or not.
If it fails, the next best method will be used.

Method	Test A (N, K < 100)			Test B ($N = 10^5$, $K = 2x10^2$)		
Result	Value rate (%)	Deliver rate (%)	Failing rate (%)	Value rate (%)	Deliver rate (%)	Failing rate (%)
Efficiency	88.38	86.57	93.62	0	0	100
Value	92.89	90.54	74.55	99.75	99.62	0
Importance	94.17	92.38	61.94	98.66	97.98	0
Quantity	93.97	93.97	29.69	97.59	97.59	0
Combine	96.78	96.05	29.26	99.75	99.62	0



efficiency, value, importance and quantity.

Combining different ways of sorting

TIME COMPLEXITY

Python default sorting algorithm is Timsort (a combination of Merge sort and Insertion sort),
→ sorting time complexity: $O(N\log N)$
Adding goods to trucks requires 2 nested for-loops, each has a time complexity of $O(NK)$
→ Adding goods time complexity: $O(NK)$
→ Time complexity: $O(NK)$
In reality, greedy algorithm (if possible) instantly give us a solution in test A and takes less than a few seconds in test B.

PROPOSED ALGORITHMS

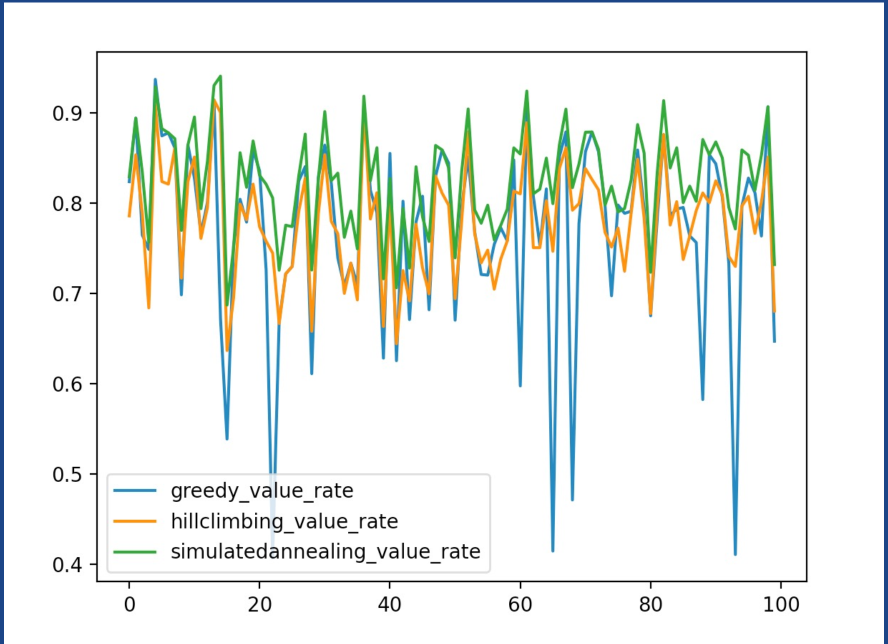
Hill climbing

Starting with sufficient point then find around itself the better solution until reach the time limit.

Hill Climbing is stuck in local maximum, plateau, or shoulder.

Simulated annealing

As recommended, temperature, cooling_rate, timelimit is 1000, 0.7, 60s respectively in defaut $p = e^{\frac{\Delta E}{k.T}}$



Quick Test: N=50, K=2

PROPOSED ALGORITHMS

The Strengths and Weaknesses of this method

Strengths: This method gives us quite fast and accurate result when the number of trucks and customers is small. The only job is to define the variables and its constraints, the rest is for the library.

Weaknesses: For large datasets, that means OR-Tools takes a long time to run. We can find many other libraries or other method to support.

PROPOSED ALGORITHMS

Import the libraries

from ortools.linear_solver import pywraplp

Creat the data

Declare the MIP solver

solver = pywraplp.Solver.CreatSolver('SCIP')

The default OR-Tools Mixed Integer Programming is SCIP - Solving Constraint Integer Programs

Creat the variables

x = [[solver.IntVar(0,1,'x[{i}][{j}]') for j in range(K)] for i in range(N)]

As in this problem, we define an array x[i][j] : size=NxK, whose value is 1 if customer[i] is served by truck[j]

PROPOSED ALGORITHMS

Define the constraints

The first constraint is each customer must be served by exactly 1 truck.

```
for i in range(N):
    solver.Add(sum(x[i]) <= 1)
```

The next constraint is the amount packed in each bin is greater than lower bound and less than upper bound.

```
for j in range(K):
    solver.Add(sum[x[i][j]*D[i] for i in range(N)] >= c1[j])
    solver.Add(sum[x[i][j]*D[i] for i in range(N)] <= c2[j])
```

Define the objective

```
obj = []
for j in range(K):
    for i in range(N):
            obj.append(x[i][j]*c[i])
solver.Maximize(sum(obj))
```

The goal is maximizing the total benefit, which is calculated by the total sum of each item value.

Call the solver and print the solution

PROPOSED ALGORITHMS

CP-SAT is another solver for linear integer programming which implements local search and meta-heuristics on top of a Constraint Programming solver.

Its performance is much higher than SCIP which was used above.

Moreover, we add an option to feed the solver with a initial solution from greedy algorithm, which improves the time required to find some first feasible solutions.
with N = 10000 and K = 50 with a variable time limit on Google Colab

Time limit	Without initial solution		With initial solution	
	Value rate (%)	Deliver rate (%)	Value rate (%)	Deliver rate (%)
30	TIMED OUT	TIMED OUT	98.83	98.46
90	91.09	90.70	98.86	98.82
120	96.82	95.59	99.06	98.98

04

EXPERIMENTS

N = 10, K = 2					N = 10, K = 5				
	Value rate (%)	Deliver rate (%)	Failing rate (%)	Exec time (s)		Value rate (%)	Deliver rate (%)	Failing rate (%)	Exec time (s)
Greedy	74.28	71.91	6.0	0.0002	Greedy	85.29	84.46	35.0	0.004
Hill Climbing	81.01	77.90	0.00	60.000	Hill Climbing	93.35	91.90	0.00	60.000
Simulated Annealing	81.01	78.00	0.00	60.000	Simulated Annealing	93.23	91.70	0.00	60.000
CP SAT	81.01	78.00	0.00	0.006	CP SAT	93.49	92.00	0.00	0.021
ILP (SCIP)	81.01	77.90	0.00	0.009	ILP (SCIP)	93.49	92.00	0.00	0.008
CP SAT + Greedy	81.01	77.90	0.00	0.005	CP SAT + Greedy	93.49	92.00	0.00	0.028

N = 50, K = 2					N = 50, K = 5				
	Value rate (%)	Deliver rate (%)	Failing rate (%)	Exec time (s)		Value rate (%)	Deliver rate (%)	Failing rate (%)	Exec time (s)
Greedy	76.99	71.24	0.00	0.0004	Greedy	85..79	83.12	0.00	0.005
Hill Climbing	77.89	74.80	0.00	60.000	Hill Climbing	91.43	88.36	0.00	60.000
Simulated Annealing	82.60	78.62	0.00	60.000	Simulated Annealing	92.12	89.06	0.00	60.000
CP SAT	82.92	79.32	0.00	1.450	CP SAT	93.44	91.22	0.00	22.445
ILP (SCIP)	82.92	79.26	0.00	0.039	ILP (SCIP)	93.47	91.22	0.00	1.383
CP SAT + Greedy	82.92	79.34	0.00	1.360	CP SAT + Greedy	93.47	91.22	0.00	22.396

N = 100, K = 2					N = 100, K = 5				
	Value rate (%)	Deliver rate (%)	Failing rate (%)	Exec time (s)		Value rate (%)	Deliver rate (%)	Failing rate (%)	Exec time (s)
Greedy	78.41	71.86	0.00	0.007	Greedy	89.69	85.74	0.00	0.007
Hill Climbing	79.07	76.15	0.00	60.000	Hill Climbing	92.50	90.11	0.00	60.000
Simulated Annealing	81.64	77.87	0.00	60.000	Simulated Annealing	92.37	90.14	0.00	60.000
CP SAT	83.11	79.25	0.00	0.171	CP SAT	93.20	93.20	0.00	11.606
ILP (SCIP)	83.11	79.27	0.00	0.098	ILP (SCIP)	93.20	93.20	0.00	1.940
CP SAT + Greedy	83.11	79.27	0.00	0.374	CP SAT + Greedy	93.20	93.20	0.00	11.973

N = 100, K = 10				
	Value rate (%)	Deliver rate (%)	Failing rate (%)	Exec time (s)
Greedy	92.14	90.68	1.00	0.009
CP SAT	97.20	96.03	0.00	2.183
ILP (SCIP)	97.20	96.03	0.00	6.492
CP SAT + Greedy	97.20	96.03	0.00	2.379

N = 100, K = 50				
	Value rate (%)	Deliver rate (%)	Failing rate (%)	Exec time (s)
Greedy	100.00	100.00	98.00	0.006
CP SAT	100.00	100.00	15.00	12.801
ILP (SCIP)	100.00	100.00	39.00	32.892
CP SAT + Greedy	100.00	100.00	13.00	11.895

N = 100, K = 100				
	Value rate (%)	Deliver rate (%)	Failing rate (%)	Exec time (s)
Greedy	0.00	0.00	100.00	0.003
CP SAT	100.00	100.00	61.00	25.848
ILP (SCIP)	100.00	100.00	74.00	15.450
CP SAT + Greedy	100.00	100.00	61.00	26.021

N = 100, K = 500				
	Value rate (%)	Deliver rate (%)	Failing rate (%)	Exec time (s)
Greedy	0.00	0.00	100.00	0.012
CP SAT	100.00	100.00	10.00	24.180
ILP (SCIP)	100.00	100.00	26.00	22.583
CP SAT + Greedy	100.00	100.00	10.00	23.803

N = 1000, K = 10				
	Value rate (%)	Deliver rate (%)	Failing rate (%)	Exec time (s)
Greedy	94.39	91.59	0.00	0.018
CP SAT	96.37	94.82	0.00	19.061
ILP (SCIP)	96.17	94.69	0.00	39.874
CP SAT + Greedy	96.36	94.82	0.00	27.999

N = 1000, K = 50				
	Value rate (%)	Deliver rate (%)	Failing rate (%)	Exec time (s)
Greedy	100.00	100.00	98.00	0.006
CP SAT	100.00	100.00	15.00	12.801
ILP (SCIP)	100.00	100.00	39.00	32.892
CP SAT + Greedy	100.00	100.00	15.00	13.021

N = 1000, K = 100				
	Value rate (%)	Deliver rate (%)	Failing rate (%)	Exec time (s)
Greedy	99.88	99.82	95.00	0.029
CP SAT	99.94	99.94	42.00	30.433
ILP (SCIP)	TIMED OUT	TIMED OUT	TIMED OUT	TIMED OUT
CP SAT + Greedy	99.93	99.93	41.00	29.586

N = 1000, K = 500				
	Value rate (%)	Deliver rate (%)	Failing rate (%)	Exec time (s)
Greedy	0.00	0.00	100	0.177
CP SAT	TIMED OUT	TIMED OUT	TIMED OUT	TIMED OUT
ILP (SCIP)	TIMED OUT	TIMED OUT	TIMED OUT	TIMED OUT
CP SAT + Greedy	TIMED OUT	TIMED OUT	TIMED OUT	TIMED OUT

N = 10000, K = 10				
	Value rate (%)	Deliver rate (%)	Failing rate (%)	Exec time (s)
Greedy	94.94	90.88	0.00	0.068
CP SAT	96.12	94.60	0.00	TIMED OUT
ILP (SCIP)	TIMED OUT	TIMED OUT	TIMED OUT	TIMED OUT
CP SAT + Greedy	96.22	94.65	0.00	TIMED OUT

N = 10000, K = 50				
	Value rate (%)	Deliver rate (%)	Failing rate (%)	Exec time (s)
Greedy	98.85	98.56	0.00	0.076
CP SAT	TIMED OUT	TIMED OUT	TIMED OUT	TIMED OUT
ILP (SCIP)	TIMED OUT	TIMED OUT	TIMED OUT	TIMED OUT
CP SAT + Greedy	98.85	98.56	0.00	TIMED OUT

N = 10000, K = 100				
	Value rate (%)	Deliver rate (%)	Failing rate (%)	Exec time (s)
Greedy	99.59	99.39	0.00	0.108
CP SAT	TIMED OUT	TIMED OUT	TIMED OUT	TIMED OUT
ILP (SCIP)	TIMED OUT	TIMED OUT	TIMED OUT	TIMED OUT
CP SAT + Greedy	99.59	99.39	0.00	TIMED OUT

N = 10000, K = 500				
	Value rate (%)	Deliver rate (%)	Failing rate (%)	Exec time (s)
Greedy	0.00	0.00	100	0.642
CP SAT	TIMED OUT	TIMED OUT	TIMED OUT	TIMED OUT
ILP (SCIP)	TIMED OUT	TIMED OUT	TIMED OUT	TIMED OUT
CP SAT + Greedy	TIMED OUT	TIMED OUT	TIMED OUT	TIMED OUT

04 SUMMARY

Conclusion

SHORTED TIME: greedy algorithms
-> its failure rate is high when N and K are close to each other.

Simulated Annealing > Hill Climbing
-> the performance gap gets narrower when K increases
-> these algorithms are implemented in Python. Their performance could be better if implemented in other programming languages.

Conclusion

when N and K are small enough: ILP method using OR-TOOLS (SCIP solver)

-> in the shortest time

when N and K are higher (N >100, K> 10): CP using OR-TOOLS (CP-SAT solver)

-> in the shorter time

when N and K are enormous, it might take a long time: CP with an initial solution if Greedy can provide us with a feasible solution.

Conclusion

methods using OR-Tools give better performance given the same time limit compared to others

when N and K are large, Greedy algorithm is the recommended method: gives us a fairly good solution in just a few seconds - even when it fails, we didn't waste too much time.

Local search methods even though might give optimal or good feasible solutions, take too much time because of their programming language.

THANKS!