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$$\begin{aligned}
 1. \text{incr } 2 &= (\lambda z. \lambda f. \lambda y. f(z f y)) (\lambda f. \lambda y. f(f y)) \\
 &\rightarrow_{\beta} (\lambda f. \lambda y. f((\lambda f. \lambda y. f(f y)) f y)) [z \rightarrow \lambda f. \lambda y. f(f y)] \\
 &\quad (\lambda z. \lambda f. \lambda y. f(z f y)) \\
 &\equiv_{\alpha} (\lambda f. \lambda y. f(\lambda t. \lambda y. t(t y)) f y) \\
 &\equiv_{\alpha} (\lambda f. \lambda y. f((\lambda t. \lambda j. t(t j)) f y)) \\
 &\rightarrow_{\beta} (\lambda f. \lambda y. f(\lambda j. f(f j) y)) [t \rightarrow f] (\lambda t. \lambda j. t(t j)) \\
 &\rightarrow_{\beta} (\lambda f. \lambda y. f(f(f y))) [j \rightarrow y] (\lambda j. f(f j))
 \end{aligned}$$

$$\text{incr } 2 = \lambda f. \lambda y. f(f(f y)) = 3$$

$$\begin{aligned}
 2a. &(((\lambda x. \lambda y. \lambda z. ((x y) z) (\lambda u. \lambda v. u)) A) B) \\
 &\rightarrow_{\beta} ((\lambda y. \lambda z. (((\lambda u. \lambda v. u) y) z) A) B) [x \rightarrow \lambda u. \lambda v. u] (\lambda x. \lambda y. \lambda z. ((x y) z)) \\
 &\rightarrow_{\beta} (\lambda z. (((\lambda u. \lambda v. u) A) z) B) [y \rightarrow A] (\lambda y. \lambda z. (((\lambda u. \lambda v. u) y) z)) \\
 &\rightarrow_{\beta} (((\lambda u. \lambda v. u) A) B) [z \rightarrow B] (\lambda z. (((\lambda u. \lambda v. u) A) z)) \\
 &\rightarrow_{\beta} (\lambda v. A) B [u \rightarrow A] (\lambda u. \lambda v. u) \\
 &\rightarrow A [v \rightarrow B] (\lambda v. A)
 \end{aligned}$$

Makes sense b.c if true, then A should be returned which is what $\lambda u. \lambda v. u$ represents.

$$\begin{aligned}
 b. &(((\lambda x. \lambda y. \lambda z. ((x y) z) (\lambda u. \lambda v. v)) A) B) \\
 &\rightarrow_{\beta} ((\lambda y. \lambda z. (((\lambda u. \lambda v. v) y) z) A) B) [x \rightarrow \lambda u. \lambda v. v] (\lambda x. \lambda y. \lambda z. ((x y) z)) \\
 &\rightarrow_{\beta} (\lambda z. (((\lambda u. \lambda v. v) A) z) B) [y \rightarrow A] (\lambda y. \lambda z. (((\lambda u. \lambda v. v) y) z)) \\
 &\rightarrow_{\beta} (((\lambda u. \lambda v. v) A) B) [z \rightarrow B] (\lambda z. (((\lambda u. \lambda v. v) A) z)) \\
 &\rightarrow_{\beta} (\lambda v. v) B [u \rightarrow A] (\lambda u. \lambda v. v) \\
 &\rightarrow B [v \rightarrow B] (\lambda v. v)
 \end{aligned}$$

Makes sense because if false, then B should be returned and $\lambda u. \lambda v. v$ represents false.