

I read the comment from last week, I apologize for this and last weeks form. I lack the time this week to make it complete. I will submit the correct/expected form starting next Homework.

## Exercise 1

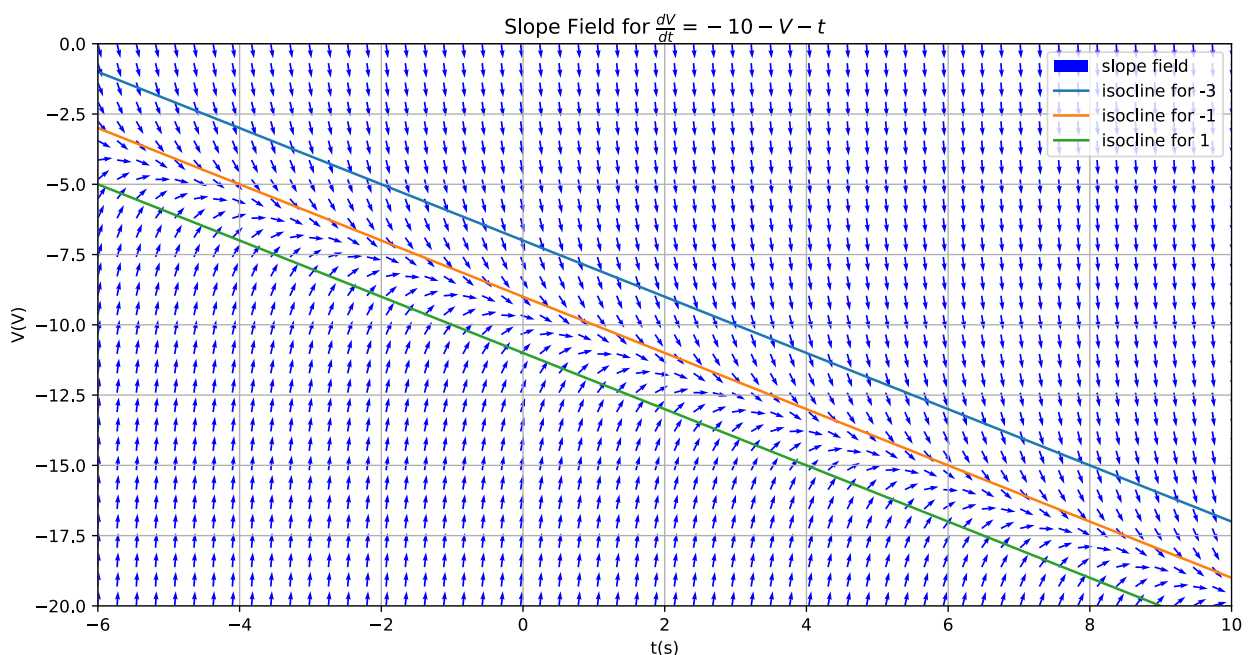
In this exercise we plot slope fields and isoclines.

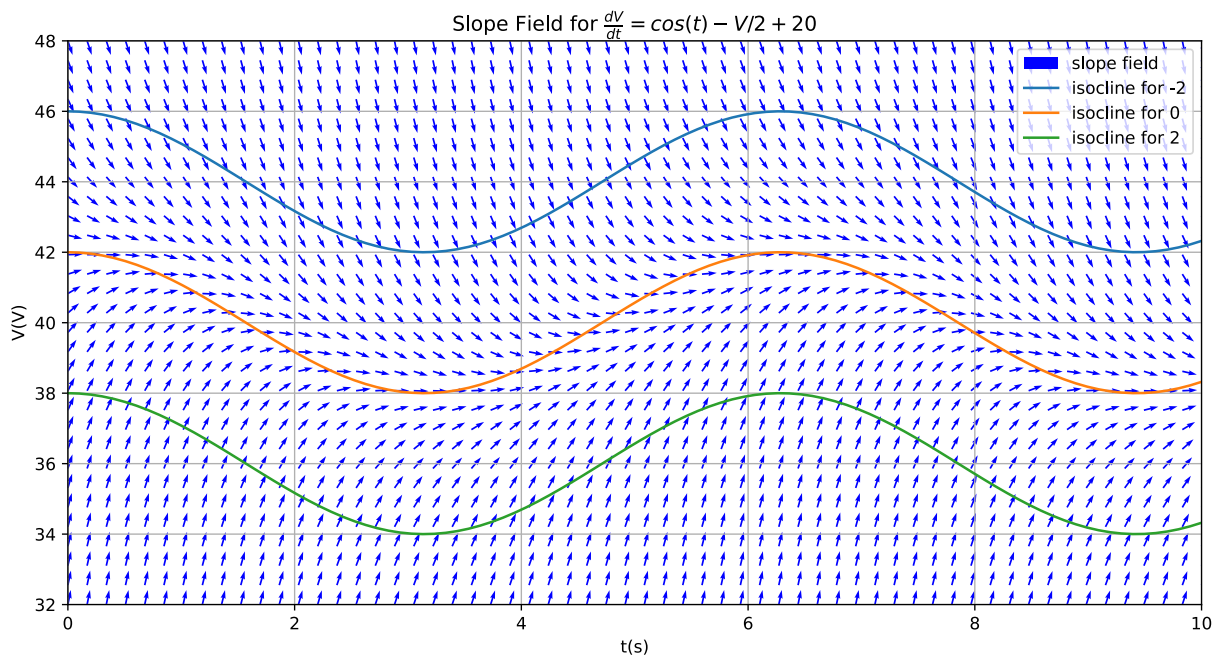
A slope field is a 2d visualation of the solution to the first order differential equation.

Isoclines are an additional outline of the slope field, and in the case here could be integrated to obtain a solution to the differential equation.

Isoclines are made by replacing the  $\frac{dV}{dt}$  with a variable and reordering the equation to  $V$ , meaning:  $\frac{dV}{dt} = f(V, t) \Rightarrow m = f(V, t)$ . The isoclines I used are copied from the solution but they make the most sense as they are the most fitting and close to the plotted field.

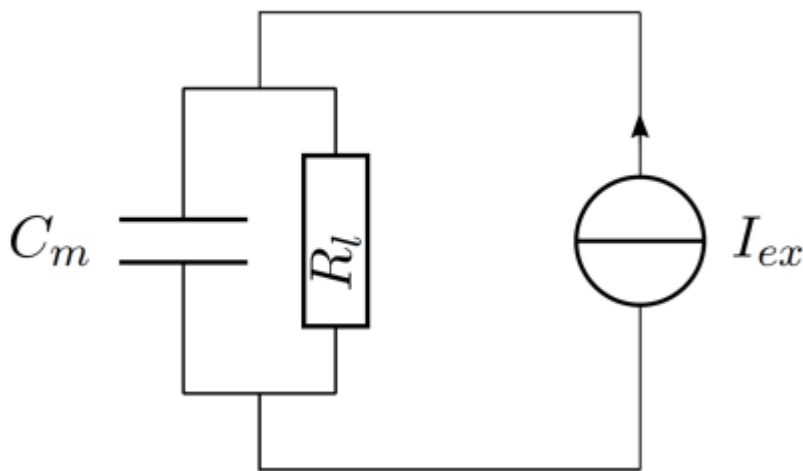
The scale is linear of course as this is a linear differential equation and grid from:to ranges are copied from the solution and also because they give the best visual insight.





## Exercise 2

Deriving the voltage equation of the Circuit modeling a simplified version of a cell:



For the external Current  $I_{ex}$  we have assumption  $I_{ex} = I_{max} \cdot \sin(t)$

What we know about the circuit is the capacitor  $C_m$  and resistor  $R_l$  are in *parallel*.

With **Kirchhoff's Voltage Law** we know that here:

$$V_R = V_C = V$$

Via **Kirchhoff's Current Law** we know that in a node the in and output are equal and the currents can be summed up. Thus we can look at the current for the capacitor and the resistor each and sum them up.

**Resistor Current:**  $I_{R_l} = \frac{V}{R}$

For the **Capacitor Current** we use the the definition

$$Q = CV$$

Further we know that  $V = RI$  and that  $I = \frac{dQ}{dt}$

We can plug in the capacitance law into the current equation

$$I_C = I = \frac{dQ}{dt} = \frac{d(CV)}{dt}$$

We assume  $C$  stays constant over time

$$I_c = C \frac{dV}{dt}$$

Now combining these with **Kirchhoff's Current Law** and reordering the equation we obtain the differential Equation in the form  $\frac{dV}{dt} = f(V, t)$  for the circuit:

$$I_{ex} = I_{R_l} + I_C$$

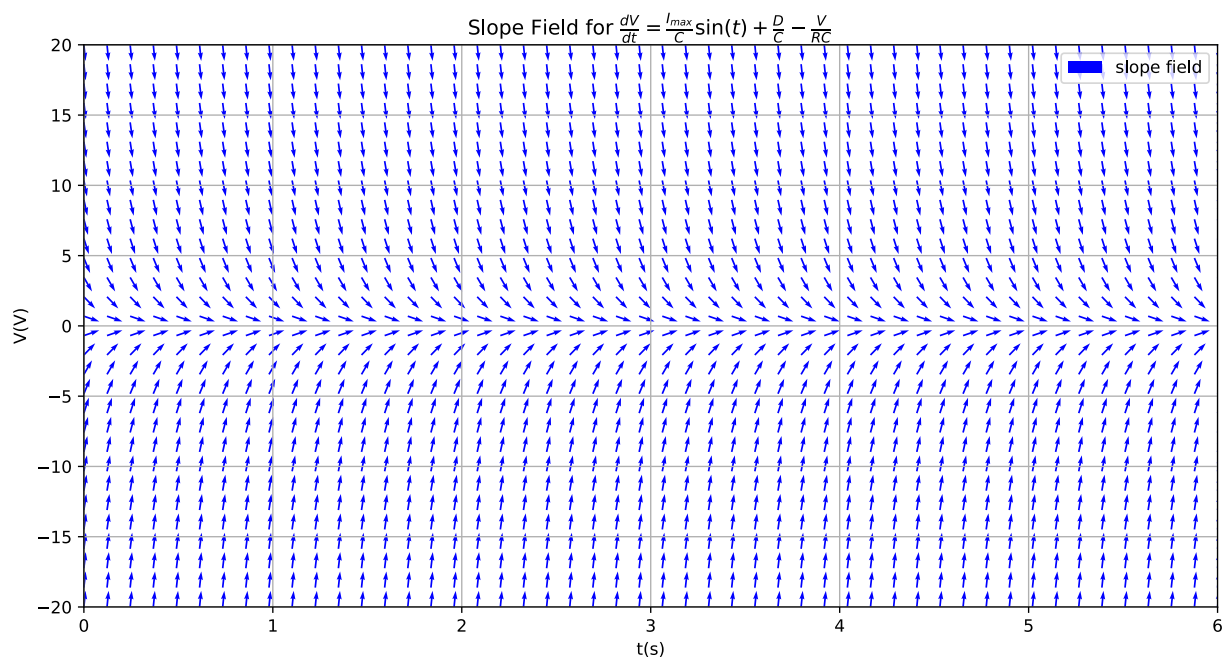
$$I_{max} \sin t = \frac{V}{R} + C \frac{dV}{dt}$$

$$C \frac{dV}{dt} = I_{max} \sin(t) - \frac{V}{R}$$

$$\frac{dV}{dt} = \frac{I_{max}}{C} \sin(t) - \frac{V}{RC}$$

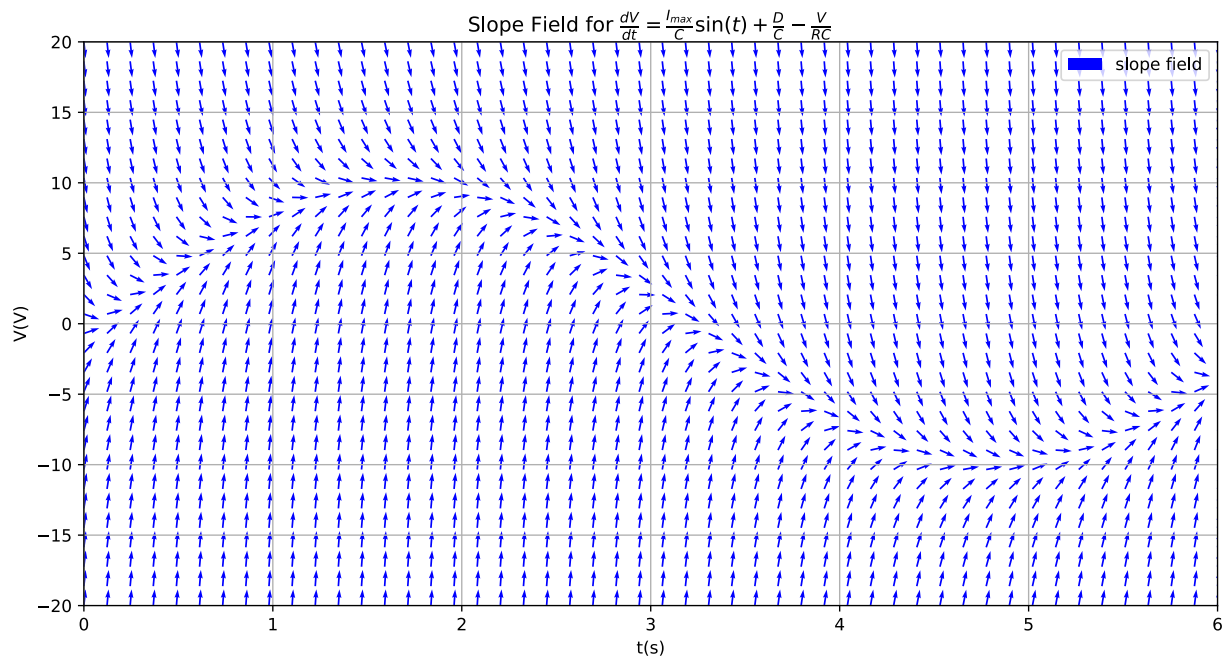
## 2.1 a)

Plotting the differential equation derived prior with the values:



$$R = 1\Omega; C = 2F; I_{max} = 0A; D = 0A$$

We see a constant voltage at 0, which makes sense from a biological perspective as we have no external current stimulus

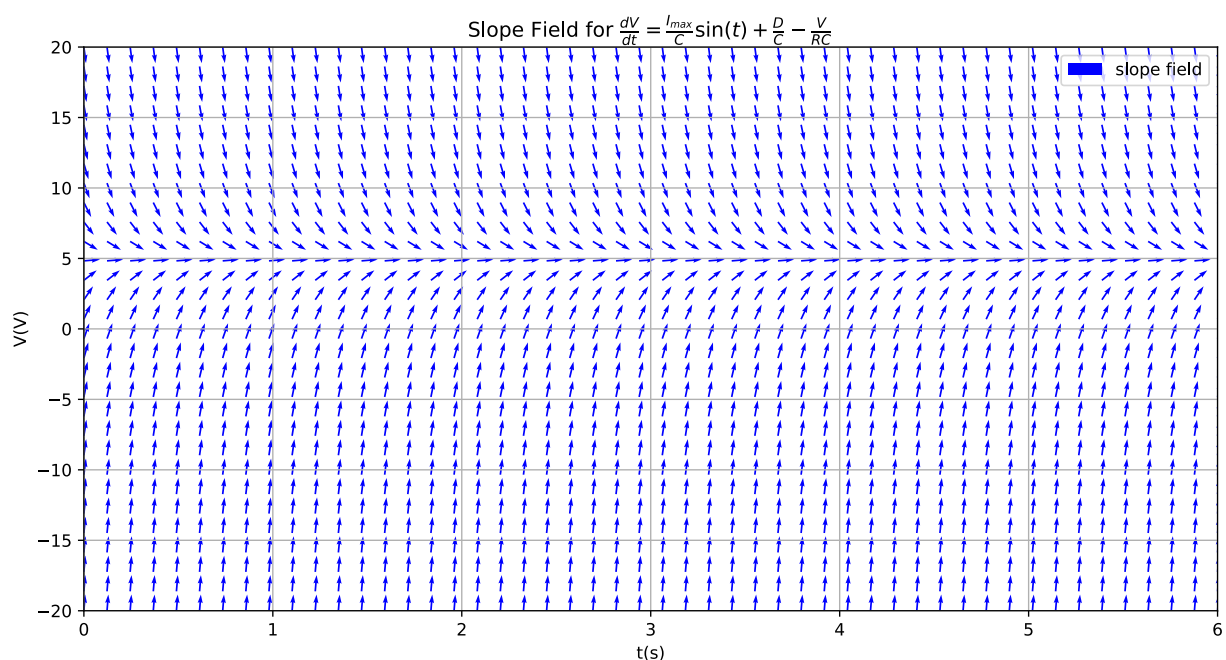


$$R = 1\Omega; C = 2F; I_{max} = 10A; D = 0A$$

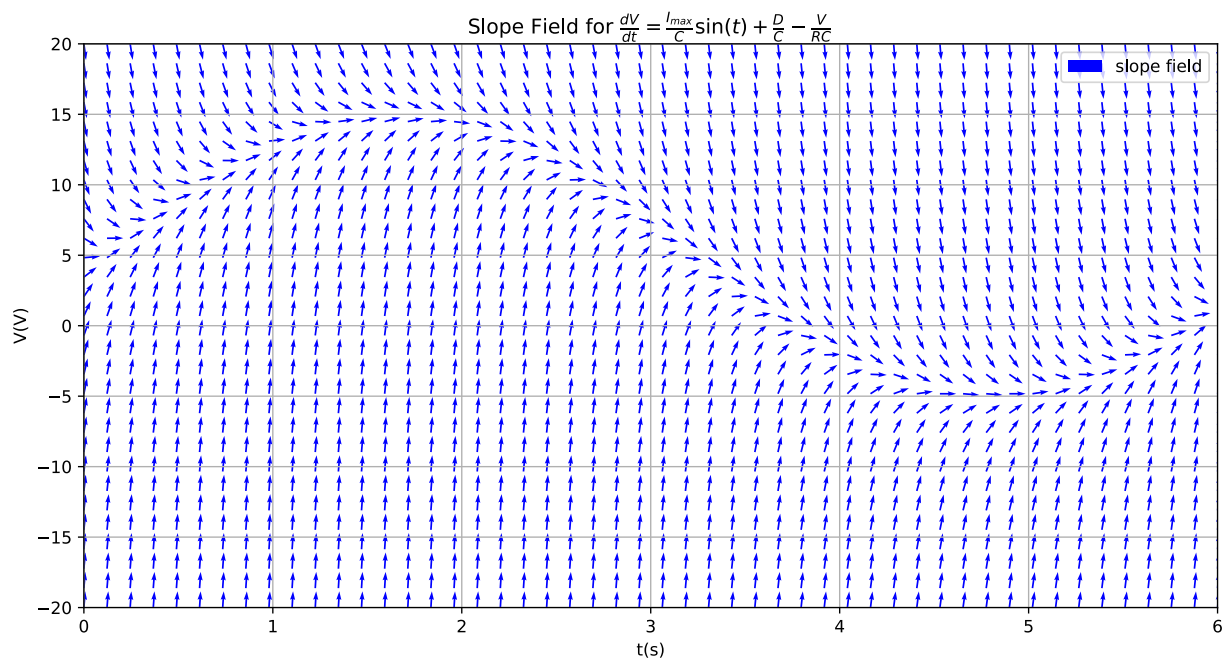
Here we see an external stimulus causing a potential.

## 2.1 b)

Here we see the same Voltage as for a) but offsetted by  $y=5$ . This can be interpreted as a constant current going through the cell.



$$R = 1\Omega; C = 2F; I_{max} = 0A; D = 5A$$



$$R = 1\Omega; C = 2F; I_{max} = 10A; D = 5A$$