

1 Problem

"2520 is the smallest number that can be divided by each of the numbers from 1 to 10 without any remainder. What is the smallest positive number that is evenly divisible by all of the numbers from 1 to 20?"

1.1 Restatement of Problem

Find the least common multiple of the numbers 1-20.

2 Solution

No programming is needed to solve this problem. I was trying to draft up an algorithm and accidentally stumbled across the answer.

1. Perform the prime factorization on each number.
2. For each distinct prime, find the factor from the list created in the previous step with the largest power.
3. Multiply these factors, this is the least common multiple.

First, perform the prime factorization on each number.

$$20 = 2^2 * 5$$

$$19 = 19$$

$$18 = 2 * 3^2$$

$$17 = 17$$

$$16 = 2^4$$

$$15 = 3 * 5$$

$$14 = 2 * 7$$

$$13 = 13$$

$$12 = 2^2 * 3$$

$$11 = 11$$

$$10 = 2 * 5$$

$$9 = 3^2$$

$$8 = 2^3$$

$$7 = 7$$

$$6 = 2 * 3$$

$$5 = 5$$

$$4 = 2^2$$

$$3 = 3$$

$$2 = 2$$

Second, for each distinct prime number, find the factor with the largest power.

$$20 = 2^2 * \mathbf{5}$$

$$19 = \mathbf{19}$$

$$18 = 2 * \mathbf{3^2}$$

$$17 = \mathbf{17}$$

$$16 = \mathbf{2^4}$$

$$15 = 3 * 5$$

$$14 = 2 * \mathbf{7}$$

$$13 = \mathbf{13}$$

$$12 = 2^2 * 3$$

$$11 = \mathbf{11}$$

$$10 = 2 * 5$$

$$9 = 3^2$$

$$8 = 2^3$$

$$7 = 7$$

$$6 = 2 * 3$$

$$5 = 5$$

$$4 = 2^2$$

$$3 = 3$$

$$2 = 2$$

Third, multiply these numbers.

$$5 * 19 * 3^2 * 17 * 2^4 * 7 * 13 * 11 = 2^4 * 3^2 * 5 * 7 * 11 * 13 * 17 * 19 = 232792560$$