My Notes on Category Theory Lecture Notes by Daniele Turi

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1 Universal property

1.1 Natural numbers in set theory and category theory

A1 is the regular Peano's definition of natural numbers. There is nothing new.

A2 is more interesting to investigate. It defines natural number set (N) to be the "initial" object of the category of all natural-number-like sets (X). The essential part of a univeral property is the unique arrow, or *factorization*. In our case, it is the $f : X \to N$. Here is an example of an X in **A2**.

Example 1.1.1. $(-1) \in X \xrightarrow{g} X$ *where* $g := a \longmapsto a - 1$. *With this case* $f := a \longmapsto -(a + 1)$.

This is straightfoward, just a demonstration of what is it about.

> The *Recursion Theorem* [1] guarantees recursively defined functions exists. Given a set X, an element of $e \in X$ and a function $g : X \to X$, the theorem states there is a unique function $f : N \to X$, such that
>
> $$f(0) = e \tag{1}$$
> $$f(n + 1) = g(f(n)) \tag{2}$$

This is essentially defines a factorization from N to X.

So then the proof of **A1** and **A2** are isomorphic.

[1] `https://en.wikipedia.org/wiki/Recursion#The_recursion_theorem`