$\Rightarrow w_i = \{ 0, \ -\lambda < z_i < \lambda$

We have:

$$\mathcal{E}(w) = \frac{1}{2}\sum_{i}^{N} (d_i - w^T x_i)^2 + \frac{\lambda}{2} ||w||^2$$

Proximal gradient descent:

$$z^{(k)} = w^{(k)} - \frac{1}{2} x_i^T (x_i w^{(k)} - d)$$ (least squares gradient descent)

$$\Rightarrow w(k+1) = \arg\min_{w} |z^{(k)} - w_i|^2 + \frac{1}{2}\lambda |w| \quad (1)$$

take derivative (1):

• $w_i > 0$: $-2(z(k) - w_i) + \frac{\lambda}{2} = 0$

$\Leftrightarrow \quad w_i = z_i(k) - \frac{\lambda}{4}$ only when $z_i(k) > \frac{\lambda}{4}$ ($w_i > 0$)

$w_i = 0$ when $z_i < \frac{\lambda}{4}$

• $w_i < 0$: $\frac{d}{dw_i}\{(z(k) - w_i)^2 + \frac{\lambda}{2}(-W)\} = 0$ (for it to be min)

$\Leftrightarrow -2(z(k) - w_i) - \frac{\lambda}{2} = 0$

$\Leftrightarrow w_i = z_i(k) + \frac{\lambda}{4}$, when $z_i(k) < -\frac{\lambda}{4}$

$w_i = 0$ when $z_i > -\frac{\lambda}{4}$

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$$\Rightarrow w_i = \begin{cases} 0, & -\frac{\lambda}{4} < z_i < \frac{\lambda}{4} \\ z_i - \frac{\lambda}{4}, & z_i > \frac{\lambda}{4} \\ z_i + \frac{\lambda}{4}, & z_i < -\frac{\lambda}{4} \end{cases}$$