Graph Visual Rhythm in Vehicular Networks using

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Abstract. *Vehicular networks comprise a set of different types of vehicles on a road. These networks do not have trivial behavior, increasing the difficulty to analyze and make decision. One traditional method to analyze vehicular networks is to model it as a graph, wherein dynamic scenarios we have different graphs representations. This work presents the Graph Visual Rhythm (GVR) for vehicular networks, providing a compact and context-enriched representation for network analysis. In this work, we use different complex network measures, such as closeness centrality, degree centrality, betweenness centrality, harmonic centrality, local efficiency, and PageRank centrality, all applied in the TAPAS Cologne scenario. We can use our results in different studies as a mechanism to analyze, make, and predict changes, such as traffic routing.*

Resumo. *Redes veiculares são formadas por conjuntos de diferentes tipos de veículos em uma via. Estes veículos não possuem um comportamento trivial resultando em uma maior dificuldade na análise para tomada de decisões. Um método tradicional para analizar redes veiculares é modelá-las como um grafo, sendo que em cenários dinâmicos existem diferentes representações. Este artigo trás o conceito de Grafo de Ritmo Visual (Graph Visual Rhythm – GVR) para redes veiculares, oferecendo uma representação compacta e contextualizada, com objetivo de auxiliar na análise destas redes. Estes grafos compactam a representação de medidas que contribuem para análise de redes. Neste artigo, as seguintes métricas foram utilizadas: centralidade de proximidade, centralidade do grau, centralidade de intermediação, centralidade harmônica, eficiência local, e centralidade de PageRank, aplicadas ao conjunto de dados TAPAS Cologne. Os resultados obtidos neste artigo podem ser utilizados como mecanismo para analizar e realizar predições de conexões, aplicadas em processos de transmissão de dados entre veículos.*

1. Introduction

Vehicular networks comprise vehicles traveling cities and highways [Alvarenga et al. 2014], and are associated with tremendous network size. Real-world datasets for dense cities with a considerable scale of traffic show that vehicular networks belong to the complex network model. Diverse hierarchical structures and node types give rise to more complex interactions with complex time-space relationship.

In the literature, vehicular networks are modeled in two forms: aggregate and temporal. In the aggregation network model, only one network is selected to represent the network over time and can ignore the time-space relationship between nodes, resulting in problems with shortest-path calculation, for example [Alvarenga et al. 2014]. The temporal model deals with the time as necessary information, given by the dynamism of these networks. One alternative way to view vehicular networks is to abstract it using graph theory, making it possible to analyze and calculate complex measures.

From the derived graphs, we can generate social network measures in order to extract patterns in the vehicular networks. The analysis of the *social* aspects of a network is the study and exploitation of the structural information present in the network, such as existence and strength of communities, node centralities, network robustness to node removal, topology evolution over time, among others.

The social network measures in vehicular networks can explore the following examples: predict where roads should be built or expanded in the future, design bridges and pavements to withstand predicted traffic loads, analyze the air quality in urban areas, and alert drivers to congestion and accidents.

The aim of this proposal is to generate graph visual rhythm generated by the extraction of complex network measures from vehicular traffic traces and generate graphs that represent vehicular traffic networks over time. After that, the goal comprises calculating a set of social network measures on each of these generated graphs: closeness centrality, degree centrality, harmonic centrality, local efficiency, betweenness centrality, and PageRank centrality. After all measures have been computed, the final goal comprises the graph visual rhythm generation with graphs being generated over time and grouped, forming a matrix where each column is a period and each row is a relative frequency histogram.

The closeness centrality of a node (vehicle) measures how close a node is to other nodes in a graph (network). The degree centrality of a node measures the number of edges connecting a node, meaning that this measure can be associated with a greater probability of short and direct information being sent between two vehicles. PageRank measures the prestige of a node based on the prestige of adjacent nodes pointing to it. We can apply the PageRank measure to verify if the most important vehicles are connected to each other, with the assumption that more important vehicles are likely to receive more connections from other vehicles.

The remainder of the work is organized as follows. Section 2 describes related works that try to analyze the traffic flow of vehicular networks. Section 3 describes the complex network measures used in this work. Section 4 describes the Graph Visual Rhythms (GVRs) used to represent the measures' behavior in vehicular networks. Section 5 describes the framework proposed in this work. Section 6 describes the analysis extracted by our proposed method. Finally, Section 7 concludes this work.

2. Related Work

In [Cunha et al. 2014], the authors analyzed two vehicular mobility dataset traces: (1) Zurich's Trace and (2) San Francisco's Trace. The authors considered different aspects of vehicular mobility to analyze different metrics at macroscopic and microscopic levels, such as distance, diameter, density, edge persistence, node degree, cluster coefficient, and closeness centrality. Some mobility parameters are being considered with the assumption to analyze the encounter of vehicles. From the analysis results, it is shown that vehicular networks exhibit social characteristics that can be exploited to enhance network performance in vehicular networks.

The social network measures can identify some features of a complex network, helping to make decisions such as choosing a route to deliver information given a set of constraints. The social network measures have been studied in vehicular networks [Alvarenga et al. 2014, Qin et al. 2016, Silva et al. 2017, Akabane et al. 2018] to analyze and get patterns of networks.

In [Alvarenga et al. 2014], the authors considered that in vehicular networks the vehicles *socialize* and share common interests, classifying scenarios according to the interactions performed, identifying common interests and related routines. The authors used temporal graphs to evaluate the behavior of vehicular networks, where the nodes are the vehicles and a connection is established if such pair of nodes respect a certain distance. They considered the degree measure, the betweenness centrality, the small-world phenomenon, and the similar contacts.

In [Qin et al. 2016], the authors studied the mobile advertising problem in vehicular networks. They examined the degree, closeness and coreness centrality measures in contact graphs derived from a set of traces. Based on their results, the authors concluded that the vehicles show clear social behavior within the network and that the vehicular social behavior is highly dynamic and has strong temporal correlations.

In [Silva et al. 2017], the authors evaluated the behavior of static and temporal centrality measures applied in different vehicular environments. The betweenness centrality, closeness centrality and degree centrality measures were selected to evaluate the vehicular networks. They considered the communication to be Vehicular-to-Vehicular (V2V), where the connection is established given by a distance limit. Using the concept of aggregation of the temporal graphs, they evaluate the behavior of the complex network measures during the simulation in different scenarios. Using only one graph, during the simulation, new edges are added respecting the distance constraint.

In [Akabane et al. 2018], the authors introduced the egocentric betweenness measure as a mechanism for the vehicular selection process. By definition, an ego-network is a local subgraph comprising a single node (ego) in addition to nodes that are connected to it (alters) and all the interconnected links among alters [Akabane et al. 2018]. The authors used this measure to select the most appropriate vehicle to carry out the tasks of information aggregation and knowledge generation.

The visual rhythm is a widely used approach for sampling videos or images. Figure 1 shows a computation of visual rhythm based on pixel values of the horizontal line found in the center of a frame. Based on this, the authors in [Rodrigues et al. 2019] introduced a generic graph visual rhythm method, dividing internally a measure (PageRank, degree centrality, betweenness centrality, eccentricity centrality, local efficiency, global efficiency, and vulnerability), into categories. The measured values were associated with distinct levels of saturation. The result of their proposal was a two-dimensional image, where the rows represent the nodes and the columns represent the time intervals. Their representation was applied in a soccer player scenario and a social network, both non-dynamic cases.

Based on the literature, we propose to use graph visual rhythm that was introduced

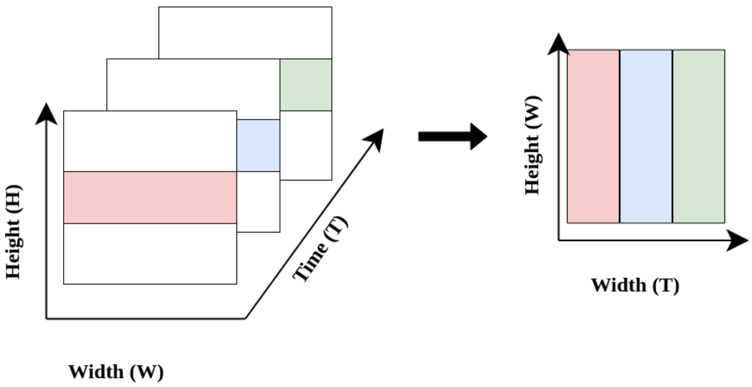

Figure 1. Example of visual rhythm by extracting the pixel values defined by the central horizontal line [Rodrigues et al. 2019].

in [Rodrigues et al. 2019] as a framework for vehicular network analysis, in order to help the research community to develop new algorithms for vehicular networks.

3. Complex Network Measures

Simple statistics such as the number of vehicles in a certain period cannot be used to reliably measure impact on routing traffic or to predict certain scenarios in vehicular networks. So, it is important to generate relevant information that contributes to network analysis.

The term *complex networks* describes a class of large networks which exhibit the following properties: a) they are sparse, that is, the number of edges is proportional to the number of nodes; b) they exhibit the small world phenomenon: almost all pairs of nodes that are in the same component are within a short distance from each other; c) clustering is present: two nodes of the network that have a common neighbor are somewhat more likely to be connected with each other; d) their degree distribution is scale-free. This means that its tail follows a power law. There has been extensive experimental evidence, suggesting that many networks that emerge in applications have a degree distribution whose tail follows a power law with an exponent between 2 and 3.

The present work used complex measurements to extract patterns from temporal graphs to represent individuals and collective behavior. In vehicular network analysis, these measures can be associated with traffic routing. We considered the following measures: degree centrality, closeness centrality, betweenness centrality, harmonic centrality, local efficiency, and PageRank centrality.

Given a graph representing the vehicular network $G = (V, E)$, where V is the set of nodes (vehicles) and E the set of undirected edges (connections). A centrality measure is a function $C : G(n) \to \mathbb{R}$, where $C_{\upsilon}(G)$ is the centrality of node υ in the social network G, and n is the number of nodes in G. A graph G can be represented by its adjacency matrix $G \in \mathbb{R}^{n\times n}$, where $G_{\upsilon,\nu} \neq 0$ represents the existence of an edge between nodes υ and ν and $G_{\upsilon,\nu} = 0$ indicates the absence of an edge between the two nodes.

3.1. Degree Centrality

The degree centrality is an important component of any attempt to determine the most important node on a vehicular network, based on the number of adjacent nodes. The degree centrality measures the number of incoming and outgoing relationships from a node, and can help us find popular nodes in temporal graphs.

The *degree centrality* [Freeman 1978] of a node υ in a undirected network G, denoted by $d_\upsilon(G) = \|\{\nu : G_{\upsilon,\nu} \neq 0\}\|$ is the number of edges (a) connecting this node to others (ν) in G. The degree centrality measure is defined by:

$$C_D(\upsilon) = \sum_{\nu} a_{\upsilon,\nu}, \tag{1}$$

where $C_D(\upsilon)$ represents the degree centrality of the node υ, $a(\upsilon, \nu)$ is an edge between υ and ν. Figure 2 shows a graph to represent the degree centrality measure, where all weights of egdes are 1. Node 2 and 13 have the highest degree centrality, because of they have more adjacent nodes connected to them.

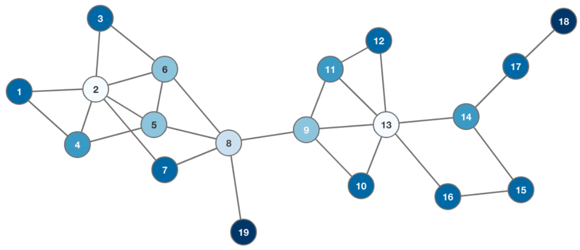

Figure 2. Degree centrality behavior.

3.2. Closeness Centrality

Closeness centrality is used to research organizational networks, where individuals with high closeness centrality are in a favorable position to control and acquire vital information and resources within the organization.

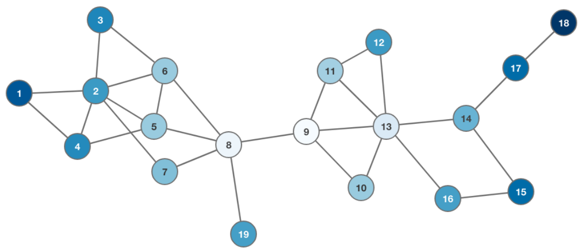

Figure 3. Closeness centrality behavior.

The *closeness centrality* [Freeman 1978] expresses how close a node is to all other nodes in a network. Nodes with a high closeness score have the shortest distances to all other nodes. The closeness centrality is defined as the inverse of the sum of the shortest distances between each node and every other node in the network. The closeness centrality measure is defined by:

$$C_C(\upsilon) = \frac{n-1}{\sum\limits_{\upsilon \neq \nu} d(\upsilon, \nu)}, \tag{2}$$

where $C_C(\upsilon)$ represents the closeness centrality of the node υ, and $d(\upsilon, \nu)$ is the shortest path distance between υ and ν. Figure 3 shows a graph representing the closeness centrality measure. The color intensity represents the measure value of a node, where a lighter color means a higher centrality, which in this example, node 9 (nine) has the highest closeness centrality when compared to the others.

3.3. Harmonic Centrality

In [Marchiori and Latora 2000], the authors introduced the harmonic centrality measure. The harmonic centrality measure is a variant of closeness centrality, that was invented to solve the problem the original formula had when dealing with unconnected graphs. As with many of the centrality algorithms, it originates from the field of social network analysis.

Harmonic centrality of a node υ is the sum of the reciprocal of the shortest path distances from all other nodes to υ.

$$C_H(\upsilon) = \sum_{\upsilon \neq \nu} \frac{1}{d(\upsilon, \nu)}, \tag{3}$$

where $d(\upsilon, \nu)$ is the shortest path distance between ν and υ. This measure reverses the sum and reciprocal operations in the definition of closeness centrality. The shortest path algorithm used to obtain all measures is the Dijkstra's algorithm.

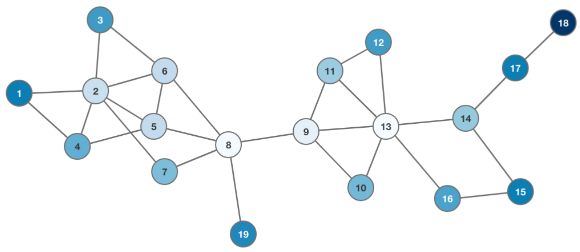

Figure 4. Harmonic centrality behavior.

3.4. Betweenness Centrality

The Betweenness centrality was introduced in [Freeman 1978], and can be used to research the network flow in, for example, a package delivery process or telecommunications network. These networks are characterized by traffic that has a known target and takes the shortest path possible. Betweenness centrality makes the assumption that all communication between nodes happens along the shortest path and with the same frequency, which is not the case in real life. Therefore, it does not give us a perfect view of the most influential nodes in a graph, but a rather good representation.

The *betweenness centrality* C_B of a node υ is the sum of the fraction of all-pairs shortest paths that pass through υ. The betweenness centrality measure is defined by:

$$C_B(\upsilon) = \sum_{s,t \in V} \frac{\sigma(s, t|\upsilon)}{\sigma(s, t)}, \tag{4}$$

when V is the set of nodes, $\sigma(s, t)$ is the number of shortest (s, t)-paths, and $\sigma(s, t|\upsilon)$ is the number of those paths passing through some node υ other than s, t. If $s = t$,

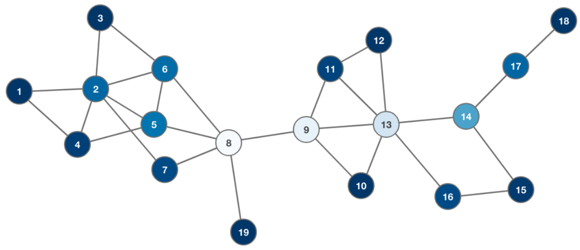

Figure 5. Betweenness centrality behavior.

$\sigma(s, t) = 1$, and if $s \in s, t, \sigma(s, t|\upsilon) = 0$. Figure 5 shows a graph to exemplify the betweenness centrality measure, where node 8 (eight) has the highest value. For the betweenness centrality, the importance of the concept of node centrality is the potential of a node for control of information flow in the network. Positions are viewed as structurally central to the degree to which they stand between others and can therefore facilitate, impede, or bias the transmission of messages.

3.5. PageRank Centrality

The PageRank centrality measure was introduced in [Page et al. 1999] to be used for Web pages, evaluating the prestige of them. PageRank counts the number and quality of links to a node, and estimates how important the node is. The underlying assumption is that pages of importance are more likely to receive a higher volume of links from other pages.

PageRank is an algorithm that measures the transitive influence or connectivity of nodes. The PageRank (P) measures the prestige of a node based on the prestige of its adjacent nodes. The PageRank value of a node υ is defined as:

$$P(\upsilon) = \frac{q}{n} + (1 - q) \times \sum_{\nu \in B_\upsilon} \frac{P(\nu)}{K(\nu)} \tag{5}$$

where n is the number of nodes, $K(\nu)$ is the outdegree of node ν, $\nu \in B_\upsilon$ and q is the damping factor, in our work we assumed 0.85 as the q value. Figure 6 shows a graph to exemplify the PageRank measure, where node 13 (thirteen) has the highest PageRank centrality.

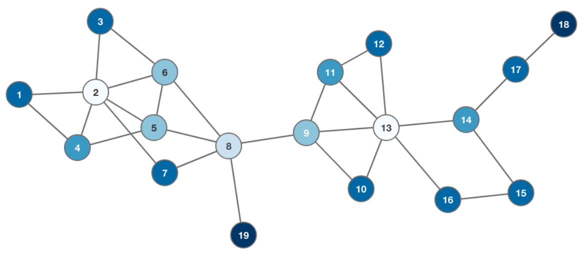

Figure 6. PageRank centrality behavior.

In vehicular networks, the PageRank centrality can predict traffic flow and human movement in specific areas. For example, the PageRank algorithm is run over a graph which contains intersections connected by roads, where the PageRank score reflects the tendency of a vehicle to go on each street.

3.6. Local Efficiency

The efficiency of a pair of nodes in a graph is the multiplicative inverse of the shortest path distance between these nodes. The local efficiency of a node in the graph is the average global efficiency of the subgraph induced by the neighbors of the node, where the global efficiency of a graph G is defined as:

$$E(G) = \frac{1}{(n(n-1))} \sum_{\upsilon \neq \nu} \frac{1}{d_{\upsilon,\nu}}, \tag{6}$$

where n is the number of nodes in G, $d_{\upsilon,\nu}$ is the shortest path distance between nodes υ and ν.

Based on the global efficiency extraction, the local efficiency is defined as [Latora and Marchiori 2001]:

$$E_L(G) = \frac{1}{n} \sum_{\upsilon \in G} E(G_\upsilon), \tag{7}$$

where G_υ is the subgraph of neighbors of node υ and n is the nodes count in G_υ. In vehicular networks this measure can be associated with vehicles' relevance for the local traffic flow. For example, the graph in Figure 6 has the local efficiency equal to 0.44, where the value is between 0 to 1.

4. Graph Visual Rhythm

Different visual properties (e.g. color and opacity) can be used to highlight graph changes. In [Rodrigues et al. 2019], the authors mapped intensity centrality scores to colors, based on their interests. The aim is to take advantage of a color pattern to associate colors with specific ranges of values. The solution introduced by authors in [Rodrigues et al. 2019], comprises a generic solution using Graph Visual Rhythm (GVR) encodes temporal network changes. The GVR can be generated using any centrality measure described in Section 3. Using the GVR representation, the authors encode the temporal graph dynamics, fostering the identification of change patterns of interest [Rodrigues et al. 2019].

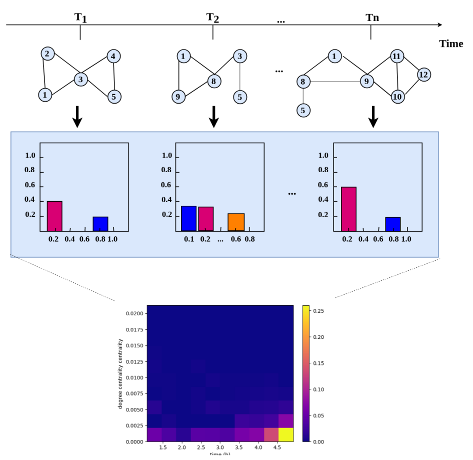

Figure 7. Flowchart showing how a GVR is computed, considering the degree centrality measure computed for each node in different temporal graphs for different timestamps.

The advantage of using GVRs is that we can extract the behavior of vehicular networks with high availability and low-cost, considering specific measures. Let $G = G_1, G_2, \ldots, G_T$ be a set of temporal graphs, where $G_t = (V_t, E_t)$ is the weighted graph at timestamp $t \in [1, T]$ composed by a set of nodes, V_t, and a set of edges, E_t. In our scenario, each node is modeled as a vehicle, and each edge is a connection between a pair of vehicles. The weights of edges are represented by the distance between a pair of nodes. Each graph G_t at timestamp t is refereed to as an instant graph. A GVR in [Rodrigues et al. 2019] is defined as:

$$GVR(t, z) = F(G_t), \tag{8}$$

where $F_{G_t} : G \to \mathbb{R}^n$ is a function that represents the instant graph $G_t \in G$ as a point in an n-dimensional space, $t \in [1, T]$ and $z \in [1, n]$. The results of this graph are represented in a two-dimensional matrix, where rows are the nodes and columns are the intervals in which the graphs were generated.

The authors in [Rodrigues et al. 2019] showed two scenarios to evaluate their proposed method. Both scenarios have a fixed number of nodes, the first one was a soccer game with eleven nodes, and the other one was a social network (1,899 nodes). In this work, we used the TAPAS Cologne network trace for this work. Its traffic is dynamic and the total number of vehicles in the 24-hour period result in GRVs much bigger than those generated by the scenarios in [Rodrigues et al. 2019], increasing the time complexity to make operations over them. To handle this problem, we propose to use relative GVR values to represent a measure at an instant, compressing the results of a specific time as a GVR, with values ranging between 0 and 1, resulting in less time to compute the results.

Figure 7 shows how a GVR is computed for temporal graphs, considering the degree centrality. In this example, the nodes are the vehicles and edges are established if the pair of nodes respects a distance limitation. First, the temporal graphs are generated in pre-determined intervals. The centrality degree of each graph is then computed. For each graph, an absolute frequency histogram is generated, and then the results are compressed in a histogram where the angle x is the intervals and angle y the centrality measures, and the centrality values are mapped to a color pattern.

5. Framework

Figure 9 shows the steps to build the framework, allowing to analyze the vehicular traffic given a scenario. Our framework was developed using Python, and it is available in [Trindade 2019]. In Figure 9, in the left side of the interface, it is possible to select the color pattern, the network measures (PageRank, degree centrality, closeness centrality, betweenness centrality, local efficiency, and harmonic centrality), the time interval in minutes $T_h = \{5, 10, 15, 30, 60, 120\}$, the start time in hours, and the end time in hours.

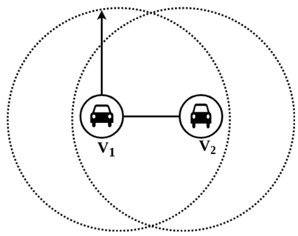

Figure 8. Connection establishment between two vehicles (nodes).

To extract the graphs, we used the Simulation of Urban MObility (SUMO) version 1.2, which is highly portable, microscopic and a continuous road traffic simulation package designed to handle large road networks. The TAPAS Cologne trace [Col] trace comprises the information of road networks imported from OSM, POIs (Patched Import of Shapes) and polygons imported from OpenStreetMap (OSM), the mapped trips for time between 6:00 and 8:00 (am), the mapped trips for time between 00:00 and 24:00,

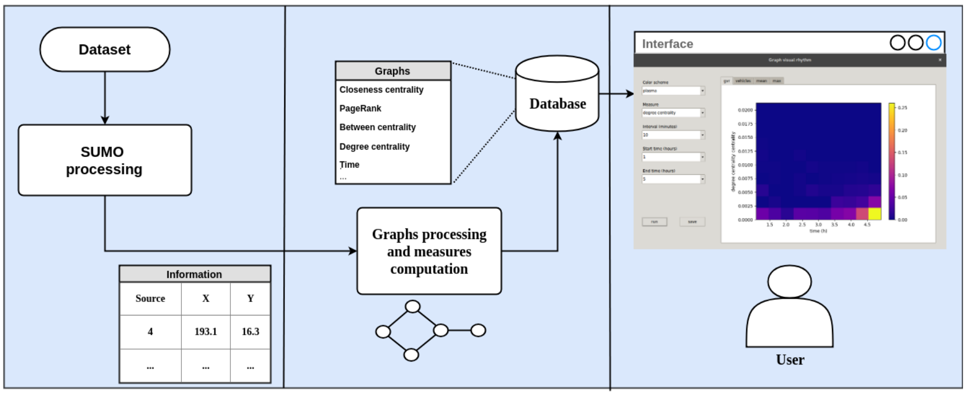

Figure 9. Flowchart representing the process to obtain the network measures as the output of the tool interface.

and the SUMO-configuration file. During the simulation, we extracted the vehicles information at specific intervals. The extracted information is then used to generate the temporal graphs in order to create GVRs.

In the graph extraction step, the connections between nodes are established depending on the distance between them, considering 200 meters as the limit (Figure 8). The position (x_v, y_v) of a vehicle is a point on the circle. The circle of radius D and the center $(x_v^{'}, y_v^{'})$, represents the distance between two cars and the location of another vehicle. The distance D is calculated as follows:

$$(x_v^{'}, y_i^{'})^2 + (x_v^{'}, y_v^{'})^2 = D^2, \tag{9}$$

The distance D will be used as the weight for the edges of the graph.

During the SUMO simulation, we extracted a set of information: index of vehicle ID and its position X and Y, and the simulation time $t_i \in T$. After that, the temporal graphs are generated establishing connections between vehicles (nodes) that are in the same time window and respecting Equation 9. For each graph, the complex measures are computed and stored in a relational database, so that in the interface only a query is executed to show the resulting data.

As a result, a set of charts are generated based on the selected measurement. Two GVRs are generated, one with raw values and another with normalized values to expand the figure area where occurs a fluctuation of centrality values. The charts for the maximum values and medium values of the centrality measure are shown in separate tabs within the tool interface. One of the charts is a plot that represents all centrality values in the selected interval. The chart with the number of vehicles in each interval also generate a chart in the framework (Figure 10). All charts generated can be saved to a file, using the $save$ button.

6. Case Study: TAPAS Cologne

To analyze the behavior of the measures in vehicular networks we used TAPAS Cologne scenario [project at CITI Lab 2019], where Figure 11 shows the Cologne map. The mobility traffic [project at CITI Lab 2019] of the Institute of Transportation Systems at the German Aerospace Center (ITS-DLR), aims at reproducing car traffic in the greater urban

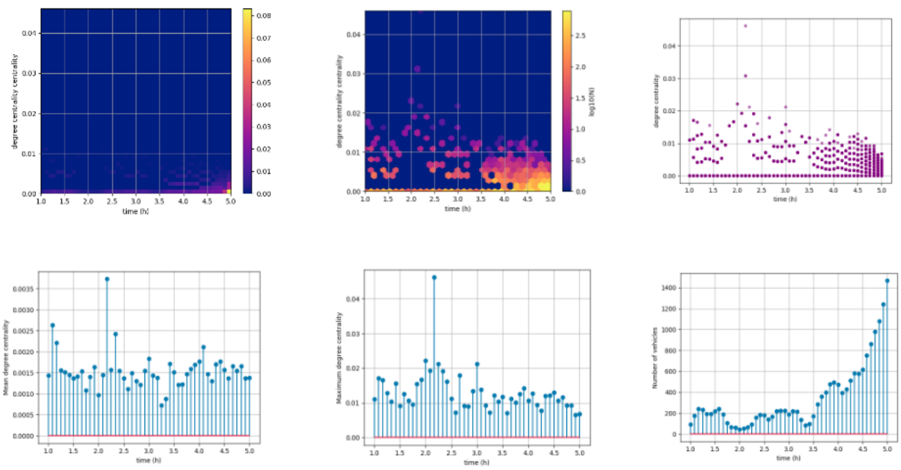

Figure 10. Measures generated from the tool interface: centrality histogram with real values, centrality histogram with values in logarithm scale, mean centrality values, maximum centrality values, and number of vehicles.

area of the city of Koln, Germany, with the highest level of realism possible. The resulting synthetic trace of the car traffic in the city of Cologne covers a region of 400 square kilometers for a period of 24 hours in a typical working day, and comprises more than 700,000 individual car trips. The analyzed data corresponds to the period between 1 to 6 (am) from the TAPAS Cologne trace.

Figure 12b and Figure 12a show the GVR representing the degree centrality of TAPAS Cologne trace. The degree centrality score evolution provides an overview of connection density of vehicles over time, where after 4 a.m the traffic increases as shown in these GVRs. As we can see the degree centrality of nodes increases as the number of vehicles over time. In this context, using GVR it is possible to identify the possibility of data transmission, where if the degree centrality of a node is zero it is impossible to establish a connection with it.

Figure 11. Map of the Cologne city [project at CITI Lab 2019] extracted of OSM.

Figure 12c and Figure 12d show the GVR representing the betweenness centrality of TAPAS Cologne scenario. This measure shows which nodes act as *bridges* between nodes in the network. It does this by identifying all the shortest paths and then counting how many times each node occurs in these paths. If all nodes have the same betweenness, it means that the network does not have a central node and can select anyone to transmit a message, considering the distance as the cost of the weight/cost of transmission.

Nodes with high betweenness values have higher possibility of receiving data. The downside is that when the traffic increases, these nodes can cause a bottleneck problem (congestion), decreasing the quality of transmission. In our scenario, over time the betweenness centrality increases, meaning that more shortest paths can be created, and ve-

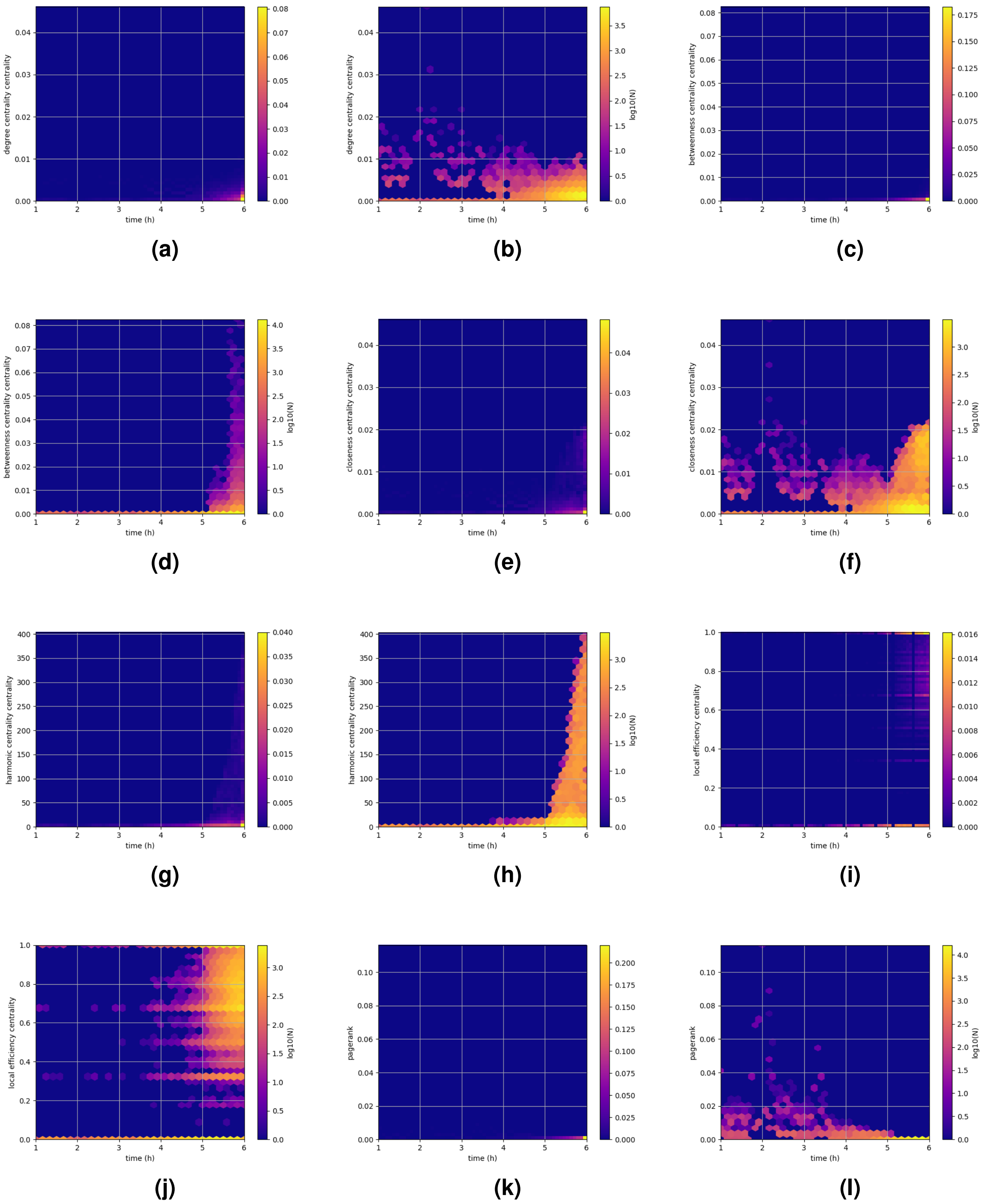

Figure 12. Graph visual rhythm representing the centrality measures of TAPAS Cologne scenario.

hicles are probably more concentrated in specific areas. This behavior can be correlated to the fact that the number of vehicles and degree centrality increased. If the betweenness of all nodes are similar and the degree centrality of all nodes are higher, it means that the vehicular network is strongly connected.

Figure 12e and Figure 12f show the GVR representing the closeness centrality of TAPAS Cologne scenario. Vehicles with high closeness centrality are in a favorable position to control and acquire vital information and resources, evaluating how much a node is close to the other nodes. The results from this measure, in our scenario, has a strong relationship with the distance between nodes and the vehicle communities formation, where the node with the highest closeness centrality can reach more nodes when a data message needs to be transmitted.

Figure 12g and Figure 12h show the GVR representing the harmonic centrality of TAPAS Cologne dataset. The harmonic centrality has a similar purpose compared to closeness centrality, but the resulting GVRs were different. For example, if we are trying to spread a message on social media we could use the algorithm to find the key influencers (vehicles) that can help us achieve that.

Figure 12i and Figure 12j show the GVR representing the local efficiency of TAPAS Cologne dataset. Local efficiency measures the ability of the vehicular network to transmit information at the global and local level. An important metric that concisely couples with network efficiency is network cost, which measures how expensive it is to build a network. In our case, the local efficiency evaluates the impact of communication between adjacent nodes (vehicles) when a node and all its connections are removed. The results show that the local efficiency started to increase after four hours, resulted by the connection degree of network increases.

Figure 12k and Figure 12l show the GVR representing the PageRank for the TAPAS Cologne scenario. The PageRank measure shows the prestige of nodes, where if the nodes measured values are closer to zero, it means that the nodes are not connected to the nodes with high influence in the vehicular network. This measure can be used to evaluate the message propagation in a network where a node with high PageRank value has more influence over the other nodes. In this case, for example, if a message of an accident occurrence needs to be propagated through the network, then selecting nodes with high centrality values can guarantee an efficient propagation. This measure also can help in link prediction, finding vehicles that will become connected soon. Figures 12k and 12l show that as the time increases, the PageRank decreases.

Considering the centrality measures, it should be noted that the shortest path makes two assumptions regarding reachability. First, the measure only works on connected graphs, since the distance between unconnected nodes is undefined. Second, taking shortest paths implies taking paths that in fact reach a particular destination—what we might call valid paths. In our scenario, there are situations where the temporal graphs have nodes unconnected, so in future works, it is necessary to study measures that work with unconnected graphs to analyze the vehicular networks more efficiently. One example that we use in this work is the harmonic centrality, which is an extension of closeness centrality to work with unconnected graphs.

7. Conclusion

In this work, we introduced graph visual rhythm to encode dynamic temporal graphs generated from vehicular networks. To simplify the visualization, we also introduced a visualization tool the graph visual rhythms and basic statistics generated from these temporal graphs. With our framework, it is possible to analyze complex network measures and some statistics, such as the maximum and medium centrality, and the number of vehicles in each period.

The results produced in this study can be used in routing algorithms to select vehicles that are more *close* to others, creating different routes when data needs to be transmitted. Moreover, the framework can work with other traces, and with different distances between vehicles to establish a connection between them.

For future works, the GVRs of 24 hours of TAPAS Cologne can be generated. More network measures and statistics could be added, such as the speed of vehicles which can help to correlate information of accident occurrence with the speed of vehicles in specific areas to develop the government's policies to decrease the accidents. Other measures can be the network clusterization, eccentricity, and eigenvector centrality. Learning algorithms can use the GVRs to predict the future traffic states in vehicular networks and do behavior analysis.

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