Show that the one-dimensional minimizer of a strongly convex quadratic function is given by

$$\alpha = \frac{-\nabla f_k^T p_k}{p_k^T Q P_k}.$$

Result: We will solve

$$\nabla f_k^T p_k = 0.$$

We have that (p. 42) $\nabla f_k = [Qx - b]$ and $Q(x_k - \alpha_k p_k) - b)^T p_k = 0$. We then get

$$\begin{aligned}
&(Qx_k + Q\alpha_k p_k - b)^T p_k \\
&= \nabla f_k^T p_k + (Q\alpha_k p_k)^T p_k \\
&= \nabla f_k^T p_k + Q^T \alpha_k p_k^T p_k = 0 \\
&\Rightarrow \alpha_k p_k^T Q^T p = -\nabla f_k^T p_k \\
&\Rightarrow \alpha = \frac{-\nabla f_k^T p_k}{p_k^T Q P_k}.
\end{aligned}$$

and its given that Q is symmetric. ◀