Forecasting Macroeconomic Time Series: LASSO-Based Approaches and Their Forecast Combinations with Dynamic Factor Models[1]

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Abstract

In a data-rich environment, forecasting economic variables amounts to extracting and organizing useful information out of a large number of predictors. So far dynamic factor model and its variants have been the most successful models for such exercises. In this paper, we investigate a category of LASSO-based approaches and evaluate their predictive abilities in forecasting twenty important macroeconomic variables. These alternative models could handle hundreds of data series simultaneously, and extract useful information for forecasting. We also show analytically and empirically that combing forecasts from LASSO-based models and those from dynamic factor models could further reduce the mean square forecast error (MSFE). Our three main findings can be summarized as follows. First, for most of the variables under investigation, all LASSO-based models outperform dynamic factor models in the out-of-sample forecast evaluations. Second, by extracting information and formulating predictors at the economically meaningful block levels, new methods greatly enhance model interpretabilities. Third, once forecasts from a LASSO-based approach and those from a dynamic factor model are combined by forecasts combination techniques, the combined forecasts are significantly better than dynamic factor model forecasts and the naïve random walk benchmark.

Keywords: High-dimensional time series; Model selection; Dynamic factor model; Combining forecasts

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1. Introduction

Forecasting macroeconomic variables plays a critical role in macroeconomic studies, financial economics and monetary policy analysis. Accurate forecasts lead to better understandings of mechanisms of economic dynamics (Bai and Ng, 2008), better portfolio management and hedging strategies (Rapach, Strauss and Zhou, 2010), and more effective monetary policies (Bernanke et al. 2005). In a data-rich environment like nowadays, a large number of economic data series are tracked by economists and policy-makers. Low-dimensional models usually incorporate a few pre-specified economic predictors, and thus are difficult to capture the complex, dynamic patterns underlying large panels of time series. Therefore, there is a daunting need to propose econometric models and analysis frameworks aimed to extend their low-dimensional counterparts for better predictions.

Over the past decade, Dynamic Factor Model (DFM; Stock and Watson, 2002a; 2002b) and its variants have been widely used to extract and organize useful information out of a large number of predictors. These methods summarize a large panel of time series by dynamic factors, make forecasts of dynamic factors, and then recover the dynamics of the original variable by its factor loadings. Motivated by dynamic factor models, Bernanke et al. (2005) proposed Factor-Augmented Vector Autoregressive (FAVAR) approach for monetary policy analysis. Moench (2008) summarized risk factors driving the pricing kernel by dynamic factor models, based on which yield curves are predicted in a no-arbitrage asset pricing framework.

Despite the analytical tractability of dynamic factor models, however, a trade-off has to be made between the loss of information and the curse of dimensionality. In other words, if only the first few principal components are used to summarize a majority of information in all time series, the remaining principal components could still explain a notable portion of the overall variation. But if more factors are included in the model, the dimensionality of the resulting model increases and the degrees-of-freedom problem will arise again. As a result, the number of factors should be limited in order to conserve the degrees-of-freedom, and the risk of losing useful information is hidden behind information compression and dimension reduction. Since such information can hardly be recovered in subsequent steps, unsatisfactory predictive ability and biased structural inference may follow. Obviously, the more time series observed and the more heterogeneous of these time series, the more severe the information loss. In some recent empirical analyses, dynamic factor models exhibit lower predictive power in forecasting some economic indicators than Bayesian shrinkage approaches (see, e.g., Korobilis, 2012).

In this paper, we propose a category of alternative forecasting methods, where a large number of predictors are accommodated simultaneously and shrinkage estimation methods are employed. Within this framework, dimension reduction is *not* carried out before forecasting, but guided by forecasting, and thus potentially important information is not discarded. Specifically, our methods depend on penalized least squares estimation, which is a generalization of ordinary least squares estimation, with an additional term that penalizes the size of regression coefficients. In doing so, it regularizes the model complexity, and avoids over-fitting that deteriorates the out-of-sample forecasting performance.

Common penalized least squares estimations include LASSO regression (Tibshirani, 1996) and ridge regression (Hoerl and Kennard, 1970), whose individual performance in forecasting economic variables were investigated by De Mol, Giannone and Reichlin (2008) in a Bayesian framework. They concluded that two methods produce highly correlated forecasts with similar predictive abilities. LASSO regressions, in particular, tend to produce estimated regression coefficients that are exactly zeros, and thus can be used for variable selections, where only predictors with nonzero estimates are considered as important ones. In macroeconomic forecasting, such property has been explored by Bai and Ng (2008) to select a subset of predictors, from which factors in dynamic factor models are constructed.

In this paper, however, we will consider several LASSO-based approaches that generalize the classic LASSO regression. First, Zou and Hastie (2005) showed that variable selection instability of LASSO is due to the parameter uncertainty in estimating large covariance matrix. By replacing the sample estimator of covariance matrix with a shrinkage estimator, they showed that the resulting regression coefficients and variable selection process are more stable. This is equivalent to imposing an additional L2 norm constraint in a LASSO regression problem. In statistics literature this method is known as elastic net, since it is like a net that catches all "big fish" for better forecasts.

Second, since predictors in economic forecasting can be divided into different blocks (Hallin and Liška, 2011; Moench et al., 2011), we impose the sparsity constraints at the block level. This is done by a two-stage procedure. In the first stage all predictors are grouped into different blocks. Then in the second stage, group LASSO (Yuan and Lin, 2005) is employed so that predictors in the same block tend to be selected together. As can be seen from the empirical analysis, all LASSO-based approaches have very similar out-of-sample forecast performance and in general outperform dynamic factor models, but elastic net regression and group LASSO regression give more consistent variable selection results over the whole out-of-sample evaluation period, leading to enhanced model interpretability.

Moreover, motivated by our results that although LASSO-based approaches have better forecast accuracy in general, dynamic factor models could gain momentum from time to time, we propose to combine forecasts of LASSO-based models and those of dynamic factor models using forecasts combination technique (Bates and Granger, 1969; Timmermann, 2006). We show analytically that in the presence of model uncertainty, the combined forecasts are associated with smaller mean square forecast error (MSFE) at the population level. Empirically, combined forecasts have significantly lower forecasting errors than those from dynamic factor models for all economic variables we predicted, and those forecasts are stabilized over time.

The advantages of these LASSO-based approaches are predictive accuracy and model interpretability. Other than model uncertainty, the forecasting gains can also be explained by the role of non-pervasive shocks. When the true data generating process is unknown, assuming common factors for all variables may ignore shocks that affect a group of variables (or non-pervasive shocks; see Luciani, 2014). As a result, LASSO-based regressions may capture the local correlation that was left behind by common factors in a factor model. As regards model interpretability, it is well known that variables selected by the classic LASSO are not stable over time, in the sense that once another observation is added into the estimation window, the estimated regression coefficients and the subset of important predictors may change dramatically. This phenomenon is observed in the statistical analysis of high-dimensional data (Fan and Li, 2001; Zou and Hastie, 2005) as well as forecasting macroeconomic time series with many predictors (De Mol, Giannone and Reichlin, 2008). Elastic net and group lasso regressions, on the other hand, could deliver relatively stable forecasts and enhanced model interpretation.

The rest of the paper is organized as follows. Section 2 introduces three versions of LASSO-based regressions and their estimation details. Section 3 presents forecasts combination technique that combines forecasts from LASSO-based methods and dynamic factor models. In Section 4 we discuss the evaluation of the statistical predictabilities of proposed methods. Section 5 presents results of an empirical analysis of more than one hundred time series, where twenty macroeconomic variables are predicted by different approaches and forecasting performances are evaluated and compared. Finally, in section 6, we provide concluding remarks.

2. **Forecasting with Shrinkage Estimation**

2.1. LASSO Regression

Assuming that J stationary time series of macroeconomic variables $\boldsymbol{x_t} = \left(x_{1,t}, \cdots x_{J,t}\right)^T, t = 1, \dots, T$ are observed with J being very large, our goal is to forecast the kth variable $x_{k,T+h}$ with forecasting model

$$x_{k,T+h} = \mu_k + \sum_{p=1}^{P} \sum_{j=1}^{J} \phi_{k,j}^{p} x_{j,T-p} + e_{k,T+h} \;, \tag{1}$$

or

$$x_{T+h} = \mu + \sum_{p=1}^{P} \sum_{j=1}^{J} \phi_{j}^{p} x_{j,T-p} + e_{T+h} \;, \tag{2}$$

after dropping subscript for simplicity, where $\phi_{k,j}^{p}, p = 1, \cdots P$ denotes the linear dependency of $x_{k,t}$ on $x_{j,t-p}$, for $j, k = 1, \cdots J$, and the error term e_{T+h} is stationary and follows a normal distribution with mean zero and variance σ_e^2.

This high-dimensional time series model with a huge number of parameters has been studied in the literature, where Bayesian shrinkage estimations and factor models were employed (Bai and Ng, 2008; De Mol, Giannone and Reichlin, 2008; Korobilis, 2012). In this study we estimate $\phi_j^p, j = 1, \dots, J, p = 1, \dots, P$ by minimizing the constrained least squares

$$\frac{1}{2} \sum_{t=1+P}^{T} \left(x_t - \mu - \sum_{p=1}^{P} \sum_{j=1}^{J} \phi_j^p x_{j,t-p}\right)^2 \quad \text{subject to} \quad \sum_{p=1}^{P} \sum_{j=1}^{J} |\phi_j^p| < s, \tag{3}$$

for some $s > 0$, which is equivalent to the following penalized least squares for some $\lambda > 0$

$$\frac{1}{2} \sum_{t=1+P}^{T} \left(x_t - \mu - \sum_{p=1}^{P} \sum_{j=1}^{J} \phi_j^p x_{j,t-p}\right)^2 + \lambda \sum_{p=1}^{P} \sum_{j=1}^{J} |\phi_j^p|. \tag{4}$$

This is the classic LASSO regression (Tibshirani, 1996). When the total number of predictors is large, the ordinary least squares estimators tend to have larger variances, and thus forecasts are highly volatile over time. This is especially a problem when we have low-frequency economic variables. With a L1 norm constraint on the regression coefficients, LASSO regression avoids over-fitting and encourages sparse solutions, in the sense that a subset of zero coefficients could be identified and the corresponding predictors are excluded from the final model.

From the perspective of Bayesian statistics, minimizing (4) is equivalent to maximizing the posterior distributions of regression coefficients in a linear regression model, where prior distributions are independent Laplace distributions (Casella and Park, 2008). De Mol, Giannone and Reichlin (2008) applied LASSO regression in forecasting using large cross sections. However, our LASSO regression differs from that in De Mol, Giannone and Reichlin (2008) in two ways. First, we select the tuning parameter λ by cross-validation, which is a data-driven method designed to maximize the expected out-of-sample predictive accuracy. Moreover, we extend the classic LASSO to a scenario with model uncertainty (see Section 3).

2.2. Elastic Net Regression

Zou and Hastie (2005) showed that in matrix notation, the objective function (4) is

$$\boldsymbol{\phi}^{\boldsymbol{T}}(\boldsymbol{X}^{T}\boldsymbol{X})\boldsymbol{\phi} - 2\boldsymbol{y}^{\boldsymbol{T}}\boldsymbol{X}\boldsymbol{\phi} + \lambda|\boldsymbol{\phi}|_1. \tag{5}$$

where $\boldsymbol{y} = (x_{1+P}, \dots, x_T)^{\boldsymbol{T}}$, $\boldsymbol{\phi} = \left(\phi_1^1, \phi_1^2, \dots, \phi_J^p\right)^{\boldsymbol{T}}$ is a vector of regression coefficients, $\boldsymbol{X}$ is the corresponding design matrix with the *t*-th row given by $(x_{1,t-1,}x_{1,t-2}, \dots, x_{J,t-P})$, $|\cdot|_1$ is the L1 norm of a vector, and λ is the same tuning parameter. Note that $\boldsymbol{X}^{T}\boldsymbol{X}$ is the sample covariance matrix of *J* time series. Since *J* is very large in economic forecasting, the number of unknown parameters in $\boldsymbol{\Sigma}$ is huge. By minimizing (5) we implicitly assume that $\widehat{\boldsymbol{\Sigma}} = \boldsymbol{X}^{T}\boldsymbol{X}$ is a reasonable estimator of its population counterpart $\boldsymbol{\Sigma}$. However, it is well known that $\widehat{\boldsymbol{\Sigma}}$ is far from the optimal estimator when a large number of assets prices or economic time series are considered (Kan and Zhou, 2007).

Alternatively, shrinkage estimators of $\boldsymbol{\Sigma}$ were proposed to replace sample estimates. In portfolio management, Ledoit and Wolf (2003; 2004) proposed to estimate the large covariance matrix by a shrinkage estimator

$$\widehat{\boldsymbol{\Sigma}}_s = (1-\gamma)\widehat{\boldsymbol{\Sigma}} + \gamma\widehat{\boldsymbol{\Sigma}}_{\text{target}}, \tag{6}$$

where $\widehat{\boldsymbol{\Sigma}}$ is the sample covariance matrix, $\widehat{\boldsymbol{\Sigma}}_{\text{target}}$ is a shrinkage target, and $0 < \gamma < 1$ is the shrinkage intensity. $\widehat{\boldsymbol{\Sigma}}_{\text{target}}$ could be an identity matrix $\boldsymbol{I}$, or the covariance matrix implied by a factor model. In particular, we would like to investigate the benefit of shrinkage estimator (6) in economic forecasting by

$$\widehat{\boldsymbol{\phi}}_{\boldsymbol{enet}} = \mathbf{argmin}_{\boldsymbol{\phi}}\, \boldsymbol{\phi}^{\boldsymbol{T}}\left((1-\gamma)\widehat{\boldsymbol{\Sigma}} + \gamma\widehat{\boldsymbol{\Sigma}}_{\text{target}}\right)\boldsymbol{\phi} - 2\boldsymbol{y}^{\boldsymbol{T}}\boldsymbol{X}\boldsymbol{\phi} + \lambda|\boldsymbol{\phi}|_1, \tag{7}$$

with $\widehat{\mathbf{\Sigma}}_{\text{target}}$ being an identity matrix. This corresponds to the elastic net shrinkage estimation proposed by Zou and Hastie (2005).

It can be shown that elastic net regression can be regarded as a linear combination of LASSO regression ($\gamma = 0$) and ridge regression ($\gamma = 1$). Since many comparisons have been made between LASSO regression and ridge regression and concluded that none of them could uniformly dominate the other in the out-of-sample forecasting exercises (Tibshirani, 1996; Fu, 1998; De Mol, Giannone and Reichlin, 2008), it is reasonable to combine this two approaches adaptively. Ridge regression alone does not zero out any predictors but shrinks all predictors. By combining it with LASSO regression, however, we could select a subset of variables using this flexible shrinkage estimation approach. Two shrinkage intensities here, γ and λ, are determined by cross-validations.

2.3. Group-LASSO Regression

It has been recognized that block structure exists in large panel of economic data, where variables within the same block are similar economic measures. Examples of blocks include variables representing employment rates or price levels. However, since both LASSO regression and elastic net regression select variables individually, interpreting the final predictive model is not straightforward. So it would be advantageous if such block structure could be used at least for better interpretations. Currently block information has been incorporated into the construction of dynamic factors (Hallin and Liška, 2011; Moench et al., 2011). Song and Bickel (2011) proposed to apply group LASSO penalty in the context of large vector autoregressions, where regression coefficients in the same group are shrunk to zero jointly. Two grouping strategies were proposed in their study: universal grouping and segmented grouping. With universal grouping, all regression coefficients using the same predictor to forecast different variables are defined as one group. In our problem of forecasting a single dependent variable, this strategy is equivalent to a simple LASSO penalty. Segmented grouping, on the other hand, takes similar strategy, but estimates all regression coefficients segment by segment, where segment has the same definition as our economically meaningful blocks. For example, in predicting a single dependent variable in one segment, this strategy is equivalent to a standard LASSO regression with two tuning parameters, one for coefficients in this segment and one for those outside.

For final model interpretability, we propose to apply group LASSO penalty on groups of predictors, so that predictors in the same economic block could either enter or leave the final model together. This is done by a two-stage procedure. In the first stage all predictors are grouped into different blocks, where either economic knowledge or statistical methods could be used. Then in the second stage sparsity constraints are imposed at the block level. In particular, we employ group LASSO penalized regression (Yuan and Lin, 2005), so that predictors in the same block tend to be selected or excluded together in the final predictive regression. In this way, it is expected that both sparsity and model interpretability are gained.

Suppose all J predictors can be partitioned into L groups, with d_l being the number of predictors in group $l = 1, \dots, L$. Then all regression coefficients $\phi_j^p, j = 1, \dots, J, p = 1, \dots, P$ can be partitioned accordingly as $\{\boldsymbol{\phi}_1^1, \dots \boldsymbol{\phi}_L^1, \dots, \boldsymbol{\phi}_L^P\}$, where d_l-dimensional vector $\boldsymbol{\phi}_l^p$ contains all regression coefficients for variables in group l at lag $p, l = 1, \dots, L, p = 1, \dots, P$. We then estimate all regression coefficients by minimizing

$$\frac{1}{2}\sum_{t=1+P}^{T}\left(x_t - \mu - \sum_{p=1}^{P}\sum_{j=1}^{J}\phi_j^p x_{j,t-p}\right)^2 + \lambda\sum_{p=1}^{P}\sum_{l=1}^{L}\sqrt{d_l}\left\|\boldsymbol{\phi}_l^p\right\|_2, \tag{8}$$

where $\left\|\boldsymbol{\phi}_l^p\right\|_2$ is the L2 norm (not squared) of vector $\boldsymbol{\phi}_l^p$. It can be shown that such penalty encourages sparsity at the block level (see Yuan and Lin, 2005 for more discussions).

2.4. Computation and Selecting Tuning Parameters

For all LASSO-based shrinkage methods, there is no direct solution. But Zou and Hastie (2005) and Yuan and Lin (2005) have shown that the elastic net regression (7) and group LASSO regression (8) can be transformed to LASSO regressions that take similar forms to (4). Several efficient algorithms have been proposed for LASSO regression, such as least angle regression (Efron et al., 2004) and coordinate descent algorithm (Fu 1998; Wu and Lange, 2008; Friedman et al., 2009). Here we will employ coordinate descent algorithm, which is suitable for problems with a large number of predictors.

Note that the strength of shrinkage in (4) and (8) depends on one tuning parameter λ, while that in (7) depends on two tuning parameters: γ and λ, with larger γ and λ corresponding to greater shrinkage on regression coefficients. In what follows we describe how to determine these tuning parameters by cross-validations, which are expected to balance a model's in-sample fit and out-of-sample predictive ability.

This data-driven method has been extensively used in statistics (Arlot and Celisse, 2010) and finance (DeMiguel et al., 2009). Given a combination of fixed values of γ and λ, say $(\gamma^{(1)}, \lambda^{(1)})$, we randomly partition the in-sample data into five pieces of roughly the same size. Then for each piece, say piece k, we estimate the model with $\gamma = \gamma^{(1)}$ and $\lambda = \lambda^{(1)}$ using all data that is not included in piece k. Suppose the estimated regression coefficients are $\widehat{\boldsymbol{\phi}}^{(\boldsymbol{k})}$. Then we go back to piece k, and make forecasts using the predictive regression with unknown parameters $\boldsymbol{\phi}$ being replaced by $\widehat{\boldsymbol{\phi}}^{(\boldsymbol{k})}$. Since $\widehat{\boldsymbol{\phi}}^{(\boldsymbol{k})}$ is estimated using data other than piece k, the forecasts of this part of data can be regarded as out-of-sample forecasts. This procedure is repeated for $k = 1, \dots, 5$ so that each observation in each of these five pieces receives its out-of-sample forecast. We calculate the mean squared prediction error (MSPE) that is conditional on $(\gamma^{(1)}, \lambda^{(1)})$. Finally the whole procedure is repeated for different values of γ and λ, and the combination that gives the lowest mean squared prediction error (MSPE) is used[3]. In order to forecast the variable of interest in the next month, we collect all the in-sample data above, and use the selected tuning parameters.

3. Forecasts Combination of LASSO-Based Approaches and Dynamic Factor Models

In general, all LASSO-based approaches and dynamic factor models are dimension reduction methods for forecasting using a large number of time series. However, the way of dimension reduction differs. While LASSO-based approaches try to identify and aggregate individual predictors that are relevant to the variable to be predicted, the dynamic factors summarize all information by $\tilde{J}$ principal components before forecasting. As a result, if only a few predictors contain information for forecasting, LASSO-based methods could eliminate irrelevant predictors from the final predictive model, and thus is robust against cumulative noises in the large data sets. However, if the variable of interest is driven by the co-movements of many time series, dynamic factor models are expected to find and characterize the underlying common factors, and outperform LASSO-based methods,

[3] The sequence of each tuning parameter is specified by taking 100 tuning parameter values between 0 and an upper bound, where the upper bound is the smallest tuning parameter value so that all regression coefficients are shrunk to zero and is found by trial and error. Empirically the upper bound has negligible impact on the estimates.

“Essentially, all models are wrong, but some are useful" (Box, 1976). This is true in economic forecasting as well. As confirmed by our empirical analyses in Section 5, although LASSO-based approaches deliver better out-of-sample forecasts in general, these methods do not dominate dynamic factor models for all variables over the full evaluation period. Therefore, we propose to resolve this model uncertainty by forecasts combination. Forecasts combination was originally proposed by Bates and Granger (1969). Based on forecasts from individual models, it formulates a linear combination of all forecasts as a new forecast. Recently forecasts combination has received lots of attentions in economics and finance due to its superior out-of-sample predictive ability and stabilized forecasts errors (Stock and Watson, 2004; Timmermann, 2006; Rapach, Strauss and Zhou, 2010; Della Corte and Tsiakas, 2011).

3.1. Theoretical Property

Aiolfi et al. (2010) considered a model uncertainty specification where x_t is assumed to be generated by one of two factor models according to a Bernoulli random variable. In this section we show analytically how combined forecast from a factor model and a sparse model can dominate two individual forecasts, where in the sparse model, only a subset of predictors have nonzero regression coefficients, and LASSO-based approaches are used to identify these important variables.

Suppose that model uncertainty takes a shift in the data generating process of the variable to be predicted, x_t, between a factor model and a sparse model:

$$x_t = \mathrm{I}_{\{S_t=1\}} L^T F_t + \mathrm{I}_{\{S_t=2\}} \Phi^T Z_{t-1} + e_t, \tag{9}$$

where $\mathrm{I}_{\{A\}}$ is an indicator function that equals to one if event A is true and zero otherwise, s_t is a Bernoulli random variable indicating two states with $\Pr(S_t = 1) = p$ and $\Pr(S_t = 2) = 1 - p, p \in (0,1)$, $F_t = (f_{1,t}, \dots, f_{m_1,t})^T$ is an m_1-dimensional vector of factors, Z_{t-1} is a vector of lagged predictors, and e_t is a random error with mean zero and variance σ_e^2. According to this dynamics, x_t is generated by two different models in two different states. In state 1, x_t is governed by a vector of factors, where L^T is a vector of factor loadings; In state 2, x_t is determined by m_2 lagged predictors $Z_{t-1} = (x_{k_1,t-1}, \dots, x_{k_{m_2},t-1})^T, k_j \in \{1, \dots, J\}, j = 1, \dots, m_2$, with Φ^T being an m_2-dimensional vector of regression coefficients. Since in this true model, Z_{t-1} only contains a subset of all predictors, it is known as a sparse model. In practice lagged predictors in Z_{t-1} are identified by LASSO-based methods.

We further assume the dynamics of F_t and Z_t

$$F_t = A_1 F_{t-1} + \epsilon_1, \tag{10}$$

and

$$Z_t = A_2 Z_{t-1} + \epsilon_2, \tag{11}$$

with $\epsilon_1 \sim N(0, \Sigma_1)$, $\epsilon_2 \sim N(0, \Sigma_2)$ and $cov(\epsilon_1, \epsilon_2) = \Sigma_{12}$. It can be shown that (see Appendix A), when a dynamic factor model is used for prediction, the associated mean squared forecast error (MSFE) is

$$\text{MSFE}\left(\hat{x}_t^{(D)}\right) = p(L^T - \beta_1^T)\Sigma_1(L - \beta_1) + (1-p)(\Phi^T \Sigma_2 \Phi + \beta_1^T \Sigma_1 \beta_1 - 2\Phi^T \Sigma_{12} \beta_1) + \sigma_e^2, \tag{12}$$

where

$$\beta_1 = pL + (1-p)\Sigma_1^{-1}\Sigma_{12}\Phi. \tag{13}$$

When a sparse model estimated by a LASSO-based approach is used for prediction, on the other hand, the associated MSFE is

$$\text{MSFE}\left(\hat{x}_t^{(L)}\right) = p(L^T \Sigma_1 L + \beta_2^T \Sigma_2 \beta_2 - 2L^T \Sigma_{12} \beta_2) + (1-p)(\Phi^T - \beta_2^T)\Sigma_2(\Phi - \beta_2) + \sigma_e^2, \tag{14}$$

where

$$\beta_2 = p\Sigma_2^{-1}\Sigma_{12}L + (1-p)\Phi. \tag{15}$$

Similarly, the MSFE of the equal-weighted, simple combined forecast $\hat{x}_t^{(C)} = \frac{1}{2}\left(\hat{x}_t^{(D)} + \hat{x}_t^{(L)}\right)$ is

$$\begin{aligned}\text{MSFE}\left(\hat{x}_t^{(C)}\right) &= p[(L - \beta_1/2)^T \Sigma_1 (L - \beta_1/2) + (\beta_2/2)^T \Sigma_2 (\beta_2/2) - (L - \beta_1/2)^T \Sigma_{12} \beta_2] \\ &\quad + (1-p)[(\Phi - \beta_2/2)^T \Sigma_2 (\Phi - \beta_2/2) + (\beta_1/2)^T \Sigma_1 (\beta_1/2) \\ &\quad -(\Phi - \beta_2/2)^T \Sigma_{12} \beta_1] + \sigma_e^2. \end{aligned} \tag{16}$$

These expressions are quite general. To gain more insights, we simplify the model to the case where $m_1 = m_2 = 1$, $A_1 = A_2 = 0$, and both L and Φ are identity matrices. This gives $F_t \sim N(0, \sigma_1^2)$, $Z_t \sim N(0, \sigma_2^2)$, and $\text{cov}(F_t, Z_{t-1}) = \sigma_{12}$. As a result, the MSFE associated with factor model forecasts is

$$\text{MSFE}\left(\hat{x}_t^{(D)}\right) = [p(1-\beta_1)^2 + (1-p)\beta_1^2]\sigma_1^2 + (1-p)\sigma_2^2 - 2(1-p)\beta_1\sigma_{12} + \sigma_e^2. \tag{17}$$

The MSFE of forecasts from the sparse model is

$$\text{MSFE}\left(\hat{x}_t^{(L)}\right) = p\sigma_1^2 + [p\beta_2^2 + (1-p)(1-\beta_2)^2]\sigma_2^2 - 2p\beta_2\sigma_{12} + \sigma_e^2. \tag{18}$$

And the MSFE of the simple combined forecast $\hat{x}_t^{(C)}$ is

$$\text{MSFE}\left(\hat{x}_t^{(C)}\right) = [p(1-\beta_1/2)^2 + (1-p)\beta_1^2/4]\sigma_1^2 + [p\beta_2^2/4 + (1-p)(1-\beta_2/2)^2]\sigma_2^2 -$$

$$[p(1-\beta_1/2)\beta_2 + (1-p)\beta_1(1-\beta_2/2)]\sigma_{12} + \sigma_e^2. \quad (19)$$

For simplicity suppose that $\sigma_{12} = 0$. Then by plugging into the definitions of β_1 and β_2, the population MSFE of the equal-weighted combined forecast, $\text{MSFE}\left(\hat{x}_t^{(C)}\right)$, will be lower than that of individual models if the following condition holds

$$\frac{1}{3}\left(\frac{p}{1-p}\right)^2 < \frac{\sigma_2^2}{\sigma_1^2} < 3\left(\frac{p}{1-p}\right)^2, \quad (20)$$

where p is the probability of being in state 1. Figure 1 shows how MSFEs of three models change with σ_2^2/σ_1^2 and p. Clearly, as long as two models have comparable predictive accuracies (all three scenarios) and the probability of being in one state is not extreme, forecasts combination has the lowest MSFE.

3.2. Empirical Implementation

In empirical applications, we estimate the combined forecast, $\hat{x}_{T+h}^{(C)}$, as the weighted average of the forecast from a LASSO-based model $\hat{x}_{T+h}^{(L)}$ and that from a dynamic factor model $\hat{x}_{T+h}^{(D)}$:

$$\hat{x}_{T+h}^{(C)} = w_{T+h}\hat{x}_{T+h}^{(L)} + (1-w_{T+h})\hat{x}_{T+h}^{(D)}, \quad (21)$$

where $0 \leq w_{T+h} \leq 1$ is the weight of LASSO-based forecast. For simple combined forecast, $w_{T+h} = 0.5$. Alternatively, w_{T+h} can be determined by the historical relative performance of LASSO-based method. Following Stock and Watson (2004), among others, we measure the past performance of each model by the inverse of its Discounted MSE (DMSE), or

$$DMSE_{T+h}^{(L)} = \sum_{q=T-\kappa}^{T-1} \theta^{T-1-q}\left(x_q - \hat{x}_q^{(L)}\right)^2, \quad (22)$$

where κ is the window size so that DMSE is evaluated based on most recent κ forecasts, θ is the discount factor, and $x_q - \hat{x}_q^{(L)}$ is the forecasting error of LASSO-based method within that window. Similarly,

$$DMSE_{T+h}^{(D)} = \sum_{q=T-\kappa}^{T-1} \theta^{T-1-q}\left(x_q - \hat{x}_q^{(D)}\right)^2. \quad (23)$$

Since the lower $DMSE_{T+h}$, the better the predictive ability of a model before time T, the weight of LASSO-based method in the combined forecast (21) is proportional to the inverse of $DMSE_{T+h}^{(L)}$:

$$w_{T+h} = \frac{\left(DMSE_{T+h}^{(L)}\right)^{-1}}{\left(DMSE_{T+h}^{(L)}\right)^{-1} + \left(DMSE_{T+h}^{(D)}\right)^{-1}}. \quad (24)$$

When the discount factor $\theta = 1$ (no discounting), the forecasts combination method is the one investigated in Bates and Granger (1969). Following Della Corte and Tsiakas (2011), in addition to simple forecasts combinations, we consider discount factors $\theta = \{1, 0.95, 0.9\}$ and evaluate the past out-of-sample performance over the last κ months, where $\kappa = \{12, 36, 60\}$.

4. **Evaluation of Statistical Predictability**

Since dynamic factor model has shown superior performance in forecasting economic variables and outperforms univariate autoregressions, small vector autoregressions, and leading indicator models (Stock and Watson 2002a), we will use dynamic factor model as a benchmark and compare the proposed strategies with dynamic factor model.

In what follows, we describe statistical criteria for comparing the out-of-sample predictive abilities of alternative approaches. First, we compute the Root Mean Squared Error (RMSE) difference statistic, $\Delta RMSE$, which were used in Welch and Goyal (2008) and Della Corte and Tsiakas (2011) for evaluating the predictability of asset pricing models. We define

$$\Delta RMSE = \sqrt{\frac{\sum_{t=T+1}^{M}(x_t - \bar{x}_t)^2}{M-T}} - \sqrt{\frac{\sum_{t=T+1}^{M}(x_t - \hat{x}_t)^2}{M-T}}, \tag{25}$$

where *M-T* is the number of out-of-sample forecasts, x_t is the observed value of variable being predicted at time $t, t = T+1, \dots, M$, $\bar{x}_t$ is the forecast from a benchmark model, and $\hat{x}_t$ is the forecast from a proposed model. A positive $\Delta RMSE$ suggests that the proposed model outperforms the benchmark, or dynamic factor model.

Other than comparing RMSE directly, statistical tests are employed to test the significance of forecasts difference. Since dynamic factor model and LASSO-based approaches are non-nested, we employ the Diebold-Mariano test (Diebold and Mariano, 1995) to compare the forecast accuracy of two methods. Specifically, if one define the difference of squared forecast errors as

$$d_t = e_{1,t}^2 - e_{2,t}^2, \; t = T+1, \dots, M, \tag{26}$$

the Diebold-Mariano test statistic is then

$$DM = \left[\frac{1}{M-T}\left(\hat{\gamma}_0 + 2\sum_{k=1}^{h-1}\hat{\gamma}_k\right)\right]^{-1/2} \frac{1}{M-T}\sum_{t=T+1}^{M} d_t \, , \tag{27}$$

where $\hat{\gamma}_k, k = 0,1,\dots,k$ is the estimated kth autocovariance of the time series d_t, and h is the forecast horizon. Under the null hypothesis of equal expected forecast performance, this statistic has an asymptotic standard normal distribution. Such standard asymptotic distribution of test statistic is advantageous here. Since the cross-validation used in selecting tuning parameters is a resampling scheme, hypothesis tests that are free of bootstrap are desired, leading to affordable computational cost in the out-of-sample evaluation.

To test the null hypothesis that forecasts combination and dynamic factor model have equal predictive ability, however, the problem is to test two nested models. We apply the recently developed testing procedure by Clark and West (2006, 2007), which acknowledges that even if two models have the same forecasting accuracy in the population level, the RMSE from a larger model is expected to be greater. This test statistic has been used in evaluating forecasting accuracy of alternative foreign exchange rate models (Corte and Tsiakas, 2012) as well as alternative asset pricing models for stock returns (Rapach, Strauss and Zhou, 2012). Specifically, given the time series of forecasting errors of two models, $e_{1,t}$ and $e_{2,t}$, we define

$$\hat{f}_t = e_{2,t}^2 - e_{1,t}^2 + \left(e_{1,t} - e_{2,t}\right)^2. \tag{28}$$

Then the test statistic MSE-t is defined as the t-statistic of the intercept in the following linear regression without independent variables

$$\hat{f}_t = \alpha + e_t. \tag{29}$$

Clark and West (2006, 2007) showed that the standard normal distribution could well approximate the distribution of the test statistic MSE-t, based on which an associated p-value can be obtained for hypothesis testing. Again, the null hypothesis is two nested models have equal predictive ability.

5. **Empirical Results**

5.1. Data and Estimation

The data set for empirical analysis contains 107 macroeconomic indicators including industrial productions, price levels, credit conditions, employment and inflation. This monthly data set has been used in Stock and Watson (2012) and can be downloaded from Mark Watson's website[4]: http://www.princeton.edu/~mwatson/publi.html. According to the common practice in macroeconomic literature, these data series have been transformed to stationary ones. The descriptions of all data series as well as their corresponding transformations are shown in the Appendix B.

[4] We are grateful to Mark Watson for making this data set available.

We examine the out-of-sample forecasting performance of the proposed forecasting strategies in a pseudo out-of-sample forecasting exercise as follows. Following Stock and Watson (2002b) and Bai and Ng (2008), the first out-of-sample forecast is based on a 10-year (120-month) estimation window. Although the original data series are from January 1959 to December 2008, by transforming variables and including up to 4 lags in predictors, the estimation window starts at July 1959, and the first set of variables to be predicted is in July 1969. Then for any month t between July 1969 and December 2008 (474 months in total), we define an in-sample period including all data series in the previous 10 years. Based on the in-sample data, we estimate a predictive regression with the maximum lag order $P = 4$. Then with the estimated model, we make one-step-ahead forecast. Finally, we drop the first observation in the rolling estimation window and include a new one at the end. The whole process is repeated until the end of the out-of-sample evaluation period.

LASSO-based predictive regressions are estimated according to discussions in the previous sections. For the group LASSO regression, in particular, we partition all 107 variables according to their economic meanings (see Appendix B). Unreported statistical tests indicate variables within each block are highly correlated, which are consistent with findings in Moench et al. (2011). In terms of dynamic factor models, we follow the specification of Bernanke et al. (2005) by considering three dynamic factors and lag order $P = 4$.

We examine the out-of-sample forecasting performance in details for twenty representative macroeconomic indicators that were carefully investigated by Bernanke et al. (2005), which include federal funds rate (FFR), industrial production (IP), consumer price index for all urban consumers (CPI) and other measures of price levels, real activities and consumptions. These variables include important indicators for the economy and monetary policies.

5.2. Predictive Accuracy

In Table 1 we report the out-of-sample Root Mean Square Error (RMSE) differentials ($\Delta RMSE$) of forecasts from LASSO-based approaches and dynamic factor models (DFM), where the former category includes LASSO regression (columns 1-2), elastic net regression (columns 3-4) and group LASSO regression (columns 5-6), and tuning parameters are selected by cross validations for each month throughout the out-of-sample evaluation period. For each comparison of each variable, Diebold-Mariano test is implemented and the associated p-value is reported, with p-values less than 0.05 indicating the rejection of equal predictive abilities at the significant level of 0.05.

By comparing forecasts from LASSO-based approaches and those from dynamic factor models, we conclude that in general LASSO-based approaches perform better out-of-sample. For example by using LASSO regressions, 18 variables out of 20 variables have positive $\Delta RMSE$, among which 12 RMSE differences are statistically significant. The other two methods give 19 positive $\Delta RMSE$ and 15 positive $\Delta RMSE$, respectively, among which more than 65% are statistically significant. Interestingly, variables whose forecasts from two models are not significantly different include interest rate related series (Federal Fund Rate, 3M Treasury Bills, 5Y Treasury Bonds), employment/unemployment rates and the index of consumer expectations. Since dynamic factors measured by the first few principal components could capture the most obvious data variations over time, we conclude that such variations provide satisfactory summaries of information useful for forecasting these variables.

Moreover, all models have comparable performance in forecasting foreign exchange rate. It has been documented that foreign exchange rate movements are disconnected with the economic fundamentals, which is known as "exchange rate disconnect puzzle" (Meese and Rogoff, 1983; Obstfeld and Rogoff, 2000) in international economics literature. So the best foreign exchange rate model at short horizons is the random walk model which predicts future foreign exchange rate by its historical mean. On the other hand, in economic forecasting literature, many works have been centered on forecasting industrial production and consumer price index (CPI). It can be seen that our new methods perform well in forecasting these two economic aggregates.

Table 2 presents similar forecast performance measures that compare forecasts combinations and dynamic factor models. Forecasts combinations combine forecasts from the dynamic factor model and forecasts from one of the LASSO-based approaches, including LASSO regression, elastic net regression and group LASSO regression, where discount factor $\theta = 0.9$ and the most recent $\kappa = 60$ forecasts are used. Since forecasts combination and dynamic factor model are nested in this scenario, the improvements of forecast accuracies are formally tested by MSE-t statistics (Clark and West; 2006, 2007), and p-values associated with the test statistics are reported. It can be seen that once forecasts are combined, $\Delta RMSE$ are positive for all variables no matter which LASSO-based approach is considered. Moreover, the improvements of forecast accuracies are statistically significant for at least 19 variables, as suggested by the MSE-t

statistics. For this data set, simple combined forecast and other specifications of θ and κ present very similar results and thus are not reported. We conclude that forecasts combination is an effective strategy in forecasting macroeconomic time series when a large number of predictors are available. Due to statistical properties of principal components, dynamic factor model is capable of capturing large variations underlying multiple time series and predicting variable by its connections to the temporal variation. LASSO-based approaches, on the other hand, are capable of identifying important predictors to recover a sparse predictive model. Once forecasts from two strategies are combined, model uncertainty is resolved and predictive power is gained. This is consistent with our analytical results in Section 3.1.

We further show the extent to which the forecasts produced by different methods differ by reporting the correlations among these forecasts (see Table 3). First of all, forecasts from LASSO regressions and elastic net regressions are highly correlated for almost all variables. Their forecasts, whoever, are less correlated with forecasts from group LASSO regressions (columns 2-3). As a result, forecasts from LASSO regressions and elastic net regressions correlate with forecasts produced by dynamic factor models to a similar extent (columns 4-5), but forecasts from group LASSO regressions and dynamic factor models are slightly less correlated (column 6). This may indicate that, group LASSO regressions and dynamic factor models provide complementary information, which explains the superior out-of-sample performance of their combined forecasts.

Sometimes it is interesting to see how predictive regressions perform over time. In Figure 2 we plot the evolution of $\Delta RMSE$ for each variable over the full sample period, where forecasts are produced by two models: LASSO regressions and dynamic factor models. At a given time point, a positive $\Delta RMSE$ implies that LASSO regression has better forecasts so far. Meanwhile, an increasing trend of $\Delta RMSE$ implies that, at this point in time the forecast from LASSO regression has smaller RMSE, and thus is better. Since these patterns are similar for all LASSO-based approaches, those for elastic net regression and group LASSO regression are not reported. In Figure 3 we further visualize the dynamics of $\Delta RMSE$ when the alternative model is the forecasts combination of LASSO regression and dynamic factor model. It can be seen that, variables with negative $\Delta RMSE$ over the full evaluation period in Table 1 usually exhibit sharp decreases at some time points in Figure 2. But once forecasts from two models are combined, the severity of such decreases is ameliorated, and $\Delta RMSE$ is smoothed over time.

Usually forecasters are concerned about the consistency of how forecasts are close to the true values. We answer this question by two measures: the maximum RMSE and the variance of RMSE. In Table 4, we report the ratio of the maximum RMSE from an alternative model to that from dynamic factor models, where alternative models include LASSO regression, elastic net regression, group LASSO regression, and the forecasts combinations of LASSO-based regressions and dynamic factor models. Ratios less than one are in bold, suggesting the alternative model produces less extreme forecasting errors. Table 5 further reports the ratio of RMSE variance from the same set of model pairs, where ratios less than one implies that the predictive ability of the alternative model is more stable over time.

Both Table 4 and Table 5 confirm that all six newly proposed strategies produce less extreme and more stable forecast errors in general. Moreover, although combined forecasts are derived from a linear function of two forecasts, empirically the forecast error instability is not increased. In fact, all forecasts combination methods have lower forecast error variances than dynamic factor models. This is consistent with applications of forecasts combination in predicting equity premium (Rapach, Strauss and Zhou, 2010), where they found combined forecasts reduce forecast volatility relative to individual forecasts. The reduction of forecast error variances is the most obvious for four data series: commodity price index, average hourly earnings, housing starts and new orders index, suggesting that large variations captured by dynamic factors have difficulties in explaining these variables.

As referees pointed out, it would be interesting to include the ridge regression and a variant of LASSO regression that considers both individual predictors and the dynamic factors simultaneously. These two shrinkage methods are implementable when forecasting with a large number of predictors, but do not assume sparse structures or model uncertainties in the data generating processes. Table 6 compares the out-of-sample Root Mean Square Error (RMSE) differentials ($\Delta RMSE$) of forecasts from alternative methods against those from dynamic factor models (DFM), where alternative methods include: (1) ridge regressions with tuning parameters determined by cross validations (Ridge); (2) the combinations of ridge regressions and dynamic factor models (Combination 4); and (3) LASSO regressions on both individual predictors and the dynamic factors simultaneously (LASSO4All). Clearly, although ridge regressions are significantly better than the dynamic factor models for forecasting many variables, their statistical gains are not as large as those from LASSO-based approaches (see, e.g., commodity price index, capacity utilization, earnings and housing starts). As a result, the combined forecasts from ridge regressions and dynamic factor models are less advantageous. LASSO4All method, on the other hand, has similar forecasting performance as LASSO regressions. But its performance is less satisfactory for forecasting exchange rates and consumptions, and the interpretation of regression coefficients is not straightforward.

5.3. Model Sparsity and Interpretability

So far we have seen similar out-of-sample forecast performance of three LASSO-based approaches. Since these approaches are built upon selecting important variables and forecasting based on the selected variables, it is interesting to see how they differ in terms of variable selections. In Table 7 we summarize the final model sparsity of LASSO regressions, elastic net regressions and group LASSO regressions, where sparsity is summarized monthly, and is defined as the proportion of independent variables used to explain the dependent variable. Therefore, the average sparsity indicates on average how many variables are used to predict the economic indicator of interest, and the standard deviation of model sparsity represents whether it is consistent over time. It can be seen that in general LASSO regression gives the most parsimonious model, and the sparsity is consistently small over the full out-of-sample period. On the other hand, elastic net regression and group LASSO regression have comparable sizes of final predictive models. Moreover, it is confirmed that predicting interest rate related variables and employment/unemployment rates requires a relatively large set of independent variables.

Although the *numbers* of variables selected by elastic net and group LASSO are relatively large, the selected variables are quite stable over time, facilitating the model interpretation. In Figure 4, we visualize the variable selection results of LASSO regression, elastic net regression as well as group LASSO regression in forecasting consumer price index (CPI). Compared with LASSO regression, elastic net regression selects more predictors in general, and these predictors could be included in a predictive regression for an extended period of time. By imposing sparsity constraint at the block level, group LASSO delivers the most interpretable results, in that selected independent variables are within the same economically meaningful block in a given month. For example, other than consumer price indices (block K in Figure 4), personal consumption indices (block J) are also strong predictors of the particular CPI time series under investigation. However, CPI is not a predictor of personal consumption (see Table 8 below), suggesting a possible lead-lag relationship between these two variables. Similar patterns are observed in forecasting other variables. To present the results in a concise manner, in Table 8 we summarize the average number of important blocks selected as well as three most frequently selected blocks in forecasting each variable. In forecasting industrial production, for example, variables showing consistent predictive power include interest rate differences (block H), housing starts (block F) and inventories and orders (block N). These findings are consistent with economic interpretations of, for example, Bernanke (1990), Harvey (1989) and Koenig (2002). In this regard, group LASSO regression is the most advantageous in producing highly interpretable results for economists and policy makers. Based on its variable selection results, a clear, dynamic network of numerous economic indicators is revealed.

5.4. Robustness

Lastly we investigate the robustness of these approaches by looking at predictive performances over sub-samples and comparing with a different benchmark. Specifically, Table 9 compares combined forecasts from group LASSO regressions and dynamic factor models with forecasts directly from dynamic factor models over two sub-samples, where the Root Mean Square Error (RMSE) differentials ($\Delta RMSE$) and MSE-t statistics are reported. This forecasts combination is the best one in Table 2. Following D'Agostino, Gambetti and Giannone (2013), among others, we separate the Great Moderation period from the whole out-of-sample evaluation period, and report the forecasting gains before and after January 1985. During the Great Moderation period, two models produce similar forecasts for dividend yield and consumer expectation. Moreover, by comparing with the full-sample results in Table 2, we find that four variables (M1, M2, employment rate and unemployment rate) are slightly more difficult to forecast using the proposed method after the mid-1980s.

Naïve forecasting models that do not include any regressors could be hard to beat. We compare three forecasts combination methods in Table 2 with naïve models, and report results in Table 10. All results are qualitatively similar, but the forecasting gains over the naïve benchmarks decrease for three variables: federal fund rates, 5-year interest rates, and Japanese yen exchange rates. This suggests that, on averages, the naïve models deliver better forecasts than the dynamic factor models for these three financial variables.

6. **Concluding Remarks**

In economics and finance, forecasting multivariate time series using a large number of predictors is an important and challenging problem. In literature efforts have been made on the model tractability and predictive ability, but model interpretation has been less addressed. This paper proposes and tests a collection of LASSO-assisted predictive regressions, where LASSO components are capable of eliminating irrelevant predictors from the predictive model based on data-driven techniques. In this way, dimension reduction is guided by the out-of-sample performance, and both predictive accuracy and model interpretability are enhanced. Although the extents to which three LASSO-based approaches improve dynamic factor model are similar, group LASSO that shrinks variables at the block level has the most interpretable results[5]. Moreover, by combining forecasts from two categories of methods, new forecasts have significantly higher predictive accuracy than those form dynamic factor models. In this regard, LASSO-based approaches could serve as an additional information source that complements dynamic factor-based approaches[6].

Using some LASSO-based approaches, Bai and Ng (2008) refined dynamic factor models by selecting important information for constructing dynamic factors. In our study forecasts combination can be regarded as another way of enhancing dynamic factor models using shrinkage estimation. These highly integrated forecasting techniques provide valuable tools for economists in the presence of complex, dynamic and high-dimensional economic variables.

[5] Note that, however, this method requires the knowledge of the group membership. Determining group membership and its effect on the stability of the results could be interesting topics for future research. We thank a Referee for pointing this out.

[6] Luciani (2014) showed that, in factor models, a variable can be decomposed into two mutually orthogonal components: a common component driven by a small number of pervasive shocks, and an idiosyncratic component driven by non-pervasive shocks. In particular, the idiosyncratic component can be described by a sparse model. Therefore, LASSO-based approaches could help capturing the cross-correlation left over when principal components are extracted. We thank a Referee for pointing this out.

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Table 1 Out-of-sample Root Mean Square Error (RMSE) differentials ($\Delta RMSE$) of forecasts from LASSO-assisted regressions and dynamic factor models (DFM). LASSO-assisted regressions include LASSO regression, elastic net regression (ENET) and group lasso regression (G-LASSO), where tuning parameters are selected by cross validations for each month throughout the out-of-sample evaluation period. The forecasting accuracies are formally tested by Diebold-Mariano test, where p-values associated with the test statistics are reported. p-values indicating significant differences of predictive abilities are in bold (significance level is 0.05).

	LASSO versus DFM		ENET versus DFM		G-LASSO versus DFM	
	$\Delta RMSE$	p-value	$\Delta RMSE$	p-value	$\Delta RMSE$	p-value
FEDERAL FUNDS RATE	0.017	0.339	0.023	0.330	-0.012	0.597
INDUSTRIAL PRODUCTION	0.054	**0.024**	0.061	**0.006**	0.031	0.161
CPI	0.114	**0.001**	0.126	**0.000**	0.129	**0.000**
3m TREASURY BILLS	-0.058	0.762	-0.034	0.693	-0.056	0.778
5y TREASURY BONDS	-0.009	0.582	0.009	0.411	-0.012	0.607
M1	0.123	**0.022**	0.050	0.288	0.073	0.206
M2	0.099	**0.024**	0.096	**0.019**	0.069	0.073
EXCHANGE RATE: JAPAN	0.024	0.153	0.017	0.245	0.034	0.085
COMMODITY PRICE INDEX	0.558	**0.000**	0.551	**0.000**	0.413	**0.000**
CAPACITY UTILIZATION	0.381	**0.000**	0.380	**0.000**	0.091	**0.000**
PERSONAL CONSUMPTION	0.112	**0.009**	0.102	**0.007**	0.120	**0.005**
DURABLE CONSUMPTION	0.068	**0.002**	0.063	**0.004**	0.068	**0.001**
NONDURABLE CONSUMPTION	0.038	0.082	0.051	**0.045**	0.118	**0.000**
UNEMPLOYMENT RATE	0.019	0.192	0.045	**0.016**	-0.002	0.533
EMPLOYMENT RATE	0.006	0.365	0.009	0.305	-0.024	0.867
AVG HOURLY EARNINGS	0.296	**0.000**	0.285	**0.000**	0.275	**0.000**
HOUSING STARTS	0.355	**0.000**	0.350	**0.000**	0.233	**0.000**
NEW ORDERS INDEX	0.228	**0.000**	0.223	**0.000**	0.215	**0.000**
PRICE/DIVIDEND RATIO	0.086	**0.002**	0.084	**0.002**	0.069	**0.009**
CONSUMER EXPECTATIONS	0.028	0.112	0.033	0.059	0.022	0.166

Table 2 Out-of-sample Root Mean Square Error (RMSE) differentials ($\Delta RMSE$) of forecasts from forecasts combinations and dynamic factor models (DFM). Forecasts combinations combine forecasts from LASSO-assisted regressions and dynamic factor models, where the former category includes LASSO regression (combination 1), elastic net regression (combination 2) and group LASSO regression (combination 3), with tuning parameters selected by cross validations, and $\theta = 0.9$ and $\kappa = 60$. The forecasting accuracies are tested by MSE-t statistics (Clark and West; 2006, 2007), where p-values associated with the test statistics are reported. p-values indicating significant differences of predictive abilities are in bold (significance level is 0.05).

	Combination 1 versus DFM		Combination 2 versus DFM		Combination 3 versus DFM	
	$\Delta RMSE$	p-value	$\Delta RMSE$	p-value	$\Delta RMSE$	p-value
FEDERAL FUNDS RATE	0.041	**0.009**	0.038	**0.016**	0.030	**0.011**
INDUSTRIAL PRODUCTION	0.068	**0.000**	0.063	**0.000**	0.062	**0.000**
CPI	0.102	**0.000**	0.107	**0.000**	0.111	**0.000**
3m TREASURY BILLS	0.012	0.077	0.017	0.068	0.022	**0.023**
5y TREASURY BONDS	0.028	**0.001**	0.033	**0.000**	0.030	**0.001**
M1	0.113	**0.004**	0.069	**0.012**	0.102	**0.016**
M2	0.084	**0.003**	0.077	**0.002**	0.067	**0.008**
EXCHANGE RATE: JAPAN	0.034	**0.000**	0.027	**0.001**	0.039	**0.000**
COMMODITY PRICE INDEX	0.530	**0.000**	0.523	**0.000**	0.378	**0.000**
CAPACITY UTILIZATION	0.368	**0.000**	0.367	**0.000**	0.185	**0.000**
PERSONAL CONSUMPTION	0.085	**0.001**	0.077	**0.000**	0.095	**0.000**
DURABLE CONSUMPTION	0.056	**0.000**	0.053	**0.000**	0.056	**0.000**
NONDURABLE CONSUMPTION	0.042	**0.000**	0.052	**0.000**	0.091	**0.000**
UNEMPLOYMENT RATE	0.038	**0.000**	0.051	**0.000**	0.033	**0.000**
EMPLOYMENT RATE	0.026	**0.000**	0.024	**0.000**	0.019	**0.000**
AVG HOURLY EARNINGS	0.245	**0.000**	0.235	**0.000**	0.228	**0.000**
HOUSING STARTS	0.339	**0.000**	0.335	**0.000**	0.253	**0.000**
NEW ORDERS INDEX	0.206	**0.000**	0.199	**0.000**	0.202	**0.000**
PRICE/DIVIDEND RATIO	0.079	**0.000**	0.077	**0.000**	0.073	**0.000**
CONSUMER EXPECTATIONS	0.033	**0.000**	0.034	**0.000**	0.026	**0.002**

Table 3 Correlations of out-of-sample forecasts from two different models. These models include LASSO regression, elastic net regression (ENET), group lasso regression (G-LASSO), dynamic factor model (DFM), and forecasts combinations that combine forecasts from LASSO-assisted regressions and dynamic factor models.

	LASSO, ENET	LASSO, GLASSO	GLASSO, ENET	LASSO, DFM	ENET, DFM	GLASSO, DFM	Comb.1，Comb.2	Comb.1，Comb.3	Comb.2，Comb.3
FEDERAL FUNDS RATE	0.936	0.799	0.762	0.726	0.758	0.612	0.978	0.953	0.943
INDUSTRIAL PRODUCTION	0.914	0.688	0.681	0.687	0.784	0.649	0.982	0.937	0.939
CPI	0.918	0.680	0.691	0.418	0.440	0.319	0.968	0.869	0.877
3m TREASURY BILLS	0.898	0.614	0.640	0.602	0.715	0.479	0.978	0.926	0.933
5y TREASURY BONDS	0.800	0.486	0.537	0.540	0.565	0.411	0.963	0.906	0.908
M1	0.561	0.454	0.846	0.415	0.616	0.407	0.875	0.819	0.950
M2	0.896	0.720	0.670	0.477	0.591	0.418	0.966	0.906	0.897
EXCHANGE RATE: JAPAN	0.763	0.481	0.473	0.338	0.465	0.270	0.949	0.880	0.883
COMMODITY PRICE INDEX	0.998	0.944	0.943	0.555	0.556	0.547	0.998	0.947	0.946
CAPACITY UTILIZATION	1.000	0.883	0.884	0.826	0.824	0.713	1.000	0.961	0.961
PERSONAL CONSUMPTION	0.937	0.700	0.720	0.508	0.575	0.472	0.982	0.907	0.907
DURABLE CONSUMPTION	0.942	0.634	0.643	0.347	0.327	0.266	0.975	0.839	0.844
NONDURABLE CONSUMPTION	0.901	0.629	0.681	0.547	0.524	0.441	0.963	0.848	0.867
UNEMPLOYMENT RATE	0.901	0.688	0.706	0.658	0.686	0.570	0.971	0.925	0.927
EMPLOYMENT RATE	0.921	0.596	0.594	0.638	0.688	0.421	0.983	0.925	0.925
AVG HOURLY EARNINGS	0.975	0.863	0.858	0.214	0.240	0.178	0.981	0.898	0.897
HOUSING STARTS	0.998	0.924	0.924	0.801	0.798	0.761	0.999	0.961	0.961
NEW ORDERS INDEX	0.993	0.955	0.949	0.805	0.818	0.773	0.996	0.976	0.973
DIVIDEND YIELD	0.999	0.996	0.995	0.963	0.964	0.961	1.000	0.999	0.999
CONSUMER EXPECTATIONS	0.752	0.546	0.512	0.151	0.333	0.221	0.936	0.888	0.872
Average	0.900	0.714	0.735	0.561	0.613	0.494	0.972	0.914	0.920

Table 4 The ratio of the maximum Root Mean Square Error (RMSE) from an alternative model and the maximum Root Mean Square Error (RMSE) from dynamic factor models (DFM). Alternative models include LASSO regression, elastic net regression, group LASSO regression, as well as their forecasts combinations with dynamic factor models. Ratios less than one are in bold, suggesting the alternative model produces less extreme forecasting errors.

	LASSO vs. DFM	ENET vs. DFM	G-LASSO vs. DFM	Comb. 1 vs. DFM	Comb. 2 vs. DFM	Comb. 3 vs. DFM
FEDERAL FUNDS RATE	1.123	1.241	1.163	1.057	1.125	1.080
INDUSTRIAL PRODUCTION	1.146	1.145	1.138	1.069	1.070	1.063
CPI	**0.893**	**0.892**	**0.974**	**0.942**	**0.942**	**0.985**
3m TREASURY BILLS	1.872	1.741	1.752	1.422	1.360	1.390
5y TREASURY BONDS	**0.996**	**0.974**	1.062	**0.998**	**0.976**	**0.988**
M1	**0.919**	**0.900**	**0.843**	**0.956**	**0.944**	**0.914**
M2	**0.755**	**0.721**	**0.818**	**0.865**	**0.845**	**0.900**
EXCHANGE RATE: JAPAN	**0.950**	**0.962**	**0.963**	**0.933**	**0.938**	**0.931**
COMMODITY PRICE INDEX	**0.496**	**0.507**	**0.589**	**0.474**	**0.469**	**0.566**
CAPACITY UTILIZATION	**0.326**	**0.298**	**0.800**	**0.329**	**0.330**	**0.676**
PERSONAL CONSUMPTION	**0.662**	**0.671**	**0.618**	**0.720**	**0.767**	**0.726**
DURABLE CONSUMPTION	**0.795**	**0.779**	**0.881**	**0.887**	**0.885**	**0.942**
NONDURABLE CONSUMPTION	**0.966**	**0.918**	**0.933**	**0.983**	**0.959**	**0.910**
UNEMPLOYMENT RATE	**0.849**	**0.774**	**0.940**	**0.908**	**0.892**	**0.879**
EMPLOYMENT RATE	1.093	1.091	1.045	1.049	1.047	1.024
AVG HOURLY EARNINGS	**0.586**	**0.591**	**0.633**	**0.736**	**0.740**	**0.770**
HOUSING STARTS	**0.579**	**0.612**	**0.713**	**0.565**	**0.572**	**0.653**
NEW ORDERS INDEX	**0.799**	**0.796**	**0.679**	**0.809**	**0.808**	**0.704**
PRICE/DIVIDEND RATIO	**0.970**	**0.970**	**0.924**	**0.967**	**0.970**	**0.951**
CONSUMER EXPECTATIONS	1.001	1.044	1.007	1.000	1.025	1.004

Table 5 The ratio of the Root Mean Square Error (RMSE) variances from an alternative model and the dynamic factor models (DFM). Alternative models include LASSO regression, elastic net regression, group LASSO regression, as well as their forecasts combinations with dynamic factor models. Ratios less than one are in bold, suggesting the predictive ability of the alternative model is more stable over time.

	LASSO vs. DFM	ENET vs. DFM	G-LASSO vs. DFM	Comb. 1 vs. DFM	Comb. 2 vs. DFM	Comb. 3 vs. DFM
FEDERAL FUNDS RATE	**0.966**	**0.954**	1.023	**0.921**	**0.926**	**0.942**
INDUSTRIAL PRODUCTION	**0.875**	**0.859**	**0.930**	**0.846**	**0.857**	**0.862**
CPI	**0.805**	**0.785**	**0.781**	**0.824**	**0.816**	**0.810**
3m TREASURY BILLS	1.118	1.069	1.111	**0.978**	**0.969**	**0.960**
5y TREASURY BONDS	1.017	**0.984**	1.023	**0.951**	**0.940**	**0.944**
M1	**0.798**	**0.915**	**0.877**	**0.814**	**0.884**	**0.831**
M2	**0.833**	**0.837**	**0.882**	**0.857**	**0.869**	**0.885**
EXCHANGE RATE: JAPAN	**0.959**	**0.972**	**0.942**	**0.942**	**0.953**	**0.933**
COMMODITY PRICE INDEX	**0.143**	**0.149**	**0.279**	**0.166**	**0.173**	**0.318**
CAPACITY UTILIZATION	**0.076**	**0.076**	**0.688**	**0.091**	**0.091**	**0.440**
PERSONAL CONSUMPTION	**0.812**	**0.827**	**0.799**	**0.855**	**0.869**	**0.839**
DURABLE CONSUMPTION	**0.863**	**0.874**	**0.863**	**0.886**	**0.893**	**0.887**
NONDURABLE CONSUMPTION	**0.933**	**0.912**	**0.800**	**0.926**	**0.909**	**0.843**
UNEMPLOYMENT RATE	**0.957**	**0.904**	1.002	**0.918**	**0.891**	**0.928**
EMPLOYMENT RATE	**0.984**	**0.978**	1.060	**0.938**	**0.943**	**0.955**
AVG HOURLY EARNINGS	**0.493**	**0.508**	**0.522**	**0.568**	**0.582**	**0.593**
HOUSING STARTS	**0.240**	**0.247**	**0.452**	**0.263**	**0.269**	**0.410**
NEW ORDERS INDEX	**0.449**	**0.458**	**0.475**	**0.493**	**0.507**	**0.501**
PRICE/DIVIDEND RATIO	**0.857**	**0.860**	**0.887**	**0.871**	**0.873**	**0.879**
CONSUMER EXPECTATIONS	**0.952**	**0.943**	**0.962**	**0.943**	**0.942**	**0.954**

Table 6 Out-of-sample Root Mean Square Error (RMSE) differentials ($\Delta RMSE$) of forecasts from alternative predictive regressions and dynamic factor models (DFM). Alternative predictive regressions include ridge regressions (Ridge), the forecasts combinations of ridge regressions and dynamic factor models (Combination 4), and LASSO regressions on both individual predictors and the dynamic factors simultaneously (LASSO4All). For ridge regressions, tuning parameters are selected by cross validations for each month throughout the out-of-sample evaluation period. The forecasting accuracies are formally tested by Diebold-Mariano test, where p-values associated with the test statistics are reported. p-values indicating significant differences of predictive abilities are in bold (significance level is 0.05).

	Ridge versus DFM		Combination 4 versus DFM		LASSO4All versus DFM	
	$\Delta RMSE$	p-value	$\Delta RMSE$	p-value	$\Delta RMSE$	p-value
FEDERAL FUNDS RATE	0.020	0.364	0.043	0.052	0.045	0.118
INDUSTRIAL PRODUCTION	0.071	**0.001**	0.064	**0.000**	0.055	**0.017**
CPI	0.100	**0.001**	0.080	**0.000**	0.110	**0.001**
3m TREASURY BILLS	-0.030	0.662	0.018	0.081	-0.059	0.797
5y TREASURY BONDS	0.014	0.373	0.037	**0.001**	-0.007	0.567
M1	0.075	0.100	0.085	**0.007**	0.101	**0.040**
M2	0.080	**0.027**	0.067	**0.004**	0.090	**0.031**
EXCHANGE RATE: JAPAN	0.028	0.108	0.031	**0.001**	0.010	0.329
COMMODITY PRICE INDEX	0.220	**0.000**	0.205	**0.000**	0.561	**0.000**
CAPACITY UTILIZATION	0.062	**0.000**	0.083	**0.000**	0.381	**0.000**
PERSONAL CONSUMPTION	0.076	**0.047**	0.062	**0.003**	0.088	**0.014**
DURABLE CONSUMPTION	0.027	0.065	0.028	**0.004**	0.059	**0.007**
NONDURABLE CONSUMPTION	0.056	**0.011**	0.045	**0.000**	0.020	0.227
UNEMPLOYMENT RATE	0.047	**0.004**	0.042	**0.000**	0.031	0.060
EMPLOYMENT RATE	0.020	0.096	0.026	**0.000**	0.014	0.177
AVG HOURLY EARNINGS	0.151	**0.000**	0.108	**0.000**	0.290	**0.000**
HOUSING STARTS	0.103	**0.000**	0.101	**0.000**	0.351	**0.000**
NEW ORDERS INDEX	0.116	**0.000**	0.096	**0.000**	0.229	**0.000**
PRICE/DIVIDEND RATIO	0.085	**0.002**	0.075	**0.000**	0.087	**0.002**
CONSUMER EXPECTATIONS	0.023	0.125	0.025	**0.002**	0.037	0.055

Table 7 Final model sparsity of LASSO regressions, elastic net regressions and group LASSO regressions, where sparsity in each month is defined as the proportion of independent variables selected to explain the dependent variable.

	LASSO		ENET		G-LASSO	
	Sparsity - mean	Sparsity - stdev	Sparsity - mean	Sparsity - stdev	Sparsity - mean	Sparsity - stdev
FEDERAL FUNDS RATE	0.152	0.112	0.609	0.357	0.459	0.235
INDUSTRIAL PRODUCTION	0.126	0.074	0.480	0.364	0.388	0.239
CPI	0.145	0.114	0.323	0.282	0.426	0.257
3m TREASURY BILLS	0.159	0.076	0.563	0.373	0.434	0.237
5y TREASURY BONDS	0.093	0.084	0.429	0.339	0.373	0.256
M1	0.125	0.094	0.293	0.266	0.451	0.296
M2	0.133	0.099	0.402	0.310	0.392	0.306
EXCHANGE RATE: JAPAN	0.058	0.077	0.440	0.398	0.187	0.180
COMMODITY PRICE INDEX	0.191	0.081	0.220	0.091	0.597	0.157
CAPACITY UTILIZATION	0.191	0.083	0.207	0.092	0.346	0.143
PERSONAL CONSUMPTION	0.099	0.082	0.185	0.180	0.328	0.260
DURABLE CONSUMPTION	0.069	0.053	0.104	0.114	0.185	0.171
NONDURABLE CONSUMPTION	0.119	0.097	0.304	0.275	0.390	0.282
UNEMPLOYMENT RATE	0.168	0.113	0.559	0.350	0.580	0.307
EMPLOYMENT RATE	0.135	0.097	0.501	0.360	0.468	0.337
AVG HOURLY EARNINGS	0.175	0.106	0.243	0.149	0.446	0.287
HOUSING STARTS	0.150	0.068	0.176	0.083	0.330	0.215
NEW ORDERS INDEX	0.208	0.073	0.279	0.118	0.495	0.228
PRICE/DIVIDEND RATIO	0.023	0.031	0.490	0.444	0.172	0.182
CONSUMER EXPECTATIONS	0.071	0.075	0.459	0.402	0.175	0.237

Table 8 Variable selection results of group LASSO regressions. For each dependent variable, some blocks of independent variables (active blocks) are selected out of 14 blocks in each month. We report the average number of active blocks over the full out-of-sample evaluation period, and three most frequently selected blocks. Three variables discussed in Section 5.3 are in bold.

	Avg. num. of active blocks	Most frequently selected blocks		
		1st	2nd	3rd
FEDERAL FUNDS RATE	6.9	Interest rates	Inventories and Orders	Stock Prices
INDUSTRIAL PRODUCTION	**6.1**	**Inventories and Orders**	**Interest rates differences**	**Housing Starts and Sales**
CPI	**6.1**	**Consumptions**	**Price Indexes**	**Money and Credit**
3m TREASURY BILLS	6.7	Interest rates	Hours	Interest rates differences
5y TREASURY BONDS	6.0	Interest rates	Stock Prices	Interest rates differences
M1	6.4	Money and Credit	Interest rates	UnEmployment
M2	5.7	Money and Credit	UnEmployment	Interest rates
EXCHANGE RATE: JAPAN	3.3	Exchange Rates	Interest rates differences	Stock Prices
COMMODITY PRICE INDEX	8.1	Price Indexes	Inventories and Orders	Interest rates differences
CAPACITY UTILIZATION	5.5	Interest rates differences	Hours	Housing Starts and Sales
PERSONAL CONSUMPTION	**5.1**	**Consumptions**	**Hourly Earnings**	**Money and Credit**
DURABLE CONSUMPTION	3.4	Consumptions	UnEmployment	Hourly Earnings
NONDURABLE CONSUMPTION	5.6	Consumptions	Hourly Earnings	Money and Credit
UNEMPLOYMENT RATE	8.4	Inventories and Orders	Interest rates differences	Hours
EMPLOYMENT RATE	6.9	Housing Starts and Sales	Inventories and Orders	UnEmployment
AVG HOURLY EARNINGS	6.8	Hourly Earnings	Hours	Consumptions
HOUSING STARTS	5.6	Housing Starts and Sales	Interest rates differences	Inventories and Orders
NEW ORDERS INDEX	7.8	Interest rates differences	Inventories and Orders	Housing Starts and Sales
PRICE/DIVIDEND RATIO	3.0	Interest rates differences	Hours	Stock Prices
CONSUMER EXPECTATIONS	2.8	Stock Prices	Consumptions	Hours

Table 9 Out-of-sample Root Mean Square Error (RMSE) differentials ($\Delta RMSE$) of forecasts from forecasts combinations and dynamic factor models (DFM) over two sub-samples. Forecasts combinations combine forecasts from group LASSO regressions (combination 3) and dynamic factor models, with tuning parameters selected by cross validations, and $\theta = 0.9$ and $\kappa = 60$. The forecasting accuracies are tested by MSE-t statistics (Clark and West; 2006, 2007), where p-values associated with the test statistics are reported. p-values indicating significant differences of predictive abilities are in bold (significance level is 0.05).

	1969 - 1984		1985 - 2008	
	$\Delta RMSE$	p-value	$\Delta RMSE$	p-value
FEDERAL FUNDS RATE	0.011	**0.012**	**0.096**	**0.000**
INDUSTRIAL PRODUCTION	0.076	**0.000**	**0.055**	**0.000**
CPI	0.191	**0.017**	**0.047**	**0.001**
3m TREASURY BILLS	-0.003	**0.000**	**0.081**	**0.000**
5y TREASURY BONDS	0.017	0.194	**0.053**	**0.000**
M1	0.016	**0.004**	**0.134**	**0.042**
M2	0.001	**0.037**	**0.099**	**0.018**
EXCHANGE RATE: JAPAN	0.028	**0.027**	**0.047**	**0.002**
COMMODITY PRICE INDEX	0.210	**0.000**	**0.490**	**0.000**
CAPACITY UTILIZATION	0.233	**0.000**	**0.190**	**0.000**
PERSONAL CONSUMPTION	0.091	**0.020**	**0.100**	**0.000**
DURABLE CONSUMPTION	0.020	**0.016**	**0.080**	**0.000**
NONDURABLE CONSUMPTION	0.087	**0.019**	**0.096**	**0.000**
UNEMPLOYMENT RATE	0.054	**0.000**	**0.018**	**0.012**
EMPLOYMENT RATE	0.034	**0.000**	**0.004**	**0.012**
AVG HOURLY EARNINGS	0.327	**0.000**	**0.126**	**0.000**
HOUSING STARTS	0.193	**0.000**	**0.279**	**0.000**
NEW ORDERS INDEX	0.177	**0.000**	**0.217**	**0.000**
PRICE/DIVIDEND RATIO	0.064	**0.003**	**0.081**	0.564
CONSUMER EXPECTATIONS	0.009	**0.030**	**0.034**	0.503

Table 10 Out-of-sample Root Mean Square Error (RMSE) differentials ($\Delta RMSE$) of forecasts from forecasts combinations and Naïve models without regressors (Naïve). Forecasts combinations combine forecasts from LASSO-assisted regressions and dynamic factor models, where the former category includes LASSO regression (combination 1), elastic net regression (combination 2) and group LASSO regression (combination 3), with tuning parameters selected by cross validations, and $\theta = 0.9$ and $\kappa = 60$. The forecasting accuracies are tested by MSE-*t* statistics (Clark and West; 2006, 2007), where p-values associated with the test statistics are reported. p-values indicating significant differences of predictive abilities are in bold (significance level is 0.05).

	Combination 1 versus Naïve		Combination 2 versus Naïve		Combination 3 versus Naïve	
	$\Delta RMSE$	p-value	$\Delta RMSE$	p-value	$\Delta RMSE$	p-value
FEDERAL FUNDS RATE	0.042	**0.001**	0.039	**0.000**	0.031	**0.004**
INDUSTRIAL PRODUCTION	0.066	**0.000**	0.063	**0.000**	0.064	**0.000**
CPI	0.101	**0.000**	0.106	**0.000**	0.110	**0.000**
3m TREASURY BILLS	0.012	**0.000**	0.018	**0.000**	0.023	**0.000**
5y TREASURY BONDS	0.030	**0.001**	0.037	**0.000**	0.033	**0.000**
M1	0.110	**0.003**	0.067	**0.044**	0.099	**0.031**
M2	0.082	**0.001**	0.075	**0.000**	0.066	**0.004**
EXCHANGE RATE: JAPAN	0.034	**0.015**	0.027	**0.002**	0.040	**0.000**
COMMODITY PRICE INDEX	0.525	**0.000**	0.518	**0.000**	0.374	**0.000**
CAPACITY UTILIZATION	0.365	**0.000**	0.365	**0.000**	0.207	**0.000**
PERSONAL CONSUMPTION	0.084	**0.000**	0.076	**0.000**	0.097	**0.000**
DURABLE CONSUMPTION	0.054	**0.000**	0.052	**0.000**	0.056	**0.000**
NONDURABLE CONSUMPTION	0.041	**0.002**	0.051	**0.000**	0.092	**0.000**
UNEMPLOYMENT RATE	0.037	**0.000**	0.049	**0.000**	0.034	**0.000**
EMPLOYMENT RATE	0.026	**0.000**	0.023	**0.000**	0.018	**0.000**
AVG HOURLY EARNINGS	0.239	**0.000**	0.229	**0.000**	0.224	**0.000**
HOUSING STARTS	0.328	**0.000**	0.325	**0.000**	0.244	**0.000**
NEW ORDERS INDEX	0.201	**0.000**	0.194	**0.000**	0.200	**0.000**
PRICE/DIVIDEND RATIO	0.077	**0.002**	0.076	**0.012**	0.074	0.260
CONSUMER EXPECTATIONS	0.032	**0.005**	0.033	**0.000**	0.026	**0.047**

Figure 1 Mean squared forecast error (MSFE) from factor model (dashed line - - -), sparse model (dotted line ⋯) and forecasts combination (blue solid line —) as a function of p.

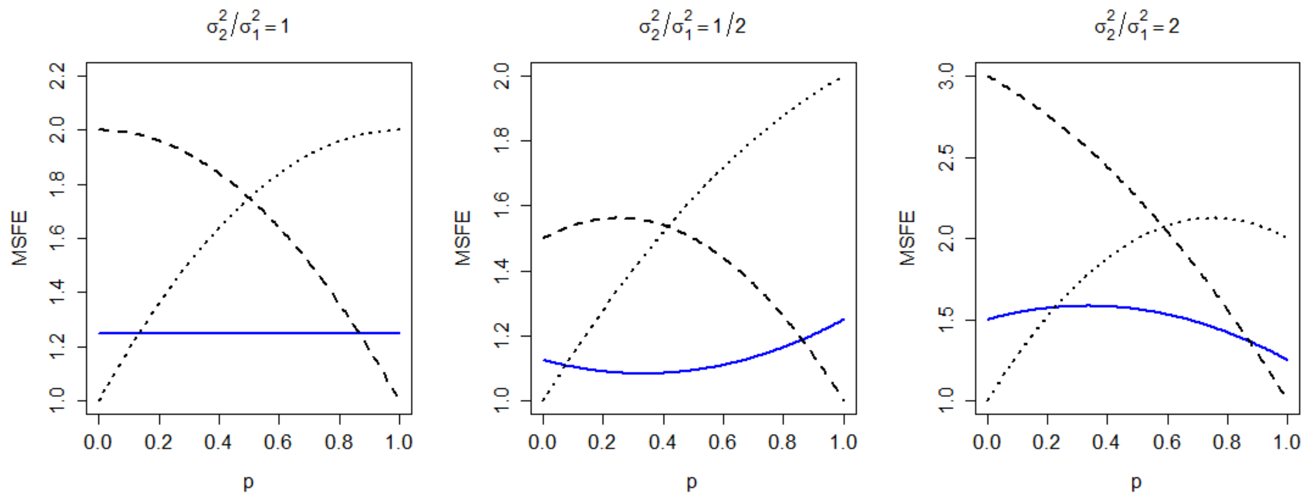


Figure 2 Out-of-sample Root Mean Square Error (RMSE) differentials ($\Delta RMSE$) over time, where forecasts are from two models: LASSO regressions and dynamic factor models (DFM).

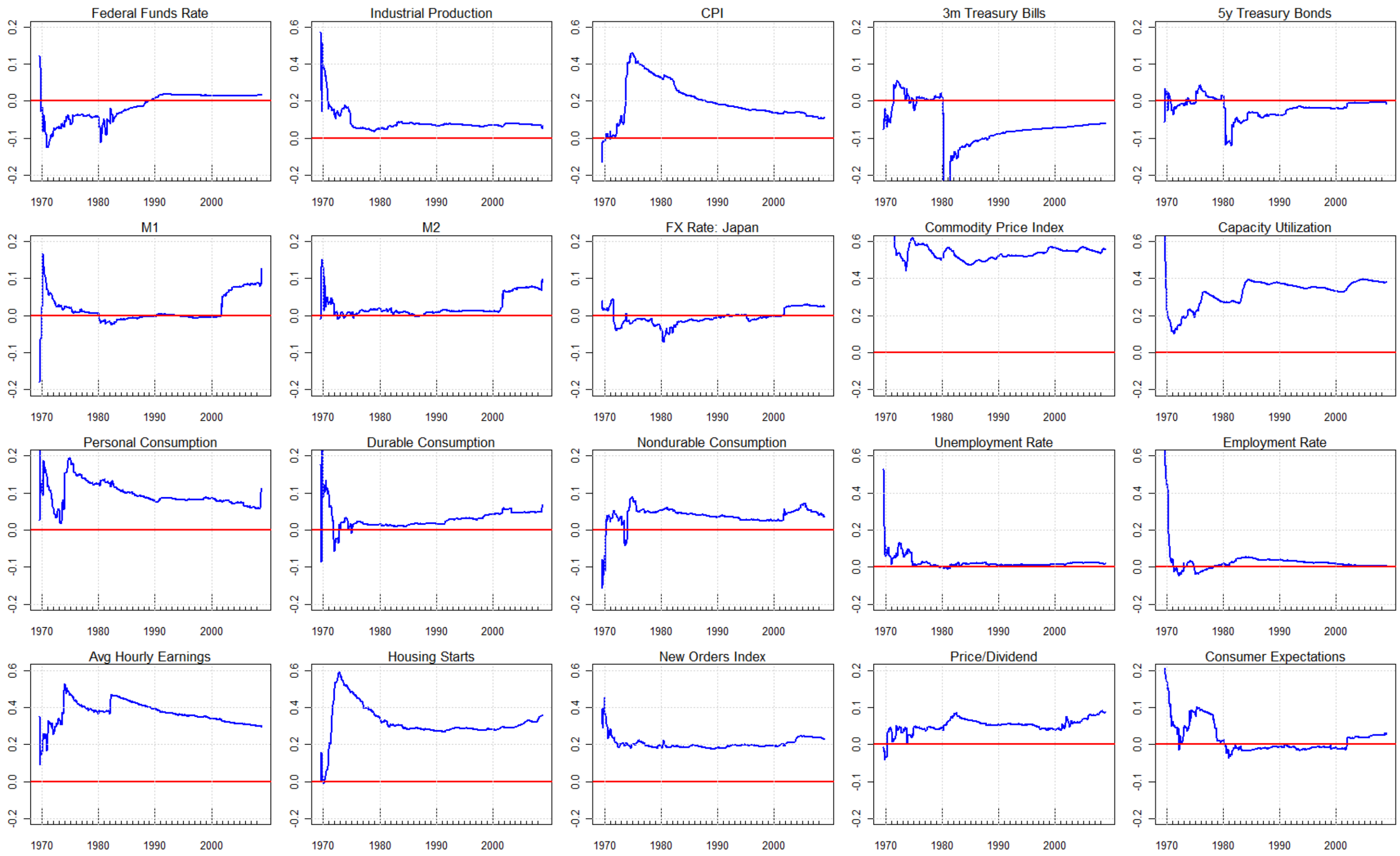


Figure 3 Out-of-sample Root Mean Square Error (RMSE) differentials ($\Delta RMSE$) over time, where forecasts are from two models: (a) the forecasts combination of LASSO regressions and dynamic factor models (DFM), and (b) dynamic factor models (DFM).

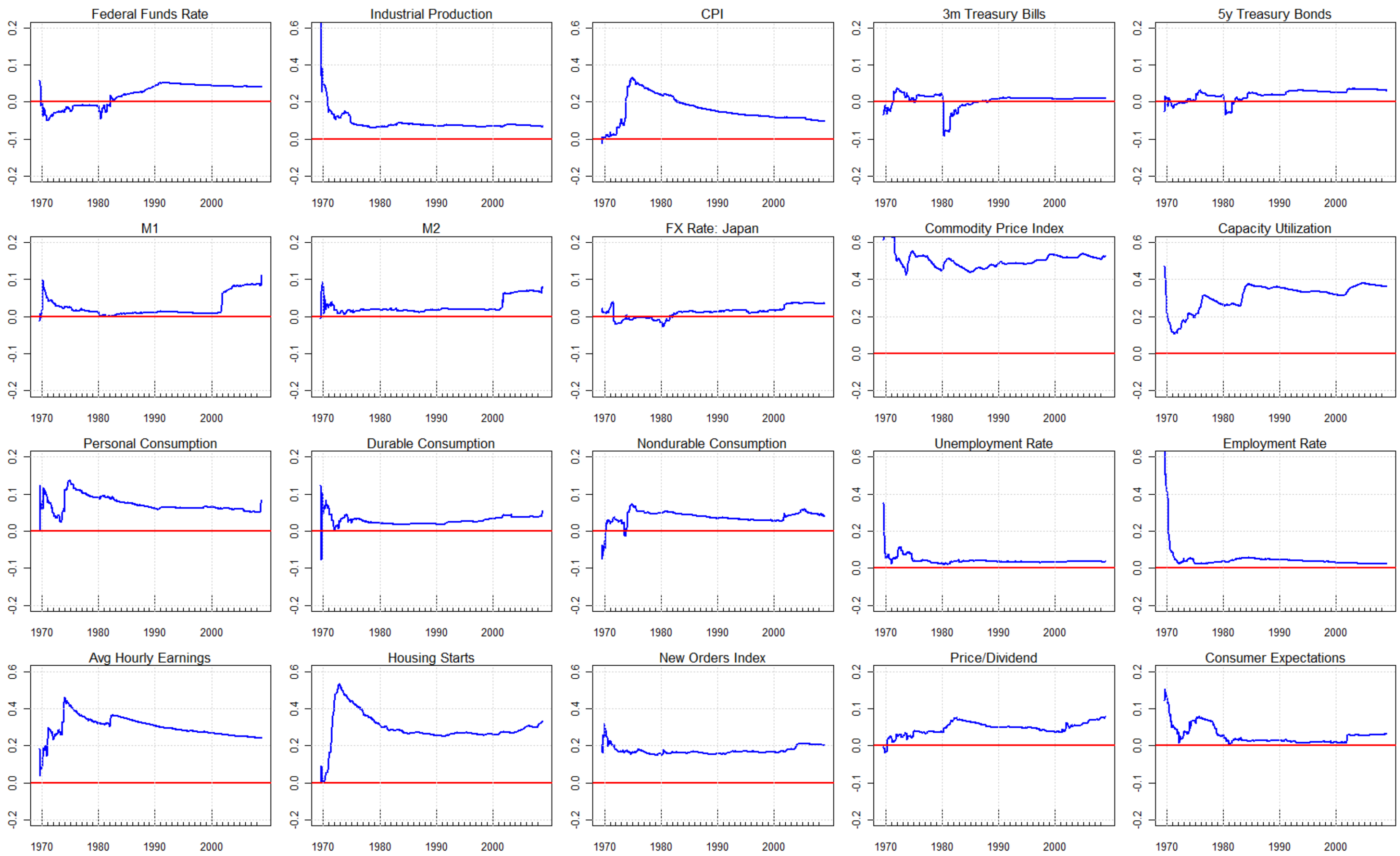
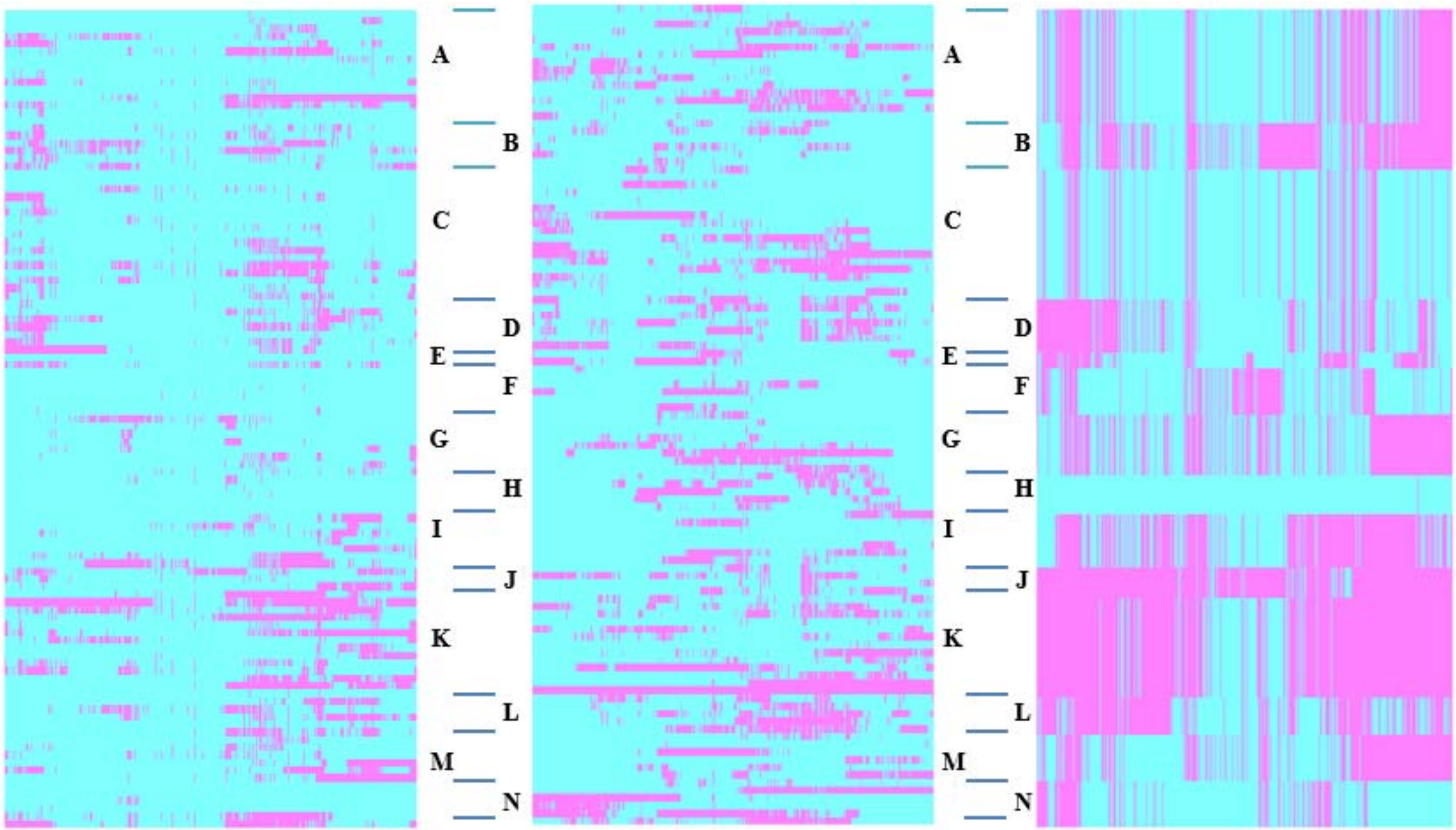


Figure 4 Variable selected for forecasting Consumer Price Index (CPI) using (a) LASSO regression, (b) ENET regression, and (c) group LASSO regression, where x-axis shows the month within the out-of-sample evaluation period, and y-axis is a total of 107 potential independent variables. Magenta color represents variables being selected and included in the predictive regression in a given month.



(a) LASSO (b) ENET (c) Group LASSO

APPENDIX A: MSFE OF COMBINED FORECASTS

Suppose that model uncertainty takes a shift in the data generating process (DGP) of the variable to be predicted, x_t, between a factor model and a sparse model:

$$x_t = \mathrm{I}_{\{S_t=1\}} L^T F_t + \mathrm{I}_{\{S_t=2\}} \Phi^T Z_{t-1} + e_t,$$

where s_t is a Bernoulli random variable indicating two states with $\Pr(S_t = 1) = p$ and $\Pr(S_t = 2) = 1 - p, p \in (0,1)$, $F_t = (f_{1,t}, \dots, f_{m_1,t})^T$ is an m_1-dimensional vector of factors, Z_{t-1} is a vector of lagged predictors, and e_t is a random error with mean zero and variance σ_e^2. When a forecast is produced by the factor model and the true DGP is $S_t = 1$, the population value of the projection coefficient of x_t on $\hat{x}_t$, $\beta_{1|S_t=1}$, is given by

$$\beta_{1|S_t=1} = \mathrm{E}\left[F_t {F_t}^T\right]^{-1} \mathrm{E}[F_t(L^T F_t + e_t)^T] = \Sigma_1^{-1}\Sigma_1 L = L.$$

When a forecast is produced by the same factor model but the true DGP is $S_t = 0$,

$$\beta_{1|S_t=0} = \mathrm{E}\left[F_t {F_t}^T\right]^{-1} \mathrm{E}[F_t(\Phi^T Z_{t-1} + e_t)^T] = \Sigma_1^{-1}\Sigma_{12}\Phi.$$

Therefore, the unconditional value of the projection coefficient is

$$\beta_1 = pL + (1-p)\Sigma_1^{-1}\Sigma_{12}\Phi,$$

and the associated mean squared forecast error (MSFE) is

$$\begin{aligned} \mathrm{MSFE}\left(\hat{x}_t^{(D)}\right) &= p\mathrm{Var}\left(x_t - \hat{x}_t^{(D,S_t=1)}\right) + (1-p)\mathrm{Var}\left(x_t - \hat{x}_t^{(D,S_t=2)}\right) \\ &= p\mathrm{Var}(L^T F_t + e_t - \beta_1^T F_t) + (1-p)\mathrm{Var}(\Phi^T Z_{t-1} + e_t - \beta_1^T F_t) \\ &= p(L^T - \beta_1^T)\Sigma_1(L - \beta_1) + (1-p)(\Phi^T \Sigma_2 \Phi + \beta_1^T \Sigma_1 \beta_1 - 2\Phi^T \Sigma_{12}\beta_1) + \sigma_e^2. \end{aligned}$$

Similarly, the MSFE associated with a forecast produced by a sparse model and the MSFE associated with a combined forecast can be derived.

APPENDIX B: DATA DESCRIPTION

All macroeconomic data series considered in the empirical study are directly taken from DRI/McGraw Hill Basic Economic Database. Four columns correspond to series number, series mnemonic, transformation code and series description. The transformation codes are 1 – no transformation; 2 – first difference; 3 – second difference; 4 – logarithm; 5 – first difference of logarithm, 6 – second difference of logarithm. Twenty variables investigated in details are in bold and marked with asterisk (*).

Real Output and Income (Block A)

1*	**IPS10**	**5**	**INDUSTRIAL PRODUCTION INDEX - TOTAL INDEX**
2	IPS11	5	INDUSTRIAL PRODUCTION INDEX - PRODUCTS, TOTAL
3	IPS299	5	INDUSTRIAL PRODUCTION INDEX - FINAL PRODUCTS

4	IPS12	5	INDUSTRIAL PRODUCTION INDEX - CONSUMER GOODS
5	IPS13	5	INDUSTRIAL PRODUCTION INDEX - DURABLE CONSUMER GOODS
6	IPS18	5	INDUSTRIAL PRODUCTION INDEX - NONDURABLE CONSUMER GOODS
7	IPS25	5	INDUSTRIAL PRODUCTION INDEX - BUSINESS EQUIPMENT
8	IPS32	5	INDUSTRIAL PRODUCTION INDEX - MATERIALS
9	IPS34	5	INDUSTRIAL PRODUCTION INDEX - DURABLE GOODS MATERIALS
10	IPS38	5	INDUSTRIAL PRODUCTION INDEX - NONDURABLE GOODS MATERIALS
11	IPS43	5	INDUSTRIAL PRODUCTION INDEX - MANUFACTURING (SIC)
12	IPS307	5	INDUSTRIAL PRODUCTION INDEX - RESIDENTIAL UTILITIES
13	IPS306	5	INDUSTRIAL PRODUCTION INDEX - FUELS
14	PMP	1	NAPM PRODUCTION INDEX (PERCENT)
15*	**UTL11**	**1**	**CAPACITY UTILIZATION - MANUFACTURING (SIC)**

Hourly Earnings (Block B)

16*	**CES275**	**6**	**AVG HRLY EARNINGS, PROD WRKRS, NONFARM - GOODS-PRODUCING**
17	CES277	6	AVG HRLY EARNINGS, PROD WRKRS, NONFARM - CONSTRUCTION
18	CES278	6	AVG HRLY EARNINGS, PROD WRKRS, NONFARM - MFG
19	CES275R	5	REAL AVG HRLY EARNINGS, PROD WRKRS, NONFARM - GOODS-PRODUCING
20	CES277R	5	REAL AVG HRLY EARNINGS, PROD WRKRS, NONFARM - CONSTRUCTION
21	CES278 R	5	REAL AVG HRLY EARNINGS, PROD WRKRS, NONFARM - MFG

Employment (Block C)

22	CES002	5	EMPLOYEES, NONFARM - TOTAL PRIVATE
23	CES003	5	EMPLOYEES, NONFARM - GOODS-PRODUCING
24	CES006	5	EMPLOYEES, NONFARM - MINING
25	CES011	5	EMPLOYEES, NONFARM - CONSTRUCTION
26	CES015	5	EMPLOYEES, NONFARM - MFG
27	CES017	5	EMPLOYEES, NONFARM - DURABLE GOODS
28	CES033	5	EMPLOYEES, NONFARM - NONDURABLE GOODS
29	CES046	5	EMPLOYEES, NONFARM - SERVICE-PROVIDING
30	CES048	5	EMPLOYEES, NONFARM - TRADE, TRANSPORT, UTILITIES
31	CES049	5	EMPLOYEES, NONFARM - WHOLESALE TRADE
32	CES053	5	EMPLOYEES, NONFARM - RETAIL TRADE
33	CES088	5	EMPLOYEES, NONFARM - FINANCIAL ACTIVITIES
34	CES140	5	EMPLOYEES, NONFARM - GOVERNMENT
35	LHEL	2	INDEX OF HELP-WANTED ADVERTISING IN NEWSPAPERS (1967=100;SA)
36	LHELX	2	EMPLOYMENT: RATIO; HELP-WANTED ADS:NO. UNEMPLOYED CLF

37*	**LHEM**	**5**	**CIVILIAN LABOR FORCE: EMPLOYED, TOTAL (THOUS.,SA)**
38	LHNAG	5	CIVILIAN LABOR FORCE: EMPLOYED, NONAGRIC.INDUSTRIES (THOUS.,SA)

Unemployment (Block D)

39*	**LHUR**	**2**	**UNEMPLOYMENT RATE: ALL WORKERS, 16 YEARS & OVER (%,SA)**
40	LHU680	2	UNEMPLOY.BY DURATION: AVERAGE(MEAN)DURATION IN WEEKS (SA)
41	LHU5	5	UNEMPLOY.BY DURATION: PERSONS UNEMPL.LESS THAN 5 WKS (THOUS.,SA)
42	LHU14	5	UNEMPLOY.BY DURATION: PERSONS UNEMPL.5 TO 14 WKS (THOUS.,SA)
43	LHU15	5	UNEMPLOY.BY DURATION: PERSONS UNEMPL.15 WKS + (THOUS.,SA)
44	LHU26	5	UNEMPLOY.BY DURATION: PERSONS UNEMPL.15 TO 26 WKS (THOUS.,SA)
45	LHU27	5	UNEMPLOY.BY DURATION: PERSONS UNEMPL.27 WKS + (THOUS,SA)

Hours (Block E)

46	CES151	1	AVG WKLY HOURS, PROD WRKRS, NONFARM - GOODS-PRODUCING
47	CES155	2	AVG WKLY OVERTIME HOURS, PROD WRKRS, NONFARM - MFG

Housing Starts and Sales (Block F)

48	HSBR	4	HOUSING AUTHORIZED: TOTAL NEW PRIV HOUSING UNITS (THOUS.,SAAR)
49*	**HSFR**	**4**	**HOUSING STARTS:TOTAL FARM&NONFARM (THOUS.U.)S.A.**
50	HSNE	4	HOUSING STARTS:NORTHEAST (THOUS.U.)S.A.
51	HSMW	4	HOUSING STARTS:MIDWEST(THOUS.U.)S.A.
52	HSSOU	4	HOUSING STARTS:SOUTH (THOUS.U.)S.A.
53	HSWST	4	HOUSING STARTS:WEST (THOUS.U.)S.A.

Interest Rates (Block G)

54*	**FYFF**	**2**	**INTEREST RATE: FEDERAL FUNDS (EFFECTIVE) (% PER ANNUM,NSA)**
55*	**FYGM3**	**2**	**INTEREST RATE: U.S.TREASURY BILLS,SEC MKT,3-MO.(% PER ANN,NSA)**
56	FYGM6	2	INTEREST RATE: U.S.TREASURY BILLS,SEC MKT,6-MO.(% PER ANN,NSA)
57	FYGT1	2	INTEREST RATE: U.S.TREASURY CONST MATURITIES,1-YR.(% PER ANN,NSA)
58*	**FYGT5**	**2**	**INTEREST RATE: U.S.TREASURY CONST MATURITIES,5-YR.(% PER ANN,NSA)**
59	FYGT10	2	INTEREST RATE: U.S.TREASURY CONST MATURITIES,10-YR.(% PER ANN,NSA)
60	FYAAAC	2	BOND YIELD: MOODY'S AAA CORPORATE (% PER ANNUM)
61	FYBAAC	2	BOND YIELD: MOODY'S BAA CORPORATE (% PER ANNUM)

Interest Rates Differences (Block H)

62	SFYGM6	1	FYGM6-FYGM3
63	SFYGT1	1	FYGT1-FYGM3
64	SFYGT10	1	FYGT10-FYGM3

65	SFYAAAC	1	FYAAAC-FYGT10
66	SFYBAAC	1	FYBAAC-FYGT10

Money and Credit Quantity Aggregates (Block I)

67*	**FM1**	**6**	**M1(CURR,TRAV.CKS,DEM DEP,OTHER CK'ABLE DEP)(BIL$,SA)**
68	MZMSL	6	MZM (SA) FRB ST. LOUIS
69*	**FM2**	**6**	**M2(M1+O'NITE RPS,EURO$,G/P&B/D MMMFS&SAV&SM TIME DEP(BIL$,SA)**
70	FMFBA	6	MONETARY BASE, ADJ FOR RESERVE REQUIREMENT CHANGES(MIL$,SA)
71	FMRRA	6	DEPOSITORY INST RESERVES:TOTAL,ADJ FOR RESERVE REQ CHGS(MIL$,SA)
72	BUSLOANS	6	COMMERCIAL AND INDUSTRIAL LOANS AT ALL COMMERCIAL BANKS (BI$,SA)
73	CCINRV	6	CONSUMER CREDIT OUTSTANDING - NONREVOLVING(G19)

Consumptions (Block J)

74*	**PI071**	**6**	**PERSONAL CONSUMPTION, PRICE INDEX (2000=100)**
75*	**PI072**	**6**	**PERSONAL CONSUMPTION - DURABLE GOODS, PRICE INDEX (2000=100)**
76*	**PI073**	**6**	**PERSONAL CONSUMPTION - NONDURABLE GOODS, PRICE INDEX (2000=100)**
77	PI074	6	PERSONAL CONSUMPTION - SERVICES, PRICE INDEX (2000=100)

Price Indices (Block K)

78*	**CPIAUCSL**	**6**	**CPI ALL ITEMS (SA) FRED**
79	CPILFESL	6	CPI LESS FOOD AND ENERGY (SA) FRED
80	PCEPILFE	6	PCE PRICE INDEX LESS FOOD AND ENERGY (SA) FRED
81	PWFSA	6	PRODUCER PRICE INDEX: FINISHED GOODS (82=100,SA)
82	PWFCSA	6	PRODUCER PRICE INDEX:FINISHED CONSUMER GOODS (82=100,SA)
83	PWIMSA	6	PRODUCER PRICE INDEX:INTERMED MAT.SUPPLIES & COMPONENTS(82=100,SA)
84	PWCMSA	6	PRODUCER PRICE INDEX:CRUDE MATERIALS (82=100,SA)
85	PWCMSAR	5	REAL PRODUCER PRICE INDEX:CRUDE MATERIALS (82=100,SA)
86	PSCCOM	6	SPOT MARKET PRICE INDEX:BLS & CRB: ALL COMMODITIES(1967=100)
87	PSCCOMR	5	REAL SPOT MARKET PRICE INDEX:BLS & CRB: ALL COMMODITIES(1967=100)
88	PW561	6	PRODUCER PRICE INDEX: CRUDE PETROLEUM (82=100,NSA)
89	PW561R	5	PPI CRUDE (RELATIVE TO CORE PCE) (PW561/PCEPILFE)
90*	**PMCP**	**1**	**NAPM COMMODITY PRICES INDEX (PERCENT)**

Exchange Rates (Block L)

91	EXRUS	5	UNITED STATES;EFFECTIVE EXCHANGE RATE(MERM)(INDEX NO.)
92	EXRSW	5	FOREIGN EXCHANGE RATE: SWITZERLAND (SWISS FRANC PER U.S.$)
93*	**EXRJAN**	**5**	**FOREIGN EXCHANGE RATE: JAPAN (YEN PER U.S.$)**
94	EXRUK	5	FOREIGN EXCHANGE RATE: UNITED KINGDOM (CENTS PER POUND)

95	EXRCAN	5	FOREIGN EXCHANGE RATE: CANADA (CANADIAN $ PER U.S.$)

Stock Prices and Miscellaneous (Block M)

96	FSPCOM	5	S&P'S COMMON STOCK PRICE INDEX: COMPOSITE (1941-43=10)
97	FSPIN	5	S&P'S COMMON STOCK PRICE INDEX: INDUSTRIALS (1941-43=10)
98*	**FSDXP**	**2**	**S&P'S COMPOSITE COMMON STOCK: PRICE/DIVIDEND RATIO (% PER ANNUM)**
99	FSPXE	2	S&P'S COMPOSITE COMMON STOCK: PRICE-EARNINGS RATIO (%,NSA)
100	FSDJ	5	COMMON STOCK PRICES: DOW JONES INDUSTRIAL AVERAGE
101*	**HHSNTN**	**2**	**U. OF MICH. INDEX OF CONSUMER EXPECTATIONS(BCD-83)**

Inventories and Orders (Block N)

102	PMI	1	PURCHASING MANAGERS' INDEX (SA)
103*	**PMNO**	**1**	**NAPM NEW ORDERS INDEX (PERCENT)**
104	PMDEL	1	NAPM VENDOR DELIVERIES INDEX (PERCENT)
105	PMNV	1	NAPM INVENTORIES INDEX (PERCENT)
106	MOCMQ	5	NEW ORDERS (NET) - CONSUMER GOODS & MATERIALS, 1996 DOLLARS (BCI)
107	MSONDQ	5	NEW ORDERS, NONDEFENSE CAPITAL GOODS, IN 1996 DOLLARS (BCI)