



Pseudo Inverse Matrix

If the columns of a matrix A are linearly independent, so $A^T \cdot A$ is invertible and we obtain with the following formula the pseudo inverse:

$$A^+ = (A^T \cdot A)^{-1} \cdot A^T$$

Here A^+ is a left inverse of A , what means: $A^+ \cdot A = E$.

However, if the rows of the matrix are linearly independent, we obtain the pseudo inverse with the formula:

$$A^+ = A^T \cdot (A \cdot A^T)^{-1}$$

This is a right inverse of A , what means: $A \cdot A^+ = E$.

If both the columns and the rows of the matrix are linearly independent, then the matrix is invertible and the pseudo inverse is equal to the inverse of the matrix.

Example:

Matrix A

=====

```
1  1  1  1
5  7  7  9
```

$A^T \cdot A$

=====

```
26  36  36  46
36  50  50  64
36  50  50  64
46  64  64  82
```

$A^T \cdot A$ is not invertible

$A \cdot A^T$

=====

```
4    28
28   204
```

$(A \cdot A^T)^{-1}$

=====

```
6,375 -0,875
-0,875 0,125
```

Right Inverse: $A^T \cdot (A \cdot A^T)^{-1}$

=====

```
2 -0,25
0,25  0
0,25  0
-1,5  0,25
```

Proof by multiplication:

1. Matrix (A)

=====

```
1  1  1  1
5  7  7  9
```

2. Matrix (A^+)

=====

```
2 -0,25
0,25  0
0,25  0
-1,5  0,25
```

Product Matrix ($A \cdot A^+$)

=====

```
1  0
0  1
```

Pop-up Menu:

Right click to open a local menu, which offers you the following functions to manage the matrix.

- *Cut Matrix* , *Copy Matrix* and *Paste Matrix*

With this you may copy the matrix to the clipboard and paste it into "Matrix multiplication".

- *Transpose Matrix*

Swaps the rows and columns of the matrix.

- *Export Matrix* and *Import Matrix*

Exports or imports the matrix in CSV format (Comma separated values), which is used to exchange data with Excel.

See also:

[Wikipedia: Moore Penrose pseudoinverse](#)