Introduction

Machine Learning

- Arthur Samuel (1959):

  "Field of study that gives computers the ability to learn without being explicitly programmed”.

- Tom Mitchell (1997):

  "A computer program is said to learn from experience E with respect to some class of tasks T and performance measure P, if its performance at tasks in T, as measured by P, improves with experience E”.

Machine Learning

- How to construct programs that automatically improve with experience.

Example

Experience

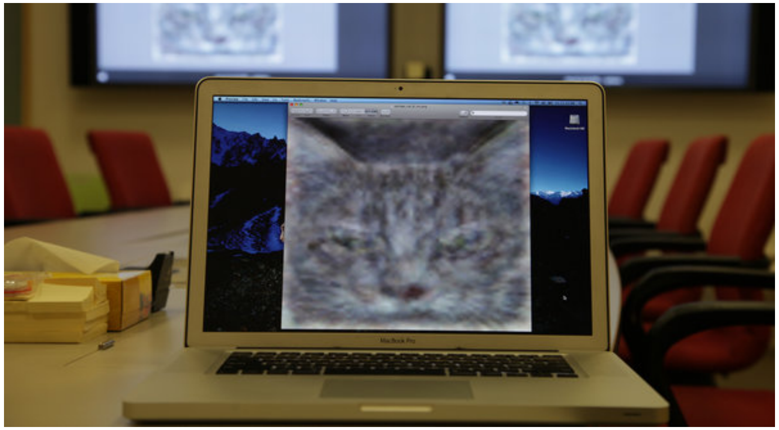
Example	GRAY?	MAMMAL?	LARGE?	VEGETARIAN?	WILD?	Elephant
1	+	+	+	+	+	+
2	+	+	+	-	+	+
3	+	+	-	+	+	- (*Mouse*)
4	-	+	+	+	+	- (*Giraffe*)
5	+	-	+	-	+	- (*Dinosaur*)
6	+	+	+	+	-	+

Prediction

7	+	+	+	-	+	?
8	+	-	+	-	+	?
9	+	+	+	-	-	?

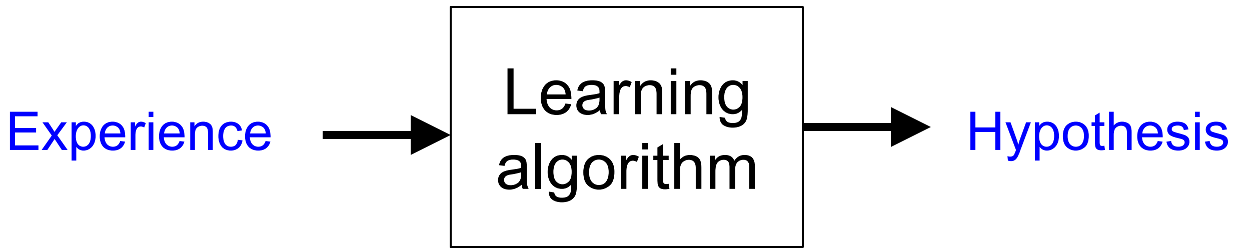
Example

- Deep learning: developed by a research group at Stanford and Google X.
- A system of 16,000 connected computer processors that can learn concepts without supervision.
- Featured in The New Youk Times in 2012.



Machine Learning

- What is learning?



Machine Learning

- Learning is an (endless) generalization or induction process.

Types of Machine Learning

- Supervised learning: the learner (learning algorithm) are trained on labelled examples, i.e., input where the desired output is known.

- Unsupervised learning: the learner operates on unlabelled examples, i.e., input where the desired output is unknown.

Types of Machine Learning

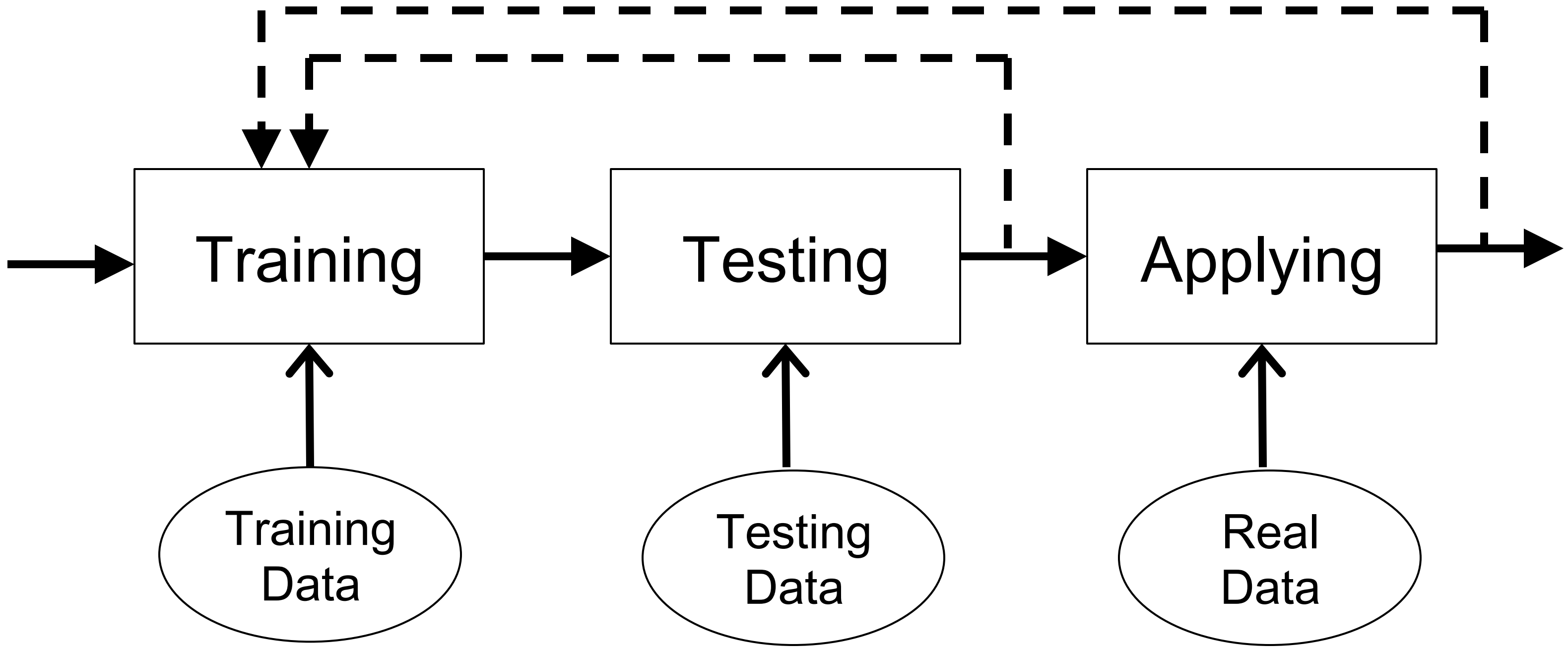
- Reinforcement learning: between supervised and unsupervised learning. It is told when an answer is wrong, but not how to correct it.

- Evolutionary learning: biological evolution can be seen as a learning process, to improve survival rates and chance of having offspring.

Types of Machine Learning

- The most common type: supervised learning.
  - Regression: to find a function whose curve passes as close as possible to all of the given data points.
  - Classification: to find the class of an instance given its selected features.

Phases of Machine Learning



Phases of Machine Learning

- K-fold cross validation:
  - Randomly partitioned k equal sized subsamples.
  - k – 1 for training and 1 for testing.
  - k times (folds) of validation and taking the average.

Phases of Machine Learning

- Statistical significance test: to reject the null-hypothesis that the two compared systems are equivalently efficient although their performance measures are different.

Phases of Machine Learning

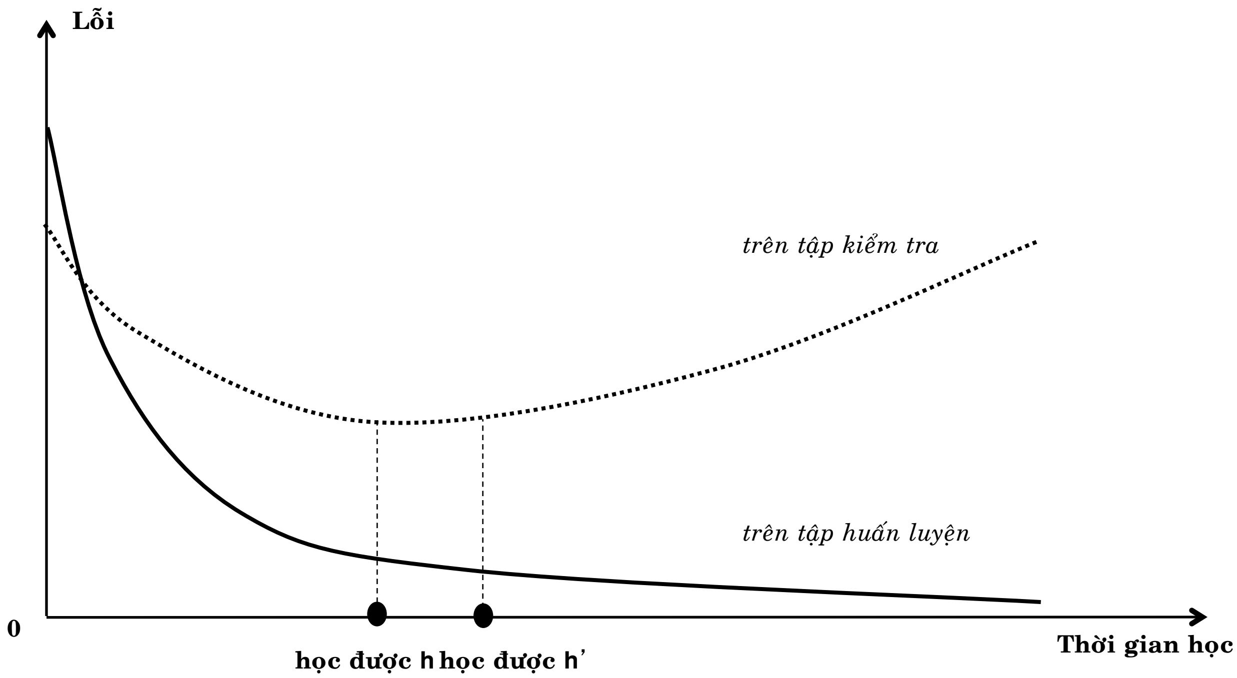
- Fisher's randomization:
  - Q testing cases.
  - $\delta = |m(A) - m(B)|$
  - $2^{|Q|}$ permutations of performances of A and B on Q cases.
  - N^+ = number of permutations whose A-B performance difference is greater than or equal to δ.
  - N^- = number of permutations whose A-B performance difference is smaller than or equal to $-\delta$.
  - two-sided p-value = $(N^+ + N^-)/2^{|Q|}$
  - $p \leq 0.05$ to reject the null-hypothesis

Phases of Machine Learning

- Overfitting: $h \in H$ is said to overfit the training data if there exists $h' \in H$, such that h has smaller error than h' over the training examples, but h' has a smaller error than h over the entire distribution of instances.

Phases of Machine Learning

- Overfitting:



Phases of Machine Learning

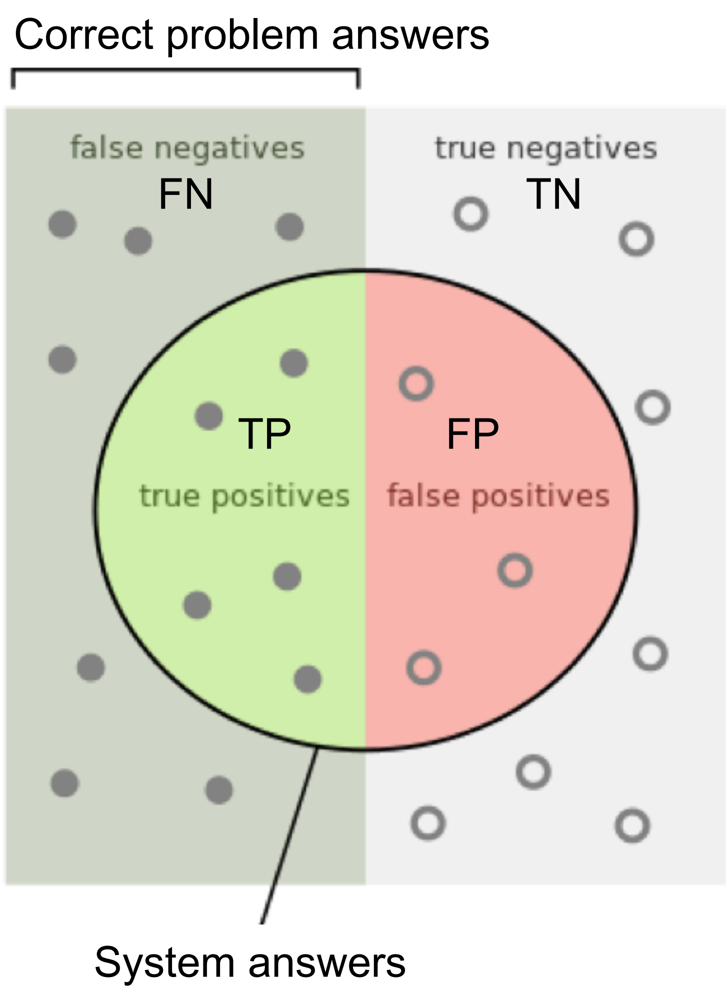
- Overfitting:
  - There is noise in the data
  - The number of training examples is too small to produce a representative sample of the target concept

Performance Measures

$$\text{Precision (P)} = \frac{\text{number of correct system answers}}{\text{number of system answers}}$$

$$\text{Recall (R)} = \frac{\text{number of correct system answers}}{\text{number of correct problem answers}}$$

Performance Measures



Precision = TP/(TP + FP)

Recall = TP/(TP + FN)

Accuracy = (TP + TN)/(TP + TN + FP + FN)

Performance Measures

- Precision (P) = $\dfrac{\text{number of correct system answers}}{\text{number of system answers}}$

- Recall (R) = $\dfrac{\text{number of correct system answers}}{\text{number of correct problem answers}}$

- F-measure (F) = 2.P.R/(P + R)

Concept Learning

- Inferring a boolean-valued function from training examples of its input (instances) and output (classifications).

Example

Experience

Example	Sky	AirTemp	Humidity	Wind	Water	Forecast	EnjoySport
1	Sunny	Warm	Normal	Strong	Warm	Same	Yes
2	Sunny	Warm	High	Strong	Warm	Same	Yes
3	Rainy	Cold	High	Strong	Warm	Change	No
4	Sunny	Warm	High	Strong	Cool	Change	Yes

Low Weak

Prediction

5	Rainy	Cold	High	Strong	Warm	Change	?
6	Sunny	Warm	Normal	Strong	Warm	Same	?
7	Sunny	Warm	Low	Strong	Cool	Same	?

Concept Learning

- Learning problem:
  - Target concept: a subset of the set of instances X

    c: $X \rightarrow \{0, 1\}$ (number of possible concepts = $2^{|X|}$)
  - Target function:

    Sky × AirTemp × Humidity × Wind × Water × Forecast $\rightarrow \{0, 1\}$
  - Hypothesis:

    Characteristics of all instances of the concept to be learned

    ≡ Constraints on instance attributes

    h: $X \rightarrow \{0, 1\}$

Concept Learning

- Satisfaction:

  $h(x) = 1$ iff x satisfies all the constraints of h

  $h(x) = 0$ otherwsie

- Consistency:

  $h(x) = c(x)$ for every instance x of the training examples

- Correctness:

  $h(x) = c(x)$ for every instance x of X

Concept Learning

- How to represent a hypothesis?

Concept Learning

- Hypothesis representation (constraints on instance attributes):

  <Sky, AirTemp, Humidity, Wind, Water, Forecast>

  - ?: any value is acceptable
  - single required value
  - $\varnothing$: no value is acceptable
  - Number of possible hypotheses = (4.3.3.3.3.3) + 1 = 973

- Example:

  h1 = <Sunny, ?, ?, Strong, ? , ?>

Concept Learning

Example	Sky	AirTemp	Humidity	Wind	Water	Forecast	EnjoySport
1	Sunny	Warm	Normal	Strong	Warm	Same	Yes
2	Sunny	Warm	High	Strong	Warm	Same	Yes
3	Rainy	Cold	High	Strong	Warm	Change	No
4	Sunny	Warm	High	Strong	Cool	Change	Yes

What is a hypothesis that is consistent with the training examples?

h = < _ , _ , _ , _ , _ , _ >

Concept Learning

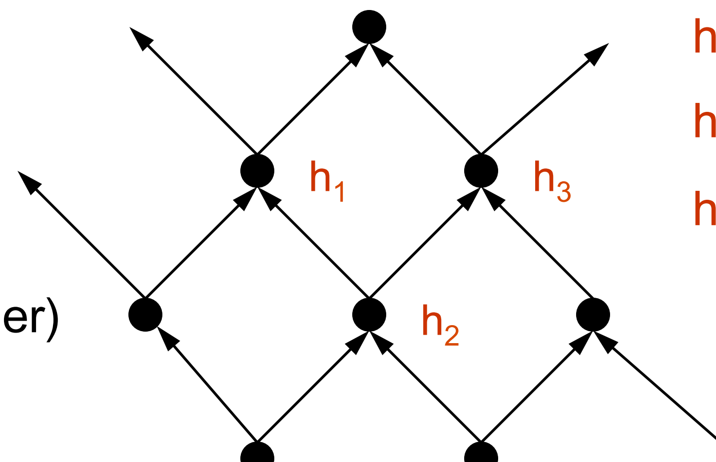
- General-to-specific ordering of hypotheses:

$h_j \geq_g h_k$ iff $\forall x \in X: h_k(x) = 1 \Rightarrow h_j(x) = 1$

Specific

Lattice
(Partial order)

General



H

h_1 = <Sunny, ?, ?, Strong, ? , ?>

h_2 = <Sunny, ?, ?, ? , ? , ?>

h_3 = <Sunny, ?, ?, ? , Cool, ?>

Concept Learning

Example	Sky	AirTemp	Humidity	Wind	Water	Forecast	EnjoySport
1	Sunny	Warm	Normal	Strong	Warm	Same	Yes
2	Sunny	Warm	High	Strong	Warm	Same	Yes
3	Rainy	Cold	High	Strong	Warm	Change	No
4	Sunny	Warm	High	Strong	Cool	Change	Yes

What is the most specific hypothesis that is consistent with the training examples?

h = < _ , _ , _ , _ , _ , _ >

FIND-S

Example	Sky	AirTemp	Humidity	Wind	Water	Forecast	EnjoySport
1	Sunny	Warm	Normal	Strong	Warm	Same	Yes
2	Sunny	Warm	High	Strong	Warm	Same	Yes
3	Rainy	Cold	High	Strong	Warm	Change	No
4	Sunny	Warm	High	Strong	Cool	Change	Yes

h = < ∅ , ∅ , ∅ , ∅ , ∅ , ∅ >

h = <Sunny, Warm, Normal, Strong, Warm, Same>

h = <Sunny, Warm, ? , Strong, Warm, Same>

h = <Sunny, Warm, ? , Strong, ? , ? >

FIND-S

- Initialize h to the most specific hypothesis in H:
- For each positive training instance x:

  For each attribute constraint a_i in h:

  If the constraint is not satisfied by x

  Then replace a_i by the next more general constraint satisfied by x

- Output hypothesis h

FIND-S

Example	Sky	AirTemp	Humidity	Wind	Water	Forecast	EnjoySport
1	Sunny	Warm	Normal	Strong	Warm	Same	Yes
2	Sunny	Warm	High	Strong	Warm	Same	Yes
3	Rainy	Cold	High	Strong	Warm	Change	No
4	Sunny	Warm	High	Strong	Cool	Change	Yes

h = <Sunny, Warm, ? , Strong, ? , ? >

Prediction

5	Rainy	Cold	High	Strong	Warm	Change	No
6	Sunny	Warm	Normal	Strong	Warm	Same	Yes
7	Sunny	Warm	Low	Strong	Cool	Same	Yes

FIND-S

- The result is consistent with the positive training examples.

- Is the result is consistent with the negative training examples?

FIND-S

Example	Sky	AirTemp	Humidity	Wind	Water	Forecast	EnjoySport
1	Sunny	Warm	Normal	Strong	Warm	Same	Yes
2	Sunny	Warm	High	Strong	Warm	Same	Yes
3	Rainy	Cold	High	Strong	Warm	Change	No
4	Sunny	Warm	High	Strong	Cool	Change	Yes
5	**Sunny**	**Warm**	**Normal**	**Strong**	**Cool**	**Change**	**No**

h = <Sunny, Warm, ? , Strong, ? , ? >

FIND-S

- The result is consistent with the negative training examples if the target concept is contained in H (and the training examples are correct).

- Sizes of the space:
  - Size of the instance space: $|X| = 3.2.2.2.2.2 = 96$
  - Size of the concept space $C = 2^{|X|} = 2^{96}$
  - Size of the hypothesis space $H = (4.3.3.3.3.3) + 1 = 973 << 2^{96}$

$\Rightarrow$ The target concept (in C) may not be contained in H.

Compact Representation of Version Space

- Version space: a set of all hypotheses that are consistent with the training examples.

Compact Representation of Version Space

- G (the generic boundary): set of the most generic hypotheses of H consistent with the training data D:

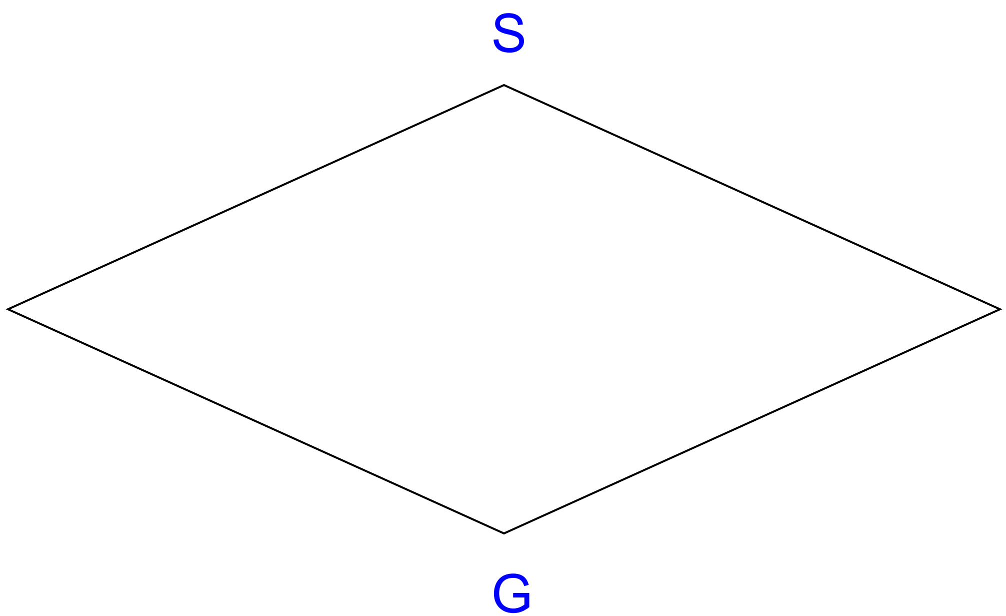
  $G = \{g \in H \mid consistent(g, D) \land \neg\exists g' \in H: g' >_g g \land consistent(g', D)\}$

- S (the specific boundary): set of the most specific hypotheses of H consistent with the training data D:

  $S = \{s \in H \mid consistent(s, D) \land \neg\exists s' \in H: s >_g s' \land consistent(s', D)\}$

Compact Representation of Version Space

- Version space = $\langle G, S \rangle = \{h \in H \mid \exists g \in G\ \exists s \in S: g \geq_g h \geq_g s\}$

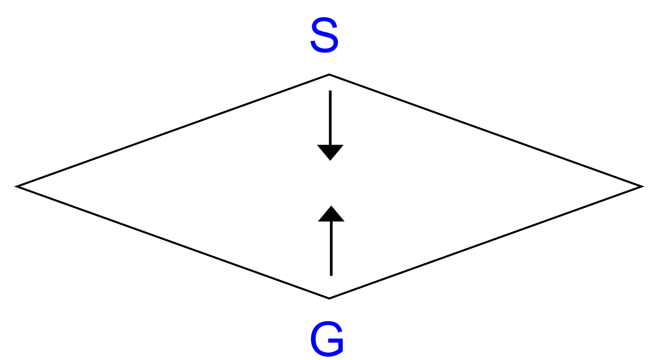


Candidate-Elimination Algorithm

Example	Sky	AirTemp	Humidity	Wind	Water	Forecast	EnjoySport
1	Sunny	Warm	Normal	Strong	Warm	Same	Yes
2	Sunny	Warm	High	Strong	Warm	Same	Yes
3	Rainy	Cold	High	Strong	Warm	Change	No
4	Sunny	Warm	High	Strong	Cool	Change	Yes

$S_0 = \{\langle \emptyset, \emptyset, \emptyset, \emptyset, \emptyset, \emptyset \rangle\}$
$G_0 = \{\langle ?, ?, ?, ?, ?, ? \rangle\}$

$S_1 = \{\langle \text{Sunny, Warm, Normal, Strong, Warm, Same} \rangle\}$
$G_1 = \{\langle ?, ?, ?, ?, ?, ? \rangle\}$



$S_2 = \{\langle \text{Sunny, Warm, ?, Strong, Warm, Same} \rangle\}$
$G_2 = \{\langle ?, ?, ?, ?, ?, ? \rangle\}$

$S_3 = \{\langle \text{Sunny, Warm, ?, Strong, Warm, Same} \rangle\}$
$G_3 = \{\langle \text{Sunny}, ?, ?, ?, ?, ? \rangle, \langle ?, \text{Warm}, ?, ?, ?, ? \rangle, \langle ?, ?, ?, ?, ?, \text{Same} \rangle\}$

$S_4 = \{\langle \text{Sunny, Warm, ?, Strong, ?, ?} \rangle\}$
$G_4 = \{\langle \text{Sunny}, ?, ?, ?, ?, ? \rangle, \langle ?, \text{Warm}, ?, ?, ?, ? \rangle\}$

Candidate-Elimination Algorithm

S4 = {<Sunny, Warm, ?, Strong, ?, ?>}

<Sunny, ?, ?, Strong, ?, ?> <Sunny, Warm, ?, ?, ?, ?> <?, Warm, ?, Strong, ?, ?>

G4 = {<Sunny, ?, ?, ?, ?, ?>, <?, Warm, ?, ?, ?, ?>}

Candidate-Elimination Algorithm

- Initialize G to the set of maximally general hypotheses in H
- Initialize S to the set of maximally specific hypotheses in H

Candidate-Elimination Algorithm

- For each positive example d:
  - Remove from G any hypothesis inconsistent with d
  - For each s in S that is inconsistent with d:

    Remove s from S

    Add to S all least generalizations h of s, such that h is consistent with d and some hypothesis in G is more general than h

    Remove from S any hypothesis that is more general than another hypothesis in S

Candidate-Elimination Algorithm

- For each negative example d:
  - Remove from S any hypothesis inconsistent with d
  - For each g in G that is inconsistent with d:

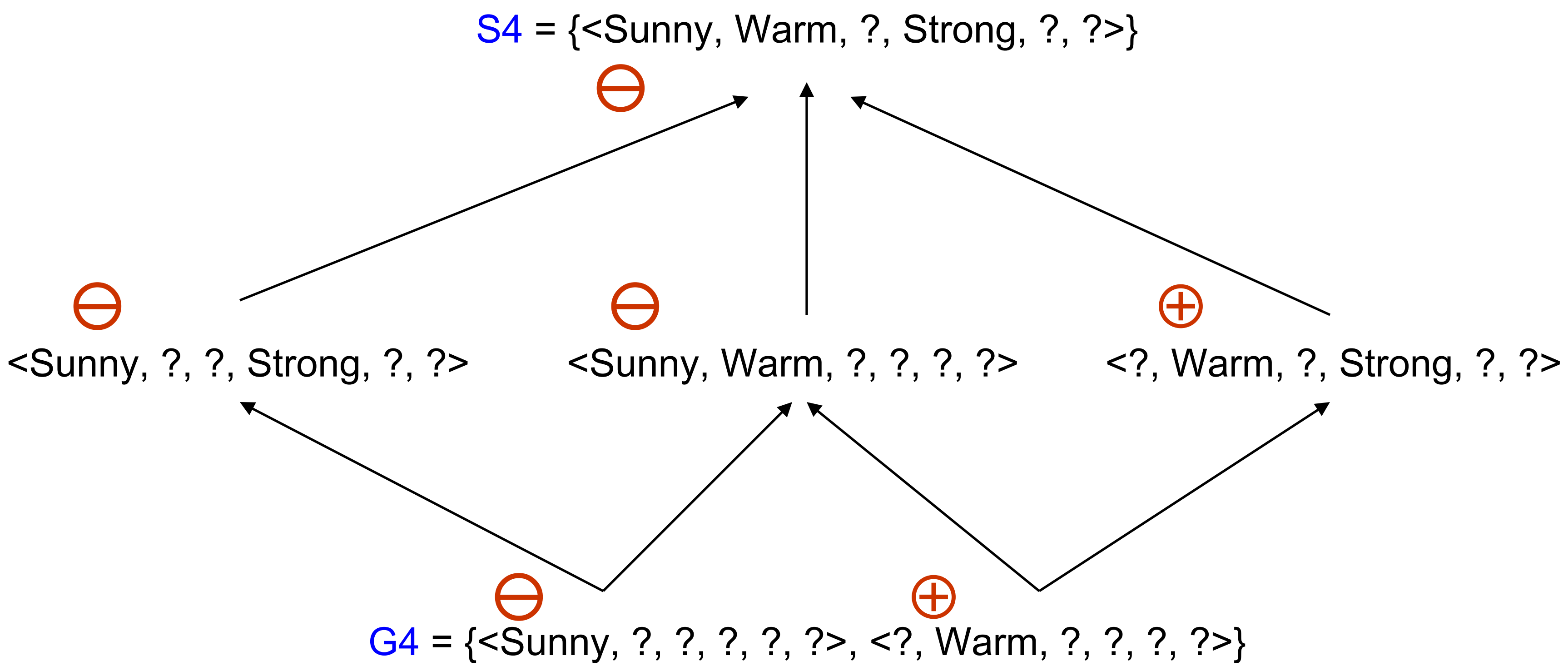
    Remove g from G

    Add to G all least specializations h of g, such that h is consistent with d and some hypothesis in S is more specific than h

    Remove from G any hypothesis that is more specific than another hypothesis in G

Candidate-Elimination Algorithm

- Partially learned concept can be used to classify new instances using the majority rule.



5	Rainy	Warm	High	Strong	Cool	Same	?

Inductive Bias

- Size of the instance space: $|X| = 3.2.2.2.2.2 = 96$
- Number of possible concepts $= 2^{|X|} = 2^{96}$
- Size of $H = (4.3.3.3.3.3) + 1 = 973 << 2^{96}$

$\Rightarrow$ a biased hypothesis space

Inductive Bias

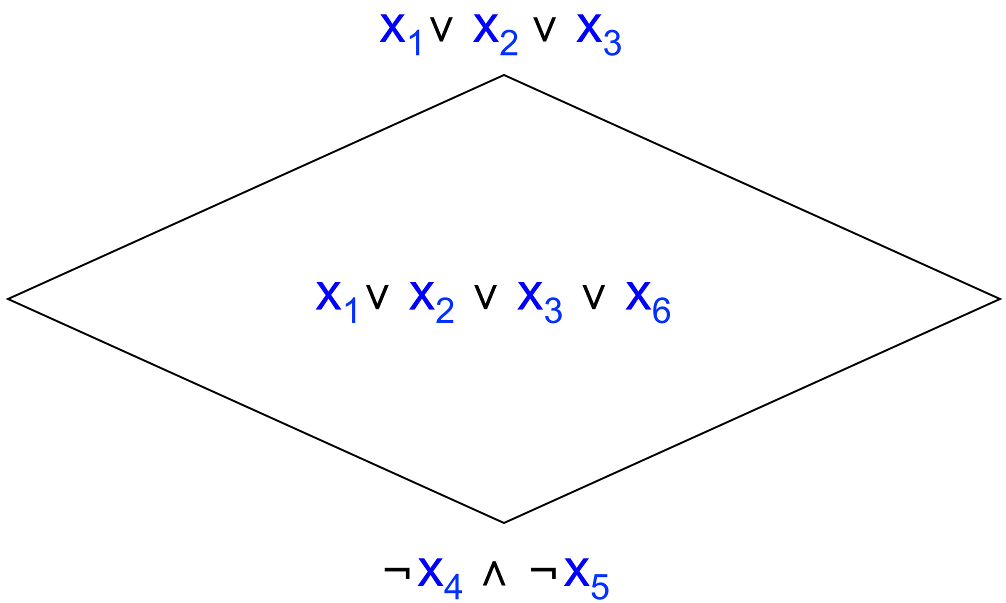
- An unbiased hypothesis space H' that can represent every subset of the instance space X: Propositional logic sentences

- Positive examples: x_1, x_2, x_3
  Negative examples: x_4, x_5

$$h(x) \equiv (x = x_1) \vee (x = x_2) \vee (x = x_3) \equiv x_1 \vee x_2 \vee x_3$$

$$h'(x) \equiv (x \neq x_4) \wedge (x \neq x_5) \equiv \neg x_4 \wedge \neg x_5$$

Inductive Bias



Any new instance x is classified positive by half of the version space, and negative by the other half

$\Rightarrow$ not classifiable

Inductive Bias

Example	Quality	Price	Buy
1	Good	Low	Yes
2	Bad	High	No

3	Good	High	?
4	Bad	Low	?

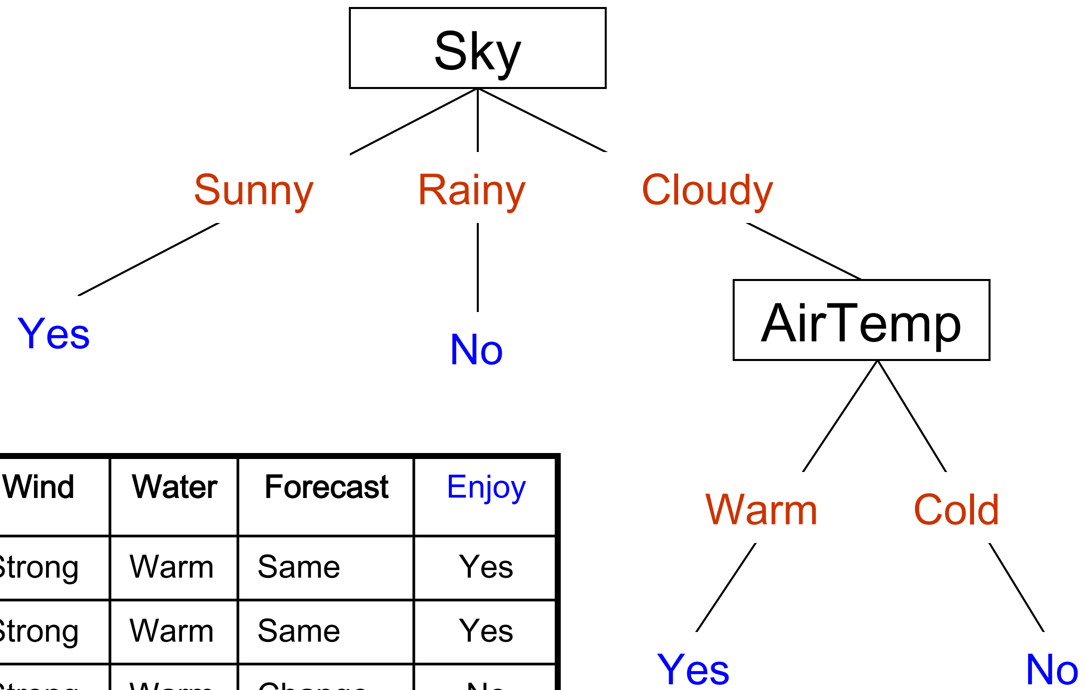
Inductive Bias

- A learner that makes no prior assumptions regarding the identity of the target concept cannot classify any unseen instances.

Decision Trees

Example	Sky	AirTemp	Humidity	Wind	Water	Forecast	EnjoySport
1	Sunny	Warm	Normal	Strong	Warm	Same	Yes
2	Sunny	Warm	High	Strong	Warm	Same	Yes
3	Rainy	Cold	High	Strong	Warm	Change	No
4	Sunny	Warm	High	Strong	Cool	Change	Yes
5	Cloudy	Warm	High	Weak	Cool	Same	Yes
6	Cloudy	Cold	High	Weak	Cool	Same	No

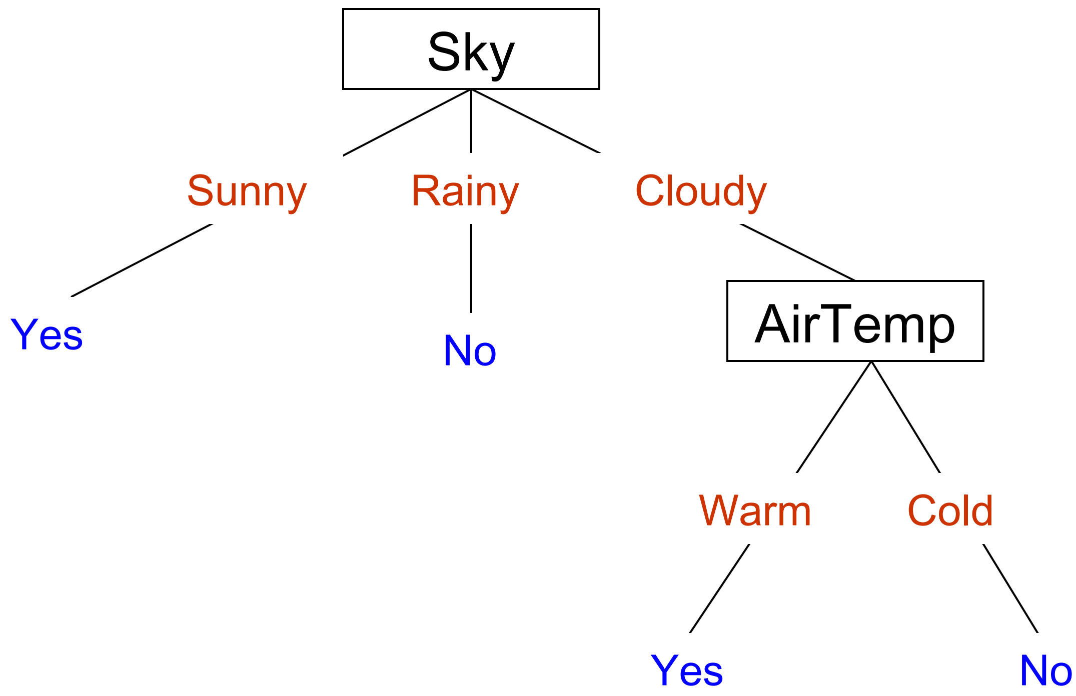
Decision Trees



No.	Sky	AirTemp	Humidity	Wind	Water	Forecast	Enjoy
1	Sunny	Warm	Normal	Strong	Warm	Same	Yes
2	Sunny	Warm	High	Strong	Warm	Same	Yes
3	Rainy	Cold	High	Strong	Warm	Change	No
4	Sunny	Warm	High	Strong	Cool	Change	Yes
5	Cloudy	Warm	High	Weak	Cool	Same	Yes
6	Cloudy	Cold	High	Weak	Cool	Same	No

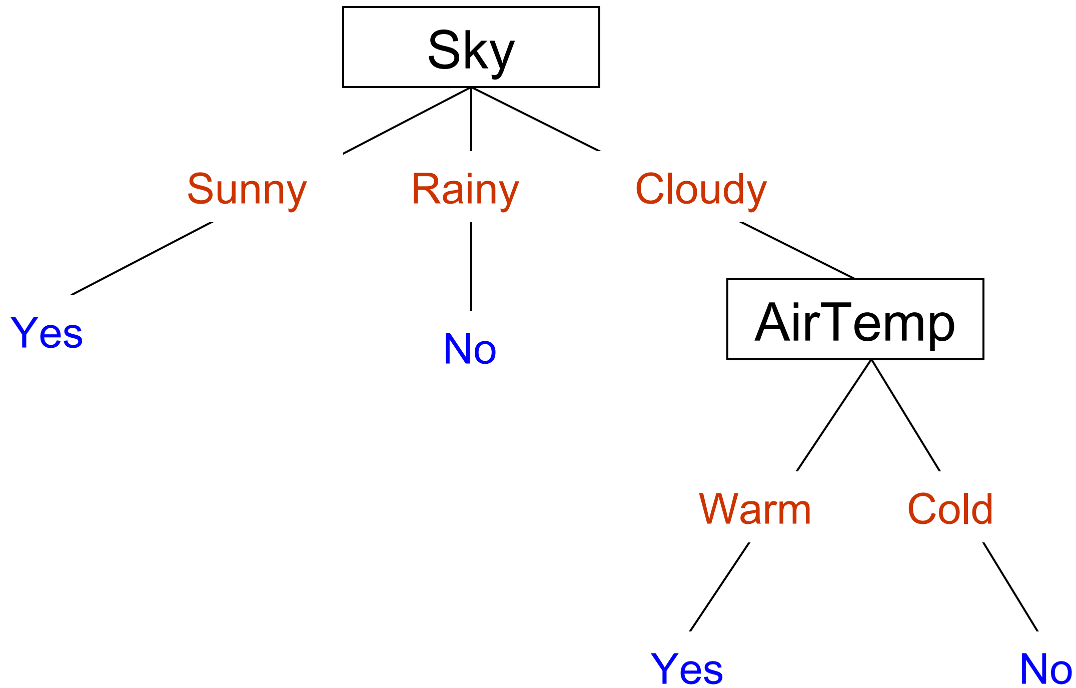
Cao Hoang Tru
CSE Faculty - HCMUT

Decision Trees



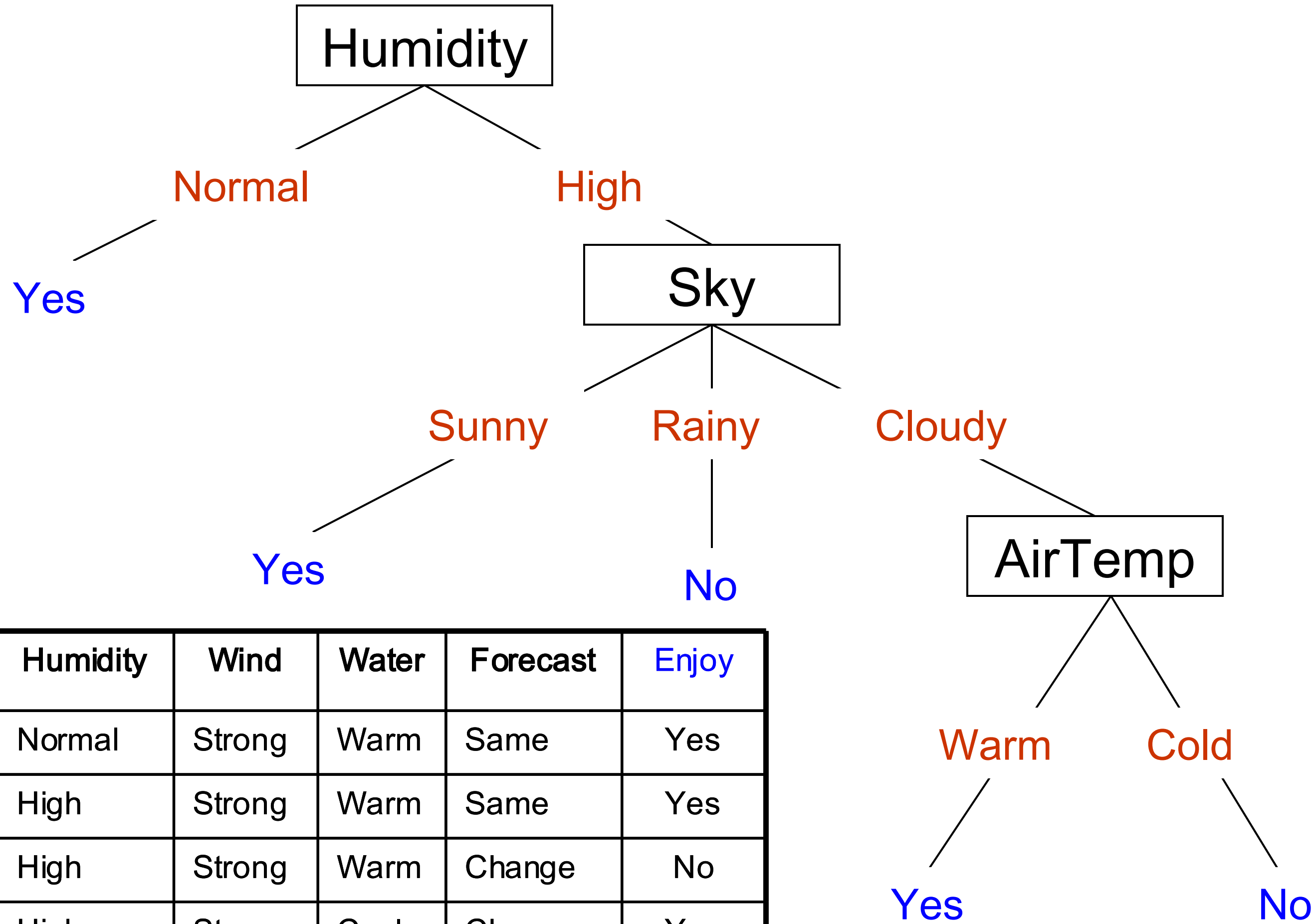
(Sky = Sunny) ∨ (Sky = Cloudy ∧ AirTemp = Warm)

Decision Trees



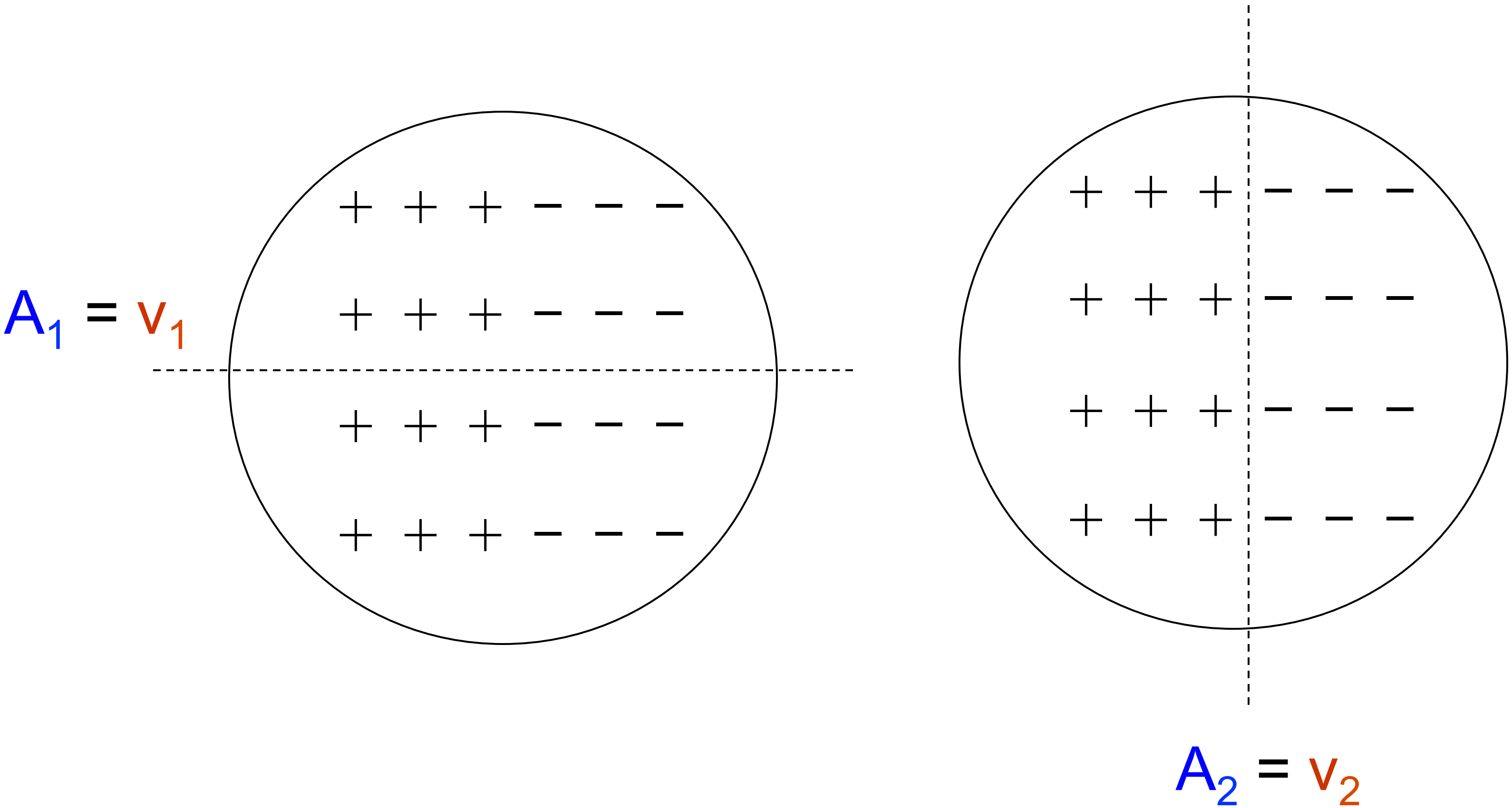
7	Rainy	Warm	Normal	Weak	Cool	Same	?
8	Cloudy	Warm	High	Strong	Cool	Change	?

Decision Trees



No.	Sky	AirTemp	Humidity	Wind	Water	Forecast	Enjoy
1	Sunny	Warm	Normal	Strong	Warm	Same	Yes
2	Sunny	Warm	High	Strong	Warm	Same	Yes
3	Rainy	Cold	High	Strong	Warm	Change	No
4	Sunny	Warm	High	Strong	Cool	Change	Yes
5	Cloudy	Warm	High	Weak	Cool	Same	Yes
6	Cloudy	Cold	High	Weak	Cool	Same	No

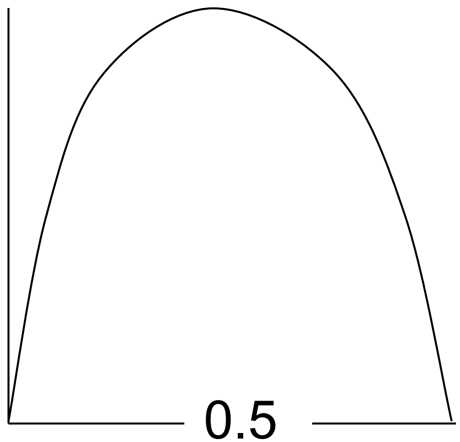
Decision Trees



Cao Hoang Tru
CSE Faculty - HCMUT

Homogenity of Examples

- Entropy(S) = $- p_+ \log_2 p_+ - p_- \log_2 p_-$

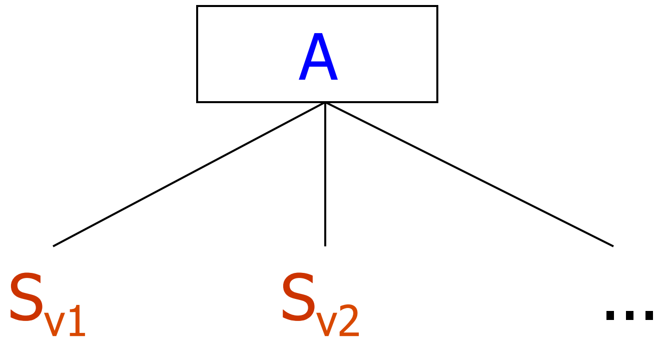


Homogenity of Examples

- Entropy(S) = $\sum_{i=1,c} - p_i \log_2 p_i$ impurity measure

Information Gain

- $Gain(S, A) = Entropy(S) - \sum_{v \in Values(A)} (|S_v|/|S|).Entropy(S_v)$



Example

- $\text{Entropy}(S) = -\,p_+\log_2 p_+ - p_-\log_2 p_- = -\,(4/6)\log_2(4/6) - (2/6)\log_2(2/6)$

  $= 0.389 + 0.528 = 0.917$

- Gain(S, Sky)

  $= \text{Entropy}(S) - \sum_{v\in\{\text{Sunny, Rainy, Cloudy}\}}(|S_v|/|S|)\text{Entropy}(S_v)$

  $= \text{Entropy}(S) - [(3/6).\text{Entropy}(S_{\text{Sunny}}) + (1/6).\text{Entropy}(S_{\text{Rainy}}) +$

  $(2/6).\text{Entropy}(S_{\text{Cloudy}})]$

  $= \text{Entropy}(S) - (2/6).\text{Entropy}(S_{\text{Cloudy}})$

  $= \text{Entropy}(S) - (2/6)[-\,(1/2)\log_2(1/2) - (1/2)\log_2(1/2)]$

  $= 0.917 - 0.333 = 0.584$

Example

- $\text{Entropy}(S) = -\,p_+\log_2 p_+ - p_-\log_2 p_- = -\,(4/6)\log_2(4/6) - (2/6)\log_2(2/6)$

  $= 0.389 + 0.528 = 0.917$

- Gain(S, Water)

  $= \text{Entropy}(S) - \sum_{v\in\{\text{Warm, Cool}\}}(|S_v|/|S|)\text{Entropy}(S_v)$

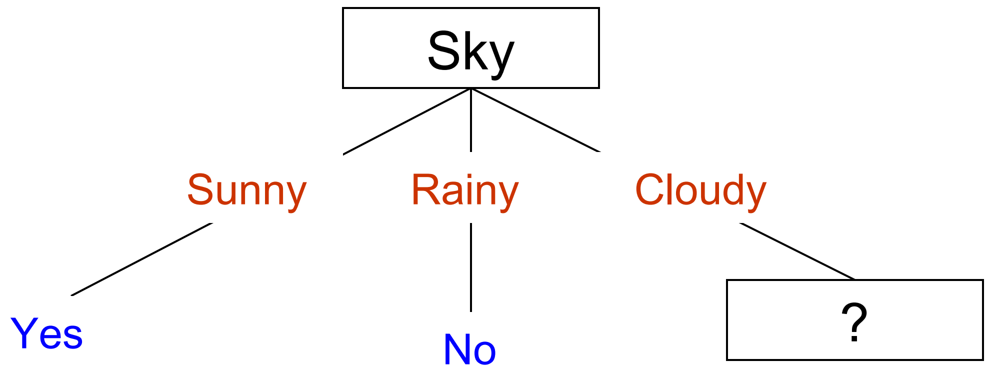
  $= \text{Entropy}(S) - [(3/6).\text{Entropy}(S_{\text{Warm}}) + (3/6).\text{Entropy}(S_{\text{Cool}})]$

  $= \text{Entropy}(S) - (3/6).2.[-\,(2/3)\log_2(2/3) - (1/3)\log_2(1/3)]$

  $= \text{Entropy}(S) - 0.389 - 0.528$

  $= 0$

Example



- Gain(S_{Cloudy}, AirTemp)
  $= Entropy(S_{Cloudy}) - \sum_{v \in \{Warm,\ Cold\}} (|S_v|/|S|) Entropy(S_v)$
  $= 1$
- Gain(S_{Cloudy}, Humidity)
  $= Entropy(S_{Cloudy}) - \sum_{v \in \{Normal,\ High\}} (|S_v|/|S|) Entropy(S_v)$
  $= 0$

Inductive Bias

- Hypothesis space: complete!

Inductive Bias

- Hypothesis space: complete!
- Shorter trees are preferred over larger trees
- Prefer the simplest hypothesis that fits the data

Cao Hoang Tru
CSE Faculty - HCMUT

Inductive Bias

- Decision Tree algorithm: searches incompletely thru a complete hypothesis space.

  ⇒ Preference bias

- Cadidate-Elimination searches completely thru an incomplete hypothesis space.

  ⇒ Restriction bias