



Higher Nationals in Computing

Unit 19: Data Structures & Algorithms ASSIGNMENT 1

Learner's name: TRINH THI DIEU HUYEN

Assessor name: **HO HAI VAN**

Class: GCS0801B.1

ID: GDD18606

Subject code: 1649

Assignment due:

Assignment submitted:





ASSIGNMENT 1 BRIEF

Qualification	BTEC Level 5 HND Diploma in Business		
Unit number	Unit 19: Data Structures and Algorithms		
Assignment title	Examine and specify ADT and DSA		
Academic Year			
Unit Tutor	Ho Hai Van		
Issue date		Submission date	
IV name and date			

Submission Format:

Format: The submission is in the form of an individual written report and a presentation. This should be written in a concise, formal business style using single spacing and font size 12. You are required to make use of headings, paragraphs and subsections as appropriate, and all work must be supported with research and referenced using the Harvard referencing system. Please also provide a bibliography using the Harvard referencing system.

Submission: Students are compulsory to submit the assignment in due date and in a way requested by the Tutors. The form of





submission will be a soft copy in PDF posted on corresponding course of http://cms.greenwich.edu.vn/

Note: The Assignment *must* be your own work, and not copied by or from another student or from books etc. If you use ideas, quotes or data (such as diagrams) from books, journals or other sources, you must reference your sources, using the Harvard style. Make sure that you know how to reference properly, and that understand the guidelines on plagiarism. *If you do not, you definitely get fail*

Assignment Brief and Guidance:

Scenario: You work as in-house software developer for Soft.net Development Ltd, a software body-shop providing network provisioning solutions. Your company is part of a collaborative service provisioning development project and your company has won the contract to design and develop a middleware solution that will interface at the front-end to multiple computer provisioning interfaces including SOAP, HTTP, JML and CLI, and the back-end telecom provisioning network via CLI.

Your account manager has assigned you a special role that is to inform your team about designing and implementing abstract data types. You have been asked to create a presentation for all collaborating partners on how ADTs can be utilised to improve software design, development and testing. Further, you have been asked to write an introductory report for distribution to all partners on how to specify abstract data types and algorithms in a formal notation.

Tasks

Part 1

You will need to prepare a presentation on how to create a design specification for data structures, explaining the valid operations that can be carried out on the structures using the example of:

- 1. A stack ADT, a concrete data structure for a First in First out (FIFO) queue.
- 2. Two sorting algorithms.





3. Two network shortest path algorithms.

Part 2

You will need to provide a formal written report that includes the following:

- 1. Explanation on how to specify an abstract data type using the example of software stack.
- 2. Explanation of the advantages of encapsulation and information hiding when using an ADT.
- 3. Discussion of imperative ADTs with regard to object orientation.





Lea	riteria		
Pass	Merit	Distinction	
LO1 Examine abstract data types, concrete			
 P1 Create a design specification for data structures explaining the valid operations that can be carried out on the structures. P2 Determine the operations of a memory stack and how it is used to implement function calls in a computer. 	 M1 Illustrate, with an example, a concrete data structure for a First in First out (FIFO) queue. M2 Compare the performance of two sorting algorithms. 	D1 Analyse the operation, using illustrations, of two network shortest path algorithms, providing an example of each.	
LO2 Specify abstract data types and algori			
P3 Using an imperative definition, specify the abstract data type for a software stack.	M3 Examine the advantages of encapsulation and information hiding when using an ADT.	D2 Discuss the view that imperative ADTs are a basis for object orientation and, with justification, state whether you agree.	





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1.2	. How to implement the ADT ?
2.	Explaining the valid operations that can be carried out on the structures
2.1	. What common operations can be carried out on the ADT ?
2.2	. How to implement the operations ?
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1.2	. What are operations on stack ?
1.3	. How to implement stacks ?
2.	How it is used to implement function calls in a computer.
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ASSIGNMENT 1 ANSWERS

LO1 Examine abstract data types, concrete data structures and algorithms

P1 Create a design specification for data structures explaining the valid operations that can be carried out on the structures.

1. Create a design specification for data structures.

1.1. What is an ADT?

ADT - refers to an Abstract Data Type that represent some data and operations on that data. It consists of Declaration of data and Declaration of operations. ADT defines a particular data structure in terms of data and operations. Besides, it also offers an interface of the objects, that is instances of an ADT. The behavior of each operation is determined by its inputs and outputs. The implementation details are hidden from the user of the ADT and protected from outside accessed, it just can be manipulated only by means of operations - This is referred to as encapsulation of data.

1.2. How to implement the ADT?

To implement the ADT needs to go from definition to implementation. The first, problem definition and identification of ADTs, mean, identify data or attributes. Consists, specify ADT interactions and specify ADT operations. Then, identify object hierarchy (if using OOP) and implement ADTs. The implementation consists of a concrete representation of the abstract data in terms of existing data types and off-the-shelf ADT operations the implementation may include program code to implements the allowable operations.

It's difficult to specify a single list Abstract Data Type (ADT) that covers both arrays and linked lists. One issue is the representation of Array position is represented by integer and Linked list position is represented by pointer. They are completely different. In case of an array with n elements, a "position" is simply an integer in the range from 0 to n-1. In linked list, a position can be a pointer to one of the nodes in the list, but there are some subtleties involved.

- 2. Explaining the valid operations that can be carried out on the structures.
 - 2.1. What common operations can be carried out on the ADT?





Table 1: Some common operations can be carried out on the ADT

Some common operations can be carried out on the ADT			
getFirst()	Returns the first element in the list		
getLast()	Returns the last element in the list		
getNext(p)	Returns the next element (here it is p) in the list		
getPrev(p)	Returns the previous element (here it is p) in the list		
get(p)	Returns the element at the specified position (here it is p) in the list		
set(p,x)	Fixes the number at position p to \boldsymbol{x}		
insert(p,x)	Add to position p the number x		
remove(p)	Removes the element at the specified position (here it is p) in the list		
removeFirst()	Remove an item from the beginning of the list		
removeLast()	Remove an item from the end of the list		
removeNext(p)	Remove an item (here it is p) from the next of the list		
removePrev(p)	Remove an item (here it is p) from the previous of the list		
find(x)	Searches the specified pattern or the expression (here it is x) in the given		
	sequence string and returns true otherwise it returns false		
size()	Return the number of objects in the list		

2.2. How to implement the operations?

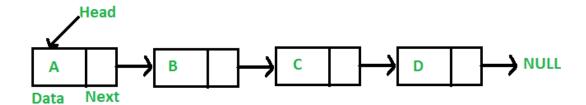


Figure 1: Linked List





A linked structure is a collection of nodes storing data and links to other nodes. In this way, nodes can be located anywhere in memory, and passing from one node of the linked structure to another is accomplished by storing the reference(s) to other node(s) in the structure. Although linked structures can be implemented in a variety of ways, the most flexible implementation is by using a separate object for each node.



Figure 2: Singly Linked List

Have two types of linked lists are Singly-linked list and Doubly-linked list. Singly linked lists node contains two data fields are info and next. Info stores information which is usable by user. Next links it to its successor in the sequence. Below is some example how to implement the operations (Singly Linked List).

```
* To change this license header, choose License Headers in Project Properties.
                                                                                        29
       * To change this template file, choose Tools | Templates
                                                                                         30
                                                                                               class MyList {
       * and open the template in the editor.
                                                                                         31
                                                                                                Node head, tail;
                                                                                         32 🖃
                                                                                                public MyList() {
                                                                                                  head = tail = null;
                                                                                         33
      package implement singly linked list;
    E /*:
                                                                                         35 📮
                                                                                                public boolean isEmpty() {
       * @author OS
                                                                                         36
                                                                                                   return (head == null);
 10
                                                                                         37
                                                                                            巨
                                                                                                 public void add(int x) {
 11
      public class Implement Singly Linked List {
                                                                                         38
 12
                                                                                         39
                                                                                                   Node q = new Node(x);
          * @param args the command line arguments
                                                                                                   if (isEmpty()) {
 14
                                                                                         41
                                                                                                     head = tail = q;
 15 public static void main(String[] args) {
                                                                                         42
           int[] a = {1, 2, 4, 5, 3, 6, 7, 8};
 16
                                                                                         43
                                                                                                   else {
          MyList ml = new MyList();
                                                                                         44
                                                                                                     tail.next = q; tail = q;
 17
          ml.addMany(a); ml.traverse();
 19
                                                                                         46
                                                                                            早
 20
                                                                                         47
                                                                                                 public void addMany(int[] a) {
21
      class Node {
                                                                                         48
                                                                                                   for (int i: a) {
        int info; Node next;
 22
                                                                                         49
                                                                                                     add(i);
 23 📮
       public Node(int info, Node next) {
                                                                                         50
 24
           this.info = info; this.next = next;
                                                                                         51
                                                                                            曱
                                                                                                public void traverse() {
 25
                                                                                         52
 26 public Node(int info) {
                                                                                         53
                                                                                                  Node p = head;
                                                                                                   while(p != null) {
 27
          this.info = info; this.next = null;
                                                                                         54
                                                                                                    System.out.print(p.info + " ");
 28
                                                                                         55
 29
                                                                                         56
                                                                                                    p = p.next;
      class MyList {
 31
        Node head, tail;
                                                                                         58
                                                                                                   System.out.println();
 32 -
        public MyList() {
                                                                                         59
33
          head = tail = null;
                                                                                         60
 34
                                                                                         61
Output - JavaApplication13 (run) X
                                                                                        Output - JavaApplication13 (run) X
1 2 4 5 3 6 7 8
                                                                                             1 2 4 5 3 6 7 8
BUILD SUCCESSFUL (total time: 0 seconds)
                                                                                             BUILD SUCCESSFUL (total time: 0 seconds)
```

Figure 3: Implement Java Code for Singly Linked List







```
this.info = info; this.next = next;
         * To change this license header, choose License Headers in Project Properties.
        * To change this template file, choose Tools | Templates * and open the template in the editor.
                                                                                                             31
                                                                                                                         this.info = info; this.next = null;
        package insertFirstSinglyLinkedlist;
                                                                                                             34
                                                                                                                    class MvList {
  8 - /**
                                                                                                                      Node head, tail;
public MyList() {head = tail = null;}
                                                                                                                      public boolean isEmpty() {return (head == null);}
public void add(int x) {
  Node q = new Node(x);
        public class InsertFirstSinglyLinkedlist {
           Node head, tail;
 14 = /**
15 | * (
                                                                                                                        if (isEmpty()) {head = tail = q;}
                                                                                                                         else {tail.next = q; tail = q;}
                                                                                                             43 public void AddMany(int[] a) {
  17 public static void main(String[] args) {
                                                                                                                         for (int i: a) {add(i);}
           int[] a = {0, 1, 2, 3, 4, 5, 6, 7};
MyList ml = new MyList();
                                                                                                                      public void Traverse() {
                                                                                                             46
             ml.AddMany(a);
ml.Traverse();
                                                                                                                        Node p = head;
while(p != null) {System.out.print(p.info + " "); p = p.next;}
     mi.InsertFirst
ml.Traverse();
}
             ml.InsertFirst(44);
                                                                                                                       System.out.println();
                                                                                                            50
51
                                                                                                                 public void InsertFirst(int x) {
                                                                                                            52
53
54
                                                                                                                        Node q = new Node(x);
if (isEmpty()) {head = tail = q;}
else {q.next = head; head = q;}
         class Node (int info; Node next;
         public Node(int info, Node next) {
             this.info = info; this.next = next;
                                                                                                            55
56
57
                                                                                                                 public void InsertFirstMany(int[] a){
  30 public Node(int info) {
                                                                                                                        for (int i: a) {InsertFirst(i);}
             this.info = info; this.next = null;
     60
        class MyList {
 Output - JavaApplication14 (run) ×
                                                                                                            Output - JavaApplication14 (run) X
| run:
| 0 1 2 3 4 5 6 7
| 44 0 1 2 3 4 5 6 7
| BUILD SUCCESSFUL (total time: 0 seconds)
                                                                                                           0 1 2 3 4 5 6 7
44 0 1 2 3 4 5 6 7
BUILD SUCCESSFUL (total time: 0 seconds)
```

Figure 4: Implement Java Code for inserting Node at the beginning of Singly Linked List

```
if (isEmpty()) {
       * To change this license header, choose License Headers in Project Properties
                                                                                                         head = tail = q;
       * To change this template file, choose Tools | Templates * and open the template in the editor.
                                                                                                         tail.next = q; tail = q;
      package insertLastSinglyLinkedList;
                                                                                             46
 8 🖵 /**
                                                                                                曱
                                                                                                    public void addMany(int[] a) {
9
10
                                                                                             48
                                                                                                       for (int i: a) {
    * @author OS
12 public class InsertLastSinglyLinkedList {
13 📮 /**
                                                                                                public void traverse() {Node p = head;
                                                                                             52
14
         * @param args the command line arguments
                                                                                                       while(p != null) {
                                                                                                         System.out.print(p.info + " "); p = p.next;
16 public static void main(String[] args) {int[] a = {0, 1, 2, 3, 4, 5, 6, 7};
          MyList ml = new MyList();
ml.addMany(a); ml.traverse();
                                                                                                       System.out.println();
18
           ml.insertLast(44); ml.traverse();
                                                                                                public void insertFirst(int x) {Node q = new Node(x);
20
                                                                                                       if (isEmpty()) {
21
                                                                                                         head = tail = q;
      class Node {
22
         int info; Node next;
                                                                                             62
                                                                                                       else (
24 📮
       public Node(int info, Node next) {
                                                                                                         q.next = head; head = q;
           this.info = info; this.next = next;
                                                                                             64
26
27 public Node(int info) {
                                                                                                public void insertFirstMany(int[] a) {
                                                                                             66
           this.info = info: this.next = null:
28
                                                                                                         insertFirst(i);
30
                                                                                             70
32
        Node head, tail;
                                                                                                public void insertLast(int x) {
                                                                                                       Node q = new Node(x);
          head = tail = null;
34
                                                                                                       if (isEmpty()) {
                                                                                                         head = tail = q:
        public boolean isEmpty() {
36 🖃
          return (head == null);
                                                                                                       else (
38
                                                                                                         tail.next = q; tail = q;
39 public void add(int x) {Node q = new Node(x);
      if (isEmpty()) {
  head = tail = q;
40
                                                                                             80
Output - InsertLastSLL (run) X
                                                                                            Output - InsertLastSLL (run) X
0 1 2 3 4 5 6 7
0 1 2 3 4 5 6 7 44
     0 1 2 3 4 5 6 7
0 1 2 3 4 5 6 7 44
BUILD SUCCESSFUL (total time: 0 seconds)
                                                                                                 BUILD SUCCESSFUL (total time: 0 seconds)
```

Figure 5: Implement Java Code for inserting Node at the end of Singly Linked List







```
/* To change this license header, choose License Headers in Project Properties.

38
39
To change this template file, choose Tools | Templates
40
                                                                                                                      public void add(int x) {
                                                                                                                        Node q = new Node(x);
if (isEmpty()) {head = tail = q;
         * and open the template in the editor.
       package deleteFirstSinglyLinkedList;
 8 P /**
                                                                                                            46
                                                                                                                         for (int i: a) {add(i);
       public class DeleteFirstSinglyLinkedList {
                                                                                                             49 🖃
                                                                                                                      public void traverse() {Node p = head;
                                                                                                                         while(p != null) {
   System.out.print(p.info + " "); p = p.next;
            * @param args the command line arguments
15 | */
16 | public static void main(String[] args) {
                                                                                                            52
53
            int[] a = {1, 2, 4, 5, 3, 6, 7, 8};
MyList ml = new MyList();
                                                                                                            54
                                                                                                                      public void insertFirst(int x) {
                                                                                                                        Node q = new Node(x);
if (isEmpty()) {head = tail = q;
            ml.addMany(a); ml.traverse();
            ml.deleteFirst(); ml.traverse();
                                                                                                            57
                                                                                                            58
59
 22
                                                                                                                         else {q.next = head; head = q;
23 class Node (int info; Node next;
24 public Node (int info, Node next) {
                                                                                                             60
            this.info = info; this.next = next;
                                                                                                            62 <u></u>
                                                                                                                      public void insertFirstMany(int[] a) {
                                                                                                            63
64
                                                                                                                        for (int i: a) {insertFirst(i);
 27
28
29
    public Node(int info) {
             this.info = info; this.next = null;
                                                                                                             65
                                                                                                                      public void insertLast(int x) {
                                                                                                                        Node q = new Node(x);
if (isEmpty()) {head = tail = q;
                                                                                                             68
          Node head, tail;
public MyList() (head = tail = null;
}
                                                                                                                        else {tail.next = q; tail = q;
35 public boolean isEmpty() {
36 return (head == null);
37
                                                                                                             73 public void deleteFirst() {
                                                                                                                         if (!isEmpty())
 38  public void add(int x) {
39  Node q = new Node(x);
          if (isEmpty()) {head = tail = q;
Output - JavaApplication58 (run) ×
                                                                                                            Output - JavaApplication58 (run) ×
                                                                                                           | Run: 12453678 | 2453678 | BUILD SUCCESSFUL (total time: 0 seconds)
     run:
1 2 4 5 3 6 7 8
2 4 5 3 6 7 8
BUILD SUCCESSFUL (total time: 0 seconds)
```

Figure 6: Implement Java Code for deleting Node at the beginning of Singly Linked List

```
public void addMany(int[] a) {
                   * To change this license header, choose License Headers in Project Properties 43 * To change this template file, choose Tools | Templates 44
                                                                                                                                                                                                                                                              for (int i: a) {add(i);
                   * and open the template in the editor.
                                                                                                                                                                                                                                    46
  6 paci
                                                                                                                                                                                                                                                        while (p != null) {
                                                                                                                                                                                                                                                                System.out.print(p.info + " "); p = p.next;
        public class DeleteLastSinglyLinkedList {
                                                                                                                                                                                                                                   53 public void insertFirst(int x) {
                                                                                                                                                                                                                                                             Node q = new Node(x);
if (isEmpty()) {head = tail = q;
                                                                                                                                                                                                                                   54
55
                          * @param args the command line arguments
else {q.next = head; head = q;
                                                                                                                                                                                                                                   59
                                                                                                                                                                                                                                                    public void insertFirstMany(int[] a) {
                                                                                                                                                                                                                                                              for (int i: a) {insertFirst(i);
21 2 3 class Node {int info; Node next; 24 public Node(int info, Node next) { this.info = info; this.next = ne; 3 public Node(int info) } 27 public Node(int info; this.next = ne; 3 public Node(int info; thi
                                                                                                                                                                                                                                   public void insertLast(int x)(Node q = new Node(x);
                                                                                                                                                                                                                                                              if (isEmpty()) {head = tail = q;
                         this.info = info; this.next = next;
                                                                                                                                                                                                                                                              else (tail.next = q; tail = q;
                          this.info = info; this.next = null;
                                                                                                                                                                                                                                   70 public void deleteFirst() {
                class MyList (Node head, tail;
public MyList() {head = tail = null;
                                                                                                                                                                                                                                                              if (!isEmpty()) {head = head.next;
public MyList() (head = tail = null;
)

public MyList() (head = tail = null;
)

public boolean isEmpty() (return (head == null);
                                                                                                                                                                                                                                                             if (!isEmpty()) {Node tmp = head;
36 public void add(int x) (Node q = new Node(x);
37 if (isEmpty()) (head = tail = q;
}
                                                                                                                                                                                                                                                                   while(tmp.next != tail) (tmp = tmp.next;
                                                                                                                                                                                                                                                                   tail = tmp; tail.next = null;
                          else {tail.next = q; tail = q;
Output - JavaApplication59 (run) X
                                                                                                                                                                                                                                  Output - JavaApplication59 (run) ×
                                                                                                                                                                                                                                              1 2 4 5 3 6 7 8
                                                                                                                                                                                                                                 BUILD SUCCESSFUL (total time: 0 seconds)
```

Figure 7: Implement Java Code for deleting Node at the end of Singly Linked List





P2 Determine the operations of a memory stack and how it is used to implement function calls in a computer.

1. Determine the operations of a memory stack.

1.1. What is a stack?

A stack is a linear data structure that can be accessed only at one of its ends for storing and retrieving data. It is a Last In/First Out (LIFO) data structure. Anything added will go to the top. Anything removed will be taken from the top and things are removed in the reverse order that they were inserted. In other words, such a stack resembles a stack of trays in a cafeteria. New trays are put on the top of the stack and taken off the top. The last tray put on the stack is the first tray removed from the stack. For this reason, a stack is called a LIFO structure: Last In/First Out.

1.2. What are operations on stack?

According to the above explanation, a tray can be taken only if there are trays on the stack, and a tray can be added to the stack only if there is enough room; that is, if the stack is not too high. Therefore, a stack is defined in terms of operations that change its status and operations that check this status. Below table is the operations on stack.

Table 2: Operations on stack

Operations on stack		
clear()	Clear the stack	
isEmpty()	Check to see if the stack is empty	
push(el)	Put the element el on the top of the stack	
pop()	Take the topmost element from the stack	
topEl()	Return the topmost element in the stack without removing it	

Operations pop and top cannot be performed if the stack is empty. If attempting the execution of pop or top on an empty stack, will throws a StackEmptyException. In addition, operations push sometimes cannot be performed if it's not enough memory. If attempting the execution of push when there is not enough memory, will throws an OutOfMemoryError.





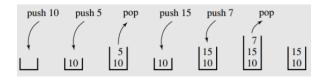


Figure 8: A series of operations executed on a stack

For example: A series of push and pop operations is shown in Figure 8. After pushing number 10 onto an empty stack, the stack contains only this number. After pushing 5 on the stack, the number is placed on top of 10 so that, when the popping operation is executed, 5 is removed from the stack, because it arrived after 10, and 10 is left on the stack. After pushing 15 and then 7, the topmost element is 7, and this number is removed when executing the popping operation, after which the stack contains 10 at the bottom and 15 above it.

1.3. How to implement stacks?

```
public ArrayStack() {
       * To change this license header, choose License Headers in Project Properties.
                                                                                                    this (50);
       ^{\star} To change this template file, choose Tools | Templates
       ^{\star} and open the template in the editor.
                                                                                          42
                                                                                          43 ⊡
                                                                                                  public boolean isEmpty() {
                                                                                                    return top == -1;
                                                                                          46 E
                                                                                                  public boolean isFull() {
 8
                                                                                          47
                                                                                                    return top == max - 1;
       * @author OS
 10
                                                                                          49 📮
                                                                                                  public void clear() {
11 = import java.util.EmptyStackException;
                                                                                          50
                                                                                                    top = -1;
12
      public class ArrayStackImp {
                                                                                          51
13
                                                                                          52
                                                                                             ₽
         * 8param args the command line arguments
                                                                                          53
                                                                                                    int max1 = max + max/2;
15
                                                                                          54
                                                                                                    Object[] al = new Object[max1];
   ₽
16
        public static void main(String[] args) {
                                                                                                    if (al == null) {
17
          decToBin(10);
                                                                                          56
                                                                                                     return false;
18
                                                                                          57
19
   public static void decToBin(int k) {
                                                                                                    System.arraycopy(a, 0, al, 0, top);
20
          ArrayStack s = new ArrayStack();
                                                                                          59
                                                                                                    a = al;
21
          System.out.println(k + " In Binary System Is: ");
                                                                                          60
                                                                                                    max = max1:
          while(k > 0) {
                                                                                          61
                                                                                                    return true;
23
            s.push(k % 2);
                                                                                          62
24
            k = k / 2;
                                                                                          63 E
                                                                                                  public void push (Object x) {
25
                                                                                          64
                                                                                                    if (isFull() && !grow()) return;
26
          while(!s.isEmpty()){
                                                                                          65
                                                                                                    a[++top] = x;
27
            System.out.print(s.pop());
                                                                                          66
28
                                                                                          67 public Object top() throws EmptyStackException (
29
          System.out.println("");
                                                                                          68
                                                                                                    if (isEmpty()) throw new EmptyStackException();
30
                                                                                          69
                                                                                                    return (a[top]);
31
                                                                                          70 -
32
      class ArrayStack {
                                                                                                 public Object pop() throws EmptyStackException {
33
        protected Object[] a;
                                                                                          72
                                                                                                    if (isEmpty()) throw new EmptyStackException();
34
         int top, max;
                                                                                          73
                                                                                                    Object x = a[top];
35
        public ArrayStack(int max) {
                                                                                          74
                                                                                                    top--:
36
          this.max = max;
                                                                                          75
                                                                                                    return x;
37
           a = new Object[max];
                                                                                          76
38
          top = -1;
                                                                                          77
39
Output - JavaApplication60 (run) ×
                                                                                         Output - JavaApplication60 (run) ×
                                                                                         D
     10 In Binary System Is:
                                                                                              10 In Binary System Is:
    BUILD SUCCESSFUL (total time: 0 seconds)
                                                                                              BUILD SUCCESSFUL (total time: 0 seconds)
```

Figure 9: Array Implementation of Stack - The demo program that can convert 10 to binary





2. How it is used to implement function calls in a computer.

2.1. Applications of stacks.

Table 3: Applications of Stacks

	Some applications of stacks
1	Any sort of nesting (such as parentheses)
2	Evaluating arithmetic expressions (and other sorts of expression)
3	Implementing function or method calls
4	Keeping track of previous choices (as in backtracking)
5	Keeping track of choices yet to be made (as in creating a maze)
6	Undo sequence in a text editor
7	Auxiliary data structure for algorithms
8	Component of other data structure
9	Any sort of nesting (such as parentheses)
10	Evaluating arithmetic expressions (and other sorts of expression)
11	Implementing function or method calls
12	Keeping track of previous choices (as in backtracking)
13	Keeping track of choices yet to be made (as in creating a maze)
14	Undo sequence in a text editor
15	Auxiliary data structure for algorithms
16	Component of other data structure





Table 4: Functionalities of the function calls

Functionalities of the function calls				
6	Object[]a	Array of Object		
Common characteristics	int top, max	Top is the number currently highest on the stack		
		Max is the number that the stack can hold		
Constructors	ArrayStack(int max)	Initialize an empty stack with a specified max		
Constructors	ArrayStack() - default max=50	Initialize an empty stack with a specified max is 50		
	isEmpty()	Check to see if the stack is empty		
	isFull()	Tests if the stack is full or not		
Common	clear()	Clear the stack		
behaviors	grow()	Increase the size of the internal array		
	push()	Pushes an item onto the top of the stack		
	top()	Return the topmost element in the stack without removing it		
	pop()	Take the topmost element from the stack		

2.2. What is method calls and how they work.

If the method has formal parameters, they have to be initialized to the values passed as actual parameters. In addition, the system has to know where to resume execution of the program after the method has finished.

The method can be called by other methods or by the main program (the method main()). The information indicating where it has been called from has to be remembered by the system. This could be done by storing the return address in main memory in a place set aside for return addresses, but we do not know in advance how much space might be needed, and allocating too much space for that purpose alone is not efficient.

For a method call, more information has to be stored than just a return address. Therefore, dynamic allocation using the run-time stack is a much better solution.





Each time a method is called, an activation record (AR) is allocated for it and this record contains the following information. *Firstly*, parameters and local variables used in the called method. *Secondly*, dynamic link - a pointer to the caller's activation record. *Thirdly*, return address to resume control by the caller - address of instruction immediately following the call. *Finally*, return value for a method not declared as void, since the size of AR may vary from one call to another so returned value is placed right above the AR of the caller.

Each new AR is placed on top of the run-time stack. When a method terminates, its AR is removed from the top of the run-time stack. Thus, the first AR placed on the stack is the last one removed.

Creating an activation record whenever a method is called allows the system to handle recursion properly. Recursion is calling a method that happens to have the same name as the caller. Therefore, a recursive call is not literally a method calling itself, but rather an instantiation of a method calling another instantiation of the same original. These invocations are represented internally by different activation records and are thus differentiated by the system.

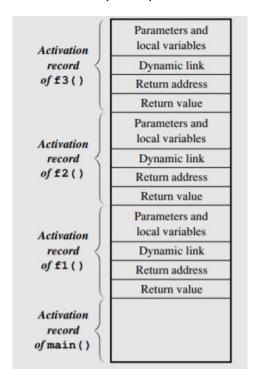


Figure 10: How to implement method calls by stack - Contents of the run-time stack when main() calls method f1(), f1() calls f2(), and f2() calls f3()





For example: Suppose that main() calls method f1(), f1() calls f2(), and f2() in turn calls f3(). If f3() is being executed, then the state of the run-time stack is as shown in Figure 10 By the nature of the stack, if the activation record for f3() is popped by moving the stack pointer right below the return value of f3(), then f2() resumes execution and now has free access to the private pool of information necessary for reactivation of its execution. On the other hand, if f3() happens to call another method f4(), then the runtime stack increases its height because the activation record for f4() is created on the stack and the activity of f3() is suspended.

M1 Illustrate, with an example, a concrete data structure for a First in First out (FIFO) queue.

FIFO is an abbreviation for first in, first out. It is a method for handling data structures where the first element is processed first and the newest element is processed last.

FIFO used in data structures, disk scheduling, communications and networking. In data structures, certain data structures like Queue and other variants of Queue uses FIFO approach for processing data. In disk scheduling, disk controllers can use the FIFO as a disk scheduling algorithm to determine the order in which to service disk I/O requests. Finally, in communications and networking, communication network bridges, switches and routers used in computer networks use FIFOs to hold data packets end route to their next destination.

Examples of queues are very much in practice. A typical example of a queue is cars lined up to enter a toll booth. Vehicles that arrive first will be charged in advance and can continue the journey. It can be seen similarly that the way the queue works is to operate first in first out. Below are pictures depicting how the queuing operation works.

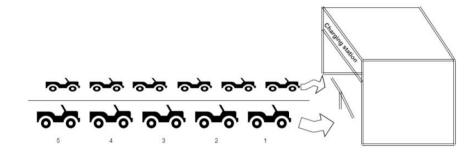


Figure 11: Simulation of queue 1 example





This is a simulated image of cars queuing up in turn to enter the toll booth. Here will number each vehicle to make it easier to track the progress. The cars numbered 1 to 5 and the vehicle numbered 1 are the cars preparing to enter the tollgate.

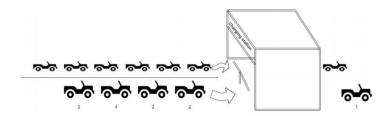


Figure 12: Simulation of queue 2 example

After paying the fee, the car No. 1 got out of the row and continued the journey. The No. 2 car continues to move to the station entrance and the process continues like that first in first out.

```
* To change this license header, choose License Headers in Project Properties.
       * To change this template file, choose Tools | Templates
       * and open the template in the editor.
      package queueExample;
   import java.util.LinkedList;
      import java.util.Queue;
    P /*
10
12
13
      // Java program to demonstrate
14
      // Working of FIFO
15
      // Using Queue interface in Java
      public class QueueExample {
16
    public static void main(String[] args) (
17
          Queue<Integer> q = new LinkedList<>();
           // Adds elements {0, 1, 2, 3, 4} to queue
20
          for (int i = 0; i < 10; i++)
21
          q.add(i);
22
            / Display contents of the queue.
          System.out.println("Elements of queue is: " + q);
23
          // To remove the head of queue.
24
           // In this the oldest element '0' will be removed
25
          int removedele = q.remove();
26
           System.out.println("Rem
                                    ved element is: " + removedele);
28
          System.out.println(q);
           // To view the head of queue
29
          int head = q.peek();
30
          System.out.println("Head of queue is: " + head);
31
           // Rest all methods of collection interface,
32
33
          // Like size and contains can be used with this
           // Implementation
35
           int size = q.size();
36
           System.out.println("Size of queue is: " + size);
37
38
      )
39
Output - JavaApplication61 (run) ×
     Elements of queue is: [0, 1, 2, 3, 4, 5, 6, 7, 8, 9]
     Removed element is: 0
(3)
     [1, 2, 3, 4, 5, 6, 7, 8, 9]
     Head of queue is: 1
2/2
     Size of queue is: 9
     BUILD SUCCESSFUL (total time: 0 seconds)
```

Figure 13: Implement Java Code for First in First out (FIFO) queue





M2 Compare the performance of two sorting algorithms.

There are many sorting algorithms, each one has its advantages and disadvantages. However, I will explain and comparison sorting algorithms Insertion sort and Merge sort.

1. Insertion sort

An insertion sort starts by considering the two first elements in an array A (0-based index). If the element A[1] is less than the element A[0], they are swapped to make them in order. Then, the third element A[2] is considered and inserted into its proper place, if it is lesser than the previous two elements then the previous elements are shifted to the right by one position and put A[2] to the shifted empty position. Making A[0] at position 1, A[1] at position 2 and A[2] at position 0. If A[2] is not less than both its predecessors, it stays in its current position. Summary, each element A[i] is inserted into its proper location j such that such that $0 \le j \le i$ and all elements greater than A[i] is shifted to the right by one position.

```
package myinsertionsort;
   - /**
 9
10
       * @author OS
11
12
      public class MyInsertionSort {
13
      //From small to big
14 public static void main(String a[]) {
15
     int[] arr1 = {10,34,2,56,7,67,88,42};
16
      int[] arr2 = doInsertionSort(arr1);
17
      for(int i:arr2){
      System.out.print(i);
18
19
      System.out.print(", ");
20
21
      System.out.println();
22
24
     for (int i = 1; i < input.length; i++) {</pre>
25
26
      for(int j = i ; j > 0 ; j--) {
27
      if(input[j] < input[j-1]){</pre>
28
      temp = input[j]; input[j] = input[j-1]; input[j-1] = temp;
29
30
      }
31
32
      return input;
33
      }
34
      }
Output - JavaApplication48 (run) ×
    2, 7, 10, 34, 42, 56, 67, 88,
    BUILD SUCCESSFUL (total time: 0 seconds)
```

Figure 14: Implement Java Code for Insertion Sort





Insertion sort runs well for partially sorted array. Because when we are considering an element A[i] with i > 0 during insertion sort, it means that all the elements from A[0...i-1] is already sorted. Initially i = 1, this means that the subarray A[0...0] is sorted. It is trivial that A[0] is sorted, then A[1] is put into its proper location, by shifting the element A[0] if it is greater than A[1] to the right by one position, making the sub-array A[0...1] sorted. Then i = 2, this means sub-array A[0...1] is sorted which is true based on the previous step. Then A[2] is put into its proper location, by shifting the elements in A[0...1] that is greater than A[2] to the right by one position, making the sub-array A[0...2] sorted. For i = j such that 2 < j < n, it follows that A[0...j-1] is already sorted and A[j] is put into its proper location making the sub-array A[0...i] sorted. From that we can see that, when we are considering an element A[i] with i > 0 during insertion sort and the sub-array A[0...i-1] is sorted, if A[i] > A[i-1] then A[i] is already in its proper location hence no swapping of elements is performed. The best case for insertion sort is when the array is already sorted. One comparison is made for each i, starting at i = 1, so there is n-1 comparison making it time complexity O(n). The worst case for insertion sort is when the array is already sorted but in reverse order. In this case, for every element A[i] where i > 0, it is always lesser than elements from A[0]...[i-1]. This means there is i comparisons for each i from i = 1 to i = n-1. Making the sum:

$$\sum_{i=1}^{n-1} i = 1 + 2 + \dots + (n-1) = \frac{n(n-1)}{2} = O(n^2)$$

The above formula gives the upper-bound for the worst-case time complexity which is O(n2). In the average-case, this depends how far away the element A[i] that we are considering from its proper location. If it is already in proper location, then only perform one compare between A[i] and A[i-1]. If it is one location away from its proper location, then two comparisons are performed between A[i], A[i-1] and A[i-2]. Summary, if it is j positions away then we do j+1 comparison. Assume that every element in the array have uniform probability of being in its proper location. So, the average number of comparisons for A[i] of iteration i is:

$$\frac{1+2+\dots+i-1}{i} = \frac{\frac{1}{2}i(i+1)}{i} = \frac{i+1}{2}$$





Next, we can calculate the total number of comparisons by summing it through all the i from 1 to n-1:

$$\sum_{i=1}^{n-1} \frac{i+1}{2} = \frac{1}{2} \sum_{i=1}^{n-1} i + \sum_{i=1}^{n-1} \frac{1}{2} = \frac{\frac{1}{2}n(n-1)}{2} + \frac{1}{2}(n-1) = \frac{n^2 + n - 2}{4}$$

The upper-bound average-case running-time for insertion O(n2) and approximately one-half of the complexity in the worst case.

2. Merge sort

Merge sort different completely from insertion sort, it works by following the divide-and-conquer paradigm. The sorting takes place by sorting two halves of the array and merge it into one. But this requires the two halves to be already sorted which can be accomplish by merging the sorted two halves of these halves. Can see that, there is a recurrence relation in merge sort.

```
public class MyMergeSort {
      private int[] array;
      private int[] tempMergArr;
 15
      private int length;
 16  public static void main(String a[]){int[] inputArr = {10,34,2,56,7,67,88,42};
     MyMergeSort mms = new MyMergeSort(); mms.sort(inputArr); for(int i:inputArr){
      System.out.print(i); System.out.print(" ");
19
20
      System.out.println();
21
 22 public void sort(int inputArr[]) {
23
      this.array = inputArr; this.length = inputArr.length;
24
      this.tempMergArr = new int[length]; doMergeSort(0, length - 1);
 25
26 private void doMergeSort(int lowerIndex, int higherIndex) {
      if (lowerIndex < higherIndex) {</pre>
      int middle = lowerIndex + (higherIndex - lowerIndex) / 2;
29
      // Below step sorts the left side of the array
      doMergeSort(lowerIndex, middle);
      // Below step sorts the right side of the array
      doMergeSort(middle + 1, higherIndex);
 32
 33
34
      mergeParts(lowerIndex, middle, higherIndex);
 35
 37 private void mergeParts(int lowerIndex, int middle, int higherIndex) {
 38
      for (int i = lowerIndex; i <= higherIndex; i++) {tempMergArr[i] = array[i];
 39
      int i = lowerIndex;
 42
      int k = lowerIndex;
      while (i <= middle && j <= higherIndex) {
if (tempMergArr[i] <= tempMergArr[j]) {</pre>
 43
 44
      array[k] = tempMergArr[i]; i++;
 46
      array[k] = tempMergArr[j]; j++;
 47
 48
51
      while (i <= middle) {
      array[k] = tempMergArr[i]; k++; i++;
52
53
Output - JavaApplication52 (run) ×
     2 7 10 34 42 56 67 88
     BUILD SUCCESSFUL (total time: 0 seconds)
```

Figure 15: Implement Java Code for Merge Sort





Suppose n is a power of 2 and T(n) denotes the worst-case running-time for merge sorting an array of n elements. Deduced following equation:

$$T(n) = T\left(\frac{n}{2}\right) + T\left(\frac{n}{2}\right) + n + n = 2T\left(\frac{n}{2}\right) + 2n$$

Based on the above equation, will get the following recurrence relation (where the constant term is the time it takes to copy the elements into the auxiliary array and merge the two halves array):

$$\begin{cases} T(1) = 1 & if \ n = 1 \\ T(n) = 2T\left(\frac{n}{2}\right) + 2n \ if \ n > 1 \end{cases}$$

Next, can calculate the worst-case time complexity as then:

$$T(n) = 2T\left(\frac{n}{2}\right) + 2n = 4T\left(\frac{n}{4}\right) + 4n = 8T\left(\frac{n}{8}\right) + 6n = \dots = 2^k T\left(\frac{n}{2^k}\right) + 2kn$$

Based on the last equation, deduced:

$$\Leftrightarrow \frac{n}{2^k} = 1 \Leftrightarrow n = 2^k \Leftrightarrow \log_2 n = k \Rightarrow 2^{\log_2 n} T(1) + 2n \log_2 n = n + 2n \log_2 n$$
$$= n(1 + 2\log_2 n) = O(n \log_2 n)$$

From that can see that, the worst-case time complexity for merge sort is O(nlog2n). The best case running-time and average running-time for merge sort is the same as its worst-case time since it also need to divide the array into its smallest sub-array size and start merging from there. However, still one drawback from merge sort it is that it requires to create additional storage for storing copy of the array which can affect a lot on its running time when sorting large array. In addition, can make merge sort more efficient by implementing it with iterative method instead of recursion and use insertion sort for small size array.

3. Compare the performance of insertion and bubble sort

Based on the given running-time of two sorting algorithms, we can come up with the following comparison table.





Table 5: Compare the performance of two sorting algorithms

Туре	Time		
7,00	Best	Average	Worst
Insertion Sort	O(n)	O(n^2)	O(n^2)
Merge Sort	O(nlog2n)	O(nlog2n)	O(nlog2n)

From the above explanations, it can be seen that: merge sort out performs insertion sort for large number n while insertion sort is better than merge sort when the array is already sorted or partially sorted. So, can combine both merge sort and insertion sort to get the best out efficiency in both algorithms. Because when sorting small size array, the merge sort time complexity is always the same for any case while insertion sort can perform better since it can be the case that the array is partially sorted so it will be more efficient. While for larger array size, merge sort certainly performs better due to it logarithmic time complexity. Though we should note that this is just the upper-bound for the algorithms which means it only really matters for large n. For small size array their running-time should be relatively as efficient.

D1 Analyze the operation, using illustrations, of two network shortest path algorithms, providing an example of each.

1. Dijkstra's Algorithm





Dijkstra's Algorithm is the widely used algorithm to find the shortest routes in a graph or a tree. Given a weighted graph and a starting (source) vertex in the graph, Dijkstra's algorithm is used to find the shortest distance from the source node to all the other nodes in the graph. As a result of the running Dijkstra's algorithm on a graph, we obtain the shortest path tree (SPT) with the source vertex as root. In Dijkstra's algorithm, we maintain two sets or lists. One contains the vertices that are a part of the shortest-path tree (SPT) and the other contains vertices that are being evaluated to be included in SPT. Hence for every iteration, we find a vertex from the second list that has the shortest path. The pseudocode for the Dijkstra's shortest path algorithm is given below.

```
procedure dijkstra(G, S)
     G-> graph; S->starting vertex
begin
    for each vertex V in G
//initialization; initial path set to infinite
        path[V] <- infinite</pre>
        previous[V] <- NULL</pre>
        If V != S, add V to Priority Queue PQueue
    path [S] <- 0
    while POueue IS NOT EMPTY
        U <- Extract MIN from POueue
         for each unvisited adjacent node V of U
             tempDistance <- path [U] + edge weight(U, V)</pre>
             if tempDistance < path [V]</pre>
                 path [V] <- tempDistance
                 previous[V] <- U</pre>
    return path[], previous[]
end
```





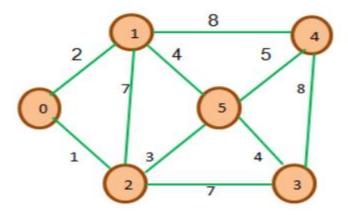


Figure 16: Sample graph the Dijkstra's shortest path algorithm

Initially, the SPT (Shortest Path Tree) set is set to infinity. So, to begin with we put the vertex 0 in sptSet. Means, $sptSet = \{0, INF, INF, INF, INF, INF, INF\}$. Next with vertex 0 in sptSet, we will explore its neighbors. Vertices 1 and 2 are two adjacent nodes of 0 with distance 2 and 1 respectively.

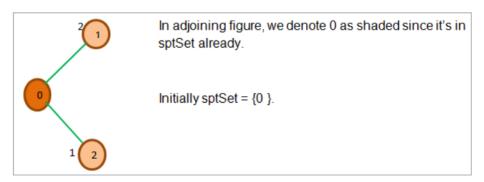


Figure 17: Analyze sample graph

In the above figure, we have also updated each adjacent vertex are 1 and 2 with their respective distance from source vertex 0. Now we see that vertex 2 has a minimum distance. So next we add vertex 2 to the <code>sptSet</code>. Also, we explore the neighbors of vertex 2.





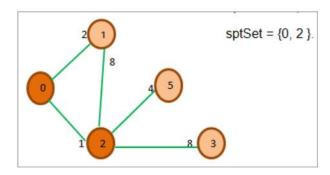


Figure 18: Analyze sample graph

Now we look for the vertex with minimum distance and those that are not there in spt. We pick vertex 1 with distance 2.

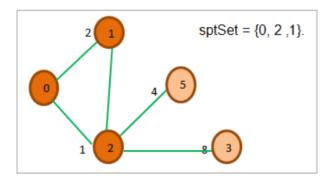


Figure 19: Analyze sample graph

As we see in the above figure, out of all the adjacent nodes of 2, 0, and 1 are already in <code>sptSet</code> so we ignore them. Out of the adjacent nodes 5 and 3, 5 have the least cost. So, we add it to the <code>sptSet</code> and explore its adjacent nodes.

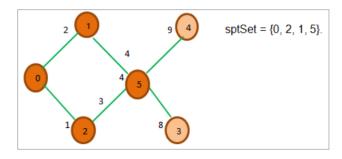


Figure 20: Analyze sample graph

In the above figure, we see that except for nodes 3 and 4, all the other nodes are in sptSet. Out of 3 and 4, node 3 has the least cost. So, we put it in sptSet.





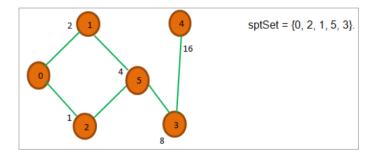


Figure 21: Analyze sample graph

As shown above, now we have only one vertex left i.e. 4 and its distance from the root node is 16. Finally, we put it in sptSet to get the final sptSet = {0, 2, 1, 5, 3, 4} that gives us the distance of each vertex from the source node 0.

Implementation of Dijkstra's shortest path algorithm in Java can be achieved using two ways. We can either use priority queues and adjacency list or we can use adjacency matrix and arrays.

```
for (int i = 0; i < adj list.get(u).size(); i++) {
          * To change this template file, choose Tools | Templates * and open the template in the editor.
                                                                                                                                                             Node v = adj_list.get(u).get(i);
// proceed only if current node
                                                                                                                                                             if (!visited.contains(v.node)) (
edgeDistance = "
 package javaapplication64;

| import java.util.ArrayList; import java.util.Comparator; import java.util.HashSet; import java.util.List; import java.util.FriorityQueue; import java.util.Set;
                                                                                                                                                                   edgeDistance = v.cost; newDistance = dist[u] + edgeDistance;
                                                                                                                                                                   if (newDistance < dist[v.node])dist[v.node] = newDistance;</pre>
                                                                                                                                                                   pqueue.add(new Node(v.node, dist[v.node]));)
10
11
12
13
14
15
16
                                                                                                                                                 public static void main(String arg[]) (int V = 6; int source = 0;
         class Graph pq {int dist[]; Set<Integer> visited;
                                                                                                                                                 List<List<Node> > adj_list = new ArrayList< >();
               PriorityQueue<Node> pqueue; int V; // Number of vertices
List<List<Node> > adj_list;
                                                                                                                                                  // Initialize adjacency list for every node in the graph
for (int i = 0; i < V; i++) {List<Node> item = new ArrayList<>();
                                                                                                                                                  adj_list.add(item);}
             public Graph_pq(int V) {this.V = V; dist = new int[V];
    visited = new HashSet<>();
                                                                                                                                                   // Input graph edges
adj_list.get(0).add(new Node(1, 5)); adj_list.get(0).add(new Node(2, 3));
                   pqueue = new PriorityQueue<>(V, (Comparator<? super Node>) new Node()); 58
20
21
22
                                                                                                                                                  adj_list.get(0).add(new Node(3, 2)); adj_list.get(0).add(new Node(4, 3)); adj_list.get(0).add(new Node(1, 2)); adj_list.get(2).add(new Node(1, 2)); adj_list.get(2).add(new Node(1, 2)); adj_list.get(2).add(new Node(3, 3));
23 P
24
25
              Graph_pq dpq = new Graph_pq(V); dpq.algo_dijkstra(adj_list, source);
                                                                                                                                                      // Print the shortest path from source node to all the nodes
System.out.println("The shorted path from source node to other nodes:");
System.out.println("Source\t\t" + "Node$\t\t" + "Distance");
for (int i = 0; i < dpq.dist.length; i++)
System.out.println(source + "\t\t" + i + "\t\t" + dpq.dist[i]);)
                   pqueue.add(new Node(src_vertex, 0));
29
30
31
                     dist[src_vertex] = 0; while (visited.size() != V) {
             // u is removed from PriorityQueue and
int u = pqueue.remove().node;
                                                                                                                                   68
69
70 ⊡
71 ⊡
72
®
74 ⊡
75
                                                                                                                                            class Node implements Comparator<Node> (public int node; public int cost;
32
                                                                                                                                              public Node() { } //empty constructor
public Node(int node, int cost) {this.node = node; this.cost = cost;}
33
34
                         visited.add(u); graph_adjacentNodes(u);)
                                                                                                                                                  @Override
                                                                                                                                                  public int compare(Node node1, Node node2)
{if (nodel.cost < node2.cost) return -1;
if (nodel.cost > node2.cost) return 1; return 0;}
            // this methods processes all neighbours of the just visited node
private void graph_adjacentNodes(int u) {
                    int edgeDistance = -1; int newDistance = -1;
// process all neighbouring nodes of u
38
                                                                                                                                  Output - JavaApplication64 (run) ×
                                                                                                                                  D
                                                                                                                                  验
2
       BUILD SUCCESSFUL (total time: 0 seconds)
                                                                                                                                         BUILD SUCCESSFUL (total time: 0 seconds)
```

Figure 22: Implementation of Dijkstra's Algorithm in Java - Using A Priority Queue

2. Bellman Ford's Algorithm

Bellman Ford's algorithm is used to find the shortest paths from the source vertex





to all other vertices in a weighted graph. It depends on the following concept, shortest path contains at most n-1 edges, because the shortest path couldn't have a cycle. Bellman-Ford do work for graphs with negative weight edges. It is also simpler than Dijkstra and suites well for distributed systems. But time complexity of Bellman-Ford is O(VE), which is more than Dijkstra.

Algorithm Steps: The outer loop traverses from $0 \to n \to 1$. Loop over all edges, check if the next node distance > current node distance + edge weight, in this case update the next node distance to "current node distance + edge weight". This algorithm depends on the relaxation principle where the shortest distance for all vertices is gradually replaced by more accurate values until eventually reaching the optimum solution. In the beginning all vertices have a distance of "Infinity", but only the distance of the source vertex = 0, then update all the connected vertices with the new distances (source vertex distance + edge weights), then apply the same concept for the new vertices with new distances and so on.

<u>For example</u>: Let the given source vertex be 0. Initialize all distances as infinite, except the distance to the source itself. Total number of vertices in the graph is 5, so all edges must be processed 4 times.

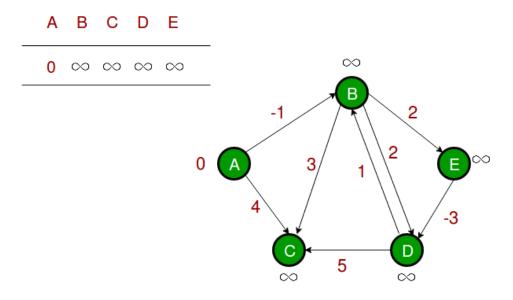


Figure 23: Sample graph the Bellman Ford's shortest path algorithm

Let all edges are processed in the following order: (B, E), (D, B), (B, D), (A, B), (A, C), (D, C), (B, C), (E, D). We get the following distances when all edges are processed the first time. The first row shows initial distances. The second row shows





distances when edges (B, E), (D, B), (B, D) and (A, B) are processed. The third row shows distances when (A, C) is processed. The fourth row shows when (D, C), (B, C) and (E, D) are processed.

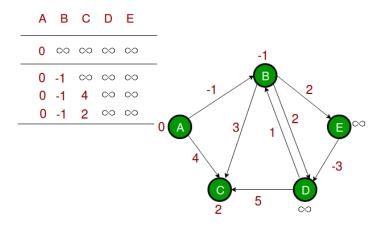


Figure 24: Analyze sample graph

The first iteration guarantees to give all shortest paths which are at most 1 edge long. We get the following distances when all edges are processed second time - The last row shows final values.

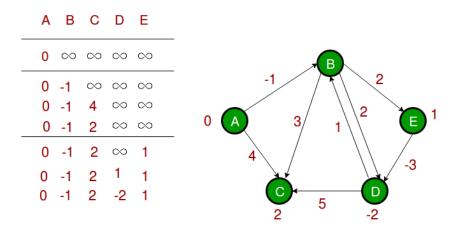


Figure 25: Analyze sample graph

The second iteration guarantees to give all shortest paths which are at most 2 edges long. The algorithm processes all edges 2 more times. The distances are minimized after the second iteration, so third and fourth iterations don't update the distances.







```
if (dist[u] != Integer.MAX_VALUE && dist[u] + weight < dist[v])</pre>
      * To change this license header, choose License Headers in Project Properties. 36
                                                                                                                                     dist[v] = dist[u] + weight;))
                                                                                       37
                                                                                             // Step 3: check for negative-weight cycles. The above
      * To change this template file, choose Tools | Templates
      * and open the template in the editor.
                                                                                       38
                                                                                            // step guarantees shortest distances if graph doesn't
 5
                                                                                            // contain negative weight cycle. If we get a shorter
      package javaapplication65;
                                                                                             // path, then there is a cycle.
                                                                                            for (int j = 0; j < E; ++j) {int u = graph.edge[j].src;</pre>
                                                                                       41
                                                                                       42
                                                                                            int v = graph.edge[j].dest; int weight = graph.edge[j].weight;
                                                                                       43
                                                                                            if (dist[u] != Integer.MAX_VALUE && dist[u] + weight < dist[v]) {</pre>
                                                                                       44
                                                                                            System.out.println("Graph contains negative weight cycle: "); return;}}
                                                                                           printArr(dist, V);}
11
                                                                                       45
      // A Java program for Bellman-Ford's single source shortest path
                                                                                             // A utility function used to print the solution
12
      // Algorithm
                                                                                       47 | void printArr(int dist[], int V) {
13
                                                                                            System.out.println("Vertex Distance from Source: "); for (int i = 0; i < V; ++i)
      // A class to represent a weighted edge in graph
                                                                                           System.out.println(i + "\t\t" + dist[i]);}
15 🖹 class Edge { int src, dest, weight;
        Edge() {src = dest = weight = 0;}};
                                                                                                Driver method to test above fu
16
                                                                                      51 public static void main(String[] args) (
17
           int V, E; Edge edge[];
                                                                                            int V = 5; // Number of vertices in graph
      // Creates a graph with V vertices and E edges
                                                                                            int E = 8; // Number of edges in graph
19 Graph(int v, int e) (V = v; E = e; edge = new Edge[e];
20 for (int i = 0; i < e; ++i) adda(i) = new Edge(v);
          for (int i = 0; i < e; ++i) edge[i] = new Edge();}
                                                                                            Graph graph = new Graph(V, E);
20
                                                                                             // add edge 0-1 (or A-B in above figure)
21
     // The main function that finds shortest distances from src
                                                                                            graph.edge[0].src = 0; graph.edge[0].dest = 1; graph.edge[0].weight = -1;
     // to all other vertices using Bellman-Ford algorithm. The
22
                                                                                             // add edge 0-2 (or A-C in above figure)
23
      // function also detects negative weight cycle
24 - void BellmanFord(Graph graph, int src) {int V = graph.V, E = graph.E;
                                                                                            graph.edge[1].src = 0; graph.edge[1].dest = 2; graph.edge[1].weight = 4;
                                                                                             // add edge 1-2 (or B-C in above figure)
25
                                        int dist[] = new int[V];
                                                                                       60
     // Step 1: Initialize distances from src to all other
                                                                                            graph.edge[2].src = 1; graph.edge[2].dest = 2; graph.edge[2].weight = 3;
26
                                                                                             // add edge 1-3 (or B-D in above figure)
      // vertices as INFINITE
                                                                                       62
                                                                                            graph.edge[3].src = 1; graph.edge[3].dest = 3; graph.edge[3].weight = 2;
      for (int i = 0; i < V; ++i) dist[i] = Integer.MAX_VALUE; dist[src] = 0;</pre>
                                                                                             // add edge 1-4 (or A-E in above figure)
     // Step 2: Relax all edges |V| - 1 times. A simple
     // shortest path from src to any other vertex can
                                                                                       64
                                                                                            graph.edge[4].src = 1; graph.edge[4].dest = 4; graph.edge[4].weight = 2;
      // have at-most |V| - 1 edges
                                                                                       65
                                                                                             // add edge 3-2 (or D-C in above figure)
                                                                                       66
                                                                                             graph.edge[5].src = 3; graph.edge[5].dest = 2; graph.edge[5].weight = 5;
32
      for (int i = 1; i < V; ++i) {for (int j = 0; j < E; ++j) {
          int u = graph.edge[j].src; int v = graph.edge[j].dest;
                                                                                       67
                                                                                             // add edge 3-1 (or D-B in above figure)
34
           int weight = graph.edge[j].weight;
                                                                                       68
                                                                                             graph.edge[6].src = 3; graph.edge[6].dest = 1; graph.edge[6].weight = 1;
35
          if (dist[u] != Integer.MAX_VALUE && dist[u] + weight < dist[v])</pre>
                                                                                       69
                                                                                             // add edge 4-3 (or E-D in above figure)
                                              dist[v] = dist[u] + weight;}}
                                                                                       70
                                                                                             graph.edge[7].src = 4; graph.edge[7].dest = 3;
36
                                                                                       71
      // Step 3: check for negative-weight cycles. The above
                                                                                             graph.edge[7].weight = -3; graph.BellmanFord(graph, 0); }
                                                                                       72
38
      // step guarantees shortest distances if graph doesn't
Output - JavaApplication65 (run) X
                                                                                      Output - JavaApplication65 (run) ×
    Vertex Distance from Source:
                                                                                           Vertex Distance from Source:
M
                                                                                      0
                                                                                      86
                                                                                      85
88
    BUILD SUCCESSFUL (total time: 0 seconds)
                                                                                           BUILD SUCCESSFUL (total time: 0 seconds)
```

Figure 26: Implementation of Bellman Ford's Algorithm in Java

LO2 Specify abstract data types and algorithms in a formal notation

P3 Using an imperative definition, specify the abstract data type for a software stack.

1. What is an imperative definition?

An abstract data type is defined as a mathematical model of the data objects that make up a data type as well as the functions that operate on these objects. There are no standard conventions for defining them. A broad division may be drawn between "imperative" and "functional" definition styles.

<u>Imperative-style definition</u>: In the philosophy of imperative programming languages, an abstract data structure is conceived as an entity that is mutable - meaning that it may be in different states at different times. Some operations may change the state of the ADT. Therefore, the order in which operations are





evaluated is important, and the same operation on the same entities may have different effects if executed at different times - just like the instructions of a computer, or the commands and procedures of an imperative language. To underscore this view, it is customary to say that the operations are executed or applied, rather than evaluated. The imperative style is often used when describing abstract algorithms.

<u>Abstract variable</u>: Imperative-style definitions of ADT often depend on the concept of an abstract variable, which may be regarded as the simplest non-trivial ADT. An abstract variable V is a mutable entity that admits two operations are store(V, x) and fetch(V). In it, store(V, x), where x is a value of unspecified nature, and fetch(V) that yields a value with the constraint that fetch(V) always returns the value x used in the most recent store(V, x) operation on the same variable V. As in so many programming languages, the operation store(V, x) is often written $V \leftarrow x$ or some similar notation, and fetch(V) is implied whenever a variable V is used in a context where a value is required.

For example: $V \leftarrow V + 1$ is commonly understood to be a shorthand for store (V, fetch(V) + 1).

In this definition, it is implicitly assumed that storing a value into a variable U has no effect on the state of a distinct variable V. To make this assumption explicit, one could add the constraint that, if U and V are distinct variables, the sequence $\{store\ (U,\ x);\ store\ (V,\ y)\}$ is equivalent to $\{store\ (V,\ y);\ store\ (U,\ x)\}$. More generally, ADT definitions often assume that any operation that changes the state of one ADT instance has no effect on the state of any other instance including other instances of the same ADT - unless the ADT axioms imply that the two instances are connected/aliased in that sense.

For example: When extending the definition of abstract variable to include abstract records, the operation that selects a field from a record variable R must yield a variable V that is aliased to that part of R.

The definition of an abstract variable V may also restrict the stored values x to members of a specific set X, called the range or type of V. As in programming languages, such restrictions may simplify the description and analysis of algorithms, and improve their readability. Note that this definition does not imply





anything about the result of evaluating fetch (V) when V is un-initialized, that is, before performing any store operation on V. An algorithm that does so is usually considered invalid, because its effect is not defined. However, there are some important algorithms whose efficiency strongly depends on the assumption that such a fetch is legal, and returns some arbitrary value in the variable's range.

<u>Instance creation</u>: Some algorithms need to create new instances of some ADT such as new variables, or new stacks. To describe such algorithms, one usually includes in the ADT definition a create() operation that yields an instance of the ADT, usually with axioms equivalent to the result of create () is distinct from any instance in use by the algorithm. This axiom may be strengthened to exclude also partial aliasing with other instances. In other words, this axiom still allows implementations of create () to yield a previously created instance that has become inaccessible to the program.

2. Using an imperative definition, specify the abstract data type for a software stack

Example about imperative for abstract stack: An imperative-style definition of an abstract stack could specify that the state of a stack S can be modified only by the operations push(S, x) and pop(S). In it, push(S, x), where x is some value of unspecified nature and pop(S) that yields a value as a result with the constraint for any value x and any abstract variable V, the sequence of operations $\{push(S, x); V \leftarrow pop(S)\}$ is equivalent to $V \leftarrow x$. Since the assignment $V \leftarrow x$, by definition, cannot change the state of S, this condition implies that $V \leftarrow pop(S)$ restores S to the state it had before the push(S, x). From this condition and from the properties of abstract variables it follows.

For example: that the sequence { push (S, x); push (S, y); $U \leftarrow \text{pop}$ (S); push (S, z); $V \leftarrow \text{pop}$ (S); $W \leftarrow \text{pop}$ (S) } where x, y, and z are any values, and U, V, W is pairwise distinct variables, is equivalent to { $U \leftarrow y$; $V \leftarrow z$; $W \leftarrow x$ }.

Here it is implicitly assumed that operations on a stack instance do not modify the state of any other ADT instance including other stacks for any values x, y, and any distinct stacks S and T, the sequence $\{push\ (S, x);\ push\ (T, y)\}$ is equivalent to $\{push\ (T, y);\ push\ (S, x)\}$. An abstract stack definition usually includes also a Boolean-valued function empty(S) and a create() operation that returns a stack instance with axioms equivalent to $create() \neq S$ for any prior stack S - a newly





created stack is distinct from all previous stacks, empty (create ()) - a newly created stack is empty, not empty(push (S, x)) - pushing something into a stack makes it non-empty.

<u>A software stack:</u> Is a collection of independent components that work together to support the execution of an application. The components, which may include an operating system, architectural layers, protocols, run-time environments, databases and function calls, are stacked one on top of each other in a hierarchy. Typically, the lower level components in the hierarchy interact with hardware, while the higher-level components in the hierarchy perform specific tasks for the end user. Components communicate directly with the application through a series of complex instructions that traverse the stack.

Three examples of software stacks: LAMP, MEAN and Apache CloudStack.

- ✓ LAMP (Linux, Apache, MySQL, PHP) a well-known software stack for web development. The lowest layer of the stack's hierarchy is the Linux operating system. The highest layer of the hierarchy is the scripting language -- in this case, PHP. Besides, the "P" may also stand for the programming languages Python or Perl. LAMP stacks are popular because the components are all open source and the stack can run on commodity hardware. Unlike monolithic software stacks, which are often tightly coupled and often built for a particular operating system, a LAMP stack is loosely coupled. This simply means that while the components were not originally designed to work together, they have proven to be complementary and are often used together. Today, LAMP components are now included in almost all Linux distributions.
- ✓ MEAN (MongoDB, Express, Angular and Node) a stack of development tools known for helping to eliminate the language barriers often experienced in software development. In a MEAN stack, the foundation is MongoDB, which is a NoSQL document datastore. The HTTP server is Express and Angular is the framework for front-end JavaScript. The highest layer of the stack is Node, a platform for server-side scripting.
- ✓ Apache CloudStack an open source cloud management stack used by large enterprise customers and service providers who provide





Infrastructure-as-a-Service (IaaS). CloudStack provides developers with multiple layers of optional services as well as support for a variety of hypervisors and application program interfaces (APIs).

M3 Examine the advantages of encapsulation and information hiding when using an ADT.

Data hiding and encapsulation both are the important concept of object-oriented programming. Encapsulation means wrapping the implementation of data member and methods inside a class. When implementation of all the data member and methods inside a class are encapsulated, the method name can only describe what action it can perform on an object of that class. Information hiding means that when someone uses that interface, they will still be able to use it without knowing any implementation details of the interface. Encapsulation and information hiding are usually come hand in hand. This is usually the case in OOP languages because of the availability of classes.

For example: In the Java language, when we are implementing a class, putting the data members and methods inside that class is essentially performing encapsulation. Furthermore, using specific keywords such as "private", "protected" means we are effectively performing information hiding. "Private" means the data member or method with this keyword will not available for access anywhere while "protected" means any inheritance of this class can see that data member or method, which is usually used for custom implementation or modification of the original class.

In addition, there are major differences between the two this concept. The encapsulation also involves the way the class is hidden from users and how the process can be reversed by unlocking the capsule using methods. In data encapsulation, the capsule and the object inside it can be classified as either private or public, unless specified or programmed by the programmer. On the other hand, data hiding is the process of hiding the details of an object or function. It is also a potent technique in programming that results in data security and less data complexity. One of the manifestations of data hiding is that it is used as a method of hiding information inside a computer code after the code is broken down and hidden from the object. All objects in the state of data hiding are in isolated units, which is the main concept of object-oriented programming. The





data inside is classified as private or non-accessible from other objects, classes, and API's in the system. The data appear as invisible to outsiders — whether objects, other classes, or users. Data encapsulation is one of the chief mechanisms of data hiding. Data hiding works by nesting the data or arranging it into capsules. Hiding the physical storage layout for data and avoiding linking to incorrect data (if a programmer does link to said data, the program will display an error to protect the content).

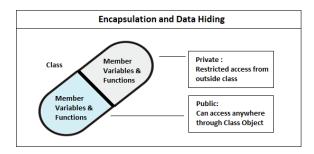


Figure 27: Encapsulation and Data Hiding

Table 6: Advantage of encapsulation and data hiding

Encapsulation	Data hiding
Improve the maintainability of the application	Restrict access to data from outside
Users can use the system in a very flexible way	Avoid the risk of data theft or information disclosure
Help application developers organize their	Hiding data reduces some system complexity by
coding better	limiting software interdependence
The packaging code is quite flexible and easily	Data hiding is also a method to help protect from
changes before new requirements	hackers
Help change a part of the code without affecting other parts of the code	Data hiding data is always private and inaccessible
Increased security and more convenient for	
application maintenance	
Abstraction hides the irrelevant details found in	
the code	
Simplify the representation of domain models	





D2 Discuss the view that imperative ADTs are a basis for object orientation and, with justification, state whether you agree.

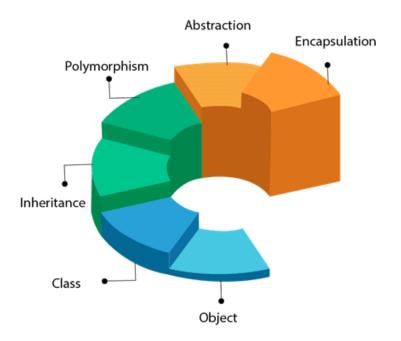


Figure 28: OOPs - Object Oriented Programming System

I disagree with the view that mandatory ADT is the basis for object orientation, although this concept influenced the design of many later languages to which object-oriented labels were attached. The basic ideas of OOP are encapsulation (local storage, state process protection and hiding), late binding in everything and the message.

According to Simula, an object is basically defined by the name of the messages it receives and the values it encapsulates. The behavior of those methods is subject to change. And you don't know what will actually be done in response to a message or a method call. While it is reasonable to expect a return value of a certain type, in Simula-style OOP, that can at least be guaranteed.

Simula is the name of two simulation programming languages, Simula I and Simula 67, developed in the 1960s at the Norwegian Computing Center in Oslo, by Ole-Johan Dahl and Kristen Nygaard. Syntactically, it is a fairly faithful superset of ALGOL 60 (Algorithmic Language 1960 was the first language implementing nested function definitions with lexical scope), it also influenced by the design of Simscript (is a free-form, English-like





general-purpose simulation language conceived by Harry Markowitz and Bernard Hausner at the RAND Corporation in 1962). Simula 67 introduced objects, classes, inheritance and subclasses, virtual procedures, coroutines, and discrete event simulation, and features garbage collection. Also, other forms of subtyping (besides inheriting subclasses) were introduced in Simula derivatives. Simula is considered the first object-oriented programming language. As its name suggests, the first Simula version by 1962 was designed for doing simulations; Simula 67 though was designed to be a general-purpose programming language and provided the framework for many of the features of object-oriented languages today. Simula has been used in a wide range of applications such as simulating very-large-scale integration (VLSI) designs, process modelina. communication protocols, algorithms, and other applications such as typesetting, computer graphics, and education. The influence of Simula is often understated, and Simula-type objects are reimplemented in C++, Object Pascal, Java, C#, and many other languages. Computer scientists such as Bjarne Stroustrup, creator of C++, and James Gosling, creator of Java, have acknowledged Simula as a major influence.

An abstract data type is determined by the values it can act on and the operations that can be performed on them. Inheritance is not part of the concept. Parametric polymorphism is required. Encapsulation is required but without the concept of an object containing its own activities, the semantics of the activities are an essential part of the definition of ADT and cannot be changed.

Java's design is influenced by CLU- a language created by one of the people who named ADT and their official definition is clearly designed to allow programming with ADT. Java's own methods, final methods, public metrics, and interfaces that allow you to effectively create things are ADT. It still doesn't give the concept of ADT differentiation - it arises from the choices you make in defining an abstract class or interface.





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