High-quality Motion debarring from a Single Image

I=L⊗f+n, (1)

I : degraded image L : latent unblurred image n : additive noise f : PSF

$I = (L' + \Delta L) \otimes (f' + \Delta f) + n = L' \otimes f' + \Delta L \otimes f' + L' \otimes \Delta f + \Delta L \otimes \Delta f + n$

$p(L, f | I) \propto p(I | L, f) p(L) p(f)$, (2)

$n = L \otimes f - I$

n_i conforms Gaussian distribution(i.i.d)

$\prod_i N(n_i | 0, \zeta_0)$ $\prod_i N(\partial_x n_i | 0, \zeta_1)$ $\zeta_1 = \sqrt{2}\zeta_0$

$$\begin{aligned} p(\mathbf{I}|\mathbf{L},\mathbf{f}) &= \prod_{\partial^* \in \Theta} \prod_i N(\partial^* n_i | 0, \zeta_{\kappa(\partial^*)}) \\ &= \prod_{\partial^* \in \Theta} \prod_i N(\partial^* I_i | \partial^* I_i^c, \zeta_{\kappa(\partial^*)}), \qquad (3) \end{aligned}$$

$$\Theta = \{\partial^0, \partial_x, \partial_y, \partial_{xx}, \partial_{xy}, \partial_{yy}\}$$

$$p(\mathbf{f}) = \prod_j e^{-\tau f_j}, \qquad f_j \geq 0,$$

$$p(\mathbf{L}) = p_g(\mathbf{L}) p_l(\mathbf{L}).$$

$\log \prod_i \sum_{j=1}^{k} \omega_j N(\partial L_i | 0, \varsigma_j)$ （根据10张natural images得到的对数梯度分布）

$$\Phi(x) = \begin{cases} -k|x| & x \leq l_t \\ -(ax^2 + b) & x > l_t \end{cases}, \qquad (4) \qquad k = 2.7,\ a = 6.1 \times 10^{-4}$$

$$p_g(\mathbf{L}) \propto \prod_i e^{\Phi(\partial L_i)}$$

（使用分段函数拟合对数梯度分布）

$$p_l(\mathbf{L}) = \prod_{i \in \Omega} N(\partial_x L_i - \partial_x I_i | 0, \sigma_1) N(\partial_y L_i - \partial_y I_i | 0, \sigma_1)$$

Ω 是标准差较小的区域

$$E(\mathbf{L},\bar{\mathbf{f}}) = -log(p(\mathbf{L},\bar{\mathbf{f}}|\mathbf{I}))$$

$$\begin{aligned} E(\mathbf{L},\mathbf{f}) \propto & \left(\sum_{\partial^* \in \Theta} w_{\kappa(\partial^*)} \|\partial^* \mathbf{L} \otimes \mathbf{f} - \partial^* \mathbf{I}\|_2^2 \right) + \\ & \lambda_1 \|\Phi(\partial_x \mathbf{L}) + \Phi(\partial_y \mathbf{L})\|_1 + \\ & \lambda_2 \left(\|\partial_x \mathbf{L} - \partial_x \mathbf{I}\|_2^2 \circ \mathbf{M} + \|\partial_y \mathbf{L} - \partial_y \mathbf{I}\|_2^2 \circ \mathbf{M} \right) + \|\mathbf{f}\|_1, \end{aligned} \quad (5)$$

$$w_{\kappa(\partial^*)} = \frac{1}{\zeta^2_{\kappa(\partial^*)}\tau}, \qquad \lambda_1 = \frac{1}{\tau}, \qquad \lambda_2 = \frac{1}{\sigma_1^2 \tau}. \qquad (6)$$

$$1/(\zeta_0^2 \cdot \tau) = 50$$

$$\begin{aligned} E_{\mathbf{L}} = & \left(\sum_{\partial^* \in \Theta} w_{\kappa(\partial^*)} \|\partial^* \mathbf{L} \otimes \mathbf{f} - \partial^* \mathbf{I}\|_2^2 \right) + \lambda_1 \|\Phi(\partial_x \mathbf{L}) + \Phi(\partial_y \mathbf{L})\|_1 \\ & + \lambda_2 \left(\|\partial_x \mathbf{L} - \partial_x \mathbf{I}\|_2^2 \circ \mathbf{M} + \|\partial_y \mathbf{L} - \partial_y \mathbf{I}\|_2^2 \circ \mathbf{M} \right). \end{aligned} \quad (7)$$

$$\begin{aligned} E_{\mathbf{L}} = & \left(\sum_{\partial^* \in \Theta} w_{\kappa(\partial^*)} \|\partial^* \mathbf{L} \otimes \mathbf{f} - \partial^* \mathbf{I}\|_2^2 \right) + \lambda_1 \|\Phi(\Psi_x) + \Phi(\Psi_y)\|_1 \\ & + \lambda_2 \left(\|\Psi_x - \partial_x \mathbf{I}\|_2^2 \circ \mathbf{M} + \|\Psi_y - \partial_y \mathbf{I}\|_2^2 \circ \mathbf{M} \right) \\ & + \gamma \left(\|\Psi_x - \partial_x \mathbf{L}\|_2^2 + \|\Psi_y - \partial_y \mathbf{L}\|_2^2 \right), \end{aligned} \quad (8)$$

（引入参变量代替L简化优化过程）

Updating Ψ. By fixing the values of $\mathbf{L}$ and $\partial^* \mathbf{L}$, (8) is simplified to

$$\begin{aligned} E'_{\Psi} = & \lambda_1 \|\Phi(\Psi_x) + \Phi(\Psi_y)\|_1 + \lambda_2 \|(\Psi_x - \partial_x \mathbf{I})\|_2^2 \circ \mathbf{M} + \\ & \lambda_2 \|(\Psi_y - \partial_y \mathbf{I})\|_2^2 \circ \mathbf{M} + \gamma \|\Psi_x - \partial_x \mathbf{L}\|_2^2 + \gamma \|\Psi_y - \partial_y \mathbf{L}\|_2^2. \end{aligned} \quad (9)$$

$$E'_{\Psi} = \sum_i \left(E'_{\psi_{i,x}} + E'_{\psi_{i,y}} \right)$$

$$E'_{\psi_{i,\nu}} = \lambda_1 |\Phi(\psi_{i,\nu})| + \lambda_2 m_i (\psi_{i,\nu} - \partial_\nu I_i)^2 + \gamma (\psi_{i,\nu} - \partial_\nu L_i)^2$$

（根据二次函数求得最小值）

Updating L

$$E'_{\mathbf{L}} = \left(\sum_{\partial^* \in \Theta} w_{\kappa(\partial^*)} \|\partial^* \mathbf{L} \otimes \mathbf{f} - \partial^* \mathbf{I}\|_2^2 \right) + \gamma \|\Psi_x - \partial_y \mathbf{L}\|_2^2 + \gamma \|\Psi_y - \partial_y \mathbf{L}\|_2^2.$$

$$E'_{\mathcal{F}(\mathbf{L})} = \left(\sum_{\partial^* \in \Theta} w_{\kappa(\partial^*)} \|\mathcal{F}(\mathbf{L}) \circ \mathcal{F}(\mathbf{f}) \circ \mathcal{F}(\partial^*) - \mathcal{F}(\mathbf{I}) \circ \mathcal{F}(\partial^*)\|_2^2 \right) + \gamma \|\mathcal{F}(\Psi_x) - \mathcal{F}(\mathbf{L}) \circ \mathcal{F}(\partial_x)\|_2^2 + \gamma \|\mathcal{F}(\Psi_y) - \mathcal{F}(\mathbf{L}) \circ \mathcal{F}(\partial_y)\|_2^2,$$

$$E'_{\mathbf{L}}|_{\mathbf{L}^*} = E'_{\mathcal{F}(\mathbf{L})}|_{\mathcal{F}(\mathbf{L}^*)}$$

$$\mathbf{L}^* = \mathcal{F}^{-1} \left(\frac{\overline{\mathcal{F}(\mathbf{f})} \circ \mathcal{F}(\mathbf{I}) \circ \Delta + \gamma \overline{\mathcal{F}(\partial_x)} \circ \mathcal{F}(\Psi_x) + \gamma \overline{\mathcal{F}(\partial_y)} \circ \mathcal{F}(\Psi_y)}{\overline{\mathcal{F}(\mathbf{f})} \circ \mathcal{F}(\mathbf{f}) \circ \Delta + \gamma \overline{\mathcal{F}(\partial_x)} \circ \mathcal{F}(\partial_x) + \gamma \overline{\mathcal{F}(\partial_y)} \circ \mathcal{F}(\partial_y)} \right)$$

$$\Delta = \sum_{\partial^* \in \Theta} w_{\kappa(\partial^*)} \overline{\mathcal{F}(\partial^*)} \circ \mathcal{F}(\partial^*) \quad (11)$$

Updating f

$$E(\mathbf{f}) = \left(\sum_{\partial^* \in \Theta} w_{\kappa(\partial^*)} \|\partial^* \mathbf{L} \otimes \mathbf{f} - \partial^* \mathbf{I}\|_2^2 \right) + \|\mathbf{f}\|_1$$

$E(\mathbf{f}) = \|\mathbf{A}\mathbf{f} - \mathbf{B}\|_2^2 + \|\mathbf{f}\|_1$ （12）（如何找出最小值对应的f?）

Algorithm 1 Image Deblurring

Require: The blurred image $\mathbf{I}$ and the initial kernel estimate.
  Compute the smooth region Ω by threshold $t = 5$.
  $\mathbf{L} \Leftarrow \mathbf{I}$ { Initialize $\mathbf{L}$ with the observed image $\mathbf{I}$}.
  repeat {Optimizing $\mathbf{L}$ and $\mathbf{f}$}
    repeat {Optimizing $\mathbf{L}$}
      Update Ψ by minimizing the energy defined in (9).
      Compute $\mathbf{L}$ according to (11).
    until $\|\Delta \mathbf{L}\|_2 < 1 \times 10^{-5}$ and $\|\Delta \Psi\|_2 < 1 \times 10^{-5}$.
    Update $\mathbf{f}$ by minimizing (12).
  until $\|\Delta \mathbf{f}\|_2 < 1 \times 10^{-5}$ or the max. iterations have been performed.
  Output: $\mathbf{L}$, $\mathbf{f}$

问题描述:

$$\text{minimize} \quad \|Ax - y\|_2^2 + \lambda \|x\|_1$$

How to derive the Lagrange dual of the problem above?