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A PPP Projects Valuation: Real Options, Competition and Anchoring Bias

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Abstract
This paper presents an integrated mathematical model for the evaluation of public-private partnership (PPP) projects, taking into account uncertainty, anchoring bias, and potential competition that may affect such investments. Before committing to a PPP project, private firms must carefully consider public opinion, as well as manage the uncertainty and possible competitive dynamics that may arise in the future. Anchoring bias is a cognitive bias in which investors have trust in a particular piece of market information, known as the anchor, when forming their investment decisions. Specifically, the paper combines the real options approach with game theory to account for competition and Natural Language Processing (NLP) techniques to capture the impact of market information disclosure. The result is an integrated model that serves as a decision-making tool for evaluating investments in PPP projects.

Keywords Options game approach · PPP projects · NLP · Competition · Anchoring bias

1 Introduction

Nowadays, public-private partnership (PPP) projects are often used for various reasons. Among the reasons is certainly the possible lack of financial resources for the government, which could lead it to transfer part of the risks to the private sector (Liu & Cheah, 2009). This may be the case of creating a public-private partnership in the form of a Build-Operate-Transfer (BOT) method, which implies that the private individual bears the cost of the investment in exchange for the revenues generated by the project itself (Ashuri et al., 2012). Consequently, undertaking these projects is not easy for the private entity involved (the concessionaire), given the enormous amount of money required and the risks that influence their performance. Zayed and Chang

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(2002) identified eight types of risks that can characterize public-private partnership projects: financial risk, revenue and market risk, promotion risk, procurement risk, development risk, construction completion risk, and operational risk. Ke et al. (2011) shows a more extensive list of risks, including political, corruption, competition, technological, etc. These characteristics could make these projects very difficult to be evaluated and pursued. Previous studies have addressed the evaluation of Public-Private Partnership (PPP) projects. Ke et al. (2008) develop a method for the fair financial evaluation of PPP (Public-Private Partnership) projects by taking into account the perspectives of key stakeholders, including the government, sponsors, and lenders. The authors propose different methodologies depending on the viewpoint considered, such as: Value for Money (VfM), Self-Liquidation Ratio (SLR), Debt Service Coverage Ratio (DSCR), Time-Interest Earned (TIE), Net Present Value (NPV), and Internal Rate of Return (IRR). These tools are applied within a value-at-risk framework, enabling the calculation of risk-adjusted indicators—SLR-at-risk, DSCR-at-risk, TIE-at-risk, NPV-at-risk, and IRR-at-risk which provide confidence levels for each metric under uncertainty. Bonnafous (2010) adopts a methodology for evaluating PPP projects based on comparing the NPV/Subsidy ratio and the coefficient of public funding scarcity. The application of these indicators is not carried out in the same way across all perspectives. In fact, Boardman and Hellowell (2017) distinguishes between two approaches based on VfM, depending on whether the goal is to minimize the present value of costs for the Treasury or to maximize overall social welfare. Suwandairi et al. (2025) analyzes the feasibility of a PPP project (Serang-Panimbang toll road project) through the NPV, the Benefit-Cost ratio, the IRR, and the payback period. Bonnafous (2025) evaluates these projects using the Socio-Economic Rate of Return (ERR), the Internal Rate of Return (IRR), and the Net Present Value (NPV).

All these studies do not focus on incorporating uncertainty into the evaluation and do not address the possibility of allowing the investor to change the investment decision. The uncertainty can be included in the project valuation by considering the managerial flexibility affecting PPP investments. The managerial flexibility value quantifies the option to take or not to take a certain investment decision during a specific time period. This managerial flexibility, or optionality, can be priced through the Real Options Approach (ROA), which can evaluate various types of options, such as the option to defer investment, the option to default during construction, the option to expand, the option to contract, the option to shut down operations, the option to abandon for salvage value, the option to switch use and corporate growth options (Trigeorgis, 1993). Concerning that point, Pellegrino et al. (2013) explained the importance and the difficulty of applying real options theory in risk allocation mechanisms in PPP projects. They explain the possibility of modeling mitigation strategies as real options to control risk and maximize project value. Various works have been done to apply the ROA to evaluate PPP projects. For example, Rakic and Radenovic (2014) use the ROA to evaluate the managerial flexibility (or option) to abandon a toll road project. They develop the real options approach through the binomial tree method of Cox et al. (1979). Stefan (2014) applied the ROA to evaluate a gold mining PPP project. Ashuri et al. (2012) applied the ROA to develop a model to price the minimum revenue guarantee and traffic revenue cap through the risk-neutral option pricing methodology. Carabonara et al. (2014) developed a real option-based model to find the optimal value of the minimum revenue guarantee, that is, government support for private investors, to balance the private and public interests. Uncertainty and managerial flexibility are not the only features to consider when evaluating PPP projects. These investments, in fact, may also be characterized by competition. The emergence of a new infrastructure capable of providing the same service as the concessionaire firm could substantially diminish the latter's profit margins (Liu et al., 2014). Some works combine the ROA with Game Theory (GT) to evaluate those projects that, in addition to being characterized by uncertainty, are also characterized by competition. The result is the development of a new methodology called Options Game Approach (OGA) (Ferreira et al., 2009; Di Bari & Villani, 2022). Biancardi et al. (2024) combined the ROA with GT to capture the operational flexibility to exercise risk-sharing mechanisms of PPP projects and the strategic interactions between government and private sector.

Investment decisions can also be influenced by external factors. For instance, Li et al. (2023) investigates the green housing market by analyzing the effects of incentive-based policies, mandatory policies, and the absence of policy measures on the decision-making processes of real estate agents and residents. The perceived urgency to purchase greenhouses may vary depending on the policy strategy implemented. Another factor that may influence the evaluation of such projects and, consequently, financial investment decisions is the anchoring bias. Anchoring bias refers to the cognitive phenomenon whereby an investor is influenced by specific information available in the market (the anchor) when making an investment decision (Wang, 2023). Turner and Schley (2016) proposes an Anchor Integration Model (AIM) to quantify the anchoring effects. This allows for obtaining quantitative judgments. The model is based on three modeled components: the prior representation, the influence of the anchor, and the posterior representation. Wilson et al. (2021) proposes a Bayesian model to analyze the relationship between hindsight bias and anchor plausibility. Differently from the anchoring bias, hindsight bias refers to the tendency for individuals to believe, after the fact, that they had accurately predicted the outcome of an event whose result is now known to them. Hindsight bias arises when individuals perceive past events as more predictable than they were (Roese & Vohs, 2012). Individuals' propensity to be influenced also manifests in other economic contexts.

These biases can affect investment decisions related to Public-Private Partnership (PPP) projects. For example, Pinilla-De La Cruz and Rabetino (2024) examines the influence of cultural context on the perception of collaboration between the public and private sectors. Previous studies have examined the impact of market information on evaluating projects characterized by uncertainty through a model known as ROBERT (Di Bari et al., 2023). This model allows us to combine the ROA theory with Sentiment Analysis, a Natural Language Processing (NLP) task used to analyze sentences in natural language. The output, also known as *sentiment* or *polarity score*, modifies the transition probability from one phase to the next. Thanks to the introduction of the *Transformers* architecture (Vaswani et al., 2017), Bidirectional Encoder Representations from Transformers (BERT) (Devlin et al., 2019), have become state-of-the-art among NLP models. Therefore, based on the aforementioned, evaluating these projects requires a model that simultaneously accounts for: uncertainty, competition, and the anchoring effect. The paper is organized as follows: Section 2 describes the basic model, which includes the NLP model used to calculate polarity, the GT binding, and the anchor effect concept; Section 3 provides a case study by using data from previous literature and news from international newspapers; finally Section 4 provides the conclusive remarks.

1.1 Contributions and Novelties

This study contributes to the scientific literature by proposing a mathematical model based on real options that incorporates competition, anchoring bias, and the uncertainty inherent in public-private partnership projects. While these factors have been considered separately in previous research, the literature lacks a mathematical model that simultaneously integrates them into a unified framework.

Specifically, Ferreira et al. (2009) and Di Bari and Villani (2022) account for demand uncertainty and competition by incorporating game theory into the real options approach for evaluating uncertain projects. These works fail to consider anchoring bias in the project valuation. Di Bari et al. (2023), on the other hand, combines real options with the impact of market information disclosure for the evaluation of broadband investments. This work fails to consider competition in the project valuation.

This study aims to address these gaps in the academic literature by providing a mathematical model to support investment decision-making for the concessionaire in PPP projects. In doing so, the model is expected to raise investor awareness of the impact of market information and competition on the valuation of such uncertainty-driven projects.

2 Methodology

We develop a mathematical framework to evaluate the PPP investments in phases. This mathematical framework is based on combining the ROA and GT that generate the OGA. As explained in the "Introduction" (Section 1), the ROA is useful for considering the uncertain performance evolution of PPP projects by including managerial flexibility. PPP agreements may include clauses that allow the private sector to abandon the project and sell it to the public sector for a salvage value (Rakic & Radenovic, 2014). Obviously, this could happen when profits become very low or negative. This option is called to abandon for a salvage value (Trigeorgis, 1993). This operational flexibility must, therefore, be priced into the project evaluation model. Thanks to the ROA, it is possible to price the possibility of changing investment decisions during the project value evolution following uncertain market conditions. However, the real options approach alone cannot control competition risk either. As explained by Ke et al. (2011), competition risk can characterize PPP projects. By embedding the GT in the ROA, we would inform the investor of the competition's impact on the PPP project valuation by allowing him to change his investment decision if the competition's presence makes the project unprofitable. The competition can exist in the subsequent investment periods once the previous one has shown a profitable project performance. For this reason, it is essential to consider the multistage nature of the valuation methodology. By passing from one phase to the subsequent one, the investor should consider the probability of positive and negative information revelation deriving from the fulfillment probability of the previous phase. The investor's susceptibility to information from the preceding stage represents a manifestation of the anchoring bias. This aspect is also included in the model through the ROBERT (Di Bari et al., 2023) idea and the Dias (2005) framework.

However, the investor may ignore the information obtained during the previous stage. Therefore, we also provide a valuation approach that does not account for the information revelation. This allows for a comparison and enables an analysis of the impact of the anchoring bias. The proposed model provides a decision-support tool for investment choices within the context of PPPs. Including the anchoring bias coefficient allows for a better understanding of the impact of information on investment decisions. Our extension of the ROA enables investors to become more aware of uncertainty, competitive dynamics, and the influence of information on the final valuation, in an economy now regulated by technologies (Sun et al., 2024).

The rest of this "section" is organized as follows. Firstly, we develop and present some characteristics of FinBERT and the newspaper articles used; then, we implement the mathematical model to evaluate the PPP investments by including the managerial flexibility of investing only if the competition does not make the project unprofitable, considering the information revelation process. Finally, we give a contextualized explanation of the anchor effect for our work.

2.1 BERT-based Setting

BERT consists of a set of transformer encoders that perform two fundamental tasks: *Masked Language Modeling* (MLM), that randomly mask the 15% of the tokens being in the corpora and the *Next Sentence Prediction* (NSP), that is the ability to predict if two sentences follow each other. Thanks to these characteristics, a BERT model can be used to determine the polarity of sentences expressed in natural language. Figure 1 represents the Transformers architecture (Vaswani et al., 2017).

For example, a model specialized in financial corpora in English is FinBERT (Araci, 2019). This model was trained on three datasets: TRC2-financial (filtering the TRC2 corpus based on financial keywords), FinancialPhraseBank (Malo et al., 2014), and FiQA Sentiment (a dataset created for the *WWW'18* Conference challenge). As reported by the author, the parameters for implementing this model and the accuracy results after a 10-fold Cross-Validation are reported in Table 1. With these parameters, FinBERT achieves a Loss of 0.37, with an accuracy of 0.86 and an F_1 -score of 0.84. The output is, for each inserted sentence of an article, a polarity index *[negative, neutral, positive]* accompanied by a numerical score $\gamma \in [-1, 1]$, which average $\bar{\gamma} = \sum \gamma / n$ represents the global polarity of the news. We can divide γ into $\gamma_{-} \in [-1, 0)$, representative of sentences with negative polarity, and $\gamma_{+} \in [0, 1]$, representative of sentences with positive polarity, hence $\gamma = \gamma_{-} \cup \gamma_{+}$. This polarity score can be used to modify the transition probabilities.

To support investment decisions in PPP projects, we can consider newspaper articles extracted from international journals and determine the polarity score, as shown in Table 2, considering each sentence as a complete unit, starting from a punctuation

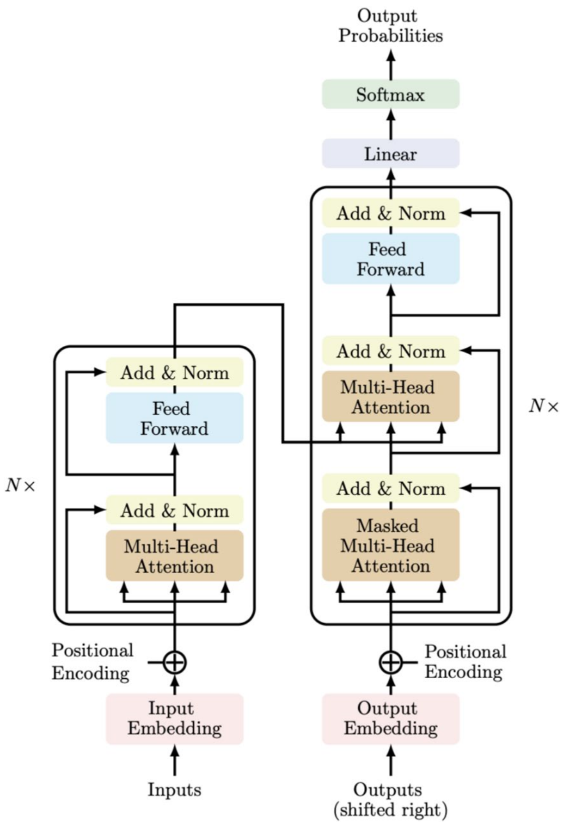

Fig. 1 Transformer architecture depicted (Vaswani et al., 2017)

Table 1 FinBERT parameters description (Araci, 2019)

Parameter	Value
Dropout (p)	0.1
Warm-up prop.	0.2
Max_Seq length	64 tokens
Learning rate	$2e-5$
Loss (Cross-Entropy)	0.37
Accuracy	0.86
F_1-score	0.84

mark (e.g., a period, question mark, or exclamation point) and ending at the next one. Although the choice seems Limited to only 4 articles, those selected cover a vast geographical extension. Furthermore, they all show a positive polarity, which can be understood as an interest (or propensity) to PPPs. The use of an excessive number of sentences can lead to the neutralization of the polarity ($\bar{\gamma}$), making it unusable, or to a lowering of the average in the case of a polarization of the newspapers.

To introduce the impact of sentiment score and anchor effect on investment decision-making, we can consider the framework developed by Dias (2005). Here, Dias considers the technical uncertainty associated with a project and defines a measure

Table 2 Articles used and related polarity (dates in US format)

Article	Structure	Keywords	$\bar{\gamma}$
Nevada city angling for state's first toll road[1]	The Bond Buyer, 469 words, 22 sentences	Bond issues, City government, Public Private Partnership, Public policy	0.19
Latvia: Rail Baltica continues work on the public-private partnership (PPP) model to reduce the burden on the state budget during construction[2]	MENA report, Al Bawaba, 818 words, 24 sentences	European Union, Government budgets, Public Private Partnership, Regulatory compliance	0.67
Uzbekistan implements ambitious program of public-private partnership - president[3]	Trend News Agency, 205 words, 6 sentences	Public Private Partnership, Allianz and partnership, Board of directors, Toll roads	0.63
Govt okays Public-Private Partnership for upgrading, tolling of Old Gwanda Road[4]	B-Metro, 834 words, 25 sentences	Cabinet offices, Public Private Partnership, Transportation department, Public administration	0.52

[1]"Nevada city angling for state's first toll road", August 26, 2024, The Bond Buyer, bondbuyer.com

[2]MENA Report, Al Bawaba "Latvia: Rail Baltica continues work on the public-private partnership (PPP) model to reduce the burden on the state budget during construction", August 29, 2024

[3]"Uzbekistan implements ambitious program of public-private partnership - president", September 30, 2024, Trend News Agency, trend.az

[4]"Govt okays Public-Private Partnership for upgrading, tolling of Old Gwanda Road", January 2, 2024, B-Metro, bmetro.co.zw

that considers the information revealed. We can start by considering two successive investment phases, modeling them as Bernoulli random variables, S_1 and S_2, characterized by the following probabilities (Dias, 2005; Di Bari et al., 2023, 2024):

$$S_1 = \begin{cases} 1 & with\, prob. \quad p \\ 0 & with\, prob. \quad 1-p \end{cases}, \quad S_2 = \begin{cases} 1 & with\, prob. \quad q \\ 0 & with\, prob. \quad 1-q \end{cases}$$

The two random variables represent the performance in one state and the next related state. To reduce uncertainty, Dias introduces a random variable associated with the *revelation distribution* $R_{S_2}(S_1) = E[S_2 \mid S_1]$. With the previous Bernoulli random variables, the expected percentage of variance reduction is (Dias, 2005; Di Bari et al., 2023, 2024):

$$\eta^2(S_2 \mid S_1) = \frac{Var[R_{S_2}]}{Var[S_2]} \tag{1}$$

a particular *learning measure* (Dias, 2005) which has properties of existence, invariance, and in the case of independence of S_1 and S_2, $\eta^2(S_2 \mid S_1) = \eta^2(S_1 \mid S_2) = 0$.

The ROBERT theory (Di Bari et al., 2023, 2024) allows us to link the learning measure to the polarity score. Considering the joint distribution between S_1 and S_2, the joint probability of success $\eta^2(S_2 \mid S_1)$ complies with the measurement conditions provided by Dias (2005) and is equal to the correlation coefficient $\eta^2(S_2 \mid S_1) = \rho^2(S_2, S_1)$. This allows us to develop revealed success probabilities as Dias (2005), Di Bari et al. (2023), and Di Bari et al. (2024):

$$q^{+} = Pr[S_2 = 1 \mid S_1 = 1] = q + \sqrt{\frac{1-p}{p}} \cdot \sqrt{q(1-q)} \cdot \rho(S_2, S_1) \tag{2}$$

$$q^{-} = Pr[S_2 = 1 \mid S_1 = 0] = q - \sqrt{\frac{p}{1-p}} \cdot \sqrt{q(1-q)} \cdot \rho(S_2, S_1) \tag{3}$$

and for negative dependence, inverting the sign after q. However, for S_1 and S_2 *exchangeable* Bernoulli variables as often happens in reality, the probabilities change as follows (Dias, 2005; Di Bari et al., 2023, 2024):

$$q^{+} = q + (1-q)\eta \tag{4}$$

$$q^{-} = q - q\eta \tag{5}$$

From Di Bari et al. (2023, 2024) we know that by imposing $\eta^2(S_2 \mid S_1) = \bar{\gamma}$ the previously defined conditions on the learning measure are still valid and, through algebraic manipulations to adjust the variation interval of $\bar{\gamma}$ to (0, 1) with the sigmoid function $\sigma(\cdot)$, we obtain (Di Bari et al., 2023, 2024):

$$\gamma_{adj} = \sigma(\bar{\gamma}) = \frac{1}{1 + \exp^{-\bar{\gamma}}} \in (0, 1) \tag{6}$$

The same procedure holds considering the overall polarity of the news $\bar{\bar{\gamma}}$, which is obtained by averaging the polarities of the individual articles $\bar{\gamma}$. In this case, $\gamma_{adj} = \sigma(\bar{\bar{\gamma}})$. Finally, the probabilities for positive dependence become (Di Bari et al., 2023, 2024):

$$q^{+} = q + (1-q)\sqrt{\gamma_{adj}} \tag{7}$$

$$q^{-} = q - q\sqrt{\gamma_{adj}} \tag{8}$$

Considering the newspaper articles in Table 2, we obtain a global polarity $\bar{\bar{\gamma}} = 0.50$, a $\gamma_{adj} = 0.62$, and a $\sqrt{\gamma_{adj}} = 0.78$.

In the framework thus defined, we can consider the anchor effect as a tool to eliminate subjectivity from using the polarity score to modify the transition probabilities. In fact, by directly using the probabilities q^{+} and q^{-}, the decision-maker completely absorbs the risk deriving from any bias contained in newspaper articles (or, in any case, from the sources chosen to calculate the polarity). We can introduce $\theta \in [0, 1]$, the anchor effect, as an indicator of the decision-maker's confidence in using polarity. This way, the "weighted" probabilities become:

$$q_{\theta}^{+/-} = q \cdot \theta + (1-\theta) \cdot q^{+/-} \tag{9}$$

If the decision-maker is perfectly anchored to external news, $\theta = 0$, then the probabilities describing the transition in investment (in PPP, for example) depend exclusively on q^+ and q^-. On the other hand, if the decision-maker does not place any kind of trust in external news $\theta = 1$, the probability q prevails.

2.2 Mathematical Setting

The proposed mathematical model aims to provide an ex-ante evaluation of a PPP project that evolves stochastically over time. Since the future trajectory of the project is unknown, we assume the project value S_0 that represents the expected cash inflows of the project can evolve along the binomial tree through up $(S_0 \cdot u)$ or down $(S_0 \cdot d)$ movements. This is because it is plausible that the value of the project may increase or decrease over time. Previous studies have demonstrated the suitability of the binomial model in accounting for the uncertainty inherent in PPP projects (Ashuri et al., 2012; Rakic & Radenovic, 2014). Regarding the time period setting, the evolution of the PPP investment process can be divided into two sequential phases: the first and the second expansion phase. It is assumed that at the end of the first phase, the government allows the concessionaire to exercise an option to abandon for a salvage value if the project becomes unprofitable. It is likely to assume that the competition increases during the second expansion period once the investor validates the investment profitability in the first operation phase. We denote as $f = 1, ..., F-1$ the phases, as t_f the final time of each phase, and as $\tau_f = t_f - t_{f-1}$ the length of each phase. In our case, we assume three time steps: $t_0 = 0$, $t_{F-2} = t_1$, $t_{F-1} = t_2$ where t_{F-1} represents the maturity (or the last time of phase t_{F-1}) and t_{F-2} represents the last time of phase $F-2$.. At each instant, the investor decides whether to make sequential investments. At time t_0 the investor decides to spend the capital to initiate and create the project, at time t_1 the investor decides to spend the capital for a first expansion investment, at time t_2 the investor decides to spend the capital for a second expansion investment.[1] We assume $u = e^{\sigma \cdot \sqrt{\Delta t}}$ and $d = e^{-\sigma \cdot \sqrt{\Delta t}}$ following the CRR (Cox, Ross and Rubinstein) model (Cox et al., 1979) with $\Delta t = 1$ (which means we consider annual variation) . The binomial model considers scenarios in which the value of the project may increase (u) or decrease (d).

The number of up movements is identified by $n_f = 0, ..., t_f$, and the down movements are identified by $t_f - n_f$. The analysis starts with the final time period t_{F-1}, referred to as the expansion phase, in which we set a game between two competitors that can invest or not invest. Thus, the project value evolution at maturity t_{F-1} along the binomial tree is:

$$S_{t_{F-1}} = S_0 \cdot u^{n_{F-1}} \cdot d^{t_{F-1} - n_{F-1}} \tag{10}$$

In the competition case, the investors A and B divide the S_{t_F} value by obtaining respectively α_A and α_B where $\alpha_A + \alpha_B = 1$. The value of α_i represents, for $i \in \{A, B\}$, the expected value of the market share allocated to each investor. We denote as $\alpha_i \cdot S_{t_{F-1}}$ the project value evolution times the market share of each inves-

[1] Note that the meaning and number of investments of the phases can change depending on the type of project being considered.

tor $i = A, B$. From $\alpha_i \cdot S_{t_{F-1}}$ the investors should subtract the second expansion investment K_2 in order to obtain the profits $\pi_{i,t_{F-1}}$:

$$\pi_{i,t_{F-1}} = \alpha_i \cdot S_{t_{F-1}} - K_2 \tag{11}$$

The profits $\pi_{i,t_{F-1}}$ assume different values according to the investment strategies of the competitors. The investors A can decide to invest (s_{A1}) or not to invest (s_{A2}) in an expansion project for a growth rate ϵ. The other investor B can decide to invest (s_{B1}) or not to invest (s_{B2}) in an expansion project for a growth rate ϵ or expansion factor. If an investor decides to undertake the project expansion while the competitor does not, the investor would capture the entire market share, whereas the competitor would retain only the value of the project without making the expansion investment. The corresponding payoffs are listed below:

- $\pi_{A,t_{F-1}}(s_{A1}, s_{B1}) = \alpha_A \cdot (1+\epsilon) \cdot S_{t_{F-1}} - K_2$
- $\pi_{B,t_{F-1}}(s_{A1}, s_{B1}) = \alpha_B \cdot (1+\epsilon) \cdot S_{t_{F-1}} - K_2$
- $\pi_{A,t_{F-1}}(s_{A1}, s_{B2}) = (\alpha_A + \alpha_B) \cdot (1+\epsilon) \cdot S_{t_{F-1}} - K_2$
- $\pi_{B,t_{F-1}}(s_{A1}, s_{B2}) = S_{t_{F-1}}$
- $\pi_{A,t_{F-1}}(s_{A2}, s_{B1}) = S_{t_{F-1}}$
- $\pi_{B,t_{F-1}}(s_{A2}, s_{B1}) = (\alpha_A + \alpha_B) \cdot (1+\epsilon) \cdot S_{t_{F-1}} - K_2$
- $\pi_{A,t_{F-1}}(s_{A2}, s_{B2}) = S_{t_{F-1}}$
- $\pi_{B,t_{F-1}}(s_{A2}, s_{B2}) = S_{t_{F-1}}$

Figure 2 inserts these payoffs in a static matrix game.

Under the assumption that $\alpha_A = \alpha_B$, the players' strategies are symmetric. In scenarios where the project value rises, the highest payoff is achieved when one player chooses to invest while the competitor abstains, as this allows the investor to capture the entire market share. Nevertheless, this outcome cannot be generalized due to the stochastic nature of the project value, which is subject to temporal fluctuations. Specifically, when the project's value falls below a certain threshold, it becomes strategically optimal for both players to forgo the expansion investment.

We apply the game matrix at the final time of the binomial tree, and the nodes assume the values of the game solution that can happen through the Nash Equilibrium (NE) results calculated through iterated elimination of strictly dominated strategies. Thus, we consider NE in pure strategies. In cases where there is no unique Nash equilibrium, namely, when no strictly dominated strategies are present, the problem is resolved by averaging the payoffs associated with the two equilibrium outcomes (Ferreira et al., 2009). Thus, we can generalize the payoffs of firms (A and B) at the final nodes of the binomial tree at t_{F-1} with $e_{i,F}$ as follows:

		Investor B	
		s_{B1}	s_{B2}
Investor A	s_{A1}	$(\pi_{A,t_{F-1}}(s_{A1}, s_{B1}), \pi_{B,t_{F-1}}(s_{A1}, s_{B1}))$	$(\pi_{A,t_{F-1}}(s_{A1}, s_{B2}), \pi_{B,t_{F-1}}(s_{A1}, s_{B2}))$
	s_{A2}	$(\pi_{A,t_{F-1}}(s_{A2}, s_{B1}), \pi_{B,t_{F-1}}(s_{A2}, s_{B1}))$	$(\pi_{A,t_{F-1}}(s_{A2}, s_{B2}), \pi_{B,t_{F-1}}(s_{A2}, s_{B2}))$

Fig. 2 Static game matrix

$$e_{i,F-1}^{(n_{F-1},t_{F-1}-n_{F-1})(u,d)} = NE_{i,F-1}^{(n_{F-1},t_{F-1}-n_{F-1})(u,d)} \tag{12}$$

Since the NE's payoffs are not certain, we let the probability of success influence them in the last stage, which depends on the revelation of information from the previous stage.

Thus, we need to include the probabilities related to positive or negative information revelation into $e_{i,F-1}^{(n_{F-1},t_{F-1}-n_{F-1})(u,d)}$ parameter depending from the performance of the previous expansion phase. In this way, we obtain a kind of expected value $\bar{e}_{i,F-1}$ that should be weighted for success and failure probabilities of the previous stage with the respective probabilities of positive and negative information revelation. We will find two NEs: the first one that considers the probabilities of positive information revelation and the second one that considers the probabilities of negative information revelation. In the case of the probability of positive/negative information revelation, the payoffs of the static matrix game will become:

- $\pi_{A,t_{F-1}}^{+/-}(s_{A1}, s_{B1}) = q^{+/-} \cdot \alpha_A \cdot (1+\epsilon) \cdot S_{t_{F-1}} - K_2$
- $\pi_{B,t_{F-1}}^{+/-}(s_{A1}, s_{B1}) = q^{+/-} \cdot \alpha_B \cdot (1+\epsilon) \cdot S_{t_{F-1}} - K_2$
- $\pi_{A,t_{F-1}}^{+/-}(s_{A1}, s_{B2}) = q^{+/-} \cdot (\alpha_A + \alpha_B) \cdot (1+\epsilon) \cdot S_{t_{F-1}} - K_2$
- $\pi_{B,t_{F-1}}^{+/-}(s_{A1}, s_{B2}) = q^{+/-} \cdot S_{t_{F-1}}$
- $\pi_{A,t_{F-1}}^{+/-}(s_{A2}, s_{B1}) = q^{+/-} \cdot S_{t_{F-1}}$
- $\pi_{B,t_{F-1}}^{+/-}(s_{A2}, s_{B1}) = q^{+/-} \cdot (\alpha_A + \alpha_B) \cdot (1+\epsilon) \cdot S_{t_{F-1}} - K_2$
- $\pi_{A,t_{F-1}}^{+/-}(s_{A2}, s_{B2}) = q^{+/-} \cdot S_{t_{F-1}}$
- $\pi_{B,t_{F-1}}^{+/-}(s_{A2}, s_{B2}) = q^{+/-} \cdot S_{t_{F-1}}$

We denote the NE under the probability of positive and negative information revelation as $NE_{i,F-1}^{+}$ and $NE_{i,F-1}^{-}$.

Investors are required to assess whether to undertake the expansion of a project based on the expected value that such an investment may generate in terms of future profits. Consequently, the strategic choices of the agents involved are influenced by several key factors: the temporal dynamics of the project's value (S_{t_f}); the degree to which investors anchor their decisions to the perceived probabilities of receiving favorable or unfavorable market information (θ); the market share allocated to each investor α_i (which, in our model, is fixed); the capital necessary to pursue the expansion project (K_2 which is also fixed). These considerations imply that, for example, when the project's value is expected to increase over time, pursuing the investment may become economically viable. Conversely, in scenarios where the value is projected to decline, investors might rationally opt to abandon the expansion plan.

At this point, we calculate $\bar{e}_{i,F-1}$ as follows:

$$\bar{e}_{i,F-1}^{(n_{F-1},t_{F-1}-n_{F-1})(u,d)} = p \cdot NE_{i,F-1}^{+} + (1-p) \cdot NE_{i,F-1}^{-} \tag{13}$$

The analysis proceeds backward considering the length of each phase $\tau_f = t_f - t_{f-1}$ with $m_f = 0, ..., \tau_f$. We apply the mathematical setting of Biancardi et al. (2021).

The backward induction process is developed by applying the risk-free rate r and the risk-neutral probability $\psi = \frac{(1+r)^{\Delta t} - d}{u-d}$ as follows:

$$\tilde{e}_{i,F-1}^{(n_{F-2},t_{F-2}-n_{F-2})(u,d)} = \frac{\sum_{m_{F-1}=0}^{\tau_{F-1}} \begin{pmatrix} \tau_{F-1} \\ m_{F-1} \end{pmatrix} \cdot \psi^{m_{F-1}} \cdot (1-\psi)^{\tau_{F-1}-m_{F-1}} \cdot \bar{e}_{F-1}^{((n_{F-2}+m_{F-1}),(t_{F-2}-n_{F-2}-m_{F-1}))(u,d)}}{(1+r)^{\tau_{F-1}}} \tag{14}$$

Successively, at the time t_{F-2} that refers to the first expansion investment (t_1), we apply a new condition. At this time, we consider as new underlying asset the value of $\tilde{e}_{F-1}$ multiplied by the expansion factor $(1+\epsilon)$, and subtract from it, the cost for the first expansion investment cost K_1. We further assume that if the project becomes unprofitable, the private sector could exercise the abandonment option for a salvage value (*SV*). The abandonment option with a salvage value serves as a risk-mitigation mechanism for private firms involved in public-private partnership (PPP) projects. Through this option, the public sector, acknowledging the inherent uncertainty of the project, grants the private investor the possibility to exit the investment and sell it either to the public entity or to another party in the market (Rakic & Radenovic, 2014).

Thus, this situation is modeled as follows:

$$c_{i,F-2}^{(n_{F-2},t_{F-2}-n_{F-2})(u,d)} = \max[SV\ ;\ (1+\epsilon) \cdot \tilde{e}_{i,F-1}^{(n_{F-2},t_{F-2}-n_{F-2})(u,d)} - K_1] \tag{15}$$

The payoff of Eq. 15 represents the payoff of a compound call real option value that means the investor should pursue the investment K_1 only if the value of revenues under competition $(1+\epsilon) \cdot \tilde{e}_{i,F}$ is higher than K_1. Otherwise, the investor should avoid that investment. By adopting the same backward induction process of Eq. 14, we proceed until time t_0:

$$\tilde{c}_{i,F-2}^{(n_{F-3},t_{F-3}-n_{F-3})(u,d)} = \frac{\sum_{m_{F-2}=0}^{\tau_{F-2}} \begin{pmatrix} \tau_{F-2} \\ m_{F-2} \end{pmatrix} \cdot \psi^{m_{F-2}} \cdot (1-\psi)^{\tau_{F-2}-m_{F-2}} \cdot c_{F-2}^{((n_{F-3}+m_{F-2}),(t_{F-3}-n_{F-3}-m_{F-2}))(u,d)}}{(1+r)^{\tau_{F-2}}} \tag{16}$$

The value of $\tilde{c}_{i,F-2}$ represents the project value at time $t_0 = 0$ and we can recall this with $c_{i,0}$. From $c_{i,0}$, the investor should subtract the first capital required to initiate the project K_0 to obtain the project valuation considering competition risk V:

$$V_{i,0} = p \cdot c_{i,0} - K_0 \tag{17}$$

A graphical exhibit to sum up the compound options game model is shown in Fig. 3. This setting includes the possibility that another investor can enter the market by reducing revenues. This way, we could have a reliable valuation methodology that includes the competition risk. The investor can choose the extent to which they rely on the information contained in the analyzed newspapers. The degree of anchoring is measured by θ, where $\theta = 0$ indicates that the investor is fully influenced by the

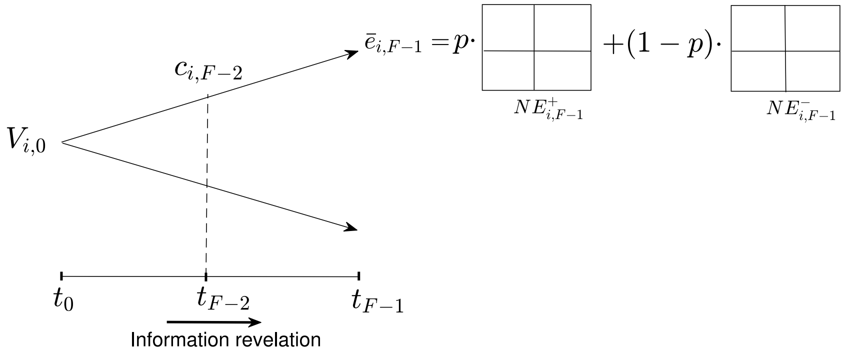

Fig. 3 Mathematical framework of PPP project valuation under uncertainty, competition and anchoring bias

market information received during the second phase, whereas $\theta = 1$ means that the investor completely disregards the market information received in the second phase, relying exclusively on their initial beliefs.

Clearly, positive or negative information does not necessarily translate into actual project performance. This approach allows us to assess the presence and extent of anchoring bias. Anchoring bias may manifest as a misjudgment that should be carefully considered during project evaluation. It should be noted that the model can be extended to multiple stages by, for example, applying the real options approach to additional phases such as $F-3$, $F-4$, and so forth.

3 Case Study

In this section, we propose a numerical example in which we assume to evaluate a toll road investment. Assume that the project value is $S_0 = €\ 100$ million.[2] Suppose that the investor should spend $K_0 = €\ 40$ million to pursue the investment at time $t_0 = 0$, $K_1 = €\ 40$ million and $K_2 = €\ 40$ million applied respectively at time $t_1 = 1$ year and $t_2 = 2$ years. We consider the case in which the public and private sectors agree on the existence of an abandonment option for a salvage value $SV = €\ 80$ million[3] that the private sector can exercise if the investment becomes unprofitable.

In the second phase, we suppose that another competitor projects could reduce the market share of the first concessionaire. This could happen if the government gives another concession to build an alternative road route that would limit the first concessionaire's revenue from tolls or other services. We chose the real options analysis parameters by analyzing the works of Rakic and Radenovic (2014) which deals with PPP projects: $\Delta t = 1$, $r = 5\%$, $\sigma = 30\%$. We suppose an expansion rate of $\epsilon = 50\%$ and the market shares in the competition case are assumed to be

[2] The project value is represented by the discounted future cash flows of the project. To make our example, we use the parameter of project value used by Rakic and Radenovic (2014).

[3] To make our example, we use the parameter of salvage value used by Rakic and Radenovic (2014).

Table 3 Summary of parameters

Parameter	Description	Value
S_0	Project value	€ 100 million
K_0	Investment to initiate the project	€ 40 million
K_1	First expansion investment	€ 40 million
K_2	Second expansion investment	€ 40 million
σ	Volatility	30 %
t_2	Time instant at time $F-1$	2
t_1	Time instant at time $F-2$	1
t_0	Time instant at time 0	0
α_A, α_B	Market shares	50%,50%
ϵ	Expansion rate	50%
q^+, q^-	Revealed success and failure probability	0.93,0.15
θ	Anchor coefficient	[0, 1]
u	Up movement of project value	1.35
d	Down movement of project value	0.74
$\sqrt{\gamma_{adj}}$	Intensity of information revelation	0.78
ψ	Risk neutral probability	0.51

$\alpha_A = 50\%$ and $\alpha_B = 50\%$.[4] We assume that the success probability of the first and the second stages are $p = 0.7$ and $q = 0.7$;[5] the intensity of information revelation $\sqrt{\gamma_{adj}} = 0.78$ that generates $q^+ = 0.93$ and $q^- = 0.15$. Table 3 explains the parameter values used in the case study. As a first step, we compute the evolution of the project along the binomial tree, as illustrated in Fig. 4. We then consider the final time step and determine the players' payoffs under each strategy, for both scenarios involving a probability of receiving positive information (q^+) and negative information (q^-). Subsequently, we identify the payoffs associated with the Nash equilibrium. In cases where multiple equilibria are present, we compute the average of the corresponding payoffs. Figure 5 shows a graphical exhibit in the case of $\theta = 0$ of how the model is implemented. For example, at the uppermost node, if Player A chooses to invest, it is optimal for Player B not to invest, since $169.46 > 89.89$. Conversely, if Player A decides not to invest, it becomes more beneficial for Player B to invest ($216.99 > 169.46$). Hence, Player B does not have a dominant strategy in this case. A similar conclusion arises when analyzing Player A's strategies, considering Player B's potential responses. At the lowest node, the Nash equilibrium is given by (51.04, 51.04), indicating that when the project's value is sufficiently low, both players find it optimal to forgo investment. In this scenario, if Player A invests, Player B will refrain ($51.04 > 1.08$), and if Player A does not invest, Player B will likewise choose not to invest ($51.04 > 39.36$). Therefore, for Player B, the strategy of not investing strictly dominates that of investing; the same holds true for Player A. Consequently, the Nash equilibrium in this case corresponds to both players choosing not to invest. Once $\bar{e}_{i,F-1}$ has been computed, the analysis proceeds backward

[4] The assumption of equal market shares was made to simplify the description and calculations. In this way the payoffs are symmetric. Naturally, other percentages can be applied. Future work could relax this assumption by conducting a more in-depth analysis of the firms involved.

[5] The success probability of each stage has been chosen considering the study of Di Bari et al. (2023).

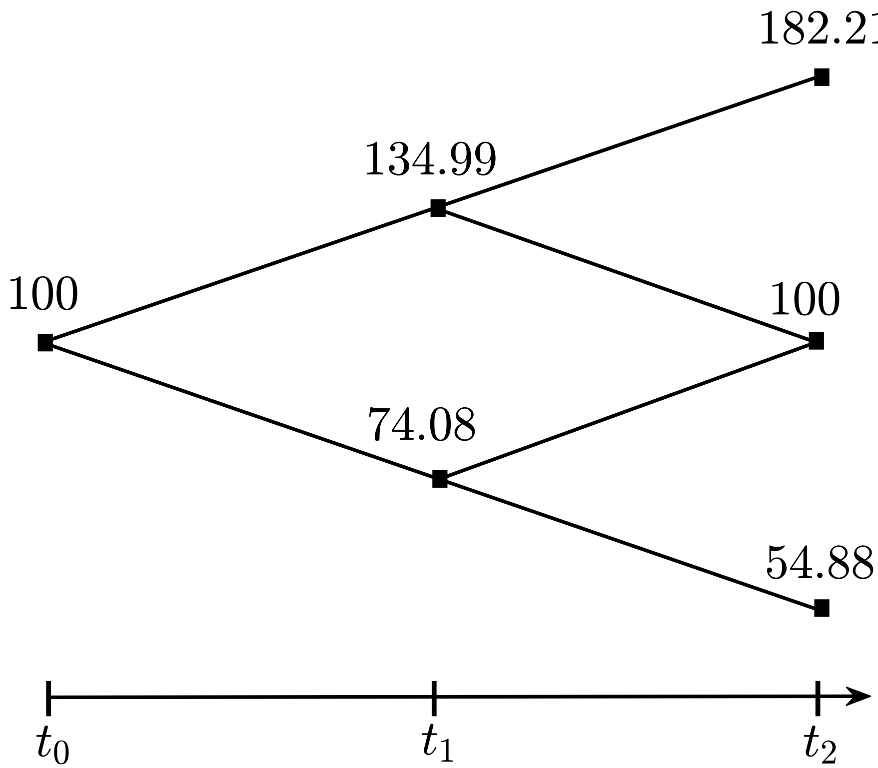

Fig. 4 Project value evolution through binomial approach

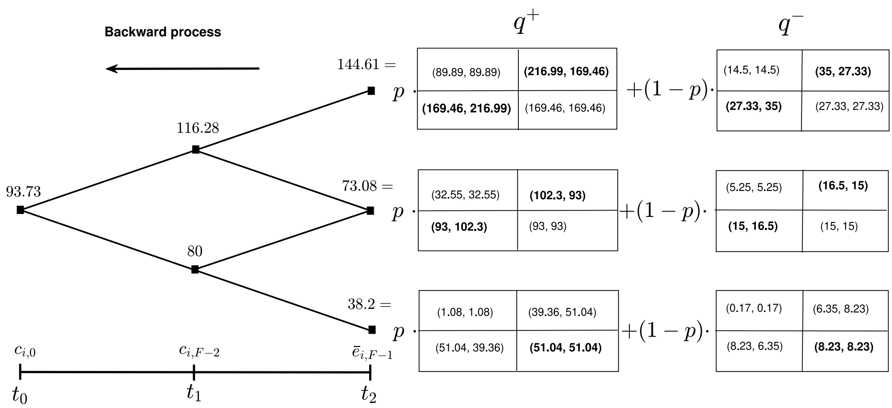

Fig. 5 Model implementation

up to time t_1 where $c_{i,F-2} = c_{i,1}$ is calculated. We proceed again backward to time 0 to calculate $c_{i,0} = 93.73$. Finally, by applying the Eq. 17, we obtain $V_{i,0} =$ € 25.61 million if $\theta = 0$, which means the investor is fully influenced by the market information received during the second phase. If we assume that the investor disregards the information contained in the newspapers during the second phase and anchors their evaluation to prior beliefs ($\theta = 1$), the project assessment will be $V_{i,0} = €\ 25.91$ million. This illustrates that, for the investor, anchoring to prior beliefs rather than to the information provided by the newspapers leads to a different project valuation. Clearly, the cases presented represent two extreme scenarios ($\theta = 0$ or $\theta = 1$). A graphical exhibit of the sensitivity analysis of the impact of θ on the value of the payoffs is presented in Figs. 6 and 7. Figure 6 illustrates how the payoffs of the upper node change with variations in θ when considering the probability of receiving positive information (q^+), while Fig. 7 shows how the payoffs of the upper node change with variations in θ when considering the probability of receiving negative information (q^-). Therefore, an increase in θ will decrease the value of the payoffs for player A and B contained in the game matrix in the case of positive information revelation (see Fig. 6). Moreover, an increase in θ will increase the value of the payoffs for player A and B contained in the game matrix in the case of negative information revelation (see Fig. 7). This happens because an increase in θ reduces the probability of receiving positive information from the market (q^+), while at the same time increasing the value of receiving negative information (q^-).

The results of this study indicate that the investor should proceed with the investment, as the project valuation remains positive both when market informa-

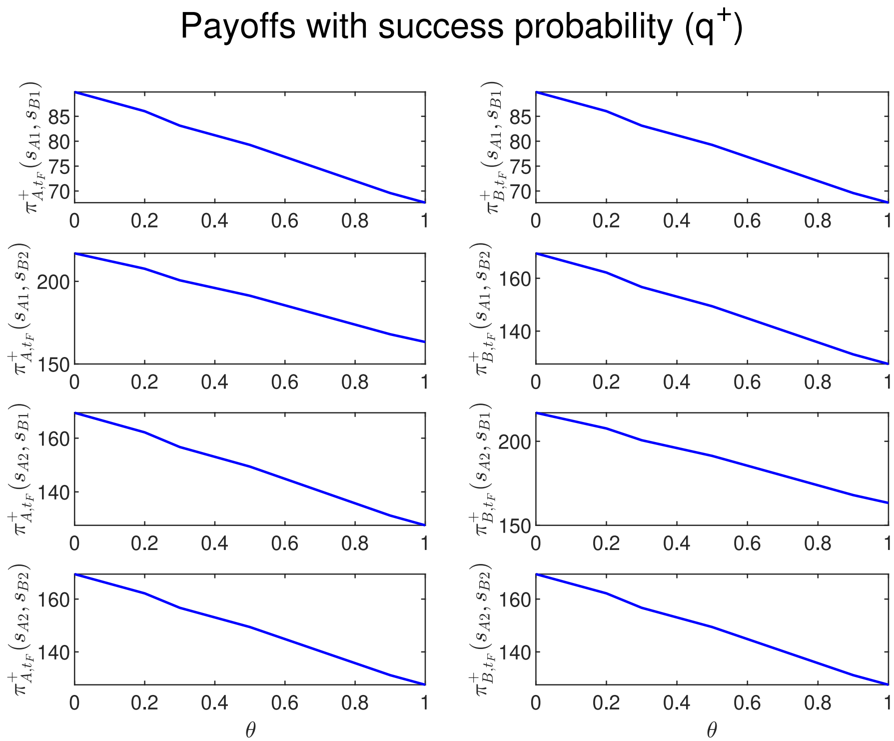

Fig. 6 Sensitivity of payoffs to changes in θ, associated with the probability of receiving positive information. The first column reports the sensitivity analysis for player A, while the second refers to player B

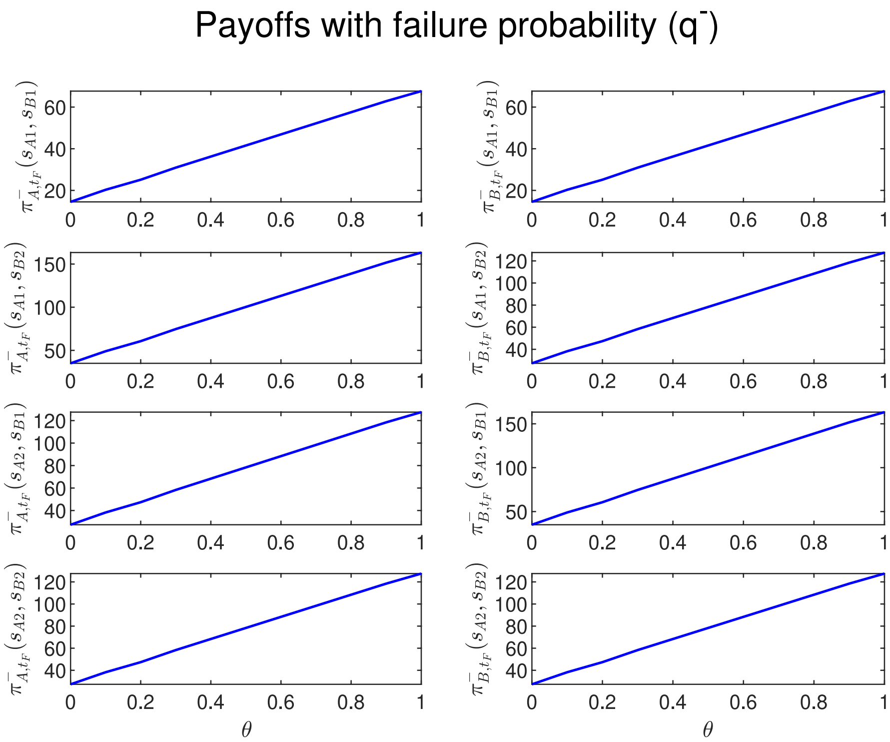

Fig. 7 Sensitivity of payoffs to changes in θ, associated with the probability of receiving negative information. The first column reports the sensitivity analysis for player A, while the second refers to player B

tion is considered and when it is not. Including the abandonment option with a salvage value further increases the project's value, which is consistent with the findings of Rakic and Radenovic (2014). Rakic and Radenovic (2014) price both American- and European-style abandonment options in the context of PPP. This option is then incorporated into the traditional NPV framework, yielding an extended NPV measure that accounts for the managerial flexibility to abandon the project in exchange for a salvage value. Although both our study and that of Rakic and Radenovic (2014) incorporate the abandonment option as an added source of value in the final evaluation, the mathematical models are different. In fact, while their study highlights the relevance of such an option in enhancing project value, their model does not account for strategic interactions, anchoring bias, or the multi-stage nature of project evaluation. Our example explicitly incorporates these aspects into the model, with the aim of providing a more reliable assessment. Moreover, although different types of options are considered, the results are also in line with those of Ashuri et al. (2012) and Biancardi et al. (2021), who demonstrate the economic profitability of Public-Private Partnership (PPP) projects and the importance of pricing the managerial flexibility.

4 Conclusions and Limitations

This work develops a mathematical model to evaluate Public-Private Partnership (PPP) projects by incorporating uncertainty, competition, and anchoring bias, three key factors characterizing such investments.

The model is based on an extension of the Real Options Approach (ROA), which is useful for capturing uncertainty through the pricing of operational flexibility. First, the model introduces the possibility of a competitor entering the market in the second phase, after the project has been constructed. This is achieved by combining real options theory with game theory, resulting in the so-called Options Game approach. Second, the model accounts for the influence of market information on investment decisions, which may lead to what is known as anchoring bias. Anchoring bias is incorporated into the model through a polarity score that reflects the sentiment of market information, whether positive or negative, which in turn affects the probability of success in the different phases of PPP projects.

Although both the Options Game approach and the analysis of how information disclosure affects ROA have been addressed in previous studies, no existing work has considered both the Options Game and the effect of anchoring bias within the same framework. This study fills this gap in the literature by developing a more robust methodology to support investment decision-making in PPPs. This work provides a case study demonstrating that anchoring to market information, such as that conveyed through newspapers, rather than ignoring such information and relying solely on prior beliefs, can lead to divergent valuations. Specifically, the analysis shows that the more an investor disregards market information, the lower the project's valuation will be in the presence of positive market signals. Conversely, the more an investor ignores market information, the higher the project's valuation will be in the presence of negative signals.

Even though this study quantifies anchoring bias, this phenomenon has not been thoroughly examined from a psychological perspective. Future research could address these aspects by exploring the complexity of behavioral biases in investment decision-making within PPP project contexts. Another limitation of this work is that we focus on evaluating the PPP project from the perspective of the firms involved, without considering social welfare. Naturally, the firms are profit-oriented, and the potential entry of another firm offering a similar service may be perceived as detrimental to their interests. Future research could address the issue of social welfare maximization: is the presence of multiple firms providing a public service ultimately beneficial or detrimental to the community? This represents one of the key challenges to be explored. The inclusion of social welfare typically represents the payoff that the government seeks to maximize. Therefore, to address this aspect, future research could incorporate the government as a third player in the game.

Moreover, other limitations concern the case study. This paper presents only a numerical example based on plausible data. Future research could focus on the specific characteristics of real-world projects by using actual parameters, thereby providing a more empirical analysis.

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