Physics 457W Section 1

Speed of Light

Version 3

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Abstract

This experiment aims at gaining experimental skills in alignment and error analysis by determining an accurate speed of light. A method using a rotating mirror developed by Foucault is used in this experiment. A light beam is reflected by a fixed mirror and a rotating mirror at a distance. A small angle difference of the rotating mirror creates a position offset that can be measured using a microscope. The speed of light is calculated using the position offset in the microscope, the rotating mirror's angular velocity, and other setup variables derived with geometry. The best result obtained in this experiment is $c = (3.011 \pm 0.067) \times 10^8\ m/s$. This result agrees with the present speed of light $c = 2.9979 \times 10^8 m/s$, with a percentage difference of 0.45% compares to the current speed of light.

Introduction

It has been proven that the speed of light is constant in a vacuum. Unlike classical particles, both the source and the observer's velocity does not affect the velocity of light. No fundamental interactions, including gravity, weak interaction, strong interaction, and electromagnetism, affect light's speed. The velocity of light is independent of its energy level, which is decided by its frequency. No object or information can travel faster than the velocity of light. The velocity of light is the velocity limit of everything. The foundation of Einstein's relativity is the speed of light stays constant in a moving system.

This experiment measures the speed of light with a method developed by Léon Foucault in 1862. This method starts by letting a light source pass through a beam splitter and hit a rotating mirror. The rotating mirror reflects the beam to a fixed mirror, and the fixed mirror reflects the beam back to the rotating mirror. The rotating mirror reflects the beam into a microscope used to measure the reflected beam's position offset. The offset occurs because it takes time for the beam to travel from the rotating mirror to the fixed mirror and back to the rotating mirror while the rotating mirror spins at high angular speed. As a result, the offset is affected by the distance between the rotating mirror and fixed mirror, the rotating mirror's angular velocity, and other variables caused by the geometry of the experiment setup.

Determining a proper speed of light and proving the speed of light is independent of classical motion is one of the fundamental elements of modern physics. The theory of special relativity was built on which the speed of light stays constant even if observable or subject moves. Special relativity proves that light's speed is the upper speed limit for both objects and information. The speed of light is one of the most important physical constants because it tells the story of the universe.

Theoretical and background

The setup of the experiment starts by setting a laser on a flat plane. The laser is focus by lenses L_1 and L_2. The laser beam then hits a rotating mirror that rotates with a high angular velocity. The reflected beam travels a long path and hits the static mirror. The stationary mirror is positioned that it reflects the beam to the rotating mirror. The rotating mirror again reflects the beam to a beam splitter that sends the beam into a microscope to measure the small offset of the beam. The visualized setup is shown in figure 1 below.

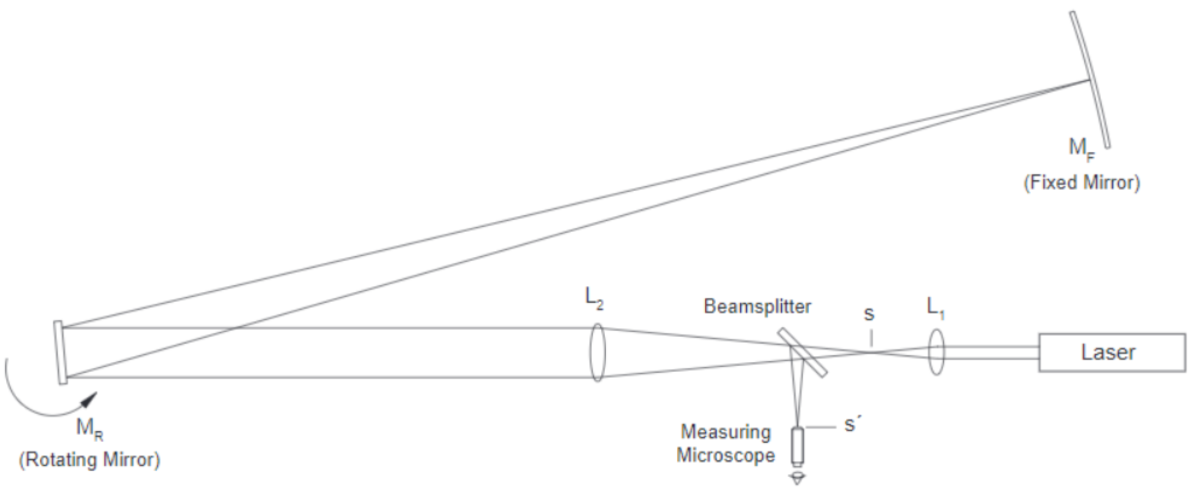

Figure 1 Instrument setup to determine Speed of Light developed by Foucault in 1862. The reflected beam offset is cause by the rotating mirror. The angle of the mirror is different when the beam hits it because light takes time to travel and the rotating mirror is rotating. As a result, the offset of beam measured in microscope depends on the angular velocity of the rotating mirror and the distance between rotating mirror and static mirror that is far from rotating mirror.

Figure 2 visualizes the variables of this experiment with a virtual image. When the angle is small, $sin\varphi \sim \varphi$. So, the offset difference of S is given by:

$$\Delta S = D\ sin(2\Delta\theta) = 2D\Delta\theta \qquad (1)$$

In figure 2, the triangle with base ΔS and height $B + D$ and triangle with base $\Delta s = \Delta s'$ and height A are similar triangles; therefore, they have a consistent ratio:

$$\frac{\Delta S}{B+D} = \frac{\Delta s\prime}{A} \qquad (2)$$

The beam angle difference $2\Delta\theta$ causes by light traveling over distance $2D$ is:

$$2\Delta\theta = \frac{2D}{c}\omega \qquad (3)$$

The variable ω is the angular velocity of the rotating mirror. An equation to determine the speed of light c can be obtained by combining equations 1, 2, and 3.

$$c = \frac{4AD^2\omega}{(B+D)\Delta s\prime} \qquad (4)$$

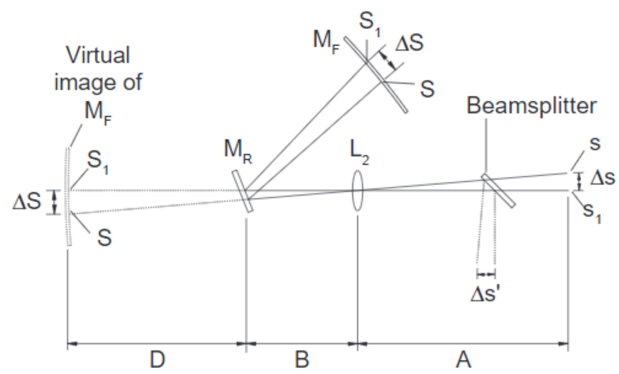

Figure 2 The angle difference of rotating mirror is $\Delta\theta$. D is the distance from rotating mirror to fixed mirror. B is the distance between rotating mirror and lens L_2. A is the distance between lens L_2 and L_1 minus the focal length of L_1. The beam offset back to the laser is Δs. The microscope is placed such a way that the image offset in microscope $\Delta s'$ is equal to the beam offset back to the laser Δs. The beam can hit the fixed mirror on two different location S and S_1.

Methods

The first equipment to place is an optics bench with a metric. The optics bench metric gives a measurement of multiple variables to calculate the speed of light. The setup on the optics bench is shown in figure 3.

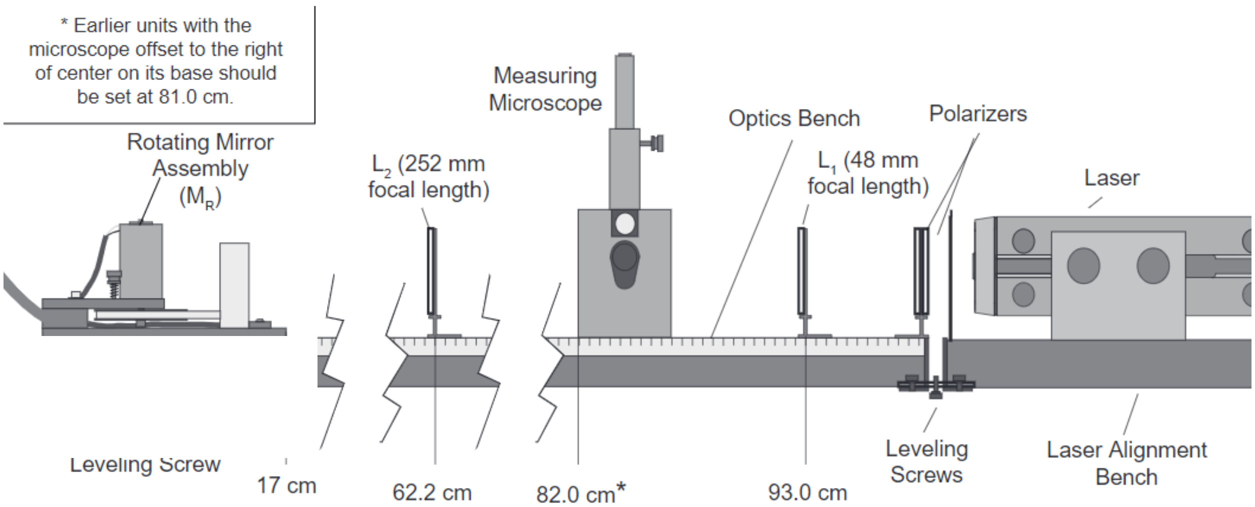

Figure 3 Equipment setting on the optics bench. A laser is mounted at the laser alignment bench. A lens L_1 with 48mm focal length is placed at about 93cm on the optic bench. A lens L_2 with 252mm focal length is placed at about 62cm on the optic bench. A microscope with a beam splitter is placed about 80cm on the optic bench. A rotating mirror is set at about 17cm on the bench. A fixed mirror is placed 5m to 15m away from the rotating mirror.

A laser should be pointed to the center of the rotating mirror with the help of placing alignment jigs on the optics bench, tweaking leveling screws of the laser bench, and sliding paper underneath the laser bench. The rotating mirror is turned to a proper angle that reflects the laser beam to the fixed mirror's direction. Then, move the fixed mirror such that the laser beam hits the center of the fixed mirror. A piece of paper should be placed on the mirror while adjusting the position of lens L_2, so it focuses the beam on the fixed mirror. Once the beam is focused, the paper on the fixed mirror can be removed. Tilt the fixed mirror by turning it and sliding pieces underneath such that it reflects the laser beam back to the rotating mirror. Move the surface under the optics bench slightly, so the laser beam reflected from the fixed mirror hit the center of the rotating mirror. A goggle or polarizer can be used to confirm if the beam hits the microscope.

Variables $A, B,$ and D need to be measured. A is the distance between the lens L_2 and L_1 minus the focal length of L_1. B is the distance between the rotating mirror and lens L_2. D is the distance from a rotating mirror to a fixed mirror. Initiate the experiment by start rotating the rotating mirror in a high angular velocity on both directions. Target the crosshair of the microscope to the laser beam to obtain data points. Angular velocity ω and microscope position s' should be recorded together as pairs.

Results

The experiment was performed in three rounds. Each round has different uncertainty due to different condition. The speed of light is calculated from the data using the equation:

$$c = \frac{4AD^2\omega}{(B+D)\Delta s'} = \frac{8\pi AD^2(f_{cw}+f_{ccw})}{(B+D)(s'_{cw}-s'_{ccw})}$$

Data set 1:

Table 1. This table shows the essential information to calculate the speed of light. The position of lens L_1, lens L_2, and mirror M_R were measured. The distance D from the rotating mirror to the fixed mirror was measured. Variables A and B were calculated. Where $A = L_1 - L_2 - focal\ lengh\ of\ L_1$ and $B = M_R - L_1$.

L_1	L_2	M_R	D	A	B
$93.0 \pm 0.1cm$	$62.0 \pm 0.1cm$	$13.6 \pm 0.2cm$	$11.36 \pm 0.03m$	$0.262 \pm 0.001m$	$0.484 \pm 0.002m$

Table 2. The offset position of beams and angular velocity of data set 1.

$f_{cw} \pm 1$	$f_{ccw} \pm 1$	$s'_{cw}\ (mm) \pm 0.005mm$	$s'_{ccw}\ (mm) \pm 0.005mm$	$c\ (\frac{m}{s})$
1515	1514	11.770	11.080	314955790.7
1512	1512	11.705	11.005	309943948.8
1514	1513	11.705	11.005	310251432.9
1513	1513	11.700	11.010	314643850.4
1514	1513	11.700	11.010	314747830.5
1514	1514	11.690	11.005	317149999.0
1514	1513	11.695	11.005	314747830.5
1513	1515	11.695	11.015	319481984.3
1515	1513	11.705	11.010	312586689.7
1514	1516	11.7	11.005	312793153.8
1513	1515	11.69	10.985	308152835.9

The average speed of light of data set 1 is $(3.136 \pm 0.047) \times 10^8\ m/s$. The standard deviation of this data set is $\sigma_{set1} = 3.3 \times 10^6 m/s$.

Data set 2:

Table 3. The position of lens L_1, lens L_2, and mirror M_R were measured. The distance D from the rotating mirror to the fixed mirror was measured. Variables A and B were calculated. Where $A = L_1 - L_2 - focal\ lengh\ of\ L_1$ and $B = M_R - L_1$.

L_1	L_2	M_R	D	A	B
$93.0 \pm 0.1cm$	$61.5 \pm 0.1cm$	$13.6 \pm 0.2cm$	$13.6 \pm 0.2cm$	$0.267 \pm 0.001m$	$0.483 \pm 0.002m$

Table 4. The offset position of beams and angular velocity of data set 2.

$f_{cw} \pm 1$	$f_{ccw} \pm 1$	$s'_{cw}\ (mm) \pm 0.005mm$	$s'_{ccw}\ (mm) \pm 0.005mm$	$c\ (\frac{m}{s})$
801	801	10.880	10.680	303822163.5
803	802	10.970	10.780	320411704.1
657	656	10.570	10.440	383096610.7
656	655	10.570	10.440	382513066.8
656	655	10.560	10.410	331511324.5
655	655	10.560	10.400	310554802.0
655	655	10.570	10.410	310554802.0
655	655	10.570	10.410	310554802.0
656	655	10.585	10.415	292509992.2
655	655	10.600	10.420	276048712.9
655	654	10.590	10.415	283719074.1
655	655	10.575	10.410	301144050.4
654	655	10.570	10.405	300914169.5

The average speed of light of data set 2 is $(3.16 \pm 0.15) \times 10^8\ m/s$. The standard deviation of this data set is $\sigma_{set2} = 3.3 \times 10^7 m/s$.

Data set 3:

Table 5. The position of lens L_1, lens L_2, and mirror M_R were measured. The distance D from the rotating mirror to the fixed mirror was measured. Variables A and B were calculated. Where $A = L_1 - L_2 - focal\ lengh\ of\ L_1$ and $B = M_R - L_1$.

L_1	L_2	M_R	D	A	B
$93.0 \pm 0.1cm$	$61.8 \pm 0.1cm$	$13.2 \pm 0.2cm$	$8.45 \pm 0.03m$	$0.264 \pm 0.001m$	$0.486 \pm 0.002m$

Table 5. The offset position of beams and angular velocity of data set 3.

$f_{cw} \pm 1$	$f_{ccw} \pm 1$	$s'_{cw}\ (mm) \pm 0.005mm$	$s'_{ccw}\ (mm) \pm 0.005mm$	$c\ (\frac{m}{s})$
1202	1200	11.585	11.155	296154674.4
1202	1201	11.575	11.160	306986811.7
1202	1202	11.585	11.155	296401264.5
1204	1202	11.575	11.155	303710898.7
1204	1201	11.600	11.175	300013083.8
1202	1202	11.600	11.175	299888338.2
1202	1202	11.600	11.180	303458437.5
1202	1204	11.605	11.180	300137829.3
1203	1201	11.600	11.185	307114563.2
1201	1202	11.600	11.175	299763592.6
1203	1203	11.600	11.175	300137829.3
1204	1202	11.595	11.170	300137829.3
1203	1202	11.565	11.145	303584668.1
1205	1204	11.555	11.140	307753320.6
1206	1203	11.565	11.140	300512066.0
1206	1204	11.575	11.145	297141034.7
1205	1204	11.560	11.140	304089590.6
1205	1205	11.565	11.130	293725620.5

The average speed of light of data 1 is $(3.011 \pm 0.067) \times 10^8\ m/s$. The standard deviation of this data set is $\sigma_{set3} = 4.0 \times 10^6 m/s$. Data set 3 denotes the best result of the speed of light among all three data sets. This average speed of light is the closest to the present speed of light. The standard deviation is the smallest compare to the other two data sets.

Error Analysis

Derive the uncertainty of A and B assuming that the uncertainty of $focal\ lengh\ of\ L_1$ is small enough that it is insignificant for ΔA.

$$A = L_1 - L_2 - focal\ lengh\ of\ L_1$$

$$B = M_R - L_1$$

$$\Delta A = \sqrt{(\Delta L_1)^2 + (\Delta L_2)^2} = \sqrt{(0.001m)^2 + (0.001m)^2} = 0.0014m$$

$$\Delta B = \sqrt{(\Delta L_1)^2 + (\Delta M_R)^2} = \sqrt{(0.001m)^2 + (0.002m)^2} = 0.0022m$$

Derive the uncertainty of speed of light c assuming $f_{cw} \sim f_{ccw}$ and $s'_{cw} \sim s'_{ccw}$

$$c = \frac{8\pi A D^2 (f_{cw} + f_{ccw})}{(B + D)(s'_{cw} - s'_{ccw})}$$

$$\Delta c = \sqrt{\left(\frac{\partial c}{\partial A}\Delta A\right)^2 + \left(\frac{\partial c}{\partial B}\Delta B\right)^2 + \left(\frac{\partial c}{\partial D}\Delta D\right)^2 + 2\left(\frac{\partial c}{\partial f}\Delta f\right)^2 + 2\left(\frac{\partial c}{\partial s'}\Delta s'\right)^2}$$

$$\frac{\partial c}{\partial A} = \frac{8\pi D^2 (f_{cw} + f_{ccw})}{(B + D)(s'_{cw} - s'_{ccw})}$$

$$\frac{\partial c}{\partial B} = -\frac{8\pi D^2 (f_{cw} + f_{ccw})}{(B + D)^2(s'_{cw} - s'_{ccw})}$$

$$\frac{\partial c}{\partial D} = \frac{8\pi D (f_{cw} + f_{ccw})(2B + D)}{(B + D)^2(s'_{cw} - s'_{ccw})}$$

$$\frac{\partial c}{\partial f} = \frac{8\pi A D^2}{(B + D)(s'_{cw} - s'_{ccw})}$$

$$\frac{\partial c}{\partial s'} = \pm\frac{8\pi A D^2 (f_{cw} + f_{ccw})}{(B + D)(s'_{cw} - s'_{ccw})^2}$$

$$\Delta c_{set1} = 4.7 \times 10^6 m/s$$

$$\Delta c_{set2} = 1.5 \times 10^7 m/s$$

$$\Delta c_{set3} = 6.7 \times 10^6 m/s$$

The uncertainty from statistical results is:

$$\sigma = \sqrt{\frac{\sum_{i=1}^{n}(x_i - \langle x \rangle)}{n - 1}}$$

$$\sigma_{set1} = 3.3 \times 10^6 m/s$$

$$\sigma_{set2} = 3.3 \times 10^7 m/s$$

$$\sigma_{set3} = 4.0 \times 10^6 m/s$$

The percentage difference to the present speed of light $2.9979 \times 10^8\ m/s$ is:

$$Percentage\ difference = \left|\frac{c_{exp} - c_{theo}}{c_{theo}}\right| \times 100\%$$

$$Diff_{set1} = 4.6\%$$

$$Diff_{set2} = 5.4\%$$

$$Diff_{set3} = 0.45\%$$

Conclusion

In this experiment, the speed of light is measured using Foucault's rotating mirror method developed in 1862. The technique uses a rotating mirror rotates in high angular velocity to reflect a beam from a light source to a fixed mirror. The beam is reflected back to the rotating mirror and to a microscope from the fixed mirror. Since it takes time for the beam to travel between the rotating mirror and the fixed mirror, the rotating mirror's angle is different when the beam reflects on it. This angle difference creates a small position offset in the microscope. The beam offset in the microscope, the angular velocity of the rotating mirror, and other equipment variables can be used to determine the speed of light using equation $c = \frac{4AD^2\omega}{(B+D)\Delta s\prime} = \frac{8\pi AD^2(f_{cw}+f_{ccw})}{(B+D)(s'_{cw}-s'_{ccw})}$. The experiment was performed under three different condition. The best result for speed of light of this experiment is $3.011 \times 10^8\ m/s$ with a systematic uncertainty of $6.7 \times 10^6 m/s$ and a standard deviation of $4.0 \times 10^6 m/s$. The result with either uncertainty agrees with the present speed of light $2.9979 \times 10^8\ m/s$. The percentage difference compares to the present speed of light is 0.45%.