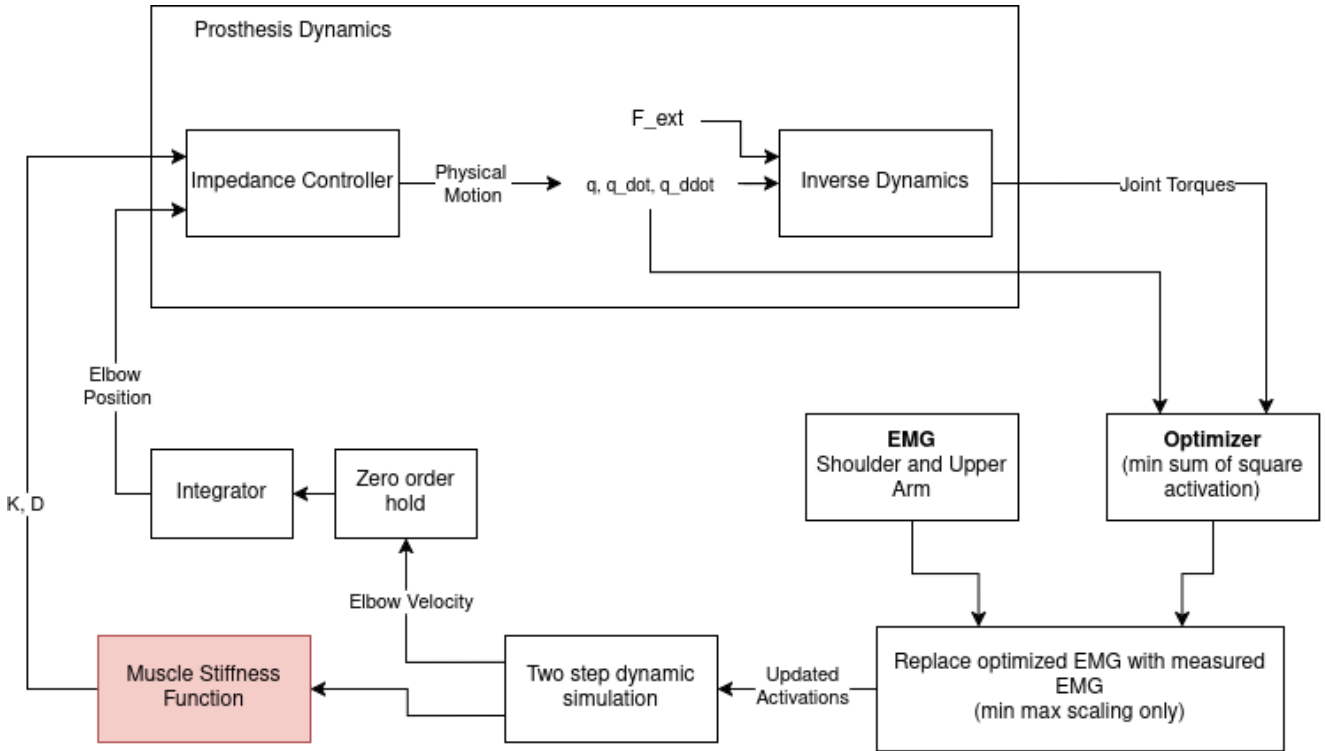


# Muscle Stiffness

## Overall setup:



At every time step of the prosthesis model, the inverse dynamics are calculated including the external forces, which gives the torques at each joint (including estimated torques at the shoulder too). The torques, positions and velocities of each joint are then sent to the Mujoco musculoskeletal model (Myosuite model), and this is set as the state that is used for the Mujoco simulation.

At this state of the arm, an optimizer is run to find the muscle activations ensuring the torque at each joint matches the torques provided by the prosthesis model.

$$\min_a \sum_i^{n_m} a_i^2$$

subject to  $\tau_q = \tau_{\text{inv dyn}}$

$$\tau_q = J_m \dot{f}_m = J_m (a F_L(L) F_V(V) + F_P)$$

Where  $L$  and  $V$  are muscle length and muscle velocity, and  $F_P$  is the passive force in the muscles.  $J_m$  is the moment arm jacobian - representing the transform from muscle forces to joint torques.

(Reference: [Mujoco muscle modelling](#))

At this step, the measured activations are not plugged in.

The measured activations are then blended with the optimized activations (in order to not allow fluctuations in the EMG measurements to suddenly affect the muscle model)

drastically).

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Myosuite Muscle model: [MyoChallenge](#) (Not using this gym workspace any more, directly using a Mujoco model)

Mujoco Model source: [Github](#)

Mujoco Model timestep: 0.002 s

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## Muscle stiffness calculation:

Muscle stiffness  $K_m = \frac{\partial F}{\partial L}$

- If the muscle is assumed to be a linear spring (or linearized about a point), then the stiffness is found as  $K_m = \frac{F}{L}$
- A constant scaling term can be applied to this  $K_m = \alpha \frac{F}{L}$   
[huMuscleShorrangeStiffness2011](#) reports this  $\alpha = 23.4$ .
- More ways to find this constant are:  
[stroevelImpedanceCharacteristicsNeuromusculoskeletal1999](#)

$$\alpha = 2 \frac{L_{opt} - L_{ce}}{L_{sh}^2}$$

$L_{opt}$  is the normalized optimal length,  $L_{sh}$  is the normalized width of the gaussian in the force length curve of the muscle.  $L_{ce}$  is the normalized actual length of the muscle at that instant.

## Muscle stiffness to joint stiffness

Since we know the jacobian from muscle forces to joint torques, we can use this to find the joint stiffness due to the muscle forces.

$$K_q = \frac{\partial(J_m f_m)}{\partial q} = J_m K_m J_m^T + \frac{\partial J_m}{\partial q} f_m$$

The second term has not been incorporated in static simulations here.

To find the endpoint stiffness, we use the jacobian from the joints to the end point.

$$K_e = J^{+T} K_q J^+$$

where  $J^+ = J^T (J J^T)^{-1}$

## EMG

- In the musculoskeletal model, there are many more muscles than the number of muscles for which EMG is measured.
- However, visual inspection showed that the activation of almost all the bicep muscles was very similar, and correlated. The muscle forces however are not because the muscle sizes of each of these muscles are different, but the muscle activations were correlated.
- Hence, the bicep EMG was assigned to all four bicep muscles (Bicep long head, short head, BRA, BRD)
  - **Question:** Some muscles may be bijoint muscles - is it still okay to let the EMG from the bicep area measurement fully correlate to all of the bicep muscles?
- Similarly for the tricep muscle.
- The blending was done as:

$$a = ka_{opt} + (1 - k)a_{EMG}$$

k was in the range of 0.1 - 0.3. This can be tuned for continuity or replaced with a more smooth function that also accounts for the activation values from previous time steps too.

**Question:** Is there a better way to blend the EMG signals - or should some part of the optimized muscle activation remain and the active measured signal be added on top of that?

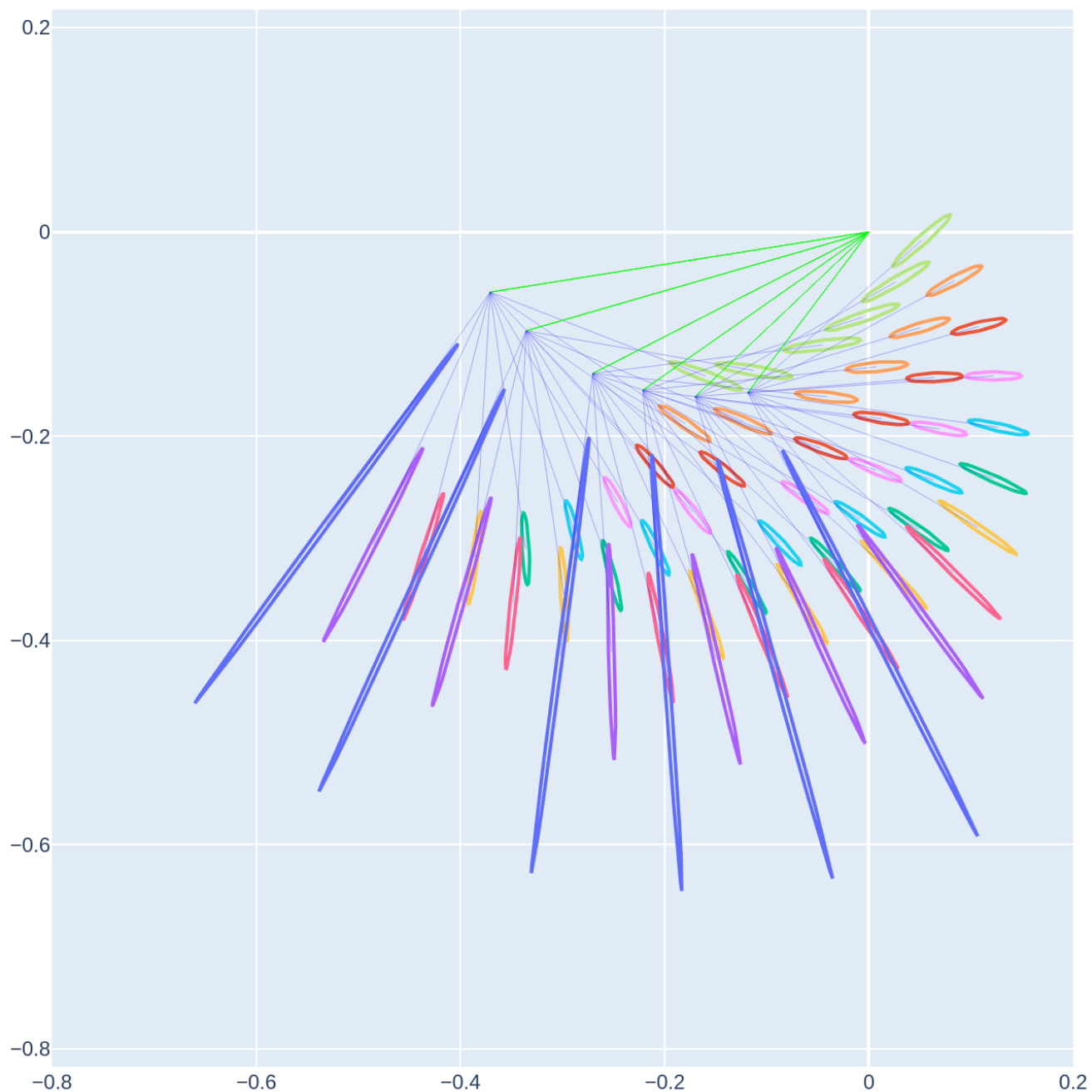
The existing weighted average may be a good solution too.

## Stiffness Plots

### Fixed Joint stiffness for all configurations

#### 1. Joint stiffness:

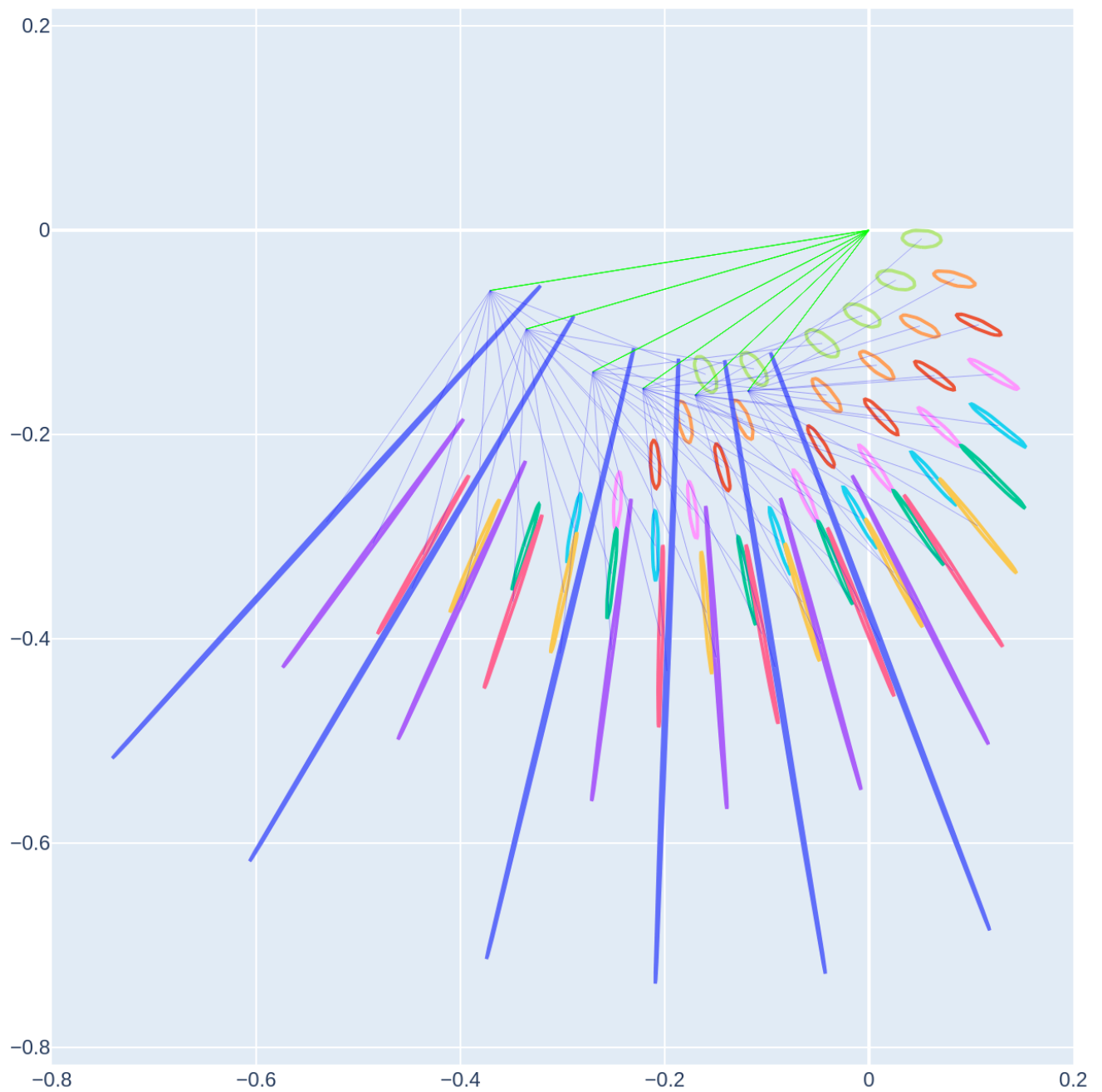
[ [22, -10.3, -4.6, 0.73],  
 [-10.3, 57, 2.5, 0.296],  
 [-4.63, 2.5, 4.28, 0.008],  
 [0.73, 0.296, 0.008, 10] ]



## 2. Joint Stiffnesses:

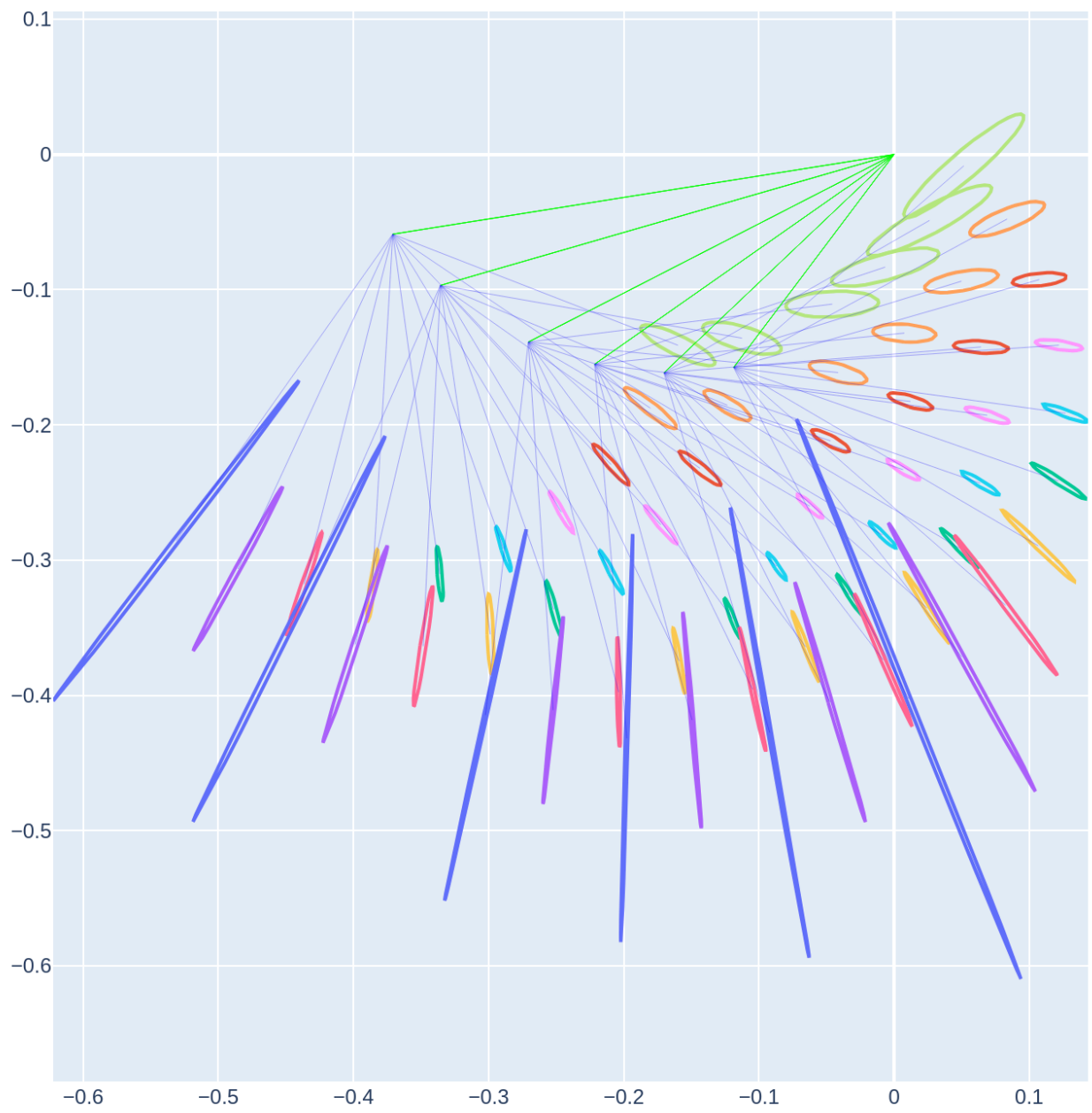
```
[ [22, -10.3, -4.6, 0.73],
  [-10.3, 57, 2.5, 0.296],
  [-4.63, 2.5, 4.28, 0.008],
  [0.73, 0.296, 0.008, 10 * 10] ]
```

(Ellipses scaled down by 3.3x compared to other plots)

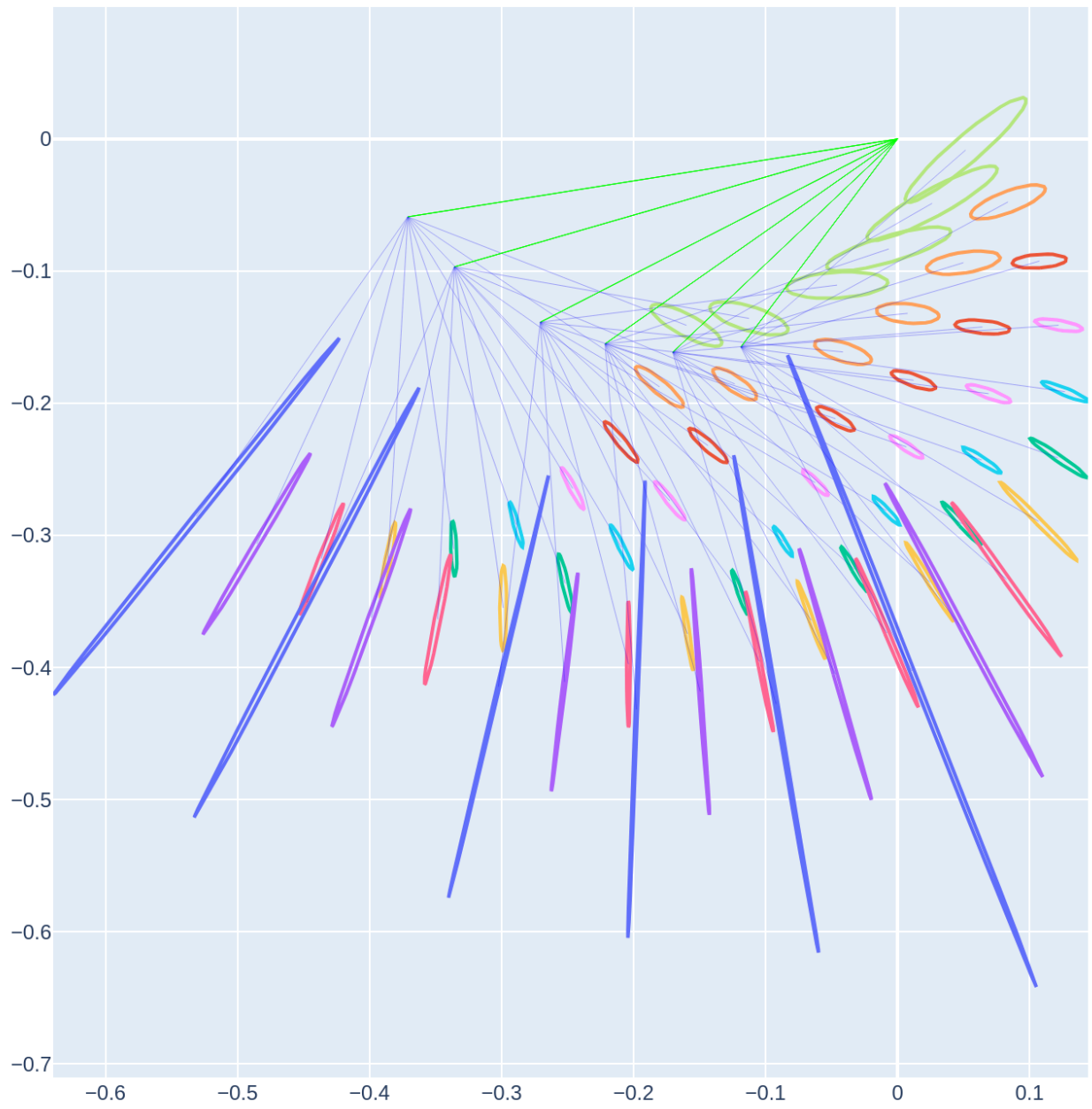


**Calculated Stiffness from muscle forces:**

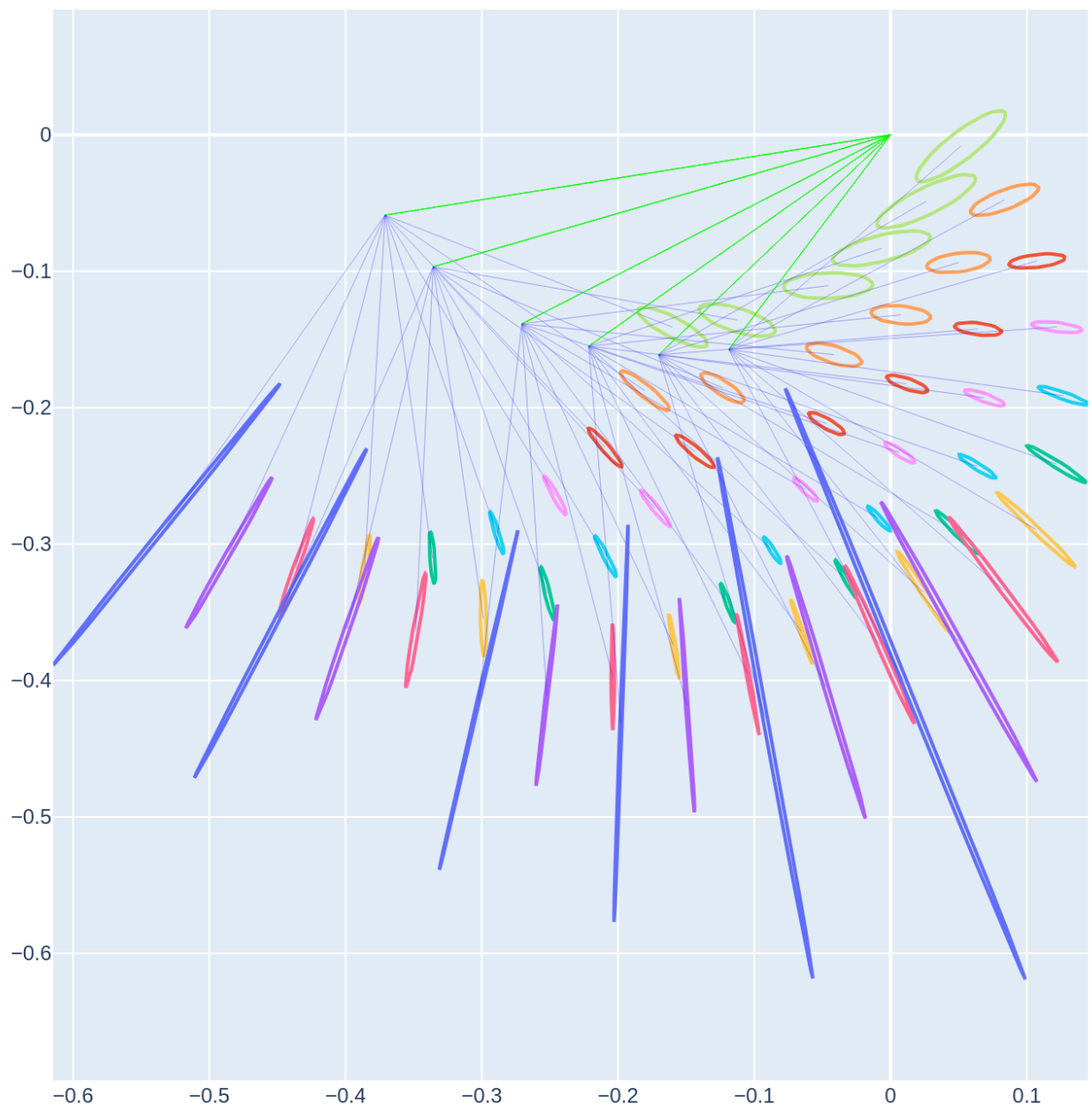
1. Torques = [-15,-1,-5,2]



2. Torques = [-15,-1,-5,5]



3. Torque = [-15,-5,-5,2]



(Note: Optimization failed in some cases where ellipses are abnormally large)