Proposta de resolução dos exercícios, podem ter erros, para comunicarem qualquer erro enviem um email para epf@isep.ipp.pt

Integração por partes

1. Resolva os seguintes integrais, utilizando a fórmula de integração por partes , $C \in R$

a) $\int (2x-1)sen(2x)dx$

Sinal	Derivar	Integrar	
+	$2x-1$	$sen(2x)$	
−	2	$-\frac{1}{2}cos(2x)$	$+(2x-1)\left(-\frac{1}{2}cos(2x)\right)$
+	0	$-\frac{1}{4}sen(2x)$	$-2\left(-\frac{1}{4}sen(2x)\right)$
			$\int (2x-1)sen(2x)dx = +(2x-1)\left(-\frac{1}{2}cos(2x)\right) - 2\left(-\frac{1}{4}sen(2x)\right) + C$

b) $\int (x+1)cos(2x)dx$

Sinal	Derivar	Integrar	
+	$x+1$	$cos(2x)$	
−	1	$\frac{1}{2}sen(2x)$	$+(x+1)\frac{1}{2}sen(2x)$
+	0	$-\frac{1}{4}cos(2x)$	$-1\left(-\frac{1}{4}cos(2x)\right)$
			$\int (x+1)cos(2x)dx = +(x+1)\frac{1}{2}sen(2x) - 1\left(-\frac{1}{4}cos(2x)\right) + C$

c) $\int e^{2x} senx dx$

	Sinal	Derivar	Integrar	
	+	e^{2x}	$sen\, x$	
	−	$2e^{2x}$	$-\cos x$	$+e^{2x}(-\cos x)$
$+\int 4e^{2x}(-\operatorname{sen} x)\,dx$	+	$4e^{2x}$	$-sen\, x$	$-2e^{2x}(-\operatorname{sen} x)$

$$\int e^{2x} senx dx = +e^{2x}(-\cos x) - 2e^{2x}(-\operatorname{sen} x) + \int 4e^{2x}(-\operatorname{sen} x)\,dx$$

$$\int e^{2x} senx dx = e^{2x}(-\cos x) - 2e^{2x}(-\operatorname{sen} x) - 4\int e^{2x} sen\, x\, dx$$

$$\int e^{2x} senx dx + 4\int e^{2x} sen\, x\, dx = e^{2x}(-\cos x) - 2e^{2x}(-\operatorname{sen} x)$$

$$\int e^{2x} sen\, x\, dx = \frac{e^{2x}(-\cos x) - 2e^{2x}(-\operatorname{sen} x)}{5} + C$$

Ou

Sinal	Derivar	Integrar
+	e^{2x}	$sen\, x$
−	$2e^{2x}$	$-\cos x$
+	$4e^{2x}$	$-sen\, x$

$$\int e^{2x} senx\, dx = -e^{2x}\cos x + 2e^{2x} senx - 4\int e^{2x} senx\, dx \Leftrightarrow$$

$$\Leftrightarrow \int e^{2x} senx\, dx + 4\int e^{2x} senx\, dx = -e^{2x}\cos x + 2e^{2x} senx \Leftrightarrow$$

$$\Leftrightarrow 5\int e^{2x} senx\, dx = -e^{2x}\cos x + 2e^{2x} senx \Leftrightarrow$$

$$\Leftrightarrow \int e^{2x} senx\, dx = \frac{2e^{2x} senx - e^{2x}\cos x}{5} + C$$

d) $\int e^{3x} sen x dx$

Sinal	Derivar	Integrar	
+	e^{3x}	$sen\ x$	$\int e^{3x} senx\,dx = e^{3x}(-\cos x) - 3e^{3x}(-senx) + 9\int e^{3x}(-senx)dx \Leftrightarrow$ $\Leftrightarrow \int e^{3x} senx\,dx + 9\int e^{3x} senx\,dx = -e^{3x}\cos x + 3e^{3x} senx \Leftrightarrow$ $\Leftrightarrow 10\int e^{3x} senx\,dx = -e^{3x}\cos x + 3e^{3x} senx \Leftrightarrow$ $\Leftrightarrow \int e^{3x} senx\,dx = \frac{3e^{3x} senx - e^{3x}\cos x}{10} + C$
−	$3e^{3x}$	$-\cos x$	
+	$9e^{3x}$	$-sen\ x$	

e) $\int x^3 ln(x) dx$

	Sinal	Derivar	Integrar	
	+	$\ln(x)$	x^3	
$-\int \frac{1}{x}\frac{x^4}{4}dx$	−	$\frac{1}{x}$	$\frac{x^4}{4}$	$+\ln(x)\frac{x^4}{4}$
$\int x^3 ln(x)dx = +\ln(x)\frac{x^4}{4} - \int \frac{1}{x}\frac{x^4}{4}dx = \ln(x)\frac{x^4}{4} - \frac{1}{4}\int x^3\,dx = \ln(x)\frac{x^4}{4} - \frac{1}{4}\frac{x^4}{4} + C$ $= \ln(x)\frac{x^4}{4} - \frac{x^4}{16} + C$				

f) $\int \sqrt{x} ln(x) dx$

Sinal	Derivar	Integrar	
+	$\ln(x)$	$x^{\frac{1}{2}}$	$\int \sqrt{x}\ln(x)dx = \frac{2}{3}x^{\frac{3}{2}}\ln x - \int \frac{2}{3}x^{\frac{3}{2}}\frac{1}{x}dx = \frac{2}{3}x^{\frac{3}{2}}\ln x - \frac{2}{3}\int x^{\frac{3}{2}-1}dx =$ $= \frac{2}{3}x^{\frac{3}{2}}\ln x - \frac{2}{3}\int x^{\frac{1}{2}}dx = \frac{2}{3}x^{\frac{3}{2}}\ln x - \frac{4}{9}x^{\frac{3}{2}} + C$
-	$\frac{1}{x}$	$\frac{2}{3}x^{\frac{3}{2}}$	

g) $\int x^{-3} ln(x)\, dx$

Sinal	Derivar	Integrar	
+	$ln(x)$	x^{-3}	$\int \frac{ln(x)}{x^3}dx = \int x^{-3}ln(x)\,dx = ln(x)\frac{x^{-2}}{-2} - \int \frac{x^{-2}}{-2}\frac{1}{x}dx =$ $= -\frac{ln(x)}{2x^2} + \frac{1}{2}\int x^{-3}\,dx = -\frac{ln(x)}{2x^2} + \frac{1}{2}\frac{x^{-2}}{-2} =$ $= -\frac{ln(x)}{2x^2} - \frac{1}{4x^2} + C$
−	$\frac{1}{x}$	$\frac{x^{-2}}{-2}$	

h) $\int x sec^2(x)\, dx$

Sinal	Derivar	Integrar	
+	x	$sec^2(x)$	$\int x sec^2(x)\, dx = x tg(x) + ln\lvert cos(x)\rvert + C$
-	1	$tg(x)$	
+	0	$-ln\lvert cos(x)\rvert$	

i) $\int \frac{ln(sen(x))}{sec(x)} dx$

Sinal	Derivar	Integrar	
+	$ln(sen(x))$	$\frac{1}{sec(x)} = cos(x)$	$\int \frac{ln(sen(x))}{sec(x)}dx = \int cos(x)\, ln(sen(x))dx =$ $= ln(sen(x))sen(x) - \int sen(x)cotg(x)\, dx =$ $= ln(sen(x))sen(x) - \int cos(x)\, dx =$ $= ln(sen(x))sen(x) - sen(x) + C$
−	$cotg(x) = \frac{cos(x)}{sen(x)}$	$sen(x)$	

j) $\int x^5 e^{x^3} dx$ como: $\int x^5 e^{x^3} dx = \int x^3 x^2 e^{x^3} dx$

Sinal	Derivar	Integrar	
+	x^3	$\int x^2 e^{x^3} dx$	$\int x^3 x^2 e^{x^3} dx = x^3 \frac{e^{x^3}}{3} - \int 3x^2 \frac{e^{x^3}}{3} dx =$ $= x^3 \frac{e^{x^3}}{3} - \frac{1}{3} \int 3x^2 e^{x^3}\, dx =$ $= x^3 \frac{e^{x^3}}{3} - \frac{1}{3} e^{x^3} + C$
$-$	$3x^2$	$\int x^2 e^{x^3} dx = \frac{1}{3} \int 3x^2\, e^{x^3} dx$ $= \frac{e^{x^3}}{3}$	

k) $\int ln\left(\sqrt{x+1}\right) dx = \int 1\, ln\left(\sqrt{x+1}\right) dx$

Sinal	Derivar	Integrar	
+	$ln\left(\sqrt{x+1}\right)$	1	$\int ln\left(\sqrt{x+1}\right) dx=$ $= xln\left(\sqrt{x+1}\right) - \int x \frac{1}{2(x+1)} dx$ $= xln\left(\sqrt{x+1}\right) - \frac{1}{2} \int \frac{x}{x+1} dx =$ $= xln\left(\sqrt{x+1}\right) - \frac{1}{2}(x - ln\|x+1\|) + C =$ $= \frac{1}{2} xln\|x+1\| - \frac{1}{2}(x - ln\|x+1\|) + C =$ $= \frac{1}{2}\left((x+1)ln\|x+1\| - x\right) + C=$ $= (x+1)ln\left(\sqrt{x+1}\right) - \frac{x}{2} + C$
-	$\frac{\frac{1}{2}(x+1)^{-\frac{1}{2}}}{(x+1)^{\frac{1}{2}}}$ $= \frac{1}{2(x+1)}$	x	

Cálculos auxiliares:

$$\int \frac{x}{x+1} dx = \int \frac{x+1-1}{x+1} dx = \int 1 - \frac{1}{x+1} dx = x - ln|x+1| + C$$

l) $\int xsen^2(x) dx$

Sinal	Derivar	Integrar	
+	x	$sen^2(x)$	$\int xsen^2(x) dx =$ $= x\frac{1}{2}\left(x - \frac{1}{2} sen(2x)\right) - \left(\frac{1}{4}x^2 + \frac{1}{8} cos(2x)\right)$ $+ C$ $= \frac{x^2}{4} - \frac{xsen(2x)}{4} - \frac{1}{8} cos(2x) + C$
-	1	$\frac{1}{2}\left(x - \frac{1}{2} sen(2x)\right)$	
+	0	$\frac{1}{4}x^2 + \frac{1}{8} cos(2x)$	

$$\int sen^2(x) dx = \int \frac{1 - cos(2x)}{2} dx = \frac{1}{2}\left(\int 1 dx - \frac{1}{2} \int 2cos(2x) dx\right) = \frac{1}{2}\left(x - \frac{1}{2} sen(2x)\right) + C$$

m) $\int x^4\, 3^x dx$

Sinal	Derivar	Integrar	
+	x^4	3^x	$\int x^4\, 3^x dx =$ $x^4 \frac{3^x}{ln3} - 4x^3 \frac{3^x}{(ln3)^2} + 12x^2 \frac{3^x}{(ln3)^3} - 24x \frac{3^x}{(ln3)^4}$ $+ 24 \frac{3^x}{(ln3)^5} + C$
-	$4x^3$	$\frac{3^x}{ln3}$	
+	$12x^2$	$\frac{3^x}{(ln3)^2}$	
-	$24x$	$\frac{3^x}{(ln3)^3}$	
+	24	$\frac{3^x}{(ln3)^4}$	
-	0	$\frac{3^x}{(ln3)^5}$	

Ou

<table>
<tr><th>Sinal</th><th>Derivar</th><th>Integrar</th><td rowspan="3">$\int x^4 3^x dx = x^4 \frac{3^x}{ln3} - \int 4x^3 \frac{3^x}{ln3} dx =$
$= x^4 \frac{3^x}{ln3} - \frac{4}{ln3} \int x^3\, 3^x dx =$
$= x^4 \frac{3^x}{ln3} - \frac{4}{ln3}\left(x^3 \frac{3^x}{ln3} - \frac{3}{ln3}\left(x^2 \frac{3^x}{ln3} - \frac{2}{ln3}\left(x \frac{3^x}{ln3} - \frac{x}{ln3}\right)\right)\right) + C$</td></tr>
<tr><td>+</td><td>x^4</td><td>3^x</td></tr>
<tr><td>-</td><td>$4x^3$</td><td>$\frac{3^x}{ln3}$</td></tr>
</table>

Cálculos auxiliares:

<table>
<tr><th>Sinal</th><th>Derivar</th><th>Integrar</th><td rowspan="3">$\int x^3\, 3^x dx = x^3 \frac{3^x}{ln3} - \int 3x^2 \frac{3^x}{ln3} dx =$
$= x^3 \frac{3^x}{ln3} - \frac{3}{ln3} \int x^2\, 3^x dx =$
$= x^3 \frac{3^x}{ln3} - \frac{3}{ln3}\left(x^2 \frac{3^x}{ln3} - \frac{2}{ln3}\left(x \frac{3^x}{ln3} - \frac{x}{ln3}\right)\right) + C$</td></tr>
<tr><td>+</td><td>x^3</td><td>3^x</td></tr>
<tr><td>-</td><td>$3x^2$</td><td>$\frac{3^x}{ln3}$</td></tr>
</table>

<table>
<tr><th>Sinal</th><th>Derivar</th><th>Integrar</th><td rowspan="3">$\int x^2\, 3^x dx = x^2 \frac{3^x}{ln3} - \int 2x \frac{3^x}{ln3} dx =$
$= x^2 \frac{3^x}{ln3} - \frac{2}{ln3} \int x\, 3^x dx =$
$= x^2 \frac{3^x}{ln3} - \frac{2}{ln3}\left(x \frac{3^x}{ln3} - \frac{x}{ln3}\right) + C$</td></tr>
<tr><td>+</td><td>x^2</td><td>3^x</td></tr>
<tr><td>-</td><td>$2x$</td><td>$\frac{3^x}{ln3}$</td></tr>
</table>

<table>
<tr><th>Sinal</th><th>Derivar</th><th>Integrar</th><td rowspan="3">$\int x\, 3^x dx = x \frac{3^x}{ln3} - \int \frac{3^x}{ln3} dx =$
$= x \frac{3^x}{ln3} - \frac{x}{ln3} + C$</td></tr>
<tr><td>+</td><td>x</td><td>3^x</td></tr>
<tr><td>-</td><td>1</td><td>$\frac{3^x}{ln3}$</td></tr>
</table>

n) $\int (x+1)^2 cos(x) dx$

<table>
<tr><th>Sinal</th><th>Derivar</th><th>Integrar</th><td rowspan="5">$\int (x+1)^2 cos(x) dx =$
$= (x+1)^2 sen(x) + 2(x+1)cos(x) - 2\, sen(x) + C$</td></tr>
<tr><td>+</td><td>$(x+1)^2$</td><td>$cos(x)$</td></tr>
<tr><td>-</td><td>$2(x+1)$</td><td>$sen(x)$</td></tr>
<tr><td>+</td><td>2</td><td>$-cos(x)$</td></tr>
<tr><td>-</td><td>0</td><td>$-sen(x)$</td></tr>
</table>

o) $\int x^3 cossec^2(x^2)\, dx$

<table>
<tr><th>Sinal</th><th>Derivar</th><th>Integrar</th><td rowspan="3">$\int x^2 x cossec^2(x^2)\, dx =$
$= -x^2 \frac{cotg(x^2)}{2} + \frac{1}{2} \int 2x cotg(x^2) dx =$
$== -x^2 \frac{cotg(x^2)}{2} + \frac{ln\lvert sen(x^2)\rvert}{2} + C$</td></tr>
<tr><td>+</td><td>x^2</td><td>$x cossec^2(x^2)$</td></tr>
<tr><td>-</td><td>$2x$</td><td>$-\frac{cotg(x^2)}{2}$</td></tr>
</table>

p) $\int ln(sen(x))sec^2(x)\,dx$

Sinal	Derivar	Integrar	
+	$ln(sen(x))$	$sec^2(x)$	$\int ln(sen(x))sec^2(x)\,dx =$ $= ln(sen(x))tg(x) - \int tg(x)cotg(x)dx =$ $= ln(sen(x))tg(x) - \int 1dx =$ $= ln(sen(x))tg(x) - x + C$
-	$cotg(x)$	$tg(x)$	

q) $\int xsen(3x)dx$

Sinal	Derivar	Integrar	
+	x	$\text{sen}\,(3x)$	$\int xsen\,(3x)dx = x\left(-\frac{1}{3}\cos\,(3x)\right) - 1\left(-\frac{1}{9}\text{sen}\,(3x)\right) + C$ $= -\frac{x}{3}\cos(3x) + \frac{1}{9}\text{sen}\,(3x) + C$
−	1	$-\frac{1}{3}\cos\,(3x)$	
+	0	$-\frac{1}{9}\text{sen}\,(3x)$	

r) $\int x^3 e^{2x} dx$

Sinal	Derivar	Integrar	
+	x^3	e^{2x}	$\int x^3 e^{2x} dx = x^3\frac{e^{2x}}{2} - 3x^2\frac{e^{2x}}{4} + 6x\frac{e^{2x}}{8} - 6\frac{e^{2x}}{16} + C$
-	$3x^2$	$\frac{e^{2x}}{2}$	
+	$6x$	$\frac{e^{2x}}{4}$	
-	6	$\frac{e^{2x}}{8}$	
+	0	$\frac{e^{2x}}{16}$	

s) $\int ln(cos(x))cosec^2(x)\,dx$

Sinal	Derivar	Integrar	
+	$ln(cos(x))$	$cosec^2(x)$	$\int ln(cos(x))cosec^2(x)\,dx =$ $= -ln(cos(x))cotg(x) - \int tg(x)cotg(x)dx =$ $= -ln(cos(x))cotg(x) - \int 1dx =$ $= -ln(cos(x))cotg(x) - x + C$
−	$-tg(x)$	$-cotg(x)$	