Exercícios elaborados no ano 2022-23 por: Eduarda Pinto Ferreira, Fernando Carvalho, Marta Pinto Ferreira

Proposta de resolução

1. Calcular as derivadas das expressões abaixo, usando as fórmulas de derivação:

a) $y = x^2 + 4x$

$$y' = \frac{dy}{dx} = (x^2+4x)' = 2x^{2-1} + 4 = 2x + 4$$

b) $f(x) = \frac{2}{x^2}$

$$y' = \frac{dy}{dx} = (2x^{-2})' = -4x^{-2-1} = -4x^{-3}$$

c) $y = \frac{x^3}{2} + \frac{3x}{2}$

$$y' = \frac{dy}{dx} = \left(\frac{1}{2}x^3 + \frac{3}{2}x\right)' = \frac{3}{2}x^{3-1} + \frac{3}{2}x^{1-1} = \frac{3}{2}x^2 + \frac{3}{2}$$

d) $y = \sqrt[3]{x}$

$$y' = \frac{dy}{dx} = \left(x^{\frac{1}{3}}\right)' = \frac{1}{3}x^{\frac{1}{3}-1} = \frac{1}{3}x^{-\frac{2}{3}} \qquad \sqrt[m]{u^n} = u^{\frac{n}{m}}$$

e) $f(x) = \left(3x + \frac{1}{x}\right)\cdot(6x-1)$

$$f'(x) = \frac{df}{dx} = \left(\left(3x+\frac{1}{x}\right)(6x-1)\right)' = (3x + x^{-1})'(6x-1) + \left(3x+\frac{1}{x}\right)(6x-1)' =$$

$$= \left(3 + (-1x^{-1-1})\right)(6x-1) + \left(3x+\frac{1}{x}\right)(6-0) =$$

$$= (3 - x^{-2})(6x-1) + 6\left(3x+\frac{1}{x}\right)$$

f) $y = \frac{x^5}{a+b} - \frac{x^2}{a-b} - x$

$$y = \frac{x^5}{a+b} - \frac{x^2}{a-b} - x$$

$$y' = \frac{dy}{dx} = \frac{d}{dx}\left(\frac{1}{a+b}x^5 - \frac{1}{a-b}x^2 - x\right) = \frac{1}{a+b}\frac{d}{dx}(x^5) - \frac{1}{a-b}\frac{d}{dx}(x^2) - \frac{d}{dx}(x) =$$

$$y' = \frac{5}{a+b}x^4 - \frac{2}{a-b}x - 1$$

g) $y = \frac{(x+1)^3}{x^{3/2}}$ R: $\frac{dy}{dx} = \frac{3(x+1)^2(x-1)}{2x^{5/2}}$

$$y' = \frac{dy}{dx} = \frac{((x+1)^3)'x^{\frac{3}{2}} - \left(x^{\frac{3}{2}}\right)'(x+1)^3}{\left(x^{\frac{3}{2}}\right)^2} = \frac{(3(x+1)^2)x^{\frac{3}{2}} - \frac{3}{2}x^{\frac{3}{2}-1}(x+1)^3}{\left(x^{\frac{3}{2}}\right)^2}$$

$$= \frac{(3(x+1)^2)x^{\frac{3}{2}} - \frac{3}{2}x^{\frac{1}{2}}(x+1)^3}{\left(x^{\frac{3}{2}}\right)^2}$$

h) $y = x(2x-1)(3x+2)$ R: $\frac{dy}{dx} = 2(9x^2 + x - 1)$

$$y' = \frac{dy}{dx} = \left(x(2x-1)(3x+2)\right)' = \left((2x^2-x)(3x+2)\right)' = (2x^2-x)'(3x+2) + (2x^2-x)(3x+2)' =$$

$$y' = (4x-1)(3x+2)+(2x^2-x)(3)=(4x-1)(3x+2)+3(2x^2-x)$$

i) $y = \dfrac{2x^4}{b^2-x^2}$ R: $\dfrac{dy}{dx} = \dfrac{4x^3\left(2b^2-x^2\right)}{\left(b^2-x^2\right)^2}$

$$\frac{dy}{dx} = \frac{\left(2x^4\right)'\left(b^2-x^2\right)-\left(b^2-x^2\right)'\left(2x^4\right)}{\left(b^2-x^2\right)^2} = \frac{8x^3\left(b^2-x^2\right)+2x\left(2x^4\right)}{\left(b^2-x^2\right)^2} = \frac{8x^3b^2-8x^5+4x^5}{\left(b^2-x^2\right)^2}$$

$$= \frac{8x^3b^2-4x^5}{\left(b^2-x^2\right)^2} = \frac{4x^3\left(2b^2-x^2\right)}{\left(b^2-x^2\right)^2}$$

j) $y = \dfrac{a-x}{a+x}$ R: $\dfrac{dy}{dx} = \dfrac{-2a}{(a+x)^2}$

$$\frac{dy}{dx} = \frac{(a-x)'(a+x)-(a+x)'(a-x)}{(a+x)^2} = \frac{-(a+x)-(a-x)}{(a+x)^2} = \frac{-2a}{(a+x)^2}$$

k) $y = \left(\dfrac{a-x}{a+x}\right)^3$ R: $\dfrac{dy}{dx} = \dfrac{-6a(a-x)^2}{(a+x)^4}$

$$\frac{dy}{dx} = \left(\left(\frac{a-x}{a+x}\right)^3\right)' = 3\left(\frac{a-x}{a+x}\right)^2\left(\frac{a-x}{a+x}\right)' = 3\frac{(a-x)^2}{(a+x)^2}\left(\frac{-2a}{(a+x)^2}\right) = \frac{-6a(a-x)^2}{(a+x)^4}$$

l) $y = \sqrt{\dfrac{1+x}{1-x}}$ R: $\dfrac{dy}{dx} = \dfrac{1}{(1-x)\sqrt{1-x^2}}$

$$y' = \left(\left(\frac{1+x}{1-x}\right)^{\frac{1}{2}}\right)' = \frac{1}{2}\left(\frac{1+x}{1-x}\right)^{\frac{1}{2}-1}\left(\frac{1+x}{1-x}\right)' = \frac{1}{2}\left(\frac{1+x}{1-x}\right)^{-\frac{1}{2}}\frac{(1+x)'(1-x)-(1+x)(1-x)'}{(1-x)^2} =$$

$$= \frac{1}{2}\left(\frac{1+x}{1-x}\right)^{-\frac{1}{2}}\frac{1(1-x)-(1+x)(-1)}{(1-x)^2} = \frac{1}{2}\left(\frac{1+x}{1-x}\right)^{-\frac{1}{2}}\frac{(1-x)+(1+x)}{(1-x)^2} = \frac{1}{2}\left(\frac{1+x}{1-x}\right)^{-\frac{1}{2}}\frac{2}{(1-x)^2} = \ldots$$

m) $y = \left(1+\sqrt[3]{x}\right)^3$ R: $\dfrac{dy}{dx} = \left(\dfrac{1}{x}+\dfrac{1}{x\sqrt[3]{x}}\right)^2$

$$\frac{dy}{dx} = 3\left(1+\sqrt[3]{x}\right)^2\left(1+\sqrt[3]{x}\right)' = 3\left(1+\sqrt[3]{x}\right)^2\left(x^{\frac{1}{3}}\right)' = 3\left(1+\sqrt[3]{x}\right)^2\frac{1}{3}\left(x^{-\frac{2}{3}}\right) =$$

$$= \left(1+\sqrt[3]{x}\right)^2\left(x^{-\frac{2}{3}}\right) = \frac{\left(1+\sqrt[3]{x}\right)^2}{\left(\sqrt[3]{x}\right)^2} = \left(\frac{1+\sqrt[3]{x}}{\sqrt[3]{x}}\right)^2 = \left(1+\frac{1}{\sqrt[3]{x}}\right)^2$$

n) $y=\dfrac{2x^2-1}{x\sqrt{1+x^2}}$

R: $\dfrac{dy}{dx}=\dfrac{1+4x^2}{x^2\sqrt{(1+x^2)^3}}$

$$\frac{dy}{dx}=\frac{(2x^2-1)'\left(x\sqrt{1+x^2}\right)-\left(x\sqrt{1+x^2}\right)'(2x^2-1)}{\left(x\sqrt{1+x^2}\right)^2}=\frac{4x\left(x\sqrt{1+x^2}\right)-\left(x'\sqrt{1+x^2}+x\left(\left(1+x^2\right)^{\frac{1}{2}}\right)'\right)(2x^2-1)}{\left(x^2(1+x^2)\right)}=$$

$$=\frac{4x\left(x\sqrt{1+x^2}\right)-(2x^2-1)\left(\sqrt{1+x^2}+x\left(\frac{1}{2}2x(1+x^2)^{-\frac{1}{2}}\right)\right)}{\left(x^2(1+x^2)\right)}=$$

$$=\frac{4x^2\sqrt{1+x^2}-(2x^2-1)\sqrt{1+x^2}-\dfrac{(2x^2-1)x^2}{\sqrt{1+x^2}}}{x^2(1+x^2)}=\frac{\dfrac{4x^2(1+x^2)-(2x^2-1)(1+x^2)-(2x^2-1)x^2}{\sqrt{1+x^2}}}{x^2(1+x^2)}=$$

$$=\frac{4x^2(1+x^2)-(2x^2-1)(1+x^2)-(2x^2-1)x^2}{x^2(1+x^2)\sqrt{1+x^2}}=\frac{4x^2+4x^4-2x^2+1-2x^4+x^2-2x^4+x^2}{x^2\sqrt{(1+x^2)(1+x^2)^2}}=$$

$$=\frac{4x^2+4x^4-2x^2+1-4x^4+2x^2}{x^2\sqrt{(1+x^2)^3}}=\frac{1+4x^2}{x^2\sqrt{(1+x^2)^3}}$$

o) $y=\left(x^2-a^2\right)^5$

R: $\dfrac{dy}{dx}=10x\left(x^2-a^2\right)^4$

$$\frac{dy}{dx}=\left(\left(x^2-a^2\right)^5\right)'=5\left(x^2-a^2\right)^4\left(x^2-a^2\right)'=5\left(x^2-a^2\right)^4 2x=10x\left(x^2-a^2\right)^4$$

p) $f(x)=arctg\left(\frac{2x}{1-x^2}\right)$

$$\left(\frac{2x}{1-x^2}\right)'=\frac{2(1-x^2)-(-2x)2x}{(1-x^2)^2}=\frac{2-2x^2+4x^2}{(1-x^2)^2}=\frac{2+2x^2}{(1-x^2)^2}=\frac{2(1+x^2)}{(1-x^2)^2}$$

$$\frac{df}{dx}=\frac{\left(\frac{2x}{1-x^2}\right)'}{1+\left(\frac{2x}{1-x^2}\right)^2}=\frac{\frac{2(1+x^2)}{(1-x^2)^2}}{1+\frac{(2x)^2}{(1-x^2)^2}}=\frac{\frac{2(1+x^2)}{(1-x^2)^2}}{\frac{(1-x^2)^2+(2x)^2}{(1-x^2)^2}}=\frac{2(1+x^2)}{1-2x^2+x^4+4x^2}=\frac{2(1+x^2)}{(1+x^2)^2}=\frac{2}{1+x^2}$$

q) $g(x)=\dfrac{arcos(x)}{x}$

$$\frac{dg}{dx}=\frac{\left(arcos(x)\right)'x-arcos(x)}{x^2}=\frac{\left(\frac{-1}{\sqrt{1-x^2}}\right)x-arcos(x)}{x^2}=\ldots$$

r) $h(x)=arcsen\left(\frac{x+1}{\sqrt{2}}\right)$

$$\left(\frac{x+1}{\sqrt{2}}\right)'=\frac{1}{\sqrt{2}}(x+1)'=\frac{1}{\sqrt{2}} \qquad \frac{dh}{dx}=\frac{\left(\frac{x+1}{\sqrt{2}}\right)'}{\sqrt{1-\left(\frac{x+1}{\sqrt{2}}\right)^2}}=\frac{\frac{1}{\sqrt{2}}}{\sqrt{1-\left(\frac{x+1}{\sqrt{2}}\right)^2}}=\cdots$$

2. Considere a função $h(t) = 2^t - 3e^t - 4^t(t+5)$

a) Determine $h'(t) = \frac{dh}{dt}$.

$$\frac{dh}{dt} = 2^t ln2 - 3e^t - (4^t ln4(t+5) + 4^t) = \cdots$$

b) Determine $h''(t) = \frac{d^2h}{d^2t}$.

$$\frac{d^2h}{d^2t} = \frac{d\left(2^t ln2 - 3e^t - (4^t ln4(t+5) + 4^t)\right)}{dt} =$$

$$= 2^t (ln2)^2 - 3e^t - (4^t (ln4)^2 (t+5) + 4^t ln4 + 4^t ln4) = \cdots$$

3. Considere as funções $f(x) = \frac{-5x^2}{2x.cos(x)}$ e $g(x) = \frac{2}{\sqrt[3]{x^3-x^2}}$ e $h(x) = e^{(3x+5)^3}$

a) Determine $\frac{df}{dx}$.

$$\frac{df}{dx} = \frac{(-5x^2)'(2x.cos(x)) - (-5x^2)(2x.cos(x))'}{(2x.cos(x))^2} =$$

$$= \frac{-10x(2x.cos(x)) - (-5x^2)(2\,cos(x) - 2xsen(x))}{(2x.cos(x))^2} =$$

$$= \frac{-20x^2 cos(x) + 10x^2\,cos(x) - 10x^3 sen(x)}{4x^2 cos^2(x)} =$$

$$= \frac{-10x^2\,cos(x) - 10x^3 sen(x)}{4x^2 cos^2(x)} = \frac{-5\,cos(x) - 5xsen(x)}{2cos^2(x)}$$

$Calc\ Aux$:

$$(2x.cos(x))' = (2x)'\,cos(x) + (2x)(cos(x))' = 2\,cos(x) - 2xsen(x)$$

b) Determine $\frac{dg}{dx}$.

$$\frac{dg}{dx} = 2\left((x^3 - x^2)^{-\frac{1}{3}}\right)' = -2\frac{1}{3}(x^3 - x^2)^{-\frac{1}{3}-1}(x^3 - x^2)' =$$

$$= -\frac{2}{3}(x^3 - x^2)^{-\frac{4}{3}}(3x^2 - 2x)$$

c) Determine $\frac{dh}{dx}$.

$$\frac{dh}{dx} = \left(e^{(3x+5)^3}\right)' = ((3x+5)^3)' e^{(3x+5)^3} = 3(3x+5)^2(3x+5)' e^{(3x+5)^3}$$

$$= 9(3x+5)^2 e^{(3x+5)^3}$$

d) Determine $\frac{d^2h}{dx^2}$

$$\frac{d^2h}{dx^2} = \left(9(3x+5)^2 e^{(3x+5)^3}\right)' = 9\left((3x+5)^2 e^{(3x+5)^3}\right)' =$$

$$= 9\left(((3x+5)^2)' e^{(3x+5)^3} + \left(e^{(3x+5)^3}\right)'(3x+5)^2\right) =$$

$$= 9\big(6(3x+5)\big)e^{(3x+5)^3} + \left(9(3x+5)^2 e^{(3x+5)^3}\right)9(3x+5)^2$$

4. Considere as funções $f(x) = e^x + x^2 - 1$; $g(x) = (sen(2x) + 3)^2$; $h(x) = ln(2x + 3)$

a) Determine $\frac{df}{dx}$

$$\frac{df}{dx} = (e^x + x^2 - 1)' = e^x + 2x$$

b) Determine $\frac{d^2f}{dx^2}$

$$\frac{d^2f}{dx^2} = (e^x + 2x)' = e^x + 2$$

c) Determine $\frac{dg}{dx}$

$$\frac{dg}{dx} = ((sen(2x) + 3)^2)' = 2(sen(2x) + 3)(2cos(2x) + 0) = 4cos(2x)(sen(2x) + 3)$$

d) Determine $\frac{dh}{dx}$

$$\frac{dh}{dx} = \big(ln(2x + 3)\big)' = \frac{2}{2x + 3}$$

5. Considere as funções $f(x) = \frac{x+senx}{x-cosx}$ e $g(x) = \sqrt[3]{2x^2 - e^{-3x}}$ e $h(t) = (t^2 + 3)^4$

a) Determine $\frac{df}{dx} = \frac{(x+senx)\prime(x-cosx)-(x-cosx)\prime(x+senx)}{(x-cosx)^2} = \frac{(1+cosx)(x-cosx)-(1+senx)(x+senx)}{(x-cosx)^2} = \ldots$

b) Determine $\frac{dg}{dx} = \left((2x^2 - e^{-3x})^{\frac{1}{3}}\right)' = \frac{1}{3}(2x^2 - e^{-3x})^{\frac{1}{3}-1}(2x^2 - e^{-3x})' =$

$$= \frac{1}{3}(2x^2 - e^{-3x})^{\frac{1}{3}-1}(4x + 3e^{-3x}) = \frac{1}{3}(2x^2 - e^{-3x})^{-\frac{2}{3}}(4x + 3e^{-3x}) =$$

...

c) Determine $\frac{dh}{dt} = 4(t^2 + 3)^3(t^2 + 3)' = 4(t^2 + 3)^3(2t) = 8t(t^2 + 3)^3$

d) Determine $\frac{d^2h}{dt^2} = (8t(t^2 + 3)^3)' = 8((t^2 + 3)^3 t)' =$

$$= 8(t'(t^2 + 3)^3) + t((t^2 + 3)^3)' = 8((t^2 + 3)^3 + 3(t^2 + 3)^2(2t)t) =$$

$$= 8(t^2 + 3)^3 + 48t^2(t^2 + 3)^2$$

6. Considere as funções $f(x) = e^x + x^2 - 1$; $g(x) = (sen(2x) + 3)^2$; $h(x) = ln(2x + 3)$

a) Determine $\frac{df}{dx} = (e^x + x^2 - 1)' = e^x + 2x$

b) Determine $\frac{d^2f}{dx^2} = (e^x + 2x)' = e^x + 2$

c) Determine $\frac{dg}{dx} = ((sen(2x) + 3)^2)' = 2(sen(2x) + 3)(2cos(2x) + 0) =$
$4cos(2x)(sen(2x) + 3)$

d) Determine $\frac{dh}{dx} = \big(ln(2x + 3)\big)' = \frac{2}{2x+3}$

7. Considere as funções $g(x) = 2sen^2(x) - 1$; $h(x) = \sqrt[4]{x^2 + 3x - 1}$, determine:

a) $\frac{dg}{dx} = (2sen^2(x) - 1)' = 2\left(sen^2(x)\right)' - 0 = 2\left(2sen(x)\left(sen(x)\right)'\right) = 4sen(x)cos(x)$

b) $\frac{d^2g}{dx^2} = \left(4sen(x)cos(x)\right)' = 4\left(sen(x)cos(x)\right)' = 4\left(\left(sen(x)\right)'cos(x) + \left(cos(x)\right)'sen(x)\right) =$

$$= 4(cos^2x - sen^2x) = 4cos(2x)$$

c) $\frac{dh}{dx} = \left((x^2 + 3x - 1)^{\frac{1}{4}}\right)' = \frac{1}{4}(x^2 + 3x - 1)^{\frac{1}{4}-1}(x^2 + 3x - 1)' = \frac{1}{4}(x^2 + 3x - 1)^{-\frac{3}{4}}(2x + 3)$

8. Considere as funções $g(x) = sin^2(x) + \sqrt{1 - 2x}$; $h(x) = e^x(x^2 + 5)^3$, determine:

a) $\frac{dg}{dx} = \left(sin^2(x) + \sqrt{1 - 2x}\right)' = 2sin(x)cos(x) + \frac{1}{2}(1 - 2x)^{\frac{1}{2}-1}(-2) =$

$$= sen(2x) - (1 - 2x)^{-\frac{1}{2}}$$

b) $\frac{dh}{dx} = (e^x(x^2 + 5)^3)' = e^x(x^2 + 5)^3 + 3(x^2 + 5)^2(2x)e^x =$

$$= e^x(x^2 + 5)^3 + 6xe^x(x^2 + 5)^2$$

c) $\frac{d^2h}{dx^2} = (e^x(x^2 + 5)^3 + 6xe^x(x^2 + 5)^2)' =$

$$= e^x(x^2 + 5)^3 + (6e^x + 6xe^x)(x^2 + 5)^2 + 2(x^2 + 5)2x(6xe^x) = \cdots$$

9. Considere as funções $g(x) = \frac{x}{(1-2x)^2} + ln(x + 1)$; $h(x) = 2^{x^2}(1 + 2x)$, determine:

a) $\frac{dg}{dx} = \left(\frac{x}{(1-2x)^2} + ln(x + 1)\right)' = \frac{1(1-2x)^2 - 2(1-2x)(-2)x}{(1-2x)^4} + \frac{1}{x+1}$

b) $\frac{dh}{dx} = \left(2^{x^2}(1 + 2x)\right)' = 2x2^{x^2}ln2(1 + 2x) + 2^{x^2}2 = ln4\left(x2^{x^2}\right)(1 + 2x) + 2^{x^2+1}$

c) $\frac{d^2h}{dx^2} = \left(2(ln2)x2^{x^2}(1 + 2x) + 2^{x^2}2\right)' =$

$$= \left(2(ln2)2^{x^2} + 2(ln2)2x2^{x^2}ln2\right)(1 + 2x) + 2(ln2)x2^{x^2}(1 + 2x)2 + 2\left(2x2^{x^2}ln2\right) =$$

$$= \left(2(ln2)2^{x^2} + 2(ln2)2x2^{x^2}ln2\right)(1 + 2x) + 2(ln2)x2^{x^2}(1 + 2x)2 + 4x2^{x^2}ln2$$

ou

$$\frac{d^2h}{dx^2} = \left(ln4\left(x2^{x^2}\right)(1 + 2x) + 2^{x^2+1}\right)' =$$

$$= \left(ln4\left(2^{x^2} + ln4\left(x2^{x^2}\right)x\right)\right)(1 + 2x) + ln4\left(x2^{x^2}\right)2 + 2x2^{x^2+1}ln2 =$$

$$= \left(ln4\left(2^{x^2} + ln4\left(x2^{x^2}\right)x\right)\right)(1 + 2x) + ln16\left(x2^{x^2}\right) + x2^{x^2+1}ln4$$

10. Considere as funções $g(x) = (x + 5)^2 + \frac{1}{\sqrt[5]{1-x^2}}$; $h(x) = ln(4x)e^{x^2}$, determine:

a) $\frac{dg}{dx} = \left((x + 5)^2 + \frac{1}{\sqrt[5]{1-x^2}}\right)' = \left((x + 5)^2 + (1 - x^2)^{-\frac{1}{5}}\right)' = 2(x + 5) - \frac{1}{5}(-2x)(1 - x^2)^{-\frac{1}{5}-1}$

b) $\frac{dh}{dx} = \left(ln(4x)e^{x^2}\right)' = \frac{4}{4x}e^{x^2} + ln(4x)2xe^{x^2} = \frac{1}{x}e^{x^2} + 2xln(4x)e^{x^2}$

c) $\frac{d^2h}{dx^2} = \left(\frac{1}{x}e^{x^2} + 2xln(4x)e^{x^2}\right)' = -\frac{1}{x^2}e^{x^2} + \frac{2x}{x}e^{x^2} + \left(2ln(4x) + 2x\frac{4}{4x}\right)e^{x^2} +$
$2xln(4x)2xe^{x^2}$

11. Considere as funções $g(x) = 2^{\sqrt{x-1}}(3x+5)^4$; $\quad h(x) = e^{x^2} ln(x)$, determine:

a) $\frac{dg}{dx} = \left(2^{\sqrt{x-1}}(3x+5)^4\right)' = 2^{(x-1)^{\frac{1}{2}}} ln2 \frac{1}{2}(x-1)^{-\frac{1}{2}}(3x+5)^4 + 12(3x+5)^3 2^{\sqrt{x-1}}$

b) $\frac{dh}{dx} = \left(e^{x^2} ln(x)\right)' = 2xe^{x^2} ln(x) + \frac{1}{x} e^{x^2}$

c) $\frac{d^2h}{dx^2} = \left(2xe^{x^2} ln(x) + \frac{1}{x} e^{x^2}\right)' = \left(2e^{x^2} + 4x^2 e^{x^2}\right) ln(x) + \left(-\frac{1}{x^2} e^{x^2} + \frac{2x}{x} e^{x^2}\right)$

12. Aplicando o teorema da derivada da função inversa, calcule $\frac{dx}{dy}$, sendo:

a) $y = \sqrt[3]{x+2}$

$$y = \sqrt[3]{x+2} = (x+2)^{\frac{1}{3}} \Rightarrow \frac{dy}{dx} = \frac{1}{3}(x+2)^{-\frac{2}{3}}$$

$$\frac{dx}{dy} = \frac{1}{\frac{1}{3}(x+2)^{-\frac{2}{3}}} = 3(x+2)^{\frac{2}{3}} \xRightarrow[x=y^3-2]{} \frac{dx}{dy} = 3(y^3-2+2)^{\frac{2}{3}} = 3(y^3)^{\frac{2}{3}} = 3y^2$$

porque $y = (x+2)^{\frac{1}{3}} \Leftrightarrow y^3 = x+2$. logo a função inversa de y é $x = y^3 - 2$

b) $y = log_5\left(\sqrt{x^3+2}\right)$

$$y = log_5\left(\sqrt{x^3+2}\right) = \frac{1}{2} log_5(x^3+2) \Rightarrow \frac{dy}{dx} = \frac{1}{2} \frac{3x^2}{ln5(x^3+2)} = \frac{3x^2}{2ln5(x^3+2)}$$

$$\frac{dx}{dy} = \frac{1}{\frac{3x^2}{2ln5(x^3+2)}} = \frac{2ln5(x^3+2)}{3x^2} \xRightarrow[x=\sqrt[3]{5^{2y}-2}]{} \frac{dx}{dy} = \frac{2ln5\left(\left(\sqrt[3]{5^{2y}-2}\right)^3+2\right)}{3\left(\sqrt[3]{5^{2y}-2}\right)^2} = \cdots$$

porque $y = log_5\left(\sqrt{x^3+2}\right) \Leftrightarrow 5^y = \sqrt{x^3+2} \Leftrightarrow 5^{2y} = x^3+2 \Leftrightarrow 5^{2y}-2 = x^3 \Leftrightarrow x = \sqrt[3]{5^{2y}-2}$

c) $y = \sqrt{e^x+1}$

$$y = \sqrt{e^x+1} = (e^x+1)^{\frac{1}{2}} \Rightarrow \frac{dy}{dx} = \frac{1}{2}(e^x+1)^{-\frac{1}{2}} e^x = \frac{e^x}{2}(e^x+1)^{-\frac{1}{2}}$$

$$\frac{dx}{dy} = \frac{1}{\frac{e^x}{2}(e^x+1)^{-\frac{1}{2}}} = \frac{2}{e^x}(e^x+1)^{\frac{1}{2}} \xRightarrow[x=ln(y^2-1)]{} \frac{dx}{dy} = \frac{2}{e^{ln(y^2-1)}}\left(e^{ln(y^2-1)}+1\right)^{\frac{1}{2}}$$

$$= \frac{2}{y^2-1}(y^2-1+1)^{\frac{1}{2}} = \frac{2y}{y^2-1}$$

porque $y = (e^x+1)^{\frac{1}{2}} \Leftrightarrow y^2 = e^x+1 \Leftrightarrow y^2-1 = e^x \Leftrightarrow x = ln(y^2-1)$

d) $y = ln(x^3+1)$

$$y = ln(x^3+1) = \Rightarrow \frac{dy}{dx} = \frac{3x^2}{x^3+1}$$

$$\frac{dx}{dy} = \frac{1}{\frac{3x^2}{x^3+1}} = \frac{x^3+1}{3x^2} \xRightarrow[x=\sqrt[3]{e^y-1}]{} \frac{dx}{dy} = \frac{\left(\sqrt[3]{e^y-1}\right)^3+1}{3\left(\sqrt[3]{e^y-1}\right)^2} = \frac{e^y-1+1}{3\left(\sqrt[3]{e^y-1}\right)^2} = \frac{e^y}{3(e^y-1)^{\frac{2}{3}}} = \cdots$$

porque $y = ln(x^3+1) \Leftrightarrow e^y = x^3+1 \Leftrightarrow e^y-1 = x^3 \Leftrightarrow x = \sqrt[3]{e^y-1}$

13. Determine y' se $y = f(x)$ é a função definida implicitamente pela equação

a) $x^3 + x^2y + y^2 = 0$

Derivamos ambos os membros da equação $x^3 + x^2y + y^2 = 0$ em ordem a x

$$\frac{d(x^3 + x^2y + y^2)}{dx} = 0 \Leftrightarrow 3x^2 + 2xy + x^2y' + 2yy' = 0 \Leftrightarrow x^2y' + 2yy' = -3x^2 - 2xy \Leftrightarrow$$

$$\Leftrightarrow (x^2 + 2y)y' = -3x^2 - 2xy \Leftrightarrow y' = -\frac{3x^2 + 2xy}{x^2 + 2y}$$

b) $ln(x) + e^{-\frac{y}{x}} = 2$

Derivamos ambos os membros da equação $ln(x) + e^{-\frac{y}{x}} = 2$ em ordem a x

$$\frac{d\left(ln(x) + e^{-\frac{y}{x}}\right)}{dx} = \frac{d(2)}{dx} \Leftrightarrow \frac{1}{x} + \left(-\frac{y}{x}\right)' e^{-\frac{y}{x}} = 0 \Leftrightarrow \frac{1}{x} - \frac{y'x - y}{x^2} e^{-\frac{y}{x}} = 0 \Leftrightarrow \frac{y'x - y}{x^2} e^{-\frac{y}{x}} = \frac{1}{x} \Leftrightarrow$$

$$\Leftrightarrow (y'x - y)e^{-\frac{y}{x}} = x \Leftrightarrow xy' - y = xe^{\frac{y}{x}} \Leftrightarrow xy' = xe^{\frac{y}{x}} + y \Leftrightarrow y' = e^{\frac{y}{x}} + \frac{y}{x}$$

c) $x^2 + y^2 - 4x - 10y + 4 = 0$

Derivamos ambos os membros da equação $x^2 + y^2 - 4x - 10y + 4 = 0$ em ordem a x

$$\frac{d(x^2 + y^2 - 4x - 10y + 4)}{dx} = 0 \Leftrightarrow 2x + 2yy' - 4 - 10y' = 0 \Leftrightarrow 2yy' - 10y' = 4 - 2x \Leftrightarrow$$

$$\Leftrightarrow yy' - 5y' = 2 - x \Leftrightarrow (y - 5)y' = 2 - x \Leftrightarrow y' = \frac{2 - x}{y - 5}$$

d) $xy^4 + xsen(y) = x^3 - y^2$

$$y^4 + 4xy^3y' + sen(y) + x\big(cos(y)\big)y' = 3x^2 - 2yy'$$

$$4xy^3y' + x\big(cos(y)\big)y' + 2yy' = 3x^2 - y^4 - sen(y)$$

$$\big(4xy^3 + x\big(cos(y)\big) + 2y\big)y' = 3x^2 - y^4 - sen(y)$$

$$y' = \frac{3x^2 - y^4 - sen(y)}{4xy^3 + x\big(cos(y)\big) + 2y}$$

e) $x^2y + 3sen^2(2y^2) + 3y = xy^2$

$$2xy + x^2y' + 6sen(2y^2)4yy'cos(2y^2) + 3y' = y^2 + 2xyy'$$

$$(-2xy + x^2 + 24ysen(2y^2)cos(2y^2) + 3)y' = y^2 - 2xy$$

$$y' = \frac{y^2 - 2xy}{x^2 + 24ysen(2y^2)cos(2y^2) + 3 - 2xy}$$

f) $xy^2 + 2y^3 = x - 2y$

$$y^2 + x2yy' + 6y^2y' = 1 - 2y'$$

$$x2yy' + 6y^2y' + 2y' = 1 - y^2$$

$$(2xy + 6y^2 + 2)y' = 1 - y^2$$

$$y' = \frac{1 - y^2}{2xy + 6y^2 + 2}$$

g) $2yx^2 + 2x^2 = x - y^2$

$$2y'x^2 + 4xy + 4x = 1 - 2yy'$$

h) $x^2y + e^xy^2 = 1$

$$2xy + x^2y' + e^xy^2 + 2ye^xy' = 0 \Leftrightarrow y'(x^2 + 2ye^x) = -2xy - e^xy^2$$

$$y' = -\frac{2xy + e^xy^2}{x^2 + 2ye^x}$$

i) $x^2y = y + e^ycos(x)$

$$2xy + x^2y' = y' + y'e^ycos(x) + e^y(-sen(x))$$

j) $xy = sin(y^2) + e^{1-2x}$

$$y + xy' = 2yy'cos(y^2) - 2\mathrm{e}^{1-2x}$$

k) $xy = y + ln\,(1 - 2x)$

$$xy + xy' = y' + \frac{-2}{1 - 2x}$$

$$\frac{dy}{dx} => \frac{d}{dx}(xy) = \frac{d}{dx}\left(y + \ln(1-2x)\right) \Leftrightarrow y + x\frac{dy}{dx} = \frac{dy}{dx} - \frac{2}{1-2x} \Leftrightarrow \frac{dy}{dx}(x-1) = -\frac{2}{1-2x} - y$$

$$\Leftrightarrow \frac{dy}{dx} = -\frac{2}{(1-2x)(x-1)} - \frac{y}{(x-1)}$$

14. Considere a função $y = f(x)$, representada por $f(x) = ln(x) + 1$ e determine a equação da reta tangente ao gráfico de $f(x)$, no ponto de abcissa 1.

$$y_0 = f(1) = ln(1) + 1 = 1$$

$$f'(x) = \frac{1}{x} \Longrightarrow m = f'(x) = \frac{1}{1} = 1$$

$$y - y_0 = m(x - x_0)$$

$$y - 1 = 1(x - 1)$$

$$y = x$$

15. Considere a função $y = f(x)$, representada por $f(x) = 3ln(1 - 2x)$, determine a equação da reta tangente ao gráfico de $f(x)$, no ponto de abcissa 0.

$$y_0 = f(0) = 3ln(1 - 0) = 0$$

$$f'(x) = 3\frac{(-2)}{1 - 2x} \Longrightarrow m = f'(0) = \frac{-6}{1} = -6$$

$$y - 0 = -6(x - 0)$$

$$y = -6x$$

16. Considere a função $y = f(x)$, representada por $f(x) = 1 - 2(x - 2)^2$ e determine a equação da reta tangente ao gráfico de $f(x)$, no ponto de abcissa 1.

$$y_0 = f(1) = 1 - 2(1 - 2)^2 = -1$$

$$f'(x) = -4(x - 2) \Longrightarrow f'(x) = -4(x - 2) = 4$$

$$y - (-1) = 4(x - 1)$$

$$y + 1 = 4(x - 1)$$

$$y = 4x - 5$$

17. Considere a função $y = f(x)$, representada por $f(x) = sen(x) + 3$, determine a equação da reta tangente ao gráfico de $f(x)$, no ponto de abcissa 0.

$$y_0 = f(0) = sen(0) + 3 = 3$$
$$f'(x) = cos(x) \Longrightarrow m = f'(0) = cos(0) = 1$$
$$y - 3 = 1(x - 0)$$
$$y = x + 3$$

18. Determine a equação da reta tangente à curva $f(x) = 2^{-x^2+2x}$ no ponto de abscissa $x = 0$.

$$y' = 2^{-x^2+2x}.\ln 2.(-2x+2)\Big|_{(0,1)} = 2^0 \ln 2.2 = 2\ln 2 = \ln 4$$
$$x = 0 \to y = 2^0 = 1 \leftrightarrow ponto = (0,1)$$
$$y - y_0 = m(x - x_0)$$
$$y - 1 = \ln 4(x - 0) \Rightarrow y = \ln 4.x + 1$$

19. Determine a equação da reta tangente à curva $f(x) = \frac{1}{2}\arcsin(1 + x)$ no ponto de abscissa $x = -1$.

$$x = -1 \to y = \frac{1}{2}\arcsin(1-1) = \frac{1}{2}\arcsin(0) = 0 \to ponto = (-1,0)$$
$$declive = m = y' = \frac{1}{2}\frac{1}{\sqrt{1-(x+1)^2}}\Big|_{(-1,0)} = \frac{1}{2}\frac{1}{\sqrt{1-(-1+1)^2}} = \frac{1}{2}$$
$$y - 0 = \frac{1}{2}(x - (-1)) \Rightarrow y = \frac{1}{2}x + \frac{1}{2}$$

20. Determine a equação da reta tangente à curva $f(x) = xe^x$ no ponto de abscissa x=0.

$$f(x) = xe^x$$
$$y_0 = f(0) = 0e^0 = 0$$
$$f'(x) = e^x + xe^x$$
$$m = f'(0) = e^0 + 0e^0 = 1$$
$$x_0 = 0 \qquad y_0 = 0 \qquad \text{m=1}$$
$$y - y_0 = m(x - x_0)$$
$$y - 0 = 1(x - 0)$$
$$y = x$$

21. Determine a equação da reta tangente à curva $f(x) = 2x^2 - 1$ no ponto de abscissa x=1.

$$y_0 = f(1) = 2(1)^2 - 1 = 1$$
$$x_0 = x = 1$$
$$f'(x) = 4x$$
$$y_0 = x_0 = 1$$
$$y - y_0 = m(x - x_0)$$
$$m = f'(x_0) = f'(1) = 4$$
$$y - 1 = 4(x - 1)$$
$$y = 4x - 3$$

22. Determine a equação da reta tangente à curva $f(x) = xe^x$ no ponto de abscissa x=0.

$$f(x) = xe^x$$
$$y_0 = f(0) = 0e^0 = 0$$
$$f'(x) = e^x + xe^x$$
$$m = f'(0) = e^0 + 0e^0 = 1$$
$$x_0 = 0 \qquad y_0 = 0 \qquad \text{m=1}$$
$$y - y_0 = m(x - x_0)$$
$$y - 0 = 1(x - 0)$$
$$y = x$$