Proposta de resolução dos exercícios, podem ter erros, para comunicarem qualquer erro enviem um email para epf@isep.ipp.pt

Integrais Imediatos – (a,b,c são constantes, $C\epsilon R$)

1) $\int(3x^5+4x+8)dx = 3\int x^5dx + 4\int xdx + \int 8dx = 3\frac{x^6}{6} + 4\frac{x^2}{2} + 8x + C = \frac{x^6}{2} + 2x^2 + 8x + C$

2) $\int\frac{dx}{x^3} = \int x^{-3}dx = \frac{x^{-3+1}}{-3+1} + C = \frac{x^{-2}}{-2} + C = -\frac{1}{2x^2} + C$

3) $\int ax^4dx = a\int x^4dx = a\frac{x^{4+1}}{4+1} + C = \frac{1}{5}ax^5 + C$

4) $\int\left(\frac{2a}{x} - \frac{b}{x^2} + 3c\right)dx = \int\frac{2a}{x}dx + \int -\frac{b}{x^2}dx + \int 3c\,dx = 2a\int\frac{1}{x}dx - b\int x^{-2}\,dx + 3c\int 1\,dx =$

$$= 2a\,ln|x| - b\frac{x^{-2+1}}{-2+1} + 3cx + C = 2a\,ln|x| + \frac{b}{x} + 3cx + C$$

5) $\int(5x-2)^{235}dx = \frac{1}{5}\int 5(5x-2)^{235}dx = \frac{1}{5}\frac{(5x-2)^{235+1}}{(235+1)} + C = \frac{(5x-2)^{236}}{1180} + C$

$u = 5x-2$

$u' = 5$

6) $\int(8x+3)^{10}dx = \frac{1}{8}\int 8(8x+3)^{10}dx = \frac{1}{8}\frac{(8x+3)^{11}}{11} + C = \frac{(8x+3)^{11}}{88} + C$

$u = 8x+3$

$u' = 8$

7) $(a-b)^2 = a^2 - 2ab + b^2$

$$\int(2x^3-4)^2dx = \int(2x^3)^2 - 2(2x^3)4 + 4^2dx = \int 4x^6 - 16x^3 + 16dx =$$

$$= 4\int x^6dx - 16\int x^3dx + \int 16dx = 4\frac{x^{6+1}}{6+1} - 16\frac{x^{3+1}}{3+1} + 16x + C = 4\frac{x^7}{7} - 4x^4 + 16x + C$$

$$\int(2x^3-4)^2dx = \frac{1}{6x^2}\int 6x^2(2x^3-4)^2dx = \textbf{ESTÁ MAL} - \textbf{ERRO GRAVE}$$

NÃO PODE MULTIPLICAR O INTEGRAL POR UMA VARIÁVEL

8) $\int(6x-2)^{25}dx = \frac{1}{6}\int 6(6x-2)^{25}dx = \frac{(6x-2)^{26}}{156} + C$

9) $\int x^2 + \frac{1}{x^2}dx = \int(x^2 + x^{-2})dx = \frac{x^3}{3} + \frac{x^{-2+1}}{-2+1} + C = \frac{x^3}{3} - \frac{1}{x} + C$

10) $\int\sqrt{x} + \frac{1}{\sqrt{x}}dx = \int x^{\frac{1}{2}} + x^{-\frac{1}{2}}\,dx = \frac{x^{\frac{1}{2}+1}}{\frac{1}{2}+1} + \frac{x^{-\frac{1}{2}+1}}{-\frac{1}{2}+1} + C = \frac{x^{\frac{3}{2}}}{\frac{3}{2}} + \frac{x^{\frac{1}{2}}}{\frac{1}{2}} + C = \frac{2}{3}\sqrt{x^3} + 2\sqrt{x} + C$

11) $\int\sqrt{2+5x}dx = \int(2+5x)^{\frac{1}{2}}dx = \frac{1}{5}\int 5(2+5x)^{\frac{1}{2}}dx = \frac{1}{5}\frac{(2+5x)^{\frac{1}{2}+1}}{\left(\frac{1}{2}+1\right)} + C = \frac{1}{5}\frac{2}{3}\sqrt{(2+5x)^3} + C = \frac{2}{15}\sqrt{(2+5x)^3} + C$

12) $\int \frac{2}{t^2}dt = 2\int t^{-2}dt = 2\frac{t^{-2+1}}{-2+1} + C = -\frac{2}{t} + C$

13) $\int (1-x)\sqrt{x}dx = \int (\sqrt{x} - x\sqrt{x})dx = \int (x^{\frac{1}{2}} - x^{\frac{3}{2}})dx = \frac{x^{\frac{1}{2}+1}}{\frac{1}{2}+1} - \frac{x^{\frac{3}{2}+1}}{\frac{3}{2}+1} + C = \frac{2}{3}\sqrt{x^3} - \frac{2}{5}\sqrt{x^5} + C$

14) $\int (2t^2+1)^2dt = \int (4t^4 + 4t^2 + 1)dt = \frac{4}{5}t^5 + \frac{4}{3}t^3 + t + C$

15)

$$\int \frac{x^3 - x^2 - 3}{x^2}dx = \int \frac{x^3}{x^2} + \frac{(-x^2)}{x^2} + \frac{(-3)}{x^2}dx = \int x - 1 - 3x^{-2}dx = \frac{x^2}{2} - x - 3\frac{x^{-2+1}}{-2+1} + C =$$
$$= \frac{x^2}{2} - x - 3\frac{x^{-1}}{-1} + C = \frac{x^2}{2} - x + 3x^{-1} + C = \frac{x^2}{2} - x + 3\frac{1}{x} + C = \frac{x^3 - 2x^2 + 6}{2x} + C$$

16) $\int \frac{x^3+5x^2-4}{x^2}dx = \int x + 5 - \frac{4}{x^2}dx = \frac{x^2}{2} + 5x - \frac{4x^{-2+1}}{-2+1} = \frac{x^2}{2} + 5x + \frac{4}{x} = \frac{x^3+10x^2+8}{2x} + C$

17) $\int \frac{t^3}{\sqrt{a^4+t^4}}dt = \int t^3(a^4+t^4)^{-\frac{1}{2}}dt = \frac{1}{4}\int 4t^3(a^4+t^4)^{-\frac{1}{2}}dt = \frac{1}{4}\frac{(a^4+t^4)^{-\frac{1}{2}+1}}{-\frac{1}{2}+1} + \mathrm{C} = \frac{1}{2}\sqrt{a^4+t^4} + C$

$u = a^4 + t^4;$ $\qquad u' = 4t^3$

18)

$$\int \frac{(a^{\frac{2}{3}} - x^{\frac{2}{3}})^3}{x^{\frac{1}{3}}}dx = \int x^{-\frac{1}{3}}(a^{\frac{2}{3}} - x^{\frac{2}{3}})^3dx = -\frac{3}{2}\int -\frac{2}{3}x^{-\frac{1}{3}}(a^{\frac{2}{3}} - x^{\frac{2}{3}})^3dx = -\frac{3}{2}\frac{(a^{\frac{2}{3}} - x^{\frac{2}{3}})^4}{4} = -\frac{3}{8}(a^{\frac{2}{3}} - x^{\frac{2}{3}})^4 + C$$

$u = a^{\frac{2}{3}} - x^{\frac{2}{3}}$

$u' = -\frac{2}{3}x^{\frac{2}{3}-1} = -\frac{2}{3}x^{-\frac{1}{3}}$

19) $\int 3ay^2dy = a\int 3y^2dy = ay^3 + C$

20) $\int \sqrt{ax}dx = \int \sqrt{a}\sqrt{x}dx = \sqrt{a}\int \sqrt{x}dx = \frac{2x\sqrt{ax}}{3} + C$

$\int \sqrt{ax}dx = \int \sqrt{a}\sqrt{x}dx = \sqrt{a}\int x^{\frac{1}{2}}dx = \sqrt{a}\frac{x^{\frac{1}{2}+1}}{\frac{1}{2}+1} + C = \sqrt{a}\frac{x^{\frac{1}{2}}x}{\frac{3}{2}} + C = \frac{2}{3}x\sqrt{a}\sqrt{x} + C = \frac{2x\sqrt{ax}}{3} + C$

21)

$$\int (\sqrt{2x} + \frac{1}{\sqrt{2x}})dx = \int (\sqrt{2}\sqrt{x} + \frac{1}{\sqrt{2}}\frac{1}{\sqrt{x}})dx = \int \sqrt{2}\sqrt{x}\,dx + \int \frac{1}{\sqrt{2}}\frac{1}{\sqrt{x}}dx =$$
$$= \sqrt{2}\int \sqrt{x}\,dx + \frac{1}{\sqrt{2}}\int \frac{1}{\sqrt{x}}dx = \frac{1}{3}(2x)^{\frac{3}{2}} + (2x)^{\frac{1}{2}} + C$$

Ou

$$\sqrt{2}\int x^{\frac{1}{2}}dx + \frac{1}{\sqrt{2}}\int x^{-\frac{1}{2}}dx = \sqrt{2}\frac{x^{\frac{1}{2}+1}}{\frac{1}{2}+1} + \frac{1}{\sqrt{2}}\frac{x^{-\frac{1}{2}+1}}{-\frac{1}{2}+1} + C$$

Ou

$$\int (2x)^{\frac{1}{2}}dx + \int (2x)^{-\frac{1}{2}}dx = \frac{1}{2}\int 2(2x)^{\frac{1}{2}}dx + \frac{1}{2}\int 2(2x)^{-\frac{1}{2}}dx = \frac{1}{2}\frac{(2x)^{\frac{1}{2}+1}}{\frac{1}{2}+1} + \frac{1}{2}\frac{(2x)^{-\frac{1}{2}+1}}{-\frac{1}{2}+1}$$

22) $\int \frac{3x}{\sqrt{1-x^2}}dx = 3\int x(1-x^2)^{-\frac{1}{2}}dx = \frac{3}{-2}\int -2x(1-x^2)^{-\frac{1}{2}}dx = -\frac{3}{2}\frac{(1-x^2)^{-\frac{1}{2}+1}}{-\frac{1}{2}+1} + C = -3\sqrt{1-x^2} + C$

23) $\int \frac{(a+bt)^2}{2}dt = \frac{1}{2}\int (a+bt)^2 dt = \frac{1}{2}\frac{1}{b}\int b(a+bt)^2 dt = \frac{(a+bt)^3}{6b} + C$

24)

$$\int \frac{(\sqrt{a}-\sqrt{x})^2}{\sqrt{x}}dx = -2\int -\frac{1}{2}\frac{1}{\sqrt{x}}(\sqrt{a}-\sqrt{x})^2 dx = -\frac{2}{3}(\sqrt{a}-\sqrt{x})^3 + C$$

$u = \sqrt{a}-\sqrt{x} = \sqrt{a} - x^{\frac{1}{2}}$

$u' = -\frac{1}{2}x^{-\frac{1}{2}} = -\frac{1}{2\sqrt{x}}$

ou

$$\int \frac{(\sqrt{a}-\sqrt{x})^2}{\sqrt{x}}dx = \int x^{-\frac{1}{2}}(a^{\frac{1}{2}} - x^{\frac{1}{2}})^2 dx = -2\int -\frac{1}{2}x^{-\frac{1}{2}}(a^{\frac{1}{2}} - x^{\frac{1}{2}})^2 dx = -2\frac{(a^{\frac{1}{2}} - x^{\frac{1}{2}})^{2+1}}{2+1} + C$$

$$= -\frac{2}{3}(\sqrt{a}-\sqrt{x})^3 + C$$

$u = a^{\frac{1}{2}} - x^{\frac{1}{2}}$

$u' = 0 - \frac{1}{2}x^{-\frac{1}{2}}$

25) $\int \sqrt{x}(3x-2)dx = \int (3x\sqrt{x} - 2\sqrt{x})dx = \int \left(3x^{\frac{3}{2}} - 2x^{\frac{1}{2}}\right)dx = \frac{3x^{\frac{3}{2}+1}}{\frac{3}{2}+1} - \frac{2x^{\frac{1}{2}+1}}{\frac{1}{2}+1} + C += \frac{6}{5}x^{\frac{5}{2}} - \frac{4}{3}x^{\frac{3}{2}} + C$

26)

$$\int t^2(1+2t^3)^{-\frac{2}{3}}dt = \frac{1}{6}\int 6t^2(1+2t^3)^{-\frac{2}{3}}dt = \frac{1}{2}(1+2t^3)^{\frac{1}{3}} + C$$

$u = 1 + 2t^3$

$u' = 6t^2$

27) $\int (\sqrt{a}-\sqrt{x})^2 dx = \int \left(a - 2\sqrt{a}\sqrt{x} + x\right)dx = ax - 2\sqrt{a}\frac{x^{\frac{1}{2}+1}}{\frac{1}{2}+1} + \frac{1}{2}x^2 + C = ax - 2\sqrt{a}\frac{x\sqrt{x}}{\frac{3}{2}} + \frac{1}{2}x^2 + C ==$

$= ax - \frac{4}{3}x\sqrt{ax} + \frac{1}{2}x^2 + C$ $\qquad \sqrt{a}\sqrt{x} = \sqrt{ax}$

28) $\int \sqrt[3]{1-x^2}\,x dx = \int x(1-x^2)^{\frac{1}{3}}dx = \frac{1}{-2}\int -2x(1-x^2)^{\frac{1}{3}}dx = -\frac{3}{8}(1-x^2)^{\frac{4}{3}} + C$

29) $\int \sqrt{x}(\sqrt{a}-\sqrt{x})^2dx = \int \sqrt{x}(a-2\sqrt{a}\sqrt{x}+x)dx = \int (a\sqrt{x}-2\sqrt{a}x+x\sqrt{x})dx = \frac{2}{3}ax^{\frac{3}{2}} - x^2\sqrt{a} + \frac{2}{5}x^{\frac{5}{2}} + C$

30) $\int \frac{x^2+1}{\sqrt{x^3+3x}}dx = \int (x^2+1)(x^3+3x)^{-\frac{1}{2}}dx = \frac{1}{3}\int 3(x^2+1)(x^3+3x)-^{\frac{1}{2}}dx =$

$$= \frac{1}{3}\int (3x^2+3)(x^3+3x)^{-\frac{1}{2}}dx = \frac{2}{3}\sqrt{x^3+3x} + C$$

$u = x^3+3x \qquad u' = 3x^2+3$

$$\ldots = \frac{1}{3}\frac{(x^3+3x)^{-\frac{1}{2}+1}}{-\frac{1}{2}+1} + C = \frac{1}{3}\frac{(x^3+3x)^{\frac{1}{2}}}{\frac{1}{2}} + C = \frac{2}{3}(x^3+3x)^{\frac{1}{2}} + C$$

31) $\int \frac{2+lnx}{x}dx = \int \frac{2}{x}dx + \int \frac{1}{x}(lnx)^1\,dx = 2ln|x| + \frac{ln^2|x|}{2} + C,$ $\qquad u = lnx \qquad u' = \frac{1}{x}$

Ou $\int \frac{2+lnx}{x}dx = \int \frac{1}{x}(2+lnx)^1dx = \frac{(2+lnx)^2}{2} + k = \frac{4+4lnx+ln^2x}{2} + k =$

$= 2 + 2ln|x| + \frac{ln^2|x|}{2} + k = 2ln|x| + \frac{ln^2|x|}{2} + C,$ $\qquad u = 2+lnx \qquad u' = \frac{1}{x}$

32) $\int (a^2+b^2x^2)^{-\frac{3}{2}}xdx = \frac{1}{2b^2}\int 2b^2x(a^2+b^2x^2)^{-\frac{3}{2}}dx = -\frac{1}{b^2\sqrt{a^2+b^2x^2}} + C$

33) $\int (2t^2+1)^2dt = \int (4t^4+4t^2+1)dt = \frac{4}{5}t^5 + \frac{4}{3}t^3 + t + C$

34) $\int a^{6x}dx = \frac{1}{6}\int 6a^{6x}dx = \frac{1}{6}\frac{a^{6x}}{ln\,a} + C = \frac{a^{6x}}{ln\,a^6} + C$

$u = 6x \qquad u' = 6$

35) $\int (e^x+1)^3e^x\,dx = \int e^x(e^x+1)^3\,dx = \frac{1}{4}(e^x+1)^4 + C$

$u = e^x+1 \qquad u' = e^x$

36) $\int 2^{2x}dx = \frac{1}{2}\int 2(2^{2x})dx = \frac{1}{2}\frac{2^{2x}}{ln\,2} + C = \frac{2^{2x}}{ln\,4} + C$

37) $\int a^{3x^2-6x}(x-1)dx = \frac{1}{6}\int 6(x-1)a^{3x^2-6x}dx = \frac{1}{6}\int (6x-6)a^{3x^2-6x}dx = \frac{a^{3x^2-6x}}{6\,ln\,a} + C$

$u = 3x^2-6x \qquad u' = 6x-6$

38) $\int e^{4x}dx = \frac{1}{4}\int 4e^{4x}dx = \frac{1}{4}e^{4x} + C$ $\qquad \int u'e^udx = e^u + C$

39) $\int e^{5x^2}xdx = \frac{1}{10}\int 10xe^{5x^2}dx = \frac{1}{10}e^{5x^2} + C$

40) $\int \frac{2x}{x^2+3}dx = ln(x^2+3) + C$ $\qquad \int \frac{u'}{u}dx = ln|u| + C$

$u = x^2+3 \qquad u' = 2x$

41) $\int \frac{x}{4x^2+1}dx = \frac{1}{8}\int \frac{8x}{4x^2+1}dx = \frac{1}{8}ln(4x^2+1) + C = ln\left((4x^2+1)^{\frac{1}{8}}\right) + C = ln\sqrt[8]{4x^2+1} + C$

$u = 4x^2+1 \qquad u' = 8x$

42) $\int \frac{x^2}{3x^3+4}dx = \frac{1}{9}\int \frac{9x^2}{3x^3+4}dx = \frac{1}{9}ln(3x^3+4)+C$

$u = 3x^3+4$

$u' = 9x^2$

43) $\int \frac{x}{x^2+4}dx = \frac{1}{2}\int \frac{2x}{x^2+4}dx = \frac{1}{2}ln(x^2+4)+C = ln\sqrt{x^2+4}+C$

$u = x^2+4$

$u' = 2x$

44) $\int \frac{e^x}{a+be^x}dx = \frac{1}{b}\int \frac{be^x}{a+be^x}dx = \frac{1}{b}ln|a+be^x|+C$

45) $\int \frac{x+2}{x+1}dx = \int \frac{x+1+1}{x+1}dx = \int \frac{x+1}{x+1}+\frac{1}{x+1}dx = \int 1+\frac{1}{x+1}dx = x+ln|x+1|+C$

$$\frac{x+2}{x+1} = \frac{x+1}{x+1}+\frac{1}{x+1} = 1+\frac{1}{x+1}$$

46) $\int \frac{x+4}{2x+3}dx = \frac{1}{2}\int \frac{x+4}{x+\frac{3}{2}}dx = \frac{1}{2}\int \frac{x+\frac{3}{2}}{x+\frac{3}{2}}+\frac{\frac{5}{2}}{x+\frac{3}{2}}dx = \frac{1}{2}\int 1+\frac{5}{2}\frac{1}{x+\frac{3}{2}}dx =$

$$= \frac{1}{2}\left(x+\frac{5}{2}ln\left|x+\frac{3}{2}\right|\right)+C = \frac{x}{2}+\frac{5}{4}ln\left|\frac{2x+3}{2}\right|+C = \frac{x}{2}+\frac{5}{4}(ln|2x+3|-ln\,2)+C_1 =$$

$$= \frac{x}{2}+\frac{5}{4}ln|2x+3|+C$$

ou

$$\frac{x+4}{2x+3} = \frac{1}{2}\left(\frac{2x+8}{2x+3}\right) = \frac{1}{2}\left(\frac{2x+3+5}{2x+3}\right) = \frac{1}{2}\left(\frac{2x+3}{2x+3}+\frac{5}{2x+3}\right) = \frac{1}{2}\left(1+\frac{5}{2x+3}\right)$$

$$\int \frac{1}{2}\left(1+\frac{5}{2x+3}\right)dx = \frac{1}{2}\left(\int 1dx+\int \frac{5}{2x+3}dx\right) = \frac{1}{2}\left(\int 1dx+\frac{1}{2}\int \frac{5}{2x+3}dx\right) = \frac{1}{2}x+\frac{5}{4}ln|2x+3|+C$$

$$\int \frac{5}{2x+3}dx = 5\int \frac{1}{2x+3}dx = 5\frac{1}{2}\int \frac{2}{2x+3}dx = \frac{5}{2}ln|2x+3|+C$$

47) $\int \frac{e^{2x}}{e^{2x}+1}dx = \frac{1}{2}\int \frac{2e^{2x}}{e^{2x}+1}dx = \frac{1}{2}ln(e^{2x}+1)+C$

48) $\int \frac{(e^x+2)}{e^x+2x}dx = ln|e^x+2x|+C$

49) $\int \frac{e^{\frac{1}{x}}}{x^2}dx = -\int -\frac{1}{x^2}e^{\frac{1}{x}}dx = -e^{\frac{1}{x}}+C$ $\qquad \int u'e^u du = e^u+C.$

$u = \frac{1}{x} = x^{-1}$ $\qquad u' = -1x^{-2}$ $\qquad u' = -\frac{1}{x^2}$

50) $\int (1+t^{-1})^2 t^{-2}dt = -\int -t^{-2}(1+t^{-1})^2 dt = -\frac{1}{3}\left(1+\frac{1}{t}\right)^3+C$

51) $\int \frac{\sqrt{1+\sqrt{x}}}{\sqrt{x}}dx = \int \frac{1}{\sqrt{x}}(1+\sqrt{x})^{\frac{1}{2}}dx = 2\int \frac{1}{2\sqrt{x}}(1+\sqrt{x})^{\frac{1}{2}}dx = 2\frac{(1+\sqrt{x})^{\frac{1}{2}+1}}{\frac{1}{2}+1} + C == 2\frac{(1+\sqrt{x})^{\frac{3}{2}}}{\frac{3}{2}} + C = \frac{4}{3}(1+\sqrt{x})^{\frac{3}{2}} + C$

$u = 1+\sqrt{x} = 1 + x^{\frac{1}{2}}$ $\quad u' = +\frac{1}{2}x^{-\frac{1}{2}} = \frac{1}{2\sqrt{x}}$

52) $\int xe^{-x^2}dx = -\frac{1}{2}\int -2xe^{-x^2}dx = -\frac{1}{2}e^{-x^2} + C$

53) $\int x(e^{x^2}+2)\,dx = \int (xe^{x^2}+2x)\,dx = \frac{1}{2}\int 2xe^{x^2}\,dx + \int 2x\,dx = \frac{1}{2}e^{x^2} + x^2 + C$

54) $\int a^x e^x dx = \int (ae)^x dx = \frac{(ae)^x}{ln(ae)} + C = \frac{a^x e^x}{ln\, e + ln\, a} + C = \frac{a^x e^x}{1+ln\, a} + C$

55) $\int sen(5x)dx = \frac{1}{5}\int 5sen(5x)dx = -\frac{1}{5}cos(5x) + C$

56) $\int x^3 sen(3x^4)dx = \frac{1}{12}\int 12x^3 sen(3x^4)dx = -\frac{1}{12}cos(3x^4) + C$

57) $\int sen(2x+1)dx = \frac{1}{2}\int 2sen(2x+1)dx = -\frac{1}{2}cos(\,2x+1) + C$

58) $\int \frac{sen\sqrt{x}}{\sqrt{x}}dx = 2\int \frac{1}{2\sqrt{x}}sen\sqrt{x}dx = -2\,cos\,\sqrt{x} + C$

$u = \sqrt{x} = x^{\frac{1}{2}}$ $\quad u' = \frac{1}{2}x^{-\frac{1}{2}} = \frac{1}{2\sqrt{x}}$

59) $\int cos(8x)\,dx = \frac{1}{8}\int 8\,cos(8x)\,dx = \frac{1}{8}sen(8x) + C$

60) $\int cos(\,2x+4)dx = \frac{1}{2}\int 2\,cos(\,2x+4)dx = \frac{1}{2}sen(2x+4) + C$

61) $\int \frac{cos\,\sqrt{x}}{\sqrt{x}}dx = 2\int \frac{1}{2\sqrt{x}}cos\,\sqrt{x}\,dx = 2sen\sqrt{x} + C$

$u = \sqrt{x} = x^{\frac{1}{2}}$

$u' = \frac{1}{2}x^{-\frac{1}{2}} = \frac{1}{2\sqrt{x}}$

62) $\int cos(\,3x-1)dx = \frac{1}{3}\int 3\,cos(\,3x-1)dx = \frac{1}{3}sen(3x-1) + C$

63) $\int x\,tg(\,5x^2)\,dx = -\frac{1}{10}\int -10x\frac{sen(5x^2)}{cos(5x^2)}\,dx = -\frac{1}{10}ln|cos(\,5x^2)| + C = \frac{1}{10}ln|cos^{-1}(\,5x^2)| + C = \frac{1}{10}ln|sec(\,5x^2)| + C$

64) $\int cot\,g\,(5x+2)dx = \frac{1}{5}\int 5\,cot\,g\,(5x+2)dx = \frac{1}{5}ln|sen(5x+2)| + C$

65) $\int cot\,g\,(5x)dx = \frac{1}{5}\int 5\,cot\,g\,(5x)dx = \frac{1}{5}ln|sen(5x)| + C$

66) $\int \frac{senx+cosx}{cos\,x}dx = \int \frac{senx}{cos\,x} + \frac{cosx}{cos\,x}dx = \int \left(\frac{senx}{cos\,x}+1\right)dx = -ln|cosx| + x + C = ln|secx| + x + C$

67) $\int \frac{dx}{sec\,x.senx} = \int \frac{dx}{\frac{1}{cos\,x}senx} = \int \frac{cos\,x}{senx}dx = \int cot\,g\,xdx = ln|senx| + C$

68) $\int sec(2x)\,dx = \frac{1}{2}\int 2\,sec(2x)\,dx = \frac{1}{2}ln|sec(2x)+tg(2x)|+C$

69) $\int \frac{sec\sqrt{x}}{\sqrt{x}}dx = 2\int \frac{1}{2\sqrt{x}}sec\sqrt{x}\,dx = 2\,ln\left|sec\sqrt{x}+tg\sqrt{x}\right|+C$

$$u=\sqrt{x}=x^{\frac{1}{2}}$$
$$u'=\frac{1}{2}x^{-\frac{1}{2}}=\frac{1}{2\sqrt{x}}$$

70) $\int cossec(3x)dx = \frac{1}{3}\int 3\,cossec(3x)dx = \frac{1}{3}ln|cossec\,3x - cot\,g\,3x|+C$

71) $\int sec^2(4x)\,dx = \frac{1}{4}\int 4\,sec^2(4x)\,dx = \frac{1}{4}tg(4x)+C$

72) $\int \frac{tg^2(x)}{sen^2(x)}dx = \int \frac{1}{sen^2(x)}tg^2(x)dx = \int \frac{1}{sen^2(x)}\frac{sen^2(x)}{cos^2(x)}dx = \int \frac{1}{cos^2(x)}dx == \int sec^2(x)\,dx = tg(x)+C$

73) $\int sec^2(2ax)\,dx = \frac{1}{2a}\int 2a\,sec^2(2ax)\,dx = \frac{1}{2a}tg(2ax)+C$

74) $\int \frac{dx}{cos^2(3x)} = \frac{1}{3}\int 3\,sec^2(3x)\,dx = \frac{1}{3}tg(3x)+C$

75) $\int \frac{dx}{1+cos\,x} = \int \frac{(1-cos\,x)}{(1+cos\,x)(1-cos\,x)}dx = \int \frac{1-cos\,x}{1-cos^2 x}dx = \int \frac{1-cos\,x}{sen^2 x}dx =$

$$= \int \frac{1}{sen^2 x} - \frac{cos\,x}{sen^2 x}dx = \int (cossec^2(x) - cot\,g\,(x)\ \ cossec(x))dx =$$
$$= -cot\,g\,(x) + cossec(x) + C$$

76) $\int \frac{dx}{sen^2(5x)} = \int cossec^2(5x)\,dx = \frac{1}{5}\int 5\,cossec^2(5x)\,dx = -\frac{1}{5}cot\,g\,(5x)+C$

77) $\int sec(2x).tg(2x)dx = \frac{1}{2}\int 2\,sec(2x).tg(2x)dx = \frac{1}{2}sec(2x)+C$

78) $\int (tg(2x)+sec(2x))^2dx = \int (tg^2(2x)+2tg(2x)sec(2x)+sec^2(2x))dx =$

$$= \int (sec^2(2x)-1+2tg(2x)sec(2x)+sec^2(2x))dx =$$
$$= \int (2sec^2(2x)-1+2tg(2x)sec(2x))dx = \ tg(2x)-x+\sec(2x)+C$$

Cálculos auxiliares $\int u'\sec u\ \ tg\,udu = \sec u + C.$

- $\int 2sec^2(2x)dx = \int 2sec^2(2x)dx = \ tg(2x)+C$
- $\int 2tg(2x)sec(2x))dx = \int 2\frac{sen(2x)}{cos(2x)}\frac{1}{cos(2x)}dx = \int 2\frac{sen(2x)}{cos^2(2x)}dx = -\int -2sen(2x)cos^{-2}(2x)dx =$

$$= -\frac{cos^{-1}(2x)}{-1} = \frac{1}{cos(2x)} = \sec(2x)+C$$

79) $\int (tg(2x)-1)^2dx = \int (tg^2(2x)-2tg(2x)+1)dx = \int (sec^2(2x)-1-2tg(2x)+1)dx =$

$$= \frac{1}{2}\int 2\,sec^2(2x)\,dx - \int 2tg(2x)dx = \frac{1}{2}tg(2x)+ln|cos(2x)|+C$$

80) $\int \frac{sen(4x)}{cos(2x)}dx = \int \frac{2sen(2x)\,cos(2x)}{cos(2x)}dx = \int 2sen(2x)dx = -cos(2x)+C$

$$sen(4x) = sen\big(2(2x)\big) = 2sen(2x)cos(2x)$$

81) $\int \frac{senx}{cos^2 x} dx = \int \frac{senx}{cos\, x} \frac{1}{cos\, x} dx = \int tgx\, sec\, x\, dx = sec\, x + C$

82) $\int cossec(2x)\, cot\, g\,(2x) dx = \frac{1}{2} \int 2\, cossec(2x)\, cot\, g\,(2x) dx = -\frac{1}{2} cossec(2x) + C$

83) $\int sen^2 x\, cos\, x\, dx = \int cos\, x\,(senx)^2 dx = \frac{1}{3} sen^3 x + C$

84) $\int \frac{1}{1-cos\, x} dx = \int \frac{(1+cos\, x)}{(1-cos\, x)(1+cos\, x)} dx = \int \frac{1+cos\, x}{1-cos^2 x} dx =$

$$= \int \frac{1+cos\, x}{sen^2 x} dx = \int \frac{1}{sen^2 x} dx + \int \frac{cos\, x}{sen^2 x} dx = \int cossec^2\, x\, dx + \int cos\, x\, sen^{-2} x\, dx =$$

$$= \int cossec^2\, x\, dx + \int cos\, x\, sen^{-2} x\, dx = -cot\, g\, x - \frac{1}{sen\, x} + C = -cot\, g\, x - cossec\, x + C$$

85) $\int \frac{1}{1+senx} dx = \int \frac{1-senx}{(1+senx)(1-senx)} dx = \int \frac{1-senx}{1-sen^2 x} dx = \int \frac{1-senx}{cos^2 x} dx =$

$$= \int \left(\frac{1}{cos^2 x} - \frac{senx}{cos^2 x} \right) dx = \int \frac{1}{cos^2 x} dx - \int \frac{senx}{cos^2 x} dx =$$

$$= \int sec^2\, x\, dx - \int \frac{1}{cos\, x} \frac{senx}{cos\, x} dx = \int sec^2\, x\, dx - \int sec\, x\, tgx\, dx = tgx - sec\, x + C$$

86) $\int \sqrt{1 - cos\, x}\,.\, senxdx = \int senx(1-cos\, x)^{\frac{1}{2}} dx = \frac{2}{3}(1-cos\, x)^{\frac{3}{2}} + C$

$u = 1 - cos\, x$

$u' = senx$

87) $\int \frac{cos(x)}{sen^3(x)} dx = \int cot\, g\,(x)\, cossec^2(x)\, dx = -\frac{1}{2} cossec^2(x) + C$

ou

$$\int \frac{cos(x)}{sen^3(x)} dx = \int cos(x)\, sen^{-3}(x)\, dx = \frac{sen^{-2}(x)}{-2} + C = -\frac{1}{2} \frac{1}{sen^2(x)} + C = -\frac{1}{2} cossec^2(x) + C$$

88) $\int e^{3\, cos(2x)} sen(2x)\, dx = -\frac{1}{6} \int -6sen(2x) e^{3\, cos(2x)}\, dx = -\frac{1}{6} e^{3\, cos(2x)} + C$

$u = 3\, cos(2x)$

$u' = -6sen(2x)$

89) $\int e^{senx} cos\, x\, dx = \int cos\, x\, e^{senx}\, dx = e^{senx} + C$

$u = senx$ $\qquad u' = cos\, x$

90) $\int e^{tgx} sec^2\, x\, dx = \int sec^2\, x\, e^{tgx}\, dx = e^{tgx} + C$

$u = tgx$ $\qquad u' = sec^2\, x$

91) $\int e^{cos(2x)} sen(2x)\, dx = -\frac{1}{2} \int -2sen(2x) e^{cos(2x)}\, dx = -\frac{1}{2} e^{cos(2x)} + C$

$u = cos(2x)$ $\qquad u' = -2sen(2x)$

92) $\int \frac{senx}{1-cos\, x} dx = ln|1 - cos\, x| + C$

$u = 1 - cos\, x$ $\qquad u' = senx$

93) $\int \frac{sec(2x)tg(2x)}{3\,sec(2x)-a}dx = \frac{1}{6}\int \frac{6\,sec(2x)tg(2x)}{3\,sec(2x)-a}dx = \frac{1}{6}ln|3\,sec\,2\,x - a| + C$

$u = 3\,sec(2x) - a$ $\quad\quad u' = 6\,sec(2x)\,tg(2x)$

94) $\int \frac{sec(x)tg(x)}{a+b\,sec(x)}dx = \frac{1}{b}\int \frac{b\,sec(x)tg(x)}{a+b\,sec(x)}dx = \frac{1}{b}ln|a + b\,sec(x)| + C$

$u = a + b\,sec(x)$ $\quad\quad u' = b\,sec(x)\,tg(x)$

95) $\int \frac{sec^2 x}{1+tgx}dx = ln|1 + tgx| + C$

$u = 1 + tgx$ $\quad\quad u' = sec^2 x$

96) $\int \frac{cos(ax)}{\sqrt{b+sen(ax)}}dx = \frac{1}{a}\int a\,cos(ax)\left(b + sen(ax)\right)^{-\frac{1}{2}}dx = \frac{1}{a}\frac{\left(b+sen(ax)\right)^{-\frac{1}{2}+1}}{-\frac{1}{2}+1} = \frac{2}{a}\sqrt{b + senax} + C$

$u = b + sen(ax)$ $\quad\quad u' = a\,cos(ax)$

97) $\int \frac{sec^2 x}{1+tg^2 x}dx = arctg(tgx) + C$

98) $\int \frac{(1+\sqrt{x})^3}{\sqrt{x}}dx = 2\int \frac{1}{2\sqrt{x}}(1 + \sqrt{x})^3\,dx = \frac{(1+\sqrt{x})^4}{2} + C$

$u = \sqrt{x} = x^{\frac{1}{2}}$ $\quad\quad u' = \frac{1}{2}x^{-\frac{1}{2}} = \frac{1}{2\sqrt{x}}$

99) $\int \frac{t-2}{(t^2-4t+3)^3}dt = \frac{1}{2}\int 2(t-2)(t^2 - 4t + 3)^{-3}\,dt =$

$$= \frac{1}{2}\int (4t-4)(t^2 - 4t + 3)^{-3}\,dt = \frac{-1}{4(t^2 - 4t + 3)^2} + C$$

$u = t^2 - 4t + 3$ $\quad\quad u' = 2t - 4$

100) $\int \frac{xdx}{x^4+3} = \int \frac{xdx}{(x^2)^2+\sqrt{3}^2} = \frac{1}{2\sqrt{3}}arctg\frac{x^2}{\sqrt{3}} + C = \frac{\sqrt{3}}{6}arctg\frac{x^2\sqrt{3}}{3} + C$

$\int \frac{u'}{u^2+a^2}dx = \frac{1}{a}arctg\frac{u}{a} + C$

101) $\int \frac{dx}{9x^2-16} = \frac{1}{24}(ln|3x-4| - ln|3x+4|) + C = \frac{1}{24}ln\left|\frac{3x-4}{3x+4}\right| + C$