1. Utilizando a sugestão de substituição indicada, determine:

a) $\int x\sqrt{8-x^2}\,dx$ fazendo $u=8-x^2$.

$$u=8-x^2$$
$$\frac{du}{dx}=-2x$$
$$du=-2xdx$$
$$dx=\frac{1}{-2x}du$$

$$\int x(8-x^2)^{\frac{1}{2}}dx=\int \cancel{x}u^{\frac{1}{2}}\frac{1}{-2\cancel{x}}du=\int u^{\frac{1}{2}}\frac{1}{-2}du=\frac{1}{-2}\int u^{\frac{1}{2}}du=\frac{1}{-2}\frac{u^{\frac{1}{2}+1}}{\frac{1}{2}+1}+C=\frac{1}{-3}(8-x^2)^{\frac{3}{2}}+C$$

b) $\int \frac{e^x}{(e^x+1)^4}dx$ fazendo $u=e^x+1$.

$$\int \frac{e^x}{(e^x+1)^4}dx$$

$$u=e^x+1$$

$$du=e^x dx$$

$$\int \frac{e^x}{(e^x+1)^4}dx=\int \frac{1}{u^4}du=\int u^{-4}du=\frac{u^{-3}}{-3}+C=\frac{(e^x+1)^{-3}}{-3}+C$$

c) $\int \frac{sen\,x}{2+\cos x}dx=$ fazendo $u=2+\cos x$.

$$u=2+\cos x$$
$$du=-senxdx$$
$$dx=\frac{1}{-senx}du$$

$$\int \frac{sen\,x}{2+\cos x}dx=\int \frac{\cancel{sen\,x}}{u}\frac{1}{-\cancel{senx}}du=\int -\frac{1}{u}du=-\,ln|u|+C=-ln|2+\cos x|+C$$

d) $\int \frac{x^2}{\sqrt{x^3+9}}dx$ fazendo $u=x^3+9$.

$$u=x^3+9$$
$$\frac{du}{dx}=3x^2$$
$$du=3x^2dx$$
$$dx=\frac{1}{3x^2}du$$

$$\int \frac{x^2}{\sqrt{x^3+9}}dx=\int \frac{x^2}{\sqrt{u}}\frac{1}{3x^2}du=\int \frac{1}{\sqrt{u}}\frac{1}{3}du=\frac{1}{3}\int \frac{1}{\sqrt{u}}du=\frac{1}{3}\int u^{-\frac{1}{2}}du=\frac{1}{3}\frac{u^{-\frac{1}{2}+1}}{-\frac{1}{2}+1}+C=\frac{2}{3}(x^3+9)^{\frac{1}{2}}+C$$

e) $\int \frac{x}{x^2+1} dx$ fazendo $u = x^2 + 1$.

$$\int \frac{x}{x^2+1} dx = \frac{1}{2} \int \frac{2x}{x^2+1} dx = \frac{1}{2} \int \frac{1}{u} du = \frac{1}{2} \ln|u| + C =$$

$$u = x^2 + 1$$

$$du = 2xdx$$

$$\int \frac{x}{x^2+1} dx = \frac{1}{2} \ln\left|x^2+1\right| + C$$

f) $\int \frac{x}{\sqrt{2x-1}} dx$ $\qquad$ $u = \sqrt{2x-1}$

$u = \sqrt{2x-1}$ $\qquad$ $2x - 1 = u^2$ $\qquad$ $x = \frac{u^2+1}{2} = \frac{u^2}{2} + \frac{1}{2}$ $\qquad$ $dx = udu$

$$\int \frac{x}{\sqrt{2x-1}} dx = \int \frac{\frac{u^2+1}{2}}{u} udu = \int \frac{u^2+1}{2u} udu = \frac{1}{2} \int (u^2+1) du = \frac{1}{2}\left(\frac{u^3}{3} + u\right) + C$$

Ou

$u = \sqrt{2x-1}$ $\qquad$ $du = \frac{2}{2\sqrt{2x-1}} = \frac{1}{\sqrt{2x-1}} dx$ $\qquad$ $dx = \sqrt{2x-1} du = udu$

$u = \sqrt{2x-1}$ $\qquad$ $u^2 = 2x - 1$ $\qquad$ $x = \frac{u^2+1}{2} = \frac{u^2}{2} + \frac{1}{2}$

$$\int \frac{x}{\sqrt{2x-1}} dx = \int \frac{\frac{u^2}{2} + \frac{1}{2}}{u} udu = \frac{1}{2} \int (u^2+1) du = \frac{1}{2}\left(\frac{u^3}{3} + u\right) + C$$

Logo,

$$\int \frac{x}{\sqrt{2x-1}} dx = \frac{1}{2}\left(\frac{\left(\sqrt{2x-1}\right)^3}{3} + \sqrt{2x-1}\right) + C$$

g) $\int \frac{x^3}{\sqrt{1+x^2}} dx$ fazendo $u = 1 + x^2$

Substituição incompleta: $\int \frac{x^3}{\sqrt{u}} \frac{1}{2x} du = \int \frac{x^2}{2\sqrt{u}} du$

$u = 1 + x^2$ $\qquad$ $x^2 = u - 1$

$\frac{du}{dx} = 2x$ $\qquad$ $du = 2xdx$ $\qquad$ $dx = \frac{1}{2x} du$

$$\int \frac{x^3}{\sqrt{1+x^2}} dx = \int \frac{x^2 x}{\sqrt{1+x^2}} dx = \int \frac{(u-1)x}{\sqrt{u}} \frac{1}{2x} du = \frac{1}{2} \int \frac{u-1}{\sqrt{u}} du =$$

$$= \frac{1}{2} \int \left(\frac{u}{\sqrt{u}} - \frac{1}{\sqrt{u}}\right) du = \frac{1}{2} \int \left(u^{\frac{1}{2}} - u^{-\frac{1}{2}}\right) du = \frac{1}{2}\left(\frac{u^{\frac{3}{2}}}{\frac{3}{2}} - \frac{u^{\frac{1}{2}}}{\frac{1}{2}}\right) + C =$$

$$= \frac{u^{\frac{3}{2}}}{3} - u^{\frac{1}{2}} + C$$

Logo, $\int \frac{x^3}{\sqrt{1+x^2}} dx = \frac{(1+x^2)^{\frac{3}{2}}}{3} - (1+x^2)^{\frac{1}{2}} + C$

h) $\int \frac{ln^2(x)-1}{x(ln(x)+1)}dx$ fazendo $u = ln(x)$

$u = ln(x)$

$\frac{du}{dx} = \frac{1}{x}$ $\qquad du = \frac{1}{x}dx$ $\qquad dx = x\,du$

$$\int \frac{ln^2(x)-1}{x(ln(x)+1)}dx = \int \frac{u^2-1}{x(u+1)}xdu = \int \frac{u^2-1}{u+1}du = \int \frac{(u+1)(u-1)}{u+1}du = \int (u-1)du = \frac{u^2}{2} - u + C$$

Logo, $\int \frac{ln^2(x)-1}{x(ln(x)+1)}dx = \frac{ln^2(x)}{2} - ln(x) + C$

i) $\int \frac{\sqrt{x^3-1}}{x^{-2}}dx$ fazendo $u = x^3 - 1$

$u = x^3 - 1$ $\qquad \frac{du}{dx} = 3x^2$ $\qquad du = 3x^2dx$ $\qquad dx = \frac{1}{3x^2}du$

$$\int \frac{\sqrt{x^3-1}}{x^{-2}}dx = \int \frac{\sqrt{u}}{x^{-2}}\frac{1}{3x^2}du = \int \frac{\sqrt{u}}{3}du = \frac{1}{3}\int u^{\frac{1}{2}}\,du = \frac{1}{3}\frac{u^{\frac{3}{2}}}{\frac{3}{2}} + C = \frac{2u^{\frac{3}{2}}}{9} + C$$

Logo,

$$\int \frac{\sqrt{x^3-1}}{x^{-2}}dx = = \frac{2(x^3-1)^{\frac{3}{2}}}{9} + C$$

j) $\int \frac{e^{2x}}{(e^x+1)^{-2}}dx$ fazendo $u = e^x$

$u = e^x$ $\qquad \frac{du}{dx} = e^x$ $\qquad du = e^x dx$ $\qquad dx = \frac{1}{e^x}du = \frac{1}{u}du$

$$\int \frac{(e^x)^2}{(e^x+1)^{-2}}dx = \int \frac{u^2}{(u+1)^{-2}}\frac{1}{u}du = \int \frac{u}{(u+1)^{-2}}du = \int u(u+1)^2du =$$

$$= \int u(u^2+2u+1)du = \int (u^3 + 2u^2 + u)du = \frac{u^4}{4} + 2\frac{u^3}{3} + \frac{u^2}{2} + C$$

Logo, $\int \frac{e^{2x}}{(e^x+1)^{-2}}dx = \frac{e^{4x}}{4} + 2\frac{e^{3x}}{3} + \frac{e^{2x}}{2} + C$

k) $\int \frac{sen(2x)}{\sqrt{1+sen^2(x)}}dx$ fazendo $u = sen(x)$

$u = sen(x)$

$\frac{du}{dx} = cos(x)$

$du = cos(x)dx$

$dx = \frac{1}{cos(x)}du$

$$\int \frac{sen(2x)}{\sqrt{1+sen^2(x)}}dx = \int \frac{2sen(x)cos(x)}{\sqrt{1+sen^2(x)}}dx = \int \frac{2ucos(x)}{\sqrt{1+u^2}}\frac{1}{cos(x)}du = \int \frac{2u}{\sqrt{1+u^2}}du =$$

$= \int 2u(1+u^2)^{-\frac{1}{2}}du = \frac{(1+u^2)^{\frac{1}{2}}}{\frac{1}{2}}$+C

Logo,

$$\int \frac{sen(2x)}{\sqrt{1+sen^2(x)}}dx = 2\sqrt{1+sen^2(x)} + C$$

l) $\int \frac{x^5}{\sqrt{1-x^2}}dx$ fazendo $u = \sqrt{1-x^2}$

$u = \sqrt{1-x^2}$ $\qquad 1 - x^2 = u^2$ $\qquad x^2 = 1 - u^2$ $\qquad x^4 = (1-u^2)^2$

$\frac{du}{dx} = \frac{-2x}{2\sqrt{1-x^2}}$ $\qquad dx = \frac{\sqrt{1-x^2}}{-x}du$

$$\int \frac{x^5}{\sqrt{1-x^2}}dx = \int \frac{x^4 x}{\sqrt{1-x^2}}dx = \int \frac{(1-u^2)^2 x}{\sqrt{1-x^2}} \frac{\sqrt{1-x^2}}{-x}du = \int -(1-u^2)^2 du$$

$$= -\int (u^4 - 2u^2 + 1)du = -\left(\frac{u^5}{5} - 2\frac{u^3}{3} + u\right) + C$$

Logo,

$$\int \frac{x^5}{\sqrt{1-x^2}}dx = -\frac{\left(\sqrt{1-x^2}\right)^5}{5} + 2\frac{\left(\sqrt{1-x^2}\right)^3}{3} - \sqrt{1-x^2} + C$$

m) $\int \sqrt{1-(x-1)^2}dx$ fazendo $x - 1 = sen(t)$

$x - 1 = sen(t)$, $\quad x = 1 + sen(t)$, $\quad dx = cos(t)dt$

$$\int \sqrt{1-(x-1)^2}dx = \int \sqrt{1-\left(sen(t)\right)^2}\, cos(t)dt = \int \sqrt{cos^2(t)}\, cos(t)\, dt =$$

$$= \int cos(t)\ cos(t)\ dt = \int cos^2(t)dt = \int \frac{1}{2}\left(1 + cos(2t)\right)dt =$$

$$= \frac{1}{2}\int 1 + cos(2t)dt = \frac{1}{2}\left(\int 1\, dt + \frac{1}{2}\int 2cos(2t)dt\right) = \frac{1}{2}\left(t + \frac{1}{2}sen(2t)\right) + C$$

$$= \frac{1}{2}arcsen(x-1) + \frac{1}{4}2(x-1)\sqrt{1-(x-1)^2} + C =$$

$$= \frac{1}{2}arcsen(x-1) + \frac{1}{2}(x-1)\sqrt{1-(x-1)^2} + C$$

Cálculos auxiliares:

$$sen^2(t) + cos^2(t) = 1 \Leftrightarrow cos^2(t) = 1 - sen^2(t)$$

$$cos^2(t) = \frac{1 + cos(2t)}{2} = \frac{1}{2}\left(1 + cos(2t)\right)$$

$$sen(t) = x - 1 \Leftrightarrow t = arcsen(x-1)$$

$$sen(2t) = 2sent\ cost = 2sent\sqrt{1 - sen^2t} = 2(x-1)\sqrt{1-(x-1)^2}$$

$$cos(t) = \sqrt{1 - sen^2t}$$