First Assignment

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February 3, 2025

1. $\lim_{n\to\infty} \frac{1}{n}\left[\tan\left(\frac{\pi}{4n}\right) + \tan\left(\frac{2\pi}{4n}\right) + \cdots + \tan\left(\frac{n\pi}{4n}\right)\right] = \frac{2}{\pi}\log 2$

2. $\lim_{n\to\infty} \left[\left(1+\frac{1^2}{n^2}\right)^{\frac{2}{n^2}} \left(1+\frac{2^2}{n^2}\right)^{\frac{4}{n^2}} \left(1+\frac{3^2}{n^2}\right)^{\frac{6}{n^2}} \cdots \left(1+\frac{n^2}{n^2}\right)^{\frac{2n}{n^2}}\right]$

3. $\lim_{n\to\infty} \frac{\pi}{2n}\left[1 + \cos\frac{\pi}{2n} + \cos\frac{2\pi}{2n} + \cdots + \cos\frac{(n-1)\pi}{2n}\right]$

4. $\lim_{n\to\infty} \left[\frac{1}{\sqrt{n^2-1^2}} + \frac{1}{\sqrt{n^2-2^2}} + \frac{1}{\sqrt{n^2-3^2}} + \cdots + \frac{1}{\sqrt{n^2-(n-1)^2}}\right]$

5. $\lim_{n\to\infty} \left[\frac{1}{n} + \frac{\sqrt{n^2-1^2}}{n^2} + \frac{\sqrt{n^2-2^2}}{n^2} + \cdots + \frac{\sqrt{n^2-(n-1)^2}}{n^2}\right]$

6. Evaluate $\lim_{n\to\infty} \frac{1^9+2^9+3^9+\cdots+n^9}{n^{10}}$.

7. Evaluate $\lim_{n\to\infty} \frac{1^m+2^m+3^m+\cdots+n^m}{n^{m+1}} \quad (m > -1).$

8. Evaluate $\lim_{n\to\infty} \frac{3}{n}\sum_{k=1}^{n}\left[\left(\frac{3k}{n}\right)^2 - 1\right]$.

9. Evaluate $\lim_{n\to\infty} \left[\frac{1}{n} + \frac{n^2}{(n+1)^3} + \frac{n^2}{(n+2^3)} + \cdots + \frac{1}{8n}\right]$.

10. Prove that , $(b-a)\sec^2 a < \tan b - \tan a < (b-a)\sec^2 b$ where $0 < a < b < \frac{\pi}{2}$.

11. $\int_0^2 x^2 \, dx$

12. **Cauchy-Schwarz Inequality for Integrals**

$$\left(\int_a^b (f(x))^2 \, dx\right) \cdot \left(\int_a^b (g(x))^2 \, dx\right) \geq \left(\int_a^b g(x)f(x) \, dx\right)^2$$

Function	**Type**
e^x	Increasing ($\uparrow$)
$-x$	Decreasing ($\downarrow$)