

First Assignment

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today

1. $\lim_{n \rightarrow \infty} \frac{1}{n} \left[\tan \left(\frac{\pi}{4n} \right) + \tan \left(\frac{2\pi}{4n} \right) + \cdots + \tan \left(\frac{n\pi}{4n} \right) \right] = \frac{2}{\pi} \log 2$

2. $\lim_{n \rightarrow \infty} \left[\left(1 + \frac{1^2}{n^2} \right)^{\frac{2}{n^2}} \left(1 + \frac{2^2}{n^2} \right)^{\frac{4}{n^2}} \left(1 + \frac{3^2}{n^2} \right)^{\frac{6}{n^2}} \cdots \left(1 + \frac{n^2}{n^2} \right)^{\frac{2n}{n^2}} \right]$

3. $\lim_{n \rightarrow \infty} \frac{\pi}{2n} \left[1 + \cos \frac{\pi}{2n} + \cos \frac{2\pi}{2n} + \cdots + \cos \frac{(n-1)\pi}{2n} \right]$

4. $\lim_{n \rightarrow \infty} \left[\frac{1}{\sqrt{n^2 - 1^2}} + \frac{1}{\sqrt{n^2 - 2^2}} + \frac{1}{\sqrt{n^2 - 3^2}} + \cdots + \frac{1}{\sqrt{n^2 - (n-1)^2}} \right]$

5. $\lim_{n \rightarrow \infty} \left[\frac{1}{n} + \frac{\sqrt{n^2 - 1^2}}{n^2} + \frac{\sqrt{n^2 - 2^2}}{n^2} + \cdots + \frac{\sqrt{n^2 - (n-1)^2}}{n^2} \right]$

6. Evaluate $\lim_{n \rightarrow \infty} \frac{1^9 + 2^9 + 3^9 + \cdots + n^9}{n^{10}}$.

7. Evaluate $\lim_{n \rightarrow \infty} \frac{1^m + 2^m + 3^m + \cdots + n^m}{n^{m+1}}$ ($m > -1$).

8. Evaluate $\lim_{n \rightarrow \infty} \frac{3}{n} \sum_{k=1}^n \left[\left(\frac{3k}{n} \right)^2 - 1 \right]$.

9. Evaluate $\lim_{n \rightarrow \infty} \left[\frac{1}{n} + \frac{n^2}{(n+1)^3} + \frac{n^2}{(n+2)^3} + \cdots + \frac{1}{8n} \right]$.

10. Prove that , $(b-a) \sec^2 a < \tan b - \tan a < (b-a) \sec^2 b$ where $0 < a < b < \frac{\pi}{2}$.

11. $\int_0^2 x^2 dx$

12. Cauchy-Schwarz Inequality for Integrals

$$\left(\int_a^b (f(x))^2 dx \right) \cdot \left(\int_a^b (g(x))^2 dx \right) \geq \left(\int_a^b g(x)f(x) dx \right)^2$$

Function	Type
e^x	Increasing (\uparrow)
$-x$	Decreasing (\downarrow)