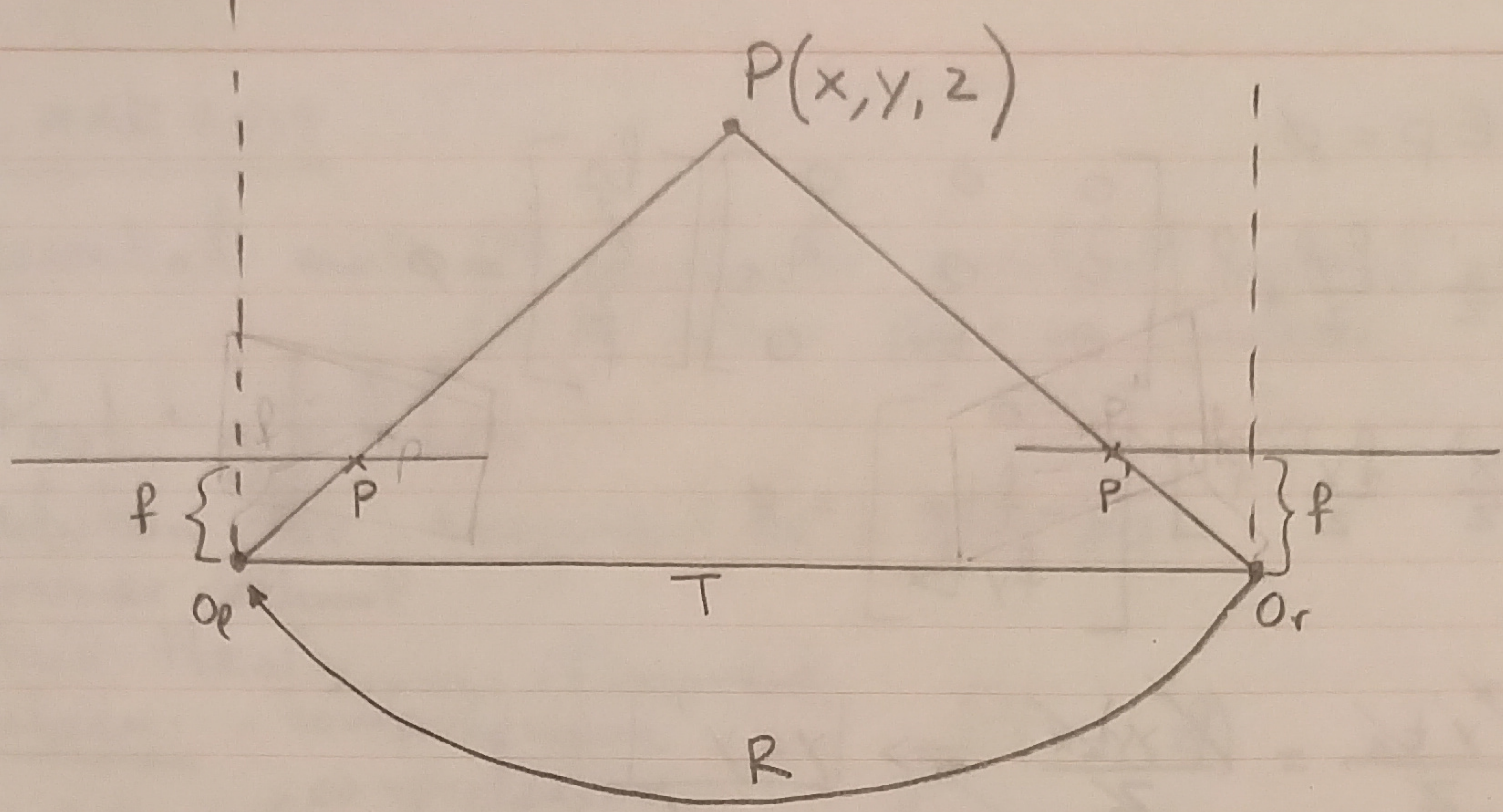
ESSENTIAL MATRIX: SPECIAL CASE - PARALLEL OPTICAL AXES



SIDE NOTE: A cross-product can be expressed as a matrix multiplication. For example: $\vec{a} \times \vec{b}$ is the same as $[a_\times]b$ where

$$[a_\times] = \begin{bmatrix} 0 & -a_3 & a_2 \\ a_3 & 0 & -a_1 \\ -a_2 & a_1 & 0 \end{bmatrix}$$

$T = [t_x, 0, 0]$ //second camera is shifted by t_x on the x-axis

$$R = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$ // parallel optical axes

$$[T_\times] = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -t_x \\ 0 & t_x & 0 \end{bmatrix}$$

$$E = [T_\times]R = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -t_x \\ 0 & t_x & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -t_x \\ 0 & t_x & 0 \end{bmatrix}$$

The projected point p is $\left[\frac{fx}{z} \ \frac{fy}{z} \ f\right]$. The projected point p' is $\left[\frac{fx'}{z} \ \frac{fy'}{z} \ f\right]$

$$p'Ep = \varnothing \Rightarrow \left[\frac{fx'}{z} \ \frac{fy'}{z} \ f\right] \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -t_x \\ 0 & t_x & 0 \end{bmatrix} \begin{bmatrix} \frac{fx}{z} \\ \frac{fy}{z} \\ f \end{bmatrix} = \varnothing$$

$$\Rightarrow \left[\frac{fx'}{z} \ \frac{fy'}{z} \ f\right] \begin{bmatrix} 0 \\ -ft_x \\ \frac{f.t_x.y}{z} \end{bmatrix} = \varnothing \Rightarrow -\frac{f^2 y' t_x}{z} + \frac{f^2 y t_x}{z} = \varnothing \Rightarrow \frac{\cancel{f^2} y \cancel{t_x}}{\cancel{z}} = \frac{\cancel{f^2} y' \cancel{t_x}}{\cancel{z}}$$

$$\Rightarrow \boxed{y' = y}$$

Every point in the left image will appear on the same scanline in the right image and vice-versa.