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[120BM0014]

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ASSIGNMENT

1) $P = 100 - 0.5Q$

$Q = q_1 + q_2$;

Cost functions: $C_1 = 5q_1$ $C_2 = 0.5q_2^2$

So, $Q = \frac{100 - P}{0.5} = 200 - 2P$

$\Rightarrow q_1 + q_2 = 200 - 2P$

$R = (100 - 0.5Q)(q_1 + q_2)$

$= (100 - \frac{Q}{2})Q$

a) Monopoly

In a molopoly 1 quantity is the whole Q.

So, $Q = q_1$ & $q_2 = 0$

So, $MR = \frac{dR}{dQ} = \frac{d(100Q - Q^2/2)}{dQ} = 100 - Q$

So, $Q = 60$ units.

$P = 100 - 60 = 40$ units.

~~Profit~~ ~~$\frac{B}{Q}$~~ :

b) Cournot Duopoly.

$$P = 100 - Q/2$$

$$R = P \cdot Q = \left(100 - \frac{Q}{2}\right)Q$$

$$= 100Q - \frac{Q^2}{2}$$

$$= 100(q_1+q_2) - \frac{(q_1+q_2)^2}{2}$$

for, $R_1 = 100q_1 - \left(\frac{q_1+q_2}{2}\right)q_1$

$$MR_1 = 100 - q_1 - \frac{q_2}{2} = 0$$

$$\Rightarrow \frac{2q_1+q_2}{2} = 100$$

$$\Rightarrow q_1 = 100 - \frac{q_2}{2} \quad \text{--- (I)}$$

By $MR_2 \longrightarrow \Rightarrow q_2 = 100 - \frac{q_1}{2}$ --- (II)

So, $q_2 = 100 - \frac{q_1}{2}$

$$= 50 - \frac{q_2}{4}$$

$$\Rightarrow q_2 = \frac{200}{3}$$

$$q_1 = 100 - \frac{100}{3}$$

$$= \frac{200}{3}$$

So, $q_1 = q_2$

$$R = 100\, q_1^2 - \frac{2q_1^2}{2}$$

$$= 100 \times \frac{200}{3} \times \frac{200}{3} - 2 \times \frac{200}{3} \times \frac{200}{3}$$

$$= 98 \times \frac{40000}{9}$$

Price $\Rightarrow$ $P = 100 - \frac{1}{2} \times \frac{200}{3}$

$$= \frac{100}{3}$$

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c) <u>Bertrand</u>

$P = 100 - Q/2$

$MR_1 = 100 - q_1 - \frac{q_2}{2} = 5$

$\Rightarrow q_1 + \frac{q_2}{2} = 100$

$\Rightarrow q_2 = 100 - \frac{q_1}{2}$

$q_1 = 95 - q_2/2$

So, $q_1 = 100/3$

$q_2 = \frac{200}{3}$

$P = \frac{100}{3}$ $\quad Z = \frac{4 \times 10^6}{9}$

d) Stackelberg

$P = 100 - \frac{Q}{2}$

$MR_1 = 100 - q_1 - \frac{q_2}{2} = 5$

$\Rightarrow q_1 + q_2/2 = 95$

$\Rightarrow q_1 = 95 - q_2/2$

$MR_2 \rightarrow q_2 = 100 - q_1/2$

$R = [100 - (\frac{q_1 + 100 - q_1/2}{2}))q_1$

$R = 50q_1 - \frac{q_1^2}{4}$

$MR_1 = 50 - \frac{q_1}{2} = 5$

$\frac{q_1}{2} = 45$

$\boxed{q_1 = 90 \quad q_2 = 55}$

So, $P = 100 - (45 + 27.5)$

$= (27.5)$

$\boxed{Z = 512.5}$

e) Cartel (Joint maximisation)

$P = 100 - Q/2$

$C_1 = 5q_1 + C_2 = 0.5q_2^2$

$\pi = TR - TC$

$\pi = P.Q - (C_a + C_b)$

$Q = q_1 + q_2$

$\frac{d\pi}{dQ} = -\frac{1}{2}(q_1 + q_2) + (100 - \frac{q_1}{2} - q_2) - 5 = 0$

$= -\frac{q_1}{2} - \frac{q_1}{2} - \frac{q_1}{2} - \frac{q_2}{2} + 95$

$= q_1 + q_2 = 95 \quad \text{--- (1)}$

Similarly

$\frac{d\pi}{dq_2} = q_1 + 2q_2 = 100 \quad \text{--- (11)}$

$q_2 = 5 \quad q_1 = 90$

So, $P = 100 - 49.5 = 52.5$

$Z = Z_1 + Z_2$

$\Rightarrow Z = 90(52.5 - 5) + 5(52.5 - 5)$

$\Rightarrow \boxed{Z = 4512.5}$

2) a) $Q = 53 - P$

$AC = MC = 5$

Let the inverse demand function be,

$P = a - bQ$

$R = P \cdot Q_2$

$= a - b(q_1 + q_2)\, q_2$

$= aq_2 - bq_1 q_2 - bq_2^2$

$MR_2 = a - bq_1 + 2bq_2$

Now, $MC = MR$ (equilibrm)

So $a - bq_1 - 2bq_2 = 5$

$\Rightarrow \boxed{q_2 = \frac{a - bq_1 - 5}{2b}}$

Price $P = a - b(q_1 + q_2)$

$= a - bq_1 - bq_2$

$TR_1 = PQ_1$

$= aq_1 - bq_1^2 - bq_1 q_2$

$= aq_1 - bq_1^2 - \cancel{b}q_1\left(\frac{a - bq_1 - 5}{2\cancel{b}}\right)$

$= aq_1 - \frac{bq_1^2}{2} + \frac{5q_1}{2}$

$$MR_1 = \frac{a}{2} - \frac{2bq_1}{2} + 5/2$$

$$\Rightarrow 5 = \frac{a}{2} - bq_1 + \frac{5}{2}$$

$$\Rightarrow \boxed{q_1 = \frac{-5 + a}{2b}} \leftarrow (ii)$$

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b)