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Remarks if any:		
Date:16/02/2022	Signature of the Instructor	Marks awarded

Medical Imaging and Image Processing-Lab (BM3704)

Fourier Analysis of Images

lab3_Solutions.pdf

Frequency domain analysis is a very important part in image processing, especially in image enhancement, noise reduction and image filtering. The processing in frequency domain simplifies the design of filters and reduces the computational complexity. Frequency domain representation gives you control over the whole images, where you can enhance (eg edges) and suppress (eg smooth shadow) different characteristics of the image very easily.

Objectives:

To

(a) find the magnitude and phase spectrum of various images.
(b) understand the importance of phase information.
(c) recognise the importance of zero padding
(d) discover the properties of 2D FFT such as rotation and shifting.

Basic Theory:

The two-dimensional discrete Fourier transform (DFT) of an image $f(x,y)$ of size M x N is represented by:

$$F(u,v) = \sum_{x=0}^{M-1}\sum_{y=0}^{N-1} f(x,y)e^{-j2\pi(ux/M+vy/N)}$$

The corresponding inverse of the above discrete Fourier transform is given by the following equation:

$$f(x,y) = \frac{1}{MN}\sum_{u=0}^{M-1}\sum_{v=0}^{N-1} F(u,v)e^{j2\pi(ux/M+vy/N)}$$

The magnitude and phase spectrum of an image $f(x, y)$ is represented by

$$F(u,v) = |F(u,v)|\, e^{j\, arg\{F(u,v)\}}$$

$$|F(u,v)| = \left[R^2(u,v) + I^2(u,v)\right]^{1/2}$$

$$\phi(u,v) = \tan^{-1}\left[\frac{I(u,v)}{R(u,v)}\right]$$

where $R(u, v)$ and $I(u, v)$ are the real and imaginary components of the spectrum $F(u, v)$. Similarly the power spectrum is represented by

$$\begin{aligned} P(u,v) &= |F(u,v)|^2 \\ &= R^2(u,v) + I^2(u,v) \end{aligned}$$

Translation Property:

$$f(x,y)(-1)^{x+y} \xleftrightarrow{\text{FT}} F(u-N/2, v-N/2)$$

Rotation Property:

$$x = r\cos\theta, \quad y = r\sin\theta, \quad u = \omega\cos\varphi, \quad v = \omega\sin\varphi$$

$$f(r, \ \theta + \theta_0) \xleftrightarrow{\text{FT}} F(\omega, \ \varphi + \varphi_0)$$

Example1: Generate an image of size 30x30 with zero magnitudes. Then create a rectangular of size 5x20 with unity magnitude. Find the magnitude spectrum and display. Use fftshift to display the spectrum in the center. Use zero padding for fine resolution. For more visibility use log of magnitude spectrum.

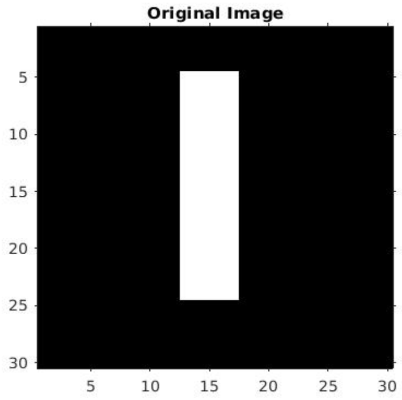
Refer **MIIP_Ex_3_1.m**

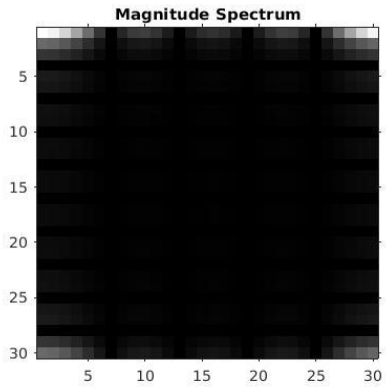
< Type the Matlab codes here >

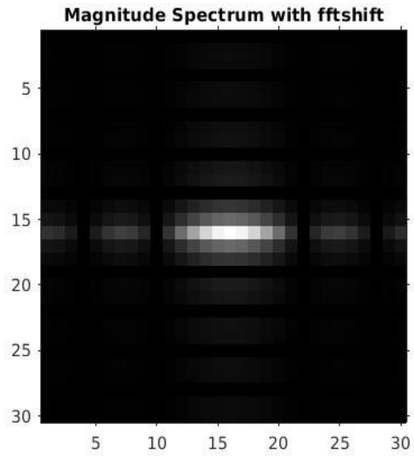
```
clear; close all; clc;

% Generate an image
im = zeros(30,30);
%%
im(5:24,13:17)=1;
%%
figure();
imshow(im); title('Original Image'); axis on
%%
% display('Spectrum of the image');
% display('Press any Key');
% pause

% Find the Spectrum using FFT
imF = fft2(im);
%%
imF_mag = abs(imF);
figure(); imshow(imF_mag,[]);title('Magnitude Spectrum'); axis on
%%
% display('Spectrum of the image with fftshift');
```

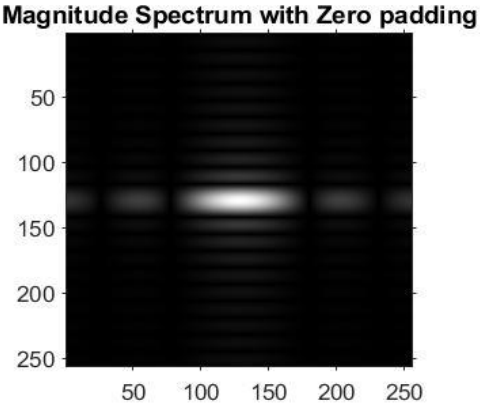
```
% display('Press any Key');
%
% pause

% The zero-frequency coefficient is displayed in the upper left
hand corner.
% To display it in the center, you can use the function fftshift.
imF_mag = fftshift(imF);

imF_mag = abs(imF_mag);
figure(); imshow(imF_mag,[]);title('Magnitude Spectrum with
fftshift'); axis on
%%
% display('Spectrum of the image with zero padding');
% display('Press any Key');
% pause

% To create a finer sampling of the Fourier transform,
% you can add zero padding to im when computing its DFT.

imF=fft2(im, 256,256);
imF_mag = abs(fftshift(imF));
figure(); imshow(imF_mag,[]); title('Magnitude Spectrum with Zero
padding'); axis on
%%
%display('Spectrum of the image with log magnitude');
%display('Press any Key');
%pause

% To brighten the display, you can use a log function
imF_log_mag=log(1+imF_mag);
figure,imshow(imF_log_mag,[]);title('Log Magnitude  Spectrum');
axis on

%display('End of the program');
```

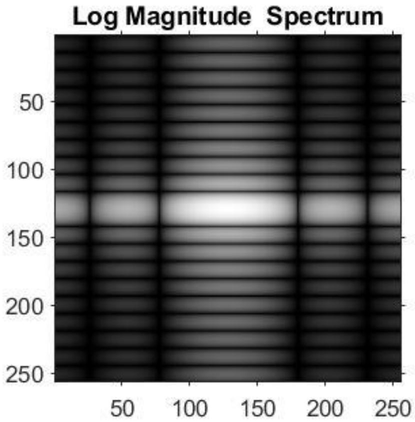
< Plot the figures and type the results here >











Example2: Find and display of magnitude and phase spectrum of a synthetic image (parameters are given in the Matlab code). Rotate the image by 45^{O}. Now reconstruct the image:

(a) Using both magnitude and phase spectrums.

(b) Using only magnitude spectrum.

(c) Using only phase spectrum.

(d) Comment on the results

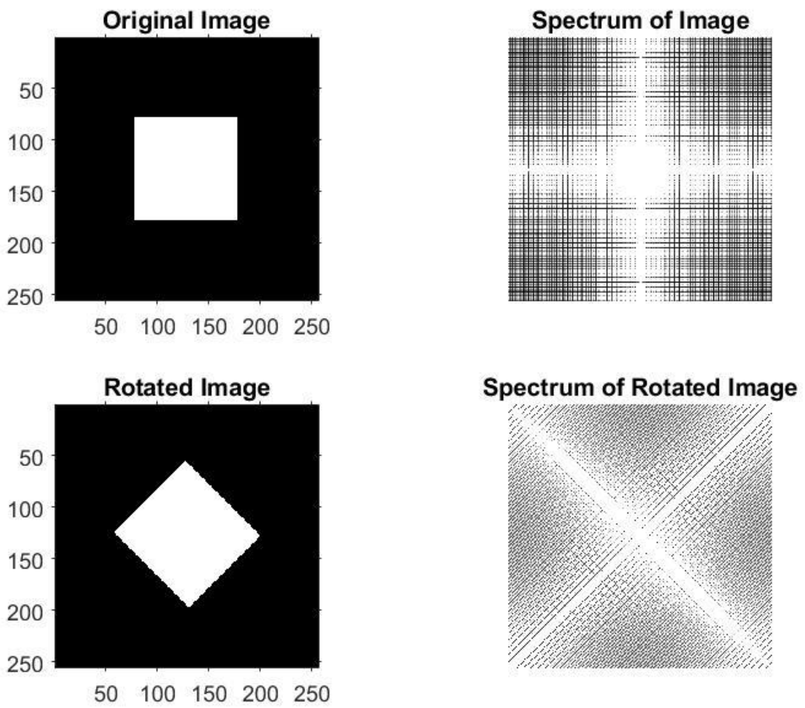
Refer **MIIP_Ex_3_2.m**

< Type the Matlab codes here >

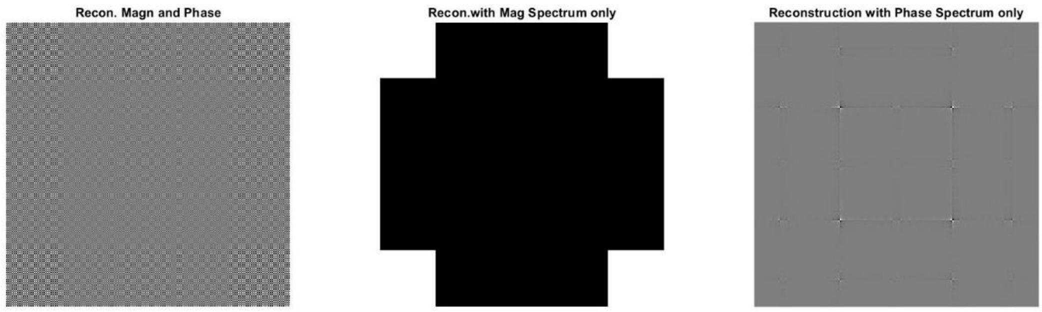
```
clear; close all; clc;
a=zeros(256,256);
a(78:178,78:178)=1;
[m n] = size(a);

figure();
subplot(2,2,1); imshow(a);title('Original Image'); axis on;
%%
af=fftshift(fft2(a));
imF_ph=angle(fft2(a));
subplot(2,2,2);imshow(abs(af));title('Spectrum of Image');
%%
% Now rotated the image by 45 degrees
[x,y] = meshgrid(1:256,1:256);
b=(x+y<329)&(x+y>182)&(x-y>-67)&(x-y<73);
subplot(2,2,3);imshow(b);title('Rotated Image');axis on;
%%
bf = abs(fftshift(fft2(b)));
imFr_ph=angle(fft2(a));
subplot(2,2,4);imshow(bf);title('Spectrum of Rotated Image');

%%
% Reconstruction

% Reconstruction by combining both magnitude and phase spectrum
imr = ifft2(af.*exp(1i*imF_ph))/(m*n);

% Reconstruction by only magnitude spectrum
imr_mag = abs(ifftshift(ifft2(af)));
% imr_mag = abs((ifft2(imF_mag)));

% Reconstruction by only phase spectrum
imr_ph = ifft2(exp(1i*imF_ph))/(m*n);


figure();
subplot(1,3,1); imshow(imr,[]);   title('Recon. Magn and Phase');
subplot(1,3,2); imshow(uint8(imr_mag),[]);  title('Recon.with Mag
Spectrum only');
subplot(1,3,3); imshow(imr_ph,[]);title('Reconstruction with Phase
Spectrum only');
%%
%%
% Reconstruction

% Reconstruction by combining both magnitude and phase spectrum
imr = ifft2(bf.*exp(1i*imFr_ph))/(m*n);
```

```
% Reconstruction by only magnitude spectrum
imr_mag = abs(ifftshift(ifft2(bf)));
% imr_mag = abs((ifft2(imF_mag)));

% Reconstruction by only phase spectrum
imr_ph = ifft2(exp(1i*imFr_ph))/(m*n);


figure();
subplot(1,3,1); imshow(imr,[]);   title('Recon. Magn and Phase of rotated image');
subplot(1,3,2); imshow(uint8(imr_mag),[]);  title('Recon.with Mag Spectrum only');
subplot(1,3,3); imshow(imr_ph,[]);title('Reconstruction with Phase Spectrum only');
```

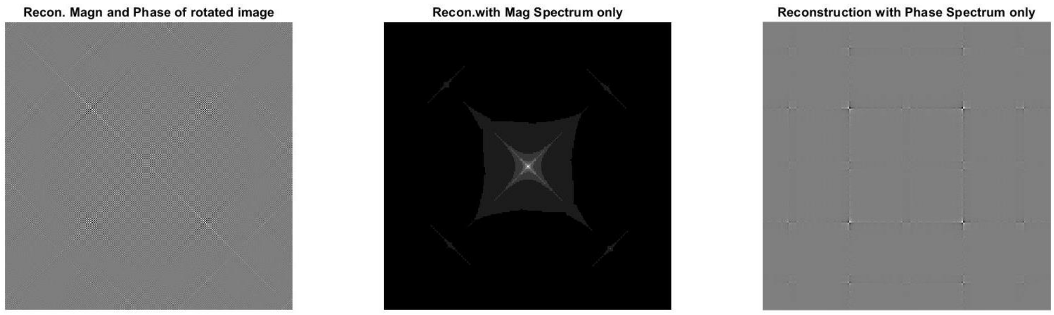
< Plot the figures and type the results here >



Reconstruction of image



Reconstruction of rotated image



Example2: Repeat the above for various images of choice.

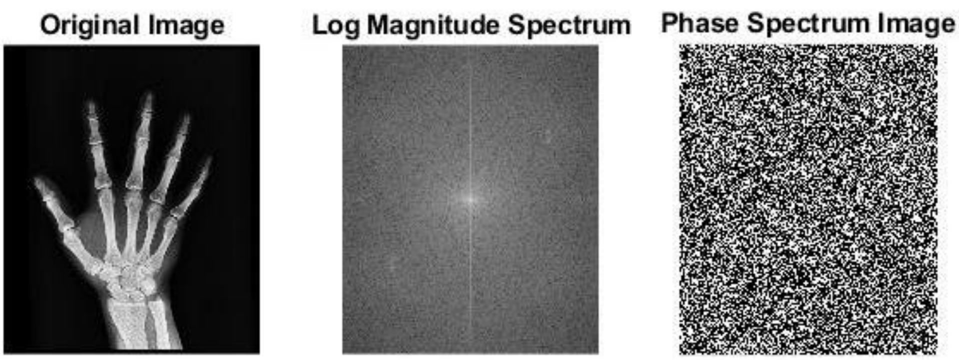
Refer `MIIP_Ex_3_3.m`

< Type the Matlab codes here >

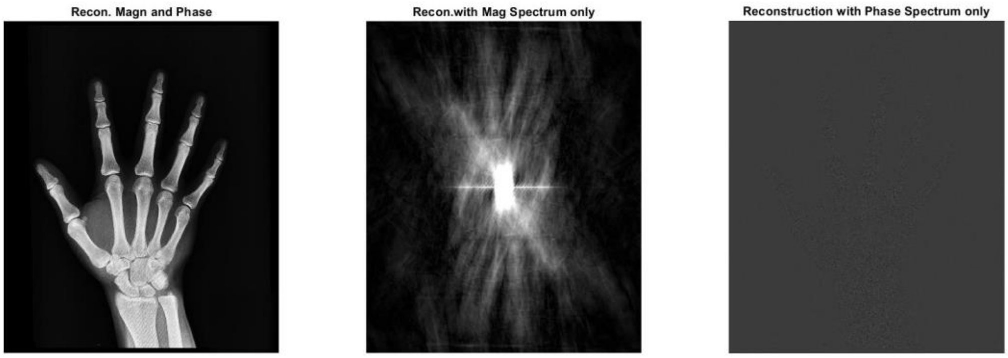
```
clear;close all;clc;

i1 = imread('hand-x-ray.jpg');
im = rgb2gray(i1);
[m n] = size(im);
%%
% Spectrum calculations
imF = fft2(im);              % 2D FFT
imF_mag = abs(imF);            % Magnitude Spectrum
s = log(1+abs(fftshift(imF)));% Log Magnitude Spectrum
imF_ph=angle(imF);       % Phase Spectrum

figure();
subplot(1,3,1); imshow(im); title('Original Image');
subplot(1,3,2); imshow(s,[]); title('Log Magnitude Spectrum');
subplot(1,3,3); imshow(imF_ph); title('Phase Spectrum Image');
%%
% Reconstruction

% Reconstruction by combining both magnitude and phase spectrum
```

```
imr = ifft2(imF_mag.*exp(1i*imF_ph))/(m*n);

% Reconstruction by only magnitude spectrum
imr_mag = abs(ifftshift(ifft2(imF_mag)));
% imr_mag = abs((ifft2(imF_mag)));

% Reconstruction by only phase spectrum
imr_ph = ifft2(exp(1i*imF_ph))/(m*n);


figure();
subplot(1,3,1); imshow(imr,[]);   title('Recon. Magn and Phase');
subplot(1,3,2); imshow(uint8(imr_mag),[]);  title('Recon.with Mag
Spectrum only');
subplot(1,3,3); imshow(imr_ph,[]);title('Reconstruction with Phase
Spectrum only');
```

< Plot the figures and type the results here >

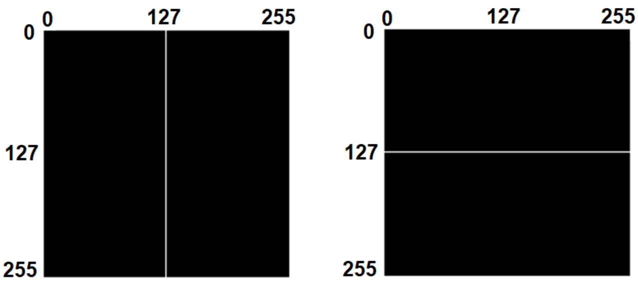


Reconstruction



Lab3-Exercise Questions

Exercise1: (a) Write a Matlab code to generate the following images. Assume that the width of the white pixel for Fig(a) and height of the white pixel Fig(b) are unity.



(a) (b)

(b) Find and display the magnitude and phase spectrums.

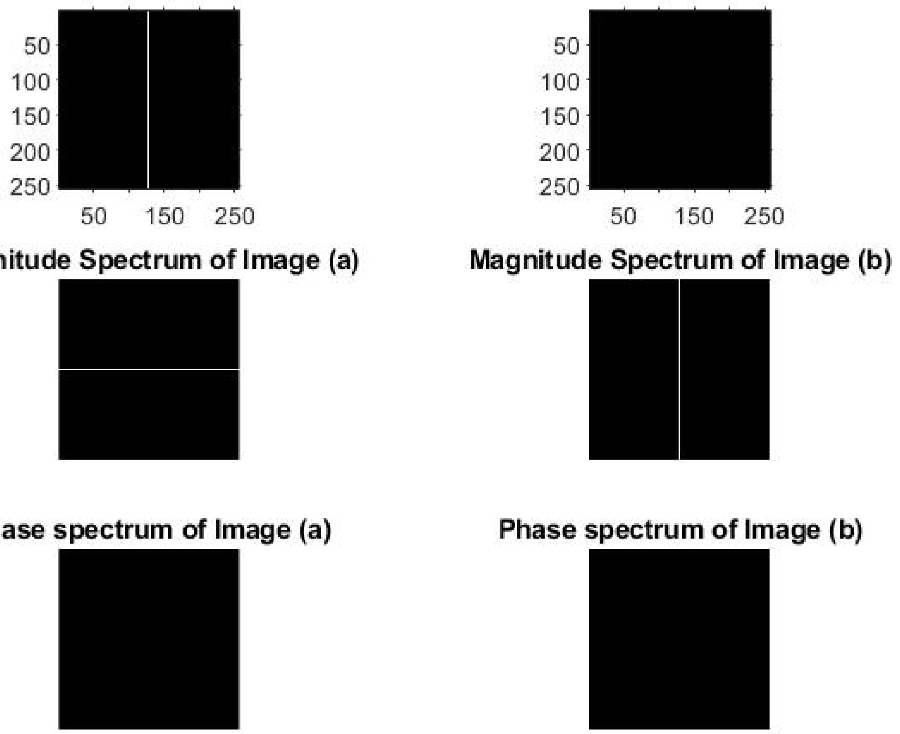
(c) Suppose the vertical line in Fig(a) and horizontal line in Fig(b) are rotated by (i) $\pm 30^{o}$, (ii) $\pm 45^{o}$ and (iii) $\pm 90^{o}$. Find and display the magnitude and phase spectrums. Comment on the results.

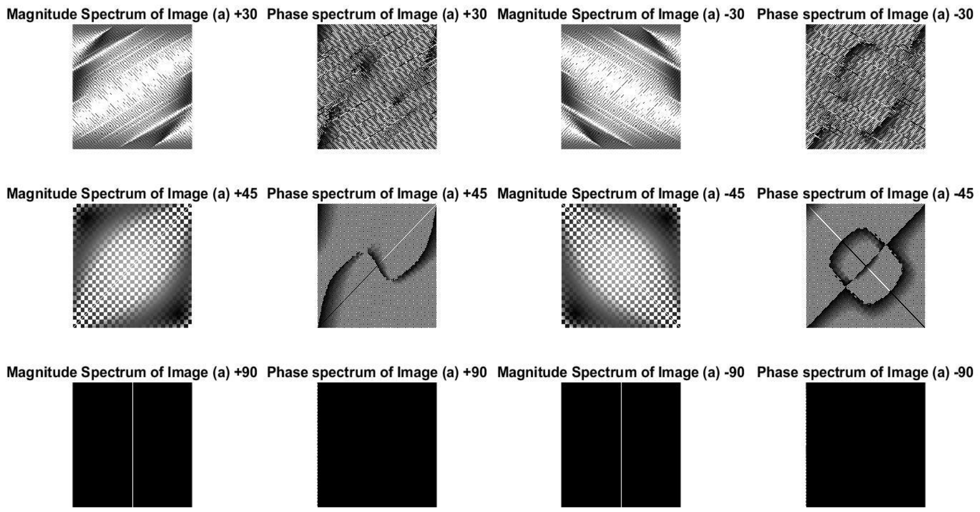
ANS:

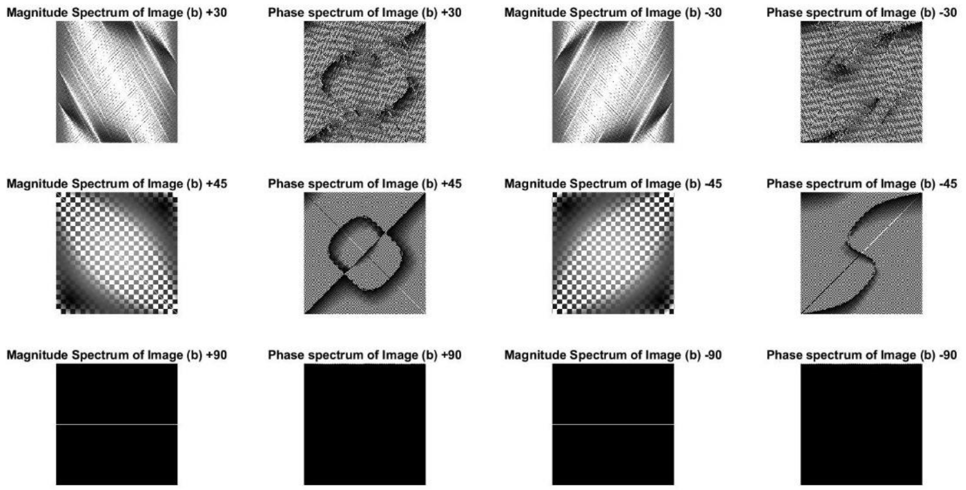
```
im1 = zeros(256,256);
im1(:,127)=1;
im2 = zeros(256,256);
im2(127,:)=1;
figure(1);
subplot(3,2,1);imshow(im1, []); axis on;
subplot(3,2,2);imshow(im2, []); axis on;
%%
% Magnitude Spectrum
a = fft2(im1);
b = fft2(im2);
af=fftshift(fft2(im1));
bf=fftshift(fft2(im2));
subplot (3,2,3);imshow(abs(af));title('Magnitude Spectrum of Image (a)');
```

```
subplot (3,2,4);imshow(abs(bf));title('Magnitude Spectrum of Image (b)');
%%
% Phase Spectrum
im_ph1 = angle(a);
im_ph2 = angle(b);
subplot (3,2,5);imshow(im_ph1);title('Phase spectrum of Image (a)');
subplot (3,2,6);imshow(im_ph2);title('Phase spectrum of Image (b)');
%%
figure(2);
im_31 = imrotate(im1, 30, 'bilinear');
im_32 = imrotate(im1, -30, 'bilinear');
subplot (3,4,1);imshow(abs(fftshift(fft2(im_31)))); title('Magnitude
Spectrum of Image (a) +30');
im_ph1 = angle(fft2(im_31));
subplot (3,4,2);imshow(im_ph1);title('Phase spectrum of Image (a) +30');
subplot (3,4,3);imshow(abs(fftshift(fft2(im_32)))); title('Magnitude
Spectrum of Image (a) -30');
im_ph1 = angle(fft2(im_32));
subplot (3,4,4);imshow(im_ph1);title('Phase spectrum of Image (a) -30');
im_31 = imrotate(im1, 45, 'bilinear');
im_32 = imrotate(im1, -45, 'bilinear');
subplot (3,4,5);imshow(abs(fftshift(fft2(im_31)))); title('Magnitude
Spectrum of Image (a) +45');
im_ph1 = angle(fft2(im_31));
subplot (3,4,6);imshow(im_ph1);title('Phase spectrum of Image (a) +45');
subplot (3,4,7);imshow(abs(fftshift(fft2(im_32)))); title('Magnitude
Spectrum of Image (a) -45');
im_ph1 = angle(fft2(im_32));
subplot (3,4,8);imshow(im_ph1);title('Phase spectrum of Image (a) -45');
im_31 = imrotate(im1, 90, 'bilinear');
im_32 = imrotate(im1, -90, 'bilinear');
subplot (3,4,9);imshow(abs(fftshift(fft2(im_31)))); title('Magnitude
Spectrum of Image (a) +90');
im_ph1 = angle(fft2(im_31));
subplot (3,4,10);imshow(im_ph1);title('Phase spectrum of Image (a) +90');
subplot (3,4,11);imshow(abs(fftshift(fft2(im_32)))); title('Magnitude
Spectrum of Image (a) -90');
im_ph1 = angle(fft2(im_32));
subplot (3,4,12);imshow(im_ph1);title('Phase spectrum of Image (a) -90');
%%
figure(3);
% img b
im_31 = imrotate(im2, 30, 'bilinear');
im_32 = imrotate(im2, -30, 'bilinear');
subplot (3,4,1);imshow(abs(fftshift(fft2(im_31)))); title('Magnitude
Spectrum of Image (b) +30');
im_ph1 = angle(fft2(im_31));
subplot (3,4,2);imshow(im_ph1);title('Phase spectrum of Image (b) +30');
subplot (3,4,3);imshow(abs(fftshift(fft2(im_32)))); title('Magnitude
Spectrum of Image (b) -30');
im_ph1 = angle(fft2(im_32));
subplot (3,4,4);imshow(im_ph1);title('Phase spectrum of Image (b) -30');
im_31 = imrotate(im2, 45, 'bilinear');
im_32 = imrotate(im2, -45, 'bilinear');
subplot (3,4,5);imshow(abs(fftshift(fft2(im_31)))); title('Magnitude
Spectrum of Image (b) +45');
im_ph1 = angle(fft2(im_31));
subplot (3,4,6);imshow(im_ph1);title('Phase spectrum of Image (b) +45');
subplot (3,4,7);imshow(abs(fftshift(fft2(im_32)))); title('Magnitude
Spectrum of Image (b) -45');
im_ph1 = angle(fft2(im_32));
```

```
subplot (3,4,8);imshow(im_ph1);title('Phase spectrum of Image (b) -45');
im_31 = imrotate(im2, 90, 'bilinear');
im_32 = imrotate(im2, -90, 'bilinear');
subplot (3,4,9);imshow(abs(fftshift(fft2(im_31)))); title('Magnitude
Spectrum of Image (b) +90');
im_ph1 = angle(fft2(im_31));
subplot (3,4,10);imshow(im_ph1);title('Phase spectrum of Image (b) +90');
subplot (3,4,11);imshow(abs(fftshift(fft2(im_32)))); title('Magnitude
Spectrum of Image (b) -90');
im_ph1 = angle(fft2(im_32));
subplot (3,4,12);imshow(im_ph1);title('Phase spectrum of Image (b) -90');
```



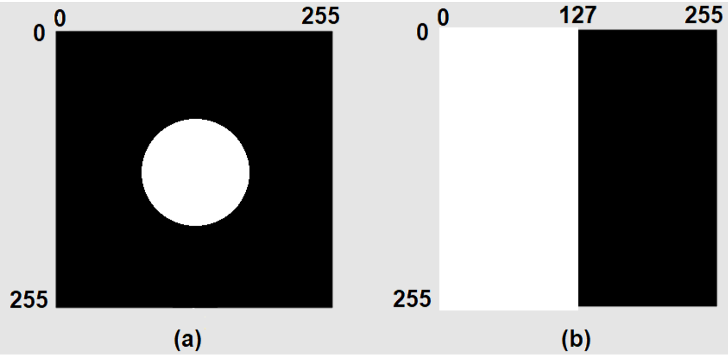




We can observe that as both images are perpendicular to each other and their magnitude spectrum is also perpendicular to each other. We can observe the pattern in phase spectrum and magnitude spectrum of rotated image.

Exercise2: (a) Write a Matlab code to generate the following images. Assume that the radius of circle is 32 for Fig(a).

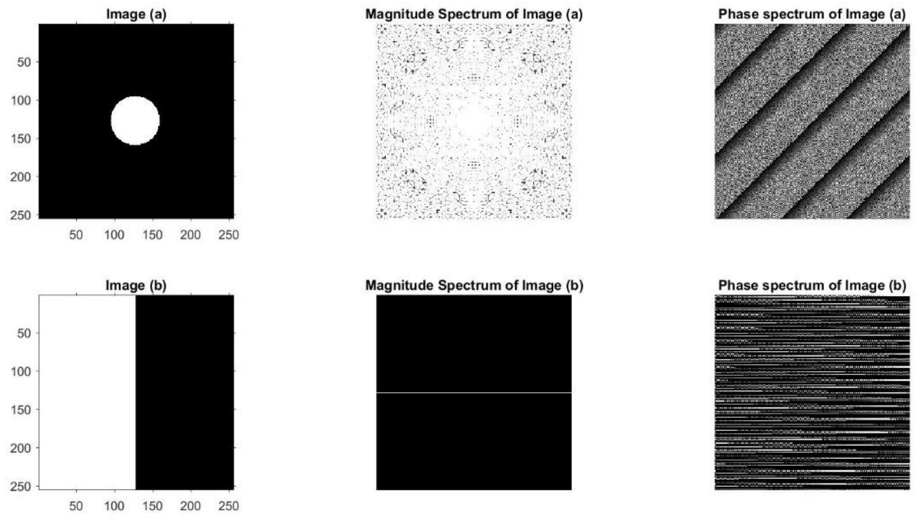
(b) Find and display the magnitude and phase spectrums.



ANS:

```
im1 = zeros(255);
for i=1:255
for j=1:255
if(((i-127)^2 + (j-127)^2)<=1024)
im1(i,j)=1;
end
end
end
figure();
subplot (2,3,1);imshow(im1, []);title('Image (a)'); axis on;
a = fft2(im1);
af=fftshift(fft2(im1));
subplot (2,3,2);imshow(abs(af));title('Magnitude Spectrum of Image (a)');
```

```
im_ph = angle(a);
subplot (2,3,3);imshow(im_ph);title('Phase spectrum of Image
(a)');
im2 = zeros(255);
for i=1:255
for j=1:127
        im2(i,j)=1;
end
end
subplot (2,3,4);imshow(im2, []);title('Image (b)'); axis on;
a1 = fft2(im2);
af1=fftshift(fft2(im2));
subplot (2,3,5);imshow(abs(af1));title('Magnitude Spectrum of
Image (b)');
im_ph1 = angle(a1);
subplot (2,3,6);imshow(im_ph1);title('Phase spectrum of Image
(b)');
```



Exercise3: Read an image of your choice.

(a) Find and display the magnitude and phase spectrum of image.

(b) Now reconstruct the image with

(i) Only magnitude spectrum

(ii) Only phase spectrum

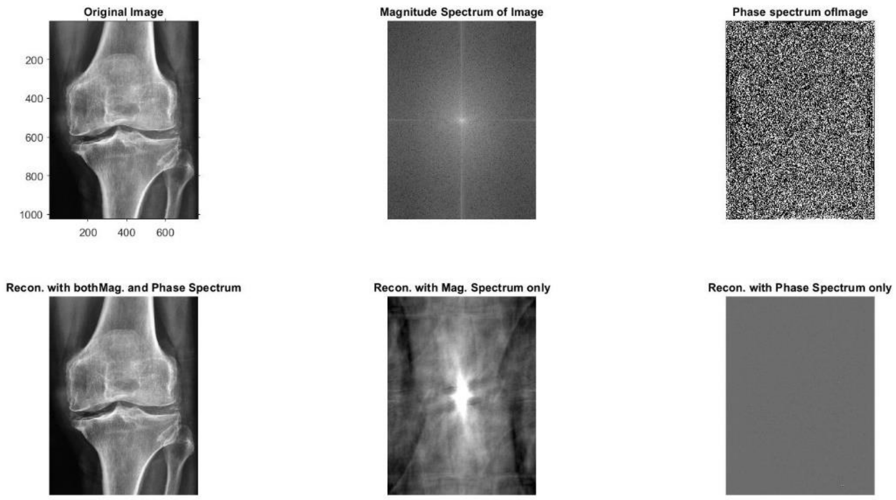
(iii) Combining the magnitude and phase spectrum.

(iv) With the above results, discuss the importance of phase information of image.

ANS:

```
I = imread('knee-x-ray.jpg');
im = rgb2gray(I);
```

```
[m n] = size(im);
figure();
subplot (2,3,1);imshow(im);title('Original Image'); axis on;
a = fft2(im);
s = log(1+fftshift(a));
subplot (2,3,2);imshow(s, []);title('Magnitude Spectrum of
Image');
im_ph = angle(a);
subplot (2,3,3);imshow(im_ph);title('Phase spectrum ofImage');
% Reconstruction by combining both magnitude and phasespectrum
imr = ifft2(abs(a).*exp(1i*im_ph))/(m*n);
% Reconstruction by only magnitude spectrum
imr_mag = abs(ifftshift(ifft2(abs(a))));
% Reconstruction by only phase spectrum
imr_ph = ifft2(exp(1i*im_ph))/(m*n);
subplot (2,3,4);imshow(imr,[]);   title('Recon. with bothMag. and
Phase Spectrum');
subplot (2,3,5);imshow(uint8(imr_mag),[]);title('Recon. with Mag.
Spectrum only');
subplot (2,3,6); imshow(imr_ph,[]);title('Recon. with Phase
Spectrum only');
```



Phase information gives the phase of transformed image and we can observe that with the help of we can reconstruct the image which gives the shape of image with a contrast info.

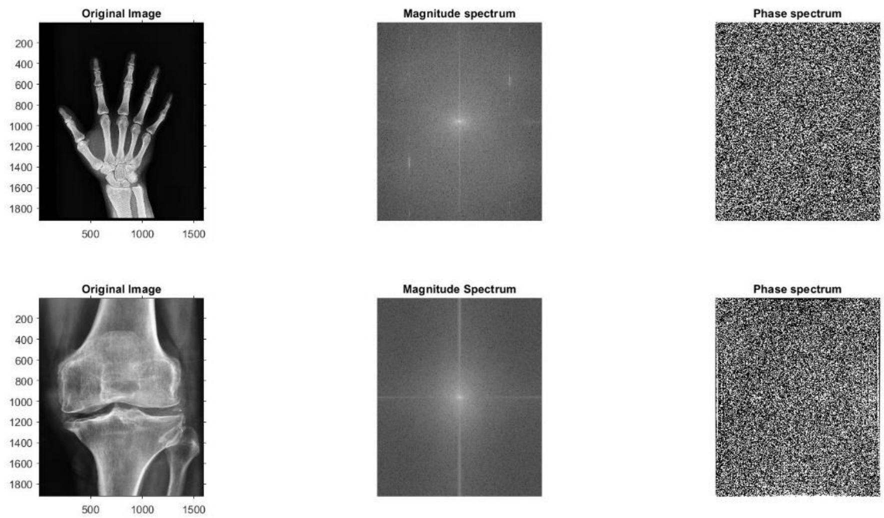
Exercise4: Read two images of your choice.

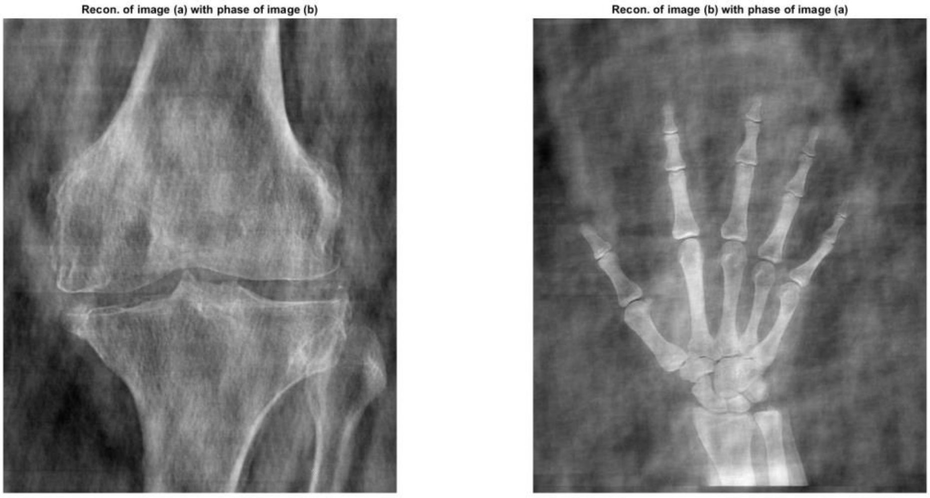
(a) Find and display the magnitude and phase spectrums of two images.

(b) Now swap the phase response of two images, while keeping the magnitude responses are same and reconstruct the two images. Based on the results illustrate the importance of the phase response.

ANS:

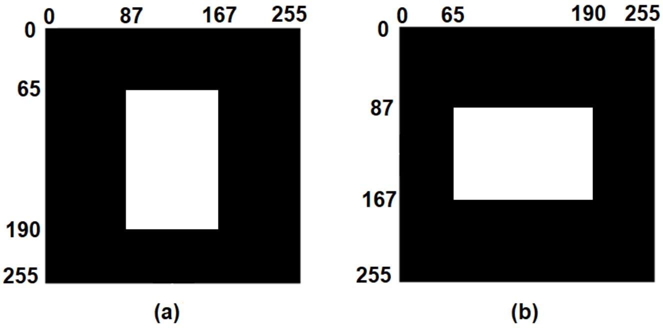
```
I = imread('hand.jpg');
im1 = rgb2gray(I);
[m n] = size(im1);
J = imread('rose.jpg');
im = rgb2gray(J);
im2 = imresize(im,[m n]);
[x y] = size(im2);
figure(1);
subplot 231;imshow(im1);title('Original Image'); axis on;
a = fft2(im1);
s = log(1+fftshift(a));
subplot 232;imshow(s, []);title('Magnitude spectrum');
im_ph = angle(a);
subplot 233;imshow(im_ph);title('Phase spectrum');
subplot 234;imshow(im2);title('Original Image'); axis on;
b = fft2(im2);
s2 = log(1+fftshift(b));
subplot 235;imshow(s2, []);title('Magnitude Spectrum');
im_ph1 = angle(b);
subplot 236;imshow(im_ph1);title('Phase spectrum');
figure(2);
imr = ifft2(abs(a).*exp(1i*im_ph1))/(m*n);
imr1 = ifft2(abs(b).*exp(1i*im_ph))/(m*n);
subplot 121;imshow(imr,[]);title('Recon. of image (a) with phase
of image (b)');
subplot 122;imshow(imr1,[]);title('Recon. of image (b) with phase
of image (a)');
```





Here we can see that by exchanging phase information we can get the exchanged image. From this we can conclude that phase information of an image contains information about shape of image.

Exercise5: (a) Write a Matlab code to generate the following images.



(b) Find and display the magnitude and phase spectrums.

(c) Suppose the white rectangular images are rotated by

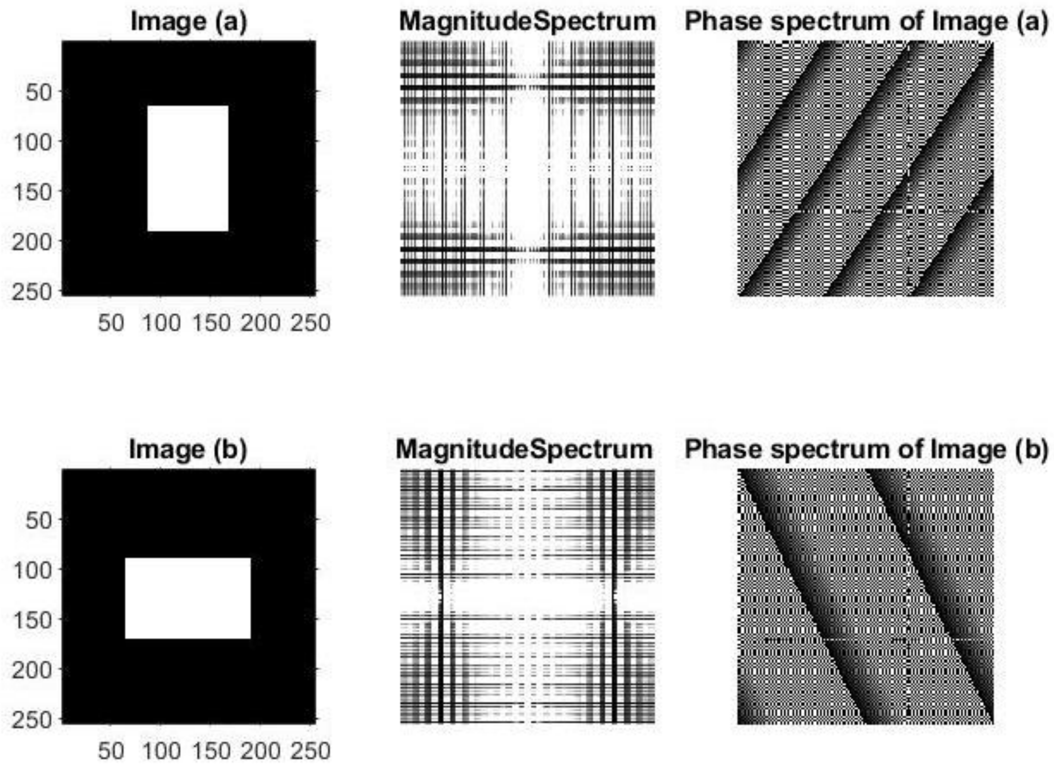
(i) ±45^{O} and (ii) ±120^{O}. Find and display the magnitude and phase spectrums. Comment on the results.

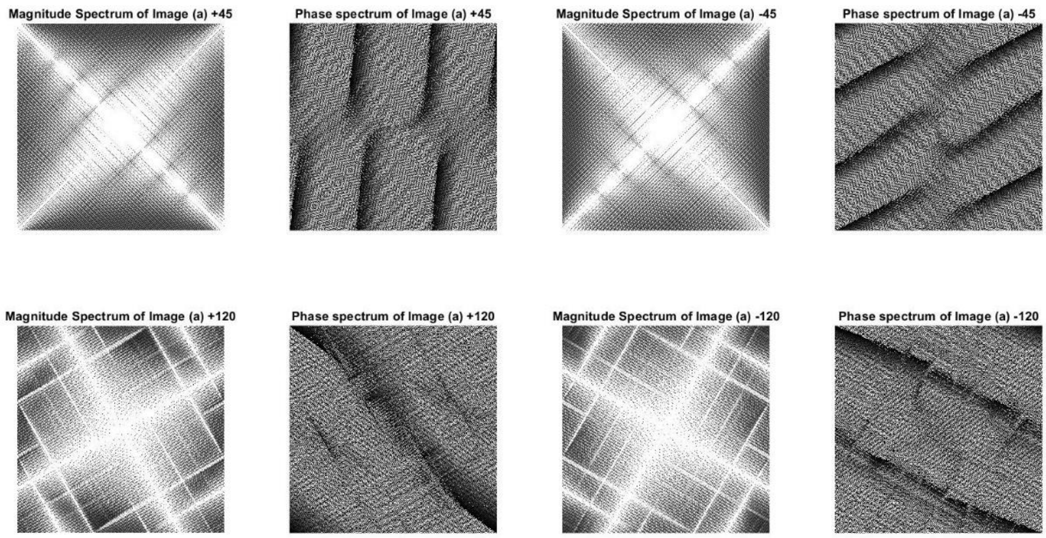
ANS:

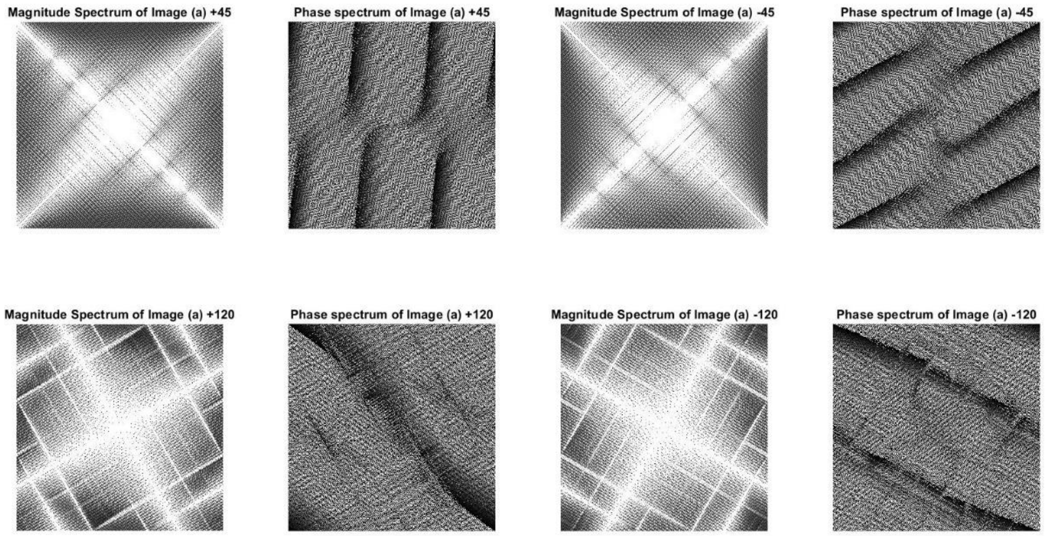
```
im = zeros(255);
im(65:190,87:167)=1;
im1 = imrotate(im, 90, 'bilinear');
%%
figure(1);
subplot 231;imshow(im, []);title('Image (a)'); axis on;
a = fft2(im);
af=fftshift(fft2(im));
subplot 232;imshow(abs(af));title('MagnitudeSpectrum');
im_ph = angle(a);
subplot 233;imshow(im_ph);title('Phase spectrum of Image (a)');
subplot 234;imshow(im1, []);title('Image (b)'); axis on;
```

```
b = fft2(im1);
bf=fftshift(fft2(im1));
subplot 235;imshow(abs(bf));title('MagnitudeSpectrum ');
im_ph1 = angle(b);
subplot 236;imshow(im_ph1);title('Phase spectrum of Image (b)');
%%
figure(2);
im_31 = imrotate(im, 45, 'bilinear');
im_32 = imrotate(im, -45, 'bilinear');
subplot 241;imshow(abs(fftshift(fft2(im_31)))); title('Magnitude
Spectrum of Image (a) +45');
im_ph = angle(fft2(im_31));
subplot 242;imshow(im_ph);title('Phase spectrum of Image (a)
+45');
subplot 243;imshow(abs(fftshift(fft2(im_32)))); title('Magnitude
Spectrum of Image (a) -45');
im_ph = angle(fft2(im_32));
subplot 244;imshow(im_ph);title('Phase spectrum of Image (a) -
45');
im_31 = imrotate(im, 120, 'bilinear');
im_32 = imrotate(im, -120, 'bilinear');
subplot 245;imshow(abs(fftshift(fft2(im_31)))); title('Magnitude
Spectrum of Image (a) +120');
im_ph = angle(fft2(im_31));
subplot 246;imshow(im_ph);title('Phase spectrum of Image (a)
+120');
subplot 247;imshow(abs(fftshift(fft2(im_32)))); title('Magnitude
Spectrum of Image (a) -120');
im_ph = angle(fft2(im_32));
subplot 248;imshow(im_ph);title('Phase spectrum of Image (a) -
120');
%%
figure(3);
im_31 = imrotate(im1, 45, 'bilinear');
im_32 = imrotate(im1, -45, 'bilinear');
subplot 241;imshow(abs(fftshift(fft2(im_31)))); title('Magnitude
Spectrum of Image (b) +45');
im_ph = angle(fft2(im_31));
subplot 242;imshow(im_ph);title('Phase spectrum of Image (b)
+45');
subplot 243;imshow(abs(fftshift(fft2(im_32)))); title('Magnitude
Spectrum of Image (b) -45');
im_ph = angle(fft2(im_32));
subplot 244;imshow(im_ph);title('Phase spectrum of Image (b) -
45');
im_31 = imrotate(im1, 120, 'bilinear');
im_32 = imrotate(im1, -120, 'bilinear');
subplot 245;imshow(abs(fftshift(fft2(im_31)))); title('Magnitude
Spectrum of Image (b) +120');
im_ph = angle(fft2(im_31));
subplot 246;imshow(im_ph);title('Phase spectrum of Image (b)
+120');
subplot 247;imshow(abs(fftshift(fft2(im_32)))); title('Magnitude
Spectrum of Image (b) -120');
im_ph = angle(fft2(im_32));
```

```
subplot 248;imshow(im_ph);title('Phase spectrum of Image (b) -
120');
```







By rotating 90 degrees all spectrum is rotating with 90 degrees.