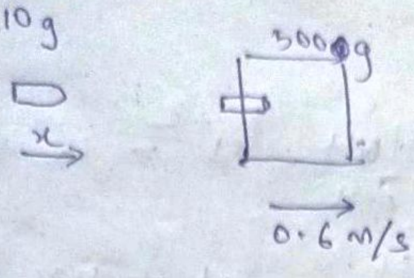
ASSIGNMENT-4

120BM0014

2) 

Conservation of momentum,

$m_1 v_1 = m_2 v_2$

$10 \times x = 5010 \times 0.6$

$x = \frac{501\cancel{0} \times 6}{1\cancel{0} \times 10}$

$\boxed{x = 300.6 \text{ m/s}}$

Ans.

3) $r = 2.5 \text{ mm} = 0.25 \text{ cm}$

$l = 20 \text{ cm}$

$P = 380 \text{ Pa}$

$\eta = 0.0027 \text{ N.s/m}^2$

$Q = \frac{\pi \Delta P r^4}{8 \eta l} = \frac{3.14 \times 380 \times 3.91 \times 10^{-10}}{8 \times 2.7 \times 10^{-3} \times 0.2}$

$\simeq 1080 \times 10^{-7} \text{ m/s}$

$\boxed{1.080 \times 10^{-4} \text{ m/s}}$

Ans.

4) $V = 3xi - 3yj$

$u = 3x$

$v = -3y$

$a_x = \frac{\partial u}{\partial x} u + \frac{\partial u}{\partial y} v$

$= 3(3x) + (0)(-3y)$

$= 9x$

$a_y = \frac{\partial v}{\partial x} u + \frac{\partial v}{\partial y} v$

$= -3 \times (-3y)$

$+9y$

$\boxed{a = 9xi + 9yj}$

Ans

5) $y' + yx = y^2$

$\frac{dy}{dx} = y^2 - yx$ $\qquad \frac{dy}{dx} + P(x)y = f(x)y^n$

$u = y^{1-n}$

$= y^{-1}$ $\qquad$ so, $-u^{-2}\frac{du}{dx} = u^{-2} - u^{-1}x$

$y = u^{-1}$

$\Rightarrow \frac{du}{dx} = -1 + ux$

$I.F = e^{\int x\,dx}$

$= e^{\frac{x^2}{2}}$

so,

$e^{\frac{x^2}{2}}\frac{du}{dx} = -e^{x^2/2} + uxe^{\frac{x^2}{2}}$

$\Rightarrow e^{\frac{x^2}{2}}\frac{du}{dx} - uxe^{\frac{x^2}{2}} = -e^{x^2/2}$

$\Rightarrow \frac{d}{dx}\left[u \cdot e^{\frac{x^2}{2}}\right] = -e^{x^2/2}$

$\Rightarrow u \cdot e^{\frac{x^2}{2}} = -\int e^{x^2/2}\,dx$

$\Rightarrow u \cdot e^{\frac{x^2}{2}} = -\int e^{x^2/2}\,dx$ $\qquad v = \frac{x^2}{2}$

$dv = x\,dx$

$= \sqrt{2v}\,dx$

$\Rightarrow u \cdot e^{\frac{x^2}{2}} = -\int \frac{e^v}{\sqrt{2v}}\,dv$ $\qquad$ Let $\sqrt{v} = t$

$v = t^2$

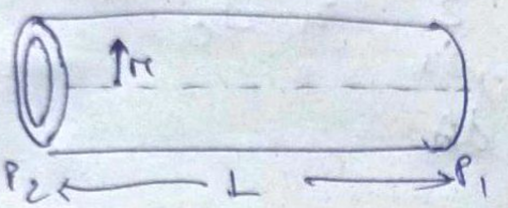
$\Rightarrow u \cdot e^{\frac{x^2}{2}} = -\sqrt{2}\int \frac{e^{t^2}}{2t} \times 2t\,dt$ $\qquad dv = 2t\,dt$

$\Rightarrow u \cdot e^{\frac{x^2}{2}} = -\sqrt{2}\, i\frac{\sqrt{\pi}}{2}\,\mathrm{erf}(ix) + c$ (using error funcn)

$\Rightarrow \boxed{\frac{e^{\frac{x^2}{2}}}{y} = \sqrt{2}\, i\frac{\sqrt{\pi}}{2}\,.\mathrm{erf}(ix) + c}$ Ans.

1) Poiseuille's law states that the flow rate through a circular pipe is proportional to the fourth power of the pipe radius & inversely proportional to the dynamic viscosity. Proportional to the pressure gradient.

Derivation.



$P = \frac{F}{A}$

$F = PA$

So, $F_D = (P_1 - P_2)\pi r^2$

$F_R = -\mu A \frac{dv}{dr} = -\mu A\, 2\pi r L \frac{dv}{dr}$

$F_R = F_D$

$-\mu A 2\pi r L \frac{dv}{dr} = (P_1 - P_2)\pi r^2$

$\frac{dv}{dr} = \frac{(P_2 - P_1) r}{2\mu L}$

$dv = \frac{\Delta P r \, dr}{2\mu L}$

$V = \frac{\Delta P r^2}{4\mu L} + C_1$

$V(R) = 0 \Rightarrow C_1 = -\frac{\Delta P R^2}{4\mu L}$

$V(r) = \frac{\Delta P}{4\mu L} \cdot (R^2 - r^2)$

$$Q = \int V_z \, dA \qquad dA = r\,dr\,d\theta$$

$$= \int_0^{2\pi}\int_0^R V(r)\, r\, dr\, d\theta$$

$$= \int_0^{2\pi}\int_0^R \frac{\Delta P}{4\mu L}(R^2 r - r^3)\, dr\, d\theta$$

$$= \int_0^{2\pi} \frac{\Delta P}{4\mu L}\left[\frac{R^2 r^2}{2} - \frac{r^4}{4}\right]_0^R d\theta$$

$$= \frac{\Delta P R^4}{\cancel{8}16\mu L} \cancel{\int_0^{2\pi}} 2\pi \, d\theta$$

$$Q = \frac{\pi R^4 \Delta P}{8\mu L}$$

$$\boxed{\Delta P = \frac{8\mu L Q}{\pi R^4} \quad \text{— Poisuillie's Law.}}$$