Transport Process Q & A

Q. Refined oil flow through copper pipes 6 cm in diameter at a velocity of 5 m/s. What must be the diameter of the pipe in order that the refined oil emerges at 30 m/s.

Given, $d_1 = 6cm = 0.06 m$.

$v_1 = 5 m/s$

$v_2 = 30 m/s$

$d_2 = ?$

we know, $A_1 V_1 = A_2 V_2$

$$\Rightarrow \frac{\pi d_1^2}{4} v_1 = \frac{\pi d_2^2}{4} v_2$$

$$\Rightarrow d_1^2 v_1 = d_2^2 v_2$$

$$\Rightarrow 36 \times 10^{-4} \times 5 = d_2^2 \times 30$$

$$\Rightarrow d_2^2 = 6 \times 10^{-4} \ m^2$$

$$\Rightarrow d_2 = 2.45 \times 10^{-2} \ m.$$

$$\Rightarrow d_2 = 0.0245 \ m. \quad \text{(Ans)}$$

Q. Explain Navier Stockes equations.

Navier Strokes Equation :-

- It is a partial differential equation that describes the flow of incompressible fluids.
- Formula :- $\rho . \frac{dv}{dt} = -\nabla P + \mu \nabla^2 v + \rho g$
- It is a non-linear PDE which is derived from Newton's second law of motion.

- It follows conservation of momentum.
- The forces include are :- (i) gravitation
  (ii) pressure
  (iii) viscous
- Time rate of change of momentum within the control volume = net rate of momentum flow into control volume + net rate of momentum flows out of control vol. − Sum of external forces acting on the control volume
- Here, $\rho \cdot \frac{dv}{dt}$ means derivative of velocity w.r.t time i.e. change in velocity or acceleration.
- $-\nabla P$ means pressure follows a diffusion process and the fluid moves from high-pressure areas to low-pressure areas in direction of largest change in pressure. This direction is called gradient.
- $\mu \nabla^2 v$ states that viscosity controls velocity diffusion. Highly viscous fluids stick together - like maple syrup. and low viscous fluids flow freely - like Gases.
- ρg indicates body force term which represents external forces that act on the fluid - gravity.
- $$\rho\left(\frac{\delta u}{\delta t} + u \cdot \frac{\delta u}{\delta x} + v \cdot \frac{\delta u}{\delta y} + w \cdot \frac{\delta u}{\delta z}\right) = -\frac{\delta P}{\delta x} + \rho g_x + \mu\left(\frac{\delta^2 u}{\delta x^2} + \frac{\delta^2 u}{\delta y^2} + \frac{\delta^2 u}{\delta z^2}\right)$$

$$\rho\left(\frac{\delta v}{\delta t} + u \cdot \frac{\delta v}{\delta x} + v \cdot \frac{\delta v}{\delta y} + w \cdot \frac{\delta v}{\delta z}\right) = -\frac{\delta P}{\delta y} + \rho g_y + \mu\left(\frac{\delta^2 v}{\delta x^2} + \frac{\delta^2 v}{\delta y^2} + \frac{\delta^2 v}{\delta z^2}\right)$$

$$\rho\left(\frac{\delta w}{\delta t} + u \cdot \frac{\delta w}{\delta x} + v \cdot \frac{\delta w}{\delta y} + w \cdot \frac{\delta w}{\delta z}\right) = -\frac{\delta P}{\delta z} + \rho \cdot g_z + \mu\left(\frac{\delta^2 w}{\delta x^2} + \frac{\delta^2 w}{\delta y^2} + \frac{\delta^2 w}{\delta z^2}\right)$$

- We can also write the Navier Strokes eq^n in stress form

i.e. $\nabla . u = 0$ (or) $\frac{\delta u}{\delta x} + \frac{\delta v}{\delta y} + \frac{\delta w}{\delta z} = 0$.

Q. The water flows through a large tube of radius 3 mm is found to be 10 cm long. The pressure across the two ends of the tube is 300 Pa. Calculate the water's average speed.

Given, $r = 3\,mm$

$l = 10\,cm$

$\Delta P = 300\,Pa$

$V_{avg} = ?$

we know, $\eta = 8.9 \times 10^{-4}\ N.s/m^2$

Using Poiseille's law i.e. $Q = \frac{\pi \Delta P r^4}{8\eta l}$ and $Q = AV$, we can write :-

$$AV = \frac{\pi \Delta P r^4}{8\eta l}$$

$$\Rightarrow \pi r^2 V = \frac{\pi \Delta P r^4}{8\eta l}$$

$$\Rightarrow V = \frac{\Delta P r^2}{8\eta l}$$

$$\Rightarrow V = \frac{300 \times 9 \times 10^{-6}}{8 \times 8.9 \times 10^{-4} \times 0.1}$$

$$\Rightarrow V = \frac{27}{0.8 \times 8.9}$$

$$\Rightarrow V = 3.792\ m/s \quad \text{(Ans)}$$

Q. What is capillarity? Derive the expression for capillary rise and capillary depression / fall.

Capillarity :-

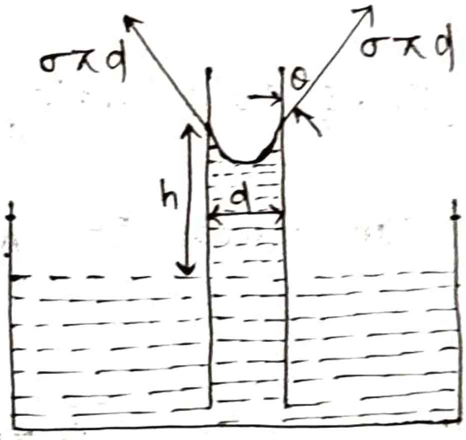
It is defined as the phenomenon of rise or fall of a liquid surface in a small tube relative to the adjacent general level of liquid when the tube is held vertically.

→ Surface tension is an important factor in the phenomenon of capillarity.

→ The rise of liquid surface is known as capillary rise

→ The fall of liquid surface is known as capillary fall or depression.

(i) Expression for capillary rise :-



Let us consider a small glass tube of diameter 'd' open at both ends and is inserted in water.

and, h = height of the liquid in tube i.e. capillary rise

σ = Surface tension

θ = angle of contact between liquid & tube.

Under equilibrium condition,

weight of liquid of height 'h' = Force at surface of liquid in the tube due to surface tension

so, weight of liquid of height 'h' $= m.g$

$= (\text{Volume} \times \text{density}) \times g$

$= (\text{Area} \times h) \times \rho g$

$= \pi r^2 h \rho g$

ie weight of liquid of height 'h' $= \frac{\pi d^2}{4} h \rho g$ ——— (1)

and vertical component of surface tensile force $= \sigma \cos\theta \times \pi d$ ——— (2)

At equilibrium, eq^n (1) = eq^n (2)

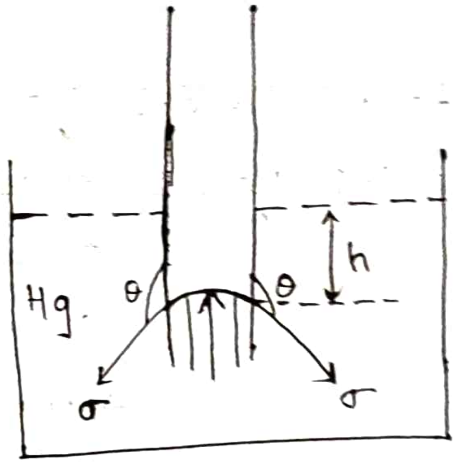
$$\Rightarrow \frac{\pi d^2}{4} h \rho g = \sigma \cos\theta \times \pi d$$

$$\Rightarrow d \cdot h \rho g = 4\sigma \cos\theta$$

$$\Rightarrow \boxed{h = \frac{4\sigma \cos\theta}{d \rho g}}$$

Expression for Capillary fall / depression :-

Here, h = height of depression



Surface tension acting in downward direction $= \pi d \times \sigma \cos\theta$ ——— (1)

Hydrostatic force acting upward direction = h × Area

$$= p \times \frac{\pi d^2}{4}$$

$$= \rho g h \times \frac{\pi d^2}{4} \quad \text{——— (2)}$$

At equilibrium, eq^n (1) = eq^n (2)

$$\Rightarrow \pi d \times \sigma \cos\theta = \rho g h \times \frac{\pi d^2}{4}$$

$$\Rightarrow \boxed{\frac{4\sigma \cos\theta}{\rho g d} = h}$$

Q. Explain the difference between

a) path line, streak line, stream line

b) 1-D, 2-D, 3-D flows

c) compressible and incompressible flows

d) Rotational and ir-rotational flows

Path line, Streak line, Stream line :-

(i) Path line :- It is the path taken by the fluid particle

(ii) Streak line :- This is the locus of the fluid particle that have passed through a fixed station or point.

(iii) Stream line :- This is drawn tangentially to the velocity vector at every point in the flow at a given instant.

1-D, 2-D, 3-D flows :-

(i) 1-D flow :- It is a flow parameter in which velocity is a function of time and one space co-ordinate only.

$\rightarrow u = f(x), \; v = 0, \; w = 0$

$u, v, w \rightarrow$ velocity components

$x, y, z \rightarrow$ directions

(ii) 2-D flow :- In this type of flow, velocity is a function of time and two rectangular co-ordinates.

$\rightarrow u = f_1(x,y) \; ; \; v = f_2(x,y) \; ; \; w = 0.$

(iii) 3-D flow :- In this, velocity is a func^n of time and three mutually perpendicular directions.

$\rightarrow u = f_1(x,y,z) \; ; \; v = f_2(x,y,z) \; ; \; w = f_3(x,y,z)$

Compressible and Incompressible flows :-

(i) Compressible flow :- Density of fluid changes from one point to another point.

→ $\rho \neq$ constant

(ii) Incompressible flow :- Density of fluid does not change from one point to another.

→ $\rho =$ constant

Rotational and Irrotational flows :-

(i) Rotational flow :- The fluid particles flow along the straight lines and also rotates about their own axis.

(ii) Irrotational flow :- The fluid particles flow along the straight lines but do not rotate about their own axis.

Q. What is counter-current heat exchange?

Counter - current heat exchange :-

- It is a mechanism in which heat is lost through conduction when arterious and venous blood flows in anti-parallel direction with small temperature gradients.
- Gradients are maintained through out the flow.
- It is a common mechanism in organisms that utilizes parallel pipes of flowing fluid in opposite directions in order to save energy.
- Example :- It helps gulls (a bird) to walk over snow in colder regions.

Q. Explain the difference between Facilitated Diffusion & Osmosis.

Facilitated diffusion :-

- It is the movement of specific molecules down the concentration gradient, passing through the membrane via a specific carrier protein.
- Each carrier protein has its own shape and only allows one molecule or one group of closely related molecules to pass through.
- Example :- Movement of glucose and amino acid through lipid bilayer.

Osmosis :-

- It is the diffusion of water through a partially permeable ie. semi-permeable membrane from a more dilute solution to a more concentrated solution.
- Differences in the relative concentration of dissolved materials in two solutions can lead to the movement of ions from one to the other.
- Example :- Absorption of water by roots of plants.

Q. A hot plate 1m x 1.5m is maintained at 300°C. Air at 20°C blows over the plate. If the convective heat transfer co-efficient is 20 W/m²°C. Calculate the rate of heat transfer.

Given, $K = 20\ W/m^2{}^\circ C$ $A = 1.5\,m^2$ $t_s = 300^\circ C$ $t_f = 20^\circ C$

we know, rate of heat transfer $= KA(t_s - t_f)$

$= 20 \times 1.5\,(300 - 20)$

$= 8400\ W$

$= 8.4\ KW$ (Ans)

Q. One side of a 2cm. thick Stainless - steel wall ($K_1 = 19\ W/m°c$) is maintained at 180°c, and another side is insulated with a layer of 5cm fibreglass ($K_2 = 0.04\ W/m°c$). The fibreglass outside is maintained at 60°c, and the heat loss through the wall is 300 W. Determine the area of the wall?

Given, $K_1 = 19\ W/m°c$

$K_2 = 0.04\ W/m°c$

$\dot{q} = 300\ W$

$x_1 = 2\,cm = 0.02\,m$

$x_2 = 5\,cm = 0.05\,m$

$T_1 = 180°c$

$T_2 = 60°c$

The resistance of the above composite,

$$R = \frac{x_1}{K_1 A} + \frac{x_2}{K_2 A}$$

$$= \frac{0.02}{19A} + \frac{0.05}{0.04A}$$

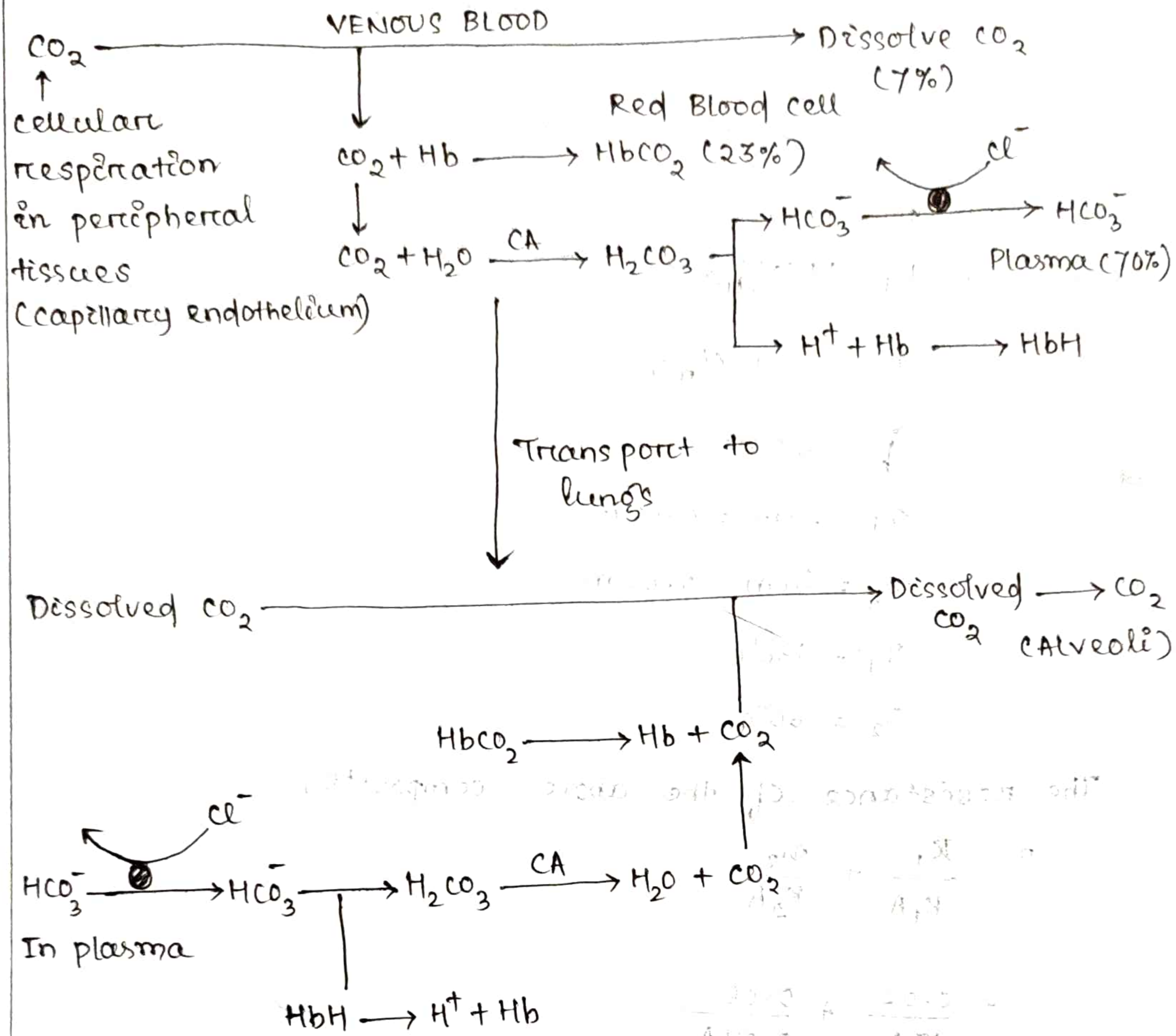
and we know, $\dot{q} = \dfrac{T_1 - T_2}{R}$

$$\Rightarrow 300 = \frac{180 - 60}{\frac{0.02}{19A} + \frac{0.05}{0.04A}}$$

$$\Rightarrow \frac{0.001 + 1.25}{A} = 0.4$$

$$\Rightarrow A = 3.127\ m^2 \text{ (Ans)}$$

Q. Explain CO_2 transport in blood with a neat diagram.



- Different ways of CO_2 transport :-
  1. Hydrogen carbonate ions (HCO_3^-) in blood plasma (70%)
  2. In combination with Haemoglobin (Hb) in RBC (23%)
  3. Dissolved CO_2 in blood plasma. (7%)
- CO_2 produced diffuses into blood plasma and RBC.
- Enzyme Carbonic Anhydrase (CA) catalyses the reaction between CO_2 & H_2O.
- Carbonic acid (H_2CO_3) then dissociates in HCO_3^- & H^+.
- Hb very readily combines with H^+ forming haemoglobinic

acid (HbH).

- As a result, Hb releases some of the oxygen it is carrying.

Q. Write a note on difference b/w Sonophoresis and Iontophoresis

Sonophoresis :-

- It is a process that exponentially increases the absorption of topical compounds (transdermal delivery) with high-frequency ultrasound.
- It occurs because ultrasound waves stimulate micro-vibrations within the skin epidermis and increase the overall kinetic energy of molecule making up topical agents.
- range of 20-100 KHZ.
- Ultrasound propably enhances drug transport by cavitation, micro-streaming, and heating.

Iontophoresis :-

- It is the delivery of a charged chemical compound across the skin membrane using an electrical field.
- It is simply defined as the use of small amounts of physiologically acceptable electric current to drive ionic (charged) drugs into the body.
- It appears to overcome the resistive properties of the skin to charged ions.
- It decreases absorption lag time while increasing delivery rate when compared with passive skin application.

Q. Explain the transdermal drug delivery system with advantages and disadvantages.

Transdermal Drug Delivery System :–

- Facilitate the passage of therapeutic quantities of drug substances through the skin and into the general circulation for their systemic effect.
- Defined as self-contained, discrete dosage forms known as 'patches'.
- when patches are applied to the intact skin, deliver the drug through the skin at a controlled rate to the systemic circulation.
- It is a dosage form designed to deliver a therapeutically effective amount of drug across a patient's skin.
- Deliver drugs into systemic circulation through skin at pre-determined rate.
- For Transdermal drug delivery, it is considered ideal if the drug penetrates through the skin to the underlying blood supply without drug build up in the dermal layers.

Advantages :–

- Avoids the stomach environment, where drug can be degraded.
- provides steady plasma levels.
- Improves bioavailability.
- Increases compliance and reduces medical costs.

Disadvantages :-

- can cause burns if they absorb heat.
- can not achieve high drug levels in blood.
- possibility of local irritation at site of application.
- Not very helpful for geriatric patients because skin undergoes various deformations & alterations.

Q. Explain ventilation-perfusion ratio.

Ventilation - perfusion ratio :-

- ventilation is the movement of air into and out of the lungs.
- Perfusion is the flow of blood in the pulmonary capillaries.
- Different alveoli may have widely differing amounts of ventilation and perfusion.
- Alveoli with increased perfusion also have increase ventilation and with decreased perfusion also have decreased ventilation.
- V/Q ratio ie. $\frac{\text{Ventilation}}{\text{Perfusion}} = 1.0$
- Key for normal gas exchange is to have matching of ventilation and perfusion for each alveolar unit.
- V/Q ratio $= 1$ (under ideal condition)

  V/Q ratio < 1 (Mixed venous blood)

  V/Q ratio $= \infty$ (Inspired air)

Q. Write a note on conduction; convection and radiation with examples.

Conduction :-

- The method of transfer of heat because of collision between neighbouring atoms or molecules.
- Occurs more rapidly in solids and liquids than in gases, due to close proximity of molecules.
- Example :- Metal rods conduct heat gradually when shown over fire.

Convection :-

- It is the transfer of heat due to bulk transfer of molecules within fluids (majorly gases and liquids).
- Example :- Boiling of water i.e. heated molecules from bottom move towards the top of the vessel.

Radiation :-

- It is the emission or transmission of energy in the form of waves or particles through space or through material medium. (x-rays, radio waves, UV etc.)
- Example :- x-rays are used for determining bone condition.

Q. Write short note on Alveolar gas equation.

Alveolar gas equation :-

- It helps us to predict the alveolar concentration of oxygen based on the alveolar conc. of CO_2.
- It is used to calculate alveolar oxygen partial pressure as it is not possible to collect gases directly from alveoli.

- Alveolar O_2 (P_AO_2) = Inspired oxygen − Oxygen consumed

ie. $P_AO_2 = P_iO_2$ − oxygen consumed.

where, P_AO_2 = alveolar oxygen partial pressure

P_iO_2 = inspired oxygen partial pressure

- It can also be written as :-

$$\boxed{P_AO_2 = F_iO_2 (P_b - P_{H_2O}) - P_aCO_2 / RQ}$$

where, P_AO_2 = alveolar oxygen partial pressure

P_b = barometric / atmospheric pressure

F_iO_2 = fraction of inspired oxygen

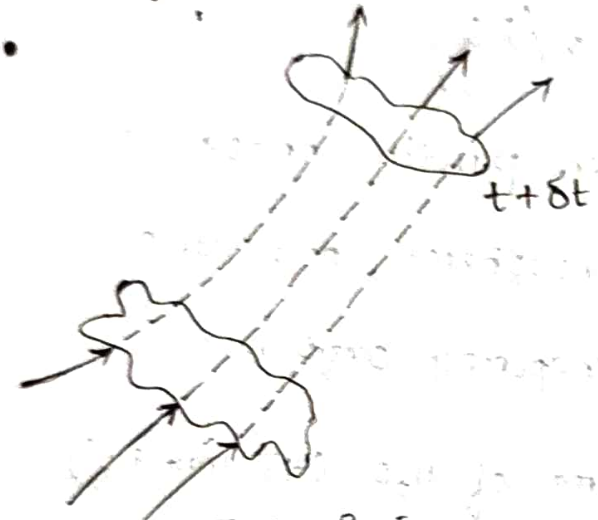
P_{H_2O} = partial pressure of H_2O (45 mm Hg)

P_aCO_2 = partial pressure of CO_2 in alveoli

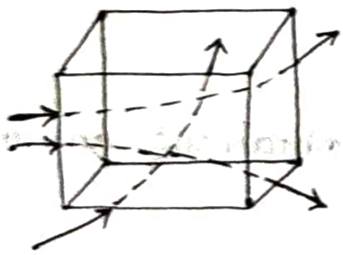
RQ = respiratory quotient, dependent on metabolic activity and diet, and is considered to be about 0.82

- The arterial P_{O_2} can be determined by obtaining an arterial blood gas.
- with the help of alveolar gas equation, the partial pressure inside the alveoli can be calculated.
- Relates the alveolar conc of oxygen F_AO_2 or equivalently partial pressure P_AO_2 to three variables ie. F_iO_2, P_ACO_2 and RQ.

Q. Explain the difference b/w
a) Lagragian and Eulerian description of fluid motion
b) local acceleration and convective acceleration
c) Behaviours of Newtonian and non-Newtonian fluids

a) Langragian Method of fluid motion :-
- It follows a single fluid particle during its motion
- 

Following the motion of the fluid element

ii) Eulerian Method of fluid motion :-
- Single fluid particle at a point.
- 

Spatially fixed volume element

b) Local acceleration :-
- It is defined as the rate of increase of velocity w.r.t time at a given point in a flow field.
- $\frac{\delta u}{\delta t}, \frac{\delta v}{\delta t}, \frac{\delta w}{\delta t}$

ii) Convective acceleration :-

- It is defined as the rate of change of velocity due to change of position of fluid particles in a fluid flow.
- $a_x = u \cdot \frac{\partial u}{\partial x} + v \cdot \frac{\partial u}{\partial y} + w \cdot \frac{\partial u}{\partial z}$
- $a_y = u \cdot \frac{\partial v}{\partial x} + v \cdot \frac{\partial v}{\partial y} + w \cdot \frac{\partial v}{\partial z}$
- $a_z = u \cdot \frac{\partial w}{\partial x} + v \cdot \frac{\partial w}{\partial y} + w \cdot \frac{\partial w}{\partial z}$

c) Newtonian fluid :-

- Obeys newton's law of motion. or viscosity.
- relation between shear stress and shear strain is linear ie. Shear stress (τ) $\propto$ rate of shear strain $\left(\frac{du}{dy}\right)$
- There is one constant of proportionality ie. dynamic viscosity of fluid (μ).
- Example :- water, air, mercury etc.

ii) Non-newtonian fluid :-

- does not obey Newton's law of viscosity.
- relation between shear stress and rate of shear strain is not linear.
- The viscosity of non-newtonian fluid is not constant at a given temperature and pressure.
- Example :- Paints, polymers, lubricants, plastics etc.

Q. Explain the effects of temperature and pressure on the viscosity.

Effect of temperature on viscosity :-

- Increase of temperature causes a decrease in the viscosity of a liquid.
- with the increase in temperature, the cohesive forces decreases with the result of decreasing viscosity.
- $\mu_t = \frac{\mu_0}{1 + At + Bt^2}$; μ_t = viscosity at $t°C$

  μ_0 = viscosity at $0°C$

- Increase of temperature causes increase in the viscosity of gases.

Effect of Pressure on viscosity :-

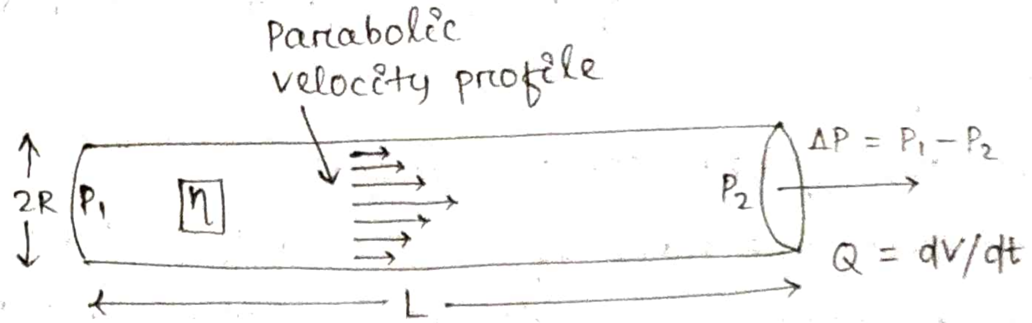
- viscosity is normally independent of pressure.
- Liquids under extreme pressure often experience an increase in viscosity.
- Since liquids are normally incompressible, an increase in pressure doesn't really bring the molecules significantly closer together.

Q. Discuss Poiseuille's law with an example

Poiseuille's law :-

- It states that the flow of liquid depends on following factors like the pressure gradient (ΔP), the length of the tube (L), radius (r) and the viscosity of the fluid (η).
- The formula of Poiseuille's law :-

$$Q = \frac{\pi \Delta P r^4}{8 \eta l}$$



- Pressure gradient (ΔP) shows the difference in the pressure between the two ends of the tube.
- Fluid always flow from high pressure (P_1) to low pressure (P_2).
- The flow rate is determined by the pressure gradient ($P_1 - P_2$).
- Radius of the tube :- The liquid flow varies directly with the radius to the power 4.
- Viscosity (η) :- The flow of the fluid varies inversely with the viscosity of fluid.
  → As the viscosity of the fluid increases, the flow decreases, vice versa.
- Length of the tube :- The liquid flow is inversely proportional to the length (l) of the tube.
- Resistance (R) :- described by $\dfrac{8\eta l}{\pi r^4}$

- Now, poiseuille's law becomes :-

$$\boxed{Q = \frac{\Delta P}{R}}$$

Example :- Airway resistance is the pressure difference between the alveoli & the mouth divided by flow rate & is determined by Poiseuille's law.

Q. Describe Fourier's law of heat conduction and derive the heat diffusion equation.

Fourier's Law of heat conduction :-

- It states that the rate of heat transfer through a material is proportional to the negative gradient in the temperature and to the area, at right angles to that gradient, through which the heat flows.
- Formula :- $\boxed{q_x = -KA.\frac{\Delta T}{L}}$

where, q_x = local heat flux density

K = Thermal conductivity

ΔT = temperature gradient

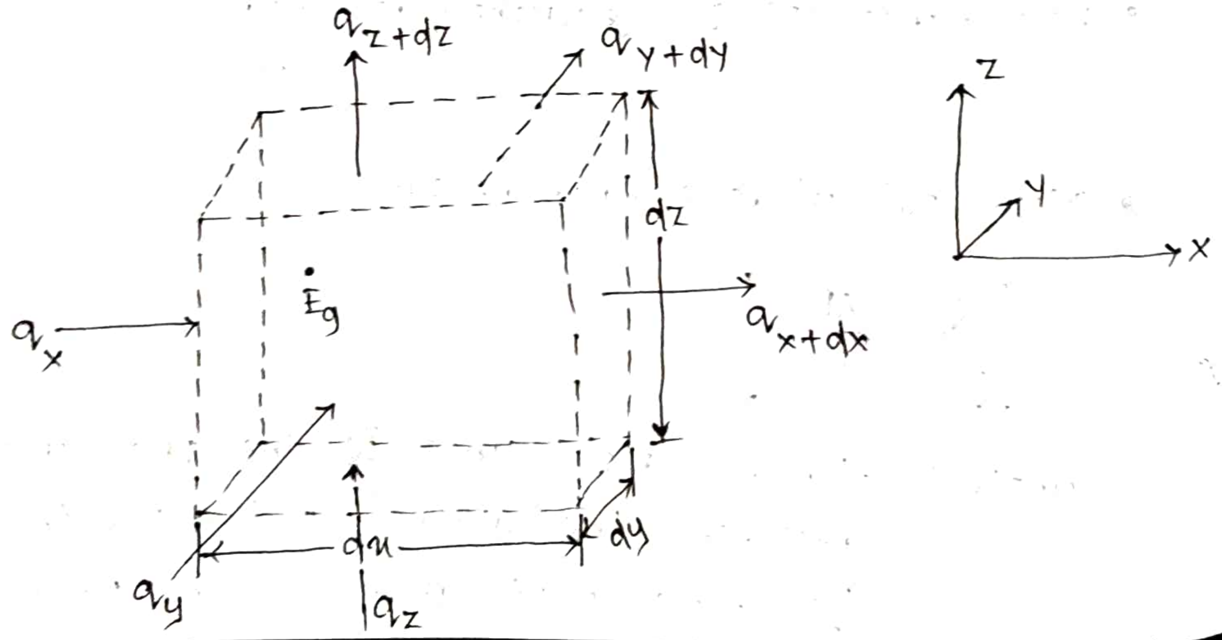
A = area

L = rod length

- Materials with large K are called conductors; those with small K are called insulators.

Heat Diffusion Equation :-

- The temperature distribution, which represents how temperature varies with position in the medium.



- The conduction heat rates perpendicular to each of the control surfaces at the x, y, z co-ordinate locations are indicated by the terms q_x, q_y and q_z respectively.
- Conduction heat rates at the opposite surfaces are :-

$$q_{x+dx} = q_x + \frac{\partial q_x}{\partial x} dx$$

$$q_{y+dy} = q_y + \frac{\partial q_y}{\partial y} dy$$

$$q_{z+dz} = q_z + \frac{\partial q_z}{\partial z} dz$$

- We know, the conservation of energy equation :-

$$\dot{E}_g + \dot{E}_{in} - \dot{E}_{out} = \dot{E}_s \quad \text{———} (1)$$

- Rate of change of energy generated is $\dot{E}_g = \dot{q}\, dx\, dy\, dz$

Now, by substituting this in eq^n (1) we will get :-

$$\dot{q}\, dx\, dy\, dz + q_x + q_y + q_z - q_{x+dx} - q_{y+dy} - q_{z+dz} = \dot{E}_s \quad \text{———} (2)$$

We know, $\dot{E}_s = \rho c_p \frac{\partial T}{\partial t} dx\, dy\, dz$

So, eq^n (2) will be :-

$$\dot{q}\, dx\, dy\, dz - \frac{\partial q_x}{\partial x} dx - \frac{\partial q_y}{\partial y} dy - \frac{\partial q_z}{\partial z} dz = \rho c_p \frac{\partial T}{\partial t} dx\, dy\, dz$$

$$\left[\because q_{x+dx} = q_x + \frac{\partial q_x}{\partial x} dx,\ q_{y+dy} = q_y + \frac{\partial q_y}{\partial y} dy,\ q_{z+dz} = q_z + \frac{\partial q_z}{\partial z} dz\right]$$

From this we can get :-

$q_x = -K\,dy\,dz\,\frac{dT}{dx}$, $q_y = -K\,dx\,dz\,\frac{dT}{dy}$ $q_z = -K\,dx\,dy\,\frac{dT}{dz}$

- At any point in the medium, the net rate of energy transfer by conduction into a unit volume plus the volumetric rate of thermal energy generation must equal to the rate of change of thermal energy stored within the volume. i.e.

$$\boxed{\frac{\partial}{\partial x}\left(K.\frac{\partial T}{\partial x}\right) + \frac{\partial}{\partial y}\left(K.\frac{\partial T}{\partial y}\right) + \frac{\partial}{\partial z}\left(K.\frac{\partial T}{\partial z}\right) + \dot{q} = \rho c.\frac{\partial T}{\partial t}}$$

For constant thermal conductivity (K) -

$$\boxed{\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} + \frac{\dot{q}}{K} = \frac{1}{\alpha}.\frac{\partial T}{\partial t}}$$

where, $\alpha = \frac{K}{\rho c}$ (thermal diffusivity)

For K = constant, Steady-state equation/condition, and no internal heat generation ($\dot{q}=0$), the above eqn can be written as :-

$$\underbrace{\boxed{\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} = 0}}_{\text{Laplace's Equation}} \quad \text{(or)} \quad \boxed{\nabla^2 T = 0}$$

- No. of boundary conditions required to solve the heat diffusion eqn is equal to the number of spatial dimensions multiplied by two.
- There is one initial condn which takes the form $T(x,y,z,0) = T_i$ (can be constant or a funcn of x, y & z)

Q. What is stream function and velocity potential ?

Stream function :—

- It is defined as the scalar function of space and time.
- It's partial derivative with respect to any direction gives the velocity component at right angle to that direction.
- $\psi = f(x,y)$, for steady-state 2-D flow.
- $v = \frac{\partial \psi}{\partial x}$ and $u = -\frac{\partial \psi}{\partial y}$

Velocity potential function :—

- It is defined as a scalar form of space and time such that its negative derivative with respect to any direction gives the velocity in that direction.
- $\phi = f(x,y,z)$ for steady state
- $u = -\frac{\partial \phi}{\partial x}$, $v = -\frac{\partial \phi}{\partial y}$ and $w = -\frac{\partial \phi}{\partial z}$
- u, v & w are the components of velocity in $x, y,$ & z direction respectively.

Q. Write the difference b/w stream func^n & velo. potential

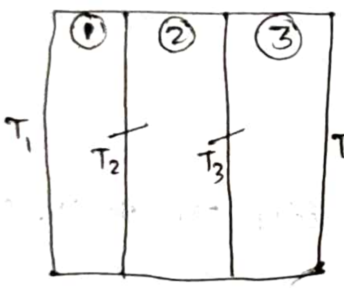
Stream function (ψ)	Velocity Potential (ϕ)
• Only 2-D flow	• all dimensional flow.
• Viscous or non-viscous flow,	• Irrotational flow i.e. inviscid or zero viscosity.
• compressible flow (only steady state)	• Compressible flow (steady or unsteady state).

Q. Calculate the rate of heat loss through the vertical walls of a boiler furnace of size 4m × 3m × 3m. The walls are constructed from an inner fire brick wall 25 cm thick of thermal conductivity 0.4 W/mK, a layer of ceramic blanket insulation of thermal conductivity 0.2 W/mK and 8cm thick, and a steel protective layer of thermal conductivity 55 W/mK and 2mm thick. The inside temp. of fire brick layer was measured at 600°c and the temp. of the outside of the insulation 60°C. Also find the interface temp. of layers.

Given, $K_1 = 0.4\, W/mK$ $\quad K_2 = 55\, W/mK$ $\quad K_3 = 0.2\, W/mK$

$x_1 = 0.25\, m$ $\quad x_2 = 0.002\, m$ $\quad x_3 = 0.08\, m$

$A = 12\, m^2$ $\quad T_1 = 600°C$, $T_4 = 60°C$



① → fire brick wall

② → Steel protective layer

③ → Ceramic blanket insulation

we know, $\dot{q}_x = -KA \cdot \frac{\Delta T}{L} = -\frac{\Delta T}{R}$ $\quad \left(\because R = \frac{x}{KA}\right)$

$\therefore R = R_1 + R_2 + R_3$

$= \frac{x_1}{K_1 A} + \frac{x_2}{K_2 A} + \frac{x_3}{K_3 A}$

$= \left(\frac{0.25}{0.4} + \frac{0.002}{55} + \frac{0.08}{0.2}\right) \times \frac{1}{12}$

$$= 1.025 \times \frac{1}{12}$$

$$= 0.085 \ K/W$$

$$\therefore \dot{q}_x = \frac{-\Delta T}{\Sigma R} = \frac{540 + \cancel{273} - \cancel{273}}{0.085} = 6353 \ W = 6.3 \ KW$$

Now, this $\dot{q}_x$ can also be written as :-

$$\dot{q}_x = \frac{T_1 - T_2}{R_1} = \frac{873 - T_2}{0.052}$$

$$\Rightarrow 873 - T_2 = 0.052 \times 6.3 \times 10^3$$

$$\Rightarrow T_2 = 545.4 \ K$$

$$\Rightarrow T_2 = 272.4^\circ C$$

Similarly, $\dot{q}_x = \frac{T_3 - T_4}{R_3}$

$$\Rightarrow 6.3 \times 10^3 = \frac{T_3 - 333}{0.033}$$

$$\Rightarrow T_3 - 333 = 207.9$$

$$\Rightarrow T_3 = 540.9 \simeq 541 \ K$$

$$\Rightarrow T_3 = 268^\circ C$$

$\therefore$ The Interface temperature of layers are :-

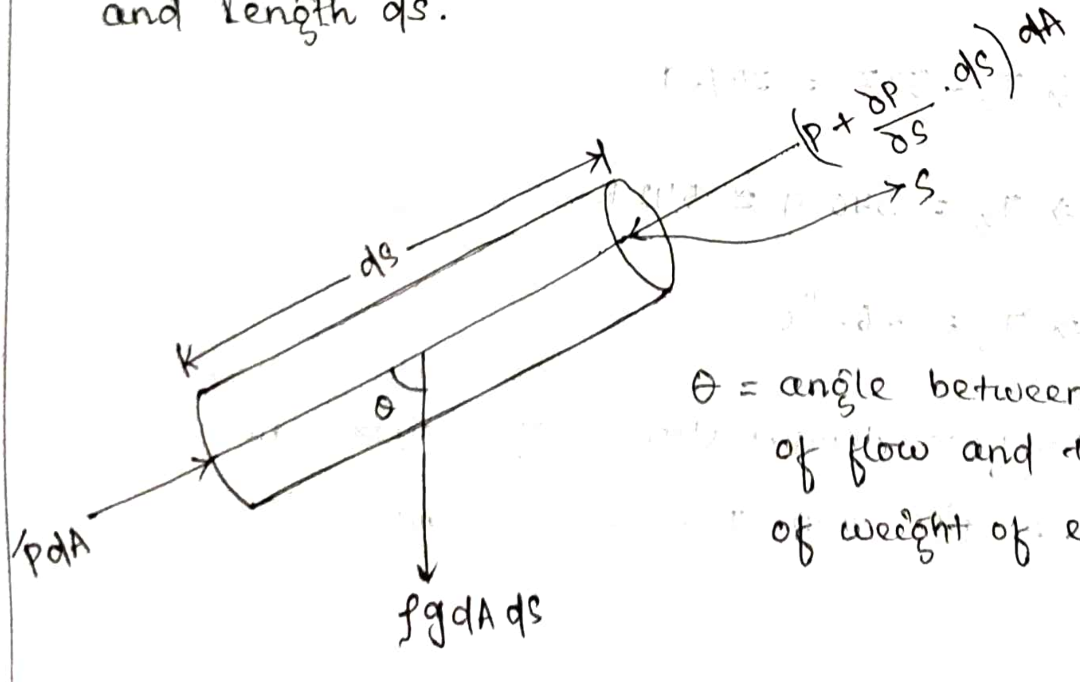
$T_2 = 272.4^\circ C$ and $T_3 = 268^\circ C$ (Ans)

Q. Derive Euler's equation of motion and hence deduce Bernoulli's equation from the same.

Euler's Equation :-

- The Euler's equation for a steady flow of an ideal fluid along a stream line is a relation between the velocity, pressure and density of a moving fluid.
- It is based on Newton's second law of motion.
- Assumptions :-
  → The fluid is non-viscous i.e. frictional losses are zero.
  → The fluid is homogeneous and incompressible i.e. the mass density of fluid is constant.
  → The flow is continuous, steady and along the stream-line.
  → The velocity of the flow is uniform over the section.
  → No energy or force (except gravity & pressure force)
- Derivation :-

Let us consider a streamline flow in 's' direction, cylindrical fluid element of cross sectional area dA and length ds.



θ = angle between the direction of flow and the line of action of weight of element.

Forces acting in the direction on the cylindrical element are :-

→ Body force or weight [$\rho g \, dA \, ds$] acting in the direction of gravitational field.

→ Pressure force [$P dA$] acting in the direction of flow

→ Pressure force [$(P + \frac{\partial P}{\partial s} \cdot ds) dA$] acting in the opposite direction of flow.

The pressure is not constant in a flow, but changes locally because the pressure differences are ultimately the reason why a flow comes about at all. Therefore, the pressure in the s-direction will change.

Now, the resultant force on the fluid element in the direction of s is :- $P dA - \left(P + \frac{\partial P}{\partial s} ds\right) dA - \rho g \, dA \, ds \cos\theta$

Euler's equation along a streamline is derived by applying Newton's 2nd law of motion. So, the resultant on the fluid element in the direction of 's' is also equals to :- mass of fluid element × acceleration

so, we can write :-

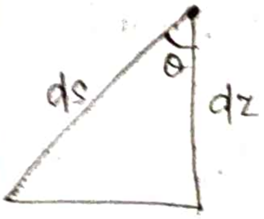
$$P dA - \left(P + \frac{\partial P}{\partial s} ds\right) dA - \rho g \, dA \, ds \cos\theta = \rho \, dA \, ds \cdot a_s$$

$$\Rightarrow \cancel{PdA} - \cancel{PdA} - \frac{\partial P}{\partial s} \cancel{ds} \cancel{dA} - \rho g \cancel{dA} \cancel{ds} \cos\theta = \rho \cancel{dA} \cancel{ds} \, a_s$$

$$\Rightarrow -\frac{\partial P}{\partial s} - \rho g \cos\theta = \rho \frac{\partial v}{\partial s} \cdot v \qquad \left(\because a_s = \frac{\partial v}{\partial s} \cdot \frac{ds}{dt} = \frac{\partial v}{\partial s} \cdot v\right)$$

$$\Rightarrow -\frac{1}{\rho} \cdot \frac{\partial P}{\partial s} - g\cos\theta = v \cdot \frac{\partial v}{\partial s}$$

$$\Rightarrow v \cdot \frac{\partial v}{\partial s} + \frac{1}{\rho} \cdot \frac{\partial P}{\partial s} + g\cos\theta = 0 \quad \text{——— (1)}$$



$$\cos\theta = \frac{dz}{ds}$$

Now, eq^n (1) can be written as :-

$$v \cdot \frac{\partial v}{\partial s} + \frac{1}{\rho} \cdot \frac{\partial p}{\partial s} + g \cdot \frac{dz}{ds} = 0$$

$$\Rightarrow \frac{1}{ds}\left(\frac{1}{\rho}\,\partial p + g\,dz + v \cdot dv\right) = 0$$

$$\Rightarrow \boxed{\frac{1}{\rho}\,dp + g\,dz + v\,dv = 0} \longrightarrow \text{Euler's equation.}$$

Now, in order to deduce the Bernoulli's equation from Euler's equation, we have to integrate the above eq^n and we will get :-

$$\frac{1}{\rho}\int dp + g\int dz + \int v\,dv = \text{constant}$$

$$\Rightarrow \frac{p}{\rho} + gz + \frac{v^2}{2} = \text{constant}$$

$$\Rightarrow \frac{p}{\rho g} + \frac{v^2}{2g} + z = \text{constant}$$

$$\Rightarrow \boxed{p + \frac{1}{2}\rho v^2 + \rho g z = \text{constant}}$$

$\Downarrow$

Bernoulli's equation for steady, incompressible, ideal and irrotational fluid flow.

- In a steady ideal flow of an incompressible fluid, the total energy at any point of the fluid is constant

Q Derive the continuity equation for three dimensional flow

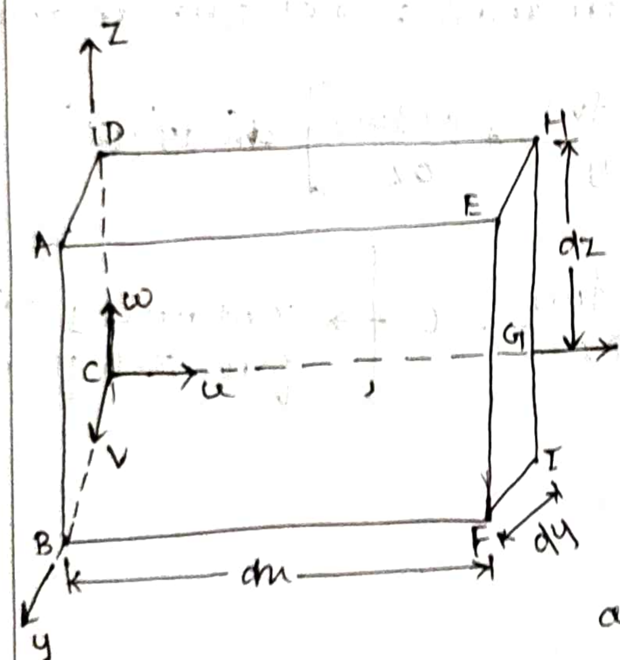
Continuity Equation :-

- In fluid mechanics, the conservation of mass relation written for a differential control volume.
- Continuity uses the conservation of mass to describe the relationship between the velocities of a fluid in different sections of a system.
- The continuity equation :-

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho V) = 0.$$

- Derivation of continuity equation for 3-D flow :-

Let us consider a fluid element of lengths dx, dy and dz in the x, y & z directions respectively.



Let u, v, w are the inlet velocity component in x, y & z direction respectively.

We know,

Mass of fluid entering ABCD :-

$\rho \times$ velocity in x-direcⁿ $\times$ Area

$= \rho \times u \times dy\, dz$

and mass of fluid leaving EFGH :-

$$\rho \times u \times dy\, dz + \frac{\partial(\rho . u . dy . dz)}{\partial x} . dx$$

So, Gain of mass in x-direction is equals to :-

Mass entering ABCD − Mass leaving EFGH

$$= -\frac{\partial(\rho . u . dy . dz)}{\partial x} . dx$$

Similarly, the gain of mass in y-direction $= -\frac{\delta(\rho v\, dx\, dz)}{\delta y} \cdot dy$

and gain of mass in z-direction $= -\frac{\delta(\rho w\, dx\, dy)}{\delta z} \cdot dz$

So, Net gain in mass of the fluid element :-

$$-\left[\frac{\delta(\rho u\, dy\, dz)}{\delta x} \cdot dx + \frac{\delta(\rho v\, dx\, dz)}{\delta y} dy + \frac{\delta(\rho w\, dx\, dy)}{\delta z} dz\right]$$

$$= -\left[\frac{\delta(\rho u)}{\delta x} + \frac{\delta(\rho v)}{\delta y} + \frac{\delta(\rho w)}{\delta z}\right] dx\, dy\, dz$$

we know, mass of the fluid element $= \rho\, dx\, dy\, dz$.

and rate of increase in mass $= \frac{\delta \rho}{\delta t}\, dx\, dy\, dz$

we know, rate of increase in mass = net gain in mass

$$\therefore \frac{\delta \rho}{\delta t} \cancel{dx\, dy\, dz} = -\left[\frac{\delta(\rho u)}{\delta x} + \frac{\delta(\rho v)}{\delta y} + \frac{\delta(\rho w)}{\delta z}\right] \cancel{dx\, dy\, dz}$$

$$\Rightarrow \boxed{\frac{\delta \rho}{\delta t} + \frac{\delta(\rho u)}{\delta x} + \frac{\delta(\rho v)}{\delta y} + \frac{\delta(\rho w)}{\delta z} = 0} \rightarrow \text{continuity eq}^n \text{ for 3-D flow.}$$

- For steady flow, $\frac{\delta \rho}{\delta t} = 0$.

So, $\frac{\delta(\rho u)}{\delta x} + \frac{\delta(\rho v)}{\delta y} + \frac{\delta(\rho w)}{\delta z} = 0$.

- If the fluid is incompressible, then ρ = constant

So, $\frac{\delta u}{\delta x} + \frac{\delta v}{\delta y} + \frac{\delta w}{\delta z} = 0$;

Q. A plate (2m x 2m), 0.25 mm distant apart from a fixed plate, moves at 40 cm/s and requires a force of 1N. Determine the dynamic viscosity of the fluid in b/w the plates.

Given, $u = 40 \text{ cm/s} = 0.4 \text{ m/s}$

$y = 0.25 \text{ mm} = 0.25 \times 10^{-3} \text{ m}.$

$F = 1 \text{ N}$

$A = 4 \text{ m}^2$

we know, Shear stress $(\tau) = F/A$

$$= \frac{1}{4}$$

$$= 0.25 \text{ N/m}^2$$

again, $\tau = \text{dynamic viscosity } (\mu) \times \frac{u}{y}$

$$\Rightarrow \mu = \tau \times \frac{y}{u}$$

$$\Rightarrow \mu = 0.25 \times \frac{0.25 \times 10^{-3}}{0.4} \text{ N.s/m}^2$$

$$\Rightarrow \mu = 1.56 \times 10^{-4} \text{ Ns/m}^2$$

$$\Rightarrow \mu = 1.56 \times 10^{-3} \text{ poise (Ans)}$$

Q. If $5 m^3$ of certain oil weighs 45 KN, calculate the specific weight, specific gravity and mass density of the oil

Given, weight of oil = 45 KN

volume = $5 m^3$

Specific weight (w) = $\frac{45}{5}$ KN/m^3

= $9 KN/m^3$

Specific gravity (S) = $\frac{\text{weight density of oil (w)}}{\text{weight density of water}}$

= $\frac{9 \, KN/m^3}{9.81 \, KN/m^3}$

= 0.917

mass density (ρ) = $\frac{\text{Specific weight (w)}}{\text{gravity (g)}}$

= $\frac{9 \times 10^3}{9.81}$ Kg/m^3

= $917.4 \, Kg/m^3$

Q. The two sides of a wall (3mm thick, with a cross-sectional area of $0.3 m^2$) are maintained at $40°C$ and $80°C$. The thermal conductivity of the wall is 1.28 $W/m°C$. Find out the rate of heat transfer through the wall.

Given, $L = 3\ mm = 3 \times 10^{-3} m$

$A = 0.3\ m^2$

$T_1 = 40°C$

$T_2 = 80°C$

$K = 1.28\ W/m°C$

We know, rate of heat transfer, $q_x = -KA \cdot \frac{dT}{L}$

$\therefore q_x = -1.28 \times 0.3 \times \frac{(40-80)}{3 \times 10^{-3}}$

$= \frac{1.28 \times 0.3 \times 40}{3 \times 10^{-3}}$

$= 5120\ W$

$= 5.12\ KW$ (Ans)

Q. A sphere, a cube and a thin circular pipe, all made of the same material and having the same mass are initially heated to the temperature of 250°C. After heating, these materials are left in air at room temperature for cooling. Explain the difference in response of sphere, cube & circular plate to cooling at room temperature

We know, that the rate of heat transfer is directly proportional to the surface area. Here, among a sphere, cube and circular plate; the circular plate has a greater surface area and the sphere has a lower surface area. Hence, circular plate will cool faster and sphere will cool the slower.

- Rate of cooling :-

  Circular plate > Cube > Sphere.

Q: What is the difference between thermal conductivity and specific heat capacity?

Thermal Conductivity (K) :-

- It is the intrinsic ability of a material to transfer or conduct heat.
- It also represents the quantity of thermal energy that flows per unit time through a unit area with a temperature gradient of 1° per unit distance.
- Units :- W/mK or Watt/meter.Kelvin
- Formula :- $K = \dfrac{QL}{A(T_2 - T_1)}$

where, K = Thermal conductivity

Q = heat flow (W)

L = Length or thickness of material (m)

A = Surface area (m^2)

$T_2 - T_1$ = Temperature gradient (K)

Specific Heat Capacity (c) :-

- It is the heat required to raise the temperature of the unit mass of a given substance by a given amount which is usually 1°.
- It is independent of the amount of substance.
- Units :- J/Kg.K or Jule/Kilogam.Kelvin
- Formula :- $Q = mC\Delta T$

where, Q = Heat energy (J)

m = mass (Kg)

c = specific heat capacity

ΔT = temperature gradient (K)

Q. Water is moving with a speed of 5.18 m/s through a pipe with a cross sectional area of 4.20 cm². Find the speed of flow of water as the pipe area increase to 7.60 cm².

Given, $v_1 = 5.18$ m/s

$A_1 = 4.2$ cm²

$A_2 = 7.6$ cm²

$v_2 = ?$

We know, $A_1 v_1 = A_2 v_2$

$\Rightarrow 4.2 \times 5.18 = 7.6 \times v_2$

$\Rightarrow v_2 = 2.86$ m/s (Ans)

Q. Explain the difference between Transepidermal route and Transfollicular route of drug penetration.

Transepidermal route of drug penetration :–

- Epidermal barrier function mainly resides in horny layer
- The viable layer may metabolize, inactivate or activate a prodrug.
- Dermal capillary contains many cappillaries, so residence time of drug is only one minute.
- Within stratum corneum molecule may penetrate either transcellularly or intercellularly.
- Intracellular region is filled with lipid rich amorphous material.

Transfollicular route of Drug penetration :–

- Fractional area available through this route is 0.1%
- Human skin contains 40-70 hair follicles, 200-250 sweat glands on every square centimeter of skin area.
- Mainly water soluable substances are diffused faster through appendages than that of other layers.
- Sweat glands and hair follicles act as a shunt ie. easy pathway for diffusion through rate limiting ST corneum.

Q. Explain the difference between Prandtl Number and Schmidt Number.

Prandtl Number :–

- It is the ratio of momentum diffusivity to heat diffusivity.

- Formula :- $Pr = \frac{\mu C_p}{K}$

where, μ = dynamic viscosity

K = thermal conductivity

C_p = Specific heat

- Significance - prandtl number is used to describe thermal boundary layer.

Schmidt Number :-

- It is the ratio of kinematic viscosity to the diffusivity.
- It can be expressed as :-

$$Sc = \frac{\mu}{\rho \cdot D}$$

where, μ = dynamic viscosity

ρ = fluid density

D = diffusivity.

Q. Describe the Four lung volumes and estimate the Total lung capacity.

The four lung volumes are :-

1. Tidal volume (TV) :-
   - The amount of gas inspired or expired with each normal breath.
   - It is about 500 ml.
2. Inspiratory Reserve Volume (IRV) :-
   - It is the maximum amount of additional air that

can be inspired from the end of a normal inspiration.

- It is about 2000 – 3000 mL.

3. Expiratory Reserve Volume (ERV):–

- It is the maximum volume of additional air that can be expired from the end of a normal expiration.
- It is about 1200 mL.

4. Residual volume (RV):–

- It is the volume of air remaining in the lungs after a maximal expiration.
- It is about 1200 mL.

Using this four basic lung volumes, we can estimate the total lung capacity. So,

Total lung capacity (TLC):–

- The volume of air contained in the lungs at the end of a maximal inspiration.
- It is the sum of 4 basic lung volumes.
- TLC = TV + IRV + ERV + RV
- It is approximately about 5000 mL.

Q. The Henry's law constant for O_2 in water at 25°C is 1.27×10^{-3} M/atm, and the mole fraction of O_2 in the atmosphere is 0.21. Calculate the solubility of O_2 in water at 25°C at an atmospheric pressure of 1 atm.

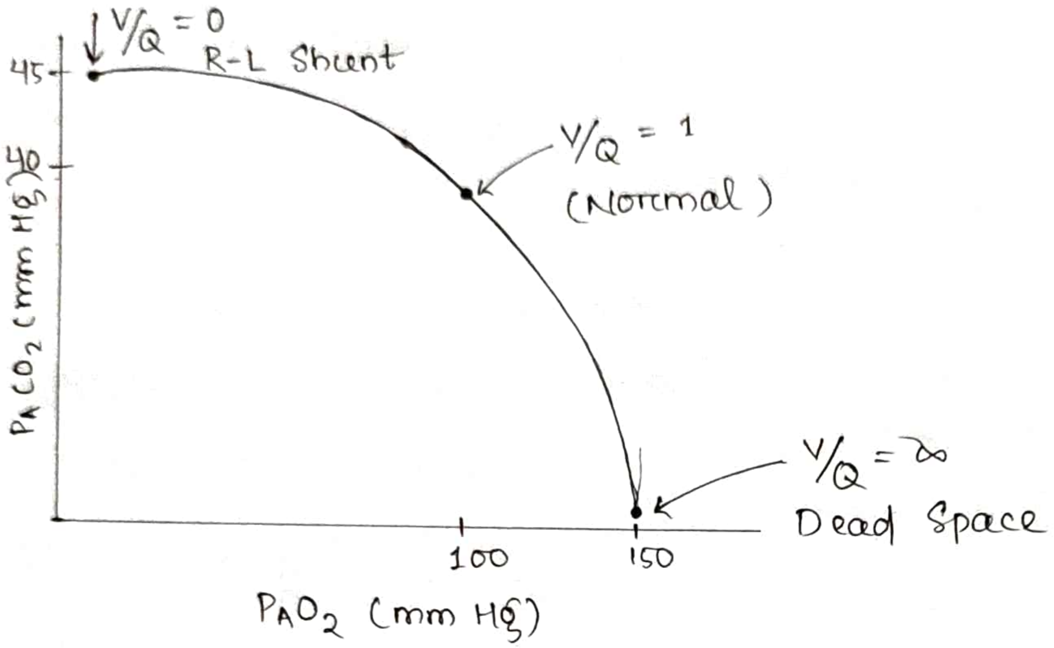
Given, $K_H = 1.27 \times 10^{-3}$ M/atm

$X_{O_2} = 0.21$

$P_{atm} = 1$ atm

Q. Draw a ventilation-perfusion ratio graph and explain.

- Ventilation-perfusion ratio graph plots the partial pressure of alveolar carbon dioxide (P_ACO_2) against that of alveolar oxygen (P_AO_2).



- $V/Q = 1$ (Normal) :-

→ At a normal V/Q ratio of 1, the alveolar gas tensions are equivalent to that of normal arterial blood.

→ As one moves to the right from this point, meaning greater ventilation to perfusion, the alveolar gas tensions approach that of inspired air.

→ Conversely, when one moves to the left of the $V/Q = 1$, meaning greater perfusion to ventilation, the alveolar gas tensions approach that of venous blood.

→ At this ratio of ventilation and perfusion, the alveolar air composition approaches a P_ACO_2 of roughly 40 mmHg and a P_AO_2 of roughly 100 mm Hg.

- $V/Q = 0$:—
  → The left most point of the curve, where the V/Q ratio equals zero.
  → This occurs when a perfused alveolus is not ventilated.
  → The alveolar ventilation (V) equals zero yet perfusion is preserved, the V/Q ratio approaches zero.
  → The P_AO_2 and P_ACO_2 approach the partial pressure for these gases in the venous blood.
  → The air within a non-ventilated alveolus can not be refreshed and thus the alveolus will be filled with gases at the same partial pressures of the blood perfusing it. Such an alveolus would effectively be contributing to a right-left (R-L) shunt of blood.

- $V/Q = \infty$ (Infinity) :—
  → The right most point on the curve indicates the scenario and occurs when perfusion to a ventilated alveolus is eliminated.
  → Because the rate of perfusion (Q) equals zero yet ventilation to the alveolus is preserved, the V/Q ratio approaches infinity.
  → As observed from the curve, the P_ACO_2 of the alveolus will equal zero ~~yet~~ whereas the P_AO_2 will approach that of external air.
  → No CO_2 will be delivered to a non-perfused alveolus and thus ventilation will eventually equilibrate alveolar air with that of external air.
  → Such an alveolus would effectively be considered part of the physiological Dead Space of the lung.

- Dead Space :-
  - → refer to those areas of lung which do not participate in gas exchange.
  - → Even lungs of healthy individuals display some dead space as this represents the volume of air that would fill the conducting airways with each breath.
  - → Because the conducting airways do not possess alveoli, any air inspired into those areas can not be used for gas exchange and thus is considered to exist in the dead space.
  - → However, in certain pathological scenarios, non-perfused alveoli may also be considered part of the dead space as many air entering non-perfused alveoli can not efficiently be used for gas exchange.

Q. Write a short-note on alveolar gas equation and its clinical significance.

<u>Alveolar Gas Equation</u> :-

- It is used to calculate the alveolar oxygen partial pressure.
- $P_AO_2 = FiO_2 (P_b - P_{H_2O}) - PaCO_2/RQ$

where, P_AO_2 = alveolar oxygen partial pressure

FiO_2 = fraction of inspired oxygen

P_b = atmospheric pressure

P_{H_2O} = partial pressure of H_2O (45 mm Hg)

$PaCO_2$ = partial pressure of CO_2 in alveoli

RQ = respiratory quotient, considered about 0.82 depends on metabolic activity and diet.

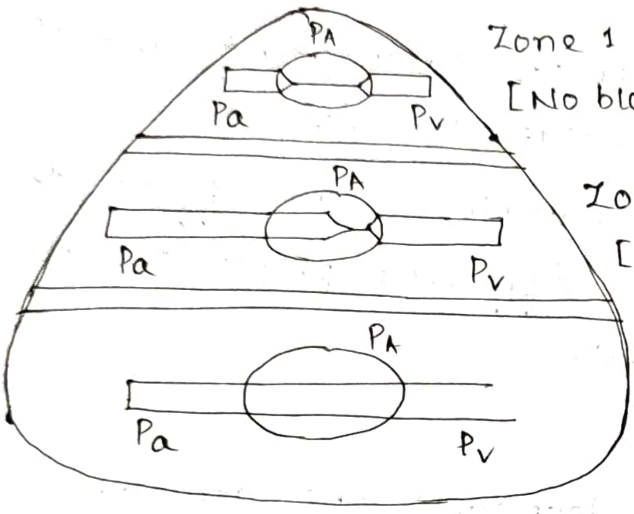
- The arterial P_{O_2} can be determined by obtaining an arterial blood gas.
- With the help of the alveolar gas equation, the partial pressure inside the alveoli can be calculated.
- Relates the alveolar conc. of oxygen F_AO_2 (or equivalently partial pressure P_AO_2) to three variable : FiO_2, P_ACO_2 and respiratory quotient (R).

Clinical Significance :-

- This equation is helpful in calculating and closely estimating the PaO_2 inside the alveoli.
- The variables in the equation can affect the $Pa\,O_2$ inside the alveoli in different physiological and pathophysiological states.
- Atmospheric pressure
- Increasing altitude, decreases the atmospheric pressure
- For any given FiO_2, there is a lower P_{O_2} in the atmosphere and a lower P_AO_2 in alveoli.
- For example, breathing 21% oxygen at sea level would result in an alveolar P_{O_2} close to 100 mm Hg, while breathing the same % oxygen at Mount Everest (at an atmospheric pressure of 263 mm Hg) would result in alveolar P_{O_2} close to 0 mm Hg.
- This can lead to hypoxemia and trigger many physiologic changes.
- At lower Patm, the PaO_2 will be lower; that's why air-plane cabins are pressurized.

Q. Explain the Pulmonary blood flow distribution.

- Blood pressure gradually increases as one travels down the lung's vertical axis.
- This vertical gradient of blood pressure can result in a nearly 20 mm Hg variation in pressure b/w the vasculature in the apex compared to that in the base of lung.



- In Zone 1, where no blood flows; the alveolar pressure (P_A) is greater than both the arterial (P_a) and venous pressure (P_v). [$P_A > P_a > P_v$].
- In Zone 2, where moderate blood flow occurs; the relevant pressure gradient is between the arterial and alveolar pressure. [$P_a > P_A > P_v$].
- In Zone 3, where the greatest blood flow occurs; the relevant pressure gradient is between the arterial and venous pressure. [$P_a > P_v > P_A$]
- <u>Zone-1</u> :-

→ Defined as those areas of the lung where alveolar pressure exceeds that of pulmonary arterial pressure.

→ Consequently, the pulmonary arteries become compressed and shut, eliminating blood flow in this zone and thus creating a zone of dead space.

→ It is likely does not exist in a healthy individual lungs as pulmonary arterial pressures just exceed that of alveolar pressure even at the top of the lung apex.

→ However, in scenarios of hypovolemia such as major hemorrhage, reductions in pulmonary arterial pressure may result in the generation of a zone 1 in the apical lung.

• Zone - 2 :—

→ Defined as those areas of the lung where the pulmonary arterial pressure exceeds that of the alveolar pressure.

→ The relevant pressure gradient determining the rate of blood flow is that between the arterial and alveolar pressures, and not between arterial and venous pressures.

→ This arises because the alveoli begins to compress the pulmonary capillaries and thus the alveolar pressure becomes a more limiting factor to blood flow than the pressure within the pulmonary veins.

→ It usually includes the top and middle portions of lung in a healthy upright individual at rest.

- **Zone - 3 :-**

→ Defined as those areas of the lung where the alveolar pressure falls below that of pulmonary venous pressure.

→ The blood pressure gradient determining blood flow is the traditional one between the pulmonary arterial and pulmonary venous pressure.

→ It is located in the bottom sections of the lung and represents the areas which receive the greatest rates of pulmonary blood flow.

Q. What is elimination half-life, volume of distribution and systemic clearance of a drug?

Elimination half-life :-

- Defined as the time taken for the amount of drug in the body as well as plasma concentration to decline by one-half or 50% its' initial value.

- Formula :- $[t_{1/2}] = 0.693/K_e$

where, K_e = First-order elimination rate constant.

- Half-life is a secondary parameter that depends upon the primary parameters, clearance and apparent volume of distribution.

- $$t_{1/2} = \frac{0.693\ [V_d]}{\text{Clearance}}$$

i.e. $$K_e = \frac{\text{Clearance}}{[V_d]}$$

Volume of distribution :-

- The volume of distribution of a drug gives information on its distribution in the body.
- The V_d is calculated as the ratio of the dose present in the body and its plasma concentration.
- Formula :- $V_d = \frac{Q}{C_0}$

where, Q = Drug dose

C_0 = plasma drug conc. at $t=0$.

Systemic Clearance :-

- It describes the efficiency with which drugs are permanently eliminated from the body.
- Both drug input rate and systemic clearance determine the average steady state plasma concentrations of a drug.
- Formula :- $C_T = C_R + C_H + \text{Others}$

where, C_T = Total systemic clearance

C_R = Renal clearance

C_H = Hepatic clearance

C_{others} = Other clearances

Q. Explain the Henderson - Hasselbach equation. Aspirin has a pK_a of 3.5 i) Calculate the ratio of ionised or unionized drug in stomach where pH is 1. (ii) calculate the ratio of ionized/unionized drug in intestine of pH 6

iii) Based on these calculations - where is aspirin absorbed within the body?

Henderson - Hasselbach equation :-

- H-H equation is used to calculate the percent ionization of a drug in cellular compartments of different pH.
- $pKa = -\log Ka$
- $pKa = pH$, when drug is 50% absorbed.
- Dissociation constant or pKa indicates the pH where 50% of the drug is ionized (water soluable) and 50% non-ionized (lipid soluable)

i) Given, $pK_a = 3.5$

pH (stomach) $= 1$

We know according to H-H equation :-

$$pH = pKa + \log [A^-]/[HA]$$

$$\therefore 1 = 3.5 + \log [A^-]/[HA]$$

$$\Rightarrow \log [A^-]/[HA] = -2.5$$

$$\Rightarrow \log [A^-]/[HA] = \log 10^{-2.5}$$

$$\Rightarrow \frac{[A^-]}{[HA]} = \frac{1}{10^{2.5}}$$

$$\Rightarrow [HA] = 316.23 \times [A^-]$$

So, [HA] is 316 fold greater than $[A^-]$.

[HA] moves from the stomach into the blood (good absorption).

(ii) Given; $pKa = 3.5$

pH (Intestine) $= 6$

Using H-H equation, $pH = pK_a + \log [A^-]/[HA]$

$\therefore 6 = 3.5 + \log [A^-]/[HA]$

$\Rightarrow \log [A^-]/[HA] = 2.5$

$\Rightarrow \log [A^-]/[HA] = \log 10^{2.5}$

$\Rightarrow \dfrac{[A^-]}{[HA]} = 10^{2.5}$

$\Rightarrow [A^-] = 316.23 \times [HA]$

So, A^- is 316 folder greater than HA.

Aspirin is readily absorbed from stomach into intestine.

(iii) Based on these calculations - in the (ii) case aspirin is absorbed within the body.

Q. Calculate density, and Specific weight of 1L of Petrol of specific gravity 0.7. Also find Specific volume

Given, Specific gravity $(S) = 0.7$

we know, Specific gravity $(S) = \dfrac{\text{Specific weight of liquid}}{\text{Specific weight of water}}$

So, Specific weight of petrol $(w) = 0.7 \times 9810 \ N/m^3$

$= 6867 \ N/m^3$

Again we know, Specific weight (w) = density (ρ) × gravity (g)

$\therefore$ density (ρ) $= \frac{w}{g}$

$= \frac{6867}{9.81}$ kg/m³

$= 700$ kg/m³

Specific volume $= \frac{1}{\rho}$

$= 1/700$

$= 1.43 \times 10^{-3}$ m³/kg

Q. Calculate the specific weight, density and specific gravity of a 1L of a liquid weight 7N.

Given, weight of liquid = 7N

Volume = 1L = 10^{-3} m³

we know, Specific weight (w) = $\frac{\text{weight of liquid}}{\text{volume of liquid}}$

$\therefore w = \frac{7}{10^{-3}}$ N/m³

$= 7000$ N/m³

again we know, Specific weight (w) = $\rho \times g$

$\therefore$ density (ρ) $= \frac{7000}{9.81}$ kg/m³

$= 713.56$ kg/m³

Specific gravity (s) $= \frac{\text{Specific weight of liquid}}{\text{Specific weight of water}}$

$= \frac{7000}{9810} = 0.713$

Q. If the velocity distribution over a plate is given by $u = \frac{2}{3}y - y^2$ in which u is the velocity in m/s at a distance y m. above the plate; determine the shear stress at $y=0$ and $y=0.15$m. Take dynamic viscosity (μ) of fluid as 8.63 poise.

Given, $\mu = 8.63$ poise $= 0.863$ N-s/m^2 ($\because$ 1 poise $= 0.1$ Ns/m^2)

velocity distribution (u) $= \frac{2}{3}y - y^2$

rate of shear strain $= \frac{du}{dy}$

$= \frac{2}{3} - 2y$

at $y=0$, $\frac{du}{dy} = \frac{2}{3} = 0.67$ s^{-1}

at $y = 0.15$, $\frac{du}{dy} = 0.37$ s^{-1}

we know, rate of shear stress $= \mu \times$ rate of shear strain

$\therefore \tau_1 = \mu \times \left.\frac{du}{dy}\right|_{y=0}$

$= 0.863 \times 0.67$

$= 0.575$ N/m^2

and $\tau_2 = \mu \times \left.\frac{du}{dy}\right|_{y=0.15}$

$= 0.863 \times 0.37 = 0.32$ N/m^2

Q. Calculate the capillary rise in a glass tube of 2.5 mm diameter, when immersed vertically in a) water and b) mercury. Given Surface tension of water (σ) = 0.0725 N/m and σ_{Hg} = 0.52 N/m in contact with air. The specific gravity of mercury is 13.6 and angle of contact is 130°

Given, $\sigma_w = 0.0725\ N/m$

$\sigma_{Hg} = 0.52\ N/m$

$d = 2.5 \times 10^{-3}\ m.$

$(\theta)_{Hg} = 130°$

$S_{Hg} = 13.6$

so, Capillary rise when immersed in water is :-

$$(h_r)_w = \frac{4\sigma \cos\theta}{d\rho g}$$

$$= \frac{4 \times 0.0725 \times 1}{2.5 \times 10^{-3} \times 1000 \times 9.81} \quad (\because \rho_w = 1000\ m^3)$$

$= 11.8\ mm$

$= 1.18\ cm.$

and capillary rise when immersed in mercury is :-

$$(h_r)_{Hg} = \frac{4 \times 0.52 \times \cos 130°}{2.5 \times 10^{-3} \times 13.6 \times 9810} \quad \left(\because S = \frac{w_l}{w_w} = \frac{\rho g}{9810}\right)$$

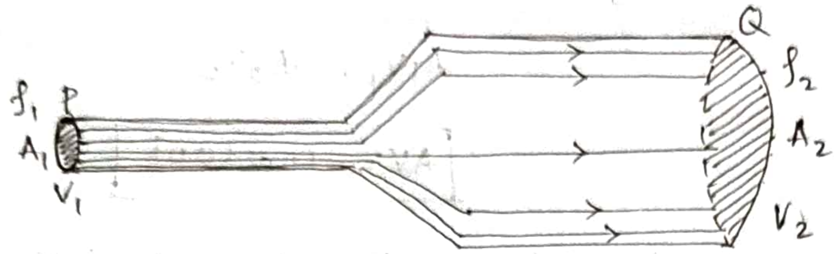
$= -4\ mm$

$= -0.4\ cm$ (capillary fall)

Q. Describe continuity eq^n for 1-D case ?

Continuity Equation in One Dimensional Case :-

Let us consider a tube of flow as :-



Here, A_1, A_2 = areas of cross-section perpendicular to fluid flow at positions P and Q is A_1 and A_2 respectively.

V_1, V_2 = velocities of fluid entering at P and leaving at Q respectively.

ρ_1, ρ_2 = densities of liquid at position P and Q respectively.

Now, the mass of the fluid (Δm_1) crossing the surface A_1 in the time interval Δt can be written as :-

$$\Delta m_1 = \rho_1 A_1 V_1 \Delta t \quad \text{——— (1)}$$

Similarly, the mass of the fluid (Δm_2) crossing the surface A_2 in the time interval Δt can be written as:-

$$\Delta m_2 = \rho_2 A_2 V_2 \Delta t \quad \text{——— (2)}$$

No fluid can leave through the walls and there is no source or sink in the above tube ie, in the control volume.

So, as per the conservation of mass we can write :-

$$\rho_1 A_1 V_1 = \rho_2 A_2 V_2$$

$$\Rightarrow \boxed{\rho AV = \text{constant}}$$

For incompressible fluid, $\rho_1 = \rho_2$

$$A_1 V_1 = A_2 V_2$$

$$\boxed{AV = \text{constant}}$$

The simple observation that the volume flow rate, must be the same throughout a system.

The volume flow rate $= \frac{\Delta V}{\Delta t} = \frac{A \Delta L}{\Delta t} = A v$ = Area x Velocity

Q. Solve the Differential equation : $y' + yx = y^2$

Bernoulli's equation form :- $\frac{dy}{dx} + P(x) \cdot y = Q(x) \cdot y^n$

Here, $P(x) = x$ and $Q(x) = 1$, $n = 2$

Given, $\frac{dy}{dx} + xy = y^2$ ——— (1)

dividing both sides by y^2 we get,

$$\frac{1}{y^2} \cdot \frac{dy}{dx} + \frac{x}{y} = 1 \quad \text{——— (2)}$$

Let $\frac{1}{y} = z$

$$\Rightarrow \frac{-1}{y^2} dy = dz$$

$$\Rightarrow \frac{-1}{y^2} \frac{dy}{dx} = \frac{dz}{dx}$$

So, Eqn (2) now can be written as :-

$$-\frac{dz}{dx} + xz = 1$$

$$\Rightarrow \frac{dz}{dx} - xz = -1 \quad \text{——— (3)}$$

Eq^n (3) is in the form $\frac{dy}{dx} + P(x) \cdot y = Q(x)$

where, $P(x) = -x$, $Q(x) = 1$

$$\therefore IF = e^{\int P(x)\,dx}$$

$$= e^{-\int x\,dx}$$

$$= e^{-x^2/2}$$

Multiplying both sides of eq^n (3) with IF, we will get :-

$$e^{-x^2/2} \cdot \frac{dz}{dx} - xze^{-x^2/2} = -e^{-x^2/2}$$

$$\Rightarrow d(z \cdot e^{-x^2/2}) = -e^{-x^2/2}\,dx$$

$$\Rightarrow ze^{-x^2/2} = -\int e^{-x^2/2}\,dx$$

$$\Rightarrow ze^{-x^2/2} = -\sqrt{2\pi} + c$$ (according to gamma function)

$$\Rightarrow \frac{1}{y} = \frac{c - \sqrt{2\pi}}{e^{-x^2/2}}$$

$$\Rightarrow y = \frac{e^{-x^2/2}}{c - \sqrt{2\pi}}$$ (Ans)

Q. A steady, 2-D velocity field is given by, $V = 3x\hat{i} - 3y\hat{j}$

Find the acceleration field.

We know, 2-D velocity will be of form :- $V = u\hat{i} + v\hat{j}$

So, here $u = 3x$ and $v = -3y$

Total acceleration = local acceleration + convective acc.

Since, the flow is in steady state so,

$$\frac{\delta u}{\delta t} = \frac{\delta v}{\delta t} = \frac{\delta w}{\delta t} = 0 \quad \text{i.e. local acceleration} = 0$$

So, Total acceleration = convective acceleration

i.e. $a_x = u \cdot \frac{\delta u}{\delta x} + v \cdot \frac{\delta u}{\delta y}$

$$= 3x \cdot \frac{\delta}{\delta x}(3x) - 3y \cdot \frac{\delta}{\delta y}(3x)$$

$$= 9x - 9y \cdot \frac{\delta x}{\delta y} \rightarrow 0$$

and $a_y = u \cdot \frac{\delta v}{\delta x} + v \cdot \frac{\delta v}{\delta y}$

$$= 3x \cdot \frac{\delta}{\delta x}(-3y) - 3y \cdot \frac{\delta}{\delta y}(-3y)$$

$$= -9x \cdot \frac{\delta y}{\delta x} + 9y \quad \rightarrow 0$$

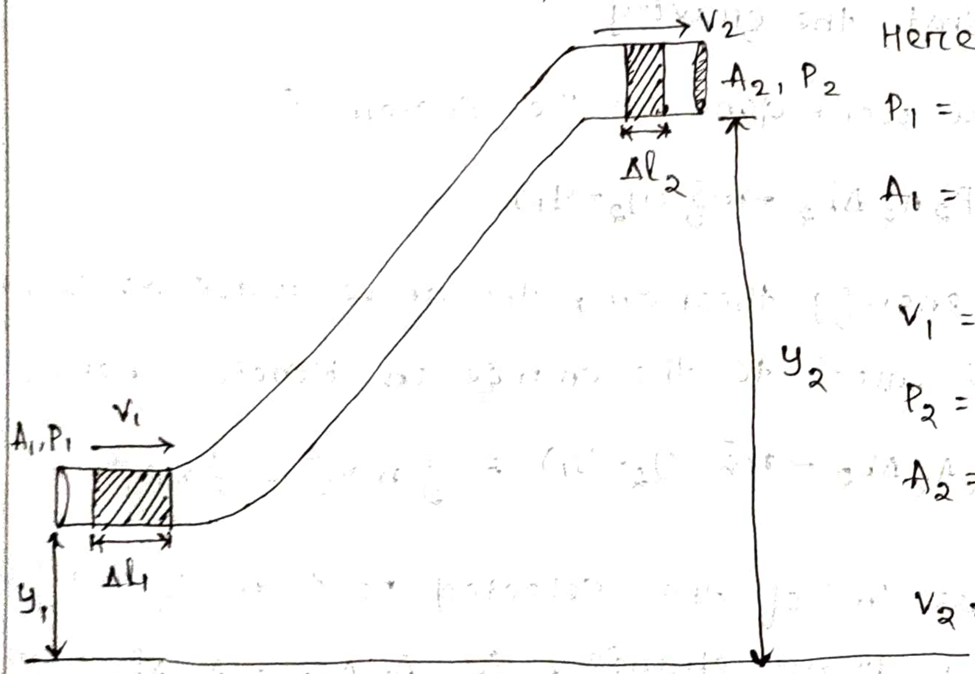
∴ Total acceleration $= 9x + 9y$

acceleration field $= 9x\hat{i} + 9y\hat{j}$

Q. Explain Bernoulli's Equation :-

Bernoulli's Equation :-

- It is a product of general work-energy theoram in fluid flow.
- According to this theoram, the work done by the resultant force acting on a system is equal to the change in kinetic energy of the system.
- For the derivation of Bernoulli's theoram, the fluid flow is considered as ideal with following conditions.
- The fluid flow should be steady, incompressible, irrotational and non-viscous.



Here,

P_1 = pressure at left side

A_1 = Area of cross section at left side

V_1 = velocity at left side

P_2 = Pressure at right

A_2 = Area of cross section at right side

V_2 = Velocity at right

let us consider a small portion of fluid over the length Δl_1, at left side having pressure P_1 and moving with a speed V_1.

when same fluid moves over to the right side, the pressure, speed and length of the fluid are P_2, V_2 and Δl_2 respectively.

The work done by the pressure force P_1A_1 on the system

$W_1 = P_1A_1\Delta l_1$

Similarly, work done by the pressure force P_2A_2 on the system is :-

$W_2 = -P_2A_2\Delta l_2$

Here, negative work implies that the work done is done by the system.

The work done on the system by gravity is :-

$W_g = -mg(y_2 - y_1)$

Negative sign implies that the work is done by the system against the gravity.

Now, the total work done on the system is :-

$W_T = P_1A_1\Delta l_1 - P_2A_2\Delta l_2 - mg(y_2 - y_1)$

As per work-energy theoram, the above work done on the system is equal to the change in kinetic energy.

So, $P_1A_1\Delta l_1 - P_2A_2\Delta l_2 - mg(y_2 - y_1) = \frac{1}{2}mv_2^2 - \frac{1}{2}mv_1^2$ ——— (1)

Since, the mass 'm' of the selected portion of the fluid and its density do not change, so $A_1\Delta l_1 = A_2\Delta l_2 = \frac{m}{\rho}$

So, now eqn (1) can be written as :-

$$(P_1 - P_2)\frac{\cancel{m}}{\rho} - \cancel{m}g(y_2 - y_1) = \frac{1}{2}\cancel{m}v_2^2 - \frac{1}{2}\cancel{m}v_1^2$$

$$\Rightarrow P_1 - P_2 - \rho g y_2 + \rho g y_1 = \frac{1}{2}\rho v_2^2 - \frac{1}{2}\rho v_1^2$$

$$\Rightarrow P_1 + \rho g y_1 + \frac{1}{2}\rho v_1^2 = P_2 + \rho g y_2 + \frac{1}{2}\rho v_2^2$$

$$\Rightarrow P + \rho g y + \frac{1}{2}\rho v^2 = \text{constant} \quad \text{——— (2)}$$

The above eq^n (2) is known as Bernoulli's equation for steady, incompressible, non-viscous, irrotational fluid flow.

For horizontal pipe, $y_1 = y_2$

Q: At 300°C the diffusion co-efficient and activation energy for Cu in Si are :- D (300°C) = 7.8×10^{-11} m²/s, $Q_d = 41.5$ KJ/mol. What is the diffusion co-efficient at 350°C ?

Given, $D = 7.8 \times 10^{-11}$ m²/s

$Q_d = 41.5$ KJ/mol

$T_1 = 300 + 273 = 573$ K

$T_2 = 350 + 273 = 623$ K

We know, $D = D_0 e^{-Q_d/RT}$

$$\therefore \frac{Q_d}{RT} = \frac{41.5 \times 10^3}{8.314 \times 573} = 8.711$$

$$\therefore e^{-Q_d/RT} = e^{-8.711} = 1.65 \times 10^{-4}$$

$$\therefore D_0 = \frac{7.8 \times 10^{-11}}{1.65 \times 10^{-4}} = 4.736 \times 10^{-7} \text{ m}^2/\text{s}$$

at $T = 623$ K, $D = 4.736 \times 10^{-7} \times e^{\frac{-41.5 \times 10^3}{8.314 \times 623}}$

$= 4.736 \times 10^{-7} \times 3.314 \times 10^{-4}$

$= 1.57 \times 10^{-10}$ m²/s (Ans)

Q. Methylene chloride is a common ingredient of paint removers. Besides being an irritant, it also may be absorbed through skin. When using this paint remover, protective gloves should be worn. If butyl rubber gloves (0.04 cm thick) are used, what is the diffusive flux of methylene chloride through the glove? Data :-

Diffusion co-efficien of butyl rubber : $D = 110 \times 10^{-8}$ cm²/s

Surface concentrations : $C_2 = 0.02$ g/cm³ $C_1 = 0.44$ g/cm³

We know, $J = -D \cdot \frac{dc}{dx}$

$$= -110 \times 10^{-8} \times \frac{0.44 - 0.02}{0.04} \ g/cm^2.s$$

$$= -11 \times 10^{-7} \times \frac{0.42}{0.04} \ g/cm^2.s$$

$$= -1.155 \times 10^{-5} \ g/cm^2.s$$

Molar mass of methylene chloride = 85 g/mol

$$\therefore J_A = \frac{-1.155 \times 10^{-5}}{85} \ mol/cm^2.s$$

$$= -1.35 \times 10^{-7} \ mol/cm^2.s \quad \text{(Ans)}$$

Q. A 0.12 M solution of a generic weak acid (HA) has a pH of 3.26. Determine the K_a.

For weak acid we know, $HA \rightleftharpoons H^+ + A^-$

So, $K_a = \frac{[H^+][A^-]}{[HA]}$

and $pH = -\log[H^+]$; $pK_a = -\log K_a$

Given, $[HA] = 0.12M$, $pH = 3.26$

So, $-\log[H^+] = 3.26$

$\log[H^+] = -3.26$

$[H^+] = 5.49 \times 10^{-4}$ M.

Because of 1:1 molar ratio, the conc. of $[A^-] = [H^+]$

So. $[A^-] = 5.49 \times 10^{-4}$ M.

Now, $K_a = \frac{[H^+][A^-]}{[HA]}$

$= \frac{(5.49 \times 10^{-4})^2}{0.12}$

$= 2.511 \times 10^{-6}$ M (Ans)

Q. A 0.128 M solution of uric acid ($HC_5H_3N_4O_3$) has a pH of 2.39. Calculate the Ka of Uric acid.

We know, For weak ~~base~~ acid :- $HA \rightleftharpoons H^+ + A^-$

Here, $HC_5H_3N_4O_3 \rightleftharpoons H^+ + C_5H_3N_4O_3^-$

and $pH = -\log[H^+]$

Given, $pH = 2.39$ i.e. $-\log[H^+] = 2.39$

So, $\log[H^+] = -2.39$

$[H^+] = 4.074 \times 10^{-3}$ M

Because of 1:1 molar ratio, $[H^+] = [C_5H_3N_4O_3^-]$

$\therefore [C_5H_3N_4O_3^-] = 4.074 \times 10^{-3}$ M

we know, $pK_a = \dfrac{[H^+][C_5H_3N_4O_3^-]}{[HC_5H_3N_4O_3]}$

$= \dfrac{(4.074 \times 10^{-3})^2}{0.128}$

$= 1.29 \times 10^{-4}$ M (Ans)

Q. $HC_9H_7O_4$ (molecular weight of 180 g/mol) is prepared by dissolving 3.6 g into a 1L solution. The pH of this solution was determined to be 2.6. What is the K_a?

Given, Molecular weight of $HC_9H_7O_4$ = 180 g/mol

dissolved amount = 3.6 g

So, no. of moles $= \dfrac{3.6}{180} = 0.02$ M

Given $pH = 2.6$ ie. $-\log[H^+] = 2.6$

$\log[H^+] = -2.6$

$[HC_9H_7O_4 \rightleftharpoons H^+ + C_9H_7O_4^-]$

$[H^+] = 2.512 \times 10^{-3}$ M.

Because of 1:1 molar ratio $[C_9H_7O_4^-] = 2.512 \times 10^{-3}$ M

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We know, $K_a = \frac{[H^+][C_9H_7O_4^-]}{[HC_9H_7O_4]}$

$$= \frac{(2.512 \times 10^{-3})^2}{0.02}$$

$$= 3.155 \times 10^{-4} \text{ M (Ans)}$$