

06. 22/02/2022

[120BM0014]

①

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TRANSPORT PROCESS

Q2)

$$\begin{aligned} u &= 3 + 3z \\ v &= 1 + 5y \\ w &= 9 - 3xz \end{aligned}$$

Q3)

Local Acceleration

1) It is the change in velocity with respect to time in a fluid flow.

2) This results when the flow is unsteady.

$$3) \boxed{a_L = \frac{\partial v}{\partial t}}$$

Convective Acceleration

1) It is the velocity at a point multiplied with the velocity gradient at that point within the fluid flow.

2) This results when the flow is non-uniform

$$3) \boxed{a_s = v \frac{\partial v}{\partial s}}$$

Q4) Stream line - A streamline is one that is drawn is tangential to the velocity vectors every point in the flow at a given instant and helps in understanding flows.

$$\frac{du}{u} = \frac{dv}{v} = \frac{dw}{w}$$

(3)

Stress form

$$\nabla \cdot \mathbf{u} = \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0$$

give 3 axes

$$\frac{\partial u}{\partial t} = \frac{\partial}{\partial t} \left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} \right)$$

$$f \left(\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} + w \frac{\partial u}{\partial z} \right) = - \frac{\partial p}{\partial x} + f g_x + \mu \left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} \right)$$

for x-axis

$$y\text{-axis} \rightarrow \text{replacing } \frac{\partial u}{\partial t} \text{ by } \frac{\partial^2 u}{\partial x^2} \text{ by } \frac{\partial v}{\partial t} \text{ by } \frac{\partial^2 v}{\partial x^2}$$

$$z\text{-axis} \rightarrow \frac{\partial u}{\partial t} \rightarrow \frac{\partial w}{\partial t} \quad f g_x \rightarrow f g_z$$

$$\frac{\partial^2 u}{\partial x^2} \rightarrow \frac{\partial^2 w}{\partial x^2}$$

Q7) Stream function (ψ) = $4x^2 - y^3$

$$u = - \frac{d\psi}{dy}$$

$$= 3y^2$$

So, at (2, 3)

$$u = 3(3)^2 = 27 \text{ m/s}$$

$$u = - \frac{d\psi}{dy}$$

$$- \int 3y^2 dy = - \psi$$

$$\Rightarrow \psi = - (3xy^2 + C)$$

ϕ = Velocity potential

$$u = - \frac{\partial \phi}{\partial x}$$

$$\begin{aligned} 27 &= - \frac{\partial \phi}{\partial x} \\ \int 27 dx &= - \phi + C \\ 27x + C &= - \phi \\ \Rightarrow \phi &= - (27x + C) \end{aligned}$$

②

Pathline - It is the line traced by a given particle.

This is generated by injecting a dye into the fluid and following its path by photography or other means.

Streakline - This concentrates on fluid particles that have gone through a fixed station or a point. At some instant of time the position of all these particles are marked a line is drawn.

So, the streamline, pathline & streakline coincide when the flow is steady. As during steady flow all the flow parameters do not change with time.

Q.5) Navier-Stokes equation, is a partial differential equation that describes the flow of incompressible fluid. This is a generalized equation of Euler.

$$\left[\frac{\partial u}{\partial t} + u \cdot \nabla u = -\frac{\nabla P}{\rho} + \nu \nabla^2 u \right]$$

This equation also expresses conservation of momentum and mass.

Continuity Equation

$$|\nabla \cdot \vec{V} = 0|$$

Partial Equation

$$\left[\rho \frac{DV}{Dt} = -\nabla P + \mu \nabla^2 V + \rho g \right]$$

change in velocity

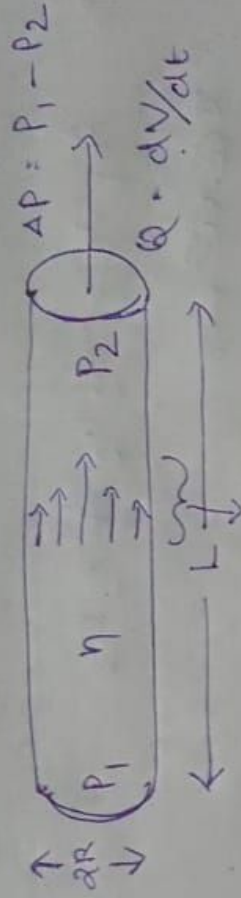
pressure flow

viscosity factor

represents external forces.

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So, the flow through the catheter.



η = viscosity

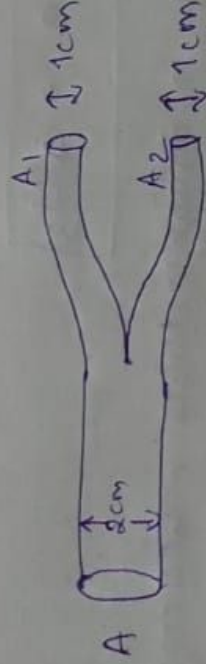
Parabolic velocity profile

This flow depends on factors like pressure gradient, length of tube, the radius & viscosity.

Q6) Average velocity = 1 m/s

diameter = 2 cm or 0.02 m

Two new vessels = 1 cm or 0.01 m diameter each.



So, discharge = area \times velocity

(parent vessel) = $\pi (0.01)^2 \times 1 \text{ m/s}$

= $3.14 \times 10^{-4} \times 1 \text{ m}^3/\text{s}$

= $3.14 \times 10^{-4} \text{ m}^3/\text{s}$

Q2)

$$u = 9 - 3x$$

$$v = 1 + 6y$$

$$w = 3 - 3z$$

For, incompressible liquid, we have the condition,

$$\frac{\partial(u)}{\partial x} + \frac{\partial(v)}{\partial y} + \frac{\partial(w)}{\partial z} = 0$$

LHS

$$\text{So, } \frac{\partial(9-3x)}{\partial x} + \frac{\partial(1+6y)}{\partial y} + \frac{\partial(3-3z)}{\partial z}$$

$$= -3 + 6 - 3$$

$$= 0 \quad (\text{RHS})$$

So, we can find that it satisfies the equation as $\text{LHS} = \text{RHS}$. So, the velocity represents incompressible flow.

Q1) the Poiseuille formula is;

$$Q = \frac{\pi \Delta P r^4}{8 \eta l}$$

⑥

As the diameter is same in the daughter tubes,

$$\text{So, } A_1 u_1 = A_2 u_2 + A_3 u_3$$

~~$$\pi (0.5)^2 \times 100$$~~

$$\pi (1)^2 \times 100 = \pi (0.5)^2 u_2 + \pi (0.5)^2 u_3$$

$$\Rightarrow 100 \cancel{\pi} = \cancel{\pi} (0.5)^2 (u_2 + u_3)$$

$$\Rightarrow \frac{100 \times 100}{5 \times 5} = u_2 + u_3$$

$$\Rightarrow 400 = u_2 + u_3$$

As, both tubes are identical, we can assume $u_2 = u_3$

$$\text{So, } u_2 + u_2 = 400$$

$$\Rightarrow \boxed{u_2 = 200 = u_3}$$

Average velocity in daughter tubes = 200 cm/s
= 2 m/s
