

Problem 1

```
import math
```

```
import time
```

```
def f(x):
```

```
    res = (4.1585*10**-5)*(1296*math.cos(0.5236*x)-1296+279.0565*x**2-31.007*x**3)-0.009
```

```
    return res
```

```
def dfdx(x):
```

```
    res = abs(0.00898*math.pi*math.sin((math.pi*x)/6)+0.0001248*((math.pi)**3)*x**2-  
0.0007485*((math.pi)**3)*x)
```

```
    return res
```

```
i = 1 # initial iteration
```

```
x0 = 1.8 # Initial guess
```

```
err = 1 #initial error
```

```
xi_1 = x0
```

```
print("Iteration: " + str(i) + ": x = " + str(x0) + ", f(x) = " +
```

```
    str(f(x0)) + "f'(x) = " + str(dfdx(x0)) + "ERROR = " + str(err))
```

```
# Iterating until either the tolerance or max iterations is met
```

```
while err > 0.0001:
```

```
    time.sleep(0.1)
```

```
    i = i + 1
```

```
    xi = xi_1-f(xi_1)/abs(dfdx(xi_1))
```

```
err = abs((xi-xi_1)/xi)
```

```
print("Iteration: " + str(i) + ": x = " + str(xi) + ", f(x) = " +
```

```
str(f(xi)) + "f'(x) = " + str(dfdx(xi)) + "ERROR = " + str(err))
```

```
#err = abs((xi-xi_1)/xi)
```

```
#print(err)
```

```
xi_1 = xi
```

```
>>> %Run 'CFD HW 3.py'
```

```
Iteration: 1: x = 1.8, f(x) = -0.0011372074972175305f'(x) = 0.006413722004639834ERROR = 1
```

```
Iteration: 2: x = 1.9773085107827948, f(x) = 8.098908431159943e-06f'(x) = 0.006498105070439908ERROR = 0.08967164699685648
```

```
Iteration: 3: x = 1.9760621615250702, f(x) = -3.028601189281055e-10f'(x) = 0.006497645246852146ERROR = 0.0006307237100085462
```

```
Iteration: 4: x = 1.9760622081358203, f(x) = 2.1913026948539027e-14f'(x) = 0.006497645264080185ERROR = 2.358769372570388e-08
```

```
>>>
```

$$y = \frac{15 \text{ kH} \cdot 3 \text{ m}}{3\pi^4 \cdot 70 \text{ GPa} \cdot 52.9 \cdot 10^{-6} \text{ m}^4} \cdot \left(48 \cdot (3 \text{ m})^3 \cos\left(\frac{\pi}{2(3 \text{ m})} x\right) - 48 \cdot (3 \text{ m})^3 + 3\pi^3 (3 \text{ m}) x^2 - \pi^3 x^3 \right) = 0.009$$

$$y = 4.1535 \cdot 10^{-5} (1296 \cos(0.5236x) - 1296 + 249.0565x^2 - 31.007x^3) - 0.009$$

$$y' = 0.00898\pi \sin\left(\frac{\pi x}{6}\right) + 0.0001243\pi^3 x^2 - 0.0007485\pi^3 x$$

$$x_{i+1} = x_i - \frac{f(x_i)}{f'(x_i)} \quad x_i = 3 \quad \epsilon = \left| \frac{x_i - x_0}{x_i} \right|$$

i	x_i	$f(x_i)$	$f'(x_i)$	ϵ_a
1	1.8	-0.001137	0.0064	100%
2	1.9773	$8.04 \cdot 10^{-6}$	0.0064	11.9%
3	1.97606	$4.297 \cdot 10^{-10}$	0.006498	0.0475%
4	1.97610		0.006498	$3.3467 \cdot 10^{-6}\%$
5				
6				

$$x_{i+1} = 1.8 - \frac{-0.001137}{0.0064} = 1.9773$$

$$\frac{1.9773 - 1.97606}{1.9773} = 0.000475$$

$$x_{\text{root}} = 1.97610$$

Problem 2

```
import time
```

```
print(" example = 3*x1+4*x2-x3 -> A*x1+B*x2-C*x3")
```

```
x11 = float(input("Equation 1 A=?"))
```

```
x12 = float(input("Equation 1 B=?"))
```

```
x13 = float(input("Equation 1 C=?"))
```

```
a1 = float(input("= ?: "))
```

```
x21 = float(input("Equation 2 A=?"))
```

```
x22 = float(input("Equation 2 B=?"))
```

```
x23 = float(input("Equation 2 C=?"))
```

```
a2 = float(input("= ?: "))
```

```
x31 = float(input("Equation 3 A=?"))
```

```
x32 = float(input("Equation 3 B=?"))
```

```
x33 = float(input("Equation 3 C=?"))
```

```
a3 = float(input("= ?: "))
```

```
print("original matrix: ")
```

```
print(x11, x12, x13, "|", a1)
```

```
print(x21, x22, x23, "|", a2)
```

```
print(x31, x32, x33, "|", a3)
```

```
time.sleep(1)
```

$$A = x_{21} - x_{11} * (x_{21} / x_{11})$$

$$B = x_{22} - x_{12} * (x_{21} / x_{11})$$

$$C = x_{23} - x_{13} * (x_{21} / x_{11})$$

$$a_2 = a_2 - a_1 * (x_{21} / x_{11})$$

$$A_1 = x_{31} - x_{11} * (x_{31} / x_{11})$$

$$B_1 = x_{32} - x_{12} * (x_{31} / x_{11})$$

$$C_1 = x_{33} - x_{13} * (x_{31} / x_{11})$$

$$a_3 = a_3 - a_1 * (x_{31} / x_{11})$$

print("first row pivoting matrix: ")

print(x₁₁, x₁₂, x₁₃, "|", a₁)

print(A, B, C, "|", a₂)

print(A₁, B₁, C₁, "|", a₃)

time.sleep(1)

$$A_1 = A_1 - A * (B_1 / B)$$

$$B_{11} = B_1 - B * (B_1 / B)$$

$$C_1 = C_1 - C * (B_1 / B)$$

$$a_3 = a_3 - a_2 * (B_1 / B)$$

print("second row pivoting matrix: ")

print(x₁₁, x₁₂, x₁₃, "|", a₁)

print(A, B, C, "|", a₂)

print(A₁, B₁₁, C₁, "|", a₃)

```
time.sleep(1)
```

```
x3 = a3/C1
```

```
x2 = (a2 - C*x3)/B
```

```
x1 = (a1 - x12*x2 - x13*x3)/x11
```

```
print("root of the matrix is: ", x1, "x2 = ", x2, "x3 = ", x3)
```

```
>>> %Run 'CFD HW 3.1.py'
    example = 3*x1+4*x2-x3 -> A*x1+B*x2-C*x3
Equation 1 A=?8
Equation 1 B=?2
Equation 1 C=?-2
= ? : -2
Equation 2 A=?10
Equation 2 B=?2
Equation 2 C=?4
= ? : 4
Equation 3 A=?12
Equation 3 B=?2
Equation 3 C=?2
= ? : 6
original matrix:
8.0 2.0 -2.0 | -2.0
10.0 2.0 4.0 | 4.0
12.0 2.0 2.0 | 6.0
first row pivoting matrix:
8.0 2.0 -2.0 | -2.0
0.0 -0.5 6.5 | 6.5
0.0 -1.0 5.0 | 9.0
second row pivoting matrix:
8.0 2.0 -2.0 | -2.0
0.0 -0.5 6.5 | 6.5
0.0 0.0 -8.0 | -4.0
root of the matrix is:  x1 =  1.5 x2 = -6.5 x3 =  0.5
```

$$\frac{10}{8} = \frac{5}{4}$$

$$\begin{aligned} 8x_1 + 2x_2 - 2x_3 &= -2 \\ 10x_1 + 2x_2 + 4x_3 &= 4 \\ 12x_1 + 2x_2 + 2x_3 &= 6 \end{aligned}$$

$$\left| \begin{array}{ccc|c} 8 & 2 & -2 & -2 \\ 10 & 2 & 4 & 4 \\ 12 & 2 & 2 & 6 \end{array} \right| \begin{array}{l} \\ R_2 - \frac{10}{8}R_1 \\ R_3 - \frac{3}{2}R_1 \end{array}$$

$$\left| \begin{array}{ccc|c} 8 & 2 & -2 & -2 \\ 0 & -0,5 & 6,5 & 6,5 \\ 0 & -1 & 5 & 9 \end{array} \right| R_3 - R_2 \cdot 2$$

$$\left| \begin{array}{ccc|c} 8 & 2 & -2 & -2 \\ 0 & -0,5 & 6,5 & 6,5 \\ 0 & 0 & -8 & -4 \end{array} \right|$$

$$-8x_3 = -4$$

$$x_3 = 0,5$$

$$-0,5x_2 + 6,5 \cdot 0,5 = 6,5$$

$$x_2 = -6,5$$

$$8x_1 + 2 \cdot (-6,5) - 2 \cdot 0,5 = -2$$

$$x_1 = 1,5$$