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1 Combinatorics

1.1 Burnside Lemma

```
1 #include<iostream>
2 #include <bits/stdc++.h>
3 #define IO ios_base::sync_with_stdio(0); cin.tie(0); cout.tie(0);
4 using namespace std;
5
6 // |Classes|=sum (k ^C(pi)) / |G|
7 // C(pi) the number of cycles in the permutation pi
8 // |G| the number of permutations
```

1.2 Catlan Numbers

```
1 void init() {
2     catalan[0] = catalan[1] = 1;
3     for (int i=2; i<=n; i++) {
4         catalan[i] = 0;
5         for (int j=0; j < i; j++) {
6             catalan[i] += (catalan[j] * catalan[i-j-1]) % MOD;
7             if (catalan[i] >= MOD) {
8                 catalan[i] -= MOD;
9             }
10        }
11    }
12 }
13 // 1- Number of correct bracket sequence consisting of n opening and n
14 //    closing brackets.
15 // 2- The number of rooted full binary trees with n+1 leaves (vertices
16 //    are not numbered).
17 // 3- The number of ways to completely parenthesize n+1 factors.
18 // 4- The number of triangulations of a convex polygon with n+2 sides
19 // 5- The number of ways to connect the 2n points on a circle to form
20 //    n disjoint chords.
```

```

18 // 6- The number of non-isomorphic full binary trees with n internal
    nodes (i.e. nodes having at least one son).
19 // 7- The number of monotonic lattice paths from point (0,0) to point
    (n,n) in a square lattice of size nxn, which do not pass above the
    main diagonal (i.e. connecting (0,0) to (n,n)).
20 // 8- Number of permutations of length n that can be stack sorted (it
    can be shown that the rearrangement is stack sorted if and only if
    there is no such index i<j<k, such that ak<ai<aj).
21 // 9- The number of non-crossing partitions of a set of n elements.
22 // 10- The number of ways to cover the ladder 1..n using n rectangles
    (The ladder consists of n columns, where ith column has a height i
    ).

```

2 Algebra

2.1 Gray Code

```

1 int g (int n) {
2     return n ^ (n >> 1);
3 }
4 int rev_g (int g) {
5     int n = 0;
6     for (; g; g >>= 1)
7         n ^= g;
8     return n;
9 }
10 int calc(int x, int y) { ///2D Gray Code
11     int a = g(x), b = g(y);
12     int res = 0;
13     f(i, 0, LG) {
14         int k1 = (a & (1 << i));
15         int k2 = (b & (1 << i));
16         res |= k1 << (i + 1);
17         res |= k2 << i;
18     }
19     return res;
20 }

```

2.2 Primitive Roots

```

1 int primitive_root (int p) {
2     vector<int> fact;
3     int phi = p - 1, n = phi;
4     for (int i = 2; i * i <= n; ++i)
5         if (n % i == 0) {
6             fact.push_back (i);
7             while (n % i == 0)
8                 n /= i;
9         }
10    if (n > 1)
11        fact.push_back (n);
12
13    for (int res = 2; res <= p; ++res) {
14        bool ok = true;
15        for (size_t i = 0; i < fact.size() && ok; ++i)
16            ok &= powmod (res, phi / fact[i], p) != 1;
17        if (ok) return res;
18    }
19    return -1;
20 }

```

2.3 Discrete Logarithm minimum x for which $a^x = b \% m$

```

1 // Returns minimum x for which a ^ x % m = b % m, a and m are coprime.
2 int solve(int a, int b, int m) {
3     a %= m, b %= m;
4     int n = sqrt(m) + 1;
5     int an = 1;
6     for (int i = 0; i < n; ++i)
7         an = (an * 1ll * a) % m;
8     unordered_map<int, int> vals;
9     for (int q = 0, cur = b; q <= n; ++q) {
10        vals[cur] = q;
11        cur = (cur * 1ll * a) % m;
12    }
13    for (int p = 1, cur = 1; p <= n; ++p) {
14        cur = (cur * 1ll * an) % m;
15        if (vals.count(cur)) {
16            int ans = n * p - vals[cur];

```

```

        return ans;
    }
    return -1;
}
//When a and m are not coprime
// Returns minimum x for which a ^ x % m = b % m.
24 int solve(int a, int b, int m) {
25     a %= m, b %= m;
26     int k = 1, add = 0, g;
27     while ((g = gcd(a, m)) > 1) {
28         if (b == k)
29             return add;
30         if (b % g)
31             return -1;
32         b /= g, m /= g, ++add;
33         k = (k * 1ll * a / g) % m;
34     }
35     int n = sqrt(m) + 1;
36     int an = 1;
37     for (int i = 0; i < n; ++i)
38         an = (an * 1ll * a) % m;
39     unordered_map<int, int> vals;
40     for (int q = 0, cur = b; q <= n; ++q) {
41         vals[cur] = q;
42         cur = (cur * 1ll * a) % m;
43     }
44     for (int p = 1, cur = k; p <= n; ++p) {
45         cur = (cur * 1ll * an) % m;
46         if (vals.count(cur)) {
47             int ans = n * p - vals[cur] + add;
48             return ans;
49         }
50     }
51     return -1;
52 }

```

2.4 Discrete Root finds all numbers x such that $x^k = a \% n$

```

1 // This program finds all numbers x such that x^k = a (mod n)
2 vector<int> discrete_root(int n, int k, int a) {
3     if (a == 0)
4         return {0};
5
6     int g = primitive_root(n);
7     // Baby-step giant-step discrete logarithm algorithm
8     int sq = (int) sqrt(n + .0) + 1;
9     vector<pair<int, int>> dec(sq);
10    for (int i = 1; i <= sq; ++i)
11        dec[i - 1] = {powmod(g, i * sq * k % (n - 1), n), i};
12    sort(dec.begin(), dec.end());
13    int any_ans = -1;
14    for (int i = 0; i < sq; ++i) {
15        int my = powmod(g, i * k % (n - 1), n) * a % n;
16        auto it = lower_bound(dec.begin(), dec.end(), make_pair(my, 0));
17        if (it != dec.end() && it->first == my) {
18            any_ans = it->second * sq - i;
19            break;
20        }
21    }
22    if (any_ans == -1) return {};
23
24    int delta = (n - 1) / __gcd(k, n - 1);
25    vector<int> ans;
26    for (int cur = any_ans % delta; cur < n - 1; cur += delta)
27        ans.push_back(powmod(g, cur, n));
28    sort(ans.begin(), ans.end());
29    return ans;
30 }

```

2.5 Factorial modulo in $p * \log(n)$ (Wilson Theroem)

```

1 int factmod(int n, int p) {
2     vector<int> f(p);
3     f[0] = 1;
4     for (int i = 1; i < p; i++)
5         f[i] = f[i-1] * i % p;
6
7     int res = 1;
8     while (n > 1) {

```

```

9         if ((n/p) % 2)
10             res = p - res;
11         res = res * f[n%p] % p;
12         n /= p;
13     }
14     return res;
15 }

```

2.6 Iteration over submasks

```

1 int s = m;
2 while (s > 0) {
3     s = (s-1) & m;
4 }

```

2.7 Totient function

```

1 void phi_1_to_n(int n) {
2     for (int i = 0; i <= n; i++)
3         phi[i] = i;
4     for (int i = 2; i <= n; i++) {
5         if (phi[i] == i) {
6             for (int j = i; j <= n; j += i)
7                 phi[j] -= phi[j] / i;
8         }
9     }
10 }

```

2.8 CRT and EEGCD

```

1 ll extended(ll a, ll b, ll &x, ll &y) {
2     if(b == 0) {
3         x = 1;
4         y = 0;
5         return a;
6     }
7     ll x0, y0;
8     ll g = extended(b, a % b, x0, y0);
9     x = y0;
10    y = x0 - a / b * y0;
11    return g;
12 }
13
14 ll de(ll a, ll b, ll c, ll &x, ll &y) {
15     ll g = extended(abs(a), abs(b), x, y);
16     if(c % g) return -1;
17     x *= c / g;
18     y *= c / g;
19     if(a < 0) x = -x;
20     if(b < 0) y = -y;
21     return g;
22 }
23
24 pair<ll, ll> CRT(vector<ll> r, vector<ll> m) {
25     ll r1 = r[0], m1 = m[0];
26     for(int i = 1; i < r.size(); i++) {
27         ll r2 = r[i], m2 = m[i];
28         ll x0, y0;
29         ll g = de(m1, -m2, r2 - r1, x0, y0);
30         if(g == -1) return {-1, -1};
31         x0 %= m2;
32         ll nr = x0 * m1 + r1;
33         ll nm = m1 / g * m2;
34         r1 = (nr % nm + nm) % nm;
35         m1 = nm;
36     }
37     return {r1, m1};
38 }

```

2.9 FFT

```

1 typedef complex<double> C;
2 typedef vector<double> vd;
3 typedef vector<int> vi;
4 typedef pair<int, int> pii;
5 void fft(vector<C>& a) {
6     int n = sz(a), L = 31 - __builtin_clz(n);
7     static vector<complex<long double>> R(2, 1);
8     static vector<C> rt(2, 1); // (^ 10% fas te r i f double)
9     for (static int k = 2; k < n; k *= 2) {
10         R.resize(n);
11         rt.resize(n);
12         auto x = polar(1.0L, acos(-1.0L) / k);

```

```

13         rep(i, k, 2 * k) rt[i] = R[i] = i & 1 ? R[i / 2] * x : R[i /
14             2];
15     }
16     vi rev(n);
17     rep(i, 0, n) rev[i] = (rev[i / 2] | (i & 1) << L) / 2;
18     rep(i, 0, n) if (i < rev[i]) swap(a[i], a[rev[i]]);
19     for (int k = 1; k < n; k *= 2)
20         for (int i = 0; i < n; i += 2 * k) rep(j, 0, k) {
21             C z = rt[j + k] * a[i + j + k]; //
22             a[i + j + k] = a[i + j] - z;
23             a[i + j] += z;
24         }
25     vd conv(const vd& a, const vd& b) {
26         if (a.empty() || b.empty()) return {};
27         vd res(sz(a) + sz(b) - 1);
28         int L = 32 - __builtin_clz(sz(res)), n = 1 << L;
29         vector<C> in(n), out(n);
30         copy(all(a), begin(in));
31         rep(i, 0, sz(b)) in[i].imag(b[i]);
32         fft(in);
33         for (C& x : in) x *= x;
34         rep(i, 0, n) out[i] = in[-i & (n - 1)] - conj(in[i]);
35         fft(out);
36         /// rep(i, 0, sz(res)) res[i] = (MOD+(ll)round(imag(out[i]) / (4 * n
37             ))) % MOD; ///in case of mod
38         rep(i, 0, sz(res)) res[i] = imag(out[i]) / (4 * n);
39         return res;
40     }
41
42 int main() {
43     //Applications
44     //1-All possible sums
45     //2-All possible scalar products
46     // We are given two arrays a[] and b[] of length n.
47     //We have to compute the products of a with every cyclic shift of
48     // b.
49     //We generate two new arrays of size 2n: We reverse a and append n
50     // zeros to it.
51     //And we just append b to itself. When we multiply these two
52     // arrays as polynomials,
53     //and look at the coefficients c[n-1], c[n], ..., c[2n-2] of the
54     // product c, we get:
55     //c[k]=sum i+j=k a[i]b[j]
56     //3-Two stripes
57     //We are given two Boolean stripes (cyclic arrays of values 0 and
58     // 1) a and b.
59     //We want to find all ways to attach the first stripe to the
60     // second one,
61     //such that at no position we have a 1 of the first stripe next to
62     // a 1 of the second stripe.

```

2.10 Fibonacci

```

1 // F(n-1) * F(n+1) - F(n)^2 = (-1)^n
2 // F(n+k) = F(k) * F(n+1) + F(k-1) * F(n)
3 // F(2*n) = F(n) * (F(n+1) + F(n-1))
4 // GCD ( F(m) , F(n) ) = F(GCD(n,m))

```

2.11 Gauss Determinant

```

1 double det(vector<vector<double>>& a) {
2     int n = sz(a); double res = 1;
3     rep(i, 0, n) {
4         int b = i;
5         rep(j, i+1, n) if (fabs(a[j][i]) > fabs(a[b][i])) b = j;
6         if (i != b) swap(a[i], a[b]), res *= -1;
7         res *= a[i][i];
8         if (res == 0) return 0;
9         rep(j, i+1, n) {
10             double v = a[j][i] / a[i][i];
11             if (v != 0) rep(k, i+1, n) a[j][k] -= v * a[i][k];
12         }
13     }
14     return res;
15 }
16 // for integers
17 const ll mod = 12345;
18 ll det(vector<vector<ll>>& a) {

```

```

19 int n = sz(a); ll ans = 1;
20 rep(i,0,n) {
21     rep(j,i+1,n) {
22         while (a[j][i] != 0) { // gcd step
23             ll t = a[i][i] / a[j][i];
24             if (t) rep(k,i,n)
25                 a[i][k] = (a[i][k] - a[j][k] * t) % mod;
26             swap(a[i], a[j]);
27             ans *= -1;
28         }
29     }
30     ans = ans * a[i][i] % mod;
31     if (!ans) return 0;
32 }
33 return (ans + mod) % mod;
34 }

```

2.12 GAUSS SLAE

```

1 const double EPS = 1e-9;
2 const int INF = 2; // it doesn't actually have to be infinity or a big
   number
3
4 int gauss (vector < vector<double> > a, vector<double> & ans) {
5     int n = (int) a.size();
6     int m = (int) a[0].size() - 1;
7
8     vector<int> where (m, -1);
9     for (int col = 0, row = 0; col < m && row < n; ++col) {
10         int sel = row;
11         for (int i = row; i < n; ++i)
12             if (abs (a[i][col]) > abs (a[sel][col]))
13                 sel = i;
14         if (abs (a[sel][col]) < EPS)
15             continue;
16         for (int i = col; i <= m; ++i)
17             swap (a[sel][i], a[row][i]);
18         where[col] = row;
19
20         for (int i = 0; i < n; ++i)
21             if (i != row) {
22                 double c = a[i][col] / a[row][col];
23                 for (int j = col; j <= m; ++j)
24                     a[i][j] -= a[row][j] * c;
25             }
26         ++row;
27     }
28
29     ans.assign (m, 0);
30     for (int i = 0; i < m; ++i)
31         if (where[i] != -1)
32             ans[i] = a[where[i]][m] / a[where[i]][i];
33     for (int i = 0; i < n; ++i) {
34         double sum = 0;
35         for (int j = 0; j < m; ++j)
36             sum += ans[j] * a[i][j];
37         if (abs (sum - a[i][m]) > EPS)
38             return 0;
39     }
40
41     for (int i = 0; i < m; ++i)
42         if (where[i] == -1)
43             return INF;
44     return 1;
45 }

```

2.13 Matrix Inverse

```

1 #define ld long double
2 vector < vector<ld> > gauss (vector < vector<ld> > a) {
3
4     int n = (int) a.size();
5     vector<vector<ld> > ans(n, vector<ld>(n, 0));
6
7     for(int i = 0; i < n; i++)
8         ans[i][i] = 1;
9     for(int i = 0; i < n; i++) {
10         for(int j = i + 1; j < n; j++)
11             if(a[j][i] > a[i][i]) {
12                 a[j].swap(a[i]);
13                 ans[j].swap(ans[i]);
14             }
15         ld val = a[i][i];
16         for(int j = 0; j < n; j++) {

```

```

17             a[i][j] /= val;
18             ans[i][j] /= val;
19         }
20         for(int j = 0; j < n; j++) {
21             if(j == i) continue;
22             val = a[j][i];
23             for(int k = 0; k < n; k++) {
24                 a[j][k] -= val * a[i][k];
25                 ans[j][k] -= val * ans[i][k];
26             }
27         }
28     }
29     return ans;
30 }

```

2.14 NTT of KACTL

```

1 const ll mod = (119 << 23) + 1, root = 62; // = 998244353
2 // For p < 2^30 there is a lso e . g . 5 << 25, 7 << 26, 479 << 21
3 // and 483 << 21 (same root) . The 1 as t two are > 10^9.
4 typedef vector<ll> vl;
5 void ntt(vl &a) {
6     int n = sz(a), L = 31 - __builtin_clz(n);
7     static vl rt(2, 1);
8     for (static int k = 2, s = 2; k < n; k *= 2, s++) {
9         rt.resize(n);
10         ll z[] = {1, modpow(root, mod >> s)};
11         rep(i,k,2*k) rt[i] = rt[i / 2] * z[i & 1] % mod;
12     }
13     vi rev(n);
14     rep(i,0,n) rev[i] = (rev[i / 2] | (i & 1) << L) / 2;
15     rep(i,0,n) if (i < rev[i]) swap(a[i], a[rev[i]]);
16     for (int k = 1; k < n; k *= 2)
17         for (int i = 0; i < n; i += 2 * k) rep(j,0,k) {
18             ll z = rt[j + k] * a[i + j + k] % mod, &ai = a[i + j];
19             a[i + j + k] = ai - z + (z > ai ? mod : 0);
20             ai += (ai + z >= mod ? z - mod : z);
21         }
22 }
23 vl conv(const vl &a, const vl &b) {
24     if (a.empty() || b.empty()) return {};
25     int s = sz(a) + sz(b) - 1, B = 32 - __builtin_clz(s), n = 1 << B;
26     int inv = modpow(n, mod - 2);
27     vl L(a), R(b), out(n);
28     L.resize(n), R.resize(n);
29     ntt(L), ntt(R);
30     rep(i,0,n) out[-i & (n - 1)] = (ll)L[i] * R[i] % mod * inv % mod;
31     ntt(out);
32     return {out.begin(), out.begin() + s};
33 }

```

3 Data Structures

3.1 2D BIT

```

1 void upd(int x, int y, int val) {
2     for(int i = x; i <= n; i += i & -i)
3         for(int j = y; j <= m; j += j & -j)
4             bit[i][j] += val;
5 }
6 int get(int x, int y) {
7     int ans = 0;
8     for(int i = x; i; i -= i & -i)
9         for(int j = y; j; j -= j & -j)
10             ans += bit[i][j];
11 }

```

3.2 2D Sparse table

```

1 /*
2     note this isn't the best cache-wise version
3     query O(1), Build O(NMlgNlgM)
4     be careful when using it and note the he build a dimension above
   another
5     i.e he builds a sparse table for each row
6     the build sparse table over each row's sparse table
7 */
8 const int N = 505, LG = 10;
9

```

```

10 int st[N][N][LG][LG];
11 int a[N][N], lg2[N];
12
13 int yo(int x1, int y1, int x2, int y2) {
14     x2++;
15     y2++;
16     int a = lg2[x2 - x1], b = lg2[y2 - y1];
17     return max(
18         max(st[x1][y1][a][b], st[x2 - (1 << a)][y1][a][b]),
19         max(st[x1][y2 - (1 << b)][a][b], st[x2 - (1 << a)][y2 - (1 << b)][a][b])
20     );
21 }
22
23 void build(int n, int m) { // 0 indexed
24     for (int i = 2; i < N; i++) lg2[i] = lg2[i >> 1] + 1;
25     for (int i = 0; i < n; i++) {
26         for (int j = 0; j < m; j++) {
27             st[i][j][0][0] = a[i][j];
28         }
29     }
30     for (int a = 0; a < LG; a++) {
31         for (int b = 0; b < LG; b++) {
32             if (a + b == 0) continue;
33             for (int i = 0; i + (1 << a) <= n; i++) {
34                 for (int j = 0; j + (1 << b) <= m; j++) {
35                     if (!a) {
36                         st[i][j][a][b] = max(st[i][j][a][b - 1], st[i][j + (1 << (b - 1))][a][b - 1]);
37                     } else {
38                         st[i][j][a][b] = max(st[i][j][a - 1][b], st[i + (1 << (a - 1))][j][a - 1][b]);
39                     }
40                 }
41             }
42         }
43     }
44 }

```

3.3 Mo's

```

1 #include <bits/stdc++.h>
2
3 int n, qq, arr[N], sz = 1000; // sz is the size of the bucket
4 int co[N], ans = 0, ansq[N];
5 int cul = 1, cur = 1;
6
7 void add(int x) {
8     co[arr[x]]++;
9     if (co[arr[x]] == 1)
10         ans++;
11     else if (co[arr[x]] == 2)
12         ans--;
13 }
14
15 void remove(int x) {
16     co[arr[x]]--;
17     if (co[arr[x]] == 1)
18         ans++;
19     else if (co[arr[x]] == 0)
20         ans--;
21 }
22
23 void solve(int l, int r, int ind) {
24     r+=1;
25     while (cul < l) remove(cul++);
26     while (cul > l) add(--cul);
27     while (cur < r) add(cur++);
28     while (cur > r) remove(--cur);
29     ansq[ind] = ans;
30 }
31
32
33 int main() {
34     FIO
35     cin >> qq;
36     // {l/sz, r}, {1, ind}
37     priority_queue<pair<pair<int, int>, pair<int, int>>, vector<pair<pair<int, int>, pair<int, int>>>, greater<pair<pair<int, int>, pair<int, int>>>> q;
38     for (int i = 0; i < qq; i++) {
39         int l, r;
40         cin >> l >> r;
41         q.push({{l / sz, r}, {l, i}});

```

```

42     }
43     while (q.size()) {
44         int ind=q.top().second.second, l=q.top().second.first, r=q.top().first.second;
45         solve(l, r, ind);
46         q.pop();
47     }
48     for (int i = 0; i < qq; i++)
49         cout << ansq[i] << endl;
50
51     return 0;
52 }
53

```

3.4 Mo With Updates

```

1 //O(N^5/3) note that the block size is not a standard size
2
3
4 #pragma GCC optimize ("O3")
5 #pragma GCC target ("sse4")
6
7 #include <bits/stdc++.h>
8
9 using namespace std;
10
11 using ll = long long;
12
13 const int N = 1e5 + 5;
14 const int M = 2 * N;
15 const int blk = 2155;
16 const int mod = 1e9 + 7;
17 struct Query{
18     int l, r, t, idx;
19     Query(int a = 0, int b = 0, int c = 0, int d = 0) {l=a, r=b, t=c, idx = d;}
20     bool operator < (Query o) {
21         if (r / blk == o.r / blk && l / blk == o.l / blk) return t < o.t;
22         if (r / blk == o.r / blk) return l < o.l;
23         return r < o.r;
24     }
25 } Q[N];
26
27 int a[N], b[N];
28 int cnt1[M], cnt2[N];
29 int L = 0, R = -1, K = -1;
30 void add(int x) { //add item to range
31     // cout << x << '\n';
32     cnt2[cnt1[x]]--;
33     cnt1[x]++;
34     cnt2[cnt1[x]]++;
35 }
36 void del(int x) { //delete item from range
37     cnt2[cnt1[x]]--;
38     cnt1[x]--;
39     cnt2[cnt1[x]]++;
40 }
41 map<int, int> id;
42 int cnt;
43 int ans[N];
44 int p[N], nxt[N];
45 int prv[N];
46 void upd(int idx) { //update item value
47     if (p[idx] >= L && p[idx] <= R)
48         del(a[p[idx]]), add(nxt[idx]);
49     a[p[idx]] = nxt[idx];
50 }
51 void err(int idx) {
52     if (p[idx] >= L && p[idx] <= R)
53         del(a[p[idx]]), add(prv[idx]);
54     a[p[idx]] = prv[idx];
55 }
56 int main() {
57
58     int n, q, l, r, tp;
59
60     scanf("%d%d", &n, &q);
61
62     for (int i = 0; i < n; i++) {
63         scanf("%d", &a[i]);
64         if (id.count(a[i]) == 0)
65             id[a[i]] = cnt++;
66         a[i] = id[a[i]];
67         b[i] = a[i];

```

```

68 }
69 int qIdx = 0;
70 int ord = 0;
71 while(q--) {
72     scanf("%d", &tp);
73     if(tp == 1) {
74         /// ADD Query
75         scanf("%d%d", &l, &r); --l, --r;
76         Q[qIdx] = Query(l, r, ord-1, qIdx); qIdx++;
77     } else {
78         /// ADD Update
79         scanf("%d%d", p + ord, nxt + ord); --p[ord];
80         if(id.count(nxt[ord]) == 0)
81             id[nxt[ord]] = cnt++;
82         nxt[ord] = id[nxt[ord]];
83         prv[ord] = b[p[ord]];
84         b[p[ord]] = nxt[ord];
85         ++ord;
86     }
87 }
88 }
89 sort(Q, Q+qIdx);
90 for(int i = 0; i < qIdx; i++) {
91     while(L < Q[i].l) del(a[L++]);
92     while(L > Q[i].l) add(a[--L]);
93     while(R < Q[i].r) add(a[++R]);
94     while(R > Q[i].r) del(a[R--]);
95     while(K < Q[i].t) upd(++K);
96     while(K > Q[i].t) err(K--);
97     ///Solve Query I
98 }
99 }
100 for(int i = 0; i < qIdx; i++)
101     printf("%d\n", ans[i]);
102
103 return 0;
104 }
105

```

3.5 Ordered Set

```

1 #include <ext/pb_ds/assoc_container.hpp>
2 #include <ext/pb_ds/tree_policy.hpp>
3 using namespace __gnu_pbds;
4
5 #define ordered_set tree<int, null_type, less<int>, rb_tree_tag,
6     tree_order_statistics_node_update>
7
8 // order_of_key(k): returns the number of elements in the set strictly
9 less than k
10 // find_by_order(k): returns an iterator to the k-th element (zero-
11 based) in the set

```

3.6 Persistent Seg Tree

```

1
2 int val[ N * 60 ], L[ N * 60 ], R[ N * 60 ], ptr, tree[N]; /// N * lgN
3 int upd(int root, int s, int e, int idx) {
4     int ret = ++ptr;
5     val[ret] = L[ret] = R[ret] = 0;
6     if (s == e) {
7         val[ret] = val[root] + 1;
8         return ret;
9     }
10    int md = (s + e) >> 1;
11    if (idx <= md) {
12        L[ret] = upd(L[root], s, md, idx), R[ret] = R[root];
13    } else {
14        R[ret] = upd(R[root], md + 1, e, idx), L[ret] = L[root];
15    }
16    val[ret] = max(val[L[ret]], val[R[ret]]);
17    return ret;
18 }
19 int qry(int node, int s, int e, int l, int r) {
20     if(r < s || e < l || !node) return 0; ///Punishment Value
21     if(l <= s && e <= r) {
22         return val[node];
23     }
24     int md = (s+e)>>1;
25     return max(qry(L[node], s, md, l, r), qry(R[node], md+1, e, l, r));
26 }
27 int merge(int x, int y, int s, int e) {
28     if(!x||!y) return x | y;

```

```

29     if(s == e) {
30         val[x] += val[y];
31         return x;
32     }
33     int md = (s + e) >> 1;
34     L[x] = merge(L[x], L[y], s, md);
35     R[x] = merge(R[x], R[y], md+1, e);
36     val[x] = val[L[x]] + val[R[x]];
37     return x;
38 }

```

3.7 Treap

```

1 mt19937_64 mrand(chrono::steady_clock::now().time_since_epoch().count
2     ());
3 struct Node {
4     int key, pri = mrand(), sz = 1;
5     int lz = 0;
6     int idx;
7     array<Node*, 2> c = {NULL, NULL};
8     Node(int key, int idx) : key(key), idx(idx) {}
9 };
10 int getsz(Node* t) {
11     return t ? t->sz : 0;
12 }
13 Node* calc(Node* t) {
14     t->sz = 1 + getsz(t->c[0]) + getsz(t->c[1]);
15     return t;
16 }
17 void prop(Node* cur) {
18     if(!cur || !cur->lz)
19         return;
20     cur->key += cur->lz;
21     if(cur->c[0])
22         cur->c[0]->lz += cur->lz;
23     if(cur->c[1])
24         cur->c[1]->lz += cur->lz;
25     cur->lz = 0;
26 }
27 array<Node*, 2> split(Node* t, int k) {
28     prop(t);
29     if(!t)
30         return {t, t};
31     if(getsz(t->c[0]) >= k) { ///answer is in left node
32         auto ret = split(t->c[0], k);
33         t->c[0] = ret[1];
34         return {ret[0], calc(t)};
35     } else { ///k > t->c[0]
36         auto ret = split(t->c[1], k - 1 - getsz(t->c[0]));
37         t->c[1] = ret[0];
38         return {calc(t), ret[1]};
39     }
40 }
41 Node* merge(Node* u, Node* v) {
42     prop(u);
43     prop(v);
44     if(!u || !v)
45         return u ? u : v;
46     if(u->pri > v->pri) {
47         u->c[1] = merge(u->c[1], v);
48         return calc(u);
49     } else {
50         v->c[0] = merge(u, v->c[0]);
51         return calc(v);
52     }
53 }
54 int cnt(Node* cur, int x) {
55     prop(cur);
56     if(!cur)
57         return 0;
58     if(cur->key <= x)
59         return getsz(cur->c[0]) + 1 + cnt(cur->c[1], x);
60     return cnt(cur->c[0], x);
61 }
62 Node* ins(Node* root, int val, int idx, int pos) {
63     auto splitted = split(root, pos);
64     root = merge(splitted[0], new Node(val, idx));
65     return merge(root, splitted[1]);
66 }

```


3.8 Wavelet Tree

```

1 // remember your array and values must be 1-based
2 struct wavelet_tree {
3     int lo, hi;
4     wavelet_tree *l, *r;
5     vector<int> b;
6
7     //nos are in range [x,y]
8     //array indices are [from, to]
9     wavelet_tree(int *from, int *to, int x, int y) {
10         lo = x, hi = y;
11         if (lo == hi or from == to)
12             return;
13         int mid = (lo + hi) / 2;
14         auto f = [mid](int x) {
15             return x <= mid;
16         };
17         b.reserve(to - from + 1);
18         b.pb(0);
19         for (auto it = from; it != to; it++)
20             b.pb(b.back() + f(*it));
21         //see how lambda function is used here
22         auto pivot = stable_partition(from, to, f);
23         l = new wavelet_tree(from, pivot, lo, mid);
24         r = new wavelet_tree(pivot, to, mid + 1, hi);
25     }
26
27     //kth smallest element in [l, r]
28     int kth(int l, int r, int k) {
29         if (l > r)
30             return 0;
31         if (lo == hi)
32             return lo;
33         int inLeft = b[r] - b[l - 1];
34         int lb = b[l - 1]; //amt of nos in first (l-1) nos that go in
35         //left
36         int rb = b[r]; //amt of nos in first (r) nos that go in left
37         if (k <= inLeft)
38             return this->l->kth(lb + 1, rb, k);
39         return this->r->kth(l - lb, r - rb, k - inLeft);
40     }
41
42     //count of nos in [l, r] Less than or equal to k
43     int LTE(int l, int r, int k) {
44         if (l > r or k < lo)
45             return 0;
46         if (hi <= k)
47             return r - l + 1;
48         int lb = b[l - 1], rb = b[r];
49         return this->l->LTE(lb + 1, rb, k) + this->r->LTE(l - lb, r -
50         rb, k);
51     }
52
53     //count of nos in [l, r] equal to k
54     int count(int l, int r, int k) {
55         if (l > r or k < lo or k > hi)
56             return 0;
57         if (lo == hi)
58             return r - l + 1;
59         int lb = b[l - 1], rb = b[r], mid = (lo + hi) / 2;
60         if (k <= mid)
61             return this->l->count(lb + 1, rb, k);
62         return this->r->count(l - lb, r - rb, k);
63     }
64 };

```

3.9 SparseTable

```

1 int S[N];
2 for (int i = 2; i < N; i++) S[i] = S[i >> 1] + 1;
3
4 for (int i = 1; i <= K; i++)
5     for (int j = 0; j + (1 << i) <= N; j++)
6         st[i][j] = f(st[i - 1][j], st[i - 1][j + (1 << (i - 1))]);
7
8 int query(int l, int r) {
9     int k = S[r - l + 1];
10    return mrg(st[k][l], st[k][r - (1 << k) + 1]);
11 }
12

```

4 DP

4.1 Dynamic Connectivity with SegTree

```

1 #define f(i, a, b) for(int i = a; i < b; i++)
2 #define all(a) a.begin(), a.end()
3 #define sz(x) (int)(x).size()
4 typedef long long ll;
5 const int N = 1e5 + 5;
6
7 struct PT {
8     ll x, y;
9     PT() {}
10    PT(ll a, ll b) : x(a), y(b) {}
11    PT operator-(const PT &o) { return PT{x - o.x, y - o.y}; }
12    bool operator<(const PT &o) const { return make_pair(x, y) <
13    make_pair(o.x, o.y); }
14 };
15
16 ll cross(PT x, PT y) {
17     return x.x * y.y - x.y * y.x;
18 }
19
20 PT val[300005];
21 bool in[300005];
22 ll qr[300005];
23 bool ask[300005];
24 ll ans[N];
25 vector<PT> t[300005 * 4]; //segment tree holding points to queries
26
27 void update(int node, int s, int e, int l, int r, PT x) {
28     if (r < s || e < l) return;
29     if (l <= s && e <= r) { //add this point to maximize it with
30     queries in this range
31         t[node].push_back(x);
32         return;
33     }
34     int md = (s + e) >> 1;
35     update(node << 1, s, md, l, r, x);
36     update(node << 1 | 1, md + 1, e, l, r, x);
37 }
38
39 vector<PT> stk;
40 inline void addPts(vector<PT> v) {
41     stk.clear(); //reset the data structure you are using
42     sort(all(v));
43     //build upper envelope
44     for (int i = 0; i < v.size(); i++) {
45         while (sz(stk) > 1 && cross(v[i] - stk.back(), stk.back() -
46         stk[stk.size() - 2]) <= 0)
47             stk.pop_back();
48         stk.push_back(v[i]);
49     }
50 }
51
52 inline ll calc(PT x, ll val) {
53     return x.x * val + x.y;
54 }
55
56 ll query(ll x) {
57     if (stk.empty())
58         return LLONG_MIN;
59     int lo = 0, hi = stk.size() - 1;
60     while (lo + 10 < hi) {
61         int md = lo + (hi - lo) / 2;
62         if (calc(stk[md + 1], x) > calc(stk[md], x))
63             lo = md + 1;
64         else
65             hi = md;
66     }
67     ll ans = LLONG_MIN;
68     for (int i = lo; i <= hi; i++)
69         ans = max(ans, calc(stk[i], x));
70     return ans;
71 }
72
73 void solve(int node, int s, int e) { //Solve queries
74     addPts(t[node]); //note that there is no need to add/delete
75     //just build for t[node]
76     f(i, s, e + 1) {
77         if (ask[i]) {
78             ans[i] = max(ans[i], query(qr[i]));
79         }
80     }
81     if (s == e) return;
82 }

```

```

72     int md = (s + e) >> 1;
73     solve(node << 1, s, md);
74     solve(node << 1 | 1, md + 1, e);
75 }
76 void doWork() {
77     int n;
78     cin >> n;
79     stk.reserve(n);
80     f(i, 1, n + 1) {
81         int tp;
82         cin >> tp;
83         if (tp == 1) {    ///Add Query
84             int x, y;
85             cin >> x >> y;
86             val[i] = PT(x, y);
87             in[i] = 1;
88         } else if (tp == 2) {    ///Delete Query
89             int x;
90             cin >> x;
91             if (in[x]) update(1, 1, n, x, i - 1, val[x]);
92             in[x] = 0;
93         } else {
94             cin >> qr[i];
95             ask[i] = true;
96         }
97     }
98     f(i, 1, n + 1)    ///Finalize Query
99     if (in[i])
100         update(1, 1, n, i, n, val[i]);
101
102     f(i, 1, n + 1) ans[i] = LLONG_MIN;
103     solve(1, 1, n);
104     f(i, 1, n + 1) if (ask[i]) {
105         if (ans[i] == LLONG_MIN)
106             cout << "EMPTY SET\n";
107         else
108             cout << ans[i] << '\n';
109     }
110 }
111 }

```

4.2 CHT Line Container

```

1  struct Line {
2      mutable ll m, b, p;
3      bool operator<(const Line &o) const { return m < o.m; }
4      bool operator<(ll x) const { return p < x; }
5  };
6
7  struct LineContainer : multiset<Line, less<>> {
8      // (for doubles, use inf = 1/.0, div(a,b) = a/b)
9      static const ll inf = LLONG_MAX;
10
11      ll div(ll db, ll dm) { // floored division
12          return db / dm - ((db ^ dm) < 0 && db % dm);
13      }
14
15      bool isect(iterator x, iterator y) {
16          if (y == end()) {
17              x->p = inf;
18              return false;
19          }
20          if (x->m == y->m)
21              x->p = x->b > y->b ? inf : -inf;
22          else
23              x->p = div(y->b - x->b, x->m - y->m);
24          return x->p >= y->p;
25      }
26
27      void add(ll m, ll b) {
28          auto z = insert({m, b, 0}), y = z++, x = y;
29          while (isect(y, z)) z = erase(z);
30          if (x != begin() && isect(--x, y))
31              isect(x, y = erase(y));
32          while ((y = x) != begin() && (--x)->p >= y->p)
33              isect(x, erase(y));
34      }
35
36      ll query(ll x) {
37          assert(!empty());
38          auto l = *lower_bound(x);
39          return l.m * x + l.b;
40      }
41  };

```

5 Geometry

5.1 Convex Hull

```

1  struct point {
2      ll x, y;
3      point(ll x, ll y) : x(x), y(y) {}
4      point operator -(point other) {
5          return point(x - other.x, y - other.y);
6      }
7      bool operator <(const point &other) const {
8          return x != other.x ? x < other.x : y < other.y;
9      }
10 };
11 ll cross(point a, point b) {
12     return a.x * b.y - a.y * b.x;
13 }
14 ll dot(point a, point b) {
15     return a.x * b.x + a.y * b.y;
16 }
17 struct sortCCW {
18     point center;
19
20     sortCCW(point center) : center(center) {}
21
22     bool operator()(point a, point b) {
23         ll res = cross(a - center, b - center);
24         if (res)
25             return res > 0;
26         return dot(a - center, a - center) < dot(b - center, b - center);
27     }
28 };
29 vector<point> hull(vector<point> v) {
30     sort(v.begin(), v.end());
31     sort(v.begin() + 1, v.end(), sortCCW(v[0]));
32     v.push_back(v[0]);
33     vector<point> ans;
34     for(auto i : v) {
35         int sz = ans.size();
36         while (sz > 1 && cross(i - ans[sz - 1], ans[sz - 2] - ans[sz - 1]) <= 0)
37             ans.pop_back(), sz--;
38         ans.push_back(i);
39     }
40     ans.pop_back();
41     return ans;
42 }

```

5.2 Geometry Template

```

1  using ptype = double; edit this first;
2  double EPS = 1e-9;
3  struct point {
4      ptype x, y;
5      point(ptype x, ptype y) : x(x), y(y) {}
6      point operator -(const point &other) const { return point(x - other.x, y - other.y); }
7      point operator +(const point &other) const { return point(x + other.x, y + other.y); }
8      point operator *(ptype c) const { return point(x * c, y * c); }
9      point operator /(ptype c) const { return point(x / c, y / c); }
10     point prep() { return point(-y, x); }
11 };
12 ptype cross(point a, point b) { return a.x * b.y - a.y * b.x; }
13 ptype dot(point a, point b) { return a.x * b.x + a.y * b.y; }
14 double abs(point a) { return sqrt(dot(a, a)); }
15
16 double angle(point a, point b) { // angle between [0, pi]
17     return acos(dot(a, b) / abs(a) / abs(b));
18 }
19 // a : point in Line, d : Line direction
20 point LineLineIntersect(point a1, point d1, point a2, point d2) {
21     return a1 + d1 * cross(a2 - a1, d2) / cross(d1, d2);
22 }
23 // Line a---b, point C
24 point ProjectPointLine(point a, point b, point c) {
25     return a + (b - a) * 1.0 * dot(c - a, b - a) / dot(b - a, b - a);
26 }

```



```

27 // segment a---b, point C
28 point ProjectPointSegment(point a, point b, point c) {
29     double r = dot(c - a, b - a) / dot(b - a, b - a);
30     if(r < 0)
31         return a;
32     if(r > 1)
33         return b;
34     return a + (b - a) * r;
35 }
36 // Line a---b, point p
37 point reflectAroundLine(point a, point b, point p) {
38     return ProjectPointLine(a, b, p) * 2 - p; // (proj-p) * 2 + p
39 }
40 // Around origin
41 point RotateCCW(point p, double t) {
42     return point(p.x * cos(t) - p.y * sin(t),
43                 p.x * sin(t) + p.y * cos(t));
44 }
45 // Line a---b
46 vector<point> CircleLineIntersect(point a, point b, point center,
47     double r) {
48     a = a - center;
49     b = b - center;
50     point p = ProjectPointLine(a, b, point(0, 0)); // project point
51     // from center to the Line
52     if(dot(p, p) > r * r)
53         return {};
54     double len = sqrt(r * r - dot(p, p));
55     if(len < EPS)
56         return {center + p};
57     point d = (a - b) / abs(a - b);
58     return {center + p + d * len, center + p - d * len};
59 }
60 vector<point> CircleCircleIntersect(point c1, ld r1, point c2, ld r2)
61 {
62     if (r1 < r2) {
63         swap(r1, r2);
64         swap(c1, c2);
65     }
66     ld d = abs(c2 - c1); // distance between c1,c2
67     if (d > r1 + r2 || d < r1 - r2 || d < EPS) // zero or infinite
68         return {};
69     ld angle = acos(min((d * d + r1 * r1 - r2 * r2) / (2 * r1 * d), (
70         ld) 1.0));
71     point p = (c2 - c1) / d * r1;
72     if (angle < EPS)
73         return {c1 + p};
74     return {c1 + RotateCCW(p, angle), c1 + RotateCCW(p, -angle)};
75 }
76 }
77 point circumcircle(point p1, point p2, point p3) {
78     return LineLineIntersect((p1 + p2) / 2, (p1 - p2).prep(),
79     (p1 + p3) / 2, (p1 - p3).prep());
80 }
81 //I : number points with integer coordinates lying strictly inside the
82 //    polygon.
83 //B : number of points lying on polygon sides by B.
84 //Area = I + B/2 - 1

```

5.3 Half Plane Intersection

```

1 // Redefine epsilon and infinity as necessary. Be mindful of precision
2 // errors.
3 const long double eps = 1e-9, inf = 1e9;
4 // Basic point/vector struct.
5 struct Point {
6
7     long double x, y;
8     explicit Point(long double x = 0, long double y = 0) : x(x), y(y)
9     {}
10
11     // Addition, subtraction, multiply by constant, cross product.
12
13     friend Point operator + (const Point& p, const Point& q) {
14         return Point(p.x + q.x, p.y + q.y);
15     }
16
17     friend Point operator - (const Point& p, const Point& q) {

```

```

17         return Point(p.x - q.x, p.y - q.y);
18     }
19
20     friend Point operator * (const Point& p, const long double& k) {
21         return Point(p.x * k, p.y * k);
22     }
23
24     friend long double cross(const Point& p, const Point& q) {
25         return p.x * q.y - p.y * q.x;
26     }
27 };
28
29 // Basic half-plane struct.
30 struct Halfplane {
31
32     // 'p' is a passing point of the line and 'pq' is the direction
33     // vector of the line.
34     Point p, pq;
35     long double angle;
36
37     Halfplane() {}
38     Halfplane(const Point& a, const Point& b) : p(a), pq(b - a) {
39         angle = atan2l(pq.y, pq.x);
40     }
41
42     // Check if point 'r' is outside this half-plane.
43     // Every half-plane allows the region to the LEFT of its line.
44     bool out(const Point& r) {
45         return cross(pq, r - p) < -eps;
46     }
47
48     // Comparator for sorting.
49     // If the angle of both half-planes is equal, the leftmost one
50     // should go first.
51     bool operator < (const Halfplane& e) const {
52         if (fabsl(angle - e.angle) < eps) return cross(pq, e.p - p) <
53             0;
54         return angle < e.angle;
55     }
56
57     // We use equal comparator for std::unique to easily remove
58     // parallel half-planes.
59     bool operator == (const Halfplane& e) const {
60         return fabsl(angle - e.angle) < eps;
61     }
62
63     // Intersection point of the lines of two half-planes. It is
64     // assumed they're never parallel.
65     friend Point inter(const Halfplane& s, const Halfplane& t) {
66         long double alpha = cross((t.p - s.p), t.pq) / cross(s.pq, t.
67         pq);
68         return s.p + (s.pq * alpha);
69     }
70 };
71
72 // Actual algorithm
73 vector<Point> hp_intersect(vector<Halfplane>& H) {
74
75     Point box[4] = { // Bounding box in CCW order
76         Point(inf, inf),
77         Point(-inf, inf),
78         Point(-inf, -inf),
79         Point(inf, -inf)
80     };
81
82     for(int i = 0; i < 4; i++) { // Add bounding box half-planes.
83         Halfplane aux(box[i], box[(i+1) % 4]);
84         H.push_back(aux);
85     }
86
87     // Sort and remove duplicates
88     sort(H.begin(), H.end());
89     H.erase(unique(H.begin(), H.end(), H.end()), H.end());
90
91     deque<Halfplane> dq;
92     int len = 0;
93     for(int i = 0; i < int(H.size()); i++) {
94
95         // Remove from the back of the deque while last half-plane is
96         // redundant
97         while (len > 1 && H[i].out(inter(dq[len-1], dq[len-2]))) {

```

```

93         dq.pop_back();
94         --len;
95     }
96
97     // Remove from the front of the deque while first half-plane
98     // is redundant
99     while (len > 1 && H[i].out(inter(dq[0], dq[1]))) {
100         dq.pop_front();
101         --len;
102     }
103
104     // Add new half-plane
105     dq.push_back(H[i]);
106     ++len;
107 }
108
109 // Final cleanup: Check half-planes at the front against the back
110 // and vice-versa
111 while (len > 2 && dq[0].out(inter(dq[len-1], dq[len-2]))) {
112     dq.pop_back();
113     --len;
114 }
115
116 while (len > 2 && dq[len-1].out(inter(dq[0], dq[1]))) {
117     dq.pop_front();
118     --len;
119 }
120
121 // Report empty intersection if necessary
122 if (len < 3) return vector<Point>();
123
124 // Reconstruct the convex polygon from the remaining half-planes.
125 vector<Point> ret(len);
126 for(int i = 0; i+1 < len; i++) {
127     ret[i] = inter(dq[i], dq[i+1]);
128 }
129 ret.back() = inter(dq[len-1], dq[0]);
130 return ret;
131 }

```

5.4 Segments Intersection

```

1  const double EPS = 1E-9;
2
3  struct pt {
4      double x, y;
5  };
6
7  struct seg {
8      pt p, q;
9      int id;
10
11      double get_y(double x) const {
12          if (abs(p.x - q.x) < EPS)
13              return p.y;
14          return p.y + (q.y - p.y) * (x - p.x) / (q.x - p.x);
15      }
16  };
17
18  bool intersectld(double l1, double r1, double l2, double r2) {
19      if (l1 > r1)
20          swap(l1, r1);
21      if (l2 > r2)
22          swap(l2, r2);
23      return max(l1, l2) <= min(r1, r2) + EPS;
24  }
25
26  int vec(const pt& a, const pt& b, const pt& c) {
27      double s = (b.x - a.x) * (c.y - a.y) - (b.y - a.y) * (c.x - a.x);
28      return abs(s) < EPS ? 0 : s > 0 ? +1 : -1;
29  }
30
31  bool intersect(const seg& a, const seg& b)
32  {
33      return intersectld(a.p.x, a.q.x, b.p.x, b.q.x) &&
34             intersectld(a.p.y, a.q.y, b.p.y, b.q.y) &&
35             vec(a.p, a.q, b.p) * vec(a.p, a.q, b.q) <= 0 &&
36             vec(b.p, b.q, a.p) * vec(b.p, b.q, a.q) <= 0;
37  }
38
39  bool operator<(const seg& a, const seg& b)
40  {
41      double x = max(min(a.p.x, a.q.x), min(b.p.x, b.q.x));
42      return a.get_y(x) < b.get_y(x) - EPS;

```

```

43 }
44
45 struct event {
46     double x;
47     int tp, id;
48
49     event() {}
50     event(double x, int tp, int id) : x(x), tp(tp), id(id) {}
51
52     bool operator<(const event& e) const {
53         if (abs(x - e.x) > EPS)
54             return x < e.x;
55         return tp > e.tp;
56     }
57 };
58
59 set<seg> s;
60 vector<set<seg>::iterator> where;
61
62 set<seg>::iterator prev(set<seg>::iterator it) {
63     return it == s.begin() ? s.end() : --it;
64 }
65
66 set<seg>::iterator next(set<seg>::iterator it) {
67     return ++it;
68 }
69
70 pair<int, int> solve(const vector<seg>& a) {
71     int n = (int)a.size();
72     vector<event> e;
73     for (int i = 0; i < n; ++i) {
74         e.push_back(event(min(a[i].p.x, a[i].q.x), +1, i));
75         e.push_back(event(max(a[i].p.x, a[i].q.x), -1, i));
76     }
77     sort(e.begin(), e.end());
78
79     s.clear();
80     where.resize(a.size());
81     for (size_t i = 0; i < e.size(); ++i) {
82         int id = e[i].id;
83         if (e[i].tp == +1) {
84             set<seg>::iterator nxt = s.lower_bound(a[id]), prv = prev(
85                 nxt);
86             if (nxt != s.end() && intersect(*nxt, a[id]))
87                 return make_pair(nxt->id, id);
88             if (prv != s.end() && intersect(*prv, a[id]))
89                 return make_pair(prv->id, id);
90             where[id] = s.insert(nxt, a[id]);
91         } else {
92             set<seg>::iterator nxt = next(where[id]), prv = prev(where
93                 [id]);
94             if (nxt != s.end() && prv != s.end() && intersect(*nxt, *
95                 prv))
96                 return make_pair(prv->id, nxt->id);
97             s.erase(where[id]);
98         }
99     }
100     return make_pair(-1, -1);

```

5.5 Rectangles Union

```

1  #include<bits/stdc++.h>
2  #define P(x,y) make_pair(x,y)
3  using namespace std;
4  class Rectangle {
5  public:
6      int x1, y1, x2, y2;
7      static Rectangle empty;
8      Rectangle() {
9          x1 = y1 = x2 = y2 = 0;
10     }
11     Rectangle(int X1, int Y1, int X2, int Y2) {
12         x1 = X1;
13         y1 = Y1;
14         x2 = X2;
15         y2 = Y2;
16     }
17 };
18 struct Event {
19     int x, y1, y2, type;

```

```

20     Event() {}
21     Event(int x, int y1, int y2, int type): x(x), y1(y1), y2(y2), type
        (type) {}
22 };
23 bool operator < (const Event&A, const Event&B) {
24     //if(A.x != B.x)
25     return A.x < B.x;
26     //if(A.y1 != B.y1) return A.y1 < B.y1;
27     //if(A.y2 != B.y2()) A.y2 < B.y2;
28 }
29 const int MX = (1 << 17);
30 struct Node {
31     int prob, sum, ans;
32     Node() {}
33     Node(int prob, int sum, int ans): prob(prob), sum(sum), ans(ans)
        {}
34 };
35 Node tree[MX * 4];
36 int interval[MX];
37 void build(int x, int a, int b) {
38     tree[x] = Node(0, 0, 0);
39     if(a == b) {
40         tree[x].sum += interval[a];
41         return;
42     }
43     build(x * 2, a, (a + b) / 2);
44     build(x * 2 + 1, (a + b) / 2 + 1, b);
45     tree[x].sum = tree[x * 2].sum + tree[x * 2 + 1].sum;
46 }
47 int ask(int x) {
48     if(tree[x].prob)
49         return tree[x].sum;
50     return tree[x].ans;
51 }
52 int st, en, V;
53 void update(int x, int a, int b) {
54     if(st > b || en < a)
55         return;
56     if(a >= st && b <= en) {
57         tree[x].prob += V;
58         return;
59     }
60     update(x * 2, a, (a + b) / 2);
61     update(x * 2 + 1, (a + b) / 2 + 1, b);
62     tree[x].ans = ask(x * 2) + ask(x * 2 + 1);
63 }
64 Rectangle Rectangle::empt = Rectangle();
65 vector < Rectangle > Rect;
66 vector < int > sorted;
67 vector < Event > sweep;
68 void compressnalc() {
69     sweep.clear();
70     sorted.clear();
71     for(auto R : Rect) {
72         sorted.push_back(R.y1);
73         sorted.push_back(R.y2);
74     }
75     sort(sorted.begin(), sorted.end());
76     sorted.erase(unique(sorted.begin(), sorted.end()), sorted.end());
77     int sz = sorted.size();
78     for(int j = 0; j < sorted.size() - 1; j++)
79         interval[j + 1] = sorted[j + 1] - sorted[j];
80     for(auto R : Rect) {
81         sweep.push_back(Event(R.x1, R.y1, R.y2, 1));
82         sweep.push_back(Event(R.x2, R.y1, R.y2, -1));
83     }
84     sort(sweep.begin(), sweep.end());
85     build(1, 1, sz - 1);
86 }
87 long long ans;
88 void Sweep() {
89     ans = 0;
90     if(sorted.empty() || sweep.empty())
91         return;
92     int last = 0, sz_ = sorted.size();
93     for(int j = 0; j < sweep.size(); j++) {
94         ans += 1ll * (sweep[j].x - last) * ask(1);
95         last = sweep[j].x;
96         V = sweep[j].type;
97         st = lower_bound(sorted.begin(), sorted.end(), sweep[j].y1) -
            sorted.begin() + 1;

```

```

98         en = lower_bound(sorted.begin(), sorted.end(), sweep[j].y2) -
            sorted.begin();
99         update(1, 1, sz_ - 1);
100     }
101 }
102 int main() {
103     // freopen("in.in", "r", stdin);
104     int n;
105     scanf("%d", &n);
106     for(int j = 1; j <= n; j++) {
107         int a, b, c, d;
108         scanf("%d %d %d %d", &a, &b, &c, &d);
109         Rect.push_back(Rectangle(a, b, c, d));
110     }
111     compressnalc();
112     Sweep();
113     cout << ans << endl;
114 }

```

6 Graphs

6.1 2 SAT

```

1  /**
2   * Description: Calculates a valid assignment to boolean variables a,
        b, c, ... to a 2-SAT problem, so that an expression of the type $(
        a\|\b)\&\&(!a\|\c)\&\&(d\|\b)\&\&...$ becomes true, or
        reports that it is unsatisfiable.
3   * Negated variables are represented by bit-inversions (\texttt{\tilde
        {x}}).
4   * Usage:
5   * TwoSat ts(number of boolean variables);
6   * ts.either(0, \tilde{3}); // Var 0 is true or var 3 is false
7   * ts.setValue(2); // Var 2 is true
8   * ts.atMostOne({0, \tilde{1}, 2}); // <= 1 of vars 0, \tilde{1} and 2 are
        true
9   * ts.solve(); // Returns true iff it is solvable
10  * ts.values[0..N-1] holds the assigned values to the vars
11  * Time: O(N+E), where N is the number of boolean variables, and E is
        the number of clauses.
12  */
13 struct TwoSat {
14     int N;
15     vector<vi> gr;
16     vi values; // 0 = false, 1 = true
17
18     TwoSat(int n = 0) : N(n), gr(2*n) {}
19
20     int addVar() { // (optional)
21         gr.emplace_back();
22         gr.emplace_back();
23         return N++;
24     }
25
26     void either(int f, int j) {
27         f = max(2*f, -1-2*f);
28         j = max(2*j, -1-2*j);
29         gr[f].push_back(j^1);
30         gr[j].push_back(f^1);
31     }
32     void setValue(int x) { either(x, x); }
33
34     void atMostOne(const vi& li) { // (optional)
35         if (sz(li) <= 1) return;
36         int cur = ~li[0];
37         rep(i, 2, sz(li)) {
38             int next = addVar();
39             either(cur, ~li[i]);
40             either(cur, next);
41             either(~li[i], next);
42             cur = ~next;
43         }
44         either(cur, ~li[1]);
45     }
46
47     vi val, comp, z; int time = 0;
48     int dfs(int i) {
49         int low = val[i] = ++time, x; z.push_back(i);
50         for(int e : gr[i]) if (!comp[e])
51             low = min(low, val[e] ? : dfs(e));
52         if (low == val[i]) do {

```

```

53     x = z.back(); z.pop_back();
54     comp[x] = low;
55     if (values[x>>1] == -1)
56         values[x>>1] = x&1;
57     } while (x != i);
58     return val[i] = low;
59 }
60
61 bool solve() {
62     values.assign(N, -1);
63     val.assign(2*N, 0); comp = val;
64     rep(i,0,2*N) if (!comp[i]) dfs(i);
65     rep(i,0,N) if (comp[2*i] == comp[2*i+1]) return 0;
66     return 1;
67 }
68 };

```

6.2 Articulation Point

```

1  vector<int> adj[N];
2  int dfsn[N], low[N], instack[N], ar_point[N], timer;
3  stack<int> st;
4
5  void dfs(int node, int par){
6      dfsn[node] = low[node] = ++timer;
7      int kam = 0;
8      for(auto i: adj[node]){
9          if(i == par) continue;
10         if(dfsn[i] == 0){
11             kam++;
12             dfs(i, node);
13             low[node] = min(low[node], low[i]);
14             if(dfsn[node] <= low[i] && par != 0) ar_point[node] = 1;
15         }
16         else low[node] = min(low[node], dfsn[i]);
17     }
18     if(par == 0 && kam > 1) ar_point[node] = 1;
19 }
20 int main(){
21     // Input
22     for(int i = 1; i <= n; i++){
23         if(dfsn[i] == 0) dfs(i, 0);
24     }
25     int c = 0;
26     for(int i = 1; i <= n; i++){
27         if(ar_point[i]) c++;
28     }
29     cout << c << '\n';
30 }

```

6.3 Bridges Tree and Diameter

```

1  #include <bits/stdc++.h>
2  #define ll long long
3  using namespace std;
4  const int N = 3e5 + 5, mod = 1e9 + 7;
5
6  vector<int> adj[N], bridge_tree[N];
7  int dfsn[N], low[N], cost[N], timer, cnt, comp_id[N], kam[N], ans;
8  stack<int> st;
9
10 void dfs(int node, int par){
11     dfsn[node] = low[node] = ++timer;
12     st.push(node);
13     for(auto i: adj[node]){
14         if(i == par) continue;
15         if(dfsn[i] == 0){
16             dfs(i, node);
17             low[node] = min(low[node], low[i]);
18         }
19         else low[node] = min(low[node], dfsn[i]);
20     }
21     if(dfsn[node] == low[node]){
22         cnt++;
23         while(1){
24             int cur = st.top();
25             st.pop();
26             comp_id[cur] = cnt;
27             if(cur == node) break;
28         }
29     }
30 }
31 }

```

```

32 void dfs2(int node, int par){
33     kam[node] = 0;
34     int mx = 0, second_mx = 0;
35     for(auto i: bridge_tree[node]){
36         if(i == par) continue;
37         dfs2(i, node);
38         kam[node] = max(kam[node], 1 + kam[i]);
39         if(kam[i] > mx){
40             second_mx = mx;
41             mx = kam[i];
42         }
43         else second_mx = max(second_mx, kam[i]);
44     }
45     ans = max(ans, kam[node]);
46     if(second_mx) ans = max(ans, 2 + mx + second_mx);
47 }
48
49 int main(){
50     ios_base::sync_with_stdio(0); cin.tie(0); cout.tie(0);
51     int n, m;
52     cin >> n >> m;
53     while(m--){
54         int u, v;
55         cin >> u >> v;
56         adj[u].push_back(v);
57         adj[v].push_back(u);
58     }
59     dfs(1, 0);
60     for(int i = 1; i <= n; i++){
61         for(auto j: adj[i]){
62             if(comp_id[i] != comp_id[j]){
63                 bridge_tree[comp_id[i]].push_back(comp_id[j]);
64             }
65         }
66     }
67     dfs2(1, 0);
68     cout << ans;
69
70     return 0;
71 }
72 }

```

6.4 Dinic With Scalling

```

1  //O(ElgFlow) on Bipartite Graphs and O(EVlgFlow) on other graphs (I
2  think)
3  struct Dinic {
4      #define vi vector<int>
5      #define rep(i,a,b) f(i,a,b)
6      struct Edge {
7          int to, rev;
8          ll c, oc;
9          int id;
10         ll flow() { return max(oc - c, 0LL); } // if you need flows
11     };
12     vi lvl, ptr, q;
13     vector<vector<Edge>> adj;
14     Dinic(int n): lvl(n), ptr(n), q(n), adj(n) {}
15     void addEdge(int a, int b, ll c, int id, ll rcap = 0) {
16         adj[a].push_back({b, sz(adj[b]), c, c, id});
17         adj[b].push_back({a, sz(adj[a]) - 1, rcap, rcap, id});
18     }
19     ll dfs(int v, int t, ll f) {
20         if (v == t || !f) return f;
21         for (int& i = ptr[v]; i < sz(adj[v]); i++) {
22             Edge& e = adj[v][i];
23             if (lvl[e.to] == lvl[v] + 1)
24                 if (ll p = dfs(e.to, t, min(f, e.c))) {
25                     e.c -= p, adj[e.to][e.rev].c += p;
26                     return p;
27                 }
28         }
29         return 0;
30     }
31     ll calc(int s, int t) {
32         ll flow = 0; q[0] = s;
33         rep(L,0,31) do { // 'int L=30' maybe faster for random data
34             lvl = ptr = vi(sz(q));
35             int qi = 0, qe = lvl[s] = 1;
36             while (qi < qe && !lvl[t]) {
37                 int v = q[qi++];

```

```

37         for (Edge e : adj[v])
38             if (!lvl[e.to] && e.c >> (30 - L))
39                 q[qe++] = e.to, lvl[e.to] = lvl[v] + 1;
40         }
41         while (ll p = dfs(s, t, LLONG_MAX)) flow += p;
42     } while (lvl[t]);
43     return flow;
44 }
45 bool leftOfMinCut(int a) { return lvl[a] != 0; }
46 };

```

6.5 Gomory Hu

```

1  /**
2   * Author: chilli, Takanori MAEHARA
3   * Date: 2020-04-03
4   * License: CC0
5   * Source: https://github.com/spaghetti-source/algorithm/blob/master/
6   * graph/gomory_hu_tree.cc#L102
7   * Description: Given a list of edges representing an undirected flow
8   * graph,
9   * returns edges of the Gomory-Hu tree. The max flow between any pair
10  * of
11  * vertices is given by minimum edge weight along the Gomory-Hu tree
12  * path.
13  * Time:  $O(V)$  Flow Computations
14  * Status: Tested on CERC 2015 J, stress-tested
15  * Details: The implementation used here is not actually the original
16  * Gomory-Hu, but Gusfield's simplified version: "Very simple methods
17  * for all
18  * pairs network flow analysis". PushRelabel is used here, but any
19  * flow
20  * implementation that supports 'leftOfMinCut' also works.
21  */
22 #pragma once
23 #include "PushRelabel.h"
24
25 typedef array<ll, 3> Edge;
26 vector<Edge> gomoryHu(int N, vector<Edge> ed) {
27     vector<Edge> tree;
28     vi par(N);
29     rep(i, 1, N) {
30         PushRelabel D(N); // Dinic also works
31         for (Edge t : ed) D.addEdge(t[0], t[1], t[2], t[2]);
32         tree.push_back({i, par[i], D.calc(i, par[i])});
33         rep(j, i+1, N)
34             if (par[j] == par[i] && D.leftOfMinCut(j)) par[j] = i;
35     }
36     return tree;
37 }

```

6.6 HopcraftKarp BPM

```

1  /**
2   * Author: Chen Xing
3   * Date: 2009-10-13
4   * License: CC0
5   * Source: N/A
6   * Description: Fast bipartite matching algorithm. Graph $g$ should be
7   * a list
8   * of neighbors of the left partition, and $btoa$ should be a vector
9   * full of
10  * -1's of the same size as the right partition. Returns the size of
11  * the matching. $btoa[i]$ will be the match for vertex $i$ on the
12  * right side,
13  * or $-1$ if it's not matched.
14  * Usage: vi btoa(m, -1); hopcraftKarp(g, btoa);
15  * Time:  $O(\sqrt{V}E)$ 
16  * Status: stress-tested by MinimumVertexCover, and tested on
17  * oldkattis.adkbipmatch and SPOJ:MATCHING
18  */
19 #pragma once
20
21 bool dfs(int a, int L, vector<vi>& g, vi& btoa, vi& A, vi& B) {
22     if (A[a] != L) return 0;
23     A[a] = -1;
24     for (int b : g[a]) if (B[b] == L + 1) {
25         B[b] = 0;
26         if (btoa[b] == -1 || dfs(btoa[b], L + 1, g, btoa, A, B))
27             return btoa[b] = a, 1;
28     }
29 }

```

```

25     return 0;
26 }
27
28 int hopcroftKarp(vector<vi>& g, vi& btoa) {
29     int res = 0;
30     vi A(g.size(), 0), B(btoa.size(), 0), cur, next;
31     for (;;) {
32         fill(all(A), 0);
33         fill(all(B), 0);
34         // Find the starting nodes for BFS (i.e. layer 0).
35         cur.clear();
36         for (int a : btoa) if (a != -1) A[a] = -1;
37         rep(a, 0, sz(g)) if (A[a] == 0) cur.push_back(a);
38         // Find all layers using bfs.
39         for (int lay = 1; lay <= g.size(); lay++) {
40             bool islast = 0;
41             next.clear();
42             for (int a : cur) for (int b : g[a]) {
43                 if (btoa[b] == -1) {
44                     B[b] = lay;
45                     islast = 1;
46                 }
47                 else if (btoa[b] != a && !B[b]) {
48                     B[b] = lay;
49                     next.push_back(btoa[b]);
50                 }
51             }
52             if (islast) break;
53             if (next.empty()) return res;
54             for (int a : next) A[a] = lay;
55             cur.swap(next);
56         }
57         // Use DFS to scan for augmenting paths.
58         rep(a, 0, sz(g))
59             res += dfs(a, 0, g, btoa, A, B);
60     }
61 }

```

6.7 Hungarian

```

1  /**
2   * Notes:
3   * note that n must be <= m
4   * so in case in your problem n >= m, just swap
5   * also note this
6   * void set(int x, int y, ll v) {a[x+1][y+1]=v;}
7   * the algorithm assumes you're using 0-index
8   * but it's using 1-based
9  */
10 struct Hungarian {
11     const ll INF = 1000000000000000000; //10^18
12     int n, m;
13     vector<vector<ll>> a;
14     vector<ll> u, v; vector<int> p, way;
15     Hungarian(int n, int m) :
16         n(n), m(m), a(n+1, vector<ll>(m+1, INF-1)), u(n+1), v(m+1), p(m+1), way(m+1) {}
17     void set(int x, int y, ll v) {a[x+1][y+1]=v;}
18     ll assign() {
19         for (int i = 1; i <= n; i++) {
20             int j0 = 0; p[0] = i;
21             vector<ll> minv(m+1, INF);
22             vector<char> used(m+1, false);
23             do {
24                 used[j0] = true;
25                 int i0 = p[j0], j1; ll delta = INF;
26                 for (int j = 1; j <= m; j++) if (!used[j]) {
27                     ll cur = a[i0][j] - u[i0] - v[j];
28                     if (cur < minv[j]) minv[j] = cur, way[j] = j0;
29                     if (minv[j] < delta) delta = minv[j], j1 = j;
30                 }
31                 for (int j = 0; j <= m; j++)
32                     if (used[j]) u[p[j]] += delta, v[j] -= delta;
33                 else minv[j] -= delta;
34                 j0 = j1;
35             } while (p[j0]);
36             do {
37                 int j1 = way[j0]; p[j0] = p[j1]; j0 = j1;
38             } while (j0);
39         }
40         return -v[0];
41     }
42 }

```

```

41     }
42     vector<int> restoreAnswer() {    ///run it after assign
43         vector<int> ans (n+1);
44         for (int j=1; j<=m; ++j)
45             ans[p[j]] = j;
46         return ans;
47     }
48 };

```

6.8 Kosaraju

```

1  /*
2  g : Adjacency List of the original graph
3  rg : Reversed Adjacency List
4  vis : A bitset to mark visited nodes
5  adj : Adjacency List of the super graph
6  stk : holds dfs ordered elements
7  cmp[i] : holds the component of node i
8  go[i] : holds the nodes inside the strongly connected component i
9  */
10
11 #define FOR(i,a,b) for(int i = a; i < b; i++)
12 #define pb push_back
13
14 const int N = 1e5+5;
15
16 vector<vector<int>>>g, rg;
17 vector<vector<int>>>go;
18 bitset<N>vis;
19 vector<vector<int>>>adj;
20 stack<int>stk;
21 int n, m, cmp[N];
22 void add_edge(int u, int v){
23     g[u].push_back(v);
24     rg[v].push_back(u);
25 }
26 void dfs(int u){
27     vis[u]=1;
28     for(auto v : g[u])if(!vis[v])dfs(v);
29     stk.push(u);
30 }
31 void rdfs(int u,int c){
32     vis[u] = 1;
33     cmp[u] = c;
34     go[c].push_back(u);
35     for(auto v : rg[u])if(!vis[v])rdfs(v,c);
36 }
37 int scc(){
38     vis.reset();
39     for(int i = 0; i < n; i++)if(!vis[i])
40         dfs(i);
41     vis.reset();
42     int c = 0;
43     while(stk.size()){
44         auto cur = stk.top();
45         stk.pop();
46         if(!vis[cur])
47             rdfs(cur,c++);
48     }
49     return c;
50 }
51 }

```

6.9 Manhattan MST

```

1  #include<bits/stdc++.h>
2  using namespace std;
3
4  const int N = 2e5 + 9;
5
6  int n;
7  vector<pair<int, int>> g[N];
8  struct PT {
9      int x, y, id;
10     bool operator < (const PT &p) const {
11         return x == p.x ? y < p.y : x < p.x;
12     }
13 } p[N];
14 struct node {
15     int val, id;
16 } t[N];
17 struct DSU {
18     int p[N];

```

```

19     void init(int n) { for (int i = 1; i <= n; i++) p[i] = i; }
20     int find(int u) { return p[u] == u ? u : p[u] = find(p[u]); }
21     void merge(int u, int v) { p[find(u)] = find(v); }
22 } dsu;
23 struct edge {
24     int u, v, w;
25     bool operator < (const edge &p) const { return w < p.w; }
26 };
27 vector<edge> edges;
28 int query(int x) {
29     int r = 2e9 + 10, id = -1;
30     for (; x <= n; x += (x & -x)) if (t[x].val < r) r = t[x].val, id = t[x].id;
31     return id;
32 }
33 void modify(int x, int w, int id) {
34     for (; x > 0; x -= (x & -x)) if (t[x].val > w) t[x].val = w, t[x].id = id;
35 }
36 int dist(PT &a, PT &b) {
37     return abs(a.x - b.x) + abs(a.y - b.y);
38 }
39 void add(int u, int v, int w) {
40     edges.push_back({u, v, w});
41 }
42 long long Kruskal() {
43     dsu.init(n);
44     sort(edges.begin(), edges.end());
45     long long ans = 0;
46     for (edge e : edges) {
47         int u = e.u, v = e.v, w = e.w;
48         if (dsu.find(u) != dsu.find(v)) {
49             ans += w;
50             g[u].push_back({v, w});
51             //g[v].push_back({u, w});
52             dsu.merge(u, v);
53         }
54     }
55     return ans;
56 }
57 void Manhattan() {
58     for (int i = 1; i <= n; ++i) p[i].id = i;
59     for (int dir = 1; dir <= 4; ++dir) {
60         if (dir == 2 || dir == 4) {
61             for (int i = 1; i <= n; ++i) swap(p[i].x, p[i].y);
62         }
63         else if (dir == 3) {
64             for (int i = 1; i <= n; ++i) p[i].x = -p[i].x;
65         }
66         sort(p + 1, p + 1 + n);
67         vector<int> v;
68         static int a[N];
69         for (int i = 1; i <= n; ++i) a[i] = p[i].y - p[i].x, v.push_back(a[i]);
70         sort(v.begin(), v.end());
71         v.erase(unique(v.begin(), v.end()), v.end());
72         for (int i = 1; i <= n; ++i) a[i] = lower_bound(v.begin(), v.end(), a[i]) - v.begin() + 1;
73         for (int i = 1; i <= n; ++i) t[i].val = 2e9 + 10, t[i].id = -1;
74         for (int i = n; i >= 1; --i) {
75             int pos = query(a[i]);
76             if (pos != -1) add(p[i].id, p[pos].id, dist(p[i], p[pos]));
77             modify(a[i], p[i].x + p[i].y, i);
78         }
79     }
80 }
81 int32_t main() {
82     ios_base::sync_with_stdio(0);
83     cin.tie(0);
84     cin >> n;
85     for (int i = 1; i <= n; i++) cin >> p[i].x >> p[i].y;
86     Manhattan();
87     cout << Kruskal() << '\n';
88     for (int u = 1; u <= n; u++) {
89         for (auto x : g[u]) cout << u - 1 << ' ' << x.first - 1 << '\n';
90     }
91     return 0;
92 }

```

6.10 Maximum Clique


```

1  ///Complexity  $O(3^{N/3})$  i.e works for 50
2  ///you can change it to maximum independent set by flipping the edges
3  0->1, 1->0
4  ///if you want to extract the nodes they are 1-bits in R
5  int g[60][60];
6  int res;
7  long long edges[60];
8  void BronKerbosch(int n, long long R, long long P, long long X) {
9      if (P == 0LL && X == 0LL) { //here we will find all possible maximal
10         cliques (not maximum) i.e. there is no node which can be
11         included in this set
12         int t = __builtin_popcountll(R);
13         res = max(res, t);
14         return;
15     }
16     int u = 0;
17     while (!(1LL << u) & (P | X)) u++;
18     for (int v = 0; v < n; v++) {
19         if (((1LL << v) & P) && !((1LL << v) & edges[u])) {
20             BronKerbosch(n, R | (1LL << v), P & edges[v], X & edges[v]);
21             P |= (1LL << v);
22             X |= (1LL << v);
23         }
24     }
25 }
26 int max_clique (int n) {
27     res = 0;
28     for (int i = 1; i <= n; i++) {
29         edges[i - 1] = 0;
30         for (int j = 1; j <= n; j++) if (g[i][j]) edges[i - 1] |= (1LL
31             << (j - 1));
32     }
33     BronKerbosch(n, 0, (1LL << n) - 1, 0);
34     return res;
35 }

```

6.11 MCMF

```

1  /*
2  3  Notes:
3  4  make sure you notice the #define int ll
4  5  focus on the data types of the max flow everythign inside is
5  6  integer
6  7  addEdge(u,v,cap,cost)
7  8  note that for min cost max flow the cost is sum of cost * flow
8  9  over all edges
9  10 */
10 11 struct Edge {
11 12     int to;
12 13     int cost;
13 14     int cap, flow, backEdge;
14 15 };
15 16 struct MCMF {
16 17     const int inf = 10000000010;
17 18     int n;
18 19     vector<vector<Edge>> g;
19 20
20 21     MCMF(int _n) {
21 22         n = _n + 1;
22 23         g.resize(n);
23 24     }
24 25
25 26     void addEdge(int u, int v, int cap, int cost) {
26 27         Edge e1 = {v, cost, cap, 0, (int) g[v].size()};
27 28         Edge e2 = {u, -cost, 0, 0, (int) g[u].size()};
28 29         g[u].push_back(e1);
29 30         g[v].push_back(e2);
30 31     }
31 32
32 33     pair<int, int> minCostMaxFlow(int s, int t) {
33 34         int flow = 0;
34 35         int cost = 0;
35 36         vector<int> state(n), from(n), from_edge(n);
36 37         vector<int> d(n);
37 38         deque<int> q;
38 39         while (true) {
39 40             for (int i = 0; i < n; i++)
40 41                 state[i] = 2, d[i] = inf, from[i] = -1;
41 42             state[s] = 1;

```

```

43         q.clear();
44         q.push_back(s);
45         d[s] = 0;
46         while (!q.empty()) {
47             int v = q.front();
48             q.pop_front();
49             state[v] = 0;
50             for (int i = 0; i < (int) g[v].size(); i++) {
51                 Edge e = g[v][i];
52                 if (e.flow >= e.cap || (d[e.to] <= d[v] + e.cost))
53                     continue;
54                 int to = e.to;
55                 d[to] = d[v] + e.cost;
56                 from[to] = v;
57                 from_edge[to] = i;
58                 if (state[to] == 1) continue;
59                 if (!state[to] || (!q.empty() && d[q.front()] > d[to]))
60                     q.push_front(to);
61                 else q.push_back(to);
62                 state[to] = 1;
63             }
64         }
65         if (d[t] == inf) break;
66         int it = t, addflow = inf;
67         while (it != s) {
68             addflow = min(addflow,
69                 g[from[it]][from_edge[it]].cap
70                 - g[from[it]][from_edge[it]].flow);
71             it = from[it];
72         }
73         it = t;
74         while (it != s) {
75             g[from[it]][from_edge[it]].flow += addflow;
76             g[it][g[from[it]][from_edge[it]].backEdge].flow -=
77                 addflow;
78             cost += g[from[it]][from_edge[it]].cost * addflow;
79             it = from[it];
80         }
81         flow += addflow;
82         return {cost, flow};
83     }
84 };

```

6.12 Minimum Arbrosce in a Graph

```

1  const int maxn = 2510, maxm = 7000000;
2  const ll maxint = 0x3f3f3f3f3f3f3f3fLL;
3
4  int n, ec, ID[maxn], pre[maxn], vis[maxn];
5  ll in[maxn];
6
7  struct edge_t {
8      int u, v;
9      ll w;
10 } edge[maxn];
11 void add(int u, int v, ll w) {
12     edge[++ec].u = u, edge[ec].v = v, edge[ec].w = w;
13 }
14
15 ll arborecence(int n, int root) {
16     ll res = 0, index;
17     while (true) {
18         for (int i = 1; i <= n; ++i) {
19             in[i] = maxint, vis[i] = -1, ID[i] = -1;
20         }
21         for (int i = 1; i <= ec; ++i) {
22             int u = edge[i].u, v = edge[i].v;
23             if (u == v || in[v] <= edge[i].w) continue;
24             in[v] = edge[i].w, pre[v] = u;
25         }
26         pre[root] = root, in[root] = 0;
27         for (int i = 1; i <= n; ++i) {
28             res += in[i];
29             if (in[i] == maxint) return -1;
30         }
31         index = 0;
32         for (int i = 1; i <= n; ++i) {
33             if (vis[i] != -1) continue;
34             int u = i, v;

```

```

35     while (vis[u] == -1) {
36         vis[u] = i;
37         u = pre[u];
38     }
39     if (vis[u] != i || u == root) continue;
40     for (v = u, u = pre[u], ++index; u != v; u = pre[u]) ID[u]
        = index;
41     ID[v] = index;
42 }
43 if (index == 0) return res;
44 for (int i = 1; i <= n; ++i) if (ID[i] == -1) ID[i] = ++index;
45 for (int i = 1; i <= ec; ++i) {
46     int u = edge[i].u, v = edge[i].v;
47     edge[i].u = ID[u], edge[i].v = ID[v];
48     edge[i].w -= in[v];
49 }
50 n = index, root = ID[root];
51 }
52 return res;
53 }

```

6.13 Minimum Vertex Cover (Bipartite)

```

1  int myrandom (int i) { return std::rand()%i;}
2
3  struct MinimumVertexCover {
4      int n, id;
5      vector<vector<int>> > g;
6      vector<int> color, m, seen;
7      vector<int> comp[2];
8      MinimumVertexCover() {}
9      MinimumVertexCover(int n, vector<vector<int>> > g) {
10
11         this->n = n;
12         this->g = g;
13         color = m = vector<int>(n, -1);
14         seen = vector<int>(n, 0);
15         makeBipartite();
16     }
17
18     void dfsBipartite(int node, int col) {
19         if (color[node] != -1) {
20             assert(color[node] == col); /* MSH BIPARTITE YA
21                                     BASHMOHANDES */
22             return;
23         }
24         color[node] = col;
25         comp[col].push_back(node);
26         for (int i = 0; i < int(g[node].size()); i++)
27             dfsBipartite(g[node][i], 1 - col);
28     }
29
30     void makeBipartite() {
31         for (int i = 0; i < n; i++)
32             if (color[i] == -1)
33                 dfsBipartite(i, 0);
34     }
35
36     // match a node
37     bool dfs(int node) {
38         random_shuffle(g[node].begin(), g[node].end());
39         for (int i = 0; i < g[node].size(); i++) {
40             int child = g[node][i];
41             if (m[child] == -1) {
42                 m[node] = child;
43                 m[child] = node;
44                 return true;
45             }
46             if (seen[child] == id)
47                 continue;
48             seen[child] = id;
49             int enemy = m[child];
50             m[node] = child;
51             m[child] = node;
52             m[enemy] = -1;
53             if (dfs(enemy))
54                 return true;
55             m[node] = -1;
56             m[child] = enemy;
57             m[enemy] = child;
58         }
59         return false;

```

```

59     }
60
61     void makeMatching() {
62         for (int j = 0; j < 5; j++)
63             random_shuffle(comp[0].begin(), comp[0].end(), myrandom);
64         for (int i = 0; i < int(comp[0].size()); i++) {
65             id++;
66             if (m[comp[0][i]] == -1)
67                 dfs(comp[0][i]);
68         }
69     }
70
71     void recurse(int node, int x, vector<int> &minCover, vector<int> &
        done) {
72         if (m[node] != -1)
73             return;
74         if (done[node]) return;
75         done[node] = 1;
76         for (int i = 0; i < int(g[node].size()); i++) {
77             int child = g[node][i];
78             int newnode = m[child];
79             if (done[child]) continue;
80             if (newnode == -1) {
81                 continue;
82             }
83             done[child] = 2;
84             minCover.push_back(child);
85             m[newnode] = -1;
86             recurse(newnode, x, minCover, done);
87         }
88     }
89
90     vector<int> getAnswer() {
91         vector<int> minCover, maxIndep;
92         vector<int> done(n, 0);
93         makeMatching();
94         for (int x = 0; x < 2; x++)
95             for (int i = 0; i < int(comp[x].size()); i++) {
96                 int node = comp[x][i];
97                 if (m[node] == -1)
98                     recurse(node, x, minCover, done);
99             }
100
101         for (int i = 0; i < int(comp[0].size()); i++)
102             if (!done[comp[0][i]]) {
103                 minCover.push_back(comp[0][i]);
104             }
105         return minCover;
106     }
107 }
108 };

```

6.14 Prufer Code

```

1  const int N = 3e5 + 9;
2  /*
3  prufer code is a sequence of length n-2 to uniquely determine a
4  labeled tree with n vertices
5  Each time take the leaf with the lowest number and add the node number
6  the leaf is connected to
7  the sequence and remove the leaf. Then break the algo after n-2
8  iterations
9  */
10 //0-indexed
11 int n;
12 vector<int> g[N];
13 int parent[N], degree[N];
14
15 void dfs (int v) {
16     for (size_t i = 0; i < g[v].size(); ++i) {
17         int to = g[v][i];
18         if (to != parent[v]) {
19             parent[to] = v;
20             dfs (to);
21         }
22     }
23 }
24
25 vector<int> prufer_code() {
26     parent[n - 1] = -1;
27     dfs (n - 1);
28     int ptr = -1;
29     for (int i = 0; i < n; ++i) {

```

```

27     degree[i] = (int) g[i].size();
28     if (degree[i] == 1 && ptr == -1) ptr = i;
29 }
30 vector<int> result;
31 int leaf = ptr;
32 for (int iter = 0; iter < n - 2; ++iter) {
33     int next = parent[leaf];
34     result.push_back(next);
35     --degree[next];
36     if (degree[next] == 1 && next < ptr) leaf = next;
37     else {
38         ++ptr;
39         while (ptr < n && degree[ptr] != 1) ++ptr;
40         leaf = ptr;
41     }
42 }
43 return result;
44 }
45 vector < pair<int, int> > prufer_to_tree(const vector<int> &
    prufer_code) {
46     int n = (int) prufer_code.size() + 2;
47     vector<int> degree(n, 1);
48     for (int i = 0; i < n - 2; ++i) ++degree[prufer_code[i]];
49
50     int ptr = 0;
51     while (ptr < n && degree[ptr] != 1) ++ptr;
52     int leaf = ptr;
53     vector < pair<int, int> > result;
54     for (int i = 0; i < n - 2; ++i) {
55         int v = prufer_code[i];
56         result.push_back(make_pair(leaf, v));
57         --degree[leaf];
58         if (--degree[v] == 1 && v < ptr) leaf = v;
59         else {
60             ++ptr;
61             while (ptr < n && degree[ptr] != 1) ++ptr;
62             leaf = ptr;
63         }
64     }
65     for (int v = 0; v < n - 1; ++v) if (degree[v] == 1) result.push_back
        (make_pair(v, n - 1));
66     return result;
67 }

```

6.15 Push Relabel Max Flow

```

1 struct edge
2 {
3     int from, to, cap, flow, index;
4     edge(int from, int to, int cap, int flow, int index):
5         from(from), to(to), cap(cap), flow(flow), index(index) {}
6 };
7
8 struct PushRelabel
9 {
10     int n;
11     vector<vector<edge> > g;
12     vector<long long> excess;
13     vector<int> height, active, count;
14     queue<int> Q;
15
16     PushRelabel(int n):
17         n(n), g(n), excess(n), height(n), active(n), count(2*n) {}
18
19     void addEdge(int from, int to, int cap)
20     {
21         g[from].push_back(edge(from, to, cap, 0, g[to].size()));
22         if (from==to)
23             g[from].back().index++;
24         g[to].push_back(edge(to, from, 0, 0, g[from].size()-1));
25     }
26
27     void enqueue(int v)
28     {
29         if (!active[v] && excess[v] > 0)
30         {
31             active[v]=true;
32             Q.push(v);
33         }
34     }
35
36     void push(edge &e)
37     {

```

```

38         int amt=(int)min(excess[e.from], (long long)e.cap - e.flow);
39         if (height[e.from]<=height[e.to] || amt==0)
40             return;
41         e.flow += amt;
42         g[e.to][e.index].flow -= amt;
43         excess[e.to] += amt;
44         excess[e.from] -= amt;
45         enqueue(e.to);
46     }
47
48     void relabel(int v)
49     {
50         count[height[v]]--;
51         int d=2*n;
52         for (auto &it:g[v])
53         {
54             if (it.cap-it.flow>0)
55                 d=min(d, height[it.to]+1);
56         }
57         height[v]=d;
58         count[height[v]]++;
59         enqueue(v);
60     }
61
62     void gap(int k)
63     {
64         for (int v=0;v<n;v++)
65         {
66             if (height[v]<k)
67                 continue;
68             count[height[v]]--;
69             height[v]=max(height[v], n+1);
70             count[height[v]]++;
71             enqueue(v);
72         }
73     }
74
75     void discharge(int v)
76     {
77         for (int i=0; excess[v]>0 && i<g[v].size(); i++)
78             push(g[v][i]);
79         if (excess[v]>0)
80         {
81             if (count[height[v]]==1)
82                 gap(height[v]);
83             else
84                 relabel(v);
85         }
86     }
87
88     long long max_flow(int source, int dest)
89     {
90         count[0] = n-1;
91         count[n] = 1;
92         height[source] = n;
93         active[source] = active[dest] = 1;
94         for (auto &it:g[source])
95         {
96             excess[source]+=it.cap;
97             push(it);
98         }
99
100         while (!Q.empty())
101         {
102             int v=Q.front();
103             Q.pop();
104             active[v]=false;
105             discharge(v);
106         }
107
108         long long max_flow=0;
109         for (auto &e:g[source])
110             max_flow+=e.flow;
111
112         return max_flow;
113     }
114 };

```

6.16 Tarjan Algo

```

1 vector< vector<int> > scc;

```

```

2 vector<int> adj[N];
3 int dfsn[N], low[N], cost[N], timer, in_stack[N];
4 stack<int> st;
5
6 // to detect all the components (cycles) in a directed graph
7 void tarjan(int node){
8     dfsn[node] = low[node] = ++timer;
9     in_stack[node] = 1;
10    st.push(node);
11    for(auto i: adj[node]){
12        if(dfsn[i] == 0){
13            tarjan(i);
14            low[node] = min(low[node], low[i]);
15        }
16        else if(in_stack[i]) low[node] = min(low[node], dfsn[i]);
17    }
18    if(dfsn[node] == low[node]){
19        scc.push_back(vector<int>());
20        while(1){
21            int cur = st.top();
22            st.pop();
23            in_stack[cur] = 0;
24            scc.back().push_back(cur);
25            if(cur == node) break;
26        }
27    }
28 }
29 int main(){
30     int m;
31     cin >> m;
32     while(m--){
33         int u, v;
34         cin >> u >> v;
35         adj[u].push_back(v);
36     }
37     for(int i = 1; i <= n; i++){
38         if(dfsn[i] == 0){
39             tarjan(i);
40         }
41     }
42     return 0;
43 }
44 }

```

6.17 Bipartite Matching

```

1 // vertex are one based
2 struct graph
3 {
4     int L, R;
5     vector<vector<int>> > adj;
6     graph(int l, int r) : L(l), R(r), adj(l+1) {}
7     void add_edge(int u, int v)
8     {
9         adj[u].push_back(v+L);
10    }
11    int maximum_matching()
12    {
13        vector<int> mate(L+R+1, -1), level(L+1);
14        function<bool (void)> levelize = [&]() {
15            queue<int> q;
16            for(int i=1; i<=L; i++){
17                level[i]=-1;
18                if(mate[i]<0)
19                    q.push(i), level[i]=0;
20            }
21            while(!q.empty()){
22                int node=q.front();
23                q.pop();
24                for(auto i: adj[node])
25                {
26                    int v=mate[i];
27                    if(v<0)
28                        return true;
29                    if(level[v]<0)
30                    {
31                        level[v]=level[node]+1;
32                        q.push(v);
33                    }
34                }
35            }
36        }

```

```

37    }
38    }
39    return false;
40 }
41 function<bool (int)> augment = [&](int node)
42 {
43     for(auto i: adj[node])
44     {
45         int v=mate[i];
46         if(v<0 || (level[v]>level[node] && augment(v)))
47         {
48             mate[node]=i;
49             mate[i]=node;
50             return true;
51         }
52     }
53     return false;
54 }
55 int match=0;
56 while(levelize())
57     for(int i=1; i<=L; i++)
58         if(mate[i] < 0 && augment(i))
59             match++;
60 return match;
61 }
62 };

```

7 Math

7.1 Sum Of Floor

```

1 // return sum_{i=0}^{n-1} floor((ai + b) / m) (mod 2^64)
2 #define ull unsigned long long
3 ull floor_sum_unsigned(ull n, ull m, ull a, ull b) {
4     ull ans = 0;
5     while (true) {
6         if (a >= m) {
7             ans += n * (n - 1) / 2 * (a / m);
8             a %= m;
9         }
10        if (b >= m) {
11            ans += n * (b / m);
12            b %= m;
13        }
14        ull y_max = a * n + b;
15        if (y_max < m) break;
16        // y_max < m * (n + 1)
17        // floor(y_max / m) <= n
18        n = (ull)(y_max / m);
19        b = (ull)(y_max % m);
20        std::swap(m, a);
21    }
22    return ans;
23 }

```

7.2 Xor With Gauss

```

1 /*
2     Some applications
3     If you want to find the maximum in xor subset
4     just ans = max(ans, ans ^ p[i]) for all i
5     if you want to count the number of subsets with a certain value
6     check all different subsets of p
7 */
8 ll p[66];
9 bool add(ll x) {
10    for(int i = 60; (~i) && x; --i) {
11        if(x >> i & 1) {
12            if(!p[i]) {
13                p[i] = x;
14                return true;
15            }
16            else {
17                x ^= p[i];
18            }
19        }
20    }
21    return false;
22 }

```

7.3 Josephus

```

1 // n = total person
2 // will kill every kth person, if k = 2, 2,4,6,...
3 // returns the mth killed person
4 ll josephus(ll n, ll k, ll m) {
5     m = n - m;
6     if (k <= 1) return n - m;
7     ll i = m;
8     while (i < n) {
9         ll r = (i - m + k - 2) / (k - 1);
10        if ((i + r) > n) r = n - i;
11        else if (!r) r = 1;
12        i += r;
13        m = (m + (r * k)) % i;
14    } return m + 1;
15 }

```

7.4 Rabin Miller Primality check

```

1 // n < 4,759,123,141          3 : 2, 7, 61
2 // n < 1,122,004,669,633      4 : 2, 13, 23, 1662803
3 // n < 3,474,749,660,383      6 : pirmes <= 13
4 // n < 3,825,123,056,413,051  9 : primes <= 23
5
6 int testPrimes[] = {2,3,5,7,11,13,17,19,23};
7
8 struct MillerRabin{
9     ///change K according to n
10    const int K = 9;
11    ll mult(ll s, ll m, ll mod){
12        if(!m) return 0;
13        ll ret = mult(s, m/2, mod);
14        ret = (ret + ret) % mod;
15        if(m & 1) ret = (ret + s) % mod;
16        return ret;
17    }
18
19    ll power(ll x, ll p, ll mod){
20        ll s = 1, m = x;
21        while(p){
22            if(p&1) s = mult(s, m, mod);
23            p >>= 1;
24            m = mult(m, m, mod);
25        }
26        return s;
27    }
28
29    bool witness(ll a, ll n, ll u, int t){
30        ll x = power(a, u, n), nx;
31        for(int i = 0; i < t; i++){
32            nx = mult(x, x, n);
33            if(nx == 1 and x != 1 and x != n-1) return 1;
34            x = nx;
35        }
36        return x != 1;
37    }
38
39    bool isPrime(ll n){ // return 1 if prime, 0 otherwise
40        if(n < 2) return 0;
41        if(!(n&1)) return n == 2;
42        for(int i = 0; i < K; i++) if(n == testPrimes[i]) return 1;
43        ll u = n-1; int t = 0;
44
45        while(u&1) u >>= 1, t++; // n-1 = u*2^t
46
47        for(int i = 0; i < K; i++) if(witness(testPrimes[i], n, u, t))
48            return 0;
49        return 1;
50    }
51 }tester;

```

8 Strings

8.1 Aho-Corasick Mostafa

```

1 struct AC_FSM {
2     #define ALPHABET_SIZE 26
3
4     struct Node {
5         int child[ALPHABET_SIZE], failure = 0, match_parent = -1;
6         vector<int> match;
7     }

```

```

8     Node() {
9         for (int i = 0; i < ALPHABET_SIZE; ++i) child[i] = -1;
10    }
11
12    vector<Node> a;
13
14    AC_FSM() {
15        a.push_back(Node());
16    }
17
18    void construct_automaton(vector<string> &words) {
19        for (int w = 0, n = 0; w < words.size(); ++w, n = 0) {
20            for (int i = 0; i < words[w].size(); ++i) {
21                if (a[n].child[words[w][i] - 'a'] == -1) {
22                    a[n].child[words[w][i] - 'a'] = a.size();
23                    a.push_back(Node());
24                }
25                n = a[n].child[words[w][i] - 'a'];
26            }
27            a[n].match.push_back(w);
28        }
29    }
30
31    queue<int> q;
32    for (int k = 0; k < ALPHABET_SIZE; ++k) {
33        if (a[0].child[k] == -1) a[0].child[k] = 0;
34        else if (a[0].child[k] > 0) {
35            a[a[0].child[k]].failure = 0;
36            q.push(a[0].child[k]);
37        }
38    }
39    while (!q.empty()) {
40        int r = q.front();
41        q.pop();
42        for (int k = 0, arck; k < ALPHABET_SIZE; ++k) {
43            if ((arck = a[r].child[k]) != -1) {
44                q.push(arck);
45                int v = a[r].failure;
46                while (a[v].child[k] == -1) v = a[v].failure;
47                a[arck].failure = a[v].child[k];
48                a[arck].match_parent = a[v].child[k];
49                while (a[arck].match_parent != -1 &&
50                    a[a[arck].match_parent].match.empty())
51                    a[arck].match_parent =
52                        a[a[arck].match_parent].match_parent;
53            }
54        }
55    }
56
57    void aho_corasick(string &sentence, vector<string> &words,
58        vector<vector<int>> &matches) {
59        matches.assign(words.size(), vector<int>());
60        int state = 0, ss = 0;
61        for (int i = 0; i < sentence.length(); ++i, ss = state) {
62            while (a[ss].child[sentence[i] - 'a'] == -1)
63                ss = a[ss].failure;
64            state = a[ss].child[sentence[i] - 'a'];
65            sentence[i] - 'a';
66            for (ss = state; ss != -1; ss = a[ss].match_parent)
67                for (int w: a[ss].match)
68                    matches[w].push_back(i + 1 - words[w].length());
69        }
70    }

```

8.2 KMP Anany

```

1 vector<int> fail(string s) {
2     int n = s.size();
3     vector<int> pi(n);
4     for(int i = 1; i < n; i++) {
5         int g = pi[i-1];
6         while(g && s[i] != s[g])
7             g = pi[g-1];
8         g += s[i] == s[g];
9         pi[i] = g;
10    }
11    return pi;
12 }
13 vector<int> KMP(string s, string t) {
14     vector<int> pi = fail(t);
15     vector<int> ret;

```

```

16   for(int i = 0, g = 0; i < s.size(); i++) {
17       while (g && s[i] != t[g])
18           g = pi[g-1];
19       g += s[i] == t[g];
20       if(g == t.size()) { ///occurrence found
21           ret.push_back(i-t.size()+1);
22           g = pi[g-1];
23       }
24   }
25   return ret;
26 }

```

8.3 Manacher Kactl

```

1  /// If the size of palindrome centered at i is x, then d1[i] stores (x
2  +1)/2.
3  vector<int> d1(n);
4  for (int i = 0, l = 0, r = -1; i < n; i++) {
5      int k = (i > r) ? 1 : min(d1[l + r - i], r - i + 1);
6      while (0 <= i - k && i + k < n && s[i - k] == s[i + k]) {
7          k++;
8      }
9      d1[i] = k--;
10     if (i + k > r) {
11         l = i - k;
12         r = i + k;
13     }
14 }
15 /// If the size of palindrome centered at i is x, then d2[i] stores x/2
16 vector<int> d2(n);
17 for (int i = 0, l = 0, r = -1; i < n; i++) {
18     int k = (i > r) ? 0 : min(d2[l + r - i + 1], r - i + 1);
19     while (0 <= i - k - 1 && i + k < n && s[i - k - 1] == s[i + k]) {
20         k++;
21     }
22     d2[i] = k--;
23     if (i + k > r) {
24         l = i - k - 1;
25         r = i + k;
26     }
27 }
28 }

```

8.4 Suffix Array Kactl

```

1  struct SuffixArray {
2      using vi = vector<int>;
3      #define rep(i,a,b) for(int i = a; i < b; i++)
4      #define all(x) begin(x), end(x)
5      /*
6       * Note this code is considers also the empty suffix
7       * so hear sa[0] = n and sa[1] is the smallest non empty suffix
8       * and sa[n] is the largest non empty suffix
9       * also LCP[i] = LCP(sa[i-1], sa[i]), meaning LCP[0] = LCP[1] =
10         0
11         if you want to get LCP(i..j) you need to build a mapping
12         between
13         sa[i] and i, and build a min sparse table to calculate the
14         minimum
15         note that this minimum should consider sa[i+1...j] since you
16         don't want
17         to consider LCP(sa[i], sa[i-1])
18         you should also print the suffix array and lcp at the
19         beginning of the contest
20         to clarify this stuff
21         */
22     vi sa, lcp;
23     SuffixArray(string& s, int lim=256) { /// or basic_string<int>
24         int n = sz(s) + 1, k = 0, a, b;
25         vi x(all(s)+1), y(n), ws(max(n, lim)), rank(n);
26         sa = lcp = y, iota(all(sa), 0);
27         for (int j = 0, p = 0; p < n; j = max(1, j * 2), lim = p) {
28             p = j, iota(all(y), n - j);
29             rep(i,0,n) if (sa[i] >= j) y[p++] = sa[i] - j;
30             fill(all(ws), 0);
31             rep(i,0,n) ws[x[i]]++;
32             rep(i,1,lim) ws[i] += ws[i - 1];
33             for (int i = n; i--;) sa[--ws[x[y[i]]]] = y[i];
34             swap(x,y), p = 1, x[sa[0]] = 0;
35             rep(i,1,n) a = sa[i - 1], b = sa[i], x[b] =

```

```

32         (y[a] == y[b] && y[a + j] == y[b + j]) ? p - 1 : p++;
33     }
34     rep(i,1,n) rank[sa[i]] = i;
35     for (int i = 0, j; i < n - 1; lcp[rank[i+1]] = k)
36         for (k && k--, j = sa[rank[i] - 1];
37             s[i + k] == s[j + k]; k++);
38 }
39 };

```

8.5 Suffix Automaton Mostafa

```

1  struct SA {
2      struct node {
3          int to[26];
4          int link, len, co = 0;
5      };
6      node() {
7          memset(to, 0, sizeof to);
8          co = 0, link = 0, len = 0;
9      }
10 };
11 int last, sz;
12 vector<node> v;
13 SA() {
14     v = vector<node>(1);
15     last = 0, sz = 1;
16 }
17 void add_letter(int c) {
18     int p = last;
19     last = sz++;
20     v.push_back({});
21     v[last].len = v[p].len + 1;
22     v[last].co = 1;
23     for (; v[p].to[c] == 0; p = v[p].link)
24         v[p].to[c] = last;
25     if (v[p].to[c] == last) {
26         v[last].link = 0;
27         return;
28     }
29     int q = v[p].to[c];
30     if (v[q].len == v[p].len + 1) {
31         v[last].link = q;
32         return;
33     }
34     int cl = sz++;
35     v.push_back(v[q]);
36     v.back().co = 0;
37     v.back().len = v[p].len + 1;
38     v[last].link = v[q].link = cl;
39     for (; v[p].to[c] == q; p = v[p].link)
40         v[p].to[c] = cl;
41 }
42 void build_co() {
43     priority_queue<pair<int, int>> q;
44     for (int i = sz - 1; i > 0; i--)
45         q.push({v[i].len, i});
46     while (q.size()) {
47         int i = q.top().second;
48         q.pop();
49         v[v[i].link].co += v[i].co;
50     }
51 }
52 };
53 };

```

8.6 Zalgo Anany

```

1  int z[N], n;
2  void Zalgo(string s) {
3      int L = 0, R = 0;
4      for (int i = 1; i < n; i++) {
5          if (i <= R && z[i-L] < R - i + 1) z[i] = z[i-L];
6          else {
7              L = i;
8              R = max(R, i);
9              while (R < n && s[R-L] == s[R]) R++;
10             z[i] = R-L; --R;
11         }
12     }
13 }

```


8.7 lexicographically smallest rotation of a string

```

1 int minRotation(string s) {
2     int a=0, N=sz(s); s += s;
3     rep(b,0,N) rep(k,0,N) {
4         if (a+k == b || s[a+k] < s[b+k]) {b += max(0, k-1); break;}
5         if (s[a+k] > s[b+k]) { a = b; break; }
6     }
7     return a;
8 }

```

9 Trees

9.1 Centroid Decomposition

```

1  /*
2  Properties:
3  1. consider path(a,b) can be decomposed to path(a,lca(a,b))
4     and path(b,lca(a,b))
5  where lca(a,b) is the lca on the centroid tree
6  2. Each one of the n^2 paths is the concatenation of two paths
7     in a set of O(n lg(n))
8  paths from a node to all its ancestors in the centroid
9  decomposition.
10 3. Ancestor of a node in the original tree is either an
11   ancestor in the CD tree or
12   a descendant
13 */
14 vector<int> adj[N]; ///adjacency list of original graph
15 int n;
16 int sz[N];
17 bool used[N];
18 int centPar[N]; ///parent in centroid
19 void init(int node, int par) { ///initialize size
20     sz[node] = 1;
21     for(auto p : adj[node])
22         if(p != par && !used[p]) {
23             init(p, node);
24             sz[node] += sz[p];
25         }
26 }
27 int centroid(int node, int par, int limit) { ///get centroid
28     for(int p : adj[node])
29         if(!used[p] && p != par && sz[p] * 2 > limit)
30             return centroid(p, node, limit);
31     return node;
32 }
33 int decompose(int node) {
34     init(node,node); ///calculate size
35     int c = centroid(node, node, sz[node]); ///get centroid
36     used[c] = true;
37     for(auto p : adj[c])if(!used[p.F]) { ///initialize parent for
38         others and decompose
39         centPar[decompose(p.F)] = c;
40     }
41     return c;
42 }
43 void update(int node, int distance, int col) {
44     int centroid = node;
45     while(centroid){
46         ///solve
47         centroid = centPar[centroid];
48     }
49 }
50 int query(int node) {
51     int ans = 0;
52     int centroid = node;
53     while(centroid) {
54         ///solve
55         centroid = centPar[centroid];
56     }
57     return ans;
58 }

```

9.2 Dsu On Trees

```

1 const int N = 1e5 + 9;
2 vector<int> adj[N];
3 int bigChild[N], sz[N];
4 void dfs(int node, int par) {
5     for(auto v : adj[node]) if(v != par){
6         dfs(v, node);
7         sz[node] += sz[v];
8         if(!bigChild[node] || sz[v] > sz[bigChild[node]]) {
9             bigChild[node] = v;
10        }
11    }
12 }
13 void add(int node, int par, int bigChild, int delta) {
14     ///modify node to data structure
15
16     for(auto v : adj[node])
17         if(v != par && v != bigChild)
18             add(v, node, bigChild, delta);
19 }
20 void dfs2(int node, int par, bool keep) {
21     for(auto v : adj[node])if(v != par && v != bigChild[node]) {
22         dfs2(v, node, 0);
23     }
24     if(bigChild[node]) {
25         dfs2(bigChild[node], node, true);
26     }
27     add(node, par, bigChild[node], 1);
28     ///process queries
29     if(!keep) {
30         add(node, par, -1, -1);
31     }
32 }
33 }
34 }

```

9.3 Heavy Light Decomposition (Along with Euler Tour)

```

1  /*
2  Notes:
3  1. 0-based
4  2. solve function iterates over segments and handles them
5     seperately
6  if you're gonna use it make sure you know what you're doing
7  3. to update/query segment in[node], out[node]
8  4. to update/query chain in[nxt[node]], in[node]
9     nxt[node]: is the head of the chain so to go to the next chain
10     node = par[nxt[node]]
11 */
12 int sz[mxN], nxt[mxN];
13 int in[N], out[N], rin[N];
14 vector<int> g[mxN];
15 int par[mxN];
16 void dfs_sz(int v = 0, int p = -1) {
17     sz[v] = 1;
18     par[v] = p;
19     for (auto &u : g[v]) {
20         if (u == p) {
21             swap(u, g[v].back());
22         }
23         if(u == p) continue;
24         dfs_sz(u,v);
25         sz[v] += sz[u];
26         if (sz[u] > sz[g[v][0]])
27             swap(u, g[v][0]);
28     }
29     if(v != 0)
30         g[v].pop_back();
31 }
32 void dfs_hld(int v = 0) {
33     in[v] = t++;
34     rin[in[v]] = v;
35     for (auto u : g[v]) {
36         nxt[u] = (u == g[v][0] ? nxt[v] : u);
37         dfs_hld(u);
38     }
39     out[v] = t;
40 }
41
42 int n;
43 bool isChild(int p, int u){

```

```

44     return in[p] <= in[u] && out[u] <= out[p];
45 }
46 int solve(int u,int v) {
47     vector<pair<int,int> > segu;
48     vector<pair<int,int> > segv;
49     if(isChild(u,v)){
50         while(nxt[u] != nxt[v]){
51             segv.push_back(make_pair(in[nxt[v]], in[v]));
52             v = par[nxt[v]];
53         }
54         segu.push_back({in[u], in[v]});
55     } else if(isChild(v,u)){
56         while(nxt[u] != nxt[v]){
57             segu.push_back(make_pair(in[nxt[u]], in[u]));
58             u = par[nxt[u]];
59         }
60         segv.push_back({in[v], in[u]});
61     } else {
62         while(u != v) {
63             if(nxt[u] == nxt[v]) {
64                 if(in[u] < in[v]) segv.push_back({in[u],in[v]}), R.push_back
65                     ({u+1,v+1});
66                 else segu.push_back({in[v],in[u]}), L.push_back({v+1,u+1});
67                 u = v;
68                 break;
69             } else if(in[u] > in[v]) {
70                 segu.push_back({in[nxt[u]],in[u]}), L.push_back({nxt[u]+1, u
71                     +1});
72                 u = par[nxt[u]];
73             } else {
74                 segv.push_back({in[nxt[v]],in[v]}), R.push_back({nxt[v]+1, v
75                     +1});
76                 v = par[nxt[v]];
77             }
78         }
79         reverse(segv.begin(),segv.end());
80         int res = 0,state = 0;
81         for(auto p : segu) {
82             qry(1,1,0,n-1,p.first,p.second,state,res);
83         }
84         for(auto p : segv) {
85             qry(0,1,0,n-1,p.first,p.second,state,res);
86         }
87     }
88     return res;
89 }

```

9.4 Mo on Trees

```

1 // Calculate the DFS order, {1, 2, 3, 3, 4, 4, 2, 5, 6, 6, 5, 1}.
2 // Let a query be (u, v), ST(u) <= ST(v), P = LCA(u,v)
3 // Case 1: P = u : the query range would be [ST(u), ST(v)]
4 // Case 2: P != u : range would be [EN(u), ST(v)] + [ST(P), ST(P)].
5 // the path will be the nodes that appears exactly once in that range

```

10 Numerical

10.1 Lagrange Polynomial

```

1 class LagrangePoly {
2 public:
3     LagrangePoly(std::vector<long long> _a) {
4         //f(i) = _a[i]
5         //interpola o vetor em um polinomio de grau y.size() - 1
6         y = _a;
7         den.resize(y.size());
8         int n = (int) y.size();
9         for(int i = 0; i < n; i++) {
10             y[i] = (y[i] % MOD + MOD) % MOD;
11             den[i] = ifat[n - i - 1] * ifat[i] % MOD;
12             if((n - i - 1) % 2 == 1) {
13                 den[i] = (MOD - den[i]) % MOD;
14             }
15         }
16     }
17
18     long long getVal(long long x) {
19         int n = (int) y.size();
20         x = (x % MOD + MOD) % MOD;
21         if(x < n) {

```

```

22             //return y[(int) x];
23         }
24         std::vector<long long> l, r;
25         l.resize(n);
26         l[0] = 1;
27         for(int i = 1; i < n; i++) {
28             l[i] = l[i - 1] * (x - (i - 1) + MOD) % MOD;
29         }
30         r.resize(n);
31         r[n - 1] = 1;
32         for(int i = n - 2; i >= 0; i--) {
33             r[i] = r[i + 1] * (x - (i + 1) + MOD) % MOD;
34         }
35         long long ans = 0;
36         for(int i = 0; i < n; i++) {
37             long long coef = l[i] * r[i] % MOD;
38             ans = (ans + coef * y[i] % MOD * den[i]) % MOD;
39         }
40         return ans;
41     }
42 private:
43     std::vector<long long> y, den;
44 };

```

11 Guide

11.1 Notes

- Don't forget to solve the problem in reverse (i.e deleting->adding or adding->deleting, ...etc)
- Max flow is just choosing the maximum number of paths between source and sink
- If you have a problem that tells you choose a[i] or b[i] (or a range) choose one of them initially and play a take or leave on the other
- If the problem tells you to do something cyclic solving it for $x + x$
- Problems that are close to NP problems sometimes have greedy solutions for large input i.e $n \leq 20-30$
- Check datatypes (if you are getting WA or TLE or RTE)
- in case of merging between sets try bitsets (i.e $i + j$ or sth)
- If you have a TLE soln using bitset might help
- If everything else fails think Brute force or randomization
- If you have a solution and you think it's wrong write it instead of doing nothing

11.2 Assignment Problems

- If you see a problem that tells you out of N choose K that has some property (think flows or aliens trick)
- If you see a problem that tells for some X choose a Y (think flows)
- If the problem tells you to choose a Y from L->R (think range flow i.e putting edges between the same layer)

11.3 XOR problems

- If the problem tells you something about choosing an XOR of a subset (think FWHT or XOR-basis)
- If the problem tells you about getting XOR of a tree path let $a[i] = \text{XOR tree from root to } i$ and solve this as an array
- If the problem tells you range XOR sth it's better to have prefix XOR and make it pairs XOR.

11.4 Subset Problems

- Problems that tells you what is the number of ways to choose X out of N that has some property (think convolution)

11.5 Decompositions

- If a problem is asking you to calculate the answer after K steps you can calculate the answer for K
- If the number of queries is significantly larger than updates or vice versa you can use square root Decompositions to give advantage to one over the other

11.6 Strings

- Longest Common Substring is easier with suffix automaton
- Problems that tell you count stuff that appears X times or count appearances (Use suffix links)
- Problems that tell you find the largest substring with some property (Use Suffix links)
- Remember suffix links are the same as aho corasic failure links (you can memoize them with dp)
- Problems that ask you to get the k-th string (can be either suffix automaton or array)
- Longest Common Prefix is mostly a (suffix automaton-array) thing
- try thinking bitsets

11.7 Trees

- For problems that ask you to count stuff in a subtree think (Euler Tour with RQ - Small to Large - DSU on Trees - PersistentSegTree)
- For Path Problems think (Centroid Decomposition - HLD)
- For a path think (HLD + Euler Tour)
- Note that the farthest node to any node in the tree is one of the two diameter heads
- In case of asking $F(\text{node}, x)$ for each node it's probably DP on Trees

11.8 Flows

- If you want to make a K-covering instead of considering lit edges consider non-lit edges
- To get mincost while maintaining a flow network (note that flows are batched together according to cost)
- If the problem asks you to choose some stuff that minimizes use Min Cut (If maximizes sum up stuff and subtract min cut)

11.9 Geometry

- In case of a set of points try scaling and translation
- Manhattan to King distance $(x,y) \rightarrow (x+y, x-y)$
- Lattice points on line: $\gcd(dx, dy) + 1$
- Pick's theorem: $A = I + \frac{B}{2} - 1$
- sine rule: $\frac{A}{\sin(a)} = \frac{B}{\sin(b)} = \frac{C}{\sin(c)}$
- cosine rule: $C^2 = A^2 + B^2 - 2AB \times \cos(c)$
- Dot product = $|A||B| \times \cos(a)$
- Cross product = $|A||B| \times \sin(a)$
- Rotation around axis: $R = (\cos(a) \times Id + \sin(a) \times \text{cross}U + (1 - \cos(a)) \times \text{outer}U)$
- Angle of regular polygon = $\frac{180 \times (n-2)}{n}$
- # Diagonals of regular polygon = $\frac{n(n-3)}{2}$
- Triangulation of n-gon = Catalan (n-2)

11.10 Area

- triangle = $\frac{B \times H}{2}$
- triangle = $\sqrt{(S \times (S - A) \times (S - B) \times (S - C))}$, S = PERIMETER/2
- triangle = $r \times S$, r = radius of inscribed circle
- circle = $R^2 \times \pi$
- ellipse = $\pi \times r_1 \times r_2$
- sector = $\frac{(r^2 \times a)}{2}$
- circular cap = $\frac{R^2 \times (a - \sin(a))}{2}$
- trapzoid = $\frac{(B1+B2)}{2} \times H$
- prsim = $perimeter(B)L + 2area(B)$
- sphere = $4\pi r^2$

11.11 Volume

- Right circular cylinder = $\pi r^2 h$
- Pyramid = $\frac{Bh}{3}$
- Right circular cone = $\frac{\pi r^2 h}{3}$
- Sphere = $\frac{4}{3}\pi r^2 h$
- Sphere sector = $\frac{2}{3}\pi r^2 h = \frac{2}{3}\pi r^3(1 - \cos(a))$
- Sphere cap = $\frac{\pi h^2(3r-h)}{3}$

11.12 Combinatorics

- Cayley formula: number of forest with k trees where first k nodes belongs to different trees = kn^{n-k-1} . Multinomial theorem for trees of given degree sequence $\binom{n}{d_i}$
- Prufer sequence (M5da calls it parent array)
- K-Cyclic permutation = $\binom{n}{k} \times (k-1)!$
- Stirling numbers $S(n, k) = k \times S(n-1, k) + S(n, k-1)$ number of way to partition n in k sets.

- Bell number $B_n = \sum_1^n (n-1, k) B_k$
- # ways to make a graph with k components connected $n^{k-2} \times \prod_{i=1}^k s_i$
- Arithmetic-geometric-progression $S_n = \frac{A_1 \times G_1 - A_{n+1} \times G_{n+1}}{1-r} + \frac{dr}{(1-r)^2} \times (G_1 - G_{n+1})$

11.13 Graph Theory

- Graph realization problem: sorted decreasing degrees: $\sum_1^k d_i = k(k-1) + \sum (k+1)^n \min(d_i, k)$ (first k form clique and all other nodes are connected to them).
- Euler formula: $v + f = e + c + 1$
- # perfect matching in bipartite graph, $DP[S][j] = DP[S][j-1] + DP[S/v][j-1]$ for all v connected to the j node.

11.14 Max flow with lower bound

- feasible flow in a network with both upper and lower capacity constraints, no source or sink: capacities are changed to upper bound - lower bound. Add a new source and a sink. let $M[v] = (\text{sum of lower bounds of ingoing edges to } v) - (\text{sum of lower bounds of outgoing edges from } v)$. For all v, if $M[v] < 0$ then add edge (S,v) with capacity M, otherwise add (v,T) with capacity -M. If all outgoing edges from S are full, then a feasible flow exists, it is the flow plus the original lower bounds.
- maximum flow in a network with both upper and lower capacity constraints, with source s and sink t: add edge (t,s) with capacity infinity. Binary search for the lower bound, check whether a feasible exists for a network WITHOUT source or sink (B).

11.15 Joseph problem

$$g(n, k) = \begin{cases} 0 & \text{if } n = 1 \\ (g(n-1, k) + k) \bmod n & \text{if } 1 < n < k \\ \left\lfloor \frac{k((g(n', k) - n \bmod k) \bmod n')}{k-1} \right\rfloor \text{ where } n' = n - \left\lfloor \frac{n}{k} \right\rfloor & \text{if } k \leq n \end{cases}$$