Continuous Distributions with R

Tripp Bishop/STAT 5255

```
library(tidyverse)
library(latex2exp)
rm(list = ls())
theme_set(theme_minimal())
```

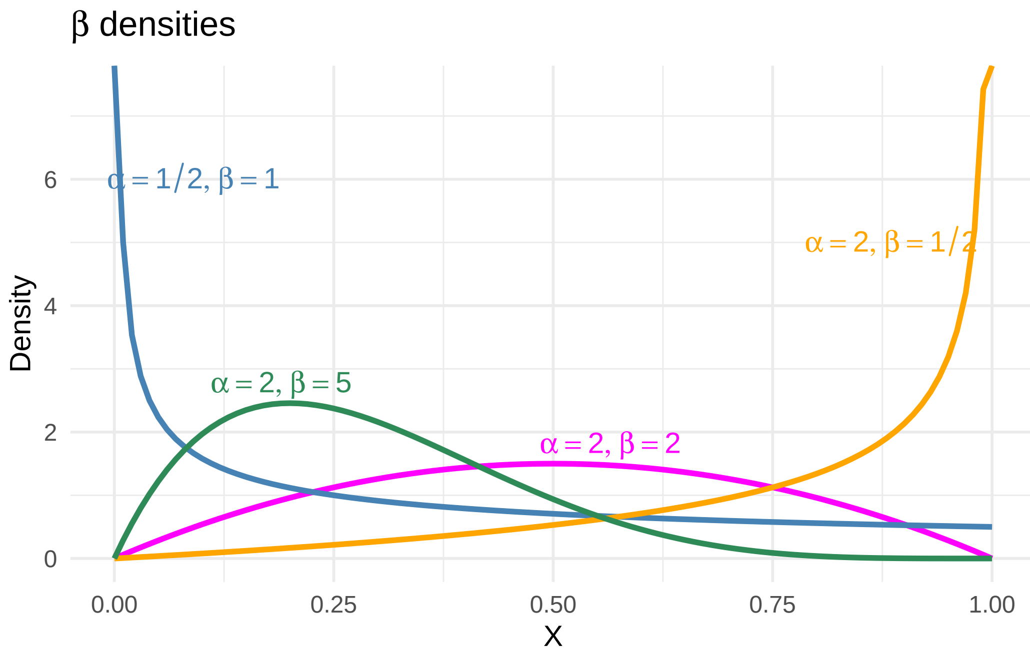
Problem 6.27

Sketch the graphs of the beta densities having
a) $\alpha = 2$, $\beta = 2$ b) $\alpha = 1/2$, $\beta = 1$ c) $\alpha = 2$, $\beta = 1/2$ d) $\alpha = 2$, $\beta = 5$

```
X <- seq(from=0,to=1,by=0.01)

Y.a <- dbeta(X, 2, 2)
Y.b <- dbeta(X, 1/2, 1)
Y.c <- dbeta(X, 2, 1/2)
Y.d <- dbeta(X, 2, 5)

tibble(X,Y.a,Y.b,Y.c,Y.d) |>
  ggplot(aes(x=X)) +
    geom_line(aes(y=Y.a), colour="magenta", linewidth=1) +
    geom_line(aes(y=Y.b), colour="steelblue", linewidth=1) +
    geom_line(aes(y=Y.c), colour="orange", linewidth=1) +
    geom_line(aes(y=Y.d), colour="seagreen", linewidth=1) +
    annotate(geom="text", label=TeX("$\\alpha=1/2, \\beta=1$"), x=0.09, y=6, colour="steelblu
    annotate(geom="text", label=TeX("$\\alpha=2, \\beta=1/2$"), x=0.885, y=5, colour="orange"
    annotate(geom="text", label=TeX("$\\alpha=2, \\beta=5$"), x=0.19, y=2.75, colour="seagree
    annotate(geom="text", label=TeX("$\\alpha=2, \\beta=2$"), x=0.565, y=1.79, colour="magent
    labs(
      title = TeX("$\\beta$ densities"),
      y = "Density"
    )
```



Problem 6.52

In a certain city, the daily consumption of electric power in millions of kilowatt-hours can be treated as a random variable having a gamma distribution with $\alpha = 3$ and $\beta = 2$. If the power plant of this city has a daily capacity of 12 million kilowatt-hours, what is the probability that this power supply will be inadequate on any given day?

What is the probability that $X > 12MWh/day$ given $X \sim \Gamma(3, 2)$?

```
(prob <- pgamma(12, shape=3, scale=2, lower.tail = FALSE))
```

```
[1] 0.0619688
```

The probability that power demand exceeds the power plant's capacity on any given day is $p = 0.062$.

Problem 6.54

The amount of time that a watch will run without having to be reset is a random variable having an exponential distribution with $\theta = 120$ days. Find the probabilities that such a watch

will
a) have to be reset in less than 24 days.

```r
(prob <- pexp(24, rate = 1/120))
```

```
[1] 0.1812692
```

The probability that the watch will have to be reset in less than 24 days is $p = 0.1813$.

b) not have to be reset in at least 180 days.

```r
(prob <- pexp(180, rate = 1/120, lower.tail = FALSE))
```

```
[1] 0.2231302
```

The probability that the watch will not have to be reset in at least 180 days is $p = 0.2231$.

Problem 6.56

The number of bad cheques that a bank receives during a 5-hour business day is a Poisson random variable with $\lambda = 2$. What is the probability that it will not receive a bad cheque on any one day during the first two hours of business?

We need to scale our rate, because λ is describing the rate of bad cheques received over a 5 hour period, but we are dealing with only the first 2 hours of the business day.

```r
(prob <- ppois(0, 4/5))
```

```
[1] 0.449329
```

The probability that the bank will not receive any bad cheques in the first two hours of business on any given day is $p = 0.4493$.

Problem 6.58

If the annual proportion of erroneous income tax returns filed with the IRS can be looked upon as a random variable having a beta distribution with $\alpha = 2$ and $\beta = 9$, what is the probability that in any given year there will be fewer than 10% erroneous returns?

```r
(prob <- pbeta(0.1, shape1=2, shape2=9))
```

```
[1] 0.2639011
```

The probability that the IRS will receive fewer than 10% erroneous returns is $p = 0.2639$.

The annual expected proportion of erroneous income tax returns is given by

$$E[X] = \frac{\alpha}{\alpha + \beta} = \frac{2}{2+9} = \frac{2}{11} \approx 0.22.$$

Problem 6.59

A certain kind of appliance requires repairs on the average once every 2 years. Assuming that the times between repairs are exponentially distributed, what is the probability that such an appliance will work at least 3 years without requiring repairs?

```r
(prob <- pexp(3, rate = 1/2, lower.tail = FALSE))
```

```
[1] 0.2231302
```

The probability that an appliance of this kind going at least 3 years without requiring repairs is $p = 0.2231$.

Problem 6.61

Suppose that the service life in hours of a semiconductor is a random variable having a Weibull distribution with $\alpha = 0.025$ and $\beta = 0.500$.
a) How long can such a semiconductor be expected to last?

```r
qweibull(0.5, shape = 0.5, scale = 1/(0.025)^2)
```

```
[1] 768.7248
```

A semiconductor can be expected to last about 768.7 hours using these distribution parameters.

b) What is the probability that such a semiconductor will still be in operating condition after 4000 hours?

```r
(prob <- pweibull(4000, shape = 0.5, scale = 1/(0.025)^2, lower.tail = FALSE))
```

```
[1] 0.2057407
```

The probability that one of these semiconductors continuing to operate beyond 4000 hours is $p = 0.2057$.

Problem 6.70

Suppose that during periods of meditation the reduction of a person's oxygen consumption is a random variable having a normal distribution with $\mu = 37.6$ cc per minute and $\sigma = 4.6$ cc per minute. Find the probabilities that during a period of meditation a person's oxygen consumption will be reduced by
a) at least 44.5 cc per minute

```r
mu <- 37.6
sigma <- 4.6

(prob <- pnorm(44.5, mean=mu, sd=sigma, lower.tail = FALSE))
```

```
[1] 0.0668072
```

The probability that during a period of meditation a person's oxygen consumption will be reduced by at least 44.5 cc per minute is $p = 0.0668$.

b) at most 35.0 cc per minute

```r
(prob <- pnorm(35.0, mean=mu, sd=sigma))
```

```
[1] 0.285963
```

The probability that during a period of meditation a person's oxygen consumption will be reduced by at most 35.0 cc per minute is $p = 0.286$.

c) anywhere from 30.0 to 40.0 cc per minute

```r
probs <- pnorm(c(30.0,40.0), mean=mu, sd=sigma)
(diff_prob <- probs[2] - probs[1])
```

```
[1] 0.6498245
```

The probability that during a period of meditation a person's oxygen consumption will be reduced by between 30.0 and 40.0 cc per minute is $p = 0.6498$.