

Continuous Distributions with R

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```
library(tidyverse)
library(latex2exp)
rm(list = ls())
theme_set(theme_minimal())
```

Problem 6.27

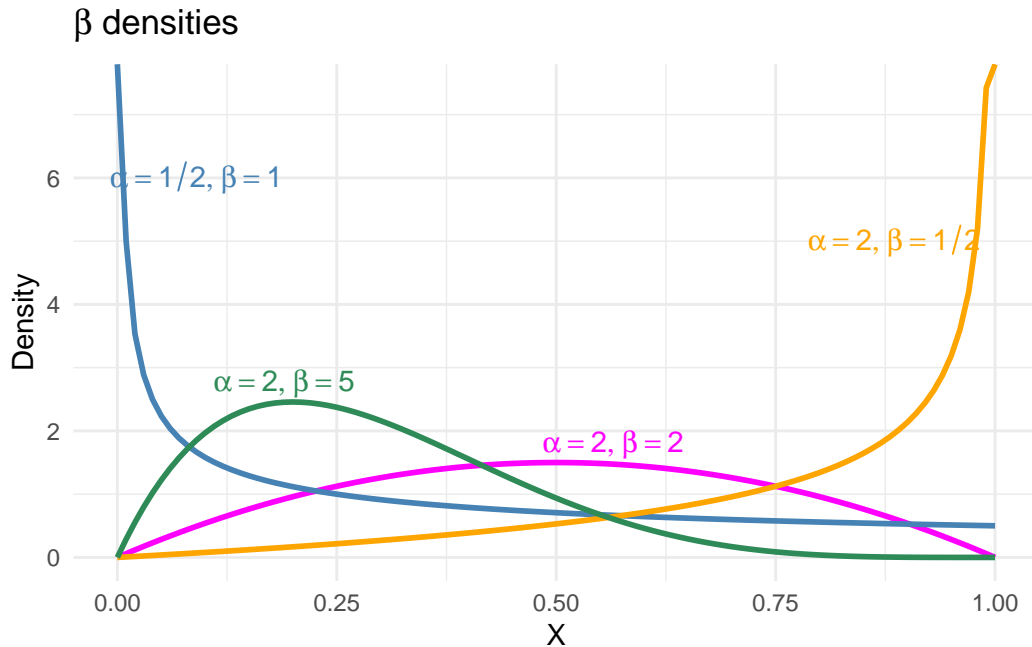
Sketch the graphs of the beta densities having

a) $\alpha = 2, \beta = 2$ b) $\alpha = 1/2, \beta = 1$ c) $\alpha = 2, \beta = 1/2$ d) $\alpha = 2, \beta = 5$

```
X <- seq(from=0,to=1,by=0.01)

Y.a <- dbeta(X, 2, 2)
Y.b <- dbeta(X, 1/2, 1)
Y.c <- dbeta(X, 2, 1/2)
Y.d <- dbeta(X, 2, 5)

tibble(X,Y.a,Y.b,Y.c,Y.d) |>
  ggplot(aes(x=X)) +
    geom_line(aes(y=Y.a), colour="magenta", linewidth=1) +
    geom_line(aes(y=Y.b), colour="steelblue", linewidth=1) +
    geom_line(aes(y=Y.c), colour="orange", linewidth=1) +
    geom_line(aes(y=Y.d), colour="seagreen", linewidth=1) +
    annotate(geom="text", label=TeX("$\\alpha=1/2, \\beta=1$"), x=0.09, y=6, colour="steelblue") +
    annotate(geom="text", label=TeX("$\\alpha=2, \\beta=1/2$"), x=0.885, y=5, colour="orange") +
    annotate(geom="text", label=TeX("$\\alpha=2, \\beta=5$"), x=0.19, y=2.75, colour="seagreen") +
    annotate(geom="text", label=TeX("$\\alpha=2, \\beta=2$"), x=0.565, y=1.79, colour="magenta") +
    labs(
      title = TeX("$\\beta$ densities"),
      y = "Density"
    )
```



Problem 6.52

In a certain city, the daily consumption of electric power in millions of kilowatt-hours can be treated as a random variable having a gamma distribution with $\alpha = 3$ and $\beta = 2$. If the power plant of this city has a daily capacity of 12 million kilowatt-hours, what is the probability that this power supply will be inadequate on any given day?

What is the probability that $X > 12MWh/day$ given $X \sim \Gamma(3, 2)$?

```
(prob <- pgamma(12, shape=3, scale=2, lower.tail = FALSE))
```

```
[1] 0.0619688
```

The probability that power demand exceeds the power plant's capacity on any given day is $p = 0.062$.

Problem 6.54

The amount of time that a watch will run without having to be reset is a random variable having an exponential distribution with $\theta = 120$ days. Find the probabilities that such a watch

will

a) have to be reset in less than 24 days.

```
(prob <- pexp(24, rate = 1/120))
```

```
[1] 0.1812692
```

The probability that the watch will have to be reset in less than 24 days is $p = 0.1813$.

b) not have to be reset in at least 180 days.

```
(prob <- pexp(180, rate = 1/120, lower.tail = FALSE))
```

```
[1] 0.2231302
```

The probability that the watch will not have to be reset in at least 180 days is $p = 0.2231$.

Problem 6.56

The number of bad cheques that a bank receives during a 5-hour business day is a Poisson random variable with $\lambda = 2$. What is the probability that it will not receive a bad cheque on any one day during the first two hours of business?

We need to scale our rate, because λ is describing the rate of bad cheques received over a 5 hour period, but we are dealing with only the first 2 hours of the business day.

```
(prob <- ppois(0, 4/5))
```

```
[1] 0.449329
```

The probability that the bank will not receive any bad cheques in the first two hours of business on any given day is $p = 0.4493$.

Problem 6.58

If the annual proportion of erroneous income tax returns filed with the IRS can be looked upon as a random variable having a beta distribution with $\alpha = 2$ and $\beta = 9$, what is the probability that in any given year there will be fewer than 10% erroneous returns?

```
(prob <- pbeta(0.1, 2, 9))
```

```
[1] 0.2639011
```

The probability that the IRS will receive fewer than 10% erroneous returns is $p = 0.2639$.