Empirical problem set 1

Problem 1

(a) Simulate from an autoregressive process of order one ($AR(1)$) i.e. simulate a data set from the data generating process

$$y_t = \beta_1 y_{t-1} + u_t \quad u_t \sim i.i.d.N(0, \sigma^2), \quad i = 1, ..., T \tag{1}$$

with $\beta_1 = 0.9$, $\sigma^2 = 2$ and $T = 100$. Use a burn-in period of 20 observations. Plot the simulated time series.

(b) Using your simulated data set, estimate the coefficient β_1 using ordinary least squares (OLS) in matrix notation.

(c) Using your simulated data set from part (a), estimate an $AR(2)$ model. Test whether the coefficient of the second lag is significantly different from zero. In order to do this, you will need the covariance matrix of the estimated parameter vector $\beta = (\beta_1, \beta_2)'$. Use significance level $\alpha = 0.05$.
Check your results using the output from the built-in functions ar.ols() and lm().
Hint: The formula for the estimated covariance matrix is

$$\widehat{\mathbb{V}}[\beta|X] = \widehat{\sigma}^2(X'X)^{-1} \tag{2}$$

with $\widehat{\sigma}^2 = \frac{1}{T-K}\widehat{u}'\widehat{u}$.

(d) (Optional) Repeat parts (a) and (b) using functions. The functions should have the following input:

- for part (a): T, β_0, β_1, σ^2 and the number of burn-in observations.
- for part (b): time series vector y, lag order p and the information whether the model should include an intercept.

Problem 2

(a) Simulate 100 observations from the bivariate VAR(1) process

$$\mathbf{y}_t = \begin{pmatrix} y_{t1} \\ y_{t2} \end{pmatrix} = \begin{pmatrix} 0.1 \\ -0.2 \end{pmatrix} + \begin{pmatrix} 4/5 & 8/3 \\ 0 & 3/10 \end{pmatrix} \mathbf{y}_{t-1} + \mathbf{u}_t \quad \mathbf{u}_t \sim i.i.d.N(0, \Sigma) \tag{3}$$

where $\Sigma = \begin{pmatrix} 1 & 0.9 \\ 0.9 & 2 \end{pmatrix}$. Use 20 burn-in observations. Initialize by the unconditional mean of the process. Convert your simulated data to a 'ts' object with quarterly data that started in 1990, Q1. Plot the bivariate time series, both in one figure and in two separate ones.

(b) Is the VAR given in equation (3) stable?

(c) Estimate the mean of the VAR(1) given in (3) and compare it to the theoretical mean. Observe whether they get closer as you increase T.