SECTION I : Introduction, Brief Description Of The Concepts and Methodology

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1 Undertaking

This is to declare that this work titled ***"N P V constrained by Quartely Return for Pre-Bid Evaluation"*** is a bonafide work done by me for participation in F & A Challenge by submission to F & A Idea Challenge Portal(http://ideabank.larsentoubro.com/fnachallenge).

All the analysis, modelling, interpretation have been my own and original creation.

All references have been included appropriately.

- Graphs have been plotted through online service of www.Desmos.org
- Document has been coded in TexStudio Interface of Latex
- Definitions has been drawn directly from Wikipedia
- Partial Derivatives have been re-verified with online services of www.WolframAlpha.com and www.Symbolab.com

2 Introduction to the Series

The Discounted Cash Flow method has become the dominant (quantitative evaluation) method (for Construction Contracts) universally(Jog and Srivastava, 1995; Payne *et al.*, 1999; Arnold and Hatzopoulos, 2000; Graham and Harvey, 2001) [1].

However, Financial Statement users make regular use of the Accounting Rate of Return (A R R) rather than the economic rate of return (I R R) to assess the performance of corporations and public sector enterprises to price financial claims such as shares.[2]

N P V is a critical parameter relied upon by L & T B U Heads in Pre-Bid Risk evaluation of Contracts. However, the quarterly performance presentation (hence evaluation) measures the performance of the Business Unit in terms of R O C E and further drills down to Days of Sales and Credit Period.Therefore, a need has been felt to also evaluate contracts at pre-bid stage on their influence to R O C E

This series is an endeavor to establish the mathematical relations that can act as a quick check at pre-bid stage to take better informed decisions.

This paper being the first in the series deals with the following:

- Establish minimum additional constraints required to be imposed over and above the Positive N P V criteria to meet threshold R O C E (Q-o-Q)
- Connect the dots back to Days of Sales and Vendor Credit Period
- Determine the Contribution (as a percentage of Sales) required to compensate an adverse collection / payment term to meet threshold R O C E

In the second part of this series, Lundberg's Inequality will be used to establish the minimum Contribution Percentage required to as a tradeoff of the uncertainty in Order Booking.(Order Inflow will be expressed as a mixed distribution of Pareto Distirbution (arrival rates) and Normal Distribution(award size))

In the third part of this series, Optimal Absorption on individual bids will be determined as a function of the L1 probability distribution function.

3 Brief Description Of the Concepts

3.1 Natural Exponent e

The number e is a mathematical constant, approximately equal to 2.71828, which appears in may different settings throughout Mathematics. It was discovered by the Swiss Mathematician Jacob Bernoulli while studying Compound Interest.
e arises as the limit of $(1+\frac{1}{n})^n$ as n aproaches Infinity.
The function $f(x) = e^x$ is called the (natural) exponential function, and is a unique exponential function of type a^x equal to its own derivative. $[f(x) = f'(x) = e^x]$[3 4]

3.2 Accumulation Factor(A F)

For t1 <= t2 we define A(t1,t2) to be the accumulation at time t2 of a balance of 1 at time t1.
The number A(t1,t2) is often called an accumulation factor, since the accumulation at time t2 of a balance of C at time t1 is, by proportion:

C * A(t1,t2)

A(n) is often used as an abbreviation for the accumulation factor A(0,n).[5]

Simple Interest is an example of Linear Accumulation.
Compound Interest is an example of Exponential Accumulation.

3.3 Force of Interest (F O I)[5]

If we consider a nominal interest rate convertible very frequently (eg every second), we are no longer thinking of a fund that suddenly acquires an interest payment at the end of each interval, but of a fund that steadily accumulates over the period as interest is earned and added. In the limiting case, the amount of the fund can be considered to be subject to a constant force causing it to grow. This leads us to the concept of a force of interest, which is the easiest way to model continuously paid interest rates mathematically.

3.3.1 Formal Definition

The Force of Interest is the instanteneous change in the balance expressed as an annualised percentage of the current balance.

$$\delta(t) = \frac{V_t{}'}{V_t}$$

it is easy to see

$$\delta(t) = d/dt(\ln V_t)$$

alternatively.

$$\frac{V_{t_2}}{V_{t_1}} = e^{\int_{t_1}^{t_2} \delta(t)dt}$$

[1] Improved Capital Budgeting Decicion Making: Evidence from Canada; Bennouna, Karim, Meredith, Geoffrey, Marchant, Teresa
[2] Economic and Accounting Rates of Return; D.W. Feenstra, H. Wang
[3] Wikipedia: https://en.wikipedia.org/wiki/E(mathematical constant)
[4] "e: The Story of a Number, by Eli Maor"
[5] ActEd Study Materials: Faculty of Actuaries and the Insititute of Actuaries

Therefore the Accumulation Factor:

$$A(0,n) = e^{\int_0^n \delta(t)dt}$$

3.4 Translation of Exponential Functions

A Translation of an exponential function has the form[6]

$$f(x) = ab^{x+c} + d$$

where the parent function $y = b^x$, b >1, is:

- shifted horizontally c units to the left
- stretched vertically by a factor | a | for | a | >0
- compressed vertically by a factor | a | for 0 <| a | <1
- shifted vertically d units
- reflected about the x-axis when a <0

4 Methodology

For every contract under bid, on success, the quarter in which the margin will start accruing will have:

- Some ongoing contracts and therefore a prevailing R O C E (in the deterministic environment, "Expected" R O C E in a probabilistic environment.)

- A dead weight of N F A (in denominator) and Fixed Expenses (in Numerator)

- Left over N W C of completed projects, not contributing margin

Therefore, the required condition to be derived is the incremental change in return over the quarter on account of the new project.

We derive the conditions assuming no change in the dead weight of N F A and Fixed Assets and also in the balances of the completed projects. These assumptions can be readily released once the conditions necessary for the new project are derived.

1. We express the Sales Accrual as an Exponential Function in terms of "Accumulation Factor" or "Force of Interest".

2. We express the Collection Terms as Time-Shift translation of Sales Accrual.

3. Similarly, we express the Payment Terms as Time-Shift translation of Cost Accrual.

4. Also, the Cost Accrual is expressed as a Scaled transformation of the Sales Accrual (under the assumption of constant contribution percentage).

5. We derive an expression of the Return over a particular period t in the life of the contract solely in terms of Collection Terms and Payment Terms as expressed above. This is the expression of interest and can form a check point for the B U for evaluation of the influence of the bid in question on prevailing Return (Prevailing Expected Return)

6. We derive a nifty expression for the sensitivity of "Return" to Collection Terms and Payment Terms.

7. Thereby we deduce the ***approximate Thumb Rule between Collection and Payment terms to keep the prevailing Return intact***.

4.1 Conserving the prevailing R O C E:

We denote the prevailing R O C E ρ as:

$$\rho = \frac{A}{B}$$

A simple manipulation yields R O C E for the individual project under bid also has to be ρ.
We show it as follows:

Let the return after acquisition of the new project be denoted by

$$\rho' >= \frac{A+a}{B+b}$$

substituting for A

$$\rho' >= \frac{\rho B + a}{B+b}$$

such that

$$\rho' B + \rho' b >= \rho B + a$$

for $\rho = \rho'$ this simplifies to the trivial solution

$$\rho' >= a/b$$

Therefore, every additional contract individually should have a return ρ' higher than the prevailing return ρ.

4.2 Cash as impulse train

Cash flows associated with a contract can be compared to discrete impulses of varying amplitude over vaying time-interval and can be modelled as Linear Time Invariant systems. The three useful signal operations are time shifting, time scaling and time reversal (inversion)[7]
A signal that is specified for a continuum of values of time t is a *continuous-time signal* and a signal that is specified only at discrete values of t is a *discrete-time signal*.

Though unit impulses of Cash can be expressed by Unit Step Function which will inherently be discontinuous, the accumulated balance can however be expressed as a continuous function with points of discontinuity only at instances of receipt. The same principle can be adopted for all accruing balances like Sales and Cost and derived balances like Accounts Receivables and Payables. The derived balances of AP and A R can further be expressed as difference of two continuous functions of Accruing Sales and Collection or Accruing Cost and Payments.

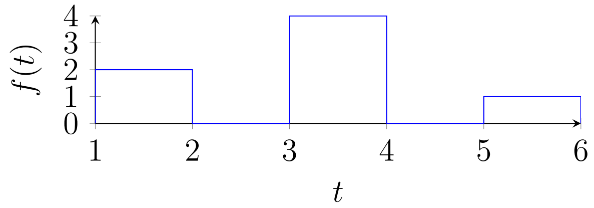
The Accrual rate over short intervals of time can be expressed in terms of time-varying ***Accumulation Factors*** with the application of the concept of ***Force of Interest*** The Collection Terms or Payment Terms can further be expressed as Time-Scaled and Time-Shifted variants of the Accruing Sales or Cost functions.

4.2.1 Unit Step Function

A unit step function is defined by :

$$f(t) = \begin{cases} 1 & t \leq 0 \\ 0 & t > 0 \end{cases}$$

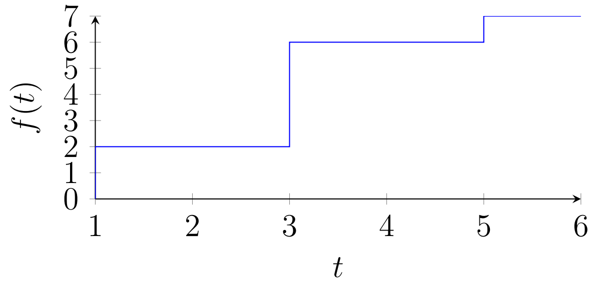
Figure 1

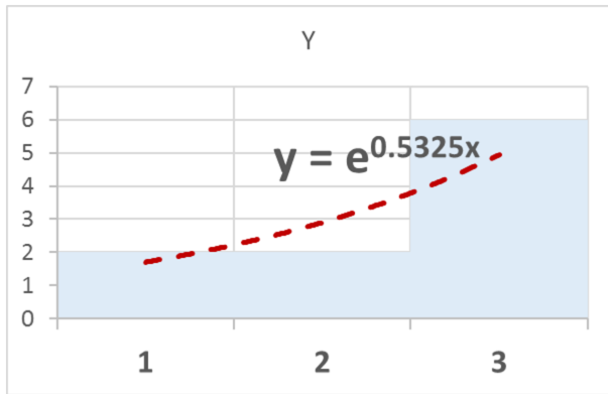


4.2.2 Accumulation of Step Function (Discrete Time)

The Figure below graphs the accumulation of same stream approximated by exponent

Figure 2





6 www.lumenlearning.com

7 Linear Systems and Signals, 2nd Edition, B P Lathi

SECTION II : Return as a function of Linear and Scaled Transformation of Exponent Function

5 Days of Sales in terms of Exp. lag

$$
\begin{aligned}
&\text{(1a)} & Sales \quad Accrual \quad (S_a) &= e^{\delta_1(\Delta t)} \\
&\text{(1b)} & Coll. \quad Accrual \quad (Coll._a) &= e^{\delta_1(\Delta t)-K} \\
&\text{(1c)} & AR \quad Accrual \quad (AR)_a &= e^{\delta_1(\Delta t)} - e^{\delta_1(\Delta t)-K} \\
&\text{(1d)} & Ave. \quad (AR)_a &= \frac{e^{\delta_1(\Delta t)} - e^{\delta_1(\Delta t)-K}}{\Delta t} \\
&\text{(1e)} & Sales/day(S_{pd}) &= \frac{e^{\delta_1(\Delta t)}}{\Delta t} \\
&\text{(1f)} & Ave. \quad (DOS) &= \frac{Ave. \quad (AR)_a}{S_{pd}} \\
& & &= \frac{e^{\delta_1(\Delta t)} - e^{\delta_1(\Delta t)-K}}{e^{\delta_1(\Delta t)}} \\
& & &= 1 - e^{-K}
\end{aligned}
$$

5.1 Plots

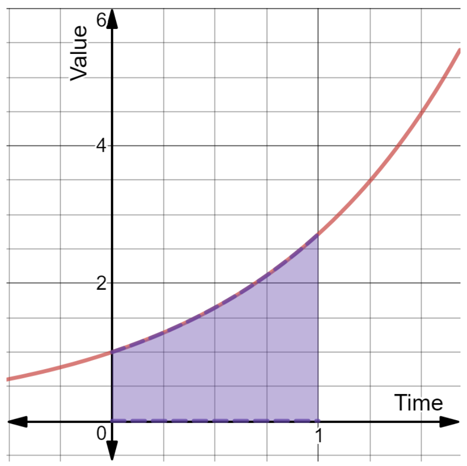


Figure 1: Plot of a Accumulated Sales as a growing function

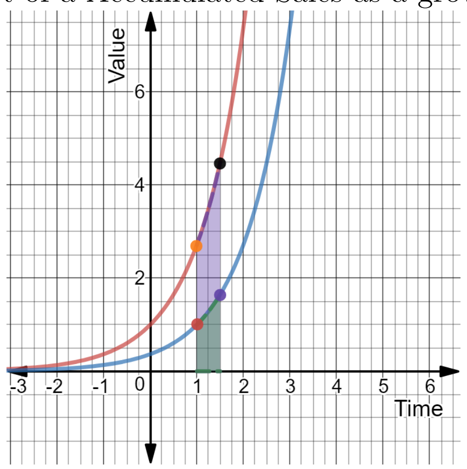


Figure 2: Plot of a A R Accrual as a function of Accrued Sales and Accrued Collection

6 Days of Purchase in terms of Exp. lag

$$
\begin{aligned}
&\text{(2a)} & Cost \quad Accrual \quad (C_a) &= e^{\delta_2(\Delta t)} \\
&\text{(2b)} & Pay. \quad Accrual \quad (P_a) &= e^{\delta_2(\Delta t)-L} \\
&\text{(2c)} & AP \quad Accrual \quad (AP)_a &= e^{\delta_2(\Delta t)} - e^{\delta_2(\Delta t)-L} \\
&\text{(2d)} & Ave. \quad (AP)_a &= \frac{e^{\delta_2(\Delta t)} - e^{\delta_2(\Delta t)-L}}{\Delta t} \\
&\text{(2e)} & Cost/day(C_{pd}) &= \frac{e^{\delta_2(\Delta t)}}{\Delta t} \\
&\text{(2f)} & Ave. \quad (DOP) &= \frac{Ave. \quad (AP)_a}{S_{pd}} \\
& & &= \frac{e^{\delta_2(\Delta t)} - e^{\delta_2(\Delta t)-L}}{e^{\delta_2(\Delta t)}} \\
& & &= 1 - e^{-L}
\end{aligned}
$$

7 Contribution in terms of scaled transformation of exponent

$$\text{(3a)} \qquad Contribution \quad M \quad = \quad e^{\delta_1(T)} - e^{\delta_2(T)}$$

However Total Cost can be expressed as a scaled transformation of Contract Value.

$$\text{(3b)} \qquad (1-M)e^{(\Delta_1 T)} \quad = \quad e^{\phi \Delta_1 T}$$

for 0 $<\phi<$1

There is no loss of generality while evaluated locally

Therefore:

$$\text{(3c)} \qquad (1-M)e^{(\delta_1 T)} \quad = \quad e^{\phi \delta_1 T}$$

Simplifying we get

$$\text{(3d)} \qquad \delta_2 t = \delta_1 t(1 + \ln(1-M))$$

for a predefined Contribution

$$\text{(3e)} \qquad \delta_2 = \delta_1 \alpha$$

7.1 Plot

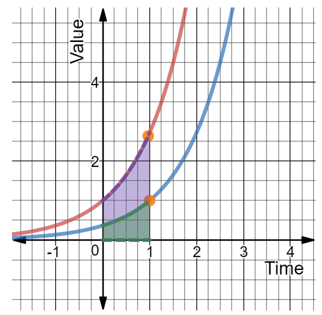


Figure 3: Plot of Contribution as a Scaled tranformation and difference. The higlighted points indicate the profit as a scaled transformation

8 Return on Capital

$$\text{(4a)} \qquad \rho = \frac{M * e^{\delta_1(\Delta t)}}{Ave. \quad (AR)_a - Ave. \quad (AP)_a}$$

Where :

$$
\begin{aligned}
Ave.(AR)_a = &\quad Ave.(DOS) * S_{pd} = \quad (1-e^{-K}) * \frac{e^{\delta_1(\Delta t)}}{\Delta t} \\
Ave.(AP)_a = &\quad Ave.(DOP) * C_{pd} = \quad (1-e^{-L}) * \frac{e^{\delta_2(\Delta t)}}{\Delta t}
\end{aligned}
$$

$$\text{(4b)} \qquad \rho(\Delta t) = \frac{M * e^{\delta_1(\Delta t)}}{(1-e^{-K})(e^{\delta_1(\Delta t)}) - (1-e^{-L})(e^{\delta_2(\Delta t)})}$$

$$\text{(4c)} \qquad = \frac{M}{(\Delta t)((1-e^{-K}) - (1-e^{-L})\alpha)}$$

α is constant (from eq. 3e)

at this point we drop the notation Δ and observe the time frame as t

$$\text{(4d)} \qquad \rho = \frac{M}{t((1-e^{-K}) - (1-e^{-L})\alpha)}$$

8.1 Perturbation of R

$$\text{(5a)} \qquad d\rho = \frac{\partial \rho}{\partial t}dt + \frac{\partial \rho}{\partial K}dK + \frac{\partial \rho}{\partial L}dL$$

For constant M and α

$$\text{(5b)} \qquad d\rho = \frac{M}{(1-e^{K}) - \alpha(1-e^{L})}(-1/t^2)dt + \frac{\partial \rho}{\partial K}dK + \frac{\partial \rho}{\partial L}dL$$

We can drop the term of t as it will approach to be negligible with an increasing time frame (viz. for a quarter or 90 days)

We recover from this

$$dρ = \frac{\partial \rho}{\partial K} dK + \frac{\partial \rho}{\partial L} dL \tag{5c}$$

$$d\rho = \frac{e^{-K}}{(e^{-K} - \alpha e^{-L})^2} dK - \frac{\alpha e^{-L}}{(e^{-K} - a e^{-L})^2} dL \tag{5d}$$

This simplifies to

$$d\rho = \frac{e^{K+2L}}{(e^{K} - \alpha e^{L})^2} dK - \frac{e^{K+2L}}{(e^{K} - \alpha e^{L})^2} dL \tag{5e}$$

Simplifying this further

$$\boxed{L - K = \ln \frac{1 - e^{2L}}{\alpha}} \tag{5f}$$

8.2 Conclusion

In this paper we derive from first principles condition required to be fulfilled additionally over and above positive N P V constraint to preserve the prevailing return. The denominator for computation of return has been considered to be comprised of only A P and A R. However, this can be readily generalised for all other Assets and Liabilities specific to the project.

8.3 Further Research

The N P V of the project will necessaily be a function of the Lag and Scale parameters used in this work. That means the minimum admissible N P V of the project to maintain a specific return (ρ) will be a "Isoperimetric Optimization Problem"[8] in terms of the lag and scale factors. With the help of the method of "Lagrange Multipliers"[9] the research can be extended to answer questions like:

- minimum Contribution required to support a given adverse Payment or Collection Term without prejudicing return
- maximum difference of lag (represented by K and L in this work) for assuring a specific return with a given Contribution percentage
- further extension to this can be to determine the minimum contribution percentage in terms of certainty equivalence in a stochastic environment where the cash flows can only be expressed in Expected Values with probability distributions.

[8] Dynamic Optimization, The Calculus of Variations and Optimal Control in Economics and Management, 2nd edition, Morton I. Kamien and Nancy L. Schwartz

[9] Cullingford and Prideaux, Isoperimetric formulation for Project Planning, 1973