Web-based supporting materials for "Identifying latent subgroups of children with developmental delay using Bayesian sequential updating and Dirichlet process mixture modelling" by P. Gilholm, K. Mengersen and H. Thompson

Please visit `https://github.com/TrishG89/Bayesian_Sequential_Updating_DPMM_supplementary` to access R code and supplementary plots for the analyses conducted in this paper. Due to confidentiality arrangements, the data for the application can not be provided but inquiries regarding access to this data can be directed to Hugh McKenzie (hugh@developingfoundation.org.au). The data used for the simulation studies is available on Github.

1 Table of all Hyperparameter combinations

Table 1: Hyperparameters for all 72 models. Rows in bold indicate the 15 models selected for stage 2 of the grid experiment. Model 26 was selected for the final model(italicised).

Model	N_0	c_0	C_0	α
1	**0.01**	**6**	$\boldsymbol{\Sigma_y}$	$\boldsymbol{Gamma(1,1)}$
2	0.01	7	Σ_y	$Gamma(1,1)$
3	0.01	6	$0.75\Sigma_y$	$Gamma(1,1)$
4	0.01	7	$0.75\Sigma_y$	$Gamma(1,1)$
5	0.01	6	$0.5\Sigma_y$	$Gamma(1,1)$
6	0.01	7	$0.5\Sigma_y$	$Gamma(1,1)$
7	0.01	6	Σ_y	$Gamma(2,2)$
8	0.01	7	Σ_y	$Gamma(2,2)$
9	0.01	6	$0.75\Sigma_y$	$Gamma(2,2)$
10	0.01	7	$0.75\Sigma_y$	$Gamma(2,2)$
11	0.01	6	$0.5\Sigma_y$	$Gamma(2,2)$
12	**0.01**	**7**	$\boldsymbol{0.5\Sigma_y}$	$\boldsymbol{Gamma(2,2)}$
13	0.05	6	Σ_y	$Gamma(1,1)$
14	**0.05**	**7**	$\boldsymbol{\Sigma_y}$	$\boldsymbol{Gamma(1,1)}$
15	0.05	6	$0.75\Sigma_y$	$Gamma(1,1)$
16	0.05	7	$0.75\Sigma_y$	$Gamma(1,1)$
17	0.05	6	$0.5\Sigma_y$	$Gamma(1,1)$
18	**0.05**	**7**	$\boldsymbol{0.5\Sigma_y}$	$\boldsymbol{Gamma(1,1)}$
19	**0.05**	**6**	$\boldsymbol{\Sigma_y}$	$\boldsymbol{Gamma(2,2)}$
20	0.05	7	Σ_y	$Gamma(2,2)$
21	0.05	6	$0.75\Sigma_y$	$Gamma(2,2)$
22	0.05	7	$0.75\Sigma_y$	$Gamma(2,2)$
23	0.05	6	$0.5\Sigma_y$	$Gamma(2,2)$
24	**0.05**	**7**	$\boldsymbol{0.5\Sigma_y}$	$\boldsymbol{Gamma(2,2)}$
25	0.1	6	Σ_y	$Gamma(1,1)$
26	***0.1***	***7***	$\boldsymbol{\Sigma_y}$	$\boldsymbol{Gamma(1,1)}$
27	0.1	6	$0.75\Sigma_y$	$Gamma(1,1)$
28	0.1	7	$0.75\Sigma_y$	$Gamma(1,1)$
29	0.1	6	$0.5\Sigma_y$	$Gamma(1,1)$

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Table 1 – continued from previous page

Model	N_0	c_0	C_0	α
30	0.1	7	$0.5\Sigma_y$	$Gamma(1,1)$
31	0.1	6	Σ_y	$Gamma(2,2)$
32	**0.1**	**7**	$\boldsymbol{\Sigma_y}$	$\boldsymbol{Gamma(2,2)}$
33	0.1	6	$0.75\Sigma_y$	$Gamma(2,2)$
34	**0.1**	**7**	$\boldsymbol{0.75\Sigma_y}$	$\boldsymbol{Gamma(2,2)}$
35	0.1	6	$0.5\Sigma_y$	$Gamma(2,2)$
36	0.1	7	$0.5\Sigma_y$	$Gamma(2,2)$
37	0.2	6	Σ_y	$Gamma(1,1)$
38	0.2	7	Σ_y	$Gamma(1,1)$
39	0.2	6	$0.75\Sigma_y$	$Gamma(1,1)$
40	**0.2**	**7**	$\boldsymbol{0.75\Sigma_y}$	$\boldsymbol{Gamma(1,1)}$
41	0.2	6	$0.5\Sigma_y$	$Gamma(1,1)$
42	0.2	7	$0.5\Sigma_y$	$Gamma(1,1)$
43	0.2	6	Σ_y	$Gamma(2,2)$
44	**0.2**	**7**	$\boldsymbol{\Sigma_y}$	$\boldsymbol{Gamma(2,2)}$
45	0.2	6	$0.75\Sigma_y$	$Gamma(2,2)$
46	0.2	7	$0.75\Sigma_y$	$Gamma(2,2)$
47	0.2	6	$0.5\Sigma_y$	$Gamma(2,2)$
48	0.2	7	$0.5\Sigma_y$	$Gamma(2,2)$
49	0.5	6	Σ_y	$Gamma(1,1)$
50	**0.5**	**7**	$\boldsymbol{\Sigma_y}$	$\boldsymbol{Gamma(1,1)}$
51	0.5	6	$0.75\Sigma_y$	$Gamma(1,1)$
52	0.5	7	$0.75\Sigma_y$	$Gamma(1,1)$
53	0.5	6	$0.5\Sigma_y$	$Gamma(1,1)$
54	0.5	7	$0.5\Sigma_y$	$Gamma(1,1)$
55	0.5	6	Σ_y	$Gamma(2,2)$
56	**0.5**	**7**	$\boldsymbol{\Sigma_y}$	$\boldsymbol{Gamma(2,2)}$
57	0.5	6	$0.75\Sigma_y$	$Gamma(2,2)$
58	0.5	7	$0.75\Sigma_y$	$Gamma(2,2)$
59	0.5	6	$0.5\Sigma_y$	$Gamma(2,2)$
60	0.5	7	$0.5\Sigma_y$	$Gamma(2,2)$
61	1	6	Σ_y	$Gamma(1,1)$
62	**1**	**7**	$\boldsymbol{\Sigma_y}$	$\boldsymbol{Gamma(1,1)}$
63	1	6	$0.75\Sigma_y$	$Gamma(1,1)$
64	1	7	$0.75\Sigma_y$	$Gamma(1,1)$
65	1	6	$0.5\Sigma_y$	$Gamma(1,1)$
66	1	7	$0.5\Sigma_y$	$Gamma(1,1)$
67	1	6	Σ_y	$Gamma(2,2)$
68	**1**	**7**	$\boldsymbol{\Sigma_y}$	$\boldsymbol{Gamma(2,2)}$
69	1	6	$0.75\Sigma_y$	$Gamma(2,2)$
70	1	7	$0.75\Sigma_y$	$Gamma(2,2)$
71	1	6	$0.5\Sigma_y$	$Gamma(2,2)$

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Table 1 – continued from previous page

Model	N_0	c_0	C_0	α
72	1	7	$0.5\Sigma_y$	$Gamma(2, 2)$

2 Effective sample size and autocorrelation for K

Table 2: Effective sample size(ESS) and autocorrelation(AC) for K for all 72 models after 100,000 iterations. Rows in bold indicate the 15 models selected for stage 2 of the grid experiment. Model 26 was chosen as the final model (italicised).

Model	ESS	AC lag 1	AC lag 5	AC lag 10	AC lag 50
1	**23.73**	**0.999**	**0.996**	**0.993**	**0.972**
2	15.50	0.999	0.998	0.996	0.984
3	12.32	0.999	0.998	0.997	0.987
4	6.31	0.999	0.999	0.999	0.994
5	5.32	0.999	0.999	0.998	0.994
6	1.68	0.999	0.999	0.999	0.998
7	15.09	0.999	0.998	0.996	0.985
8	9.02	0.999	0.999	0.998	0.991
9	10.94	0.999	0.998	0.997	0.987
10	3.77	0.999	0.999	0.999	0.996
11	1.79	0.999	0.999	0.999	0.998
12	**19.17**	**0.999**	**0.998**	**0.996**	**0.980**
13	31.41	0.998	0.994	0.989	0.963
14	**46.76**	**0.998**	**0.991**	**0.985**	**0.946**
15	13.40	0.999	0.996	0.994	0.980
16	7.25	0.999	0.998	0.997	0.991
17	non-convergence				
18	**72.57**	**0.997**	**0.989**	**0.980**	**0.926**
19	**36.32**	**0.998**	**0.992**	**0.987**	**0.954**
20	13.19	0.999	0.997	0.995	0.983
21	9.49	0.999	0.998	0.997	0.989
22	12. 03	0.999	0.997	0.996	0.985
23	14.37	0.999	0.997	0.995	0.983
24	**74.60**	**0.997**	**0.989**	**0.980**	**0.925**
25	19.80	0.998	0.994	0.991	0.974
26	***181.87***	***0.990***	***0.962***	***0.936***	***0.804***
27	22.09	0.998	0.994	0.990	0.971
28	173.26	0.992	0.968	0.944	0.819
29	non-convergence				
30	146.49	0.994	0.976	0.956	0.847
31	26.82	0.998	0.993	0.988	0.964
32	**150.78**	**0.992**	**0.968**	**0.944**	**0.827**
33	32.35	0.997	0.991	0.985	0.956
34	**153.33**	**0.992**	**0.969**	**0.946**	**0.824**
35	non-convergence				

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Table 2 – continued from previous page

Model	ESS	AC lag 1	AC lag 5	AC lag 10	AC lag 50
36	148.84	0.995	0.979	0.962	0.861
37	67.71	0.994	0.980	0.968	0.913
38	327.24	0.984	0.937	0.895	0.699
39			non-convergence		
40	**374.32**	**0.984**	**0.934**	**0.887**	**0.665**
41	164.06	0.990	0.961	0.933	0.808
42			non-convergence		
43	56.12	0.995	0.983	0.973	0.926
44	**362.99**	**0.980**	**0.924**	**0.873**	**0.650**
45	109.24	0.992	0.971	0.952	0.867
46	322.05	0.984	0.935	0.891	0.687
47			non-convergence		
48			non-convergence		
49	515.52	0.968	0.890	0.827	0.574
50	**704.41**	**0.964**	**0.873**	**0.796**	**0.483**
51	523.86	0.973	0.903	0.842	0.579
52	619.48	0.970	0.890	0.820	0.517
53	343.68	0.980	0.922	0.871	0.653
54	356.18	0.981	0.927	0.877	0.666
55	518.78	0.969	0.892	0.828	0.565
56	**674.10**	**0.966**	**0.879**	**0.805**	**0.501**
57	432.51	0.975	0.910	0.855	0.622
58	563.02	0.971	0.894	0.827	0.543
59			non-convergence		
60			non-convergence		
61	662.02	0.961	0.871	0.799	0.507
62	**929.18**	**0.953**	**0.840**	**0.747**	**0.390**
63	565.91	0.965	0.876	0.804	0.520
64	798.95	0.961	0.865	0.782	0.455
65	437.72	0.975	0.909	0.853	0.610
66	439.76	0.975	0.908	0.848	0.599
67	773.98	0.959	0.865	0.789	0.471
68	**1015.70**	**0.954**	**0.841**	**0.751**	**0.384**
69	628.91	0.965	0.880	0.810	0.524
70	749.55	0.961	0.862	0.779	0.463
71	438.27	0.975	0.909	0.854	0.618
72	364.87	0.977	0.913	0.858	0.635

3 Effective sample size and autocorrelation for α

Table 3: Effective sample size(ESS) and autocorrelation(AC) for α for all 72 models after 100,000 iterations. Rows in bold indicate the 15 models selected for stage 2 of the grid experiment. Model 26 was chosen as the final model (italicised).

Model	ESS	AC lag 1	AC lag 5	AC lag 10	AC lag 50
1	**1308.17**	**0.154**	**0.153**	**0.152**	**0.147**
2	476.90	0.251	0.247	0.246	0.242
3	281.09	0.303	0.309	0.303	0.307
4	95.36	0.465	0.463	0.460	0.464
5	98.93	0.463	0.465	0.463	0.463
6	53.00	0.563	0.559	0.561	0.561
7	331.68	0.293	0.290	0.294	0.201
8	104.80	0.468	0.466	0.463	0.460
9	361.13	0.273	0.271	0.276	0.268
10	67.66	0.522	0.522	0.522	0.521
11	51.01	0.578	0.577	0.577	0.574
12	**579.02**	**0.224**	**0.230**	**0.225**	**0.225**
13	409.02	0.272	0.269	0.276	0.268
14	**621.57**	**0.239**	**0.239**	**0.240**	**0.228**
15	147.58	0.423	0.419	0.417	0.415
16	32.71	0.714	0.712	0.712	0.705
17	non-convergence				
18	**599.43**	**0.259**	**0.260**	**0.256**	**0.239**
19	**876.39**	**0.188**	**0.192**	**0.190**	**0.187**
20	88.23	0.518	0.512	0.514	0.508
21	68.80	0.544	0.540	0.540	0.540
22	57.82	0.597	0.596	0.593	0.589
23	124.26	0.473	0.474	0.469	0.462
24	**547.43**	**0.246**	**0.266**	**0.263**	**0.248**
25	94.49	0.539	0.539	0.539	0.526
26	***1251.31***	***0.219***	***0.217***	***0.208***	***0.179***
27	99.86	0.536	0.535	0.531	0.521
28	843.01	0.273	0.271	0.264	0.227
29	non-convergence				
30	603.25	0.323	0.319	0.312	0.275
31	148.97	0.459	0.459	0.455	0.422
32	**925.98**	**0.257**	**0.255**	**0.244**	**0.216**
33	188.56	0.427	0.429	0.422	0.412
34	**871.14**	**0.276**	**0.273**	**0.264**	**0.230**
35	non-convergence				

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Table 3 – continued from previous page

Model	ESS	AC lag 1	AC lag 5	AC lag 10	AC lag 50
36	510.77	0.350	0.345	0.336	0.303
37	280.36	0.439	0.430	0.425	0.393
38	1191.52	0.295	0.288	0.270	0.209
39			non-convergence		
40	**1328.22**	**0.304**	**0.293**	**0.276**	**0.203**
41	846.44	0.302	0.293	0.285	0.239
42			non-convergence		
43	219.82	0.468	0.462	0.455	0.429
44	**1558.80**	**0.270**	**0.260**	**0.242**	**0.182**
45	453.65	0.375	0.367	0.359	0.326
46	1250.88	0.298	0.287	0.279	0.208
47			non-convergence		
48			non-convergence		
49	2044.40	0.259	0.242	0.222	0.154
50	**2283.61**	**0.302**	**0.282**	**0.256**	**0.149**
51	1694.30	0.282	0.272	0.245	0.161
52	1963.11	0.310	0.297	0.272	0.168
53	1491.91	0.291	0.278	0.263	0.191
54	1159.67	0.337	0.320	0.305	0.228
55	2104.50	0.263	0.244	0.224	0. 146
56	**2136.88**	**0.315**	**0.295**	**0.268**	**0.162**
57	1663.99	0.290	0.278	0.258	0.183
58	1770.63	0.322	0.303	0.277	0.177
59			non-convergence		
60			non-convergence		
61	2304.06	0.285	0.268	0.244	0.148
62	**3020.09**	**0.300**	**0.272**	**0.240**	**0.118**
63	2143.54	0.289	0.267	0.242	0.157
64	2505.92	0.301	0.281	0.247	0.142
65	1556.50	0.313	0.291	0.276	0.193
66	1495.52	0.320	0.304	0.286	0.199
67	2586.93	0.288	0.269	0.237	0.140
68	**3260.57**	**0.303**	**0.275**	**0.242**	**0.118**
69	2059.52	0.298	0.275	0.260	0.160
70	2570.59	0.294	0.270	0.244	0.142
71	1453.71	0.313	0.301	0.275	0.200
72	1283.49	0.334	0.316	0.294	0.218

4 Traceplots for K and α for the 15 selected models

The traceplots for the models are located on **Github**

5 Convergence statistics for the 15 selected models

Table 4: Gelman Rubin (GR) statistic for K and α, maximum Posterior Expected Adjusted Rand Index (PEAR) and the corresponding optimal number of clusters for each chain within each model. Models in bold indicate that the model converged. The selected model is italicised.

Model	Chain	GR K	GR α	PEAR	No. of clusters
1		1.25	1		
	1			0.737	3
	2			0.746	2
	3			0.847	2
2		2.42	1.05		
	1			0.859	3
	2			0.921	3
	3			0.936	3
3		**1.02**	**1**		
	1			0.710	3
	2			0.741	3
	3			0.728	3
4		2.56	2		
	1			0.442	12
	2			0.506	4
	3			0.834	3
5		1.15	1.01		
	1			0.731	3
	2			0.713	4
	3				model failed
6		1.59	1.43		
	1			0.347	9
	2			0.454	4
	3			0.831	3
7		***1.01***	***1***		
	1			0.427	9
	2			0.426	11
	3			0.444	10

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Table 4 – continued from previous page

Model	Chain	GR K	GR α	PEAR	No. of clusters
8		**1.1**	**1.02**		
	1			0.488	13
	2			0.385	9
	3			0.452	12
9		**1**	**1**		
	1			0.526	14
	2			0.514	13
	3			0.525	16
10		**1**	**1**		
	1			0.480	15
	2			0.478	19
	3			0.476	16
11		**1**	**1**		
	1			0.450	13
	2			0.448	13
	3			0.455	15
12		**1**	**1**		
	1			0.404	14
	2			0.410	19
	3			0.403	18
13		**1**	**1**		
	1			0.406	19
	2			0.413	18
	3			0.411	16
14		**1**	**1**		
	1			0.357	16
	2			0.361	14
	3			0.357	18
15		**1**	**1**		
	1			0.363	19
	2			0.366	16
	3			0.362	15

Alluvial plots that display the consistency of the clusterings across the three chains within each converged model can be found on**Github**.

6 Plots of individual cumulative sums of achieved milestones for each group

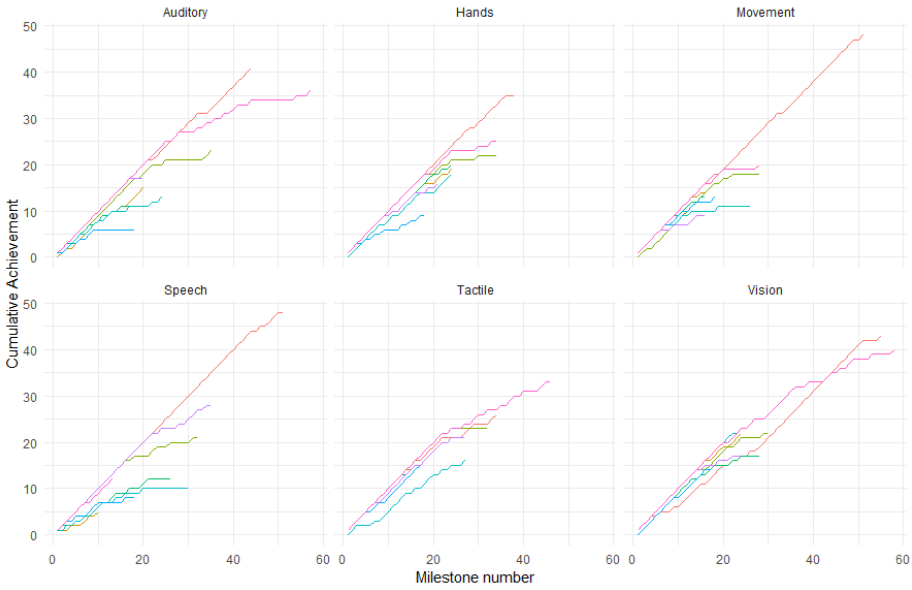

Figure 1: Cumulative sum of achieved milestones for all individuals in Group 2

This is the cumulative sum of achieved milestones for all individuals in Group 2. Different colours indicate different children. This group was characterised by large delays in all functional domains. The plots for all remaining groups can be found on **Github**

7 Sensitivity Analysis - Scenario 1: Overlapping clusters

Table 5: Gelman Rubin (GR) statistic for K and α; the average PEAR (and standard deviation) for the 3 chains within a model, the optimal number of clusters and the average percentage of correct classifications (and standard deviation) for the 3 chains within a model for simulated well separated, adjacent and overlapping clusters

Scenario	Model	GR K	GR α	Average PEAR	Optimal no. of clusters	Average classification accuracy (%)
Well-separated	1	1.00	1.00	1.000(0.000)	3	100.00(0.00)
	2	1.00	1.00	1.000(0.001)	3	100.00(0.00)
	3	1.00	1.00	1.000(0.000)	3	100.00(0.00)
	4	1.00	1.00	0.999(0.001)	3	100.00(0.00)
	5	1.00	1.00	1.000(0.000)	3	100.00(0.00)
	6	1.00	1.00	0.999(0.000)	3	100.00(0.00)
	7	1.00	1.00	0.999(0.000)	3	100.00(0.00)
	8†	1.00	1.00	0.999(0.000)	3	100.00(0.00)
	9	1.00	1.00	0.999(0.000)	3	100.00(0.00)
	10	1.00	1.00	0.999(0.000)	3	100.00(0.00)
	11	1.00	1.00	0.999(0.000)	3	100.00(0.00)
	12	1.00	1.00	0.999(0.000)	3	100.00(0.00)
	13	1.00	1.00	0.999(0.000)	3	100.00(0.00)
	14	1.00	1.00	0.999(0.000)	3	100.00(0.00)
	15	1.00	1.00	0.999(0.000)	3	100.00(0.00)
Adjacent	1	1.00	1.00	0.932(0.000)	3	100.00(0.00)
	2	1.00	1.00	0.924(0.001)	3	100.00(0.00)
	3	1.00	1.00	0.925(0.001)	3	100.00(0.00)
	4	1.00	1.00	0.877(0.002)	3	100.00(0.00)
	5	1.00	1.00	0.929(0.000)	3	100.00(0.00)
	6	1.00	1.00	0.877(0.002)	3	100.00(0.00)
	7	1.00	1.00	0.917(0.000)	3	100.00(0.00)
	8	1.00	1.00	0.917(0.001)	3	100.00(0.00)
	9	1.00	1.00	0.901(0.001)	3	100.00(0.00)
	10	1.00	1.00	0.889(0.000)	3	100.00(0.00)
	11	1.00	1.00	0.908(0.000)	3	100.00(0.00)
	12	1.00	1.00	0.901(0.000)	3	100.00(0.00)
	13	1.00	1.00	0.901(0.001)	3	100.00(0.00)
	14	1.00	1.00	0.898(0.000)	3	100.00(0.00)
	15	1.00	1.00	0.898(0.000)	3	100.00(0.00)
Overlapping	1	1.01	1.00	0.683(0.004)	2	65.33(0.00)

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Table 5 – continued from previous page

Scenario	Model	GR K	GR α	Average PEAR	Optimal no. of clusters	Average classification accuracy (%)
	2	1.00	1.00	0.580(0.005)	3	74.00(0.00)
	3	1.00	1.00	0.529(0.002)	3	81.56(2.70)
	4	1.00	1.00	0.524(0.003)	3	76.67(0.00)
	5	1.00	1.00	0.622(0.010)	2	66.00(0.00)
	6	1.00	1.00	0.521(0.002)	3‡	76.45(0.39)
	7	1.00	1.00	0.518(0.002)	3	83.11(3.42)
	8	1.00	1.00	0.518(0.000)	3	87.56(4.82)
	9	1.00	1.00	0.511(0.003)	3	80.67(1.76)
	10	1.00	1.00	0.494(0.002)	3	88.22(4.53)
	11	1.00	1.00	0.516(0.001)	3	90.67(0.67)
	12	1.00	1.00	0.514(0.003)	3	92.00(1.33)
	13	1.00	1.00	0.515(0.003)	3	90.45(0.39)
	14	1.00	1.00	0.521(0.001)	3	77.78(2.04)
	15	1.00	1.00	0.520(0.004)	3	76.00(1.15)

† Chain 3 failed to run, so results are based on the average for Chain 1 and 2
‡ Chain 3 returned 4 as the optimal number of clusters

Traceplots for K and α are available on **Github**

8 Alternative methods for assessing convergence and post-processing chains for Scenario 2

Due to the computational challenges in the simulation study for scenario 2 outlined in Section 4, the large sample size was assessed by running each chain for segments of 10,000 iterations. Each segment (except the first) was initialised using the values from the last iteration of the preceding segment. The chains were assessed for convergence each time a new segment was added. As the number of iterations used for this analysis was substantially smaller than that used for the application or the first simulation study, 6 chains were run for each model (initialised at K = 2, 4, 5, 6, 10 and 15 clusters) in order to be more certain that the models had converged. After the chains had run for 40,000 iterations, the majority of the models had reached convergence for K and all the models had converged for α based on the Gelman-Rubin statistic. The first 20,000 iterations were discarded for burn-in before processing the chains.

It was computationally intensive to post-process the chains using the PEAR method, as it took on average 50hr:36min and 23.9GB of RAM per model to process 10,000 iterations. This would increase exponentially with an increase in iterations, therefore, an alternative method was performed which still involves processing the posterior similarity matrix. The posterior similarity matrix was obtained for every chain for all models, which was then converted into a dissimilarity matrix (i.e., dissimilarity matrix = 1 - posterior similarity matrix) and clustered using agglomerative hierarchical clustering. The resulting dendrogram was then cut at a range of different cluster sizes (clusters = 2 to 20) and the average silhouette width for all cluster sizes was calculated. The number of clusters that had the largest silhouette width was deemed optimal. All of the chains for all of the models returned three clusters as the optimal number, based on the silhouette width. The average silhouette width for three clusters and the average classification accuracy for each model is displayed in *Table 6* below. These results were calculated by averaging across the six chains within a model. In order to make accurate comparisons with the small and medium sample sizes, the models processed for these scenarios were also run for 40,000 iterations and were processed using the hierarchical clustering method. The results for small and medium sample sizes are displayed also in *Table 6* below.

9 Sensitivity Analysis - Scenario 2: Sample size

Table 6: Gelman Rubin (GR) statistic for K and α; the average silhouette width (and standard deviation) for the 6 chains within a model and the average percentage of correct classifications (and standard deviation) for the 6 chains within a model for simulated small ($N = 150$), medium ($N = 1\ 500$) and large sample sizes ($N = 15\ 000$).

Scenario	Model	GR K	GR α	Average silhouette width	Average classification accuracy (%)
Small	1	1.01	1.00	0.861(.004)	94.67(.000)
	2	1.03	1.01	0.797(.022)	95.00(.361)
	3	1.00	1.00	0.786(.007)	95.33(.000)
	4	1.01	1.00	0.626(.016)	95.00(.361)
	5	1.02	1.00	0.840(.008)	95.00(.361)
	6	1.02	1.01	0.623(.026)	94.56(.501)
	7	1.00	1.00	0.755(.011)	95.22(.269)
	8	1.00	1.00	0.744(.010)	95.33(.000)
	9	1.00	1.00	0.651(.011)	94.56(.501)
	10	1.00	1.00	0.596(.013)	94.89(.341)
	11	1.01	1.00	0.686(.014)	94.78(.269)
	12	1.00	1.00	0.656(.015)	94.78(.269)
	13	1.00	1.00	0.649(.007)	94.67(.000)
	14	1.00	1.00	0.674(.007)	94.67(.000)
	15	1.00	1.00	0.666(.007)	94.67(.000)
Medium	1	1.02	1.00	0.904(.002)	96.42(.140)
	2	1.09	1.01	0.876(.020)	96.27(.318)
	3	1.11	1.03	0.838(.036)	96.18(.258)
	4	1.07	1.03	0.762(.039)	96.38(.156)
	5	1.04	1.00	0.893(.008)	96.29(.130)
	6	1.04	1.02	0.754(.026)	96.43(.173)
	7	1.10	1.03	0.837(.030)	96.30(.130)
	8	1.01	1.00	0.829(.012)	96.29(.245)
	9	1.03	1.01	0.778(.021)	96.47(.250)
	10	1.06	1.04	0.692(.046)	96.29(.417)
	11	1.04	1.01	0.805(.024)	96.33(.180)
	12	1.05	1.02	0.806(.032)	96.36(.110)
	13	1.03	1.02	0.795(.024)	96.47(.211)
	14	1.01	1.00	0.839(.012)	96.38(.200)
	15	1.03	1.01	0.834(.024)	96.18(.170)
Large	1	1.25	1.00	0.958(.013)	98.57(.029)
	2	1.05	1.00	0.959(.006)	98.58(.020)
	3	1.10	1.01	0.964(.001)	98.59(.039)
	4	1.12	1.02	0.950(.015)	98.57(.034)

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Table 6 – continued from previous page

Scenario	Model	GR K	GR α	Average silhouette width	Average classification accuracy (%)
	5	1.05	1.00	0.959(.008)	98.57(.015)
	6	1.14	1.04	0.946(.014)	98.57(.029)
	7	1.15	1.03	0.933(.025)	98.59(.016)
	8	1.15	1.04	0.912(.048)	98.58(.036)
	9	1.07	1.02	0.935(.030)	98.57(.033)
	10	1.11	1.06	0.900(.044)	98.58(.020)
	11	1.21	1.07	0.927(.033)	98.58(.016)
	12	1.04	1.01	0.934(.020)	98.54(.060)
	13	1.06	1.02	0.921(.021)	98.56(.028)
	14	1.07	1.02	0.929(.025)	98.58(.030)
	15	1.04	1.02	0.922(.018)	98.55(.025)

Traceplots for K and α are available on **Github**