Assignment-3

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Problem 3.1. *(UAG 5.1) A rgular function on $\mathbb{P}^1$ is constant. Deduce that there are no non-constant morphisms $\mathbb{P}^1 \to \mathbb{A}^m$ for $m \geq 1$.*

Solution. Suppose $f \in k(\mathbb{P}^1)$ be a rational function, which is regular everywhere. If we restrict it to the affine piece $\mathbb{A}_{(0)}$, we get $f(x, 1) = p(x) \in k[x]$ (as for the case of affine variety $\operatorname{dom} f = V$ iff $f \in k[V]$). Similarly, we can restrict f to another affine piece $\mathbb{A}_\infty$. We get, $f(1, y) = f(1/y, 1) = p(1/y) \in k[y]$. It is possible iff p is constant.

Any morphisms $\mathbb{P}^1 \to \mathbb{A}^m$ can be given by $(f_1, \cdots, f_m)$ where f_i are regular on $\mathbb{P}^1$. Thus the function f is constant by the previous part. ■

Problem 3.2. *(UAG 5.7) Let $\varphi : \mathbb{P}^1 \to \mathbb{P}^1$ be an isomorphism; identify φ as subvariety of $\mathbb{P}^1 \times \mathbb{P}^1 \subset \mathbb{P}^3$. Now do the same if $\varphi : \mathbb{P}^1 \to \mathbb{P}^1$ is given by map $(X, Y) \mapsto (X^2, Y^2)$.*

Solution