# Assignment-5

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## Problem 5.14

We will begin with the assumption, the underlying field $k$ is infinte and algebraically closed (according to contexts). The property of lines passing through points is a projective property. So we can take a suitable projective transformation so that $P_1 = [0 : 0 : 1]$. Thus, any line passing through this looks like $ax + by = 0$ where $a, b \in k$. The set of lines passing through $P_1$ is

$$A = \{x + my : m \in k\} \cup \{y = 0\}$$

Since, the field is infinite, there is infinitely many elements in $A$. Given two points in $\mathbb{P}^2$ there is a unique line passing through $P_1$ and that point. Thus the set of lines

$$L = \{\ell \text{ pass through } P_1 \text{ and } P_i : 2 \leq i \leq n\} \subset A$$

is finite. So there are only finitely many line in the above set. But in $A$ there are infinitely elements. So, there are infinitely many elements in $A \setminus L$.

Since $P_1$ is a simple point of $F$, there is a tangent $T$ at $P$ so that the tangent $T$ don't contained in $V(F)$ (or $F$). From the problem 5.12 we can say,

$$\sum I(P; F \cap T) = n$$

where $n = \deg F$. Thus, If we take $P_2, \cdots, P_m$ be the other intersection points (here $m \leq n$) of $T$ and $F$, by the previous calculation we can say there exists infinitely many lines through $P$ don't intersect $F$ at $P_i$ $(i > 1)$. These lines are transversal to $F$. ∎