B.Math II-Ordinary Differential Equations

Assignment 3

Question 1: Consider the following ODE

$$y' = \frac{x}{2} + y, \qquad y(0) = 1.$$

1. Apply the following numerical techniques to approximate the solution by using increment of size $h = 0.1$ on the domain $[0, 0.5]$.

   (a) Euler method,

   (b) Euler modified method,

   (c) Taylor's second order method,

   (d) Trapezoidal method,

   (e) Runge-Kutta 4th order method.

2. Evaluate the following relative error for each of the above methods

$$E_k = \frac{|y(x_k) - y_k|}{|y(x_k)|}.$$

3. Find out which method better approximates the solution of the given ODE.

Question 2: Consider the following ODE

$$y' = f(x, y), \qquad y(x_0) = y_0,$$

defined on a real bounded domain I. Let y_0 be bounded, and f is analytic function on I. Then, check the convergence of the following numerical approximation for this ODE.

1. $y_{k+1} = y_k + \frac{h}{2}(f(x_k, y_k) + f(x_{k+1}, y_k + hf(x_k, y_k)))$.

2. $y_{k+1} = y_k + \frac{h}{2}(f(x_k, y_k) + f(x_k + \frac{h}{2}, y_k + \frac{h}{2}f(x_k, y_k)))$.

And, if they are convergent, find out the rate of convergence for them.