Statistics-III

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Assignment-5

Problem 1

Let there is some C for which, CY is unbiased estimator of β.

$$\begin{aligned}
&\therefore E(CY) = \beta \\
&\Rightarrow CE(Y) = \beta \\
&\Rightarrow CX\beta = \beta \\
&\Rightarrow (CX - I_P)\,\beta = 0
\end{aligned}$$

The above equation is true for arbitrary $\beta \in \mathbb{R}^p$. So, $CX = I_P$ Let, $X = [X_1, X_2, \ldots, X_p]$. Since $\sum_{i=1}^{P} \alpha_i X_i = 0 \Rightarrow \sum_{i=1}^{P} \alpha_i C X_i = \sum_{i=1}^{p} \alpha_i e_i = 0 \Rightarrow \alpha_i = 0.\quad \forall i = 1, \ldots .p$. But, we know, all of i cant be zero. And hence a contradiction. ■

Problem 2

In Gauss Markov model Let, X be a hep matrix and $a'\beta$ is estimable for all $a \in \mathbb{R}^p$.

$$\begin{aligned}
a'\beta \text{ estimable } \forall a' \in \mathbb{R}^p &\Leftrightarrow a \in M_c\,(X') \quad \forall a \in \mathbb{R}^p \\
&\Leftrightarrow \text{ Column Rank}\,(X') = p \\
&\Leftrightarrow \text{Row Rank}\,(X) = p \\
&\Leftrightarrow \text{ Column Rank}(X) = p \\
&\Leftrightarrow \text{ all the Column of } X \text{ are linearly independent}
\end{aligned}$$

■

Problem 3

$a_1'\beta, a_2'\beta, \ldots, a_n'\beta$ are estimable. So, we can write, $a_1, ., a_n \in M_c\,(X')$ and hence, $a_1 = X'b_1, \ldots, a_n = X'b_n$,for some $b_1, \cdots, b_n$ belongs to $\mathbb{R}^p$. Let, $a' = \sum t_i a_i'$ a linear Combination of $a_1', \ldots, a_n'\cdot$ clearly, $a = X'\left(\sum t_i b_i\right) \Rightarrow a' \in M_C\,(X')$ and hence, $a'\beta$ is estimable., So, any linear Combination of $a_1'\beta, ., a_n'\beta$ is estimable.

■

Problem 4

We know $a'\beta$ is estimable iff $a \in M_C(X') = M_C\left(X'X\right)$ Let, $a = X'Xb$. So,

$$\begin{aligned} a'\left(X'X\right)^{-}X'X &= b'\left(X'X\right)\left(X'X\right)^{-}\left(X'X\right) \\ &= b'\left(X'X\right) \\ &= a \end{aligned}$$

Conversely if, $a'\left(X'X\right)^{-}X'X = a$ then. We can see that,

$$\begin{aligned} a &= X'X\left(a'(X'X)^{-1}\right)' \\ \Rightarrow a &\in M_C(X'X) \end{aligned}$$

Hence $a'\beta$ is estimable. ■