Worksheet

Particle Physics Group Studies 2024

2024-01-11

A few questions demonstrating possible behaviour you might come across during your group studies. The problems are not accessed. The questions are divided by your four sub-groups.

1 Sensors

1.1 *Size of Depletion Region*

This exercise will show you how to derive the size of the depletion region inside a sensor. Consider the sensor PN junction as one dimensional, with the two types joined at the origin ($x = 0$). The doping counts in the depleted n- and p-type regions are N_n and N_p.

Assume that when a voltage V_0 is applied, the depletion region spans $-a < x < 0$ in the p-type silicon and $0 < x < b$ in the p-type silicon. Use Poisson's equation to derive the electric field as a function of x. Remember that the electric field must be continuous at $x = 0$. Sketch the electric field.

For convenience, the Poisson equation is

$$\frac{d^2V}{dx^2} = -\frac{\rho}{\epsilon}, \tag{1}$$

where ρ is the *charge* density and ϵ is the dielectric constant. You can assume the same dielectric constant for both p-type and n-type.

Integrate the electric field to find the potential as a function of x. The magnitude of the potential equals to the bias potential V_0. Sketch the potential.

Take the limit $N_n \gg N_p$ and show that the depletion region has size d,

$$d \approx \sqrt{\frac{2\epsilon V_0}{eN_p}}. \tag{2}$$

Treating the sensor as a parallel plate capacitor, with plate (cathode/anode) area A, show that the depletion region can be measured via a CV curve, with

$$\frac{1}{C^2} \propto V. \tag{3}$$

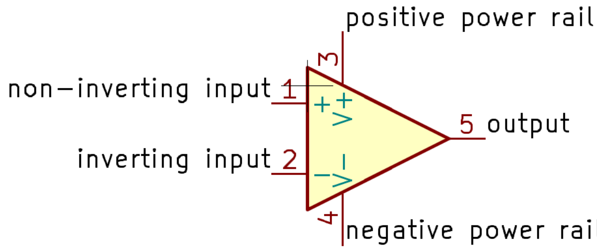

Figure 1: A symbol for an operational feedback. Common names for the different pins are labeled.

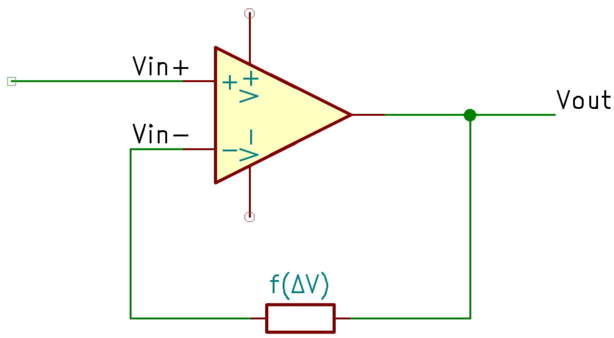

Figure 2: An Operational Amplifier connected in a negative feedback configuration with a generic circuit in the feedback connection. The square corresponds to a black box, not a resistor.

2 Front-End

2.1 *Negative Feedback*

This exercise will teach you about rules of an Operational Amplifiers present in a circuit. The situation of a generalized negative feedback configuration will be analyzed.

An OpAmp consists of three pins; positive and negative inputs (+, -) and the output. You also need to provide power and ground connections. An example symbol is shown in figure 1. The rules for analyzing an OpAmp in a circuit are as follows:

- No current flow in or out of the positive and negative inputs.
- The output voltage is proportional the difference between the input voltages; $V_{\text{out}} = A\left(V_{\text{in},+} - V_{\text{in},-}\right)$.

The value for the gain (A) can be found to be in the specific OpAmp datasheet. However for most use-cases, it is considered to be "large". This is useful when analyzing negative feedback configurations.

A generalized negative feedback configuration is shown in figure 2. The $f\left(\Delta V\right))$ represents an unknown circuit whose behaviour depends on the output and negative input voltages.

Use the above rules to show that in the limit of very large A, the following is true.

$$V_{\text{in},-} = V_{\text{in},+}. \tag{4}$$

Now consider a single current to voltage converter shown in figure 3. Show that the output voltage is

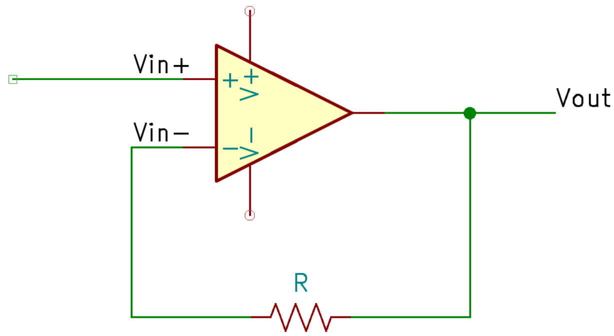

Figure 3: An current to voltage converter built using an operational amplifier.

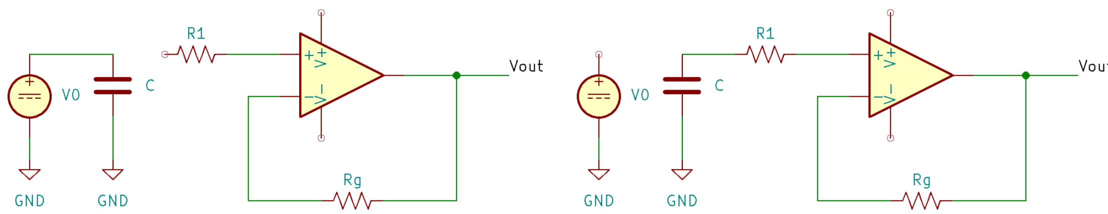

Figure 4: A charge injection circuit that can be used to calibrate a front-end. Left shows the first step, where a capacitor is charged to a known voltage. Right shows the second step, where the charge on the capacitor is injected into the circuit.

$$V_{\text{out}} = -iR. \tag{5}$$

2.2 *Calibration Circuit*

In this exercise, you will derive the waveform of a charge injection circuit as it goes through the current to voltage converter. The charge injection circuit is shown in figure 4.

Consider the case where the capacitor is charged to voltage V_q and then connected to the front-end at time $t_0 = 0$. Show that the corresponding pulse is given by

$$V_{\text{out}} = -\frac{R_g}{R_1} V_q \exp\left(-\frac{t}{R_1 C}\right). \tag{6}$$

3 DAQ and Software

3.1 *Calibration Procedure*

In this exercise, you will show how you can measure the noise of a binary front-end using a charge injection circuit.

Assume that you have a front-end that converts injected charge q into a signal pulse with a voltage peak of V_q. The signal is then fed into a discriminator with a tune-able threshold of V_{thr} and outputs a 1 if $V_q > V_{\text{thr}}$.

1. Consider a fixed injected charge q that creates a pulse with height $V_{\mathrm{sig}} = G \times q$. The pulse height is further smeared by noise (gaussian) with standard deviation σ as it goes through the front-end. Draw the distribution of V_{sig}.

2. The comparator with a threshold V_{thr} accepts only charge injections that result in a pulse height with $V_{\mathrm{sig}} \geq V_{\mathrm{thr}}$. Draw the fraction of signal injections accepted as a function of V_{thr}. What shape does it follow?

3. Describe how you can measure the gain, G, by varying the injected charge q.

4 System Simulation

In this exercise, you will learn about the effect that the material has on the trajectory of a charged particle. As the particle transverses a detector, it can interact with the electric field of its nuclei and scatter. This has a negative impact on the resolution of the reconstructed trajectory and extracted quantities (ie: momentum, origin vertex). This is why simulations including material (ie: using GEANT4) are important to understand the performance of a detector.

The amount of scaterring through a material can be approximated using the Highland equation,

$$\theta_0 = \frac{14.1\ \mathrm{MeV}}{pv} z \sqrt{\frac{L}{L_R}} \left[1 + \frac{1}{9} \log_1 0 \left(\frac{L}{L_R}\right)\right] \ \mathrm{rad}. \tag{7}$$

The θ_0 is the standard deviation of the angular change of a particle's trajectory as it exits a material of length L. The p and v are the particle momenta and velocity. The L_R and z represent the material via is the radiation length and atomic number.

Consider a detector module made up of 1 mm silicon (sensor) 2x50 μm copper (2 copper layers in PCB) and 1 mm FR4 (PCB material). The a common energy of cosmic muons is 1-10 GeV. Calculate the scattering angles that you expect.

You can find radiation lengths of different materials on the Particle Data Group website. `https://pdg.lbl.gov/2009/AtomicNuclearProperties/`. FR4 radiation length is provided at `http://personalpages.to.infn.it/~tosello/EngMeet/ITSmat/SDD/SDD_G10FR4.html`.