Human Movement Mapping

Topology of Asana

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Informatics of Human Movement

> *Communication... not only involves written and oral speech, but also music, the pictorial arts, the theatre, the ballet, and in fact all human behavior.*

Claude Shannon
The Mathematical Theory of Communication

Difference and Repetition

The 1968 *Difference and Repetition* was the principal doctoral thesis of Gilles Deluze. He presents the ideas of difference and repetition as conceptual substitutes for Hegel's dialectic of identity and negation. In this scheme, difference and repetition are logically prior to identity and negation.

Systems may be characterized looking at the relationships between things rather than identifying the things in themselves.

Difference and Repetition

Human motion contains information like written or spoken language. Contemporary camera and computer technologies capture this information for gaming, animation, medical diagnostics and robotic control. By comparing a body to itself over time we are able to extract the informational content of movement without a priori knowledge.

Strategy

Beginning with a kinematic chain model of the human body we generate a metric for comparing different states of skeletal articulation. Applying this measure over motion data time series generates similarity spectra from which we identify and characterize body motions.

Applications

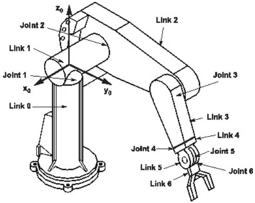
- **Human Computer Interfaces** The use of gestural control as a means of interacting and controlling a computer device. Settings include home entertainment, industrial, military, or commercial settings.
- **Computer Animation** The use of human motion databases for use in computer animation for video games and movies.
- **Anthropological Databasing** The development of database systems for recording and cataloging lexicons of human movement. This could include dance techniques, athletics, martial arts.
- **Human Motion Analysis** The techniques developed can be used for the analysis of pathological motion in human gait or workplace design.

Previous Work

- **Pullen** *Motion Capture Assisted Animation: Texturing and Synthesis.*PhD thesis, Stanford, 2002.
- **Brand and Hertzmann** *Style machines.* SIGGRAPH, 2000.
- **Arikan and Forsyth** *Interactive motion generation from examples.* SIGGRAPH, 2002.
- **Reitsma and Pollard** *Evaluating Motion Graphs for Character Animation.* ACM Transactions on Graphics, 2007.

Kinematic Chains

A kinematic chain is a system of rigid body links connected by fixed joints. It is generally used in robotics research to articulated robotic controllers. Each link in the chain is fixed in its own coordinate system while adjacent links, or kinematic pairs, are connected to one another by joints which allow rotational freedom.



Robotic Arm

Mathematics of Kinematic Chains

The position and orientation of each link measured in the global coordinate system, A, can be represented by the product of euclidean transformations. Using homogenous coordinates, we represent the rotational and translational portions of each transformation, g, using a quaternion, Q, and vector, $\overrightarrow{p}$, together in a matrix.

$$\begin{pmatrix} \overrightarrow{r}_A \\ 1 \end{pmatrix} = g_{AB} \begin{pmatrix} \overrightarrow{r}_B \\ 1 \end{pmatrix} \qquad \begin{pmatrix} \overrightarrow{r}_A \\ 1 \end{pmatrix} = g_{AC} \begin{pmatrix} \overrightarrow{r}_C \\ 1 \end{pmatrix}$$

$$g_{AB} = \begin{pmatrix} Q_{AB} & \overrightarrow{p}_{AB} \\ 0 & 1 \end{pmatrix} \qquad g_{AC} = g_{AB} g_{BC}$$

Quaternions I

Quaternions are a 4-tuple consisting of a real component q_0 and three imaginary components $q_1 i + q_2 j + q_3 k$. The algebra of the imaginary components follows.

$$ii = jj = kk = ijk = -1$$

The imaginary components may be written as a vector $\vec{q}$ so that the overall quaternion Q can be written $Q = \begin{pmatrix} q_0 \\ \vec{q} \end{pmatrix}$ Quaternion multiplication between a quaternion $Q = \begin{pmatrix} q_0 \\ \vec{q} \end{pmatrix}$ and $G = \begin{pmatrix} g_0 \\ \vec{g} \end{pmatrix}$ is given.

$$QG = Q \cdot G = \begin{pmatrix} q_0 \\ \vec{q} \end{pmatrix} \cdot \begin{pmatrix} g_0 \\ \vec{g} \end{pmatrix} = \begin{pmatrix} q_0 g_0 - \vec{q} \cdot \vec{g} \\ q_0 \vec{g} + g_0 \vec{q} + \vec{q} \times \vec{g} \end{pmatrix}$$

Quaternions II

Quaternions maybe be used to represent rotations if we use $q_0 = \cos(\theta/2)$ and $\overrightarrow{q} = \sin(\theta/2)\hat{\omega}$. Here $\hat{\omega}$ is the unit vector around which the rotation occurs. Clearly the quaternion representing the inverse rotation, Q^*, would be obtained by flipping the sign of θ. $Q^* = \begin{pmatrix} q_0 \\ -\overrightarrow{q} \end{pmatrix}$.

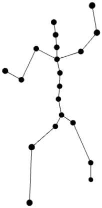
$$QQ^* = Q \cdot Q^* = \begin{pmatrix} q_0 \\ \overrightarrow{q} \end{pmatrix} \cdot \begin{pmatrix} q_0 \\ -\overrightarrow{q} \end{pmatrix} = \begin{pmatrix} q_0^2 + \overrightarrow{q} \cdot \overrightarrow{q} \\ \overrightarrow{0}_3 \end{pmatrix} = \begin{pmatrix} 1 \\ \overrightarrow{0}_3 \end{pmatrix}$$

Rotation of a vector is achieved though quaternion action on a vector $\overrightarrow{r}$ as follows.

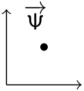
$$Q\overrightarrow{r} = Q \cdot \begin{pmatrix} 0 \\ \overrightarrow{r} \end{pmatrix} \cdot Q^* = \overrightarrow{r} + 2q_0\overrightarrow{q} \times \overrightarrow{r} + 2\overrightarrow{q} \times \overrightarrow{q} \times \overrightarrow{r}$$

Configuration Space

Given a kinematic chain with set link lengths we can describe the state of that chain with a vector. We will call this a state or configuration vector $\overrightarrow{\Psi}$. Each element of the vector is a quaternion Q_i determining the rotational state of joint i. The set of rotational transformations constitutes a manifold. This manifold is called the configuration space, or $\mathcal{C}$, of the kinematic chain. The state vector $\overrightarrow{\Psi}$ spans $\mathcal{C}$. Due to limits on range of motion of each joint and conditions of self collision not all $\overrightarrow{\Psi}$ are physically posible. $\mathcal{C}_{free}$ is the accessible part of the manifold. It is the set of all physically realizable configurations of $\overrightarrow{\Psi}$.

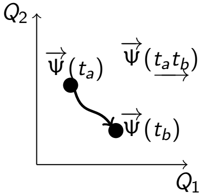
$\overrightarrow{\Psi} =$ 

$$\overrightarrow{\Psi} = \begin{pmatrix} Q_1 \\ Q_2 \\ \vdots \\ Q_n \end{pmatrix}$$



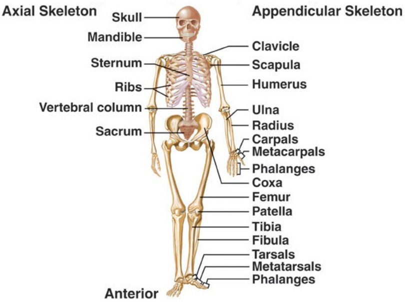
Motion Paths

We consider the configuration state as a function of time $\overrightarrow{\Psi}(t_i)$. The configuration states over a period of time t_a to t_b carve a continuous path through $\mathcal{C}$ from $\overrightarrow{\Psi}(t_a)$ to $\overrightarrow{\Psi}(t_b)$. We notate this motion path as $\overrightarrow{\Psi}(\underrightarrow{t_a t_b})$.



Appendicular/Axial Decomposition

We begin by splitting $\vec{\Psi}$ into three separate components. These are the axial component $\vec{\Theta}$, consisting of the Q values for the axial skeleton, and the appendicular components $\vec{\Gamma}_{left}$ and $\vec{\Gamma}_{right}$, consisting of the Q values for the appendicular skeleton.

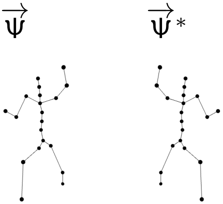


$$\vec{\Psi} = \begin{pmatrix} \vec{\Theta} \\ \vec{\Gamma}_{left} \\ \vec{\Gamma}_{right} \end{pmatrix}$$

$$\vec{\Theta} = \begin{pmatrix} Q_{root} \\ Q_{lowerback} \\ Q_{upperback} \\ Q_{thorax} \\ Q_{lowerneck} \\ Q_{upperneck} \\ Q_{head} \end{pmatrix}$$

$$\vec{\Gamma} = \begin{pmatrix} Q_{femur} \\ Q_{tibia} \\ Q_{humerus} \\ Q_{radius} \end{pmatrix}$$

Symmetry Basis

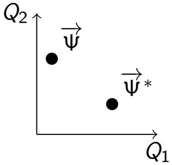


The reflection operator $\mathbf{W}$ acts on the configuration state vector. $\mathbf{W}\overrightarrow{\Psi} = \overrightarrow{\Psi}^*$

This not only reflects the individual quaternions but switches left and right of the appendicular vectors.

$$\overrightarrow{\Psi} = \begin{pmatrix} \overrightarrow{\Theta} \\ \overrightarrow{\Gamma}_{left} \\ \overrightarrow{\Gamma}_{right} \end{pmatrix}$$

$$\overrightarrow{\Psi}^* = \begin{pmatrix} \overrightarrow{\Theta}^* \\ \overrightarrow{\Gamma}^*_{right} \\ \overrightarrow{\Gamma}^*_{left} \end{pmatrix}$$



We assume a symmetrically capable body so that if $\overrightarrow{\Psi}$ is an element of $\mathcal{C}_{free}$ then $\overrightarrow{\Psi}^*$ is also assumed to be an element of $\mathcal{C}_{free}$.

Reflection and Symmetry Basis

We define an alternate basis of the conformation $\overrightarrow{\Psi}$ as follows.

$$\overrightarrow{S} = \frac{\overrightarrow{\Psi} + \overrightarrow{\Psi}^*}{2} \qquad \overrightarrow{A} = \frac{\overrightarrow{\Psi} - \overrightarrow{\Psi}^*}{2}$$

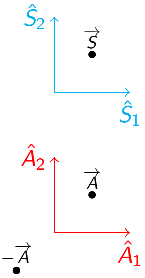
$$\mathbf{W}\overrightarrow{S} = \overrightarrow{S}$$
$$\mathbf{W}\overrightarrow{A} = -\overrightarrow{A}$$

$\overrightarrow{S}$ is invariant under reflection while $\overrightarrow{A}$ reverses sign. $\overrightarrow{S}$ and $\overrightarrow{A}$ are the symmetric and antisymmetric component, respectively.

We can reconstruct $\overrightarrow{\Psi}$ and $\overrightarrow{\Psi}^*$ from the components $\overrightarrow{S}$ and $\overrightarrow{A}$ as follows.

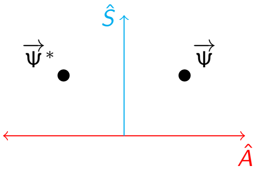
$$\overrightarrow{\Psi} = \overrightarrow{S} + \overrightarrow{A}$$
$$\overrightarrow{\Psi}^* = \overrightarrow{S} - \overrightarrow{A}$$

Configuration Space in Symmetry Basis



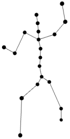
- **Symmetry Basis** Here the configuration space $\mathcal{C}$ is shown as two subspaces created by the symmetric and antisymmetric components. In this view each state $\overrightarrow{\Psi}$ is represented by one point in the $\mathcal{S}$ subspace and one point in the $\mathcal{A}$ subspace. The points corresponding to $\overrightarrow{\Psi}$ and $\overrightarrow{\Psi}^*$ are identical in $\mathcal{S}$ and reflected about the origin in $\mathcal{A}$.

- **Combined Symmetry Basis** This combines both subspaces together in the same graph, while keeping them orthogonal. In the combined symmetry basis $\overrightarrow{\Psi}$ and $\overrightarrow{\Psi}^*$ are reflected trough the symmetry axis, as shown on the right.

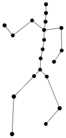


Distance Metrics I

$\vec{\Psi}_a$



$\vec{\Psi}_b$



In order to compare two configuration states, $\vec{\Psi}_a$ and $\vec{\Psi}_b$, we need to define a distance function or metric. We follow the usual definition for metrics as a map $D : \mathcal{C} \times \mathcal{C} \rightarrow \mathbb{R}$ satisfying the following four axioms.

1. $D(a, b) \geq 0$ (non-negativity)
2. $D(a, b) = 0 \Leftrightarrow a = b$ (indiscernability)
3. $D(a, b) = D(b, a)$ for $a, b \in \mathcal{C}$ (symmetry)
4. $D(a, c) \leq D(a, b) + D(b, c)$ for $a, b, c \in \mathcal{C}$ (subadditivity)

Distance Metrics II

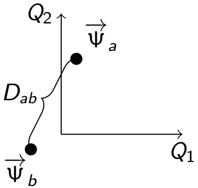
The first metric uses the inner product of quaternions. We define the inner product operator for quaternions as follows.

$$Q_a \circ Q_b = q_{a0}q_{0b} + q_{1a}q_{1b} + q_{2a}q_{2b} + q_{a3}q_{b3}$$

We use the inner product of quaternions to derive a distance metric based solely on the rotational states of a particular rigid body element.

$$D(Q_a, Q_b) = 1 - Q_a \circ Q_b$$

In considering metrics between whole body configurations we can simply take the weighted sum of metrics for individual links.

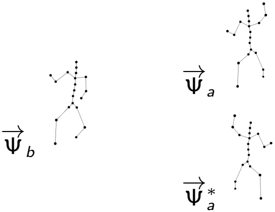


$$D(\overrightarrow{\Psi}_a, \overrightarrow{\Psi}_b) = \Sigma_i w_i D(Q_{ai}, Q_{bi})$$

If we weight all of the quaternions equally we get $D_{ab} = 1 - \overrightarrow{\Psi}_a \circ \overrightarrow{\Psi}_b$.

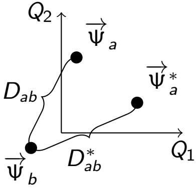
Double Metric

This metric has two components. Given the configuration states, $\overrightarrow{\Psi}_a$ and $\overrightarrow{\Psi}_b$, the double metric is the distance between the two states, $D(\overrightarrow{\Psi}_a, \overrightarrow{\Psi}_b)$, and that distance when one state is reflected, $D(\overrightarrow{\Psi}^*_a, \overrightarrow{\Psi}_b)$.



$$D_{ab} = 1 - \overrightarrow{\Psi}_a \circ \overrightarrow{\Psi}_b$$

$$D^*_{ab} = 1 - \overrightarrow{\Psi}^*_a \circ \overrightarrow{\Psi}_b$$



Symmetry Considerations

Since $\overrightarrow{A}$ and $\overrightarrow{S}$ are orthogonal, representing $\overrightarrow{\Psi}$ in the symmetry basis yields the following identities.

$$\overrightarrow{\Psi}_a \circ \overrightarrow{\Psi}_b = \overrightarrow{S}_a \circ \overrightarrow{S}_b + \overrightarrow{A}_a \circ \overrightarrow{A}_b$$

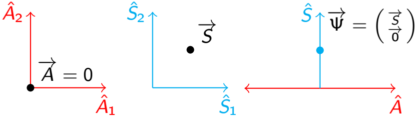
$$\overrightarrow{\Psi}^*_a \circ \overrightarrow{\Psi}_b = \overrightarrow{S}_a \circ \overrightarrow{S}_b - \overrightarrow{A}_a \circ \overrightarrow{A}_b$$

$\overrightarrow{\Psi}_a$

The states are normalized, $\overrightarrow{\Psi}_a \circ \overrightarrow{\Psi}_a = 1$. Therefore if a state has unity inner product with it's mirror image then the state is identical to it's mirror image and that state is symmetric.

$$\overrightarrow{\Psi}^*_a \circ \overrightarrow{\Psi}_a = 1 \longrightarrow \overrightarrow{\Psi}^*_a = \overrightarrow{\Psi}_a, \overrightarrow{A}_a = 0, \overrightarrow{\Psi}_a = \overrightarrow{S}_a$$

Since the state has no anti-symmetric component it "lives" completely in the symmetric subspace which is mapped to the vertical axis in the combined symmetry basis.



Similarity Function and Matrix

Z, the similarity function, is decaying exponential mappings of the distance function between two states.

$$Z_{ab} = Z(\overrightarrow{\Psi}_a, \overrightarrow{\Psi}_b) = e^{-\beta D_{ab}} = e^{-\beta(1-\overrightarrow{\Psi}_a \circ \overrightarrow{\Psi}_b)} \approx (\overrightarrow{\Psi}_a \circ \overrightarrow{\Psi}_b)^{\beta}$$

Given a time series $\overrightarrow{\Psi}(t)$, the elements of the matrix Z_{ij} are the similarity functions between each state of the motion path.

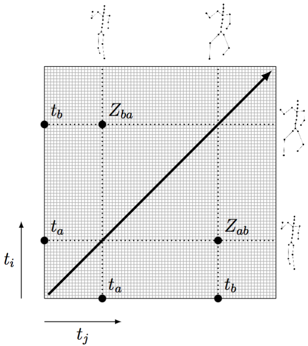
	Z_{31}	Z_{32}	$Z_{33} = 1$
$t_i \uparrow$	Z_{21}	$Z_{22} = 1$	Z_{23}
	$Z_{11} = 1$	Z_{12}	Z_{13}
	$t_j \rightarrow$		

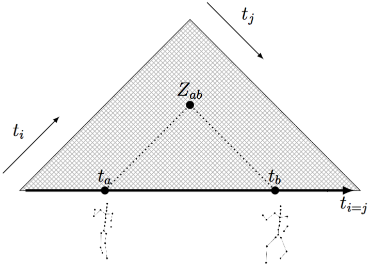
Similarity Matrix

$D_{aa} = 0$ (indiscernability) requires the diagonal elements are one. We call this the $i = j$ elements the identity axis. $D_{ab} = D_{ba}$ (symmetry) requires the similarity matrix is symmetric. These properties of the similarity matrix make it more compact to show only the $i > j$ terms and rotated so the identity axis is horizontal.

$$Z_{ii} = 1$$

$$Z_{ij} = Z_{ji}$$

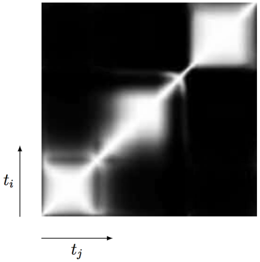


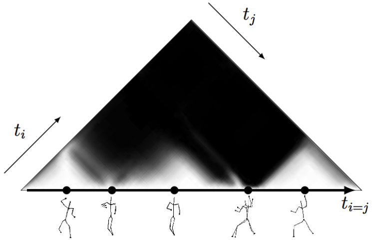


Similarity Plots / Motion Spectra

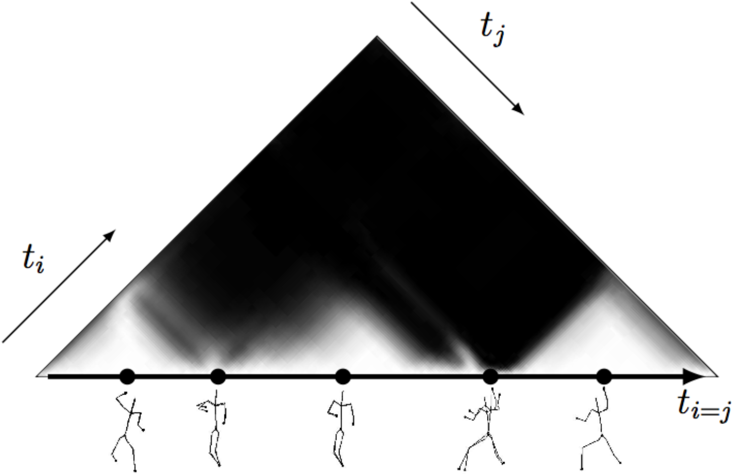
By mapping the values of the similarity matrix Z_{ij} to greyscale we generate an image. To distinguish it we call it a similarity plot or a Z plot. We refer to light areas of the plot as proximity signals.

Physical qualities may be determined by looking by at the plots. Rapidity, for example is related to the thickness proximity signal along the identity axis. A thin signal means fast motion.





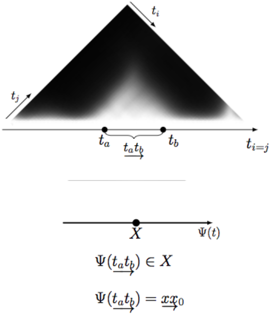
Rapidity



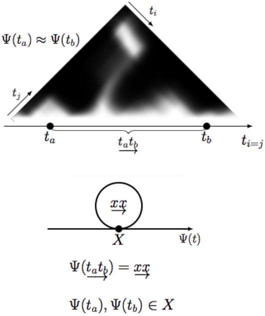
Model poses approximately 150ms apart are superimposed.

Spectral Motifs

We will go over basic motifs from Z plots and construct the corresponding motion graphs in $\mathcal{C}$.

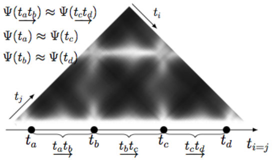


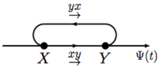
Presence



State Repetition

Spectral Motifs



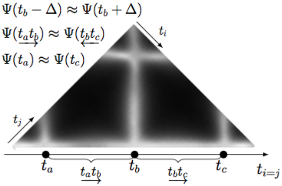


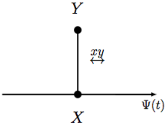
$\Psi(\underrightarrow{t_a t_b}), \Psi(\underrightarrow{t_c t_d}) \in \underrightarrow{xy}$

$\Psi(t_b), \Psi(t_d) \in Y$

$\Psi(t_a), \Psi(t_c) \in X$

Path Repetition





$\Psi(\underrightarrow{t_a t_b}), \Psi(\underrightarrow{t_b t_c}) \in \underleftrightarrow{xy}$

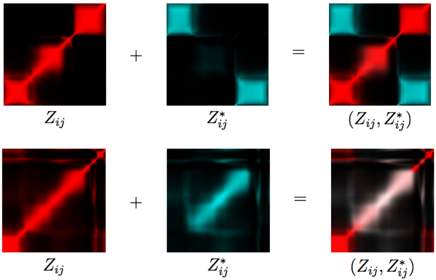
$\Psi(t_b) \in Y$

$\Psi(t_a), \Psi(t_c) \in X$

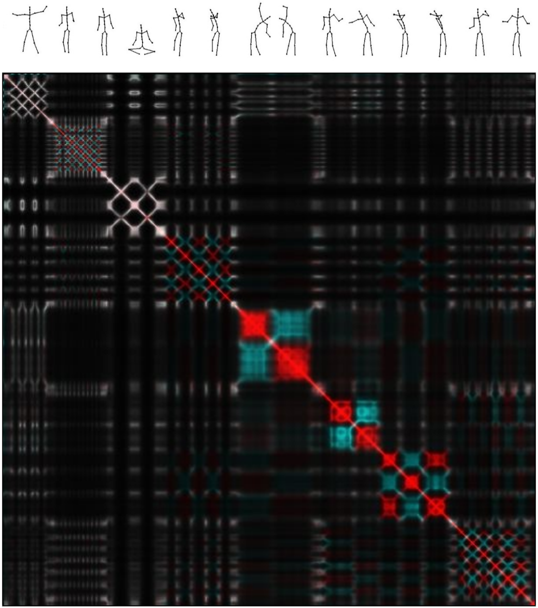
Palendromic Repetition

Bilateral Motion Spectra

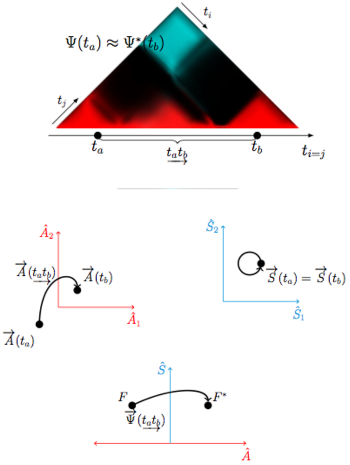
Z^* plots are generated through measuring the proximity between a state and it's mirror image. Since we are reflecting one of the states we no longer have a proper distance metric. Specifically the feature of indiscernability is lifted so the diagonal elements need not be one. If Z^*_{ii} are one then the corresponding state the same as it's reflection and the state is symmetric. Overlapping Z in red and Z^* in cyan yields the following.



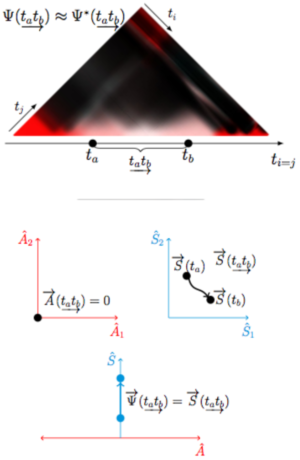
Example



Bilateral Spectral Motifs

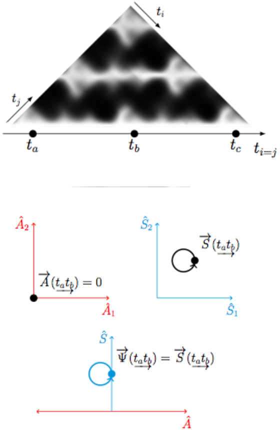


Reflected State Repetition

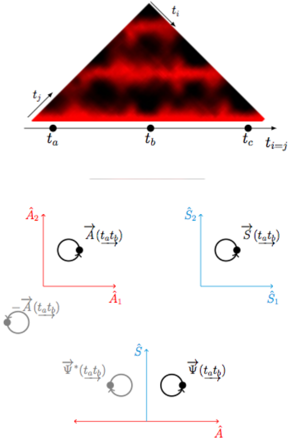


Symmetric Path

Bilateral Spectral Motifs

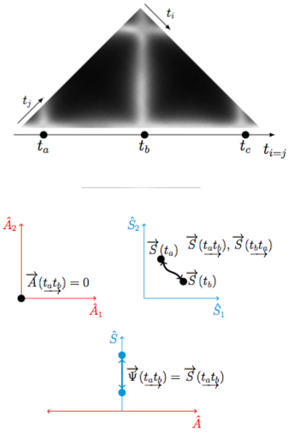


Symmetric Path Repetition

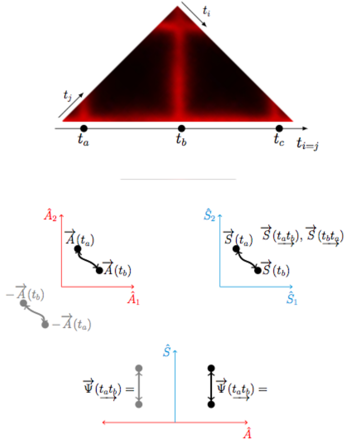


Asymmetric Path Repetition

Bilateral Spectral Motifs

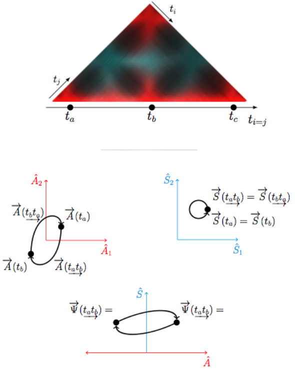


Symmetric Palindrome

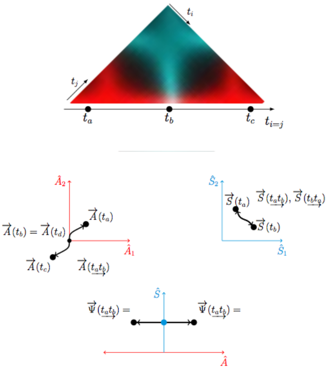


Asymmetric Palindrome

Bilateral Spectral Motifs

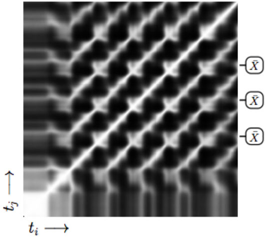


Antiphase Motion



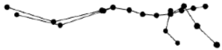
Antipalindromic Motion

Example: Butterfly Stroke



(Z_{ij}, Z^*_{ij})

$\mathcal{C}_{free}$

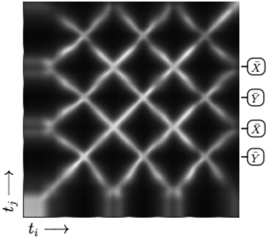


$\bar{X}$

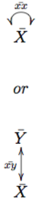
$\underrightarrow{\bar{x}x}$

(Symmetric, Cyclic)

Example: Jumping Jacks

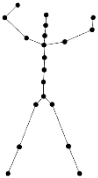


(Z_{ij}, Z^*_{ij})

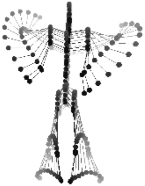


$\mathcal{C}_{free}$

$\bar{X}$



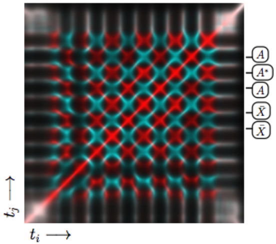
$\bar{Y}$



$\overrightarrow{\bar{x}\bar{y}}$

(Symmetric, Palendromic)

Example: Jogging in Place



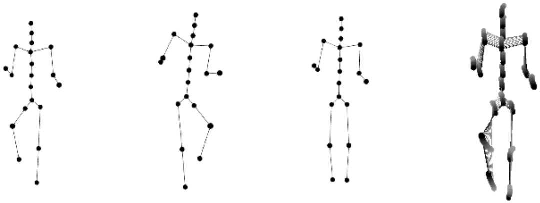
(Z_{ij}, Z^*_{ij})

$$A \xleftrightarrow{ax} \bar{X} \xleftrightarrow{(ax)^*} A^*$$

or

$$A \longleftarrow \bar{X} \longrightarrow A^*$$

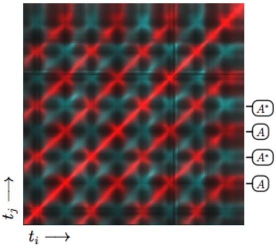
$\mathcal{C}_{free}$



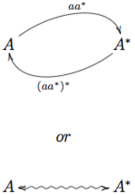
A $\quad A^*$ $\quad \bar{X}$ $\quad \overset{ax}{\leftrightarrow}$

(Palindromic, Antiphase Cyclic)

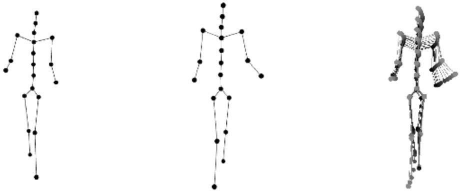
Example: Walking



(Z_{ij}, Z^*_{ij})



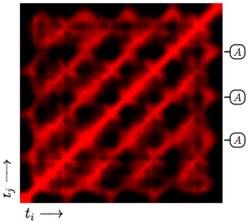
$\mathcal{C}_{free}$



A A^* $\underrightarrow{aa^*}$

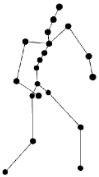
(Antiphase Cyclic)

Example: Limping



(Z_{ij}, Z^*_{ij})

$\mathcal{C}_{free}$

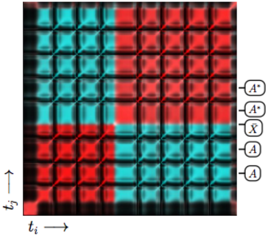


A

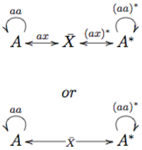
$\overrightarrow{aa}$

(Asymmetric Cyclic)

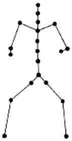
Side Kicks



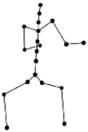
(Z_{ij}, Z^*_{ij})



$\mathcal{C}_{free}$



$\bar{X}$

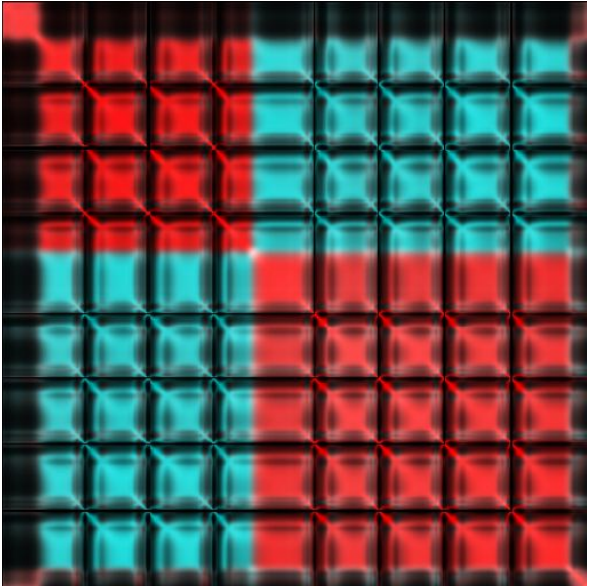


A

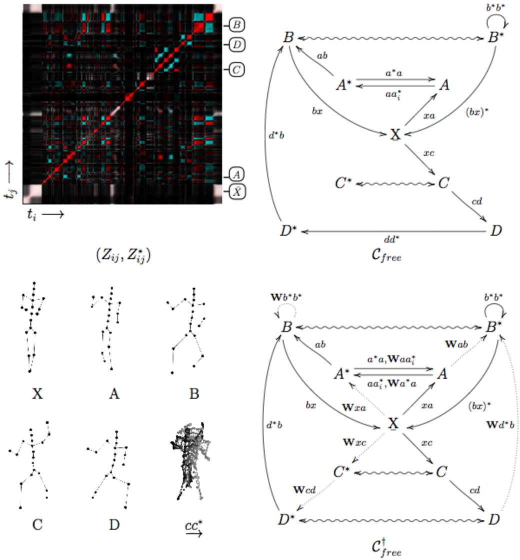
$\underrightarrow{aa}$

$\underleftrightarrow{ax}$

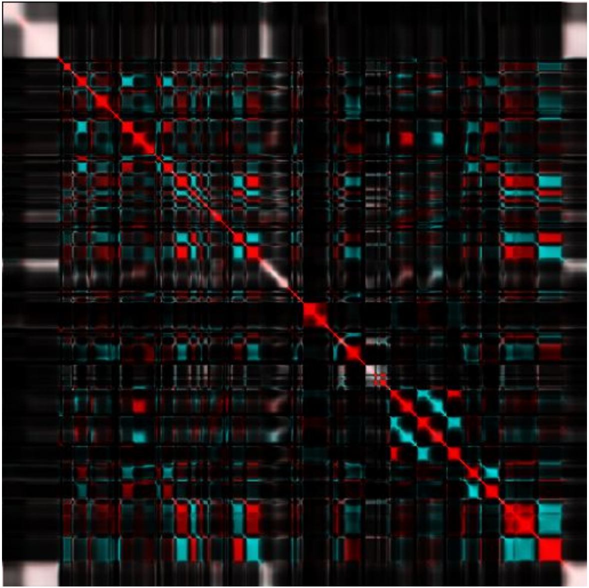
Side Kicks



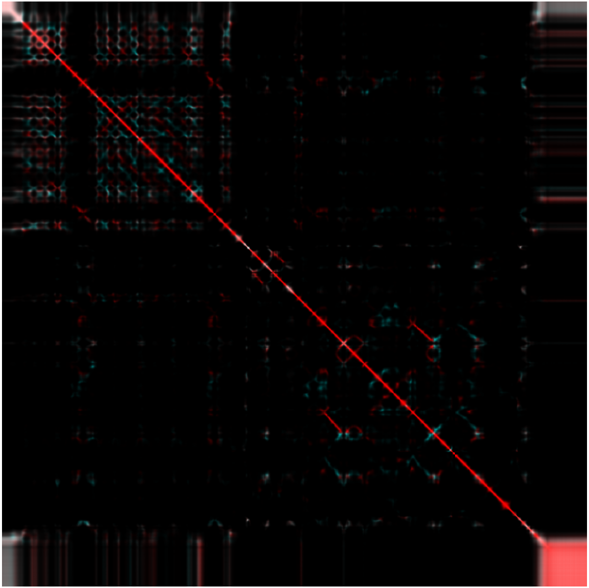
Karate (Bassai Dai Kata)



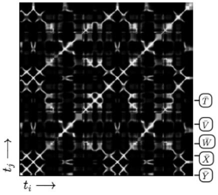
Karate (Bassai Dai Kata)



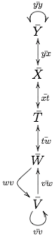
Karate (Breakdance)



Sun Salutation



(Z_{ij}, Z^*_{ij})

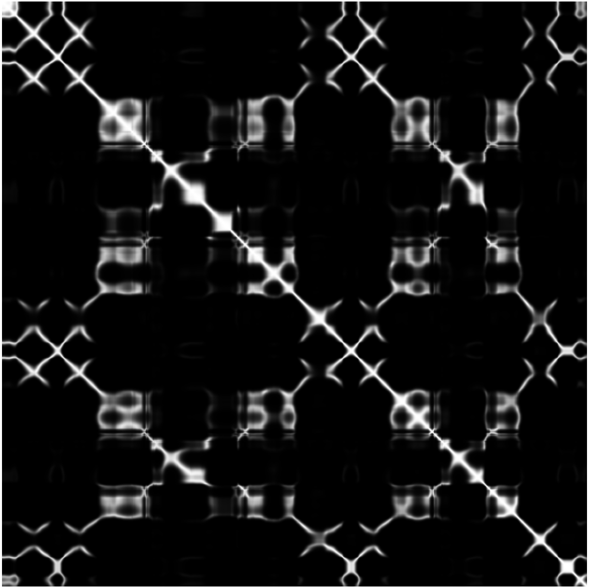


$\mathcal{C}_{free}$

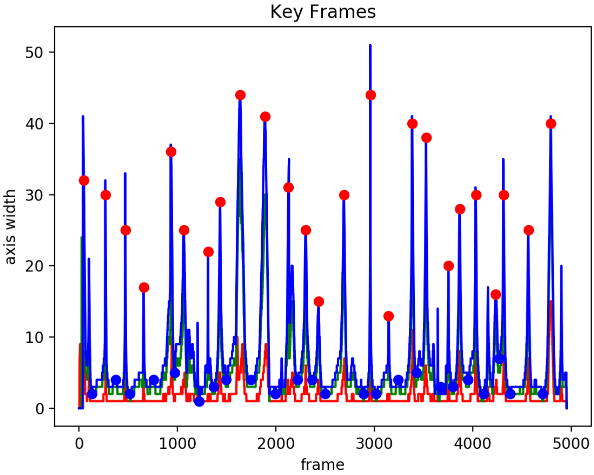
$\bar{\mathrm{Y}}$ $\bar{\mathrm{X}}$ $\bar{\mathrm{T}}$ $\bar{\mathrm{W}}$ $\bar{\mathrm{V}}$

$\underleftrightarrow{\bar{x}y}$ $\underleftrightarrow{\bar{y}y}$ $\underleftrightarrow{\bar{x}t}$ $\underleftrightarrow{\bar{t}w}$ $\underrightarrow{wy}$ $\underrightarrow{\bar{v}y}$ $\underrightarrow{\bar{v}w}$

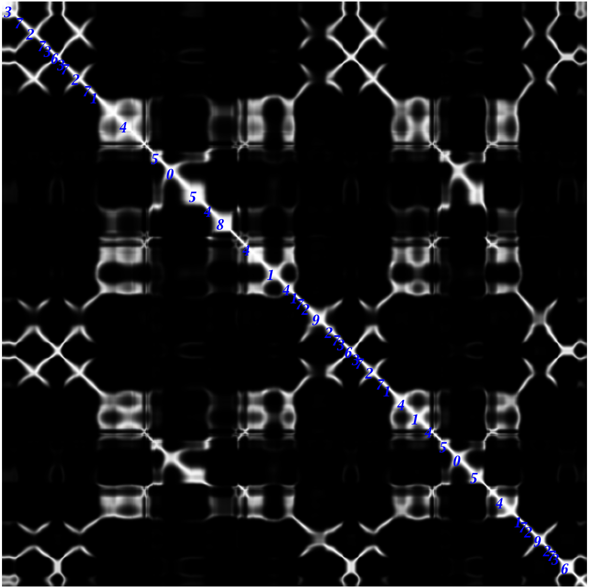
Sun Salutation



Sun Salutation Key Frames



Sun Salutation Key Frames



The End

Thank you

Questions

tony.schultz@gmail.com