

I THINK THAT MODERN PHYSICS HAS DEFINITELY DECIDED IN FAVOR OF PLATO. IN FACT THE SMALLEST UNITS OF MATTER ARE NOT PHYSICAL OBJECTS IN THE ORDINARY SENSE; THEY ARE FORMS, IDEAS WHICH CAN BE EXPRESSED UNAMBIGUOUSLY ONLY IN MATHEMATICAL LANGUAGE.

WERNER HEISENBERG

THE ONLY SHIBBOLETH THE WEST HAS IS SCIENCE. IT IS THE PREMISE OF MODERNITY AND IT DEFINES ITSELF AS A RATIONALITY CAPABLE OF, INDEED REQUIRING SEPARATION FROM POLITICS, RELIGION AND REALLY, SOCIETY. MODERNISATION IS TO WORK TOWARDS THIS... IF ONE LOOKS AT THE WORKS OF NEWTON TO EINSTEIN, THEY WERE NEVER SCIENTISTS IN THE WAY MODERNITY UNDERSTANDS THE TERM.

BRUNO LATOUR

THE BOUNDARY BETWEEN SCIENCE FICTION AND SOCIAL REALITY IS AN OPTICAL ILLUSION.

DONNA HARAWAY

DR. DOEG

INTRODUCTION TO MECHANICS

THE INVISIBLE COLLEGE

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*Dedicated to the ghosts of the college and
the spectral bodies of physics.*

Note

This physics text is an OpenSource academic project developed in abstraction by the Invisible College. The manuscript is written in L^AT_EX and makes use of the tufte-book and tufte-handout document classes.

<http://latex-project.org/ftp.html>

<https://git-scm.com/downloads>

Introduction

The limits of my language means the limits of my world.

-Ludwig Wittgenstein

PHYSICS IS EMBODIED through the process of measurement. Within physics any meaningful statement relates to some observable quantity. The process of observation requires measurement using physical instrumentation. In this way, physics is not only about the world but made of the world.

PHYSICS IS SYMBOLIC in the application of mathematics to the working world. Observable quantities are represented by algebraic variables and . Once quantities are abstracted as mathematical variables, physics becomes a language game.

PHYSICS IS VISUAL in both the measurement and the representation.

PHYSICS IS SOCIAL. In the end mathematics and physical observation are human activities. As is Soylent Green, physics too is made of people. People! This does not mean we can not gain insights beyond ourselves. Hopefully it means physicists will have jobs.

Quantities

A **PHYSICAL QUANTITY** is a physical property of a phenomenon, body, or substance, that can be quantified by measurement. We can represent a physical quantity using an algebraic variable. A physical quantity requires a value and unit of measure.

$$t = \underbrace{153}_{\text{Value}} \underbrace{\text{seconds}}_{\text{Unit}}$$

Consider the physical quantity, time. In the above equation we use t as the algebraic variable, seconds as the unit of measure and 153 as

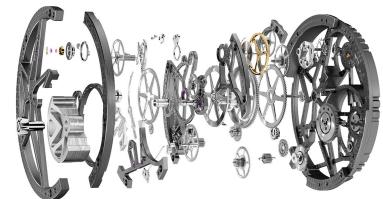


Figure 1: Exploded clock assembly

$$E = hf$$

Figure 2: Mathematical equation relating the energy and oscillatory frequency of a radiant particle.

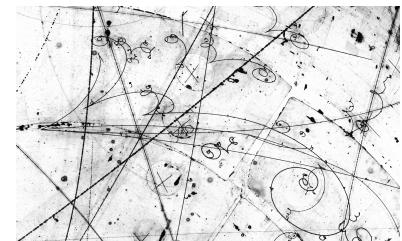


Figure 3: Bubble chamber image of a neutrino particle interaction event from the Fermi National Accelerator Laboratory.

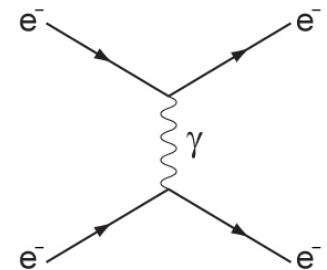
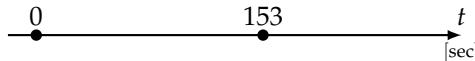


Figure 4: Feynman diagram representing photon mediated electron-electron scattering.

the value or magnitude of that unit.



The same information in the above equation may be encoded graphically using an appropriately labeled number line. Here the quantity's magnitude may be represented a length measured by a point and the origin.

Scientific Notation

Scientific notation is a way of writing numbers that are too big or too small to be conveniently written in decimal form. In scientific notation the numerical value 153 would be written as 1.53×10^2 .

$$153 = \underbrace{1.53}_{\text{significant figures}} \times \underbrace{10^2}_{\text{order of magnitude}}$$

In this format only one non-zero digit is to the left of the decimal place and to the right of the decimal place go the remaining significant figures. The order of magnitude is the exponent power of ten. In this example there are three significant figures and an order of magnitude of two. Scientific notation enables simpler order-of-magnitude comparisons.

Approximation

MAKING ORDER OF MAGNITUDE APPROXIMATIONS involves using these 10^n representations. Consider for example a deep breathing yogi. How many breaths do they take per day? The time scale for a breath is on the order of 10 or (10^1) seconds while a day is on the order of 10^5 seconds. Therefore the number of breaths per day would be on the order of 10^4 , or ten thousand breaths.

$$\frac{10^5}{10^1} = 10^{5-1} = 10^4$$

Said another way, the time for the rotation of the earth is four orders of magnitude greater than the cycle of the breath.

So Simon Peter climbed back into the boat and dragged the net ashore. It was full of large fish, 153, but even with so many the net was not torn. - John 21:11

The **accuracy** of a measurement system is the degree of closeness of measurements of a quantity to that quantity's true value. The **precision** of a measurement system, related to reproducibility and repeatability, is the degree to which repeated measurements under unchanged conditions show the same results.

Length	Meters
Observable Universe	10^{26}
Milky Way	10^{21}
Solar System	10^{13}
Earth to Sun	10^{11}
Earth	10^7
Football Field	10^2
Human	10^0
Cell	10^{-5}
Hydrogen Atom	10^{-10}
Proton	10^{-15}

Table 1: A list of length scales.

Time	Seconds
Age Universe	10^{17}
Age Earth	10^{17}
Life on Earth	10^{17}
Year	10^7
Month	10^6
Day	10^5
Sunlight to Earth	10^2
Heartbeat	10^0
Audible Latency	10^{-2}
Pion Lifetime	10^{-8}

Table 2: A list of time scales.

Units and Dimensions

meter The meter is the length of the path travelled by light in vacuum during a time interval of $1/299,792,458$ of a second.

kilogram The kilogram is the unit of mass; it is equal to the mass of the international prototype of the kilogram. This international prototype is made of platinum-iridium and is kept at the International Bureau of Weights and Measures in France.

second The second is the duration of $9,192,631,770$ periods of the radiation corresponding to the transition between the two hyperfine levels of the ground state of the cesium-133 atom.

ampere The ampere is that constant current which, if maintained in two straight parallel conductors of infinite length, of negligible circular cross-section, and placed one meter apart in vacuum, would produce between these conductors a force equal to 2×10^{-7} newton per meter of length.

The kelvin, unit of thermodynamic temperature, is the fraction $1/273.16$ of the thermodynamic temperature of the triple point of water.

mole The mole is the amount of substance of a system which contains as many elementary entities as there are atoms in 0.012 kilogram of carbon 12. When the mole is used, the elementary entities must be specified and may be atoms, molecules, ions, electrons, other particles or specified groups of such particles. In this definition, it is understood that the carbon 12 atoms are unbound, at rest and in their ground state.

candela The candela is the luminous intensity, in a given direction, of a source that emits monochromatic radiation of a frequency 540×10^{12} Hertz and has a radiant intensity in that direction of $1/683$ watt per steradian.

The *International System of Units (SI)* is the modern form of the metric system and is the world's most widely used system of measurement, used in both commerce and science. It comprises a coherent system of units of measurement built on seven base units. It defines twenty-two named units, and includes many more unnamed coherent derived units. The system also establishes a set of twenty prefixes to the unit names and unit symbols that may be used when specifying multiples and fractions of the units.

Prefix	Symbol	Value
yotta	Y	10^{24}
zetta	Z	10^{21}
exa	E	10^{18}
peta	P	10^{15}
tera	T	10^{12}
giga	G	10^9
mega	M	10^6
kilo	k	10^3
hecto	h	10^2
deca	da	10^1
deci	d	10^{-1}
centi	c	10^{-2}
milli	m	10^{-3}
micro	μ	10^{-6}
nano	n	10^{-9}
pico	p	10^{-12}
femto	f	10^{-15}
atto	a	10^{-18}
zepto	z	10^{-21}
yocto	y	10^{-24}

Table 3: A list of metric prefixes.

Quantity	SI Unit	SI Symbol	Variable	Dimension
Fundamental				
distance	meter	m	l	L
mass	kilogram	kg	m	M
time	second	s	t	T
electrical current	ampere	A	I	I
temperature	kelvin	K	T	Θ
number particles	mole	mol	n	N
luminous intensity	candela	cd	J	J
Derived				
angle	radian	rad	θ	1
frequency	hertz	$\text{Hz} = 1/\text{s}$	f	T^{-1}
force	newton	$\text{N} = \text{kg}\cdot\text{m}/\text{s}^2$	F	MLT^{-2}
pressure	pascal	$\text{Pa} = \text{N}/\text{m}^2$	P	$\text{ML}^{-1}\text{T}^{-2}$
energy	joule	$\text{J} = \text{N}\cdot\text{m}$	E	ML^2T^{-2}
power	watt	$\text{W} = \text{J}/\text{s}$	P	ML^2T^{-3}
electric charge	coulomb	$C = \text{A}\cdot\text{s}$	q	IT
electric potential	volt	$V = \text{J}/\text{C}$	V	$\text{ML}^2\text{T}^{-3}\text{I}^{-1}$
capacitance	farad	$F = \text{C}/\text{V}$	C	$\text{M}^{-1}\text{L}^{-2}\text{T}^4\text{I}^2$
resistance	ohm	$\Omega = \text{V}/\text{A}$	R	$\text{ML}^2\text{T}^{-3}\text{I}^{-2}$
magnetic field	tesla	$\text{T} = \text{V}\cdot\text{s}/\text{m}^2$	B	$\text{MT}^{-2}\text{I}^{-1}$
inductance	henry	$H = \text{V}\cdot\text{s}/\text{A}$	L	$\text{ML}^2\text{T}^{-2}\text{I}^{-2}$
radioactivity	becquerel	$\text{Bq} = 1/\text{s}$	A	T^{-1}

ANY PHYSICAL QUANTITY Q is proportional to a product of fundamental quantities. The critical exponents always being integer values.

$$Q = Cl^\alpha m^\beta t^\gamma I^\delta T^\epsilon n^\zeta J^\eta$$

The dimension of the quantity, $\dim Q$, is a product of the dimensions of the constituent quantity factors.

$$\dim Q = \text{L}^\alpha \text{M}^\beta \text{T}^\gamma \text{I}^\delta \Theta^\epsilon \text{N}^\zeta \text{J}^\eta$$

This constitutes what is called and Abelian group.

Algebra and Similitude A sum or difference of two commensurate quantities (having the same dimensions) is a physically meaningful expression.

$$402 \text{ meter} - 137 \text{ meter} = (402 - 137) \text{ meter} = 265 \text{ meter}$$

A product or quotient of any quantities can be a physically meaningful expression.

$$\frac{265 \text{ meter}}{153 \text{ second}} = \frac{265}{153} \frac{\text{meter}}{\text{second}} = 1.73 \text{ m/s}$$

Unit Conversion The dimensions of a conversion factor are unity. Any physically meaningful equality can be turned into one of two conversion factors.

$$1 \text{ mile} = 1609 \text{ meters} \iff \frac{1609 \text{ meters}}{1 \text{ mile}} = \frac{1 \text{ mile}}{1609 \text{ meters}} = 1$$

Table 4: A list of physical quantities with SI units and dimensions.

The concept of physical dimension was introduced by Joseph Fourier in 1822. Physical quantities that are commensurable have the same dimension; if they have different dimensions, they are incommensurable. For example, it is meaningless to ask whether a kilogram is less, the same, or more than an hour. Any physically meaningful equation will have the same dimensions on the left and right sides, a property known as "dimensional homogeneity".

Here $\alpha, \beta, \gamma, \delta, \epsilon, \zeta, \eta$ are all positive or negative integers

The conversion of a quarter mile into meters.

$$0.25 \text{ mi} \underbrace{\left(\frac{1609 \text{ m}}{1 \text{ mi}} \right)}_1 = \frac{0.25 \cdot 1609}{1} \frac{\cancel{\text{mi}}}{\cancel{\text{mi}}} \quad 0.25 \text{ mi} = 402 \text{ m}$$

English \iff Metric

$$\begin{aligned} 1 \text{ mile} &= 5280 \text{ feet} = 1609 \text{ meters} \\ 1 \text{ gallon} &= 3.785 \text{ liters} = 4.000 \text{ quarts} \\ 1 \text{ cm}^3 &= 1 \text{ milliliter} \\ 1 \text{ hp} &= 746 \text{ Watts} \\ 1 \text{ lb} &= 4.45 \text{ Newtons} \end{aligned}$$

Table 5: English and Metric equivalences

Constants

A **PHYSICAL CONSTANT** is a physical quantity that is generally believed to be both universal in nature and constant in time. It can be contrasted with a mathematical constant, which is a fixed numerical value, but does not directly involve any physical measurement.

Earth	Sun & Moon	Water
$g = 9.81 \text{ m/s}^2$	$m_{\text{sun}} = 1.99 \times 10^{30} \text{ kg}$	$c_{\text{vapor}} = 2.08 \times 10^3 \text{ J/kg}\cdot\text{K}$
$B_{\text{earth}} = 5.0 \times 10^{-5} \text{ T}$	$R_{\text{sun}} = 6.96 \times 10^8 \text{ m}$	$c_{\text{water}} = 4.18 \times 10^3 \text{ J/kg}\cdot\text{K}$
$m_{\text{earth}} = 5.98 \times 10^{24} \text{ kg}$	$m_{\text{moon}} = 7.36 \times 10^{22} \text{ kg}$	$c_{\text{ice}} = 2.11 \times 10^3 \text{ J/kg}\cdot\text{K}$
$R_{\text{earth}} = 6.38 \times 10^6 \text{ m}$	$R_{\text{moon}} = 1.74 \times 10^6 \text{ m}$	$L_{\text{fusion}} = 3.3 \times 10^5 \text{ J/kg}$
$r_{\text{earth}} = 1.50 \times 10^{11} \text{ m}$	$r_{\text{moon}} = 3.84 \times 10^8 \text{ m}$	$L_{\text{vapor}} = 2.1 \times 10^6 \text{ J/kg}$
$T_{\text{earth}} = 365.24 \text{ days}$	$T_{\text{moon}} = 27.3 \text{ days}$	$\rho_{\text{water}} = 1.00 \times 10^3 \text{ kg/m}^3$

Two tables are given here with common constants. Above is a table of data solar system and thermodynamic properties of water.

Description	Symbol	Quantity
Gravitational Constant	G	$6.67 \times 10^{-11} \text{ N}\cdot\text{m}^2/\text{kg}^2$
Electrostatic Constant	k_e	$8.99 \times 10^9 \text{ N}\cdot\text{m}^2/\text{C}^2$
Boltzmann's Constant	k_B	$1.38 \times 10^{-23} \text{ J/K}$
Avogadro's Number	N_A	6.02×10^{23}
Plank's Constant	h	$6.63 \times 10^{-34} \text{ J}\cdot\text{s}$
Speed of Light	c	$3.0 \times 10^8 \text{ m/s}$
Fundamental Charge	e	$1.6 \times 10^{-19} \text{ C}$
Mass of the Electron	m_e	$9.1 \times 10^{-31} \text{ kg}$
Mass of Proton	m_p	$1.7 \times 10^{-27} \text{ kg}$
Gas Constant	R	$8.31 \text{ J/mole}\cdot\text{K}$
Vacuum Permittivity	ϵ_0	$8.85 \times 10^{-12} \text{ F/m}$
Vacuum Permeability	μ_0	$4\pi \times 10^{-7} \text{ T}\cdot\text{m/A}$
Bohr Radius	a_0	$0.53 \times 10^{-10} \text{ m}$
Fine Structure Constant	α	$1/137$

Table 6: A list of physical quantities with SI units and dimensions.

Table 7: A list of physical quantities with SI units and dimensions.

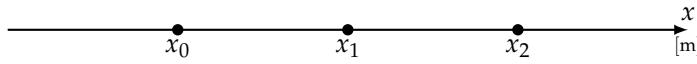
$$\begin{aligned} R &= N_A k_B \\ k_e &= 1/4\pi\epsilon_0 \\ \mu_0 c &= 1/\epsilon_0 c \\ \alpha &= \frac{\mu_0 c e^2}{2h} \\ a_0 &= \frac{\hbar^2}{m_e k_e e^2} = \frac{\hbar}{m_e c \alpha} \end{aligned}$$

Sets, Sequences & Series

A set of N values of x may be ordered into a sequence using an index i running from 0 to $N - 1$.

$$\{x_0, x_1, x_2, \dots, x_{N-1}\} = \{x_i\}$$

They can represent various points on the number line.



A sum of values is known as a series. Series may be represented using sigma notation.

$$x_0 + x_1 + x_2 + \dots + x_M = \sum_{i=0}^M x_i$$

A **set** is a collection without a sequential order while a **sequence** is an ordered set. An index subscript is used to order the sequence. A **series** is a summation of a sequence of numbers. Sigma notation is your friend in representing .

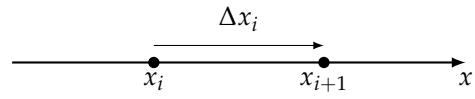
$\langle x \rangle$ is the mean, or average value of a set. σ is the uncertainty or standard deviation.

$$\begin{aligned} \langle x \rangle &= \frac{x_0 + x_1 + x_2 + \dots}{N} \\ \langle x^2 \rangle &= \frac{x_0^2 + x_1^2 + x_2^2 + \dots}{N} \\ \sigma^2 &= \langle x^2 \rangle - \langle x \rangle^2 \end{aligned}$$

Difference

The delta operator Δ indicates difference. Δx is a change in x from $x = a$ to $x = b$. $\Delta x = b - a$, or indexed $\Delta x_i = x_{i+1} - x_i$.

$$\{\Delta x_i\} = \{x_1 - x_0, x_2 - x_1, \dots, x_N - x_{N-1}\}$$



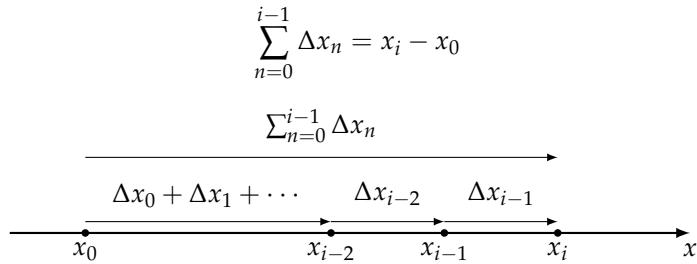
For any sequence x_i with N elements we can generate a sequence Δx_i of changes. This is a mapping of one sequence to another.

```
for (i=0, i<N-1, i=i+1)
    dx[i]=x[i+1]-x[i]
```

$$\{x_i\} \xrightarrow{\Delta} \{\Delta x_i\}$$

Accumulation

Accumulation is the inverse of difference. Rather than subtract successive terms we add them. It is the summation of a sequence as a series. The summation of Δx_i returns us to the sequence x_i .



For any sequence Δx_i of changes we can generate a sequence x_i (up to a constant). This is the reverse mapping.

```
for (i=0, i<N-1, i=i+1)
    x[i+1]=x[i]+dx[i]
```

$$\{\Delta x_i\} \xrightarrow{\Sigma} \{x_i\}$$

Functions

Given a sequence of discrete ordered pairs $\{x_i, y_i\}$ we can graph points on a 2-dimensional graph using two number lines. We can model the relationship using a continuous function, $y = f(x)$.

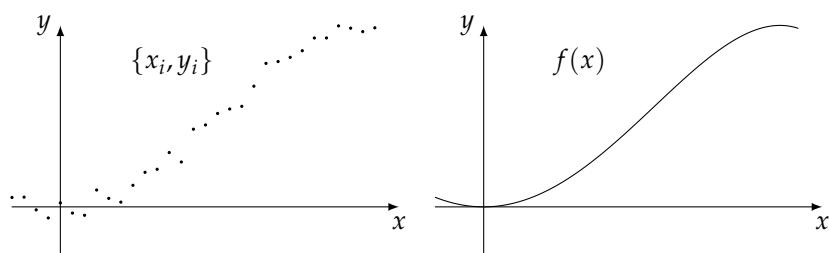


Figure 5: Ordered pairs $\{x_i, y_i\}$ and continuous function $f(x)$

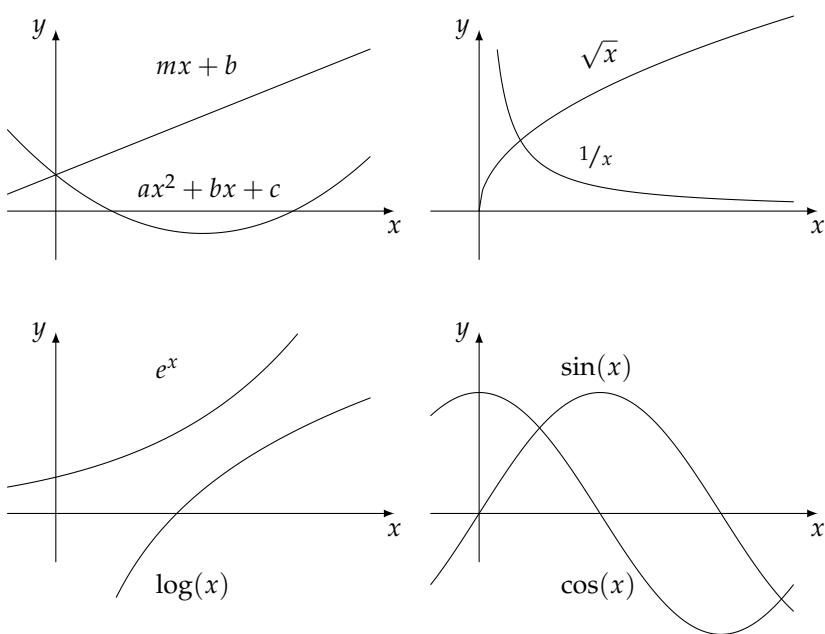


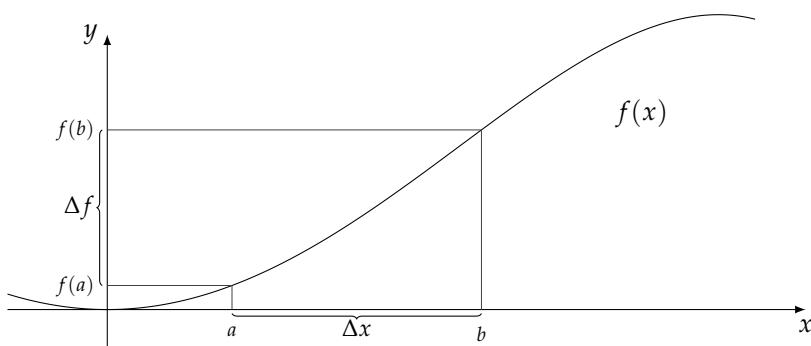
Figure 6: Common functions

Function	Formula
Linear	$y = mx + b$
Quadratic	$y = ax^2 + bx + c$
Reciprocal	$y = \frac{1}{x}$
Power	$y = x^n$
Exponential	$y = a^x$
Logarithmic	$y = \log_a b$
Sine	$y = \sin x$
Cosine	$y = \cos x$
Tangent	$y = \tan x$
Arc sine	$y = \sin^{-1} x$
Arc cosine	$y = \cos^{-1} x$
Arc tangent	$y = \tan^{-1} x$

Table 8: A list of common functions

Changing Functions

CHANGES IN THE VALUE OF A FUNCTION generate difference in $f(x)$. There is a relationship between Δf and Δx .

Figure 7: Difference on a continuous function $f(x)$

$$\begin{aligned}\Delta f &= f(b) - f(a) \\ \Delta f &= f(a + \Delta x) - f(a)\end{aligned}$$

Difference is the object of a practical affirmation inseparable from essence and constitutive of existence.- Gilles Deleuze

Slope, Tangent Line & Slope Function

The **slope** is a ratio of difference between two points on the function graph. The construction of the right triangle with sides Δx and Δy yields a tangent which is equivalent to the slope. In the limit of small Δ we call the slope a **tangent line**.

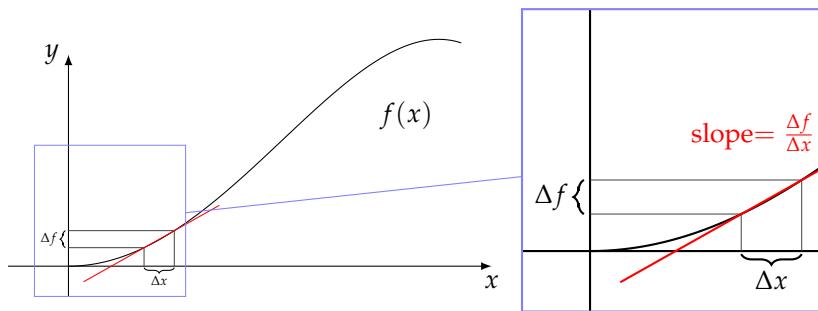
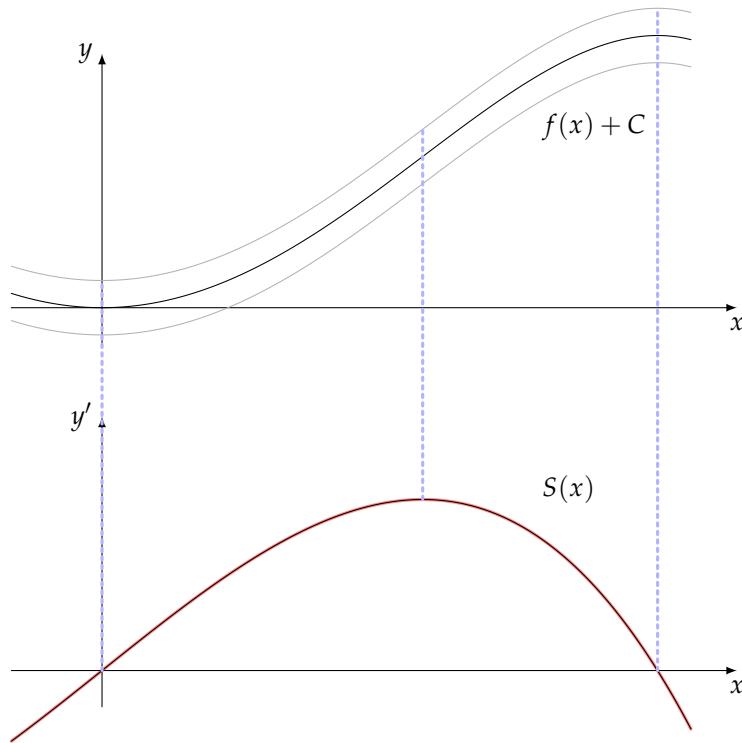


Figure 8: Tangent lines and the slope function $S(x)$ for a function $f(x)$

$$\text{slope} = \frac{\text{rise}}{\text{run}} = \frac{\Delta f}{\Delta x}$$



This specific function $f(x)$ is

$$y = x^2 - \frac{x^4}{12}$$

$$S(x) = \text{slope of tangent line} = \lim_{\Delta x \rightarrow 0} \frac{\Delta f}{\Delta x}$$

This specific slope function $S(x)$ is

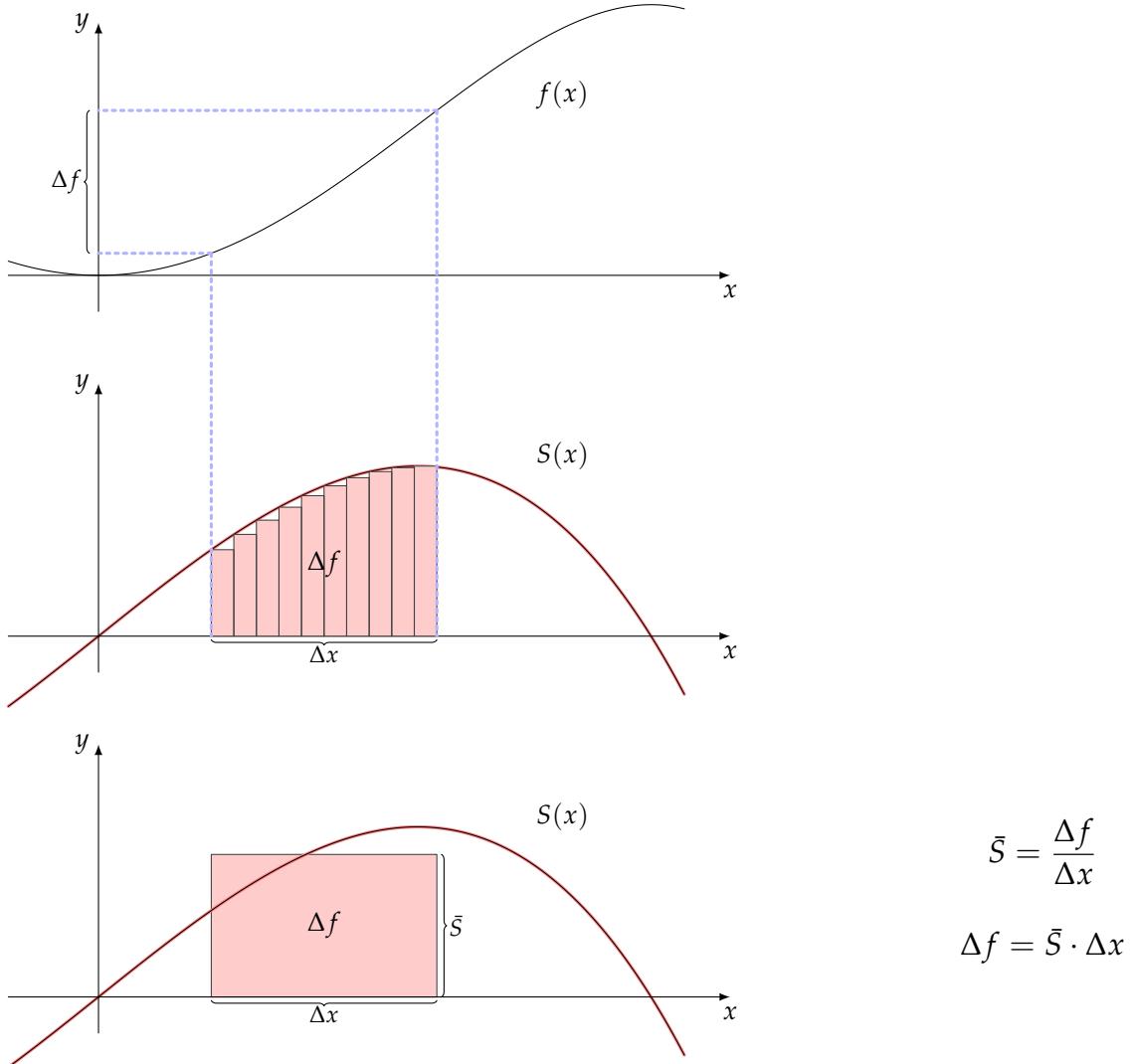
$$y = 2x - \frac{x^3}{3}$$

The **slope function** $S(x)$ represents the slope of the tangent line at any point x .

Accumulation of a Slope Function

Over some range Δx the area beneath the slope function represents some accumulation of $S(x)$. The total area represents Δf over the range Δx .

Figure 9: Area underneath a slope function $S(x)$



If this area representing Δf is represented as a rectangle with base Δx the height of the rectangle is the average value \bar{S} .

Scalars and Vectors

A **scalar** is a one-dimensional physical quantity, i.e. one that can be described by a single real number (signed, with units). It is a physical quantity that only has magnitude but no direction. A **vector** is a geometric object that has magnitude (or length) and direction and can be added to other vectors according to vector algebra. A vector can be represented by a set of scalars.

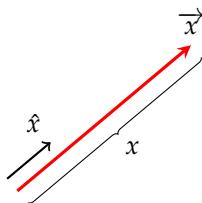
\vec{x} is a vector. x is the magnitude (or length) of the vector \vec{x} . The vector \hat{x} is the directional vector of \vec{x} . It has a magnitude of 1 and points in the same direction as \vec{x} .

$$\vec{x} = x\hat{x}$$

$$|\vec{x}| = x$$

$$\hat{x} = \frac{\vec{x}}{x}$$

$$|\hat{x}| = 1$$



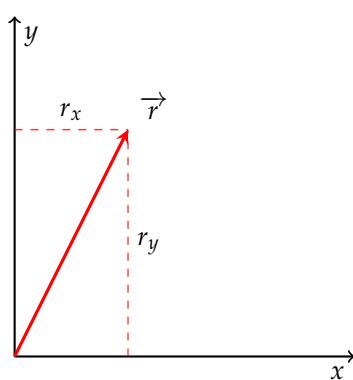
Spatial Coordinate Systems

2D Cartesian (x, y)

In two dimensions we use the vector representation \vec{r} with components x and y . This can be represented as a column vector or using the unit vectors \hat{x} and \hat{y} . The magnitude of the vector \vec{r} is r . We can calculate r given the x and y components using the Pythagorean theorem.

x (or r_x) and y (or r_y) are called orthogonal components of \vec{r} meaning \hat{x} and \hat{y} do not overlap at all, namely they meet at 90 degrees.

$$\vec{r} = x\hat{x} + y\hat{y}$$



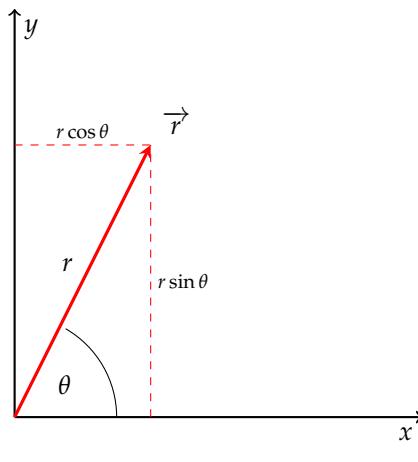
$$\vec{r} = \begin{pmatrix} x \\ y \end{pmatrix}$$

$$r = \sqrt{x^2 + y^2}$$

$$\hat{r} = \begin{pmatrix} x/\sqrt{x^2+y^2} \\ y/\sqrt{x^2+y^2} \end{pmatrix}$$

Polar Coordinates (r, θ)

In polar coordinates we parameterize the vector \vec{r} in terms of its length and direction rather than in terms of the x and y components. The length of the vector \vec{r} is r and the direction is expressed in terms of θ . The vector \vec{r} makes an angle θ with the unit vector \hat{x} . We can easily convert between polar coordinates (r, θ) and 2-D cartesian coordinates (x, y) .



$$\vec{r} = r\hat{r}$$

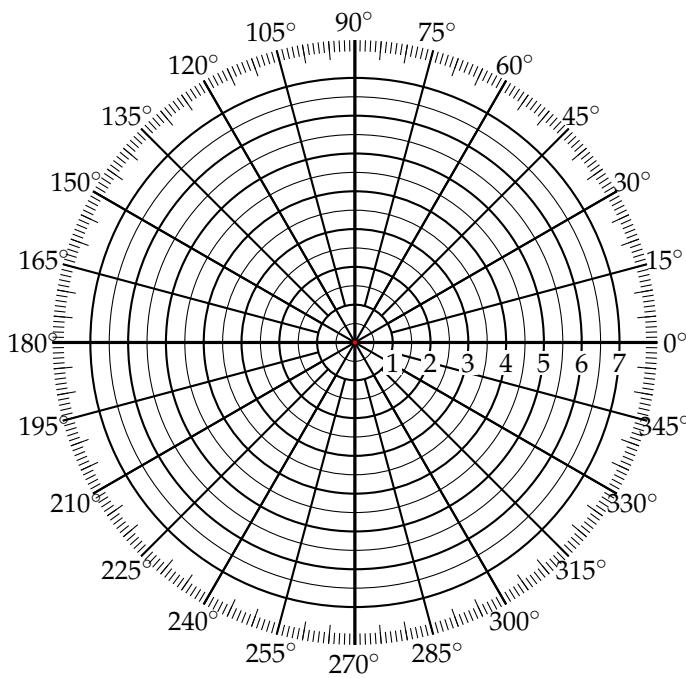
$$r = \sqrt{x^2 + y^2}$$

$$\theta = \tan^{-1}\left(\frac{y}{x}\right)$$

$$x = r \cos \theta$$

$$y = r \sin \theta$$

$$\hat{r} = \begin{pmatrix} \cos \theta \\ \sin \theta \end{pmatrix}$$



In mathematics, the polar coordinate system is a two-dimensional coordinate system in which each point on a plane is determined by a distance from a reference point and an angle from a reference direction.

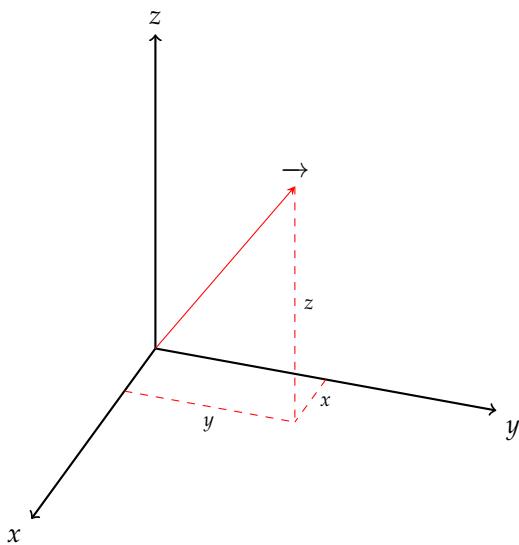
The reference point (analogous to the origin of a Cartesian system) is called the pole, and the ray from the pole in the reference direction is the polar axis. The distance from the pole is called the radial coordinate or radius, and the angle is the angular coordinate, polar angle, or azimuth.

The concepts of angle and radius were already used by ancient peoples of the 1st millennium BC. The Greek astronomer and astrologer Hipparchus (190-120 BC) created a table of chord functions giving the length of the chord for each angle, and there are references to his using polar coordinates in establishing stellar positions. In On Spirals, Archimedes describes the Archimedean spiral, a function whose radius depends on the angle. The Greek work, however, did not extend to a full coordinate system.

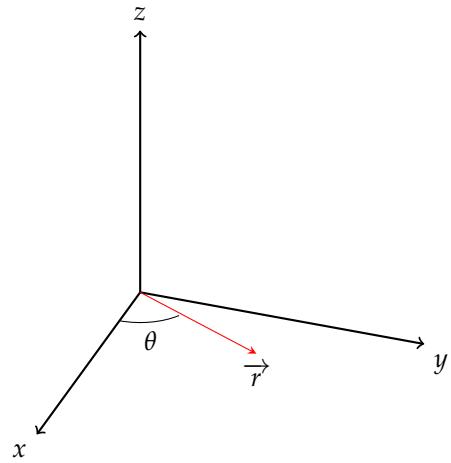
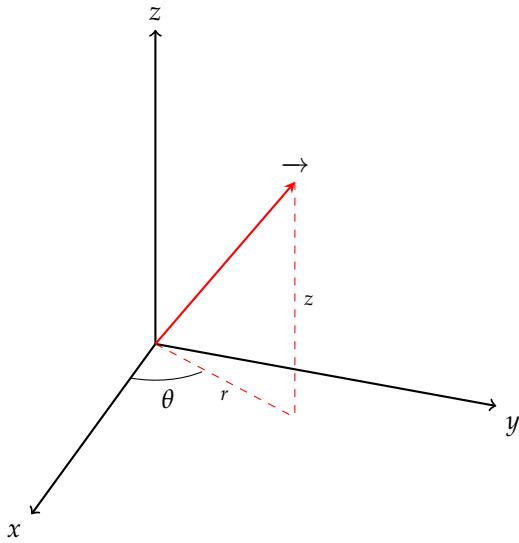
3D Cartesian (x, y, z)

$$\vec{r} = \vec{x} + \vec{y} + \vec{z} = x\hat{x} + y\hat{y} + z\hat{z} =_x \hat{x} +_y \hat{y} + \hat{z}$$

$$\vec{r} = \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} x \\ y \\ z \end{pmatrix}$$

**Cylindrical (r, θ, z)**

$$\vec{r} = \vec{r}' + \vec{z} = r\hat{r} + z\hat{z} =_r \hat{r} + \hat{z}$$



In three dimensions a vector can be described by three orthogonal components (x, y, z) . It is the 3D cartesian coordinate system. These form a right handed coordinate system so that if you look at the standard (x, y) plane the z -axis comes straight out of the page. The magnitude of \vec{r} is determined using the Pythagorean Theorem (in 3D).

$$= \sqrt{x^2 + y^2 + z^2}$$

The concept of Cartesian coordinates generalizes to allow axes that are not perpendicular to each other, and/or different units along each axis. In that case, each coordinate is obtained by projecting the point onto one axis along a direction that is parallel to the other axis (or, in general, to the hyperplane defined by all the other axes). In such an oblique coordinate system the computations of distances and angles must be modified from that in standard Cartesian systems, and many standard formulas (such as the Pythagorean formula for the distance) do not hold.

We use the \vec{r} to notation the vector in 3D and reserve \vec{r}' to represent the component in the xy -plane. In cylindrical coordinates the vector \vec{r} is represented using polar coordinates for \vec{r}' and an orthogonal z -component. Converting between cylindrical and 3D cartesian coordinates is identical to the polar and 2D cartesian conversion.

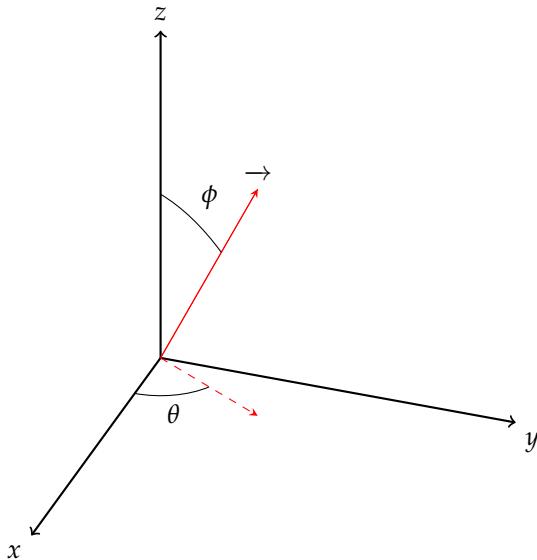
Spherical (r, θ, ϕ)

$$\rightarrow = \hat{}$$

$$= \sqrt{x^2 + y^2 + z^2}$$

$$\theta = \tan^{-1} \left(\frac{y}{x} \right)$$

$$\phi = \tan^{-1} \left(\frac{r}{z} \right)$$



In a spherical coordinate system we represent the 3D vector \vec{r} in terms of its magnitude (length) r , azimuthal angle θ and polar angle ϕ . The conversion between 3D cartesian or cylindrical coordinates and spherical coordinates are given.

$$\hat{r} = \begin{pmatrix} \sin \phi \cos \theta \\ \sin \phi \sin \theta \\ \cos \phi \end{pmatrix}$$

A spherical coordinate system is a coordinate system for three-dimensional space where the position of a point is specified by three numbers: the radial distance of that point from a fixed origin, its polar angle measured from a fixed zenith direction, and the azimuth angle of its orthogonal projection on a reference plane that passes through the origin and is orthogonal to the zenith, measured from a fixed reference direction on that plane.

The radial distance is also called the radius or radial coordinate. The polar angle may be called co-latitude, zenith angle, normal angle, or inclination angle.

The mathematics used to describe electron distributions around atoms uses spherical coordinates. The symmetry features of this mathematics gives rise to the structure of the periodic table.

$$\begin{aligned} r &= \sin \phi \\ z &= \cos \phi \\ x &= \sin \phi \cos \theta \\ y &= \sin \phi \sin \theta \end{aligned}$$

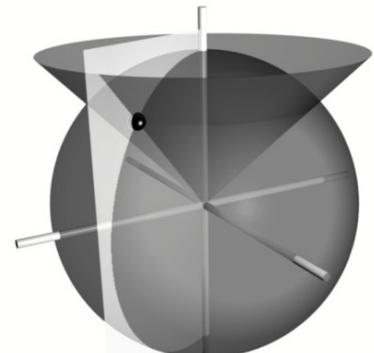


Figure 10: Spherical coordinates

Vector Operations

Vector Scaling

$$a \vec{p} = a \begin{pmatrix} p_x \\ p_y \\ p_z \end{pmatrix} = \begin{pmatrix} ap_x \\ ap_y \\ ap_z \end{pmatrix}$$

Vector Addition

$$\vec{p} + \vec{q} = \begin{pmatrix} p_x \\ p_y \\ p_z \end{pmatrix} + \begin{pmatrix} q_x \\ q_y \\ q_z \end{pmatrix} = \begin{pmatrix} p_x + q_x \\ p_y + q_y \\ p_z + q_z \end{pmatrix}$$

Dot Product

The dot product, or scalar product, or inner product, is an algebraic operation that takes two equal-length sequences of numbers (usually coordinate vectors) and returns a single number. This operation can be defined either algebraically or geometrically.

$$\vec{p} \cdot \vec{q} = \begin{pmatrix} p_x \\ p_y \\ p_z \end{pmatrix} \cdot \begin{pmatrix} q_x \\ q_y \\ q_z \end{pmatrix} = p_x q_x + p_y q_y + p_z q_z$$

$$|\vec{p}| = p = \sqrt{\vec{p} \cdot \vec{p}} = \sqrt{p_x^2 + p_y^2 + p_z^2}$$

$$\vec{p} \cdot \vec{q} = pq \cos \gamma$$

Cross Product

The cross product, or vector product, $\vec{p} \times \vec{q}$ is the very small displacement caused by rotation of the vector \vec{p} around the vector \vec{q} by an angle q . It yields a vector perpendicular to both \vec{p} and \vec{q} .

$$\vec{p} \times \vec{q} = \begin{pmatrix} p_y q_z - p_z q_y \\ p_z q_x - p_x q_z \\ p_x q_y - p_y q_x \end{pmatrix}$$

$$|\vec{p} \times \vec{q}| = pq \sin \gamma$$

$$\vec{p} \times \vec{q} = -\vec{q} \times \vec{p}$$

$$\hat{x} \times \hat{x} = \hat{y} \times \hat{y} = \hat{z} \times \hat{z} = 0$$

$$\hat{x} \times \hat{y} = \hat{z} \quad \hat{y} \times \hat{x} = \hat{x} \quad \hat{z} \times \hat{x} = \hat{y}$$

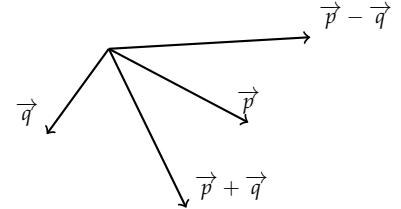


Figure 11: Vector Addition and Subtraction



Figure 12: Dot product is proportional to the overlap of the two vectors

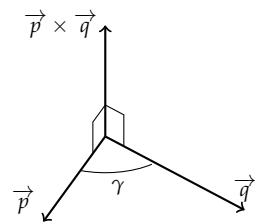


Figure 13: Cross product is proportional to the perpendicularity of the two vectors

Forces and Frames

The people... are the motive force.

- Mao Tse Tung



Figure 14: *Frames of Reference* is a 1960 educational film by Physical Sciences Study Committee.

Principia

Philosophiae Naturalis Principia Mathematica, Latin for "Mathematical Principles of Natural Philosophy", is often referred to simply as the Principia. The work consists of three books by Sir Isaac Newton, in Latin, first published in 1687. The Principia states Newton's laws of motion, forming the foundation of classical mechanics, also Newton's law of universal gravitation, and a mathematical derivation of Kepler's empirically derived laws of planetary motion. The Principia is justly regarded as one of the most important works in the history of science.

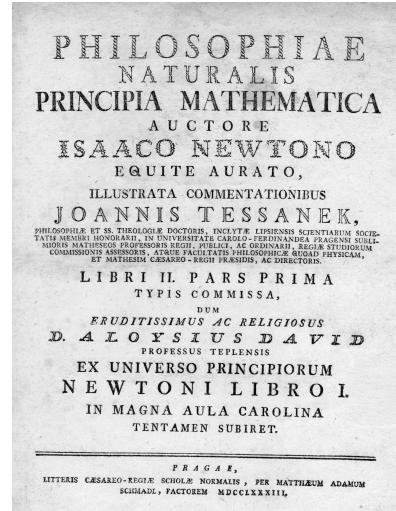
Newton's First Law

Every body perseveres in its state of being at rest or of moving uniformly straight forward, except insofar as it is compelled to change its state by forces impressed. - *Principia*

$$\sum \vec{F} = 0 \rightarrow \vec{a} = 0$$

IN AN INERTIAL FRAME, an object at rest tends to stay at rest and an object in motion tends to stay in motion unless an external net force acts upon it. Newton's first law is often called the Law of Inertia.

An inertial frame of reference is in a state of constant, rectilinear motion. It is peaceful and non-accelerating. Inertial is synonymous with Newtonian and Galilean. Newton's laws are only applicable in an inertial frame of reference.



Inertia

INERTIA IS THE RESISTANCE of any physical object to any change in its state of motion including changes to its speed and direction or the state of rest. It is the tendency of objects to keep moving in a straight line at constant velocity. The principle of inertia is one of the fundamental principles of classical physics that are used to describe the motion of objects and how they are affected by applied forces. Inertia comes from the Latin word, *iners*, meaning idle, sluggish. Inertia is one of the primary manifestations of mass. Matter is the occupancy of mass over space therefore matter has inertia.

Space and Time Uniformity

Homogeneity A homogeneous system has the same properties at every point; it is uniform without irregularities. The laws of physics must be invariant (unchanging) to translations in space. The location of the coordinate system does not change physics.

Isotropy A isotropic system has the same properties in all directions. The laws of physics must be invariant to rotations in space. The orientation of the coordinate system does not change physics.

Time-Independence A time-independent system has the same properties throughout time. The laws of physics must be invariant to translations in time. The start time of the physicist's watch does not change physics.

Newton's Second Law

Newton's second law states the sum of forces on an object is equal to the product of mass and acceleration of the object.

$$F_{net} = ma$$

Stated another way, the sum of forces is equal to the time rate of change of momentum.

$$\sum \vec{F} = \lim_{\Delta \rightarrow 0} \frac{\Delta \vec{p}}{\Delta t} = \lim_{\Delta \rightarrow 0} \frac{\Delta(m \vec{v})}{\Delta t} = m \lim_{\Delta \rightarrow 0} \frac{\Delta \vec{v}}{\Delta t} = m \vec{a}$$

MOMENTUM IS DEFINED as the product of mass and velocity. Though Newton does not use the term he describes a *quantitas motus*, "quantity of motion", as "arising from the velocity and quantity of matter conjointly", which identifies it as momentum. Thus, in the second

Inherent force of matter is the power of resisting by which every body, so far as it is able, perseveres in its state either of resting or of moving uniformly straight forward.
- *Principia*



Figure 16: Isaac Newton doing his thing

A change in motion is proportional to the motive force impressed and takes place along the straight line in which that force is impressed.
- *Principia*

law, when he refers to *mutatio motus* being proportional to the force impressed, he is generally taken to mean momentum again. It remained only to assign a standard term to the quantity of motion. The first use of "momentum" in its proper mathematical sense is not known but Jenning's 1721 *Miscellanea* defines momentum, or "quantity of motion", as "a rectangle", the product of Q and V, where Q is "quantity of material" and V is "velocity".

$$\underbrace{\vec{p}}_{\text{momentum}} = \overbrace{m}^{\text{inertial mass}} \underbrace{\vec{v}}_{\text{velocity}}$$

GALILEAN INVARIANCE or Galilean relativity states that the laws of motion are the same in all inertial frames. Galileo Galilei first described this principle in 1632 in his *Dialogue Concerning the Two Chief World Systems* using the example of a ship travelling at constant velocity, without rocking, on a smooth sea; any observer doing experiments below the deck would not be able to tell whether the ship was moving or stationary. Newton's second law is invariant to change in the velocity of the frame.

Newton's Third Law

THE THIRD LAW STATES that all forces between two objects exist in equal magnitude and opposite direction: if object one exerts a force $\vec{F}_{1 \rightarrow 2}$ on object two, then object two simultaneously exerts a force $\vec{F}_{2 \rightarrow 1}$ on object one, and the two forces are equal in magnitude but opposite in direction.

$$\vec{F}_{1 \rightarrow 2} = -\vec{F}_{2 \rightarrow 1}$$

The third law means that all forces are interactions between different bodies, and thus that there is no such thing as a unidirectional force or a force that acts on only one body. This law is sometimes referred to as the action-reaction law, with $\vec{F}_{1 \rightarrow 2}$ called the "action" and $\vec{F}_{2 \rightarrow 1}$ the "reaction". The action and the reaction are simultaneous, and it does not matter which is called the action and which is called reaction; both forces are part of a single interaction, and neither force exists without the other.

THE CONTEMPORARY MANTRA for Newton's third law is uttered as follows.

Every action has an equal and opposite reaction.

The quantity of matter is that which arises conjointly from its density and magnitude. A body twice as dense in double the space is quadruple in quantity. This quantity I designate by the name of body or of mass.
- Principia



Figure 17: If you wakeup from sleeping on a train moving at constant velocity, there is nothing about the physics in the train that tells you it's moving. The constant velocity train is inertial and a legit frame of reference from which to do physics.

To any action there is always an opposite and equal reaction; in other words, the actions of two bodies upon each other are always equal and always opposite in direction.
- Principia



Figure 18: Action/Reaction was a 2014 performance art work by Rahman Hak-Hagir and Francesca Lolli.

Forces

Forces are vector quantities as they had magnitudes and direction in space. Summation of forces follows vector addition. The unit of force is the Newton, or N. This is one kilogram meter per square second.

$$1 \text{ Newton} = \frac{\text{kg} \cdot \text{m}}{\text{s}^2}$$

Statics

STATICS IS THE BRANCH OF MECHANICS that is concerned with the analysis of force loads on physical systems in static equilibrium, that is, in an inertial state where the relative positions of subsystems do not vary over time. When in static equilibrium, the system is not being accelerated. Therefore, by Newton's first law, the vector sum of the forces is zero.

Dynamics

DYNAMICS IS THE BRANCH OF MECHANICS concerned with the study of forces and their effect on motion. In a dynamic situation the net force is non-zero and there is a resultant acceleration. Newton's second law is invoked when analyzing a dynamic system.

Free Body Diagrams

A **FREE BODY DIAGRAM** IS A GRAPHICAL ILLUSTRATION used to visualize and conceive of the applied forces on a body. In the current treatment we represent particles with a dot and draw all forces incident on the particle as a vector diagram with all force vectors with their tail at the point.

The English pound is a unit of force equivalent to 4.45 Newtons.

$$1 \text{ lbs} = 4.45 \text{ N}$$

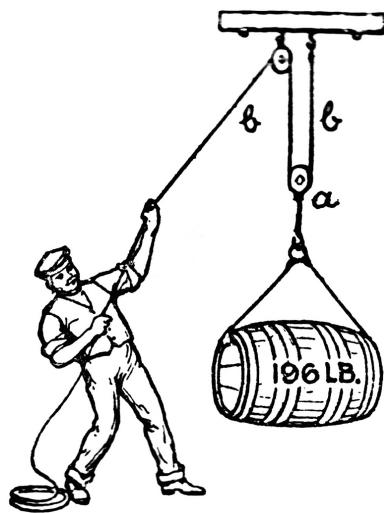
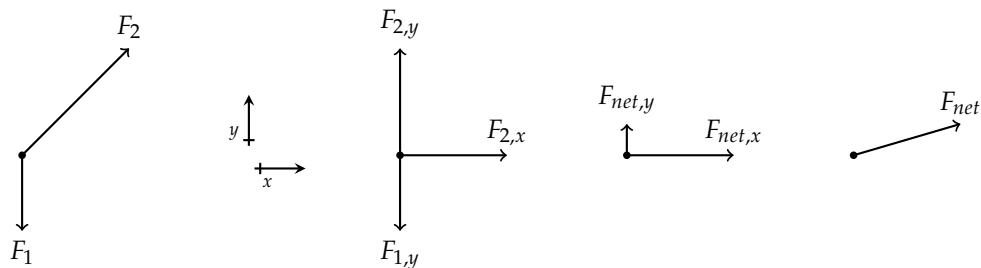


Figure 19: This could be a static or dynamic system, depending on the balance of forces.

Once a coordinate system is established all forces can be decomposed in terms of components and summed across the component directions. The resultant force determines the acceleration. In some cases first knowing the acceleration gives an understanding of the constituent forces.

$$\vec{F}_{net} = \vec{F}_1 + \vec{F}_2 = \left(\begin{array}{c} \vec{F}_{2,x} \\ \vec{F}_{1,y} + \vec{F}_{2,y} \end{array} \right) = m \vec{a} \implies \vec{a} = \frac{1}{m} \left(\begin{array}{c} \vec{F}_{2,x} \\ \vec{F}_{1,y} + \vec{F}_{2,y} \end{array} \right)$$



Weight

GRAVITY IS A NATURAL PHENOMENON by which all things with mass are brought towards (or 'gravitate' towards) one another including stars, planets, galaxies and even light and sub-atomic particles. Gravity is responsible for the complexity in the universe, by creating spheres of hydrogen, igniting them under pressure to form stars and grouping them into galaxies. Without gravity, the universe would be an uncomplicated one, existing without thermal energy and composed only of equally spaced particles. On Earth, gravity gives weight to physical objects and causes the tides. Gravity has an infinite range, and it cannot be absorbed, transformed, or shielded against.

Up until now mass has only been considered in its inertial properties. Gravity is another feature of mass. At the surface of the Earth, massive objects accelerate at a rate of $g = 9.81 \frac{\text{m}}{\text{s}^2}$. Therefore by Newton's second law it follows that, at the surface of the Earth, the force due to gravity, otherwise known as the **weight**, is equal to mg .

Modern work on gravitational theory began with the work of Galileo Galilei in the late 16th and early 17th centuries. In his famous (though possibly apocrypha) experiment dropping balls from the Tower of Pisa, and later with careful measurements of balls rolling down inclines, Galileo showed that gravity accelerates all objects at the same rate. This was a major departure from Aristotle's belief that heavier objects accelerate faster. Galileo postulated air resistance as the reason that lighter objects may fall more slowly in an atmosphere. Galileo's work set the stage for the formulation of Newton's theory of gravity.

Weight is the gravity force on a body.

Constant Weight and Acceleration

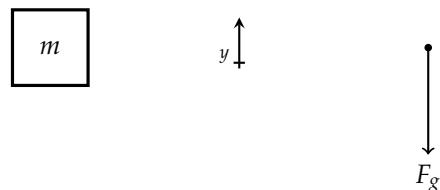
$$g = 9.81 \frac{\text{m}}{\text{s}^2}$$

$$F_g = mg$$

$$\vec{F}_{\text{net}} = \vec{F}_g = -\overbrace{m}^{\text{Gravitational Mass}} g \hat{y} = \begin{pmatrix} 0 \\ -mg \end{pmatrix} = m \vec{a}$$

The constant g is the magnitude of acceleration due to gravity at the surface of the earth. The direction of the acceleration is toward the center of the earth, namely down.

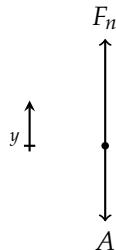
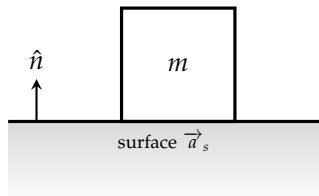
The force of gravity, or weight, is notated \vec{F}_g .



$$\vec{a} = \begin{pmatrix} 0 \\ -g \end{pmatrix}$$

Normal Force

$$\vec{F}_{net} = \vec{A} + \vec{F}_n = (F_n - A)\hat{y} = m\vec{a}$$



The normal force \vec{F}_n is the contact force between an object and a surface. The direction of the normal force is directly out of the surface. If there is no surface contact there is no normal force.

$$\vec{a} = \frac{F_n - A}{m}\hat{y}$$

$$F_n = A + ma_y$$

surface contact requires $\vec{a} \cdot \hat{n} = \vec{a}_s \cdot \hat{n}$ and $(\vec{F}_n \cdot \hat{y}) > 0$
for a non-accelerating surface $\vec{a}_s \cdot \hat{n} = 0 \implies F_{net} = 0$ and $F_n = A$

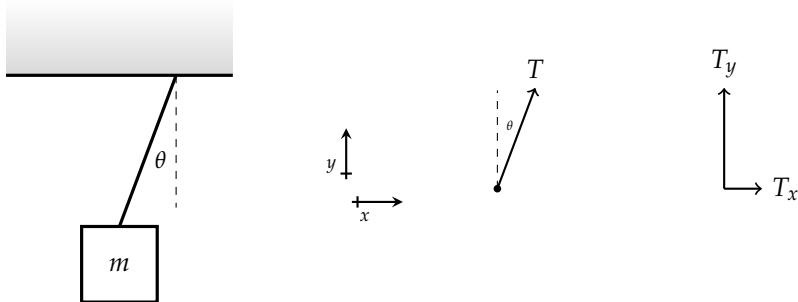
As this free body diagram is drawn the normal force exceeds the applied force \vec{A} , therefore the object must be accelerating in the direction of the normal force. This situation would arise in an upwardly accelerating elevator where the floor exerts a normal force on a passenger in excess of their weight.

Tension Force

For simplicity this system is weightless.

$$\vec{F}_{net} = \vec{T} = \begin{pmatrix} T_x \\ T_y \end{pmatrix} = \begin{pmatrix} T \sin \theta \\ T \cos \theta \end{pmatrix} = m\vec{a}$$

Tension describes the pulling force exerted by each end of a string, cable, chain, or similar one-dimensional continuous object, or by each end of a rod, truss member, or similar three dimensional object.



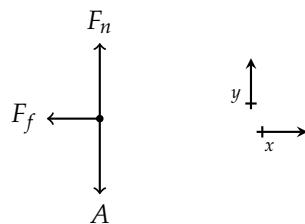
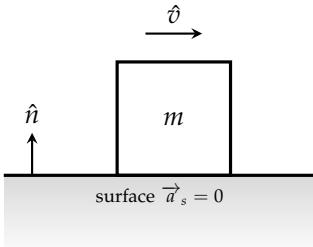
For simplicity this system is weightless. The tension is drawn with horizontal and vertical components. This will correspond to horizontal and vertical components of acceleration.

Friction Forces

Kinetic Friction

$$\vec{F}_f = -\mu_k F_n \hat{v}$$

$$\vec{F}_{net} = \vec{A} + \vec{F}_n + \vec{F}_f = \begin{pmatrix} -F_f \\ F_n - A \end{pmatrix} = m \vec{a}$$



$$\vec{a} = \frac{1}{m} \begin{pmatrix} -F_f \\ F_n - A \end{pmatrix}$$

for a non-accelerating surface $\vec{a}_s \cdot \hat{n} = 0 \implies F_n = A$

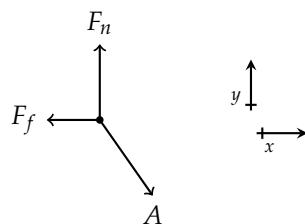
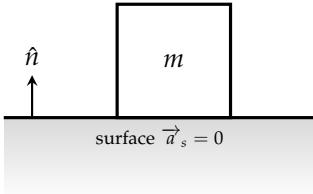
$$F_f = \mu_k F_n = \mu_k A$$

$$\vec{a} = \frac{1}{m} \begin{pmatrix} -\mu_k A \\ 0 \end{pmatrix}$$

Static Friction

$$F_{f,max} = \mu_s F_n$$

$$\vec{F}_{net} = \vec{A} + \vec{F}_n + \vec{F}_f = \begin{pmatrix} A_x - F_f \\ F_n - A_y \end{pmatrix} = m \vec{a} = 0$$



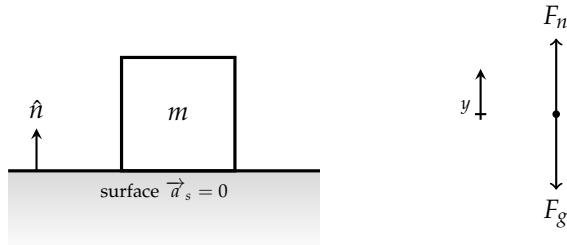
if $A_x > F_{f,max} \implies$ kinetic

Friction is the force resisting the relative motion of solid surfaces sliding against each other. Friction is not itself a fundamental force. Dry friction arises from a combination of inter-surface adhesion, surface roughness, surface deformation, and surface contamination. The complexity of these interactions makes the calculation of friction from first principles impractical and necessitates the use of empirical methods.

Kinetic friction is modeled as a force directly proportional to the normal force and in the opposite direction of sliding. The constant of proportionality is termed μ_k .

Static friction is modeled as a force resisting sliding equal to the total applied shear force. It keeps the object still until the static friction force is overcome. The maximum magnitude of the friction force is modeled as directly proportional to the normal force. The constant of proportionality is μ_s .

Sitting Under Gravity



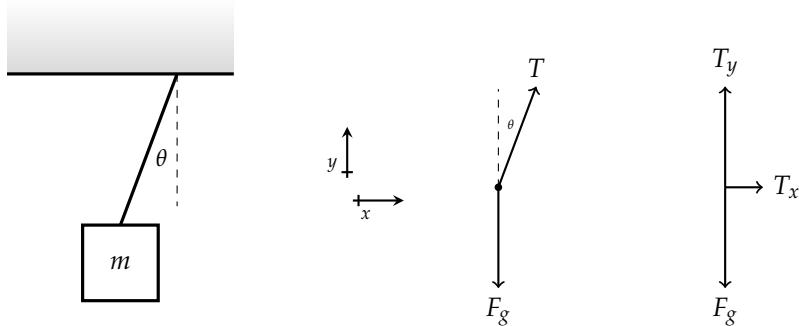
This system is pretty boring. Static. Two forces, equal and opposite

$$\vec{F}_{net} = \vec{F}_n + \vec{F}_g = \begin{pmatrix} 0 \\ F_n - mg \end{pmatrix} = m \vec{a} = 0$$

$$F_n = mg$$

Hanging Under Gravity and Horizontal Acceleration

$$\vec{F}_{net} = \vec{T} + \vec{F}_g = \begin{pmatrix} T_x \\ T_y - mg \end{pmatrix} = \begin{pmatrix} T \sin \theta \\ T \cos \theta - mg \end{pmatrix} = m \vec{a} = \begin{pmatrix} ma_x \\ 0 \end{pmatrix}$$



This system is the jam! It's a mix of statics and kinetics, a veritable physics explosion. There is no acceleration in the vertical direction, only in the horizontal.

$$T \cos \theta = mg$$

$$T \sin \theta = ma_x$$

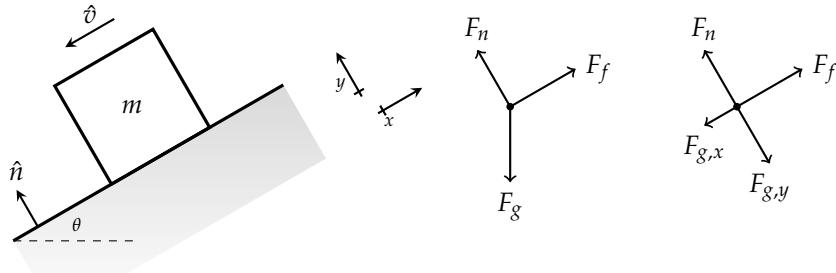
Given the angle we can determine the acceleration in the horizontal direction. Fuzzy dice hanging from the rearview window are effectively an accelerometer.

$$T = m\sqrt{g^2 + a_x^2}$$

$$\theta = \tan^{-1} \left(\frac{a_x}{g} \right)$$

Inclined Plane with Friction

$$\vec{F}_{net} = \vec{F}_g + \vec{F}_n + \vec{F}_f = \begin{pmatrix} F_f - F_{g,x} \\ F_n - F_{g,y} \end{pmatrix} = \begin{pmatrix} F_f - mg \sin \theta \\ F_n - mg \cos \theta \end{pmatrix} = m \vec{a} = \begin{pmatrix} ma_x \\ 0 \end{pmatrix}$$



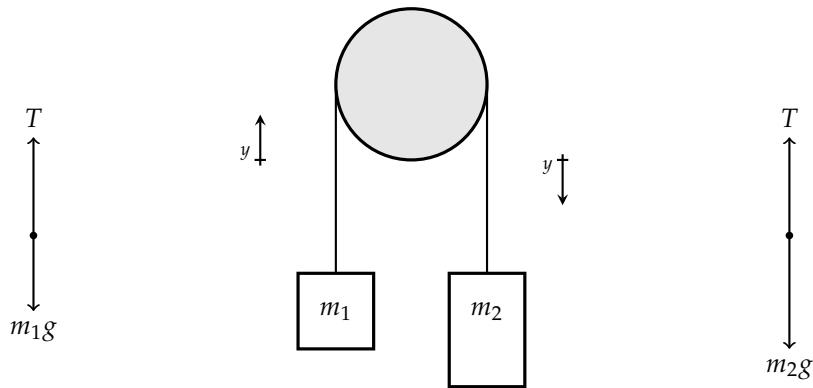
This system is best modeled by applying a rotated coordinate system so that x is in the direction of motion and y is normal to the surface of the inclined plane.

$$F_n = mg \cos \theta$$

$$F_f = \mu mg \cos \theta$$

$$a_x = \mu g \cos \theta - g \sin \theta$$

Atwood's Machine



$$1: \vec{F}_{1,net} = \vec{F}_{1,g} + \vec{T}_1 = (-T + m_1 g)\hat{y} = m_1 \vec{a}$$

$$2: \vec{F}_{2,net} = \vec{F}_{2,g} + \vec{T}_2 = (T - m_2 g)\hat{y} = m_2 \vec{a}$$

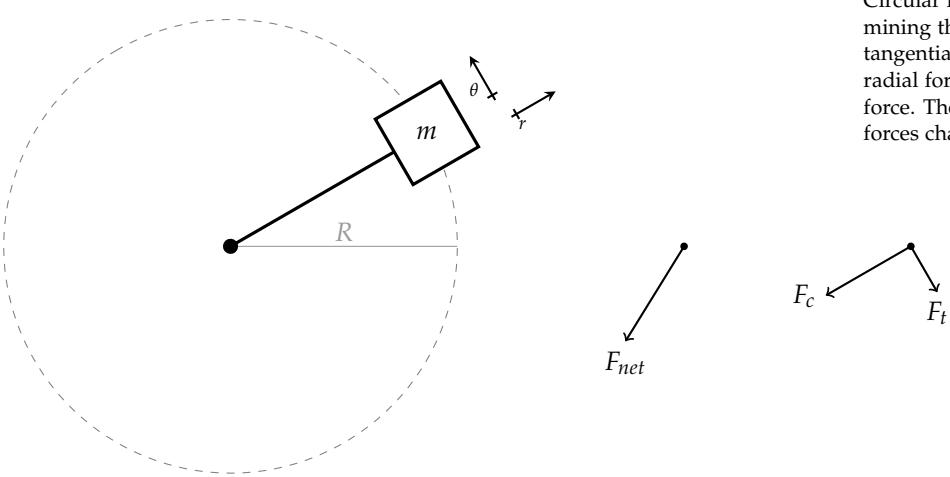
The Atwood machine (or Atwood's machine) was invented in 1784 by the English mathematician George Atwood as a laboratory experiment to verify the mechanical laws of motion with constant acceleration. Atwood's machine is a common classroom demonstration used to illustrate principles of classical mechanics. The ideal Atwood Machine consists of two objects of mass m_1 and m_2 , connected by an inextensible massless string over an ideal massless pulley.

$$T - m_1 g = m_1 a$$

$$-T + m_2 g = m_2 a$$

$$a = \frac{g(m_2 - m_1)}{m_1 + m_2} \quad \text{and} \quad T = \frac{(m_1 - m_2)a + (m_1 + m_2)g}{2} = \frac{2g m_1 m_2}{m_1 + m_2}$$

Circular Motion



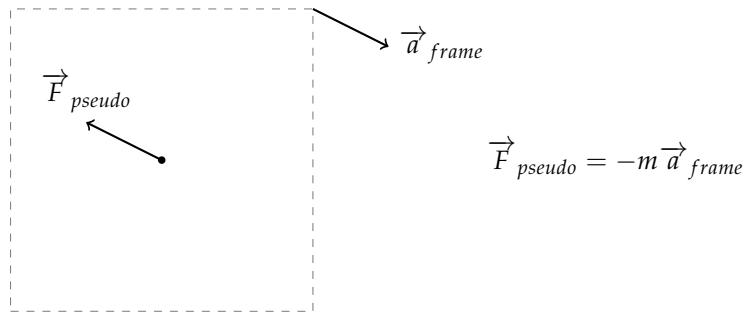
Circular motion is best analyzed determining the components of force in the tangential and radial direction. The net radial force functions as the centripetal force. The tangential components of the forces change the speed of the mass.

$$\vec{F}_{net} = m \vec{a} = mr\ddot{\theta}\hat{\theta} - mr\dot{\theta}^2\hat{r}$$

$$F_t = mr\ddot{\theta} = mr\alpha = ma_t$$

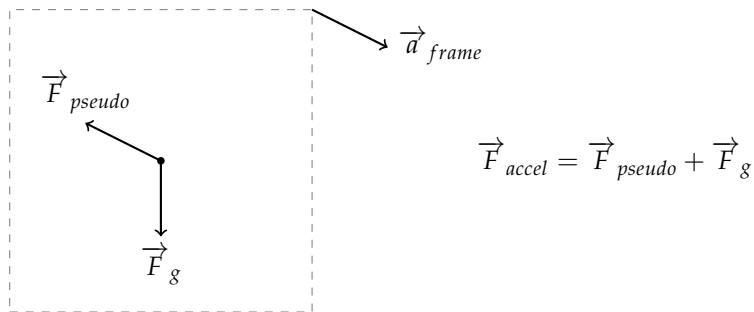
$$F_c = mr\omega^2 = m\frac{v^2}{r} = ma_c$$

Non-Inertial Frames



In a non-inertial frame a fictional force is observed in the opposite direction of the acceleration of the frame. For example, a frame accelerated centripetally will exhibit a centrifugal pseudo-force. The Coriolis effect is another example of pseudo-forces in action.

Accelerometers



Universal Gravitational Force

$$F_g = \frac{Gm_1m_2}{r^2} \quad G = 6.674 \times 10^{-11} \frac{\text{N} \cdot \text{m}^2}{\text{kg}^2}$$

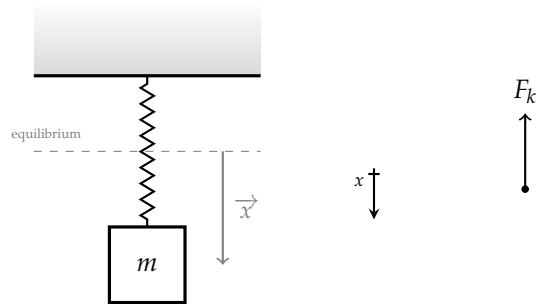
$$\vec{F}_{g1 \rightarrow 2} = -\frac{Gm_1m_2}{r^2} \hat{r} = -\vec{F}_{g2 \rightarrow 1}$$

Newton's law of universal gravitation states that any two bodies in the universe attract each other with a force that is directly proportional to the product of their masses and inversely proportional to the square of the distance between them.



Spring Force

$$\begin{aligned}\vec{F}_k &= -k \vec{x} \\ \vec{F}_{net} &= \vec{F}_k = -kx \hat{x} = m \vec{a}\end{aligned}$$



Hooke's law states that the force needed to extend or compress a spring by some distance is directly proportional to that distance. The proportionality constant, k , is known as the spring constant. The law is named after 17th century British physicist Robert Hooke. Hooke's equation in fact holds (to some extent) in many other situations where an elastic body is deformed, such as wind blowing on a tall building, a musician plucking a string of a guitar, or the filling of a party balloon. An elastic body or material for which this equation can be assumed is said to be linear-elastic or Hookean.

Time Dependent Forces

$$\vec{F} = \vec{F}(t)$$

- Propulsion, Wind, Tide, etc

Space Dependent Forces

$$\vec{F} = \vec{F}(\rightarrow)$$

- Spring, Universal Gravity, Electromagnetic Forces, etc

**SPECIAL
FORCES**

Velocity Dependent Forces

$$\vec{F} = \vec{F}(\vec{v})$$

- Drag, Magnetic Force, etc